Beam Optics in the Cooling Channel

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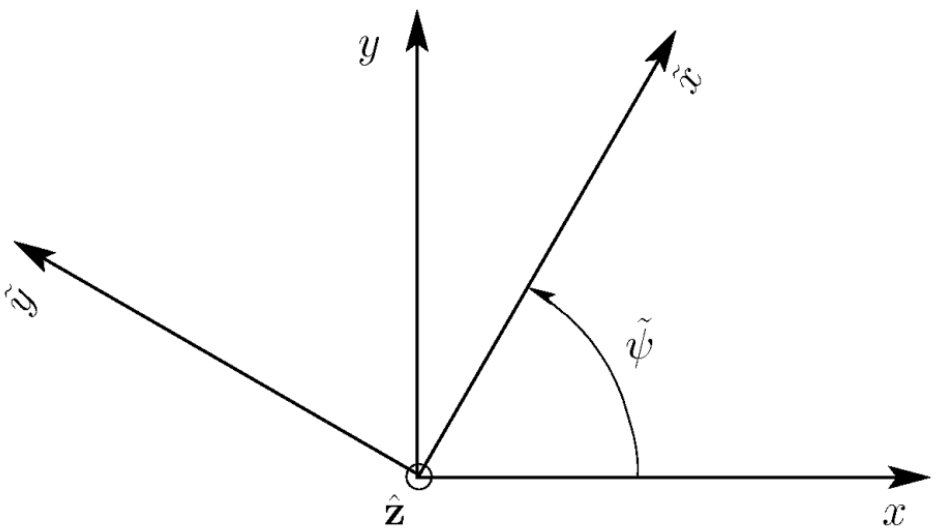
Overview

- Spent some time studying the beam phase-space in the cooling channel
- Transformed particles in the beam into their rotating Larmor frame to decouple the x and y dynamics
- Further transformed xpx and ypy phase space into normalised coordinates
- Started testing a mismatch model for emittance growth
- Used hybrid MC truth from data run 10448, 3 mm, 140 MeV/c, flip mode, no absorber

Larmor Angle



$\tilde{...}$ used to denote rotating frame variables

$$\tilde{x} = x\cos\tilde{\psi}(s) + y\sin\tilde{\psi}(s)$$

$$\tilde{y} = -x\sin\tilde{\psi}(s) + y\cos\tilde{\psi}(s)$$

$$\tilde{\psi}(s) = -\int_{s_i}^{s} d\bar{s}\; k_L(\bar{s})$$

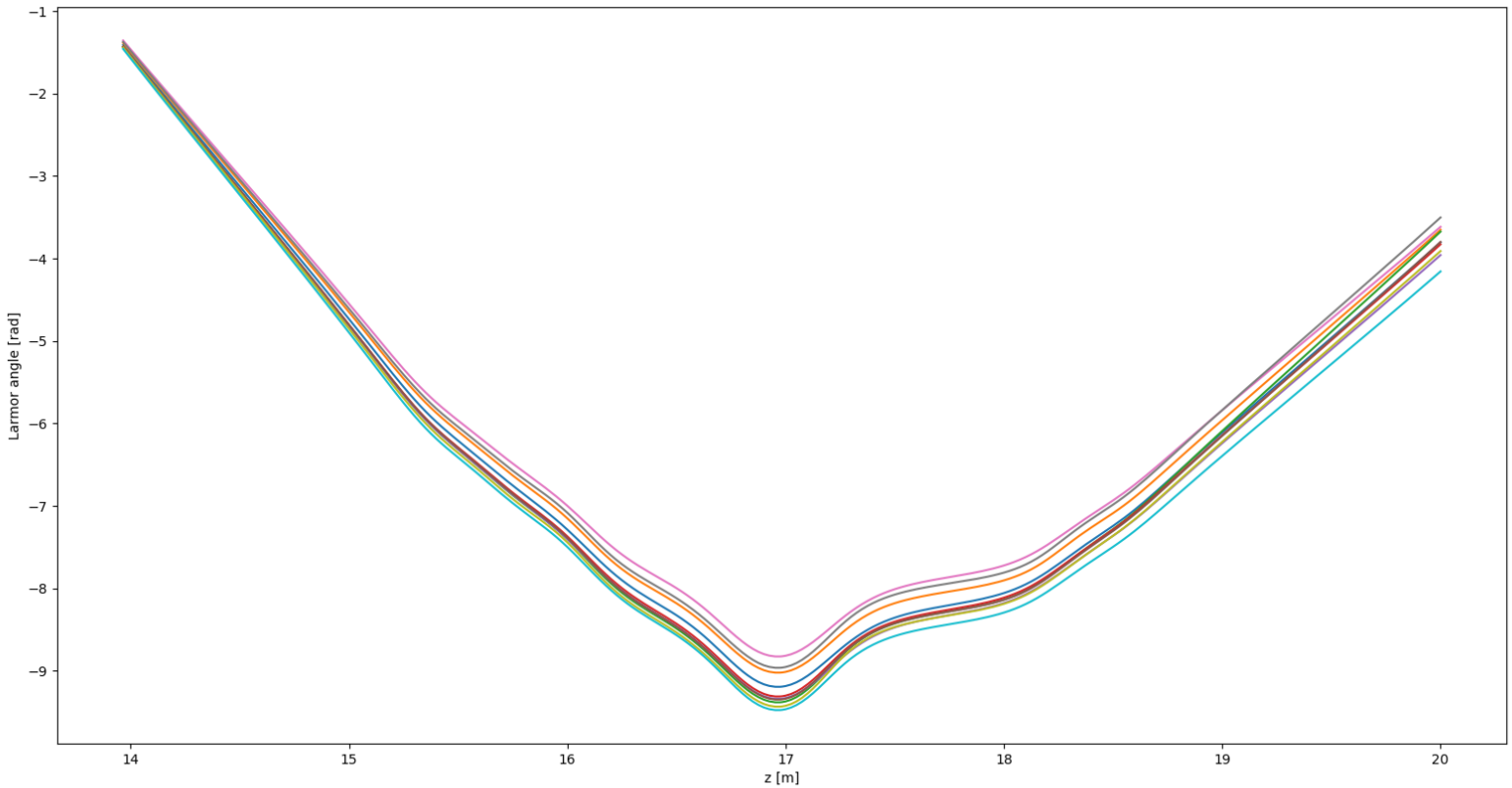
$$k_L(s) \equiv \frac{B_{z0}(s)}{2[B\rho]} = \frac{\omega_c(s)}{2\gamma_b\beta_b c}$$

