Effective field theory methods for chiral plasmas

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Outline

1. Real vs imaginary time formalism.
2. Propagators in real time.
3. Hard thermal loops (HTL).
4. Chiral On-Shell Effective Field Theory (COSEFT).
5. Polarization tensor.
6. Chiral magnetic effect (CME).

01

Special Relativity + Thermodynamics + Quantum Mechanics = QTFT.

02

Allowed to understand the Electro-Weak phase transition in the Standard Model and later the phase transition in Quantum Chromodynamics from hadrons to quark-gluon plasma.

03

The techniques of QTFT can be applied to any quantum field theory, in particular QED.

What is quantum thermal field theory ?

1. Real vs Imaginary time formalism.

1) IMAGINARY TIME FORMALISM:

- Perform a Wick-Rotation to imaginary time.
- Integrals in energy become sums over Matsubara frequencies.

2) REAL TIME FORMALISM:

- Use real time (Integrals in energy).
- Has a big problem: pinch singularities.
- Better if we want to apply EFT techniques.
- Non-equilibrium situation.

2. Propagators in real time.

$$\Delta^{T=0}(x,y) = \langle 0|\mathcal{T}\phi(x)\phi(y)|0\rangle \longrightarrow \Delta^{T\neq 0}(x,y) = \langle \mathcal{T}\phi(x)\phi(y)\rangle_{\beta}$$

Vacuum expectation value. Thermal Average.

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Time ordering

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$$\langle A\rangle_{\beta} = \mathrm{tr}\,(\rho A) \qquad \rho = \frac{e^{-\beta H}}{Z}.$$

Density matrix.

Trace

$$Z = \mathrm{tr}\left(e^{-\beta H}\right)$$

Partition function.

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. Partition function.

Introduce a solution in terms of creation and annihilation operators.

$$\phi(x) = \int \frac{d^3p}{\sqrt{2(2\pi)^3\omega_p}}\left(a(\boldsymbol{p})e^{-ipx} + a^{\dagger}(\boldsymbol{p})e^{ipx}\right)$$

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To get: $$i\Delta(k) = \frac{i}{k^2 - m^2 + i\epsilon} + 2\pi n_B(k_0)\delta(k^2 - m^2)$$

Bose-Einstein.

$$n_B(k_0) = \frac{1}{e^{\beta|k_0|} - 1}.$$

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On-Shell

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Perform the same process with a fermionic field.

2. Propagators in real time.

And find: $$iS(k) = (\not{k} + m)\left[\frac{i}{k^2 - m^2 + i\epsilon} - 2\pi n_F(k_0)\delta(k^2 - m^2)\right]$$

2. Propagators in real time.

Fermi-Dirac: $n_F(k_0) = \frac{1}{e^{\beta|k_0|}+1}$

And find: $iS(k) = (\not{k}+m)\left[\frac{i}{k^2-m^2+i\epsilon} - 2\pi n_F(k_0)\delta(k^2-m^2)\right]$

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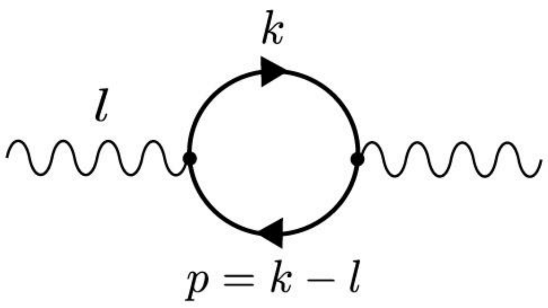

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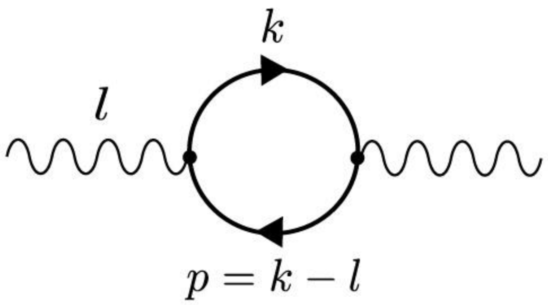

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Pinch singularity.

A solution: doubling of degrees of freedom (DDF).

