Workshop on Forward Physics and QCD at the LHC
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IMPACT-PARAMETER DEPENDENCE OF COLINEARLY IMPROVED BALITSKY-KOVCHEGOV EVOLUTION

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Based on Phys. Rev. D 99, 051502 and Phys. Rev. D 100, 054015

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OUTLINE

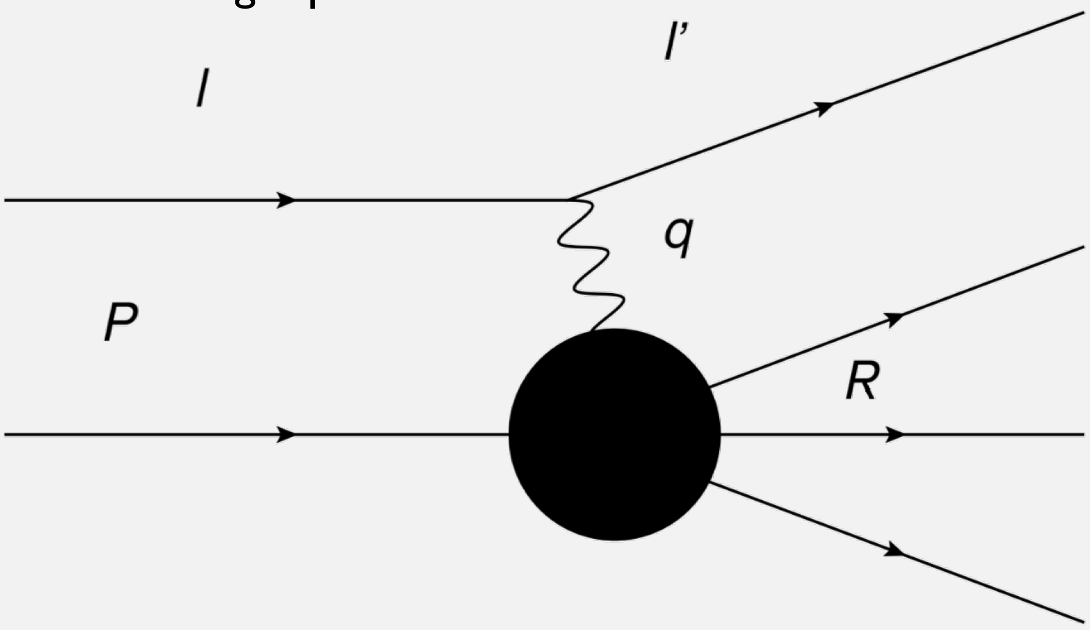
I. Balitsky-Kovchegov evolution equation
II. Impact-parameter-dependent computation
III. The problem of Coulomb tails
IV. Collinearly improved BK equation and suppression of large daughter dipoles
V. Results
VI. Conclusions

BK EQUATION

DEEP INELASTIC SCATTERING

The electron-proton collisions are considered to happen as:

1. The incoming electron emits a virtual photon.
2. The virtual photon interacts with the target proton
3. The proton breaks apart.

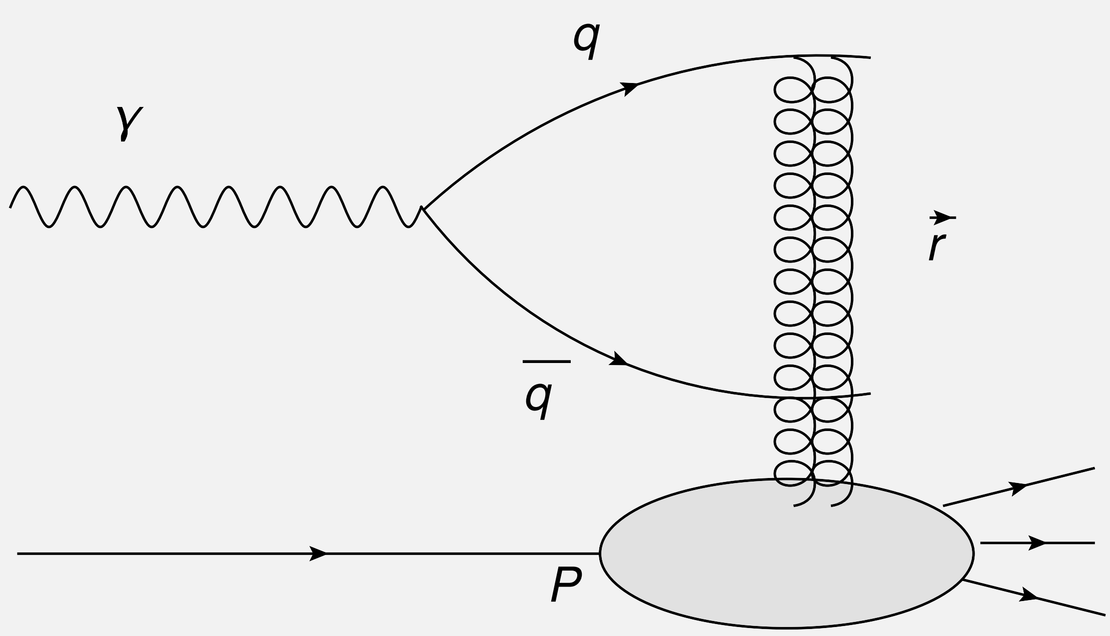


HOW DOES A PHOTON INTERACT WITH A PROTON?

DIPOLE MODEL

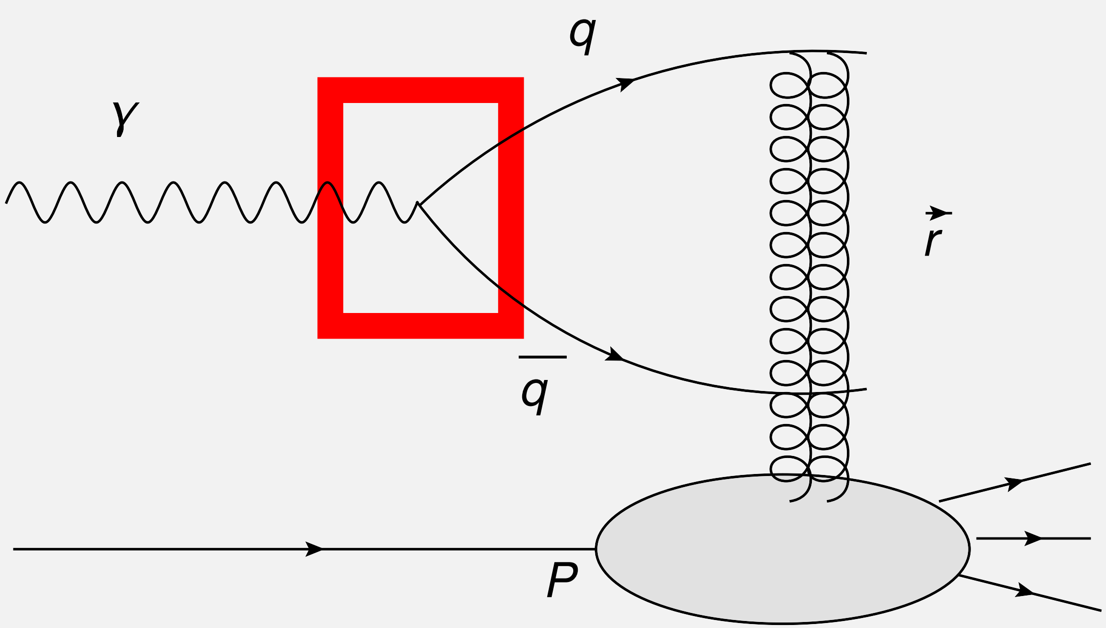
The photon must interact strongly with the target proton, how is that possible?

1. The virtual photon first fluctuates into a quark-antiquark pair
2. Then it exchanges an object with vacuum quantum numbers with the proton



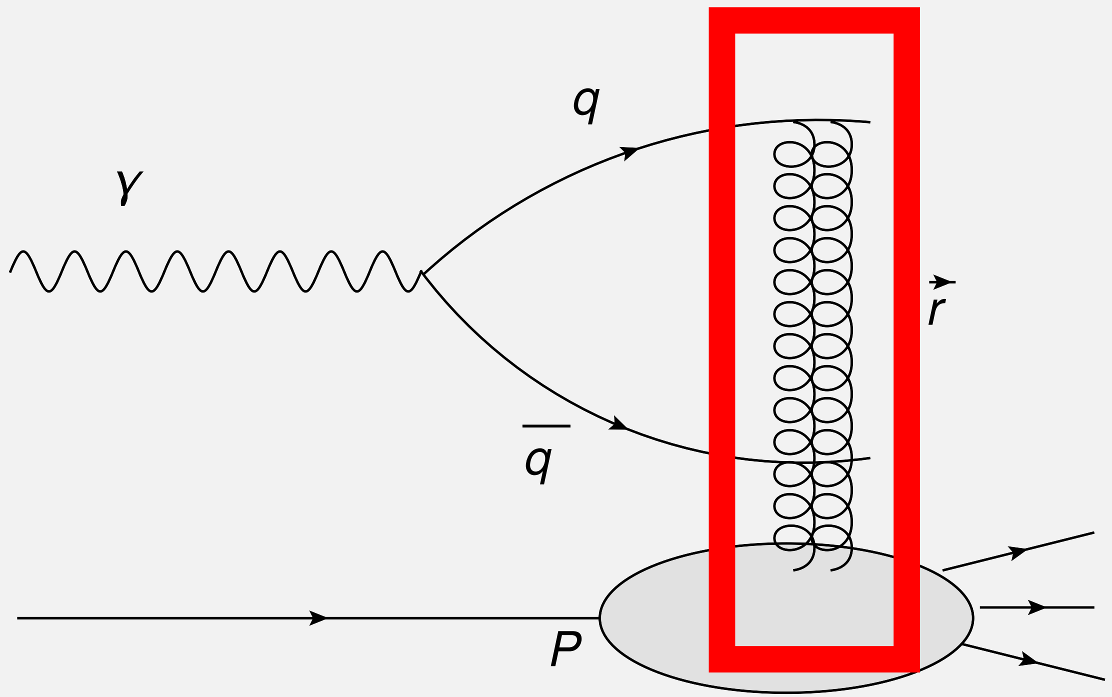
DIPOLE MODEL

The probability of a photon splitting to a quark-antiquark pair is computed from QFT.