$= \text{Larmor wave number}$

$s = s_i$ defines initial condition

Larmor Angle

- Computed the Larmor angle for all the particles in the sample

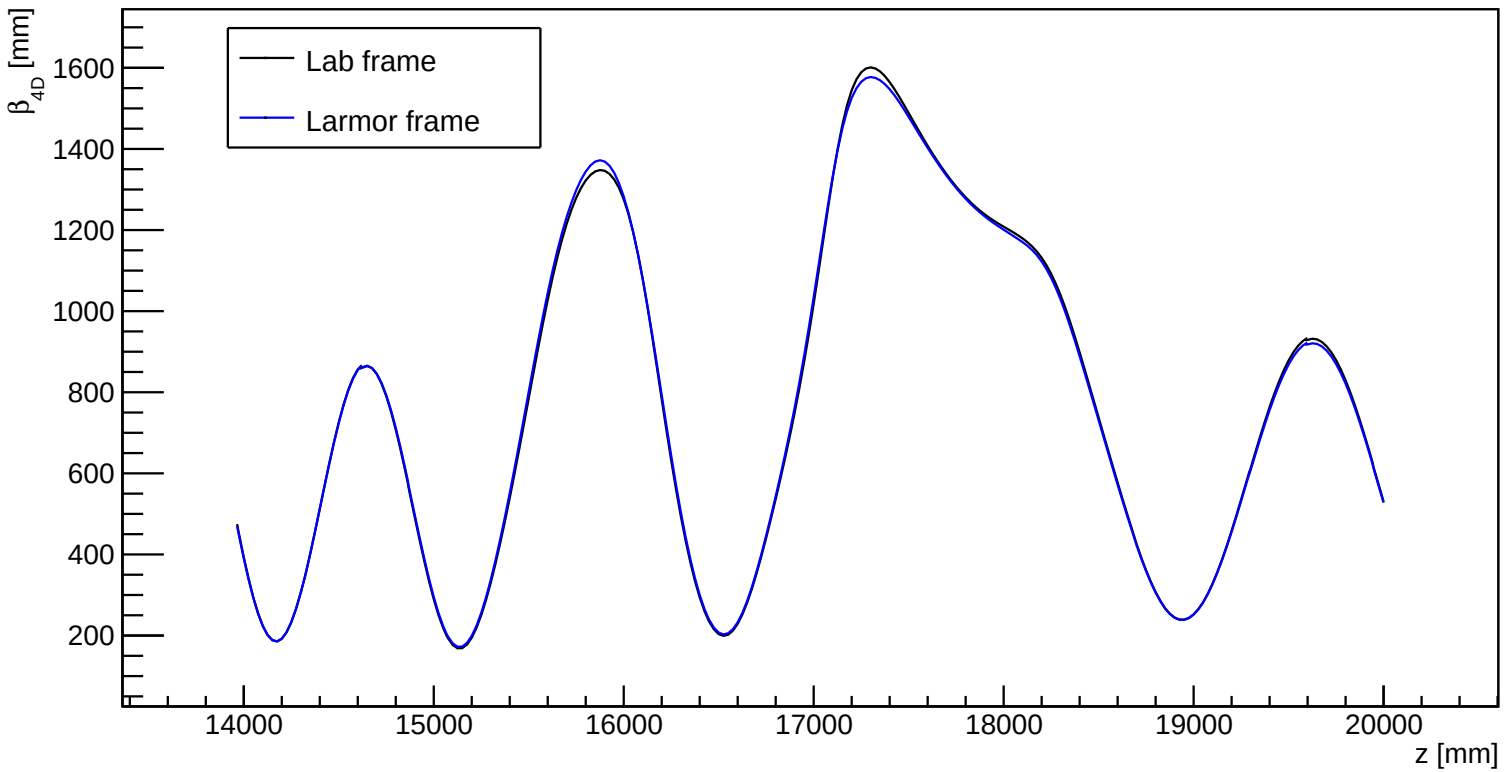


Transformation to Larmor Frame

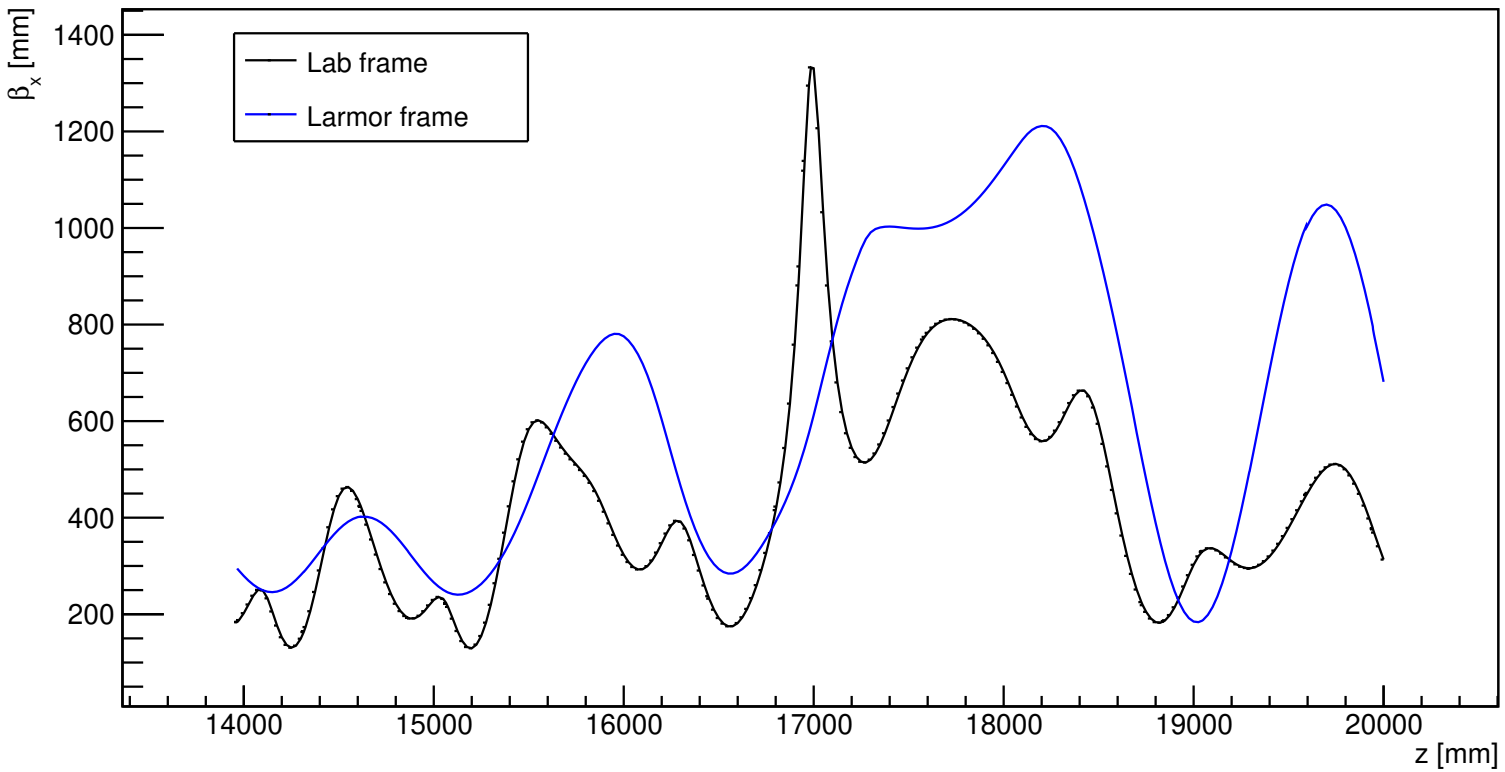
$$\begin{bmatrix} \tilde{x} \\ \tilde{x}' \\ \tilde{y} \\ \tilde{y}' \end{bmatrix} = \tilde{\mathbf{M}}_r^{-1}(s|s_i) \cdot \begin{bmatrix} x \\ x' \\ y \\ y' \end{bmatrix}$$

$$\tilde{\mathbf{M}}_r^{-1}(s|s_i) = \begin{bmatrix} \cos\tilde{\psi} & 0 & \sin\tilde{\psi} & 0 \\ k_L \sin\tilde{\psi} & \cos\tilde{\psi} & -k_L \cos\tilde{\psi} & \sin\tilde{\psi} \\ -\sin\tilde{\psi} & 0 & \cos\tilde{\psi} & 0 \\ k_L \cos\tilde{\psi} & -\sin\tilde{\psi} & k_L \sin\tilde{\psi} & \cos\tilde{\psi} \end{bmatrix}$$