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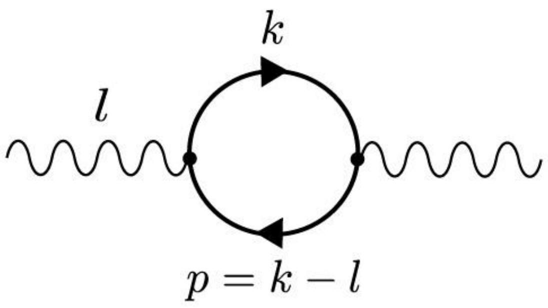

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→ Based on KMS relation. $D(t_1, t_2) = D(t_2, t_1 + i\beta)$

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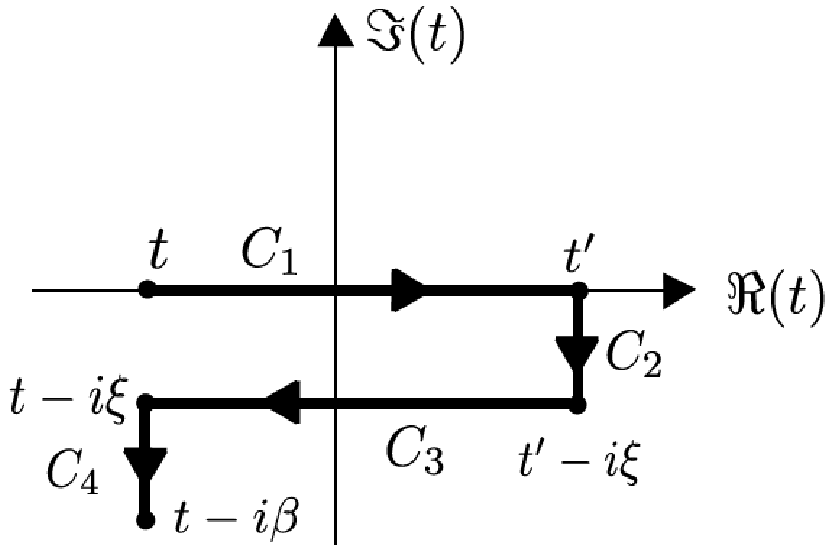

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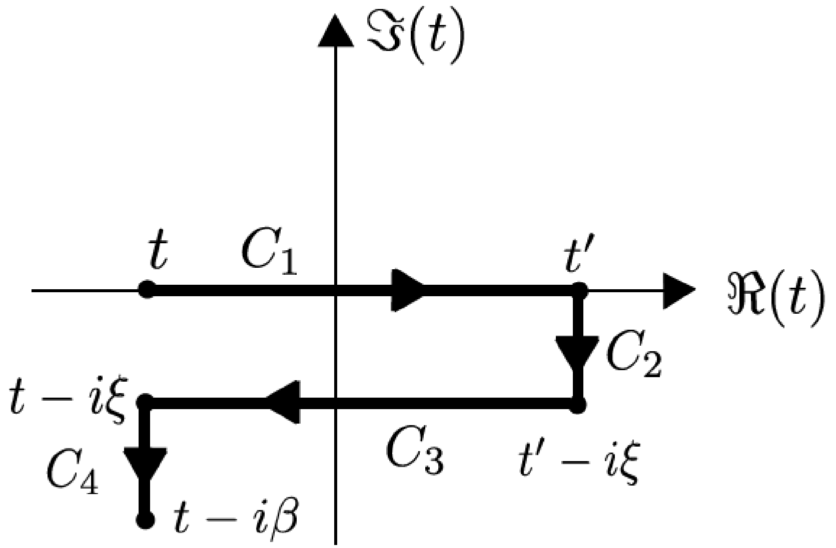

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Then the propagator becomes
a 2x2 matrix in the space spanned by particle/thermal ghosts.

$$S(q) = (\not{q} + m)\left[\begin{pmatrix} \frac{1}{q^2+i\epsilon} & 0 \\ 0 & -\frac{1}{q^2-i\epsilon} \end{pmatrix} + i2\pi\delta(q^2)\begin{pmatrix} -n_F(q_0) & \theta(-q_0) - n_F(q_0) \\ \theta(q_0) - n_F(q_0) & -n_F(q_0) \end{pmatrix}\right]$$