DIPOLE MODEL

To compute the cross section of the interaction, we are missing the $\sigma_{\text{dipole-proton}}$



HOW DO WE OBTAIN THE DIPOLE-PROTON CROSS SECTION?

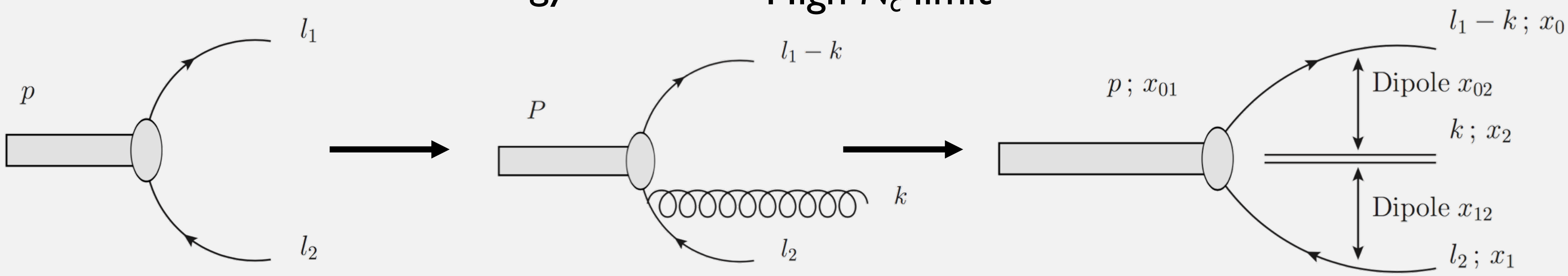
BK EQUATION

BK equation describes the dressing of a color-dipole under the evolution towards higher energies.

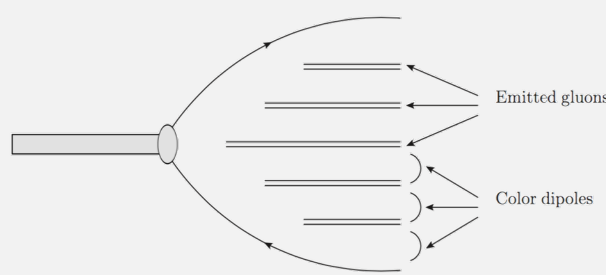
It has been used to predict structure functions, vector meson production, as well as transverse momentum distributions of partons in hadrons.

Add a bit of energy

High N_c limit



After some time, the initial dipole becomes dressed.



I. Balitsky, Nucl. Phys. B 463, 99 (1996)
Y. V. Kovchegov, Phys. Rev. D 60, 034008 (1999)

BK EQUATION

The Balitsky-Kovchegov equation describes the evolution of a color dipole scattering amplitude $N(\vec{r},\vec{b},Y)$ in rapidity

$$\frac{\partial N(\vec{r},\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K(r,r_1,r_2)(N(\vec{r_1},\vec{b_1},Y) + N(\vec{r_2},\vec{b_2},Y) - N(\vec{r},\vec{b},Y) - N(\vec{r_1},\vec{b_1},Y)N(\vec{r_2},\vec{b_2},Y))$$

given by $Y = \ln \frac{x_0}{x}$.

Since the process of gluon emission can be computed under different approximations, we have a number of kernels derived such as

Running coupling kernel:

$$K^{run}(r,r_1,r_2) = \frac{N_c \alpha_s(r^2)}{2\pi^2}\left(\frac{r^2}{r_1^2 r_2^2} + \frac{1}{r_1^2}\left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1\right) + \frac{1}{r_2^2}\left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1\right)\right)$$

Collinearly improved kernel:

$$K^{col}(r,r_1,r_2) = \frac{\overline{\alpha}_s}{2\pi}\frac{r^2}{r_1^2 r_2^2}\left[\frac{r^2}{\min(r_1^2,r_2^2)}\right]^{\pm\overline{\alpha}_s A_1} K_{DLA}(\sqrt{L_{r_1 r} L_{r_2 r}})$$

I. Balitsky, Nucl. Phys. B 463, 99 (1996)
Y. V. Kovchegov, Phys. Rev. D 60, 034008 (1999)
J. L. Albacete at al, Phys.Rev. D75 (2007) 125021
E. Iancu at al, Phys. Let. B. (2015) 1507.03651

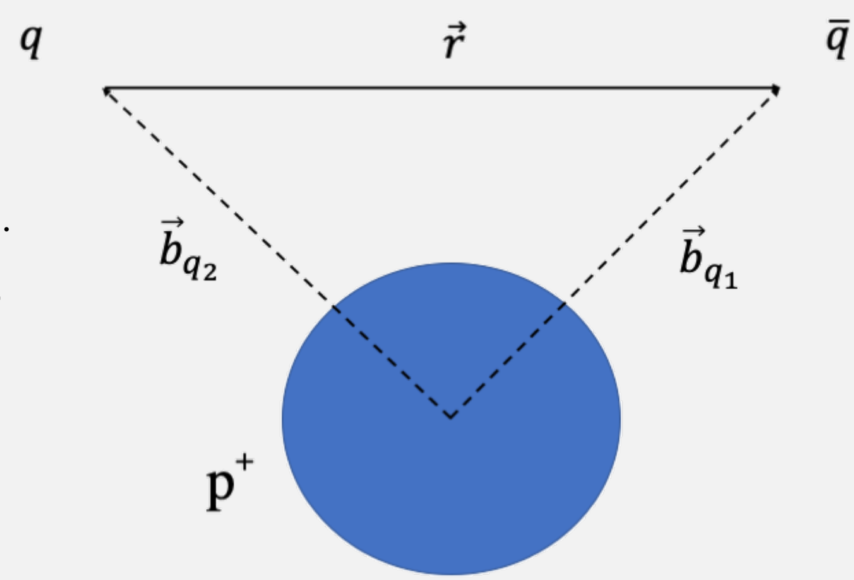
BK EQUATION

For solving this equation numerically, we choose an initial condition

$$N(r,b,Y=0) = 1 - \exp\left(-\frac{1}{2}\frac{Q_s^2}{4}r^2 T(b_{q_1}, b_{q_2})\right), \qquad \text{where} \quad T(b_{q_1}, b_{q_2}) = \left[\exp\left(-\frac{b_{q_1}^2}{2B}\right) + \exp\left(-\frac{b_{q_2}^2}{2B}\right)\right].$$

There are two free parameters; saturation scale $Q_s^2 = 0.49\ \mathrm{GeV}^2$ and variance of the profile distribution $B_G = 3.22\ \mathrm{GeV}^{-2}$.

- The *r* behavior mimics that of the GBW model.
- The *b* behavior exhibits the exponential fall-off calculated for the individual quarks.



K. J. Golec-Biernat and M. Wusthoff, Phys. Rev. D59, 014017 (1998)
J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 99, 051502

IMPACT-PARAMETER DEPENDENCE OF THE BK EQUATION

IMPACT-PARAMETER DEPENDENT BK

There are two main options for treating the impact-parameter dependence of the scattering amplitude:

Option a) Factorizing the impact-parameter dependence. $N(\vec{r}, \vec{b}, x) \cong T(\vec{b}) N(\vec{r}, x)$

If we factorize the impact-parameter dependence, we can integrate over it and replace it with a multiplicative factor.

$$\sigma^{q\bar{q}}(\vec{r}, x) = \int d\vec{b} N(\vec{r}, \vec{b}, x) = \sigma_0 N(x, \vec{r})$$

This factor then stays the same for all energies and dipole sizes and is usually fit to data.

Option b) Solving the equation with the impact-parameter dependence on rapidity.