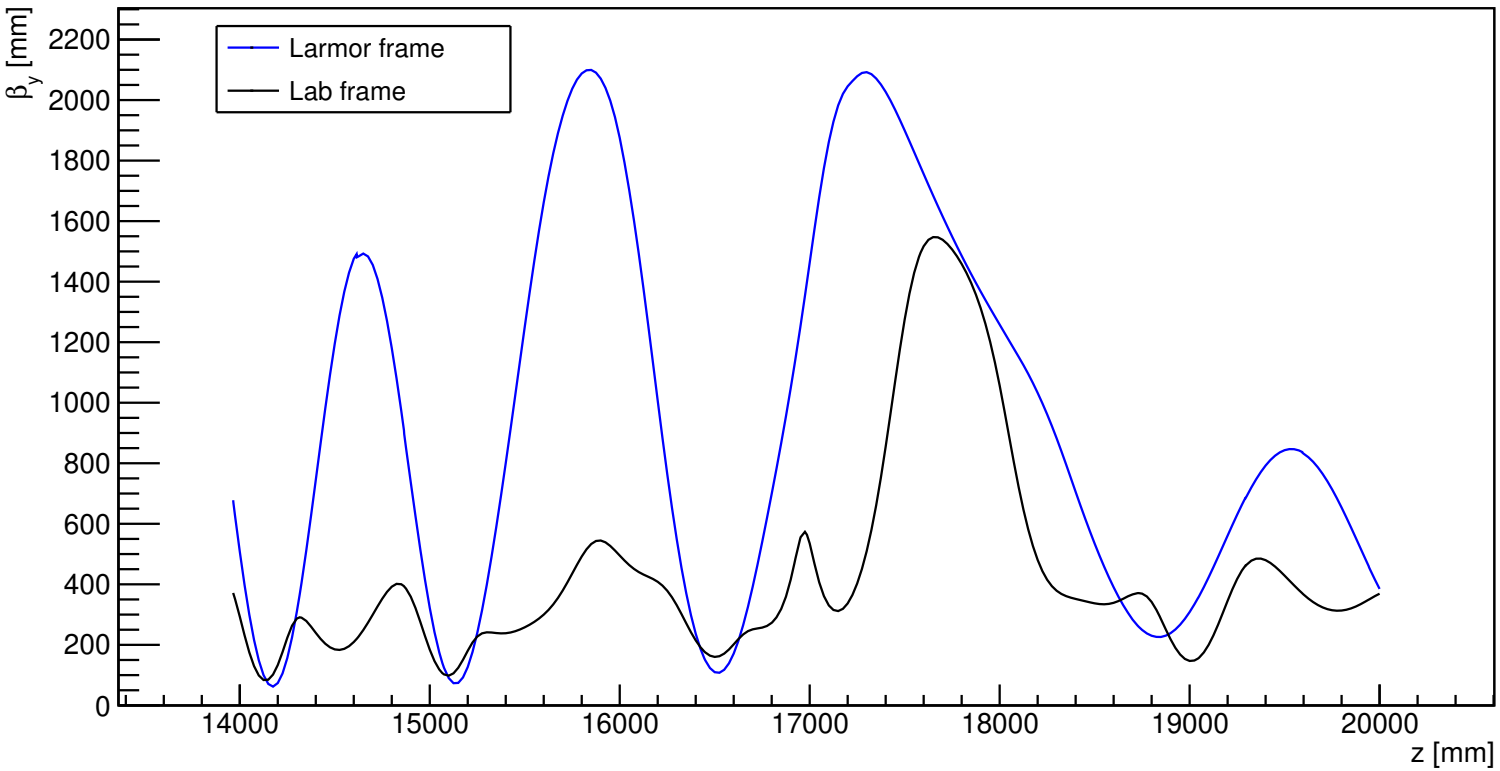
Twiss parameters: β_{4D}



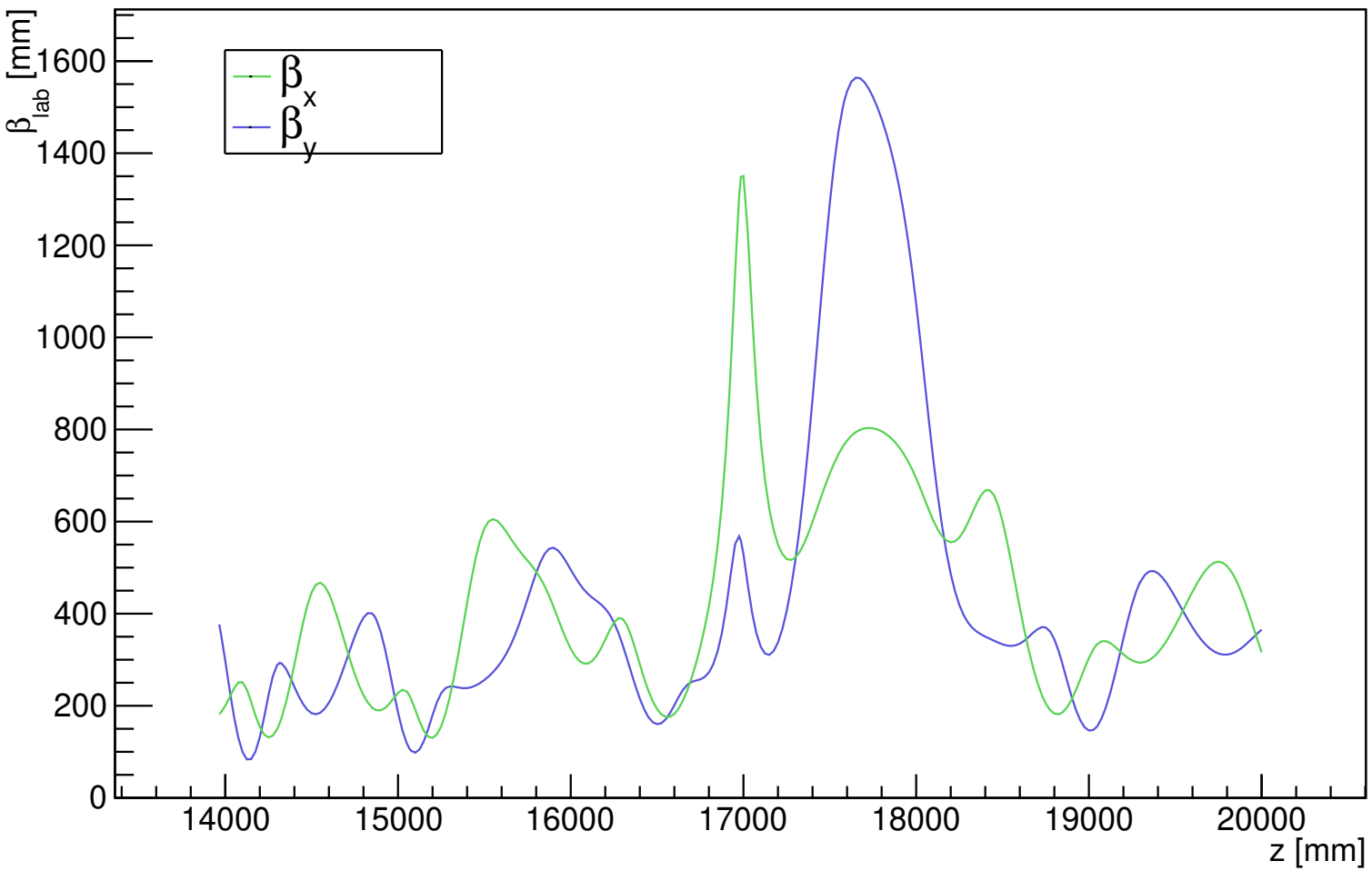
Twiss parameters: β_x



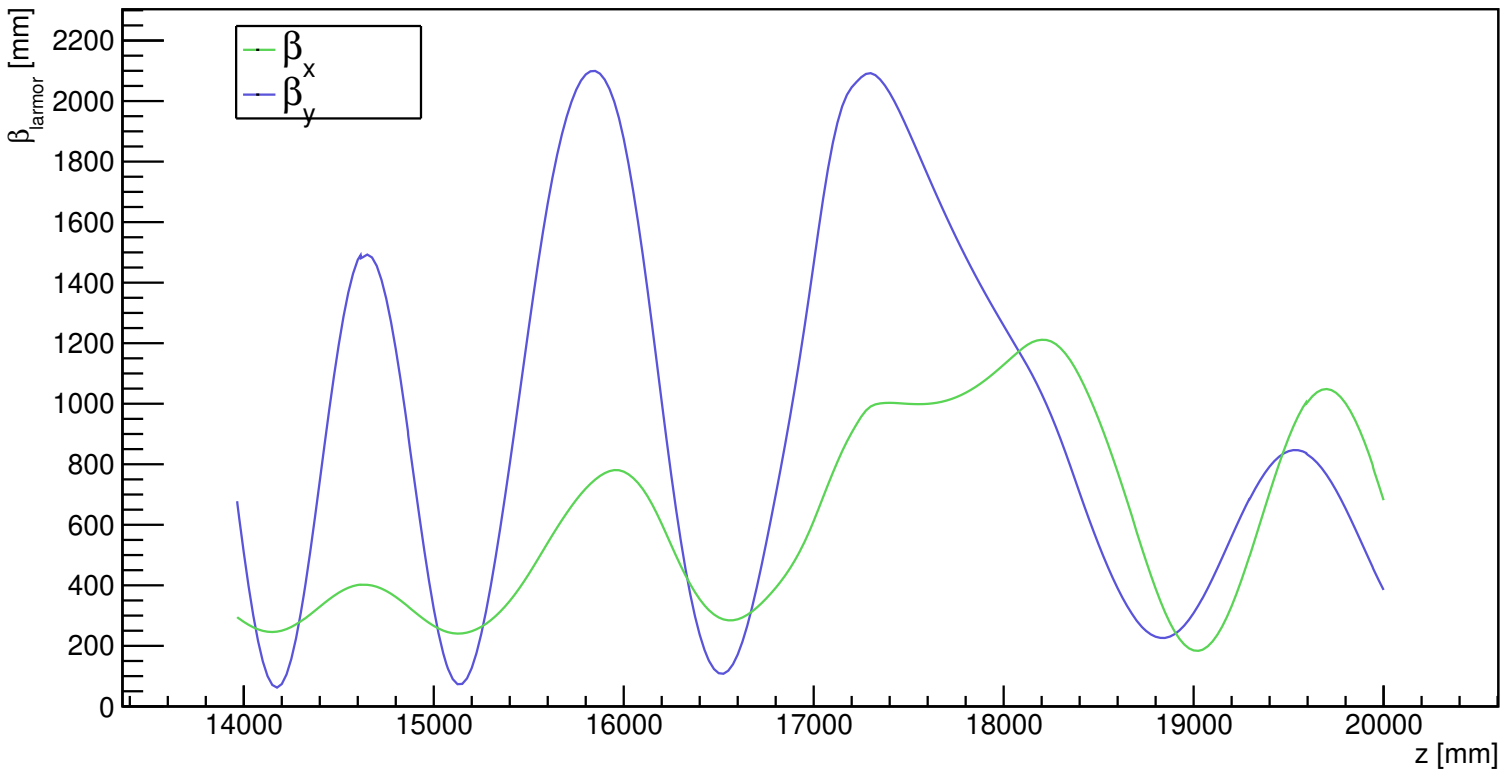
Twiss parameters: β_y



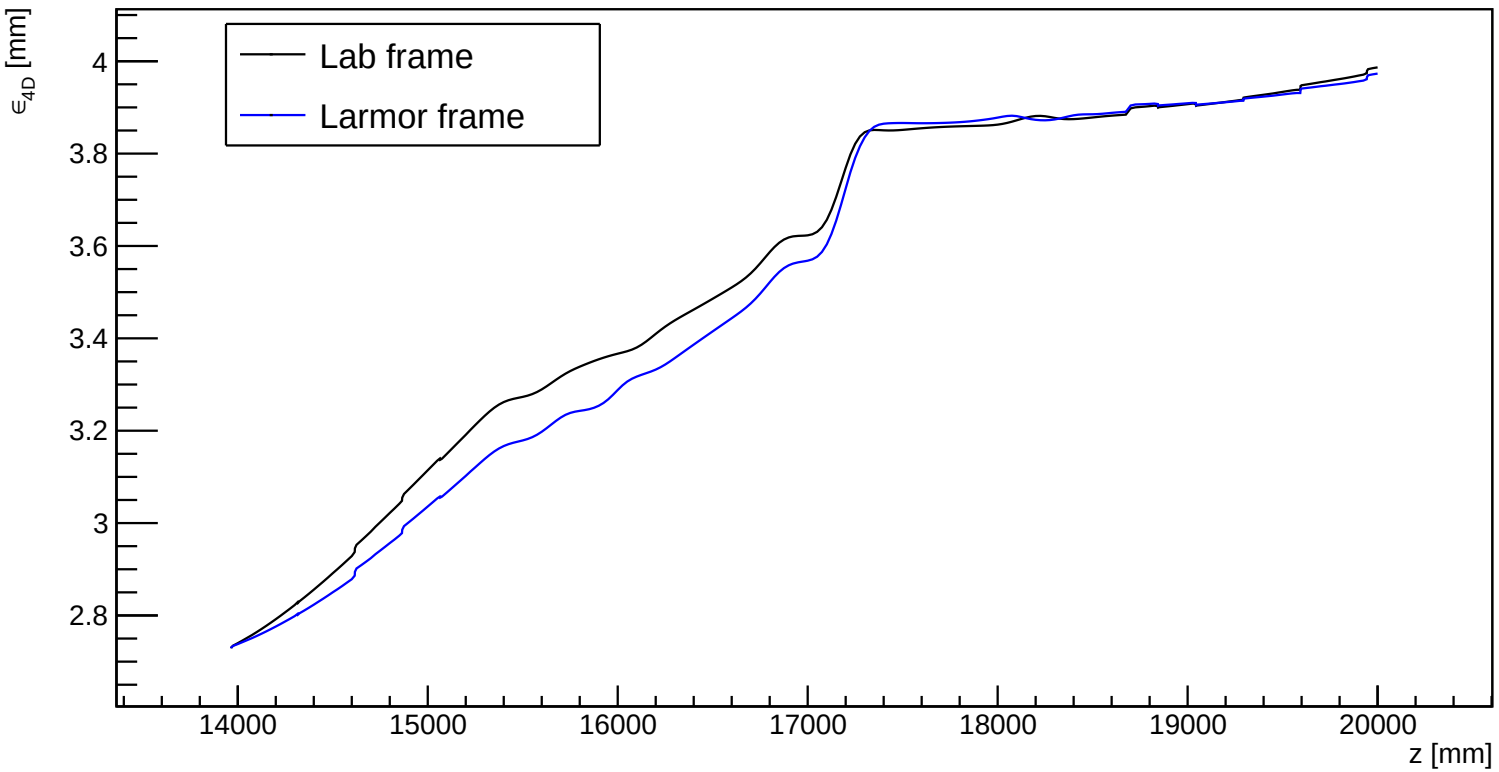
Twiss parameters: β_x vs β_y lab frame



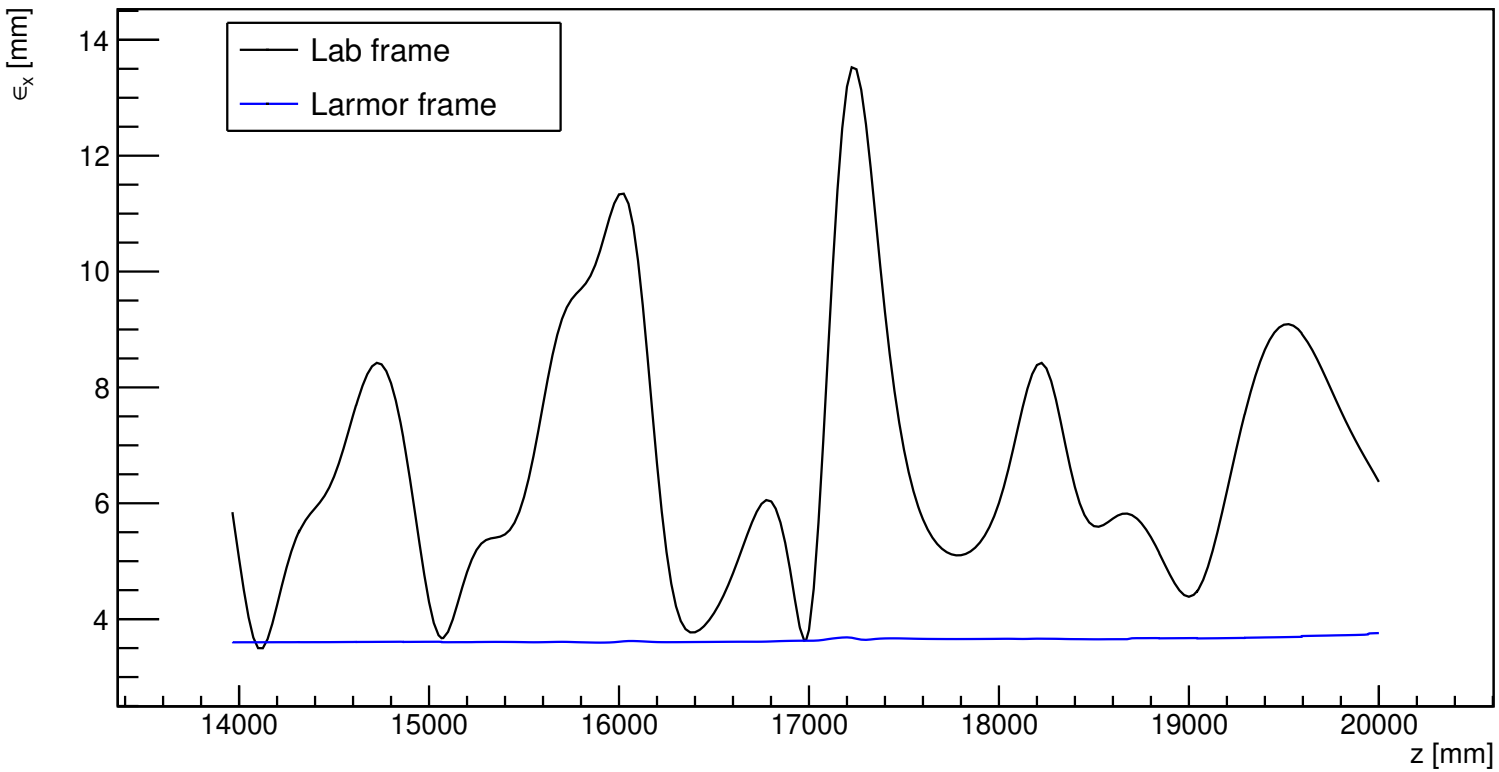
Twiss parameters: β_x vs β_y Larmor frame