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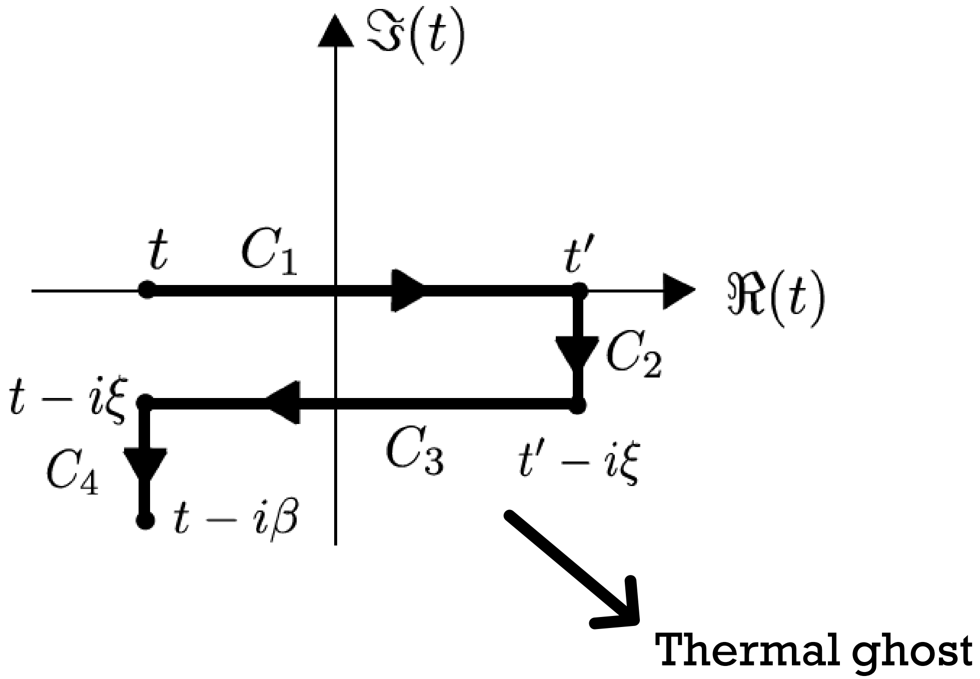

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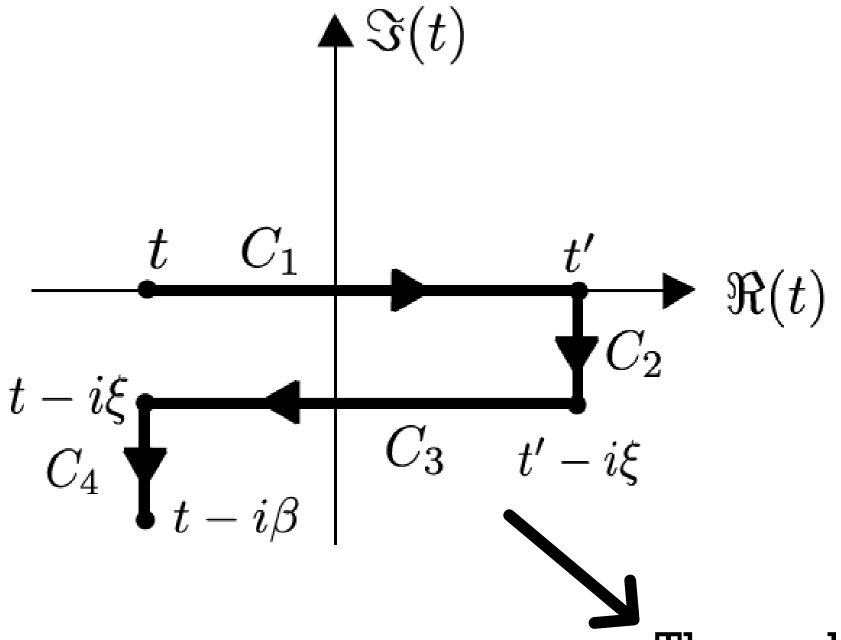

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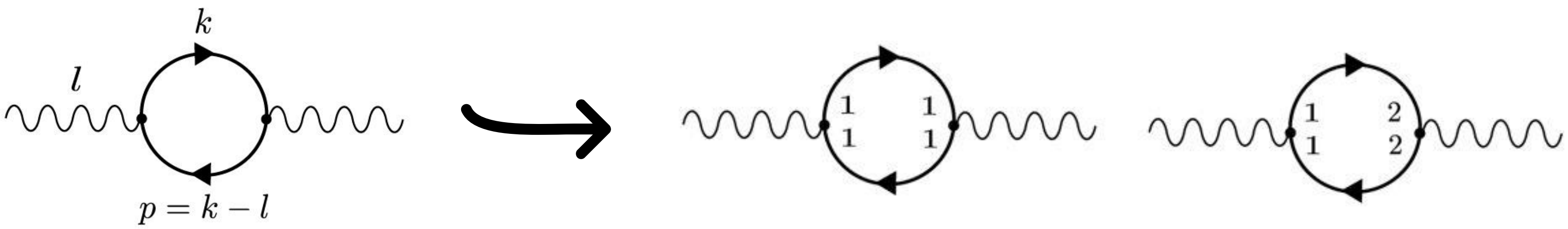

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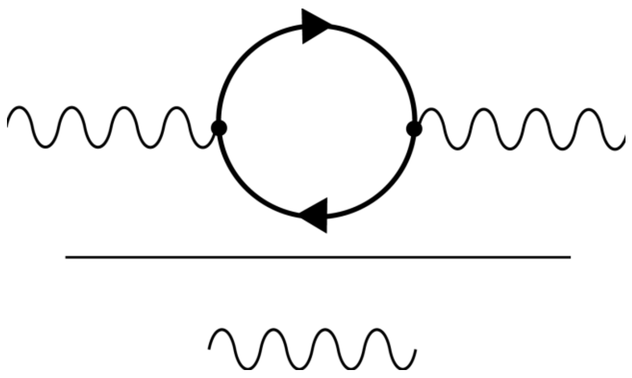
The physical is the retarded self-energy.