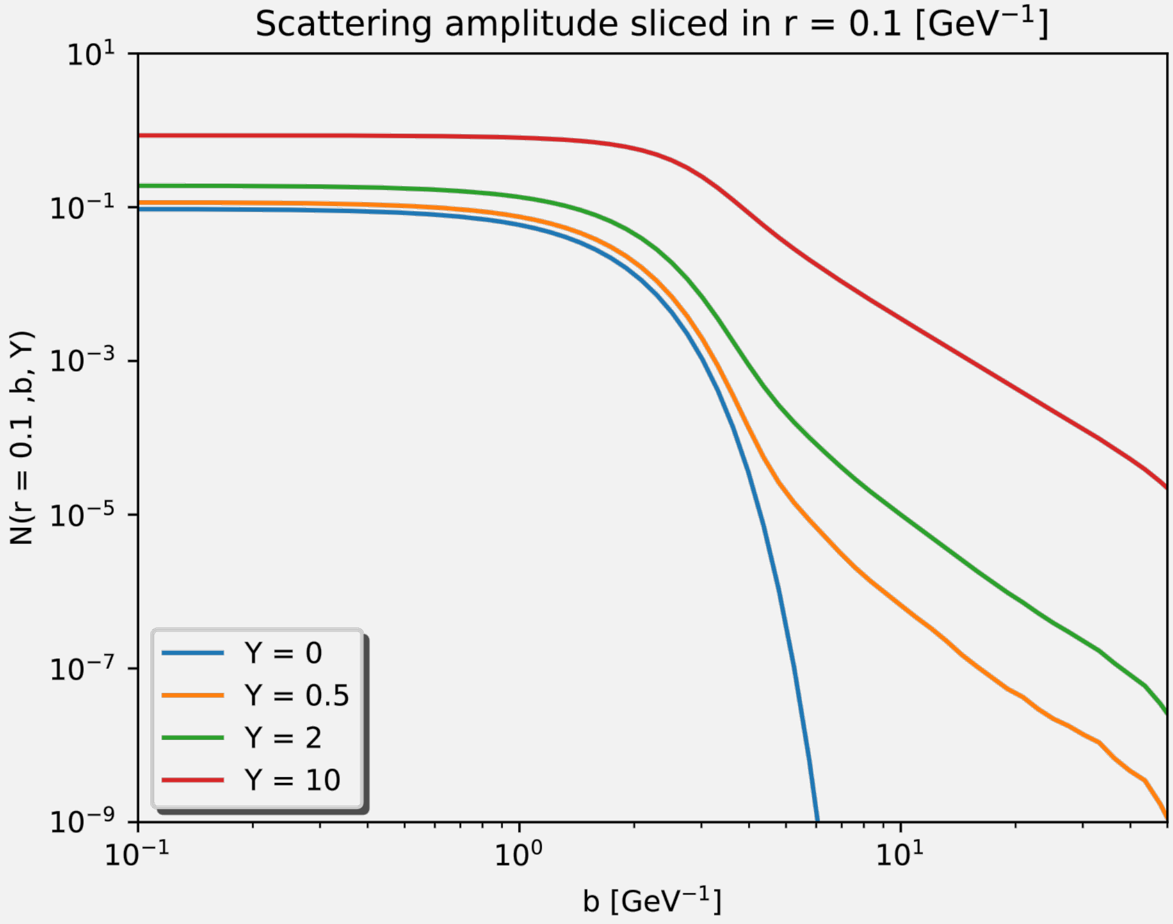
This adds two additional dimensions to the computation. The usual grid size in these two dimensions is 225x20. Which in turn means, that the CPU time gets increased with a factor of 4500.

This is not the only problem. When one tries to run the computation with the usual choice of kernels, one encounters the problems of Coulomb tails.

THE PROBLEM OF COULOMB TAILS

THE PROBLEM

If we start with an exponentially falling initial condition and the usual running coupling kernel.



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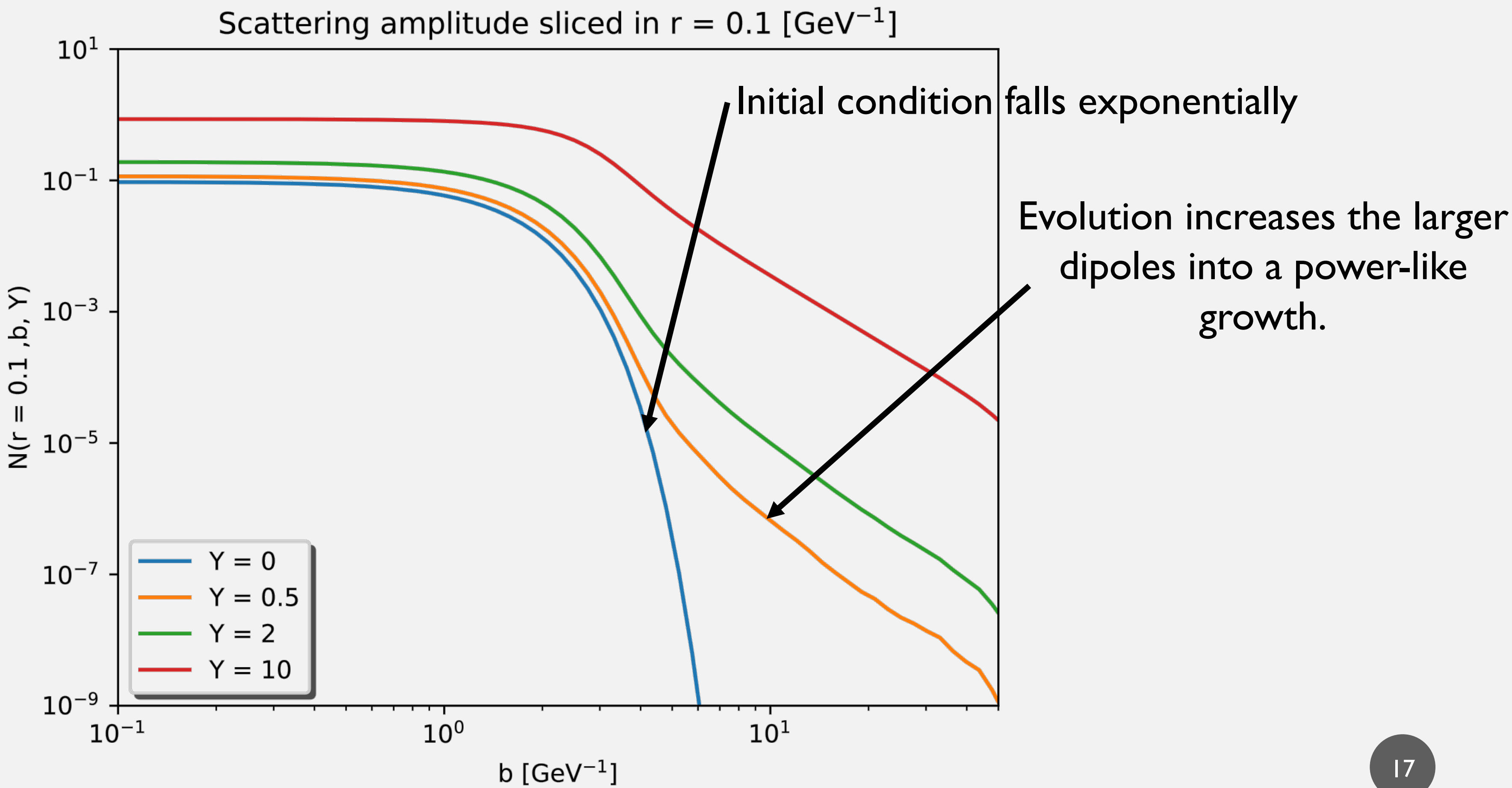

THE PROBLEM

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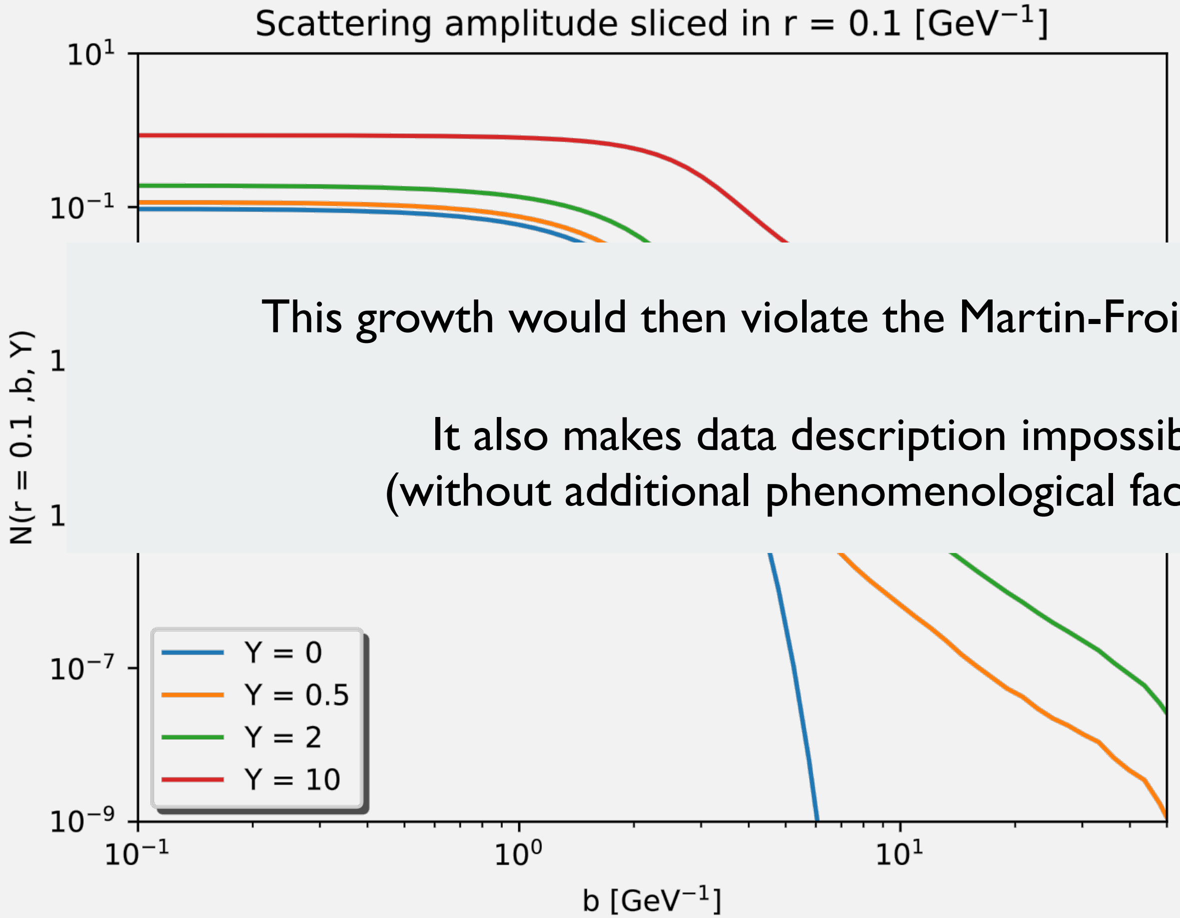


This growth would then violate the Martin-Froisart bound.

It also makes data description impossible.
(without additional phenomenological factors)

HIGH-b SUPPRESSION

The kernel itself does not depend on b. We can however tame the growth in b by suppressing evolution at big sizes of daughter dipoles.