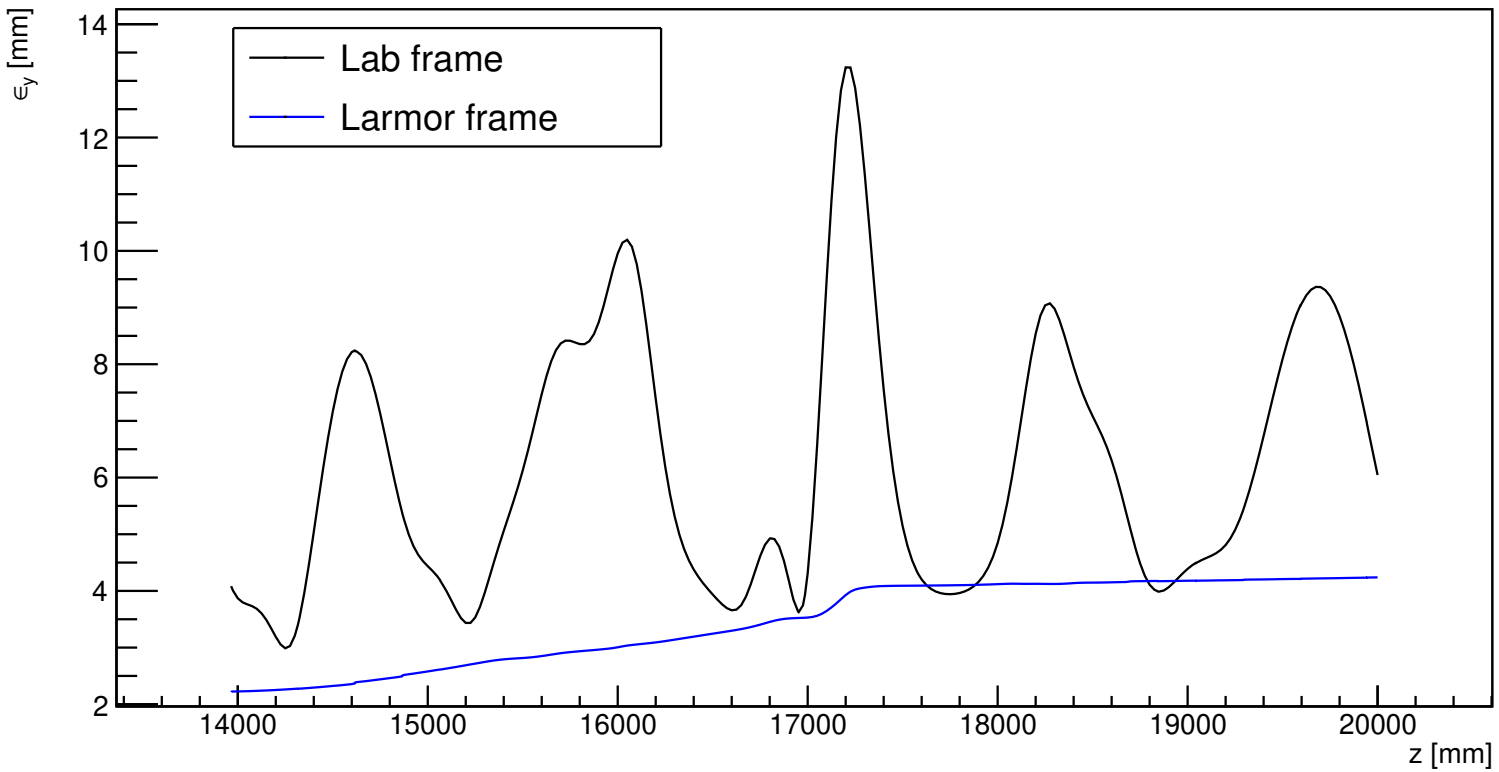
Emittance: ϵ_{4D}



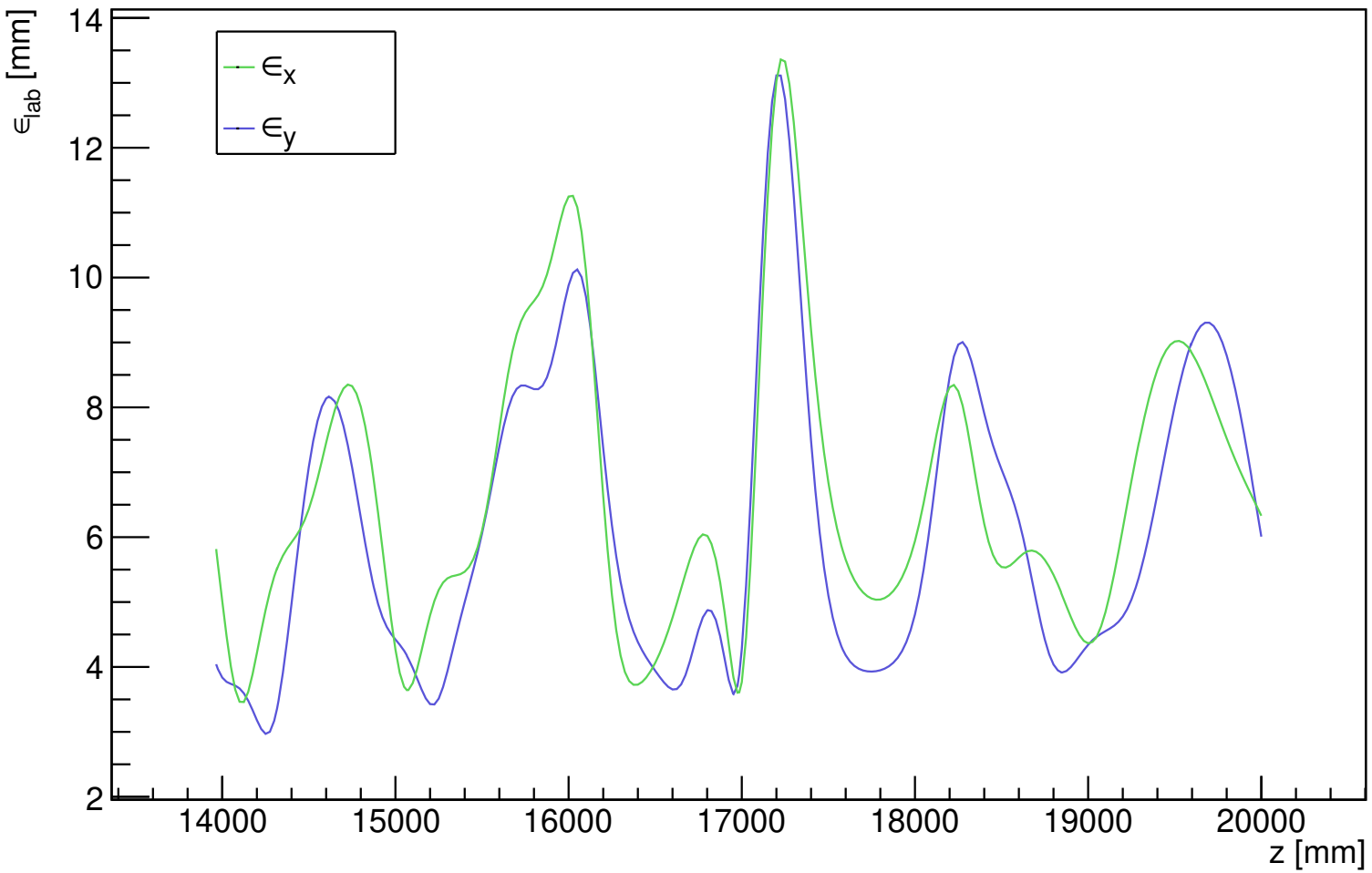
Emittance: ϵ_x



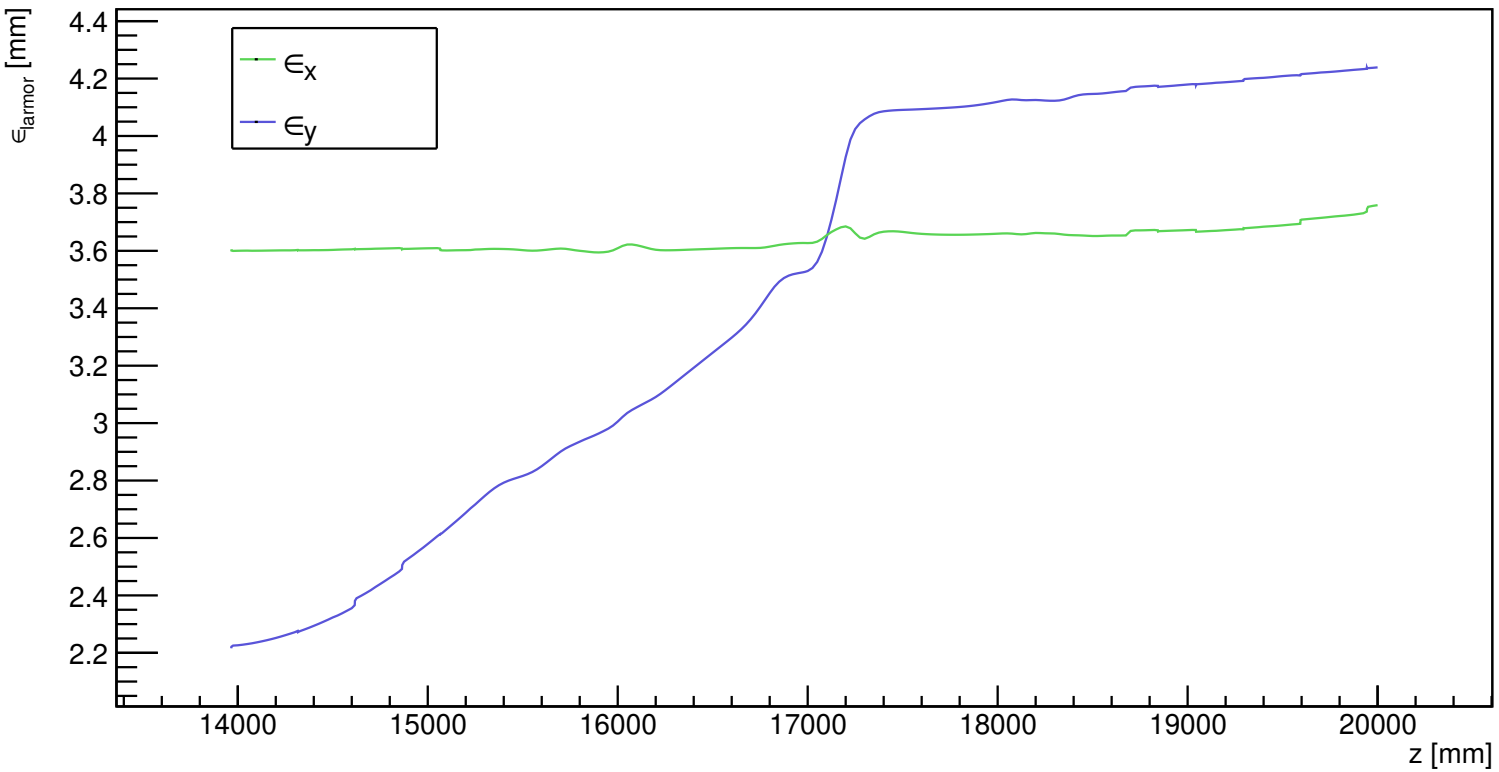
Emittance: ϵ_y



Emittance: ϵ_x vs ϵ_y lab frame

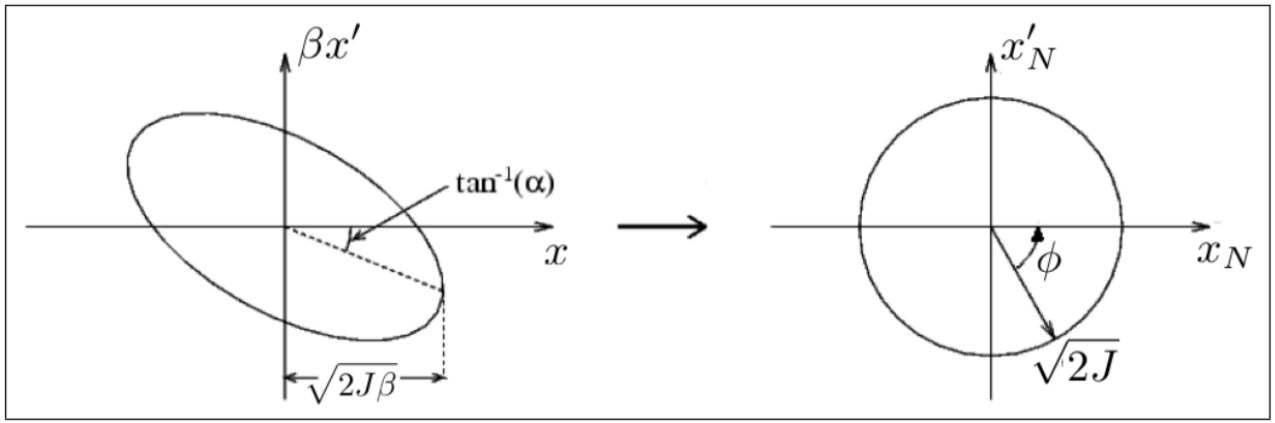


Emittance: ϵ_x vs ϵ_y Larmor frame



Normalised Coordinates

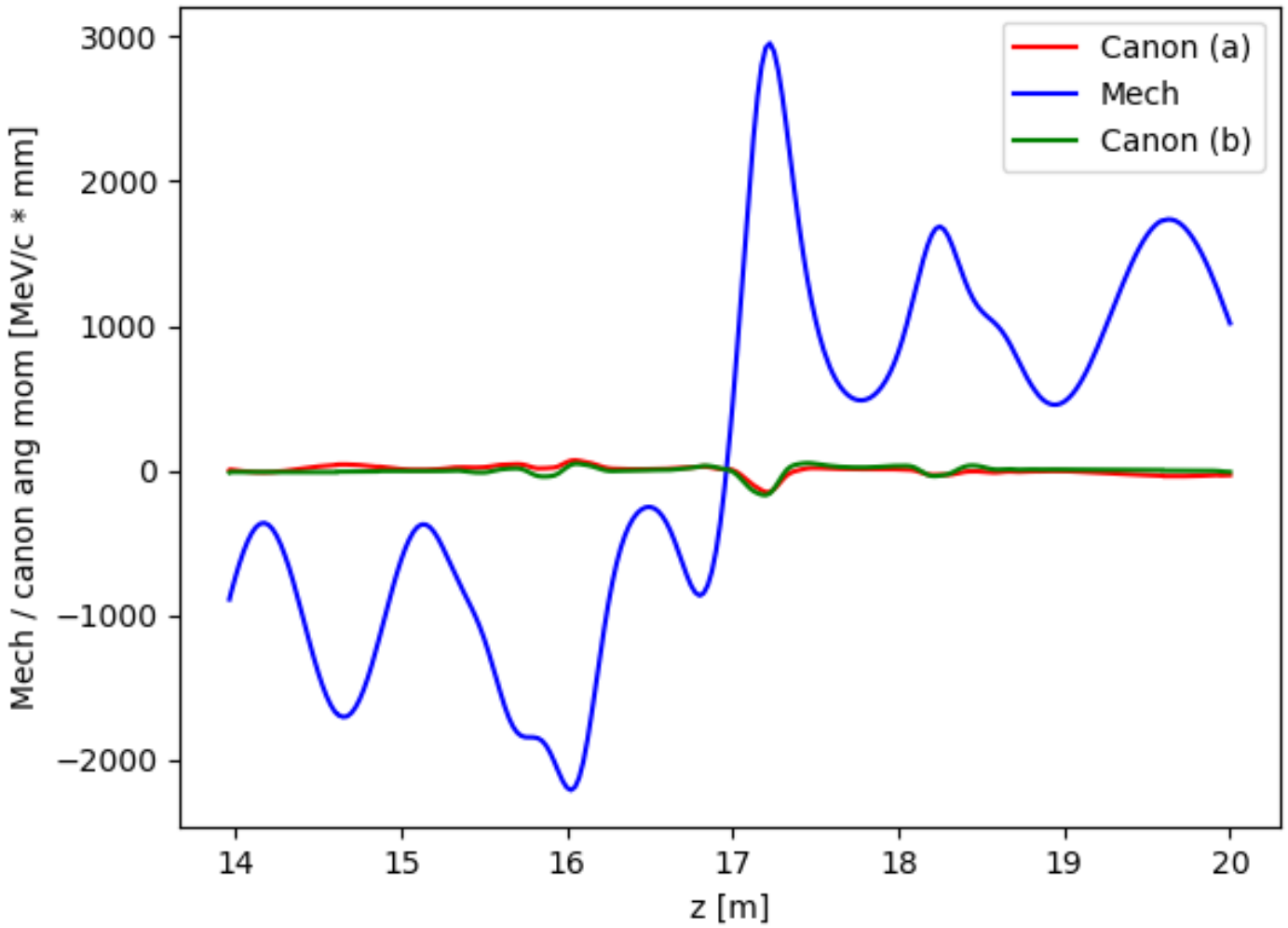