3. Hard thermal Loops (HTL).

Another difficulty when turning on the temperature:



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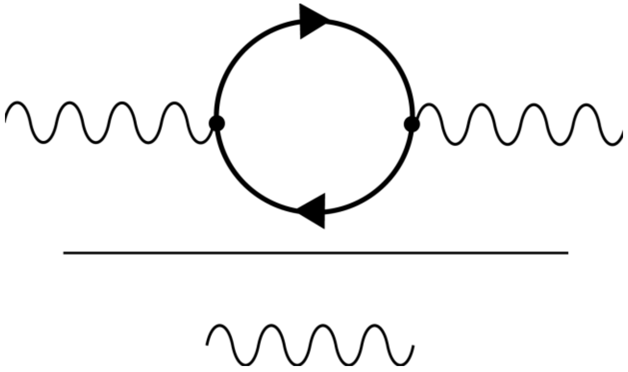

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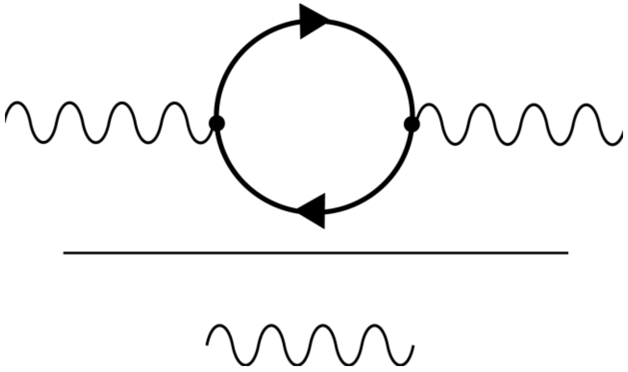

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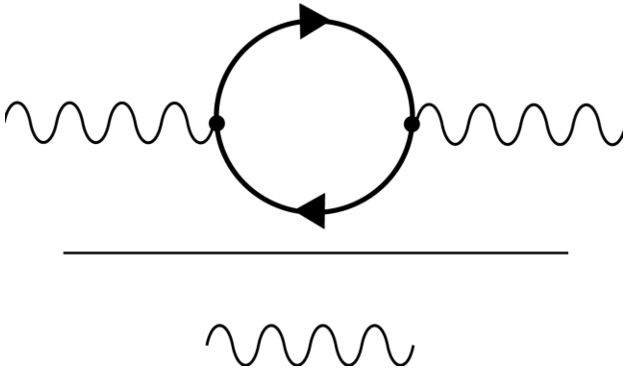

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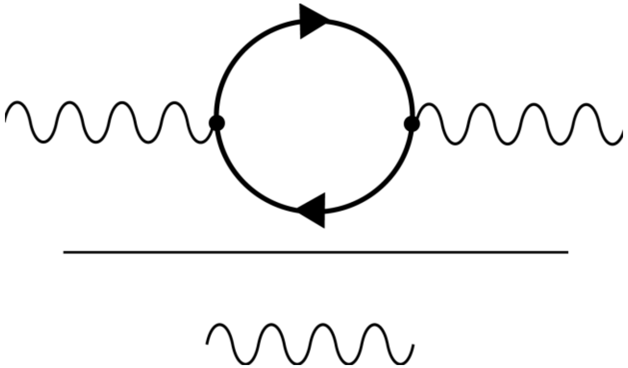

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It turns out that in a hot plasma of massless weakly interacting particles:

$$e \ll 1 \qquad T \uparrow \quad \begin{cases} T & \text{Hard} \\ eT & \text{Soft} \end{cases} \longrightarrow \text{Scales}$$

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Hierarchy of scales implies:

$$l_0, |\boldsymbol{l}| \ll |\boldsymbol{k}| \quad \Rightarrow \quad \frac{l_0}{|\boldsymbol{k}|}, \frac{|\boldsymbol{l}|}{|\boldsymbol{k}|} \ll 1 \quad \Rightarrow \quad \frac{l}{|\boldsymbol{k}|} \ll 1$$