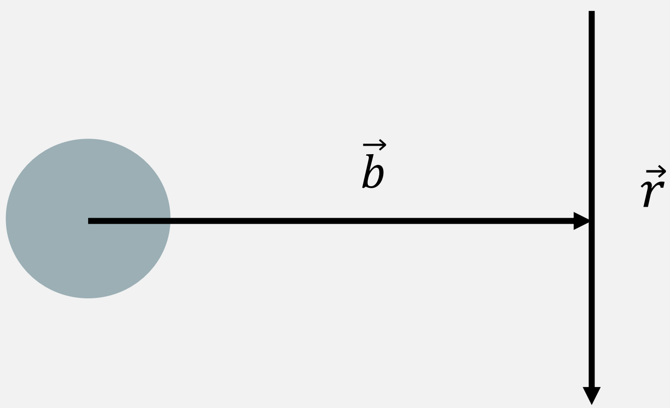
Why?

HIGH-b SUPPRESSION

For high-*b*, the scattering amplitude is exponentially suppressed at the initial condition.

~0

$$\frac{\partial N(\vec{r},\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K(r,r_1,r_2)(N(\vec{r_1},\vec{b_1},Y) + N(\vec{r_2},\vec{b_2},Y) - N(\vec{r},\vec{b},Y) - N(\vec{r_1},\vec{b_1},Y)N(\vec{r_2},\vec{b_2},Y))$$

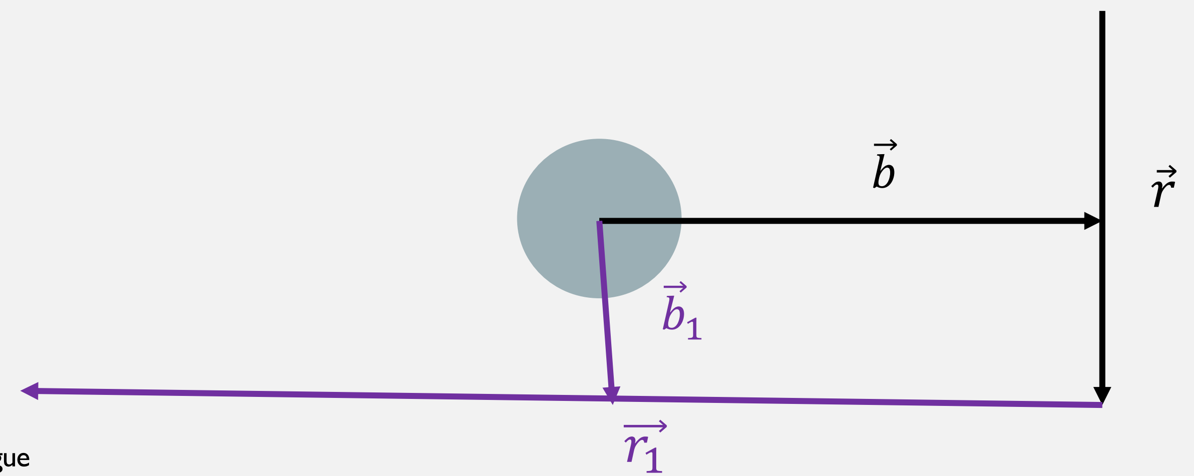


HIGH-b SUPPRESSION

For high-*b*, the scattering amplitude is exponentially suppressed at the initial condition.

~0 ~0 ~0

$$\frac{\partial N(\vec{r},\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K(r,r_1,r_2)(N(\vec{r_1},\vec{b_1},Y) + N(\vec{r_2},\vec{b_2},Y) - N(\vec{r},\vec{b},Y) - N(\vec{r_1},\vec{b_1},Y)N(\vec{r_2},\vec{b_2},Y))$$



HIGH-b SUPPRESSION

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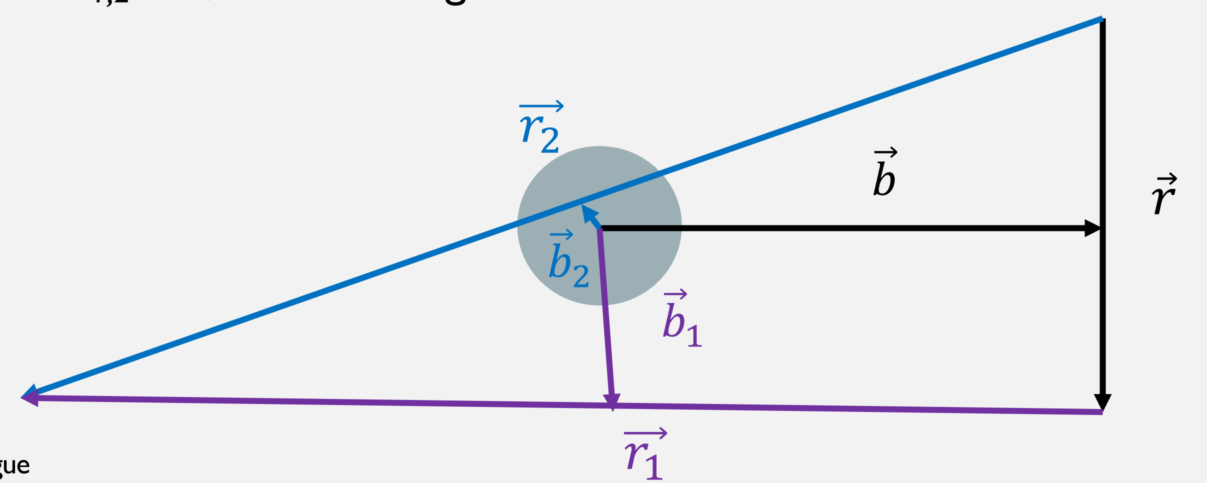
$$\frac{\partial N(\vec{r},\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K(r,r_1,r_2)(N(\vec{r_1},\vec{b_1},Y) + N(\vec{r_2},\vec{b_2},Y) - N(\vec{r},\vec{b},Y) - N(\vec{r_1},\vec{b_1},Y)N(\vec{r_2},\vec{b_2},Y))$$

~0 ~1 ~0 ~0 ~1

The only amplitudes that could be non-zero are those with small impact parameter.

These have $r_{1,2}$~$2b$, which is large.



HIGH-b SUPPRESSION

For high-*b*, the scattering amplitude is exponentially suppressed at the initial condition.

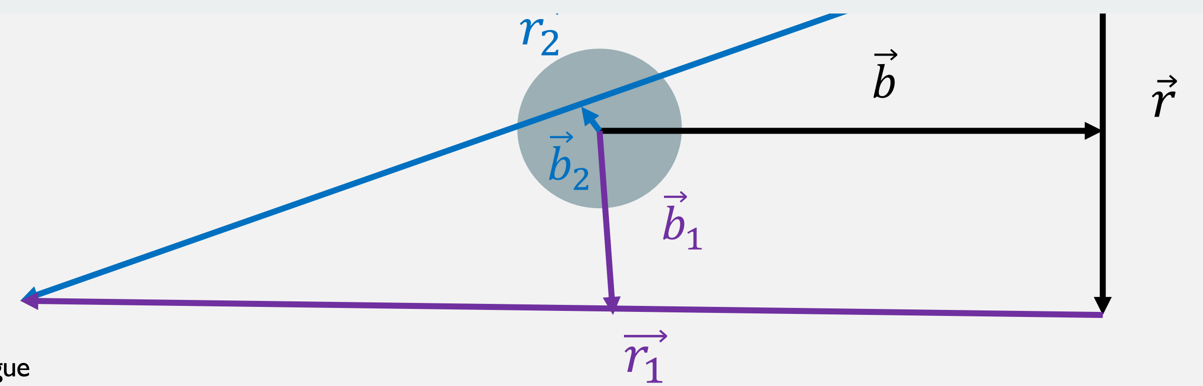
~0 ~1 ~0 ~0 ~1

$$\frac{\partial N(\vec{r},\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K(r,r_1,r_2)(N(\vec{r_1},\vec{b_1},Y) + N(\vec{r_2},\vec{b_2},Y) - N(\vec{r},\vec{b},Y) - N(\vec{r_1},\vec{b_1},Y)N(\vec{r_2},\vec{b_2},Y))$$

T … er.

T

Therefore if we suppress kernel at high r_1 and r_2, we suppress the evolution at high-*b* and maintain the exponential falloff of the scattering amplitude.



HOW TO SUPPRESS LARGE DAUGHTER DIPOLES

KERNEL CUTOFF

One possible solution to this problem is that we can cut the kernel, so that dipoles, that are too big would not contribute to the evolution.

$$\frac{\partial N(r,\vec{b},Y)}{\partial Y} = \int d\vec{r_1} K^{run}(r,r_1,r_2)\Theta\left(\frac{1}{m^2} - r_1^2\right)\Theta\left(\frac{1}{m^2} - r_2^2\right)$$
$$(N(r_1,\vec{b_1},Y) + N(r_2,\vec{b_2},Y) - N(r,\vec{b},Y) - N(r_1,\vec{b_1},Y)N(r_2,\vec{b_2},Y))$$