$$\begin{pmatrix} x_N \\ x'_N \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \frac{\alpha}{\sqrt{\beta}} & \sqrt{\beta} \end{pmatrix} \begin{pmatrix} x \\ x' \end{pmatrix}$$



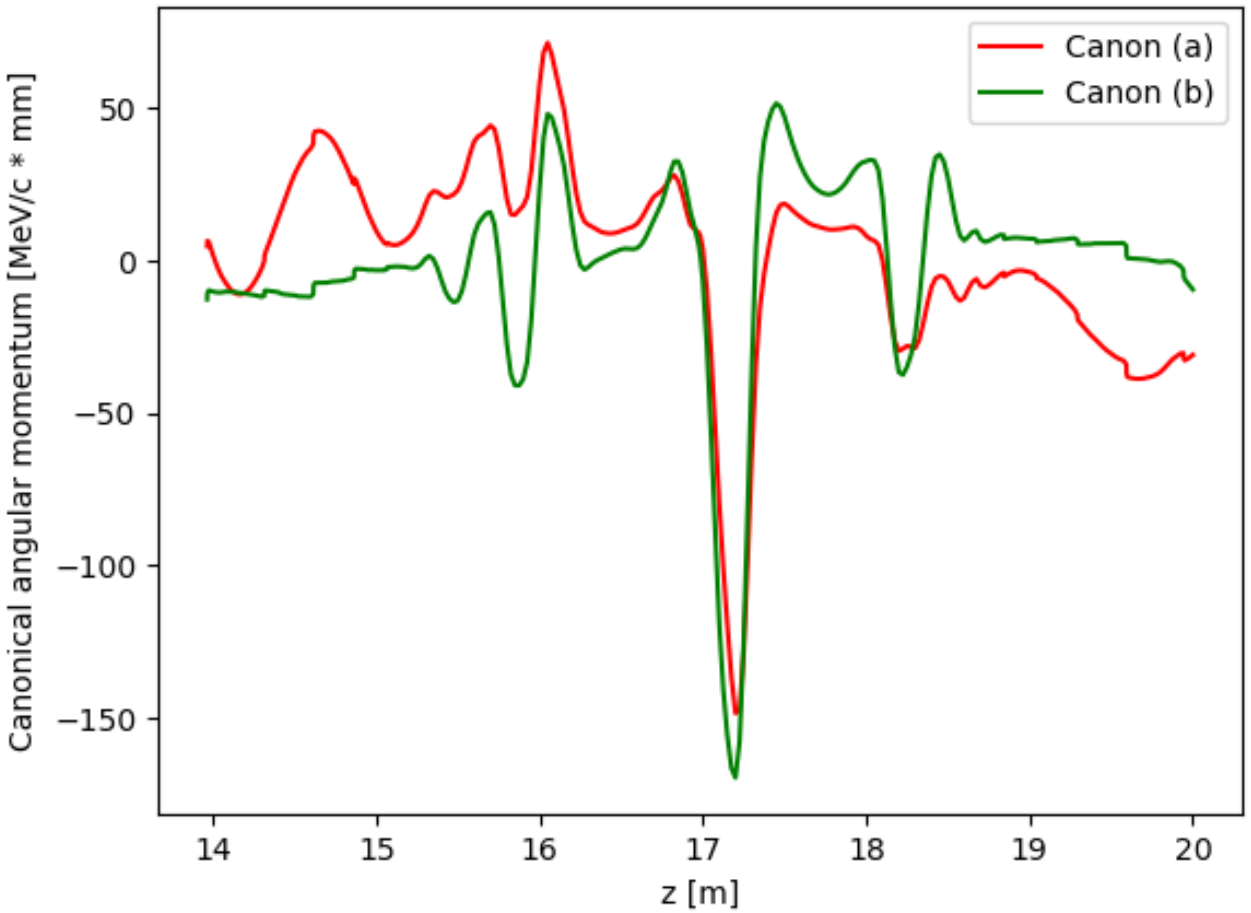
- Nonlinearities are expected to deform the circles in the normalised coordinates phase space

Videos with the phase space evolution in the cooling channel.

