
$$D^0 \rightarrow P_1^+ P_2^- \ell^+ \ell^-:$$

SM prediction and windows on NP

Oscar Catà

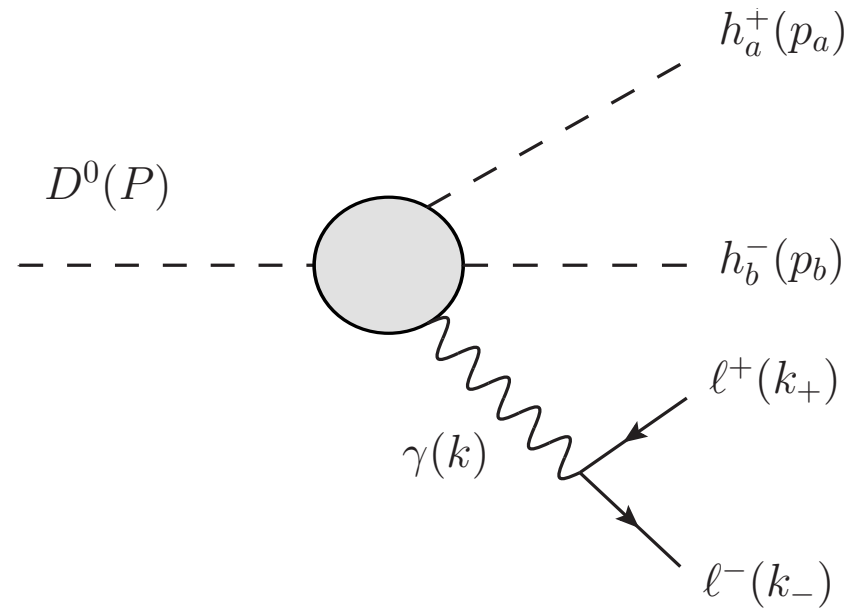


Jahrestreffen LHCb 2019, Rostock, October 1st, 2019

Motivation

- FCNC processes suppressed in the SM: window to new physics. D physics has a very efficient GIM suppression but long-distance dominated.
- As opposed to B and K physics, no meaningful expansion works. EFT language not very useful: estimation of hadronic form factors through lattice or sum rules.
- Many intermediate states allowed by phase space. Resonant contributions dominate.
- Rich kinematics.
- Some of the channels have already been measured. [\[arXiv:1510.08367; 1707.08377; 1808.09680\]](#)

Long distances



- Bremsstrahlung: pure QED effects, calculable with Low's theorem:

$$\mathcal{M}_b(D^0 \rightarrow h_1^+ h_2^- \gamma) = 2e \left[\frac{p_1 \cdot \epsilon}{2p_1 \cdot q + q^2} - \frac{p_2 \cdot \epsilon}{2p_2 \cdot q + q^2} \right] \mathcal{M}(D^0 \rightarrow h_1^+ h_2^-)$$

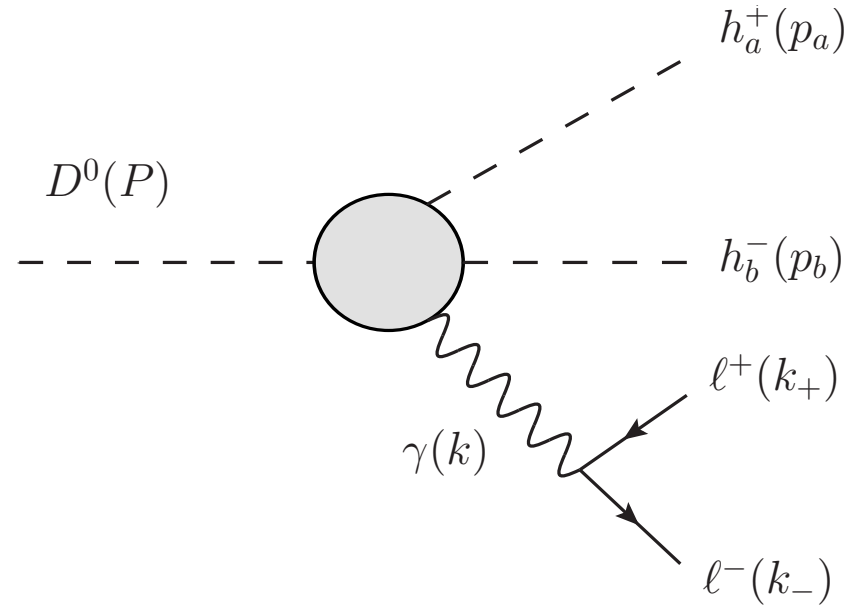
- Resonant contributions (dominant effects, estimated at $\sim 10^{-6}$).
For comparison, short distance SM contribution estimated at $\sim 10^{-9}$.

[Burdman et al'02].

[Bigi et al'11].

- Charge radius (suppressed, not discussed in this talk).