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Result:

$$\Pi_R^{00}(l) = \Pi_R^L(l) \qquad \Pi_R^{0i}(l) = \frac{l_0 l^i}{|\boldsymbol{l}|^2}\Pi_R^L(l)$$

$$\Pi_R^{ij}(l) = \left(\delta^{ij} - \frac{l^i l^j}{|\boldsymbol{l}|^2}\right)\Pi_R^T(l) + \frac{l^i l^j}{|\boldsymbol{l}|^2}\frac{l_0^2}{|\boldsymbol{l}|^2}\Pi_R^L(l)$$

$$\Pi_R^L(l) = -m_D^2\left[1 - \frac{l_0}{2|\boldsymbol{l}|}\left(\ln\left|\frac{l_0+|\boldsymbol{l}|}{l_0-|\boldsymbol{l}|}\right| - i\pi\theta(\boldsymbol{l}^2 - l_0^2)\right)\right]$$

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$$m_D^2 = \frac{e^2T^2}{3}$$

Debye mass

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4.Chiral on-shell effective field theory. (COSEFT)

Motivation:

- Exploit hierarchy of scales in a hot plasma of massless weakly interacting particles.
- The fact that HTL are dominated by almost on-shell degrees of freedom.
- Reproduce the HTL contributions easily.
- Go beyond HTL.
- Study the consequences of chiral imbalance.
- Derive kinetic theory.

4.Chiral on-shell effective field theory. (COSEFT)

The momentum of a nearly on-shell fermion with chemical potential may be written as:

$$q^{\mu}_{\chi} = p^{\mu}_{\chi} + k^{\mu} \quad \text{where} \quad p^{\mu}_{\chi} = (-\mu_{\chi} + |\boldsymbol{p}|)v^{\mu} + \mu_{\chi}\delta^{\mu}_{j}v^{j} \quad j \in \{1,3\}$$

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Then the dependence on hard momenta in the Dirac field can be factored out:

$$\psi_\chi(x) = e^{-ip_\chi \cdot x}\left(P_v \chi_v^\chi + P_{\tilde{v}} H_{\tilde{v}}^{(1),\chi}\right) + e^{-i\tilde{p}_\chi \cdot x}\left(P_{\tilde{v}} \xi_{\tilde{v}}^\chi + P_v H_v^{(2),\chi}\right)$$

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Residual — Hard — Residual — Hard

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Residual | Hard | Residual | Hard

Particle/antiparticle projectors:

$$P_v = \frac{\not{v}\gamma^0}{2} \qquad P_{\tilde{v}} = \frac{\not{\tilde{v}}\gamma^0}{2}.$$

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And after tedious algebra, we get:

$$\mathcal{L}_\chi = \overline{\chi}_v^\chi i\gamma^0 (v\cdot D)\chi_v^\chi + \overline{H}_{\tilde{v}}^{(1),\chi}\gamma^0(2|\boldsymbol{p}| + i\tilde{v}\cdot D)H_{\tilde{v}}^{(1),\chi}$$

$$+\overline{\xi}_{\tilde{v}}^\chi i\gamma^0(\tilde{v}\cdot D)\xi_{\tilde{v}}^\chi + \overline{H}_v^{(2),\chi}\gamma^0(-2|\boldsymbol{p}| + iv\cdot D)H_v^{(2),\chi}$$

$$+\overline{\chi}_v^\chi i\not{D}_\perp H_{\tilde{v}}^{(1),\chi} + \overline{H}_{\tilde{v}}^{(1),\chi} i\not{D}_\perp \chi_v^\chi + \overline{\xi}_{\tilde{v}}^\chi i\not{D}_\perp H_v^{(2),\chi} + \overline{H}_v^{(2),\chi} i\not{D}_\perp \xi_{\tilde{v}}^\chi .$$

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$$+ \overline{\chi}_v^\chi i\not{D}_\perp H_{\tilde{v}}^{(1),\chi} + \overline{H}_{\tilde{v}}^{(1),\chi} i\not{D}_\perp \chi_v^\chi + \overline{\xi}_{\tilde{v}}^\chi i\not{D}_\perp H_v^{(2),\chi} + \overline{H}_v^{(2),\chi} i\not{D}_\perp \xi_{\tilde{v}}^\chi .$$

$$\not{D}_\perp = P_\perp^{\mu\nu} \gamma_\mu D_\nu$$

$$P_\perp^{\mu\nu} = \eta^{\mu\nu} - \frac{1}{2}(v^\mu \tilde{v}^\nu + \tilde{v}^\mu v^\nu)$$

4.Chiral on-shell effective field theory. (COSEFT)

Now, take the expression for the Dirac field and plug it into the chiral QED Lagrangian.

$$\psi_\chi \longrightarrow \quad \mathcal{L} = \sum_\chi \mathcal{L}_\chi \quad \text{with} \quad \mathcal{L}_\chi = \overline{\psi}_\chi (i\not{D} + \gamma^0 \mu_\chi)\psi_\chi .$$