Canonical vs Mechanical angular momentum



Canonical angular momentum



$(a) Red : \langle L_{canon} \rangle = \langle L_{mech} \rangle + 2mc\epsilon_{4D}\beta_{4D}\langle \kappa \rangle \, (lab\ frame)$

(b) Green : $\langle L_{mech} \rangle = \langle xP_y \rangle - \langle yP_x \rangle \, (Larmor\ frame)$

Emittance growth due to beta functions mismatch

If the beam is mismatched to the lattice and the beam filaments, the rms emittance will increase according to:

$$\epsilon_f = B_{mag}\epsilon$$

where

$$B_{mag} = \frac{1}{2}[\frac{\beta^*}{\beta} + \frac{\beta}{\beta^*} + (\alpha\sqrt{\frac{\beta^*}{\beta}} - \alpha^*\sqrt{\frac{\beta}{\beta^*}})^2]$$

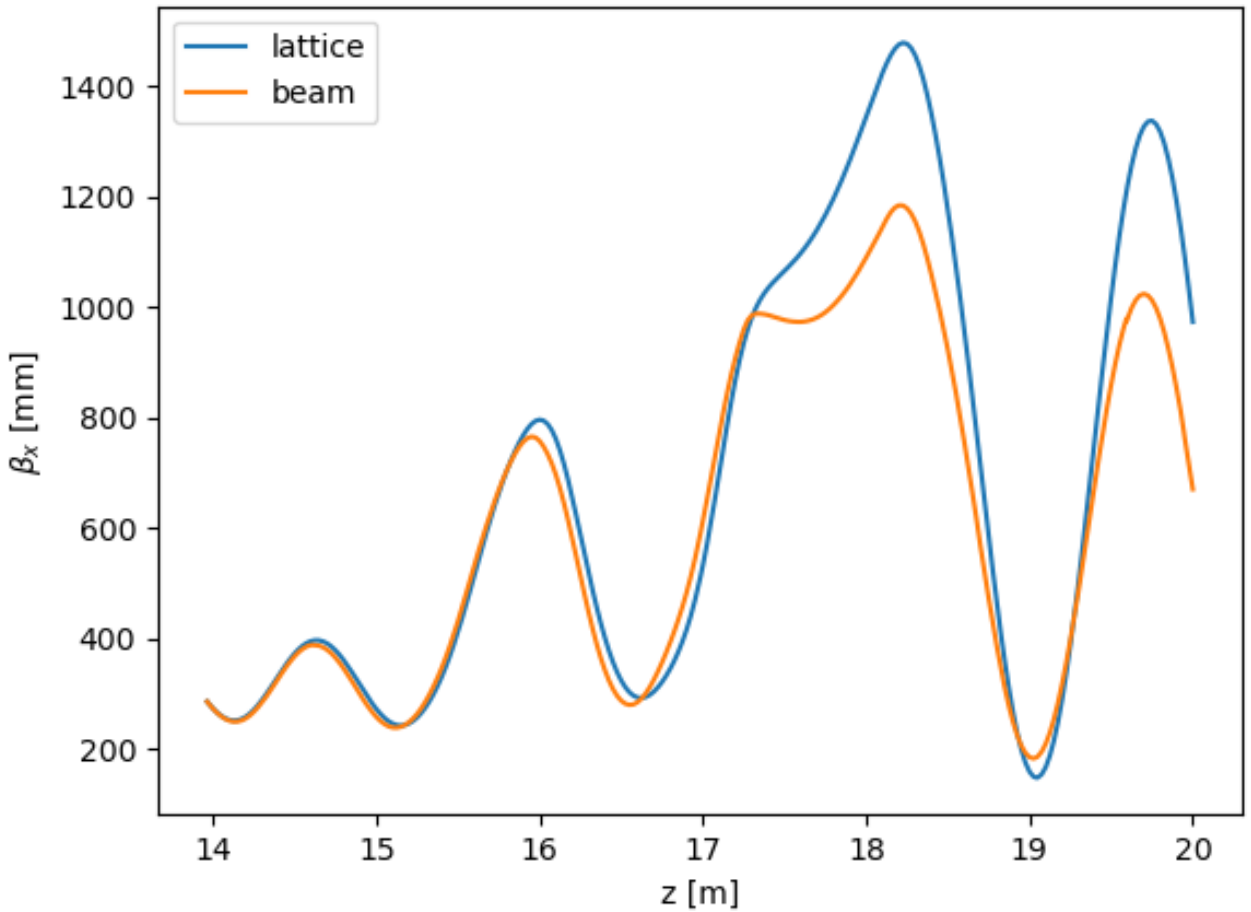
where the * quantities represent the beam. Lattice Twiss functions computed using:

$$\frac{1}{2}\beta\beta'' - \frac{1}{4}\beta'^2 + \frac{\kappa^2}{4}\beta^2 = 1$$