Long distances



Generic kinematical parametrization:

$$\mathcal{M}_{ab} = \frac{e}{k^2} [\bar{u}(k_-) \gamma^\mu v(k_+)] H_{ab}^\mu(p_a, p_b; k)$$

with

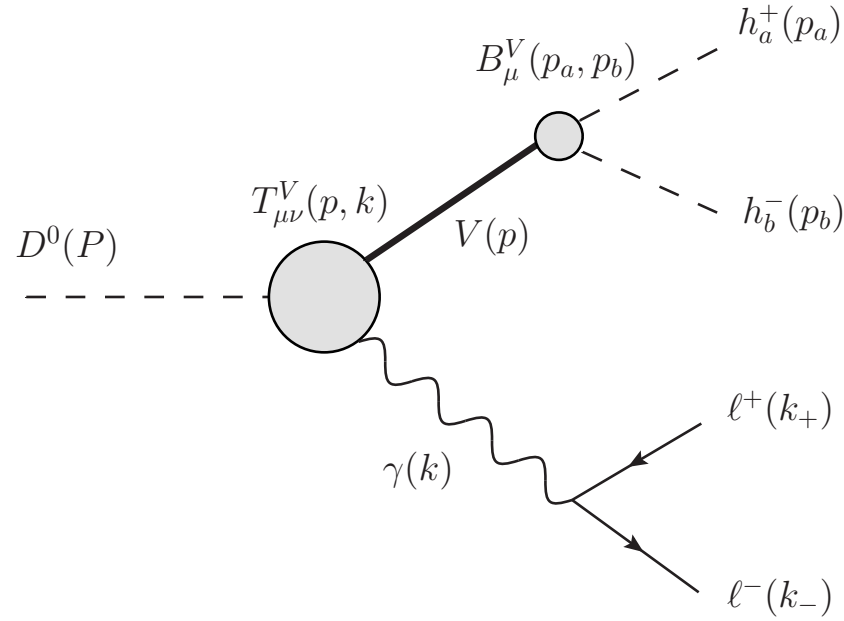
$$H_{ab}^\mu(p_a, p_b; k) = F_1^{(ab)} p_a^\mu + F_2^{(ab)} p_b^\mu + F_3^{(ab)} \epsilon^{\mu\nu\lambda\rho} p_{a\nu} p_{b\lambda} k_\rho$$

Matrix elements:

$$\sum_{\text{spins}} |\mathcal{M}_{ab}|^2 = \frac{2e^2}{q^4} \left[\sum_i^3 |F_i^{(ab)}|^2 T_{ii} + 2\text{Re} \sum_{i < j}^3 (F_i^{(ab)})^* F_j^{(ab)} T_{ij} \right]$$

Resonant contribution

[Cappiello, OC, D'Ambrosio'12]



$$H_{ab}^\mu(p_a, p_b; k) = \langle h_a^+ h_b^- \gamma^* | \mathcal{H} | D^0 \rangle \varepsilon_\gamma^\mu = \sum_k \langle h_a^+ h_b^- | \mathcal{H} | V_k \rangle \frac{\varepsilon_\gamma^\mu}{P_k(p^2)} \langle V_k \gamma^* | \mathcal{H} | D^0 \rangle$$

where

$$\langle h_a^+ h_b^- | \mathcal{H} | V \rangle \equiv B_V^\mu(p_a, p_b) \varepsilon_\mu^V(p); \quad \langle V \gamma^* | \mathcal{H} | D^0 \rangle \equiv T_V^{\mu\nu}(p, k) \varepsilon_\mu^{V*}(p) \varepsilon_\nu^{\gamma*}(k)$$

Relevant kinematic invariants:

$$B_V^\mu(p_a, p_b) = b^V(p^2)(p_a - p_b)^\mu; \quad T_{\mu\nu}^V(p, k) = t_1^V g_{\mu\nu} + t_2^V k_\mu p_\nu + t_3^V \epsilon_{\mu\nu\lambda\rho} p_\lambda k_\rho$$

Weak form factors

At hadronic scales,

$$\mathcal{H}_{\Delta c=1} = \sum_{j=1,2} \left[\mathcal{H}_j^{CF} + \mathcal{H}_j^{SCS} + \mathcal{H}_j^{DCS} \right]$$

where

$$\begin{aligned} \mathcal{H}_j^{CF} &= \frac{G_F}{\sqrt{2}} \left[\lambda_{sd} C_j^{(sd)} Q_{sd}^{(j)} \right] && (K^- \pi^+) \\ \mathcal{H}_j^{SCS} &= \frac{G_F}{\sqrt{2}} \left[\lambda_d C_j^{(d)} Q_d^{(j)} + \lambda_s C_j^{(s)} Q_s^{(j)} \right] && (\pi^+ \pi^-; K^+ K^-) \\ \mathcal{H}_j^{DCS} &= \frac{G_F}{\sqrt{2}} \left[\lambda_{ds} C_j^{(ds)} Q_{ds}^{(j)} \right] && (K^+ \pi^-) \end{aligned}$$