And after tedious algebra, we get:

$$\mathcal{L}_\chi = \overline{\chi}_v^\chi i\gamma^0 (v\cdot D)\chi_v^\chi + \overline{H}_{\tilde{v}}^{(1),\chi} \gamma^0 (2|\mathbf{p}| + i\tilde{v}\cdot D) H_{\tilde{v}}^{(1),\chi}$$

$$+\overline{\xi}_{\tilde{v}}^\chi i\gamma^0 (\tilde{v}\cdot D)\xi_{\tilde{v}}^\chi + \overline{H}_v^{(2),\chi} \gamma^0 (-2|\mathbf{p}| + iv\cdot D) H_v^{(2),\chi}$$

$$+\overline{\chi}_v^\chi i\not{D}_\perp H_{\tilde{v}}^{(1),\chi} + \overline{H}_{\tilde{v}}^{(1),\chi} i\not{D}_\perp \chi_v^\chi + \overline{\xi}_{\tilde{v}}^\chi i\not{D}_\perp H_v^{(2),\chi} + \overline{H}_v^{(2),\chi} i\not{D}_\perp \xi_{\tilde{v}}^\chi .$$

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Now we use E.O.M to integrate out the fields. $\longrightarrow$ $H_v^{(1),\chi}, H_v^{(2),\chi}$

4.Chiral on-shell effective field theory. (COSEFT)

So finally we get the COSEFT Lagrangian:

$$\mathcal{L}_\chi = \overline{\chi}_v^\chi \gamma^0 \left(iv \cdot D + i\not{D}_\perp \frac{1}{2|\boldsymbol{p}| + i\tilde{v} \cdot D} i\not{D}_\perp \right) \chi_v^\chi$$

$$+ \overline{\xi}_{\tilde{v}}^\chi \gamma^0 \left(i\tilde{v} \cdot D + i\not{D}_\perp \frac{1}{-2|\boldsymbol{p}| + iv \cdot D} i\not{D}_\perp \right) \xi_{\tilde{v}}^\chi .$$

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Expand in powers of 1/p.

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- Particle term (first line) → Particle; its denominator → Expand in powers of 1/p.
- Antiparticle term (second line) → Antiparticle; its denominator → Expand in powers of 1/p.

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→ Particle

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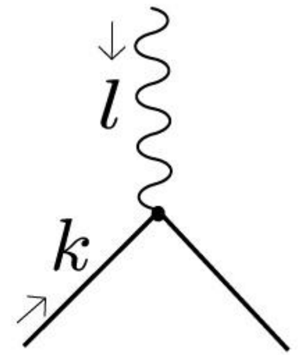
→ Antiparticle

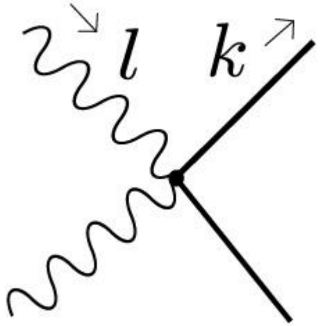
Expand in powers of 1/p.

Then we can derive vertices, propagators... up to the desired order in the energy expansion.

4.Chiral on-shell effective field theory. (COSEFT)



Vertex	Feynman rule
$V^{\mu}_{(0)}(k,l)$	$ev^{\mu}\gamma^{0}$
$V^{\mu}_{(1)}(k,l)$	$\frac{e\gamma^{0}}{\lvert\boldsymbol{p}\rvert}\left(k^{\mu}_{\perp}+\frac{l^{\mu}_{\perp}}{2}-\frac{i}{2}\sigma^{\mu\nu}_{\perp}l_{\nu}\right)$

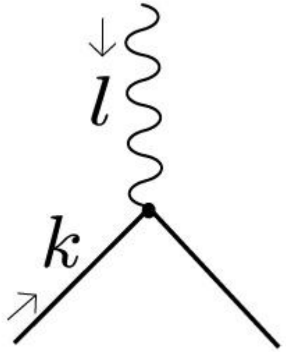


Vertex	Feynman rule
$W^{\mu\nu}_{<(0)}(k,l)$	–
$W^{\mu\nu}_{<(1)}(k,l)$	$\frac{ie^{2}\gamma^{0}}{\lvert\boldsymbol{p}\rvert}P^{\mu\nu}_{\perp}$
$W^{\mu\nu}_{<(2)}(k,l)$	$-\frac{ie^{2}\gamma^{0}}{8\lvert\boldsymbol{p}\rvert^{2}}\left\{\left[\sigma^{\mu\alpha}_{\perp}\tilde{V}^{\nu}-\sigma^{\nu\alpha}_{\perp}\tilde{V}^{\mu}-2v^{i}\left(\sigma^{\mu\alpha}_{\perp}\delta^{\nu}_{i}-\sigma^{\nu\alpha}_{\perp}\delta^{\mu}_{i}\right)\right]l_{\alpha}-2(\tilde{v}\cdot l)\sigma^{\mu\nu}_{\perp}\right\}$

$$\tilde{V}^\mu = \delta_0^\mu - v^i \delta_i^\mu$$

$$\sigma_\perp^{\mu\nu} = P_\perp^{\mu\alpha} P_\perp^{\nu\beta} \sigma_{\alpha\beta}$$