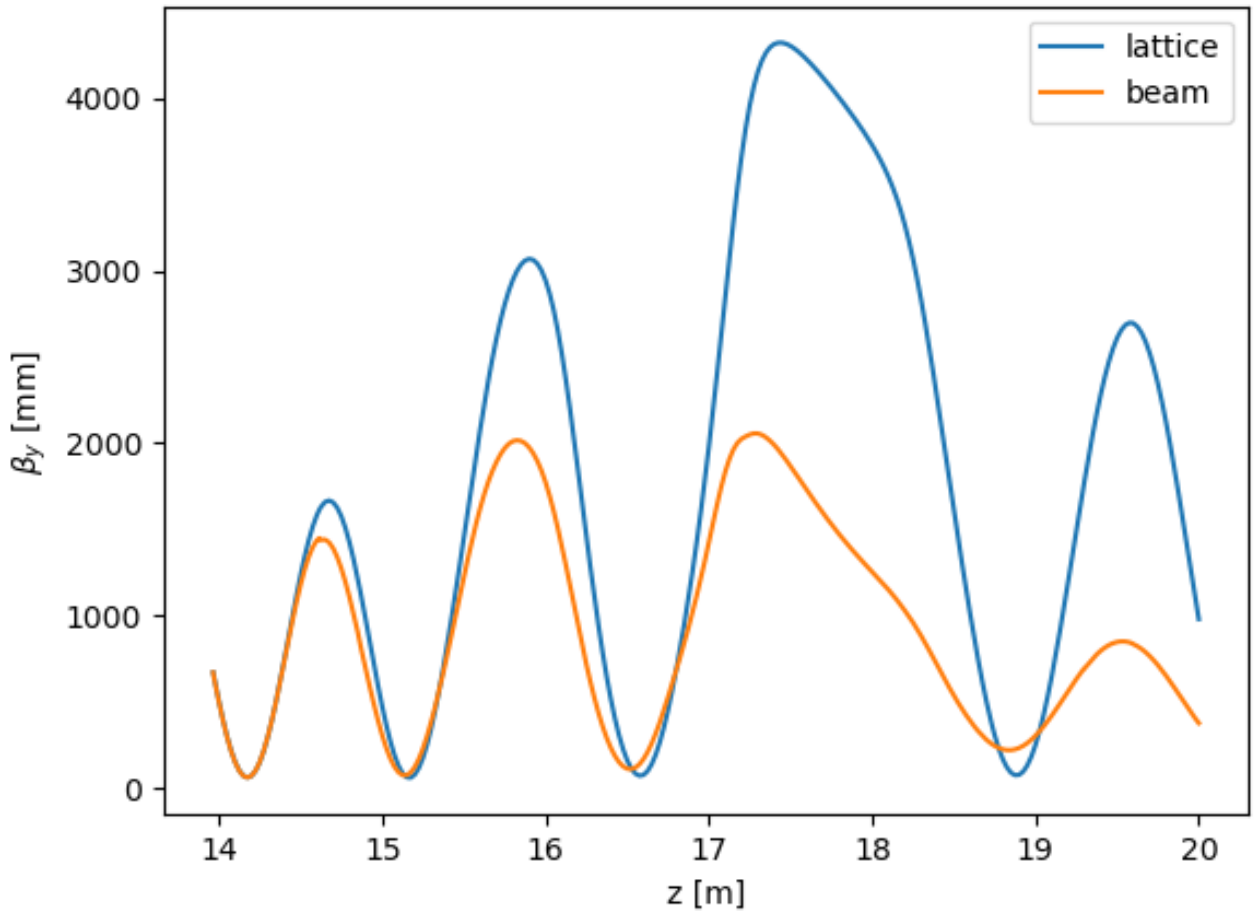
and

$$\alpha = -\frac{\beta'}{2}$$

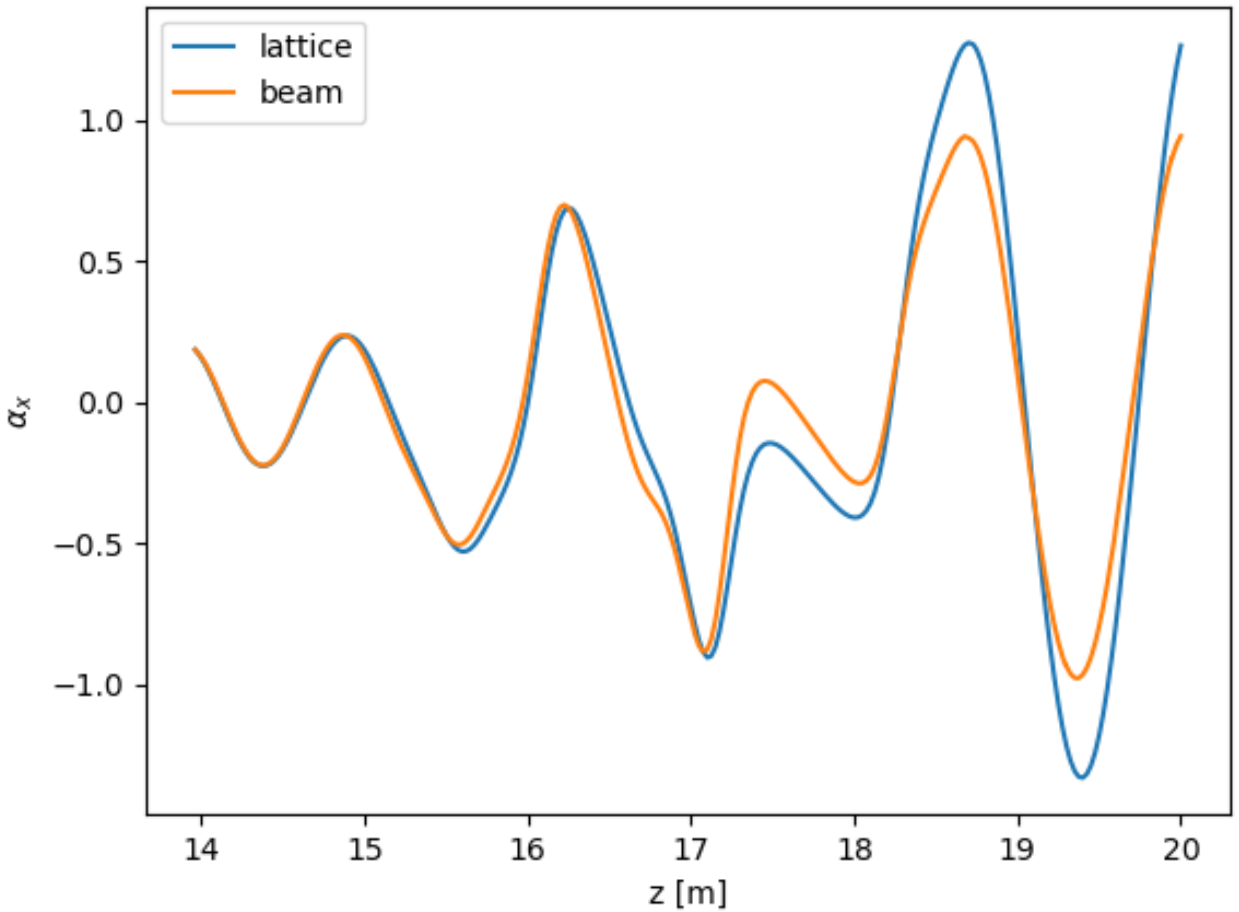
β_x Beam VS Lattice



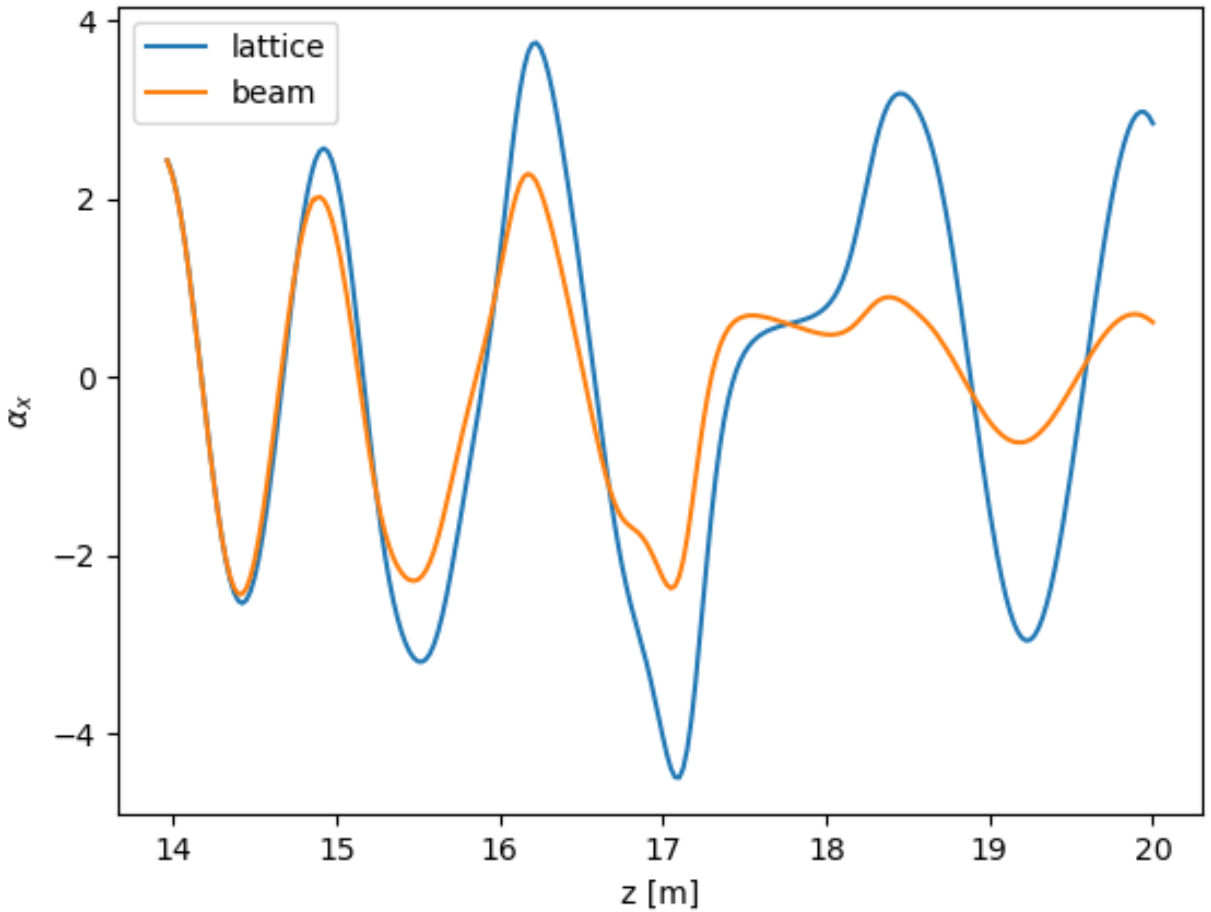
β_y Beam VS Lattice



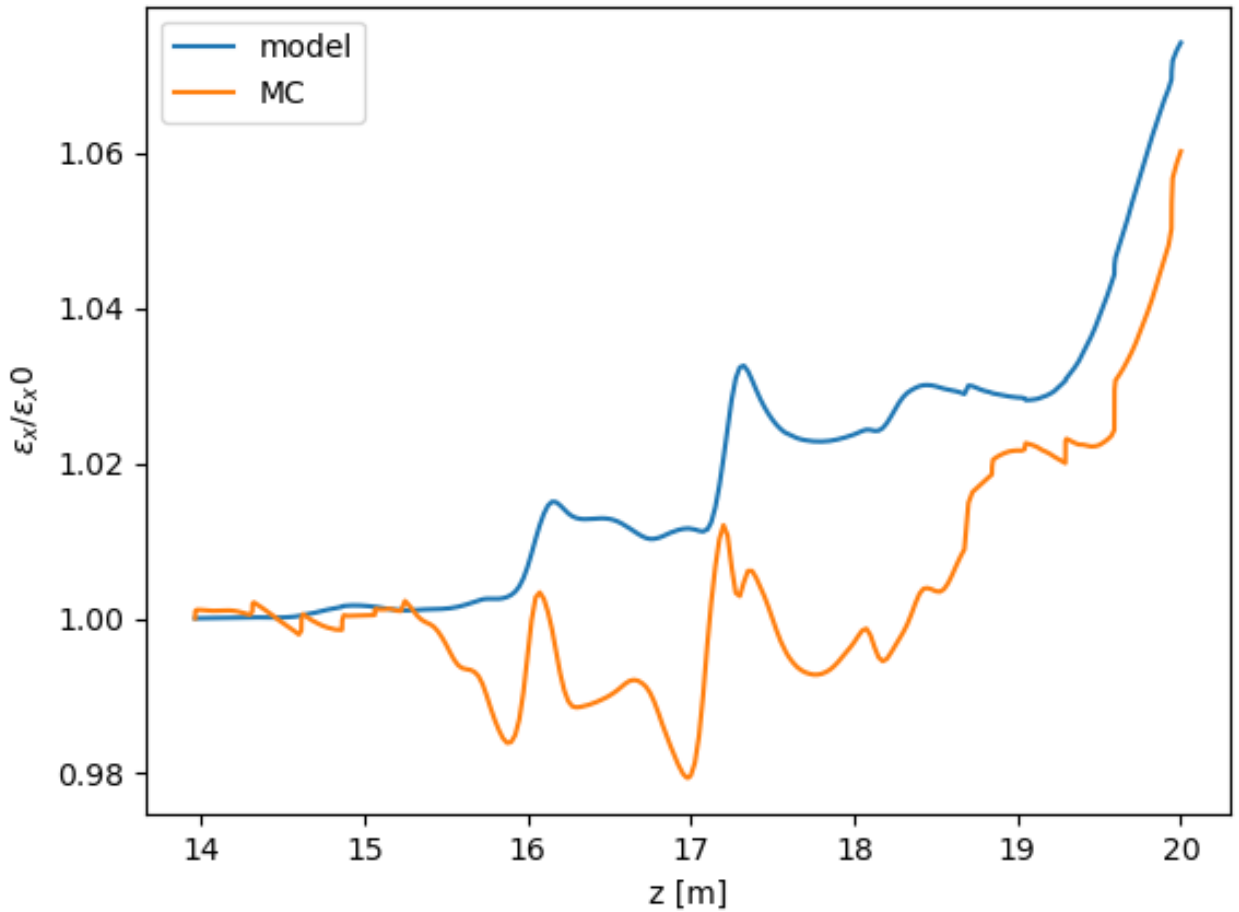
α_x Beam VS Lattice



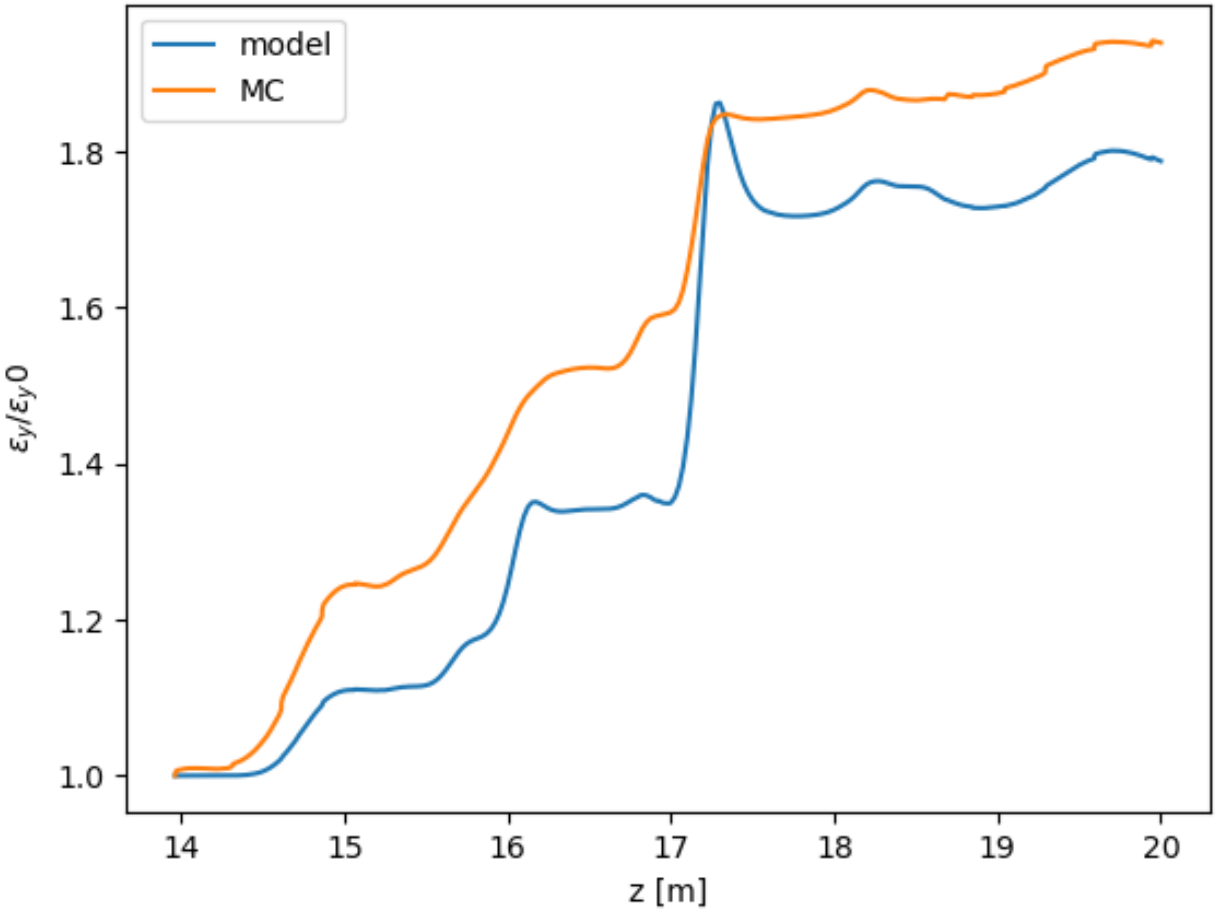
α_y Beam VS Lattice



ϵ_x/ϵ_{x0} MC VS Filamentation model



ϵ_y/ϵ_{y0} MC VS Filamentation model



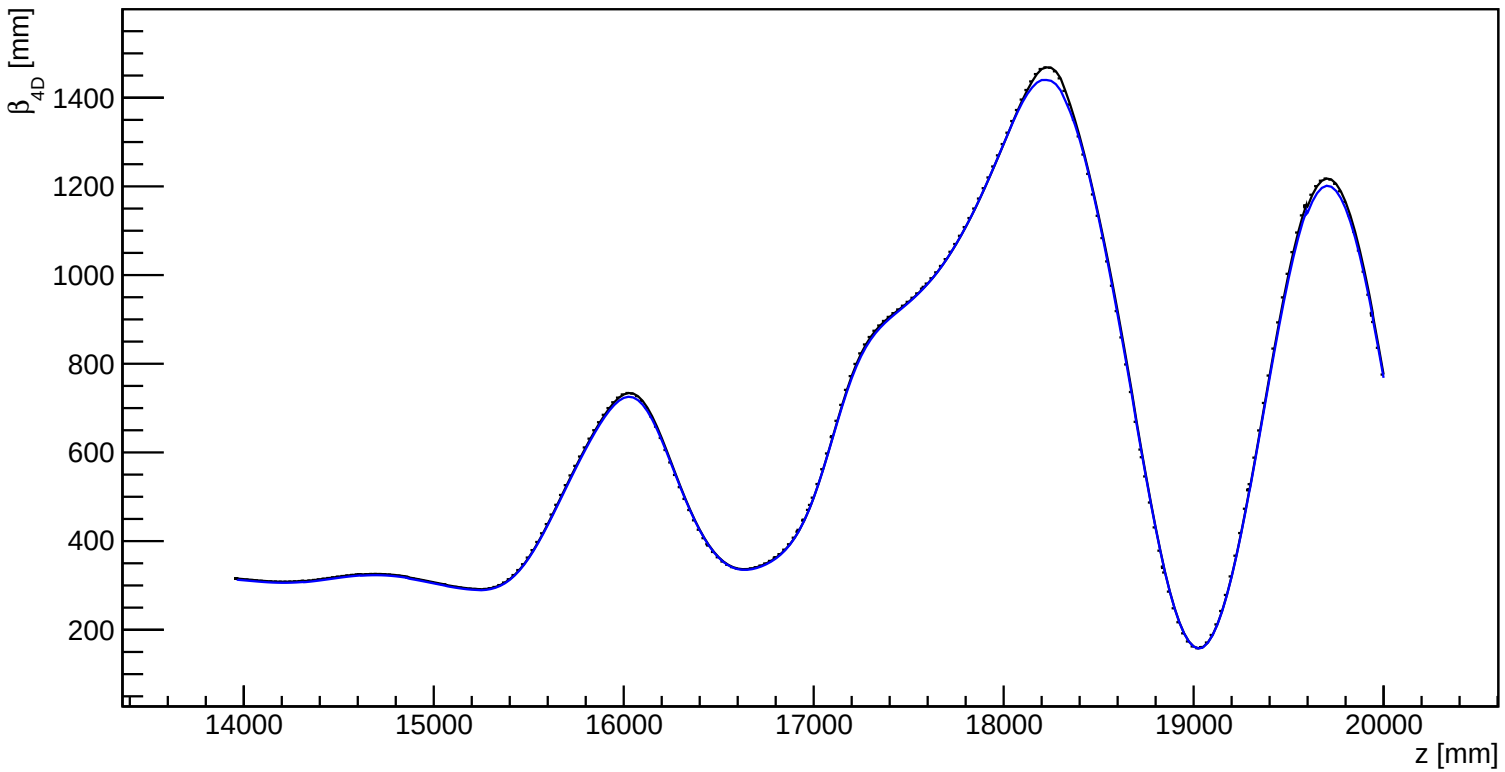
Summary

- Decoupled x and y dynamics by transforming the particles into the rotating Larmor frame
- x dynamics close to the matching condition and thus suffers less emittance growth
- Transformation into the Larmor frame and normalised coordinates reveal filamentation of the beam