Jeffrey Berger, Anna M. Stasto, Phys.Rev. D84 (2011) 094022

KERNEL CUTOFF

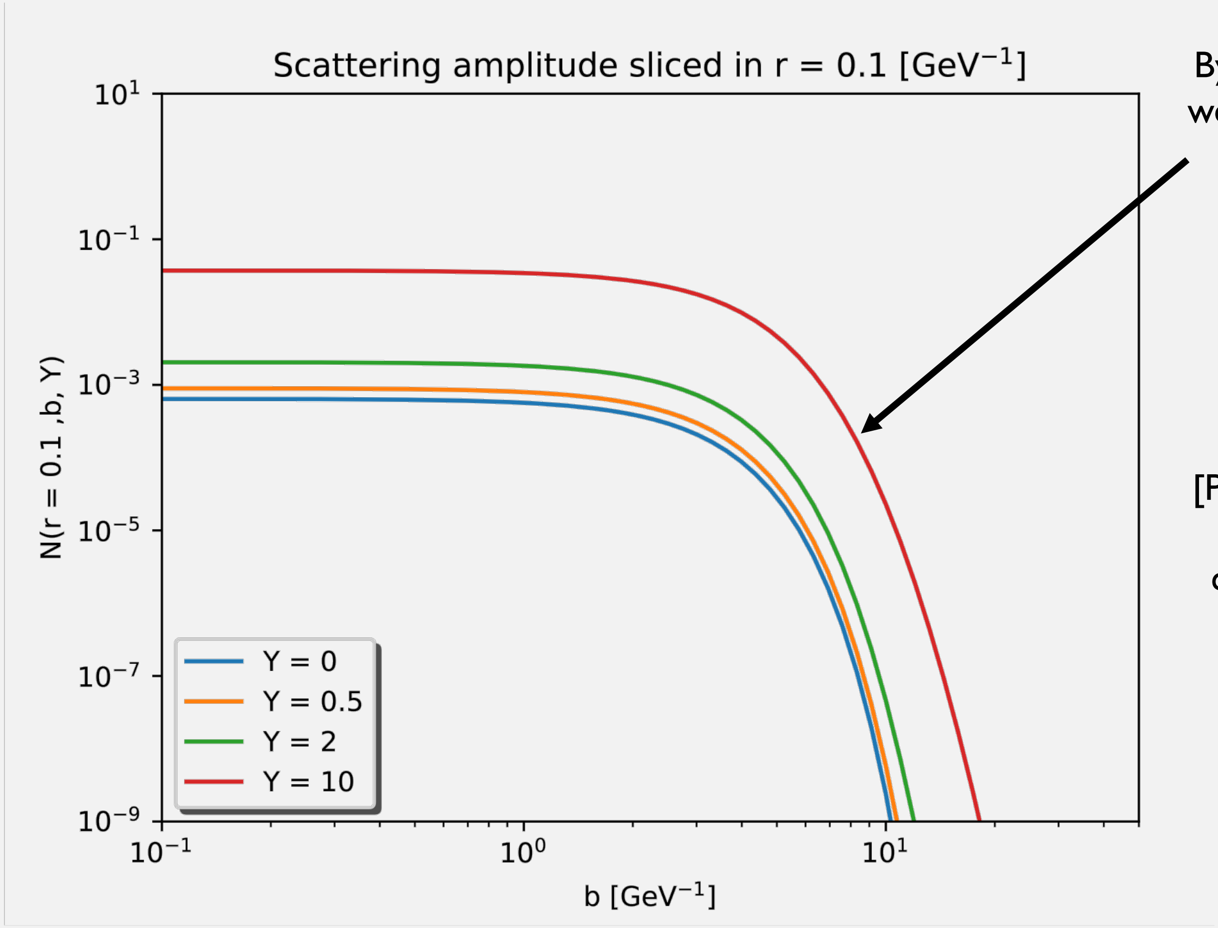
One possible solution to this problem is that we can cut the kernel, so that dipoles, that are too big would not contribute to the evolution.

$$\frac{\partial N(r,\vec{b},Y)}{\partial Y} = \int d\vec{r}_1 K^{run}(r,r_1,r_2)\Theta\left(\frac{1}{m^2}-r_1^2\right)\Theta\left(\frac{1}{m^2}-r_2^2\right)$$
$$(N(r_1,\vec{b}_1,Y)+N(r_2,\vec{b}_2,Y)-N(r,\vec{b},Y)-N(r_1,\vec{b}_1,Y)N(r_2,\vec{b}_2,Y))$$

Mass of the emitted gluon is a free parameter, that is fitted to data.

Jeffrey Berger, Anna M. Stasto, Phys.Rev. D84 (2011) 094022

KERNEL CUTOFF



By imposing the cutoff of the kernel, we maintain the exponential falloff of the scattering amplitude.

However, as was shown in [Phys. Rev. D84(2011)094022], we still cannot describe the data, since the cutoff is too strong and we need to impose new phenomenological constants to cure this.

Jeffrey Berger, Anna M. Stasto, Phys.Rev. D84 (2011) 094022

KERNEL CUTOFF

The recently proposed collinearly improved kernel is by its nature suppressed at high $r_{1,2}$ and does not require additional dimensional parameters.

$$K^{col}(r, r_1, r_2) = \frac{\overline{\alpha}_s}{2\pi} \frac{r^2}{r_1^2 r_2^2} \left[\frac{r^2}{\min(r_1^2, r_2^2)} \right]^{\pm \overline{\alpha}_s A_1} K_{DLA}(\sqrt{L_{r_1 r} L_{r_2 r}})$$

where $K_{DLA}(\rho) = \frac{J_1(2\sqrt{\overline{\alpha}_s \rho^2})}{\sqrt{\overline{\alpha}_s}\rho}$ with $L_{r_i r} = \ln\left(\frac{r_i^2}{r^2}\right)$

$\pm \overline{\alpha}_s A_1$ is positive when r is smaller than the daughter dipoles and negative otherwise and $A_1 = 11/12$

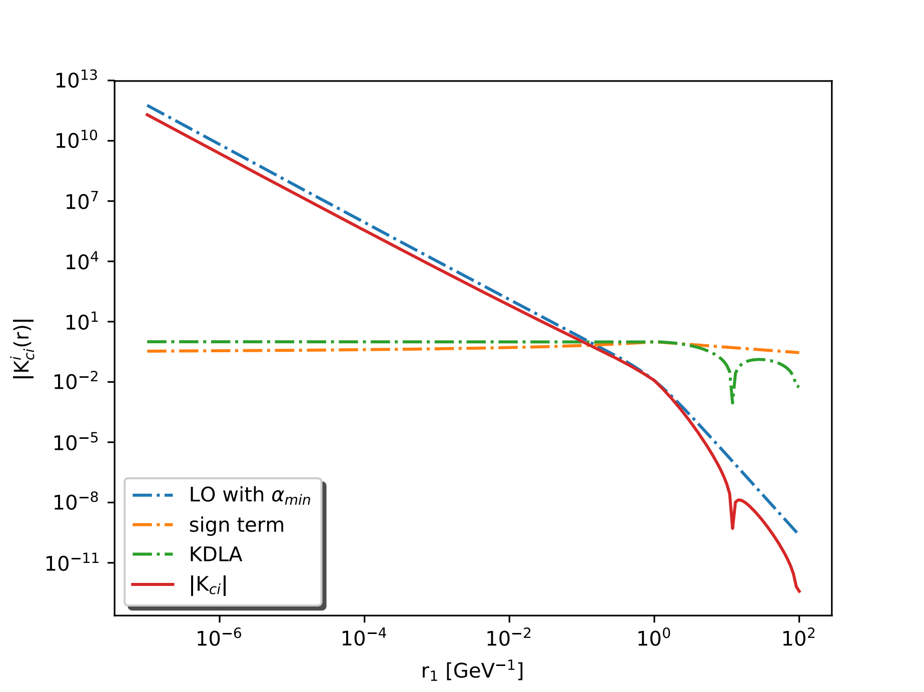
Running coupling is of the usual scheme for the BK computations as in [J. L. Albacete at al, Eur.Phys.J. C71 (2011) 1705] at the minimal scale given by

$\overline{\alpha}_s = \alpha_s \frac{N_c}{\pi}$ $\alpha_s = \alpha_s(r_{\min})$ $r_{\min} = \min(r_1, r_2, r)$ with C = 9.

The factor in square brackets represents the contribution of single collinear logarithms and DLA term resums double collinear logarithms to all orders.

E. Iancu at al, Phys. Let. B. (2015) 1507.03651

KERNEL CUTOFF



r = 1 GeV^{-1}, $\theta_{rr1} = \pi/2$

$$K^1_{\rm ci} = \frac{\overline{\alpha}_s}{2\pi}\frac{r^2}{r_1^2 r_2^2},$$

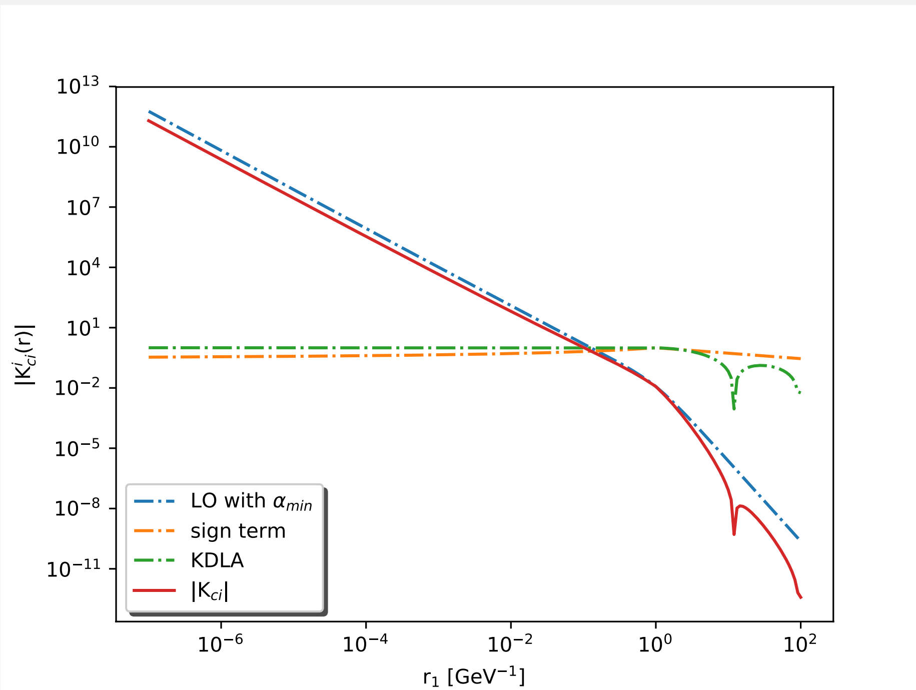
$$K^2_{\rm ci} = \left[\frac{r^2}{\min(r_1^2, r_2^2)}\right]^{\pm\overline{\alpha}_s A_1},$$