and

$$\begin{aligned} Q_{sd}^{(1)} &= (\bar{s} \gamma^\mu c)_L (\bar{u} \gamma_\mu d)_L; & Q_{sd}^{(2)} &= (\bar{u} \gamma_\mu c)_L (\bar{s} \gamma^\mu d)_L \\ Q_d^{(1)} &= (\bar{d} \gamma^\mu c)_L (\bar{u} \gamma_\mu d)_L; & Q_d^{(2)} &= (\bar{u} \gamma_\mu c)_L (\bar{d} \gamma^\mu d)_L \\ Q_s^{(1)} &= (\bar{s} \gamma^\mu c)_L (\bar{u} \gamma_\mu s)_L; & Q_s^{(2)} &= (\bar{u} \gamma_\mu c)_L (\bar{s} \gamma^\mu s)_L \\ Q_{ds}^{(1)} &= (\bar{d} \gamma^\mu c)_L (\bar{u} \gamma_\mu s)_L; & Q_{ds}^{(2)} &= (\bar{u} \gamma_\mu c)_L (\bar{d} \gamma^\mu s)_L \end{aligned}$$

Penguin and semileptonic operator contributions not considered (expected to be subdominant).

Weak form factors

MAIN ASSUMPTIONS:

(i) Photon leg dominated by (lowest-lying) vector exchange:

$$\langle R_i \gamma^*(k) | \mathcal{H} | R_j \rangle = \sum_{V=\rho,\omega,\phi} \frac{\langle \gamma^* | \mathcal{H}_{\text{EM}} | V \rangle}{P_V(k^2)} \langle R_i V | \mathcal{H} | R_j \rangle$$

(ii) Factorization of weak matrix elements (current-current operators)

$$\langle R^+ R^- | J_{ik}^\mu J_\mu^{jc} | D^0 \rangle = \langle R^+ | J_{ik}^\mu | 0 \rangle \langle R^- | J_\mu^{jc} | D^0 \rangle + \langle R^- | J_{ik}^\mu | 0 \rangle \langle R^+ | J_\mu^{jc} | D^0 \rangle$$

The weak tensor $T_{\mu\nu}$ can then be related to the $D^0 \rightarrow V$ transitions:

$$\begin{aligned} \langle V(p) | J_\mu^{\bar{u}c} | D^0(P) \rangle = & P_0^V(k^2) k \cdot \varepsilon^* \frac{(m_D^2 - m_V^2)}{k^2} k^\mu + A_1^V(k^2) (m_D^2 - m_V^2) \left[\varepsilon^{*\mu} - \frac{k \cdot \varepsilon^*}{k^2} k^\mu \right] \\ & + A_2^V(k^2) k \cdot \varepsilon^* \left[p_+^\mu - \frac{(m_D^2 - m_V^2)}{k^2} k^\mu \right] + i V^V(k^2) \epsilon^{\mu\nu\lambda\rho} p_{+\nu} k_\lambda \varepsilon_\rho^* \end{aligned}$$

The form factors are determined using single pole exchange, e.g.

[Wirbel et al'85]

$$V(k^2) \sim \frac{h_{V1} m_{V1}^2}{m_{V1}^2 - k^2}$$

with $m_{V1} = 2110$ MeV and residues measured from $D^0 \rightarrow K^*$.

Form factors

One eventually finds

$$t_1^V(p^2, k^2) = -ie\xi_2^V(m_D + m_\rho) \left[J^V(k^2) \hat{A}_1(p^2) + \delta_{V,\rho} W(k^2) \hat{A}_1(k^2) \right]$$

$$t_2^V(p^2, k^2) = \frac{2ie\xi_2^V}{m_D + m_\rho} \left[J^V(k^2) \hat{A}_2(p^2) + \delta_{V,\rho} W(k^2) \hat{A}_2(k^2) \right]$$

$$t_3^V(p^2, k^2) = -\frac{2e\xi_2^V}{m_D + m_\rho} \left[J^V(k^2) \hat{V}(p^2) + \delta_{V,\rho} W(k^2) \hat{V}(k^2) \right]$$

with

$$J^V(k^2) = k^2 \left(\frac{f_\rho}{m_\rho P_\rho(k^2)} + \frac{f_\omega}{3m_\omega P_\omega(k^2)} \right) f_V m_V$$

$$W(k^2) = k^2 \left(\frac{f_\rho^2}{P_\rho(k^2)} + \frac{f_\omega^2}{3P_\omega(k^2)} - \frac{\xi_2^s}{\xi_2^d} \frac{\sqrt{2}f_\phi^2}{3P_\phi(k^2)} \right)$$

The strong parameters b_V can be determined from $V \rightarrow e^+e^-$ decay (experimental input).