4.Chiral on-shell effective field theory. (COSEFT)



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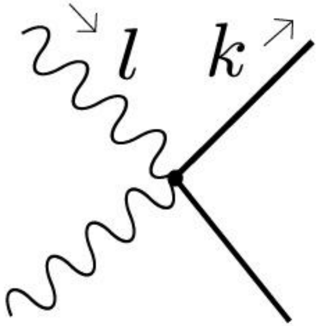

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$W_{<(2)}^{\mu\nu}(k,l)$	$-\frac{ie^2\gamma^0}{8\lvert\boldsymbol{p}\rvert^2}\left\{\left[\sigma_\perp^{\mu\alpha}\tilde{V}^\nu - \sigma_\perp^{\nu\alpha}\tilde{V}^\mu - 2v^i\left(\sigma_\perp^{\mu\alpha}\delta_i^\nu - \sigma_\perp^{\nu\alpha}\delta_i^\mu\right)\right]l_\alpha - 2(\tilde{v}\cdot l)\sigma_\perp^{\mu\nu}\right\}$

4.Chiral on-shell effective field theory. (COSEFT)

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$$S(k) = \frac{P_\chi P_v \gamma^0}{k_0 - f(\boldsymbol{k}) + i\epsilon}.$$

$$f(\boldsymbol{k}) = k_{||} + \frac{\boldsymbol{k}_\perp^2}{2|\boldsymbol{p}|}.$$

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What about the propagator of the COSEFT?

Chiral projector

$$S(k) = \frac{P_\chi P_v \gamma^0}{k_0 - f(\boldsymbol{k}) + i\epsilon}. \longrightarrow P_\chi = \frac{1+\chi\gamma_5}{2}$$

$$f(\boldsymbol{k}) = k_{\|} + \frac{\boldsymbol{k}_\perp^2}{2|\boldsymbol{p}|}. \longrightarrow k_\perp^2 = P_\perp^{\mu\alpha} P^{\nu}_{\perp\alpha} k_\mu k_\nu = -\boldsymbol{k}_\perp^2$$

$$k_{\|} = \boldsymbol{k}\cdot\boldsymbol{v}$$

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$$k_{\|} = \boldsymbol{k} \cdot \boldsymbol{v}$$

Now we need to turn on the temperature (DDF) !

4.Chiral on-shell effective field theory. (COSEFT)

Finally the propagators in the Keldysh representation read:

$$S^{\chi}_{A,R}(k) = \frac{P_\chi P_v \gamma^0}{k_0 - f(\boldsymbol{k}) \mp i\epsilon}$$

$$S^{\chi}_{S}(k) = -i2\pi P_\chi P_v \gamma^0 \delta(k_0 - f(\boldsymbol{k}))[1 - 2n_\chi(|\boldsymbol{p}| + k_0)]$$

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Where now the Fermi-Dirac distributions read:

Fermion:

$$n_{\chi}(q_0) = \frac{1}{e^{\beta(|q_0| - \chi\mu_{\chi})} + 1}$$

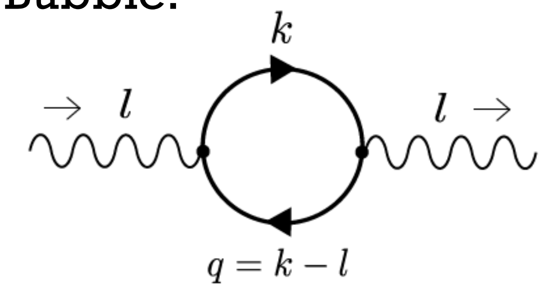
Antifermions:

$$\tilde{n}_{\chi}(q_0) = \frac{1}{e^{\beta(|q_0| + \chi\mu_{\chi})} + 1}.$$

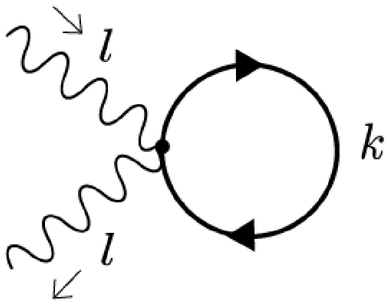
5. Polarization tensor.