$$K^3_{\rm ci} = K_{\rm DLA}(\sqrt{L_{r_1 r} L_{r_2 r}}).$$

$$K_{\rm ci} = K^1_{\rm ci} K^2_{\rm ci} K^3_{\rm ci}$$

D. Bendova, J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 100, 054015

KERNEL CUTOFF



r = 1 GeV^{-1}, $\theta_{rr1} = \pi/2$

This term is present already at the LO

Resums single collinear logarithms

$$K^1_{\rm ci} = \frac{\overline{\alpha}_s}{2\pi}\frac{r^2}{r_1^2 r_2^2},$$

$$K^2_{\rm ci} = \left[\frac{r^2}{\min(r_1^2, r_2^2)}\right]^{\pm\overline{\alpha}_s A_1},$$

$$K^3_{\rm ci} = K_{\rm DLA}(\sqrt{L_{r_1 r} L_{r_2 r}}).$$

Resums double collinear logarithms

$$K_{\rm ci} = K^1_{\rm ci} K^2_{\rm ci} K^3_{\rm ci}$$

D. Bendova, J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 100, 054015

KERNEL CUTOFF

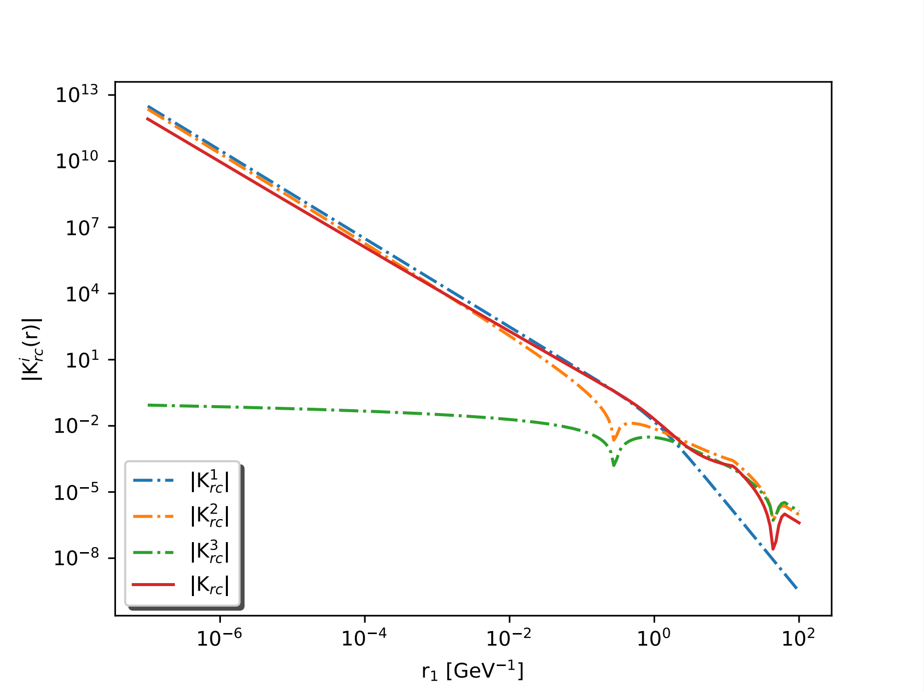
Here we compare the value of the collinearly improved kernel with the running coupling kernel versus r_1.

$$K_{\mathrm{rc}}^{1} = \frac{N_c \alpha_s(r^2)}{2\pi^2} \frac{r^2}{r_1^2 r_2^2},$$

$$K_{\mathrm{rc}}^{2} = \frac{N_c \alpha_s(r^2)}{2\pi^2} \frac{1}{r_1^2} \left(\frac{\alpha_s(r_1^2)}{\alpha_s(r_2^2)} - 1 \right),$$

$$K_{\mathrm{rc}}^{3} = \frac{N_c \alpha_s(r^2)}{2\pi^2} \frac{1}{r_2^2} \left(\frac{\alpha_s(r_2^2)}{\alpha_s(r_1^2)} - 1 \right),$$

$$K_{\mathrm{rc}} = K_{\mathrm{rc}}^{1} + K_{\mathrm{rc}}^{2} + K_{\mathrm{rc}}^{3}$$

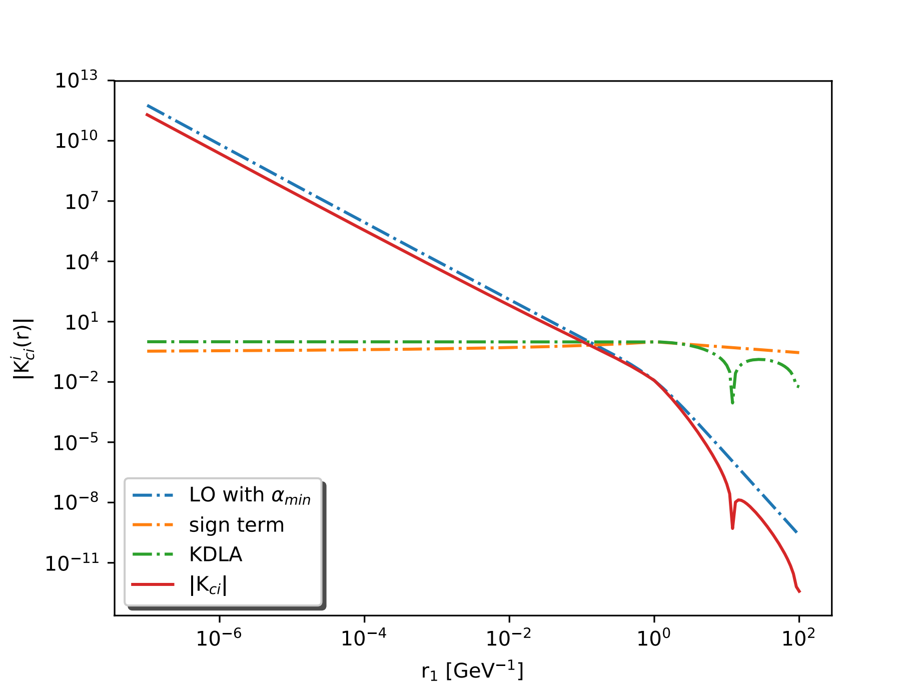


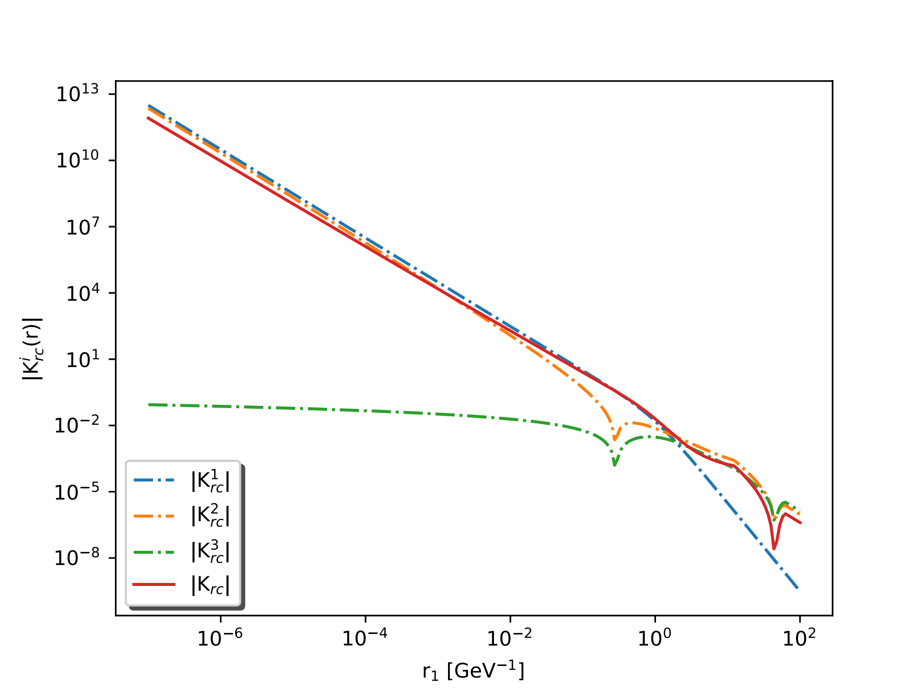
$r = 1$ GeV^{-1}, $\theta_{rr1} = \pi/2$

D. Bendova, J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 100, 054015

KERNEL CUTOFF

Here we compare the value of the collinearly improved kernel with the running coupling kernel versus r_l.





r = 1 GeV^{-1}, $\theta_{rr1} = \pi/2$

$$K_{\rm ci} = K^1_{\rm ci} K^2_{\rm ci} K^3_{\rm ci} \qquad K_{\rm rc} = K^1_{\rm rc} + K^2_{\rm rc} + K^3_{\rm rc}$$

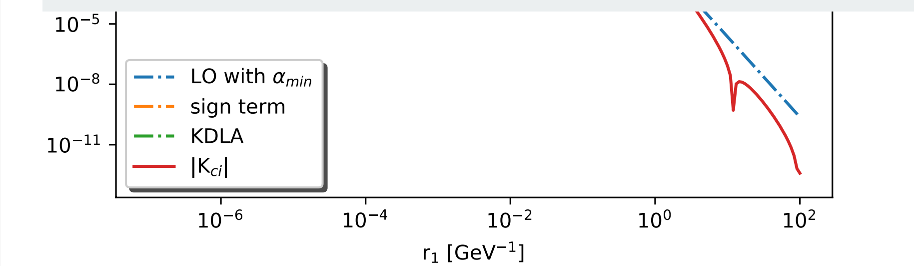
D. Bendova, J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 100, 054015

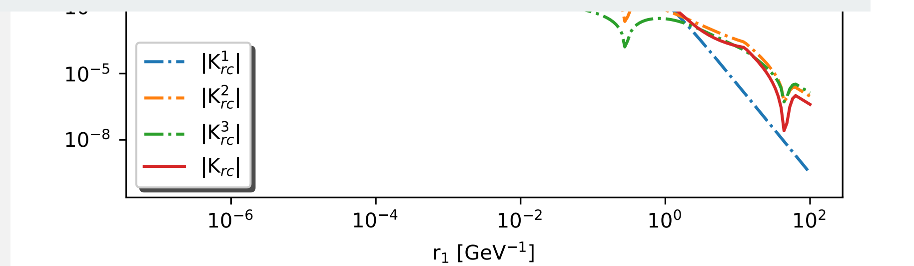
KERNEL CUTOFF

Here we compare the value of the collinearly improved kernel with the running coupling kernel versus r_1.