$$b_\rho \sim 6.0; \quad b_{K^*} \sim 5.4; \quad b_\phi \sim 4.5$$

Form factors

In terms of the hadronic form factors:

$$F_{1V}^{(ab)}(t, b) = \left[(t_2^V [k \cdot (p_b - p_a)] - t_1^V) + \frac{m_a^2 - m_b^2}{m_V^2} (t_2^V p \cdot k + t_1^V) \right] \frac{b_V}{P_V(p^2)}$$

$$F_{2V}^{(ab)}(t, b) = \left[(t_2^V [k \cdot (p_b - p_a)] + t_1^V) + \frac{m_a^2 - m_b^2}{m_V^2} (t_2^V p \cdot k + t_1^V) \right] \frac{b_V}{P_V(p^2)}$$

$$F_{3V}^{(ab)}(t, b) = 2t_3^V \frac{b_V}{P_V(p^2)}$$

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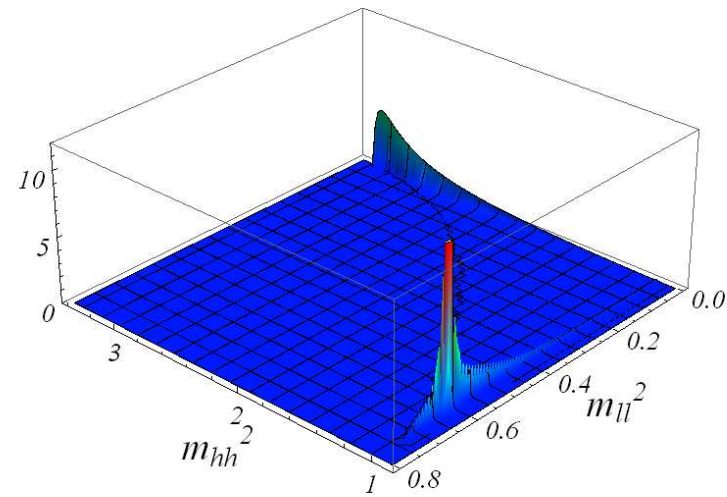
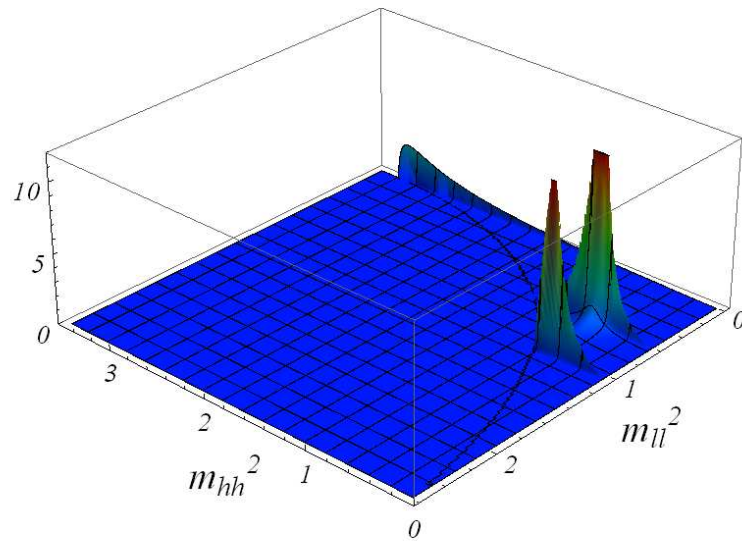
- t_i from $D^0 \rightarrow V$ transitions.
- b_V from $V \rightarrow e^+e^-$ decays.

Dalitz plots

$$F_1^{(\pi\pi)} = \frac{2ie}{2q \cdot p_1 + q^2} \mathcal{M}_{(D \rightarrow \pi\pi)} + b_\rho \frac{t_2^\rho [q \cdot (p_1 - p_2)] + t_1^\rho}{P_\rho(p^2)}$$

$$F_2^{(\pi\pi)} = -\frac{2ie}{2q \cdot p_2 + q^2} \mathcal{M}_{(D \rightarrow \pi\pi)} + b_\rho \frac{t_2^\rho [q \cdot (p_1 - p_2)] - t_1^\rho}{P_\rho(p^2)}$$

$$F_3^{(\pi\pi)} = -2b_\rho \frac{t_3^\rho}{P_\rho(p^2)}$$



Bremsstrahlung and resonant regions far apart: interference negligible.