- Mismatch model for emittance growth prediction vaguely(?) agrees with simulation, but does not encompass all the contributions to the growth observed
- Next steps:
  - Further testing the routines
  - Apply to beams matched using beam selection
  - Use beam selection to study the emittance growth as a function of lattice and initial beam optics; compare with theoretical predictions

Backup: Toy Model

- Matched cylindrically symmetric beam
- $\beta_0 = 311$ mm
- $\alpha_0 = 0.0$
- $L_0 = 1.1$
- $\epsilon_0 = 3$ mm

Twiss parameters: β_{4D}



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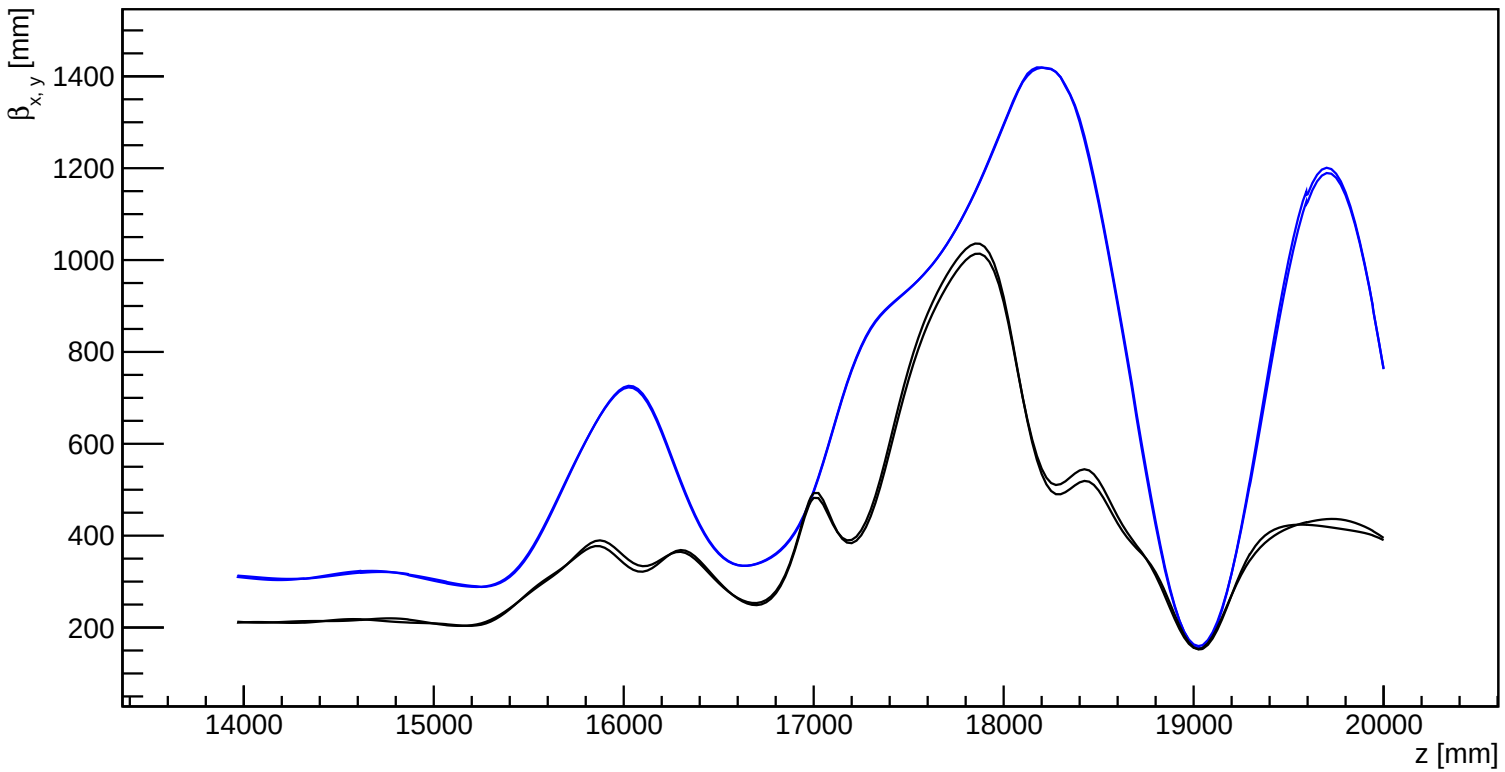

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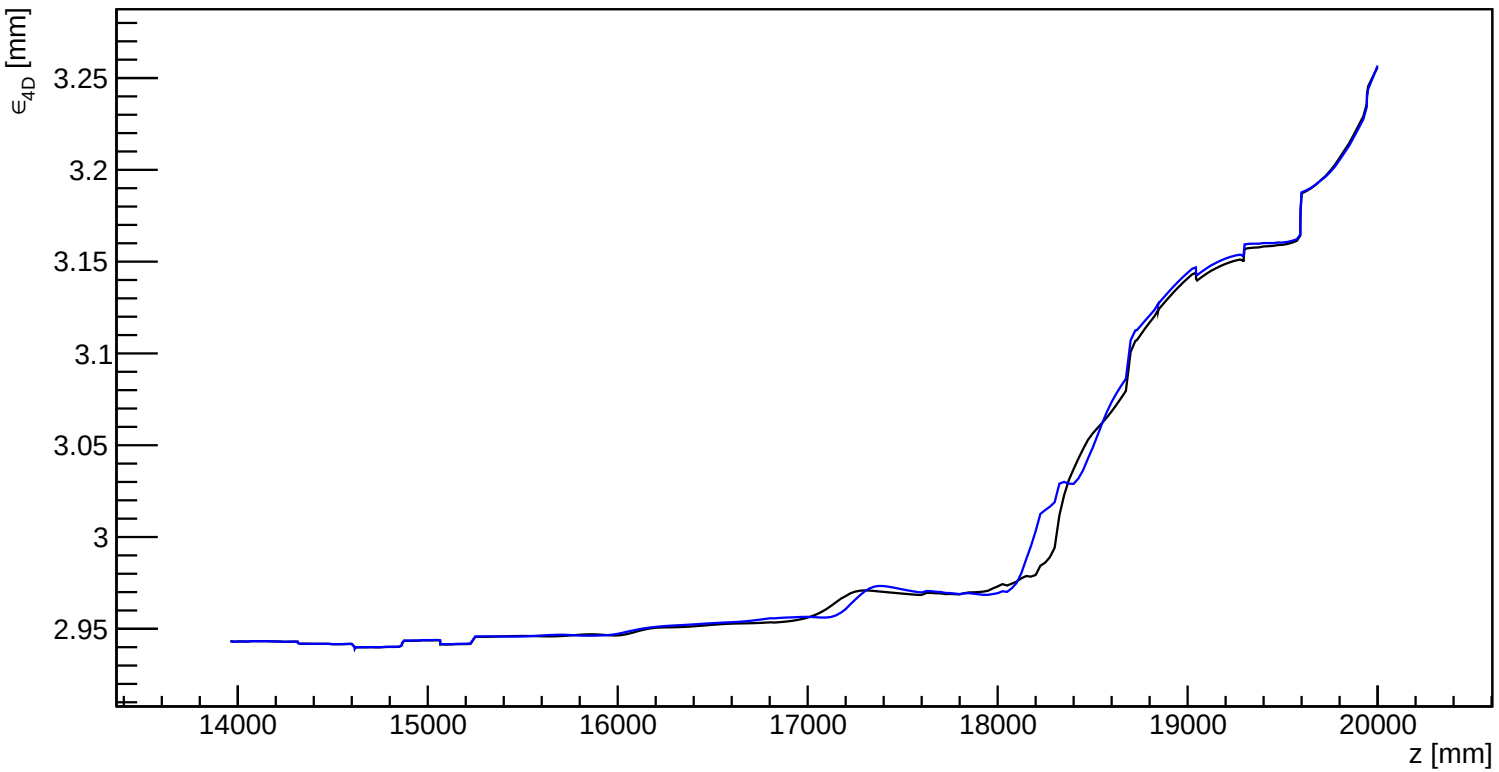

Emittance: ϵ_x vs ϵ_y