The suppression can be traced back to the fact that large daughter dipoles do not follow the time-ordering prescription built in when setting up the resummation that leads to the collinearly improved kernel.

They would live longer than the parent dipole.





r = 1 GeV^{-1}, $\theta_{rr1} = \pi/2$

$$K_{ci} = K^1_{ci} K^2_{ci} K^3_{ci} \qquad K_{rc} = K^1_{rc} + K^2_{rc} + K^3_{rc}$$

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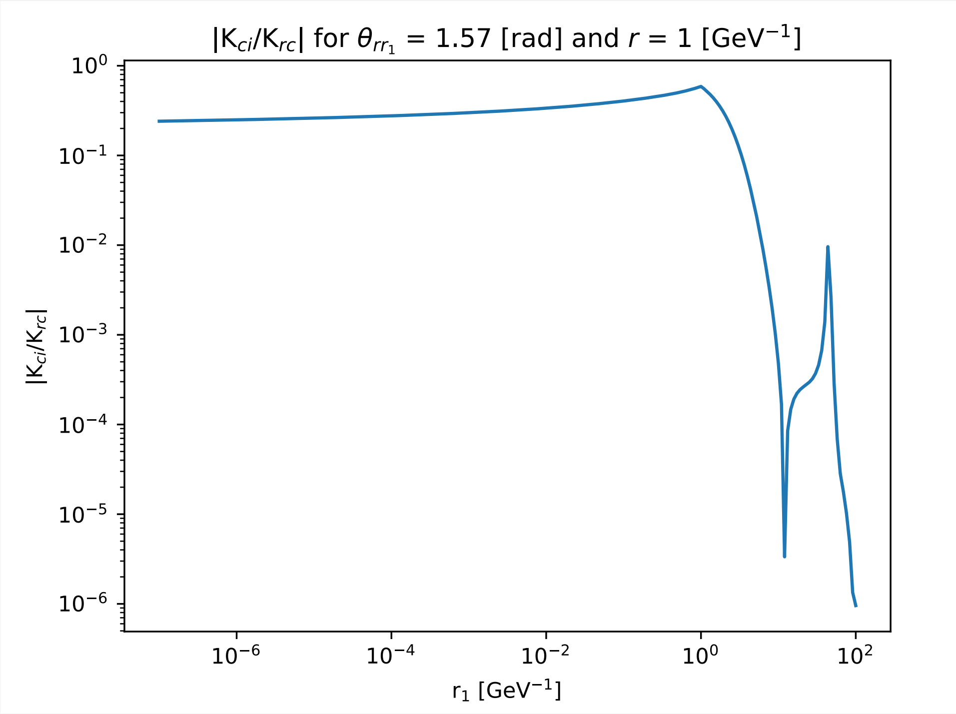
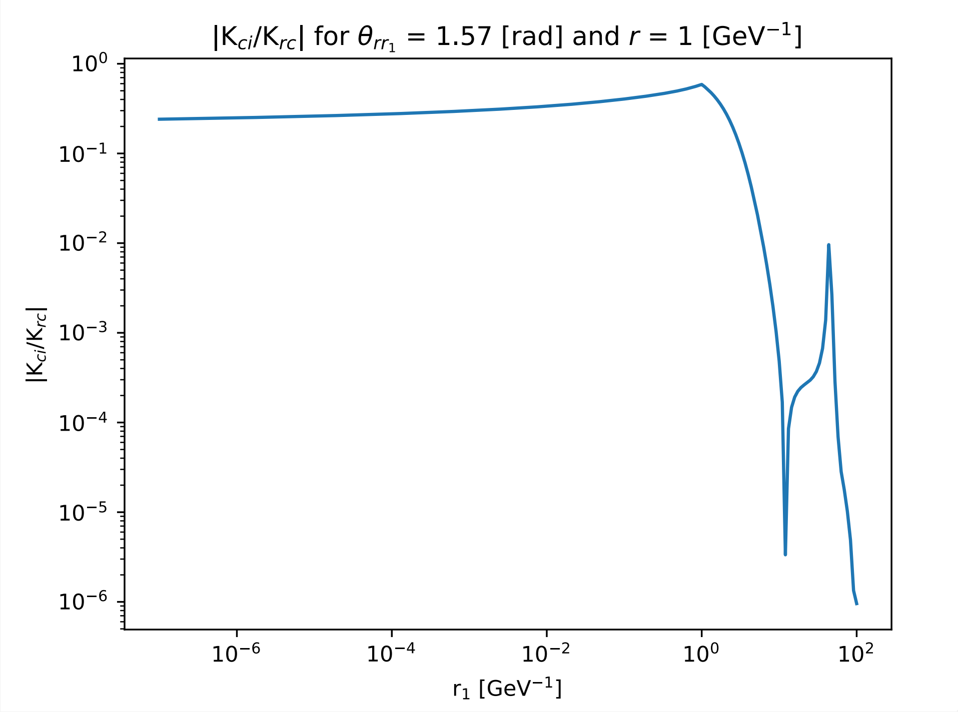
KERNEL CUTOFF

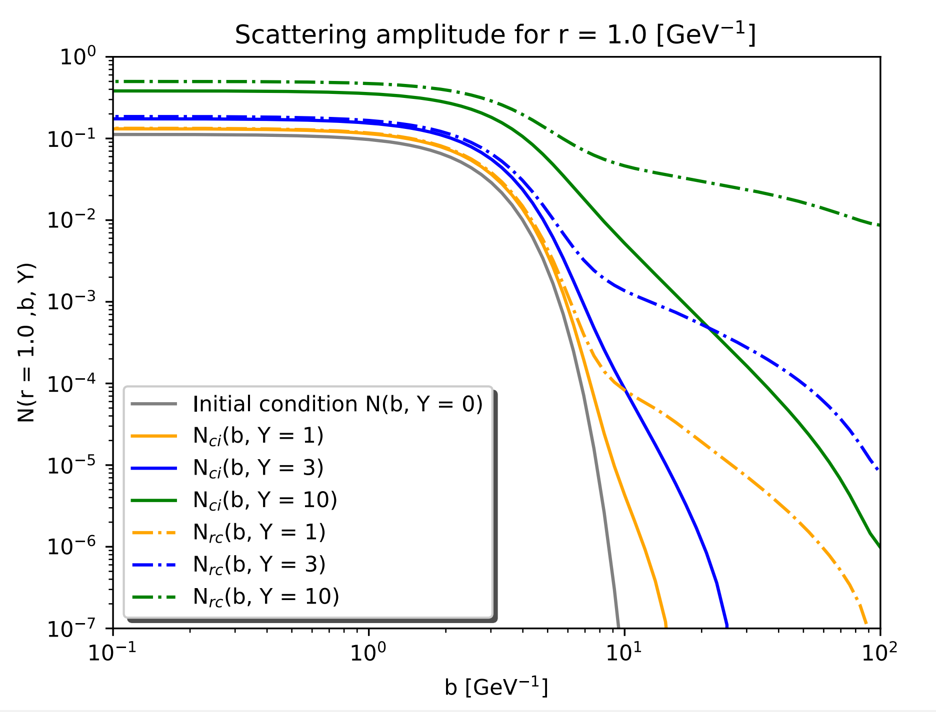
Here we compare the value of the collinearly improved kernel with the running coupling kernel versus r_1.



KERNEL CUTOFF

Here we compare the value of the collinearly improved kernel with the running coupling kernel versus r_l.





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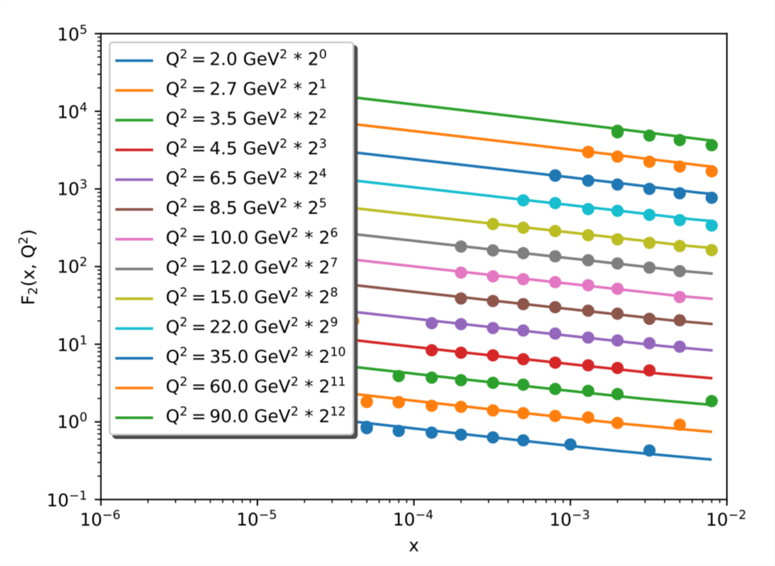
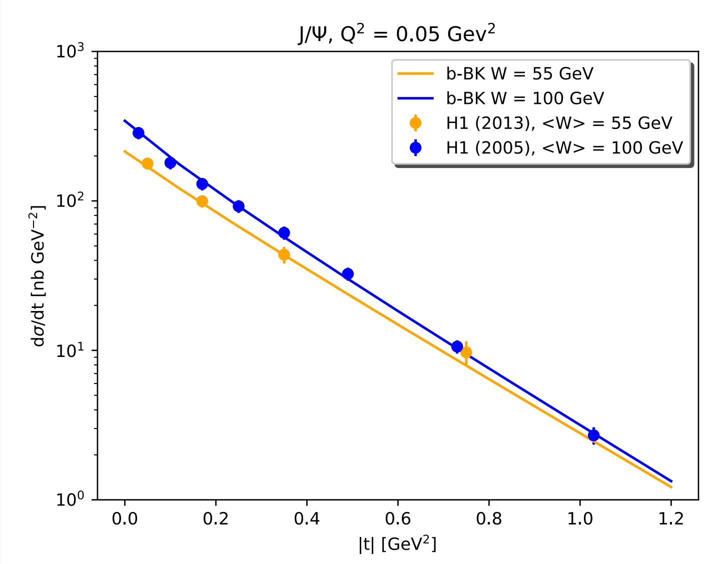
COMPARISON TO DATA