Branching ratios

Decay mode	Bremsstrahlung	Direct emission (E)	Direct emission (M)
$D^0 \rightarrow K^- \pi^+ e^+ e^-$	$9.9 \cdot 10^{-6}$	$6.2 \cdot 10^{-6}$	$4.8 \cdot 10^{-7}$
$D^0 \rightarrow \pi^+ \pi^- e^+ e^-$	$5.3 \cdot 10^{-7}$	$1.3 \cdot 10^{-6}$	$1.3 \cdot 10^{-7}$
$D^0 \rightarrow K^+ K^- e^+ e^-$	$5.4 \cdot 10^{-7}$	$1.1 \cdot 10^{-7}$	$5.0 \cdot 10^{-9}$
$D^0 \rightarrow K^+ \pi^- e^+ e^-$	$3.7 \cdot 10^{-8}$	$1.7 \cdot 10^{-8}$	$1.3 \cdot 10^{-9}$
$D^0 \rightarrow K^- \pi^+ \mu^+ \mu^-$	$8.6 \cdot 10^{-8}$	$6.2 \cdot 10^{-6}$	$4.8 \cdot 10^{-7}$
$D^0 \rightarrow \pi^+ \pi^- \mu^+ \mu^-$	$5.6 \cdot 10^{-9}$	$1.3 \cdot 10^{-6}$	$1.3 \cdot 10^{-7}$
$D^0 \rightarrow K^+ K^- \mu^+ \mu^-$	$3.3 \cdot 10^{-9}$	$1.1 \cdot 10^{-7}$	$5.0 \cdot 10^{-9}$
$D^0 \rightarrow K^+ \pi^- \mu^+ \mu^-$	$3.3 \cdot 10^{-10}$	$1.7 \cdot 10^{-8}$	$1.3 \cdot 10^{-9}$

- Experimental results (LHCb and BaBar): [\[arXiv:1510.08367; arXiv:1707.08377; arXiv:1808.09680\]](#)

$$\mathcal{B}(D^0 \rightarrow K^- \pi^+ \mu^+ \mu^-) = (4.17 \pm 0.42) \cdot 10^{-6}$$

$$\mathcal{B}(D^0 \rightarrow \pi^+ \pi^- \mu^+ \mu^-) = (9.64 \pm 0.48 \pm 0.51 \pm 0.97) \cdot 10^{-7}$$

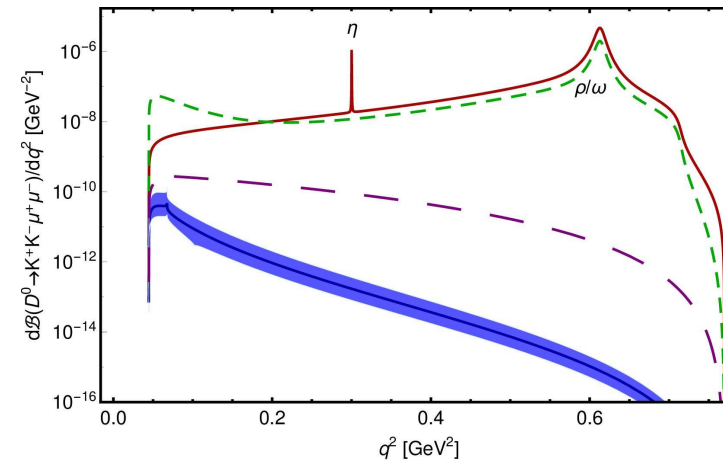
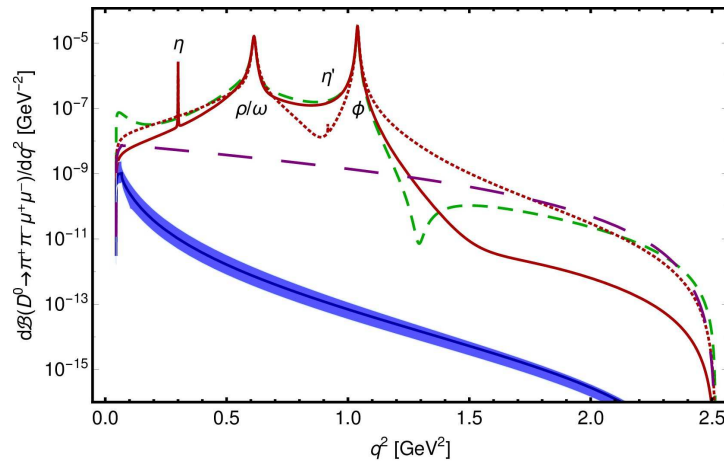
$$\mathcal{B}(D^0 \rightarrow K^+ K^- \mu^+ \mu^-) = (1.54 \pm 0.27 \pm 0.09 \pm 0.16) \cdot 10^{-7}$$

$$\mathcal{B}(D^0 \rightarrow K^- \pi^+ e^+ e^-)_{\text{res}} = (4.0 \pm 0.5 \pm 0.2 \pm 0.1) \cdot 10^{-6}$$

- Good agreement overall. Approximations capture the bulk of the effect.