There are two diagrams that contribute to the self-energy tensor:

Bubble:



Tadpole:



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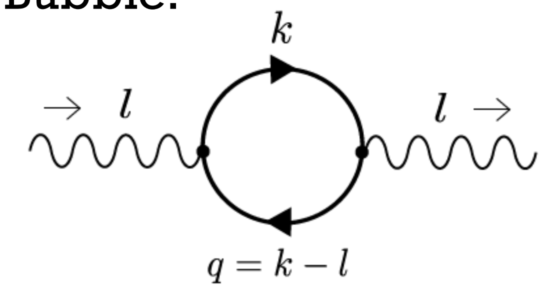

$$\Pi^{\mu\nu}_{b,T}(l) = \Pi^{\mu\nu}_{b,R}(l) + \tilde{\Pi}^{\mu\nu}_{b,R}(l) + \Pi^{\mu\nu}_{b,L}(l) + \tilde{\Pi}^{\mu\nu}_{b,L}(l).$$

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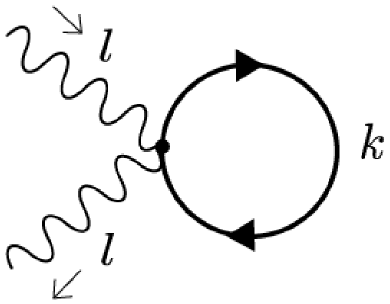

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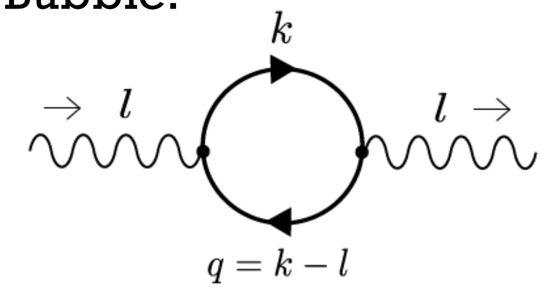

Antiparticles

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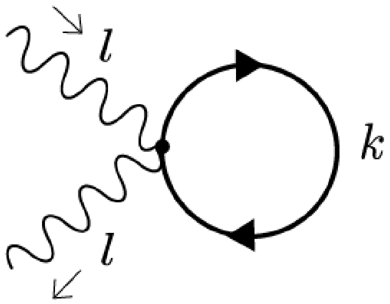
Particles

Tadpole:



$$\Pi^{\mu\nu}_{t,T}(l) = \Pi^{\mu\nu}_{t,R}(l) + \tilde{\Pi}^{\mu\nu}_{t,R}(l) + \Pi^{\mu\nu}_{t,L}(l) + \tilde{\Pi}^{\mu\nu}_{t,L}(l).$$

$$\longrightarrow \quad \Pi^{\mu\nu}_{T} = \Pi^{\mu\nu}_{b,T} + \Pi^{\mu\nu}_{t,T}$$

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Leading order: e^2T and $e^2\mu_\chi$.

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Leading order: $e^2 T$ **and** $e^2 \mu_\chi$.

$$\Pi_T^{\mu\nu}(l) = -m_D^2 \left(\delta_0^\mu \delta_0^\nu - l_0 \int_v \frac{v_{\boldsymbol{q}}^\mu v_{\boldsymbol{q}}^\nu}{v_{\boldsymbol{q}} \cdot l} \right) \qquad m_D^2 = e^2 \left(\frac{T^2}{3} + \frac{\mu_R^2 + \mu_L^2}{2\pi^2} \right)$$

5. Polarization tensor.

Angular integration gives exactly HTL. !

Leading order: $e^2 T$ and $e^2 \mu_\chi$.

Debye mass.

$$\Pi_T^{\mu\nu}(l) = -m_D^2 \left(\delta_0^\mu \delta_0^\nu - l_0 \int_v \frac{v_{\boldsymbol{q}}^\mu v_{\boldsymbol{q}}^\nu}{v_{\boldsymbol{q}} \cdot l} \right) \qquad m_D^2 = e^2 \left(\frac{T^2}{3} + \frac{\mu_R^2 + \mu_L^2}{2\pi^2} \right)$$

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$$\Pi_T^{00}(l) = 0 \qquad \Pi_T^{0i}(l) = 0$$

$$\Pi_T^{ij}(l) = \frac{i e^2 \mu_5}{4\pi^2} \left(\epsilon^{ijk} l_k + l_0 \int_v \frac{\hat{q}^i \epsilon^{jkl} - \hat{q}^j \epsilon^{ikl}}{v_{\boldsymbol{q}} \cdot l} \hat{q}_k l_l \right)$$

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Symmetric !

Anti-symmetric !

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Only in chiral imbalance!

Parallel to B !

Summary:

- Learn the techniques of QTFT.
- Differences between real and imaginary time formalism.
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Future work and applications:

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