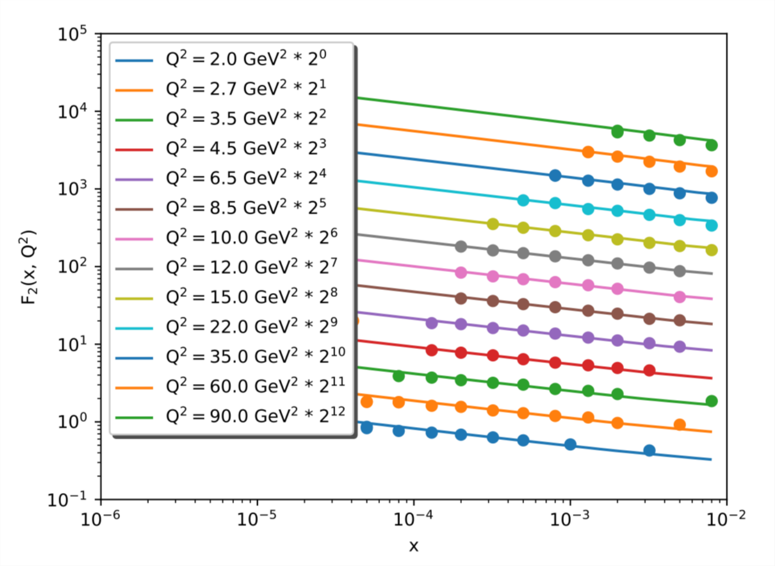
RESULTS



J. Cepila, J. G. Contreras, M. Matas; Phys. Rev. D 99, 051502

RESULTS





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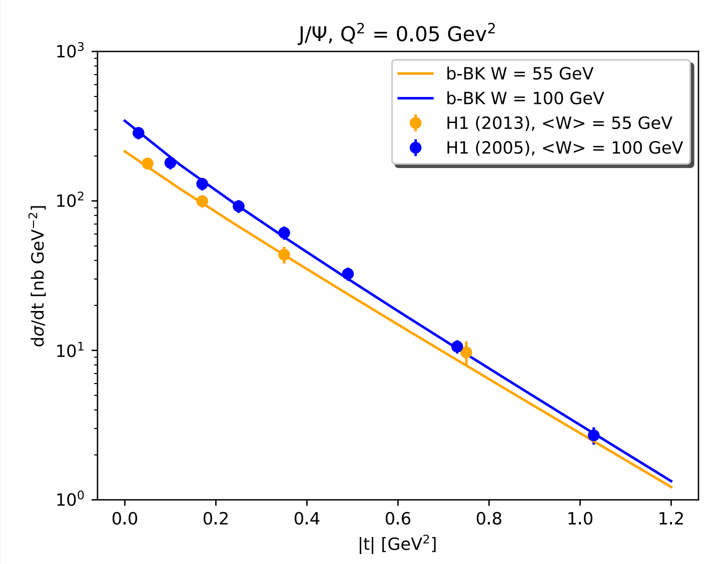
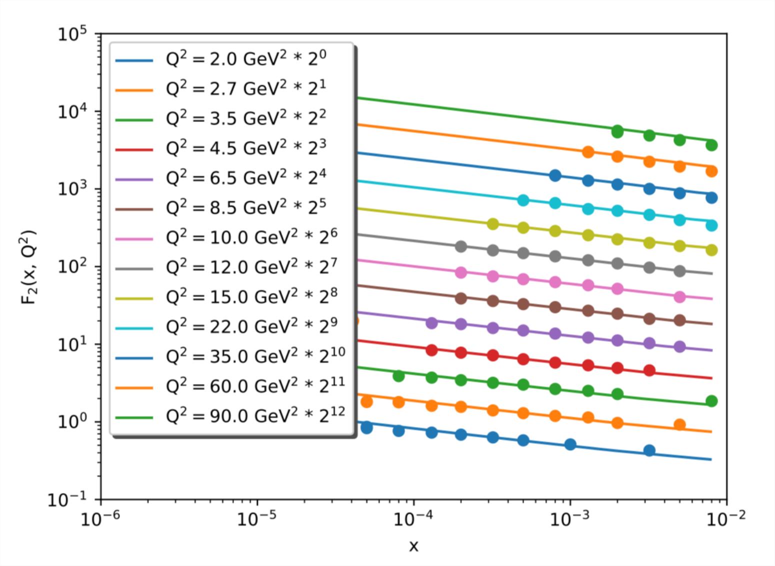
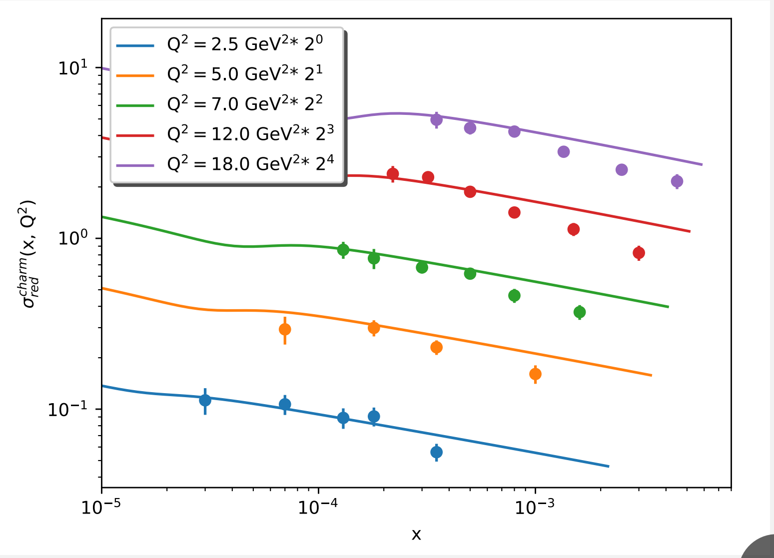
RESULTS







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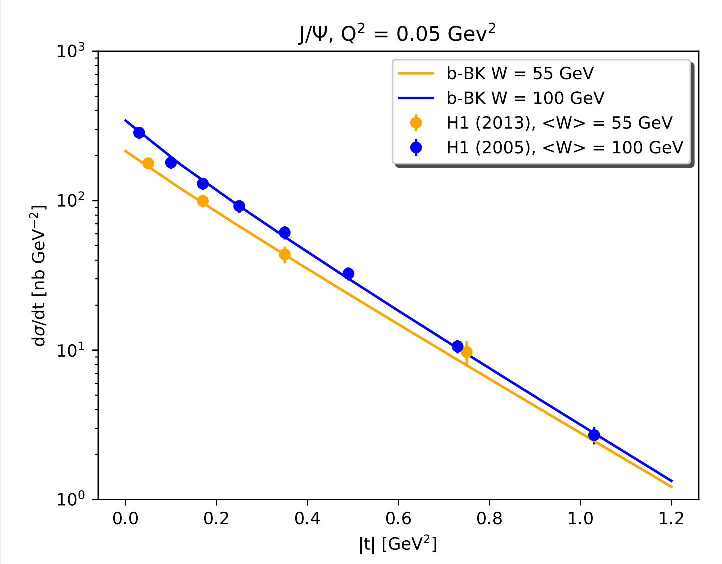
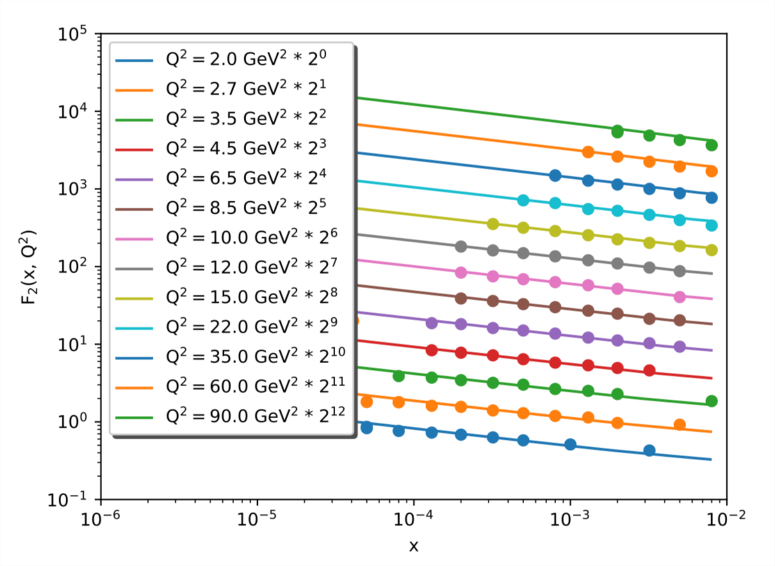