Branching ratios

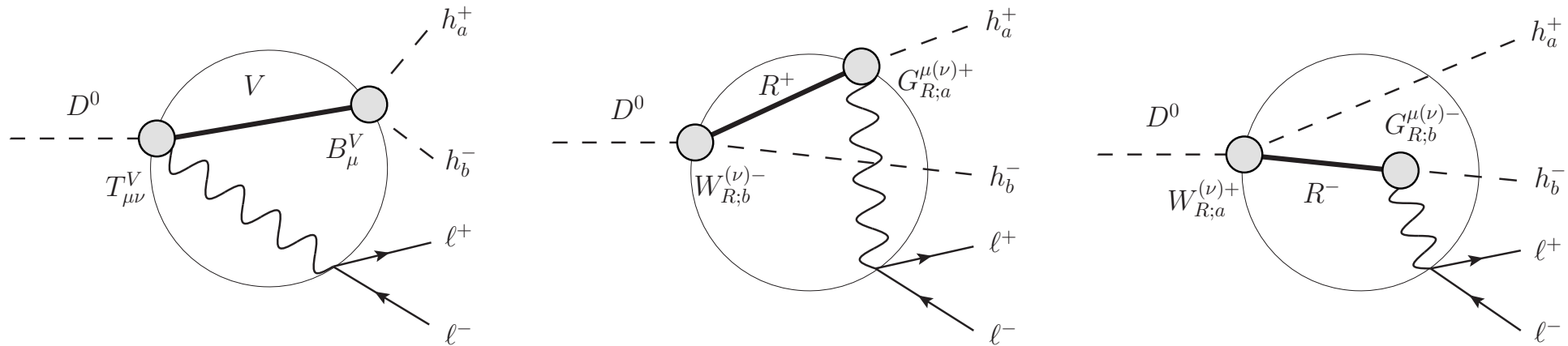
Numbers for SCS modes and dilepton invariant mass distribution confirmed: [de Boer, Hiller'18]



Similar techniques are employed but different numerical input. Nonresonant effects also estimated (negligible).

Additional resonant contributions: the role of axials

[OC, D'Ambrosio, in preparation]

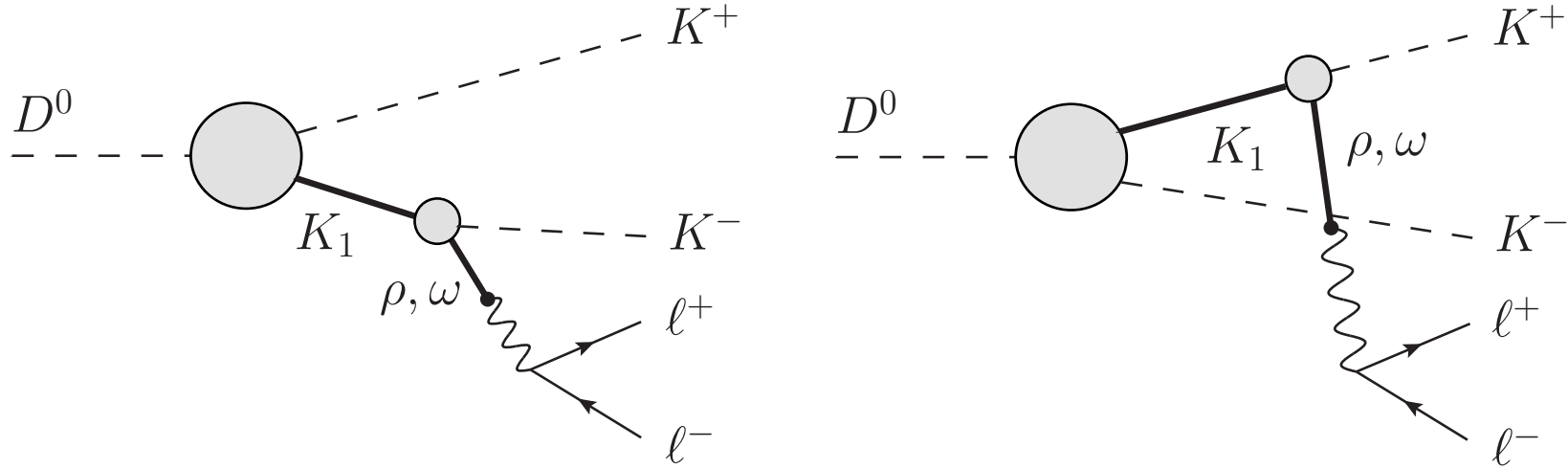


- Extra topologies have intermediate axials ($a_1(1230)$ and $K_1(1272)$). A priori sizeable.

Decay mode	Bremsstrahlung	Resonant V. (E)	Resonant A. (E)
$D^0 \rightarrow K^+ K^- \mu^+ \mu^-$	$3.3 \cdot 10^{-9}$	$8.2 \cdot 10^{-8}$	$2.5 \cdot 10^{-8}$

Preliminary estimate ($\sim 30\%$ of the vector contribution). Interference not negligible.

Extraction of form factors

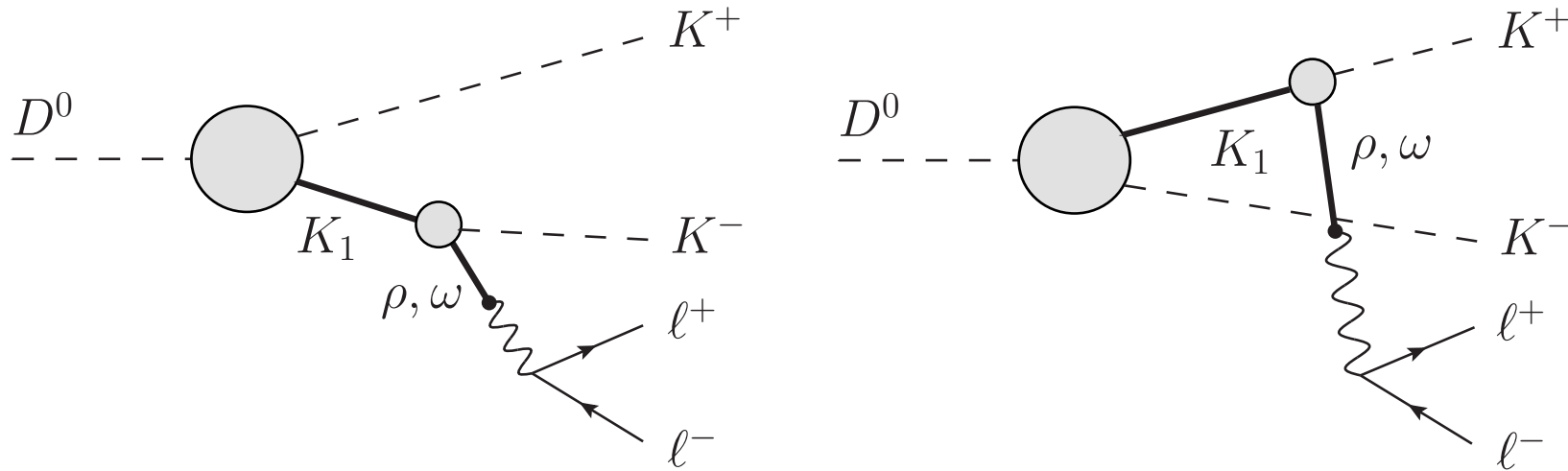


$$F_{1A}^{(ab)} = \frac{w_1^{A;a}}{P_A(q_+^2)} g_{1A;b}^- + \frac{w_1^{A;b}}{P_A(q_-^2)} \left[g_{2A;a}^+ \left(p_b \cdot k - \frac{(k \cdot q_-)(p_b \cdot q_-)}{m_A^2} \right) - g_{1A;a}^+ \frac{p_b \cdot q_-}{m_A^2} \right]$$

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$$F_{3V}^{(ab)} = \frac{w_1^{V;a}}{P_V(q_+^2)} g_{3V;b}^- - \frac{w_1^{V;b}}{P_V(q_-^2)} g_{3V;a}^+$$

Extraction of form factors



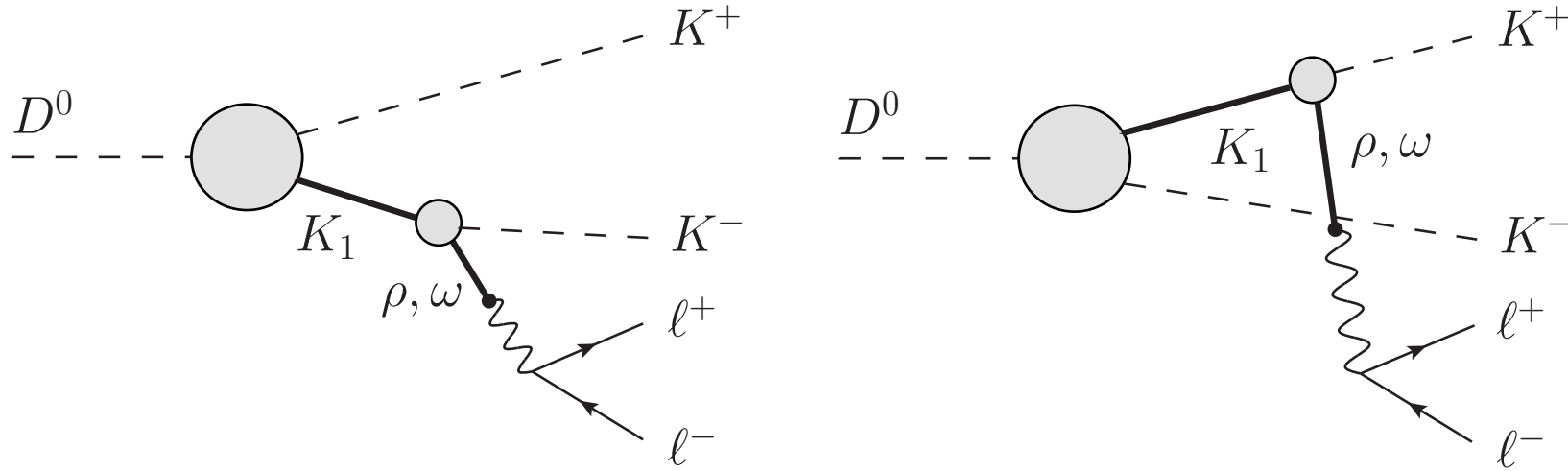
$$F_{1A}^{(ab)} = \frac{w_1^{A;a}}{P_A(q_+^2)} g_{1A;b}^- + \frac{w_1^{A;b}}{P_A(q_-^2)} \left[g_{2A;a}^+ \left(p_b \cdot k - \frac{(k \cdot q_-)(p_b \cdot q_-)}{m_A^2} \right) - g_{1A;a}^+ \frac{p_b \cdot q_-}{m_A^2} \right]$$

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- w_1 from $D^0 \rightarrow P, A$ transitions (applying factorization). $D^0 \rightarrow P$ well-known, $D^0 \rightarrow A$ not measured, so only an educated estimate is possible.

Extraction of form factors



$$F_{1A}^{(ab)} = \frac{w_1^{A;a}}{P_A(q_+^2)} g_{1A;b}^- + \frac{w_1^{A;b}}{P_A(q_-^2)} \left[g_{2A;a}^+ \left(p_b \cdot k - \frac{(k \cdot q_-)(p_b \cdot q_-)}{m_A^2} \right) - g_{1A;a}^+ \frac{p_b \cdot q_-}{m_A^2} \right]$$

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- w_1 from $D^0 \rightarrow P, A$ transitions (applying factorization). $D^0 \rightarrow P$ well-known, $D^0 \rightarrow A$ not measured, so only an educated estimate is possible.
- g_i from VAP correlator [Moussallam'98, Knecht et al'01], once properly LSZ-reduced.

Short distances

- Most general angular distribution in terms of 9 structures:

$$\begin{aligned}\frac{d^5\Gamma}{dxdy} = & \mathcal{A}_1(x) + \mathcal{A}_2(x)s_\ell^2 + \mathcal{A}_3(x)s_\ell^2c_\phi^2 + \mathcal{A}_4(x)s_{2\ell}c_\phi \\ & + \mathcal{A}_5(x)s_\ell c_\phi + \mathcal{A}_6(x)c_\ell + \mathcal{A}_7(x)s_\ell s_\phi \\ & + \mathcal{A}_8(x)s_{2\ell}s_\phi + \mathcal{A}_9(x)s_\ell^2s_{2\phi}\end{aligned}$$

Only the first line contributes to the decay width. Sensitivity to remaining structures through angular asymmetries.

- Null tests especially interesting. Examples:

$$\begin{aligned}A_\phi = \langle \text{sgn}(s_\phi c_\phi) \rangle &= \frac{1}{\Gamma} \int_0^{2\pi} \frac{d\Gamma}{d\phi} d\phi^*, \quad \int_0^{2\pi} d\phi^* \equiv \left[\int_0^{\pi/2} - \int_{\pi/2}^\pi + \int_\pi^{3\pi/2} - \int_{3\pi/2}^{2\pi} \right] d\phi \\ A_{FB} = \langle \text{sgn}(c_\ell) \rangle &= \frac{1}{\Gamma} \left[\int_0^1 dy \frac{d\Gamma}{dy} - \int_{-1}^0 dy \frac{d\Gamma}{dy} \right]\end{aligned}$$

Recently measured:

[LHCb, arXiv:1806.10793]

$$\begin{aligned}A_{CP}(\pi\pi) &= (4.9 \pm 3.8 \pm 0.7)\%, & A_{CP}(KK) &= (0 \pm 11 \pm 2)\% \\ A_\phi(\pi\pi) &= (-0.6 \pm 3.7 \pm 0.6)\%, & A_\phi(KK) &= (9 \pm 11 \pm 1)\% \\ A_{FB}(\pi\pi) &= (3.3 \pm 3.7 \pm 0.6)\%, & A_{FB}(KK) &= (0 \pm 11 \pm 2)\%\end{aligned}$$

- Rather exhaustive analysis of null tests recently performed.

[de Boer, Hiller'18]

Conclusions

- Long-distance estimates of $D^0 \rightarrow V(h^+h^-)\ell^+\ell^-$ refined with axial states (sizeable effect). Good agreement with experiment remains.
- A priori vector and axial resonances could be disentangled experimentally.
- Determination of the DCS mode $D^0 \rightarrow K^+\pi^-\mu^+\mu^-$ and the modes decaying into electrons interesting to test the consistency of the theoretical approximations.
- Short distances: probes of new physics not restricted to charge asymmetry. Clean angular asymmetries exist. Distributions are peaked at the resonant poles. Reanalysis of short distances with axials in mind important.
- Role of resonant axial states also interesting in $B \rightarrow P_1P_2\ell^+\ell^-$ decays, where there is better theoretical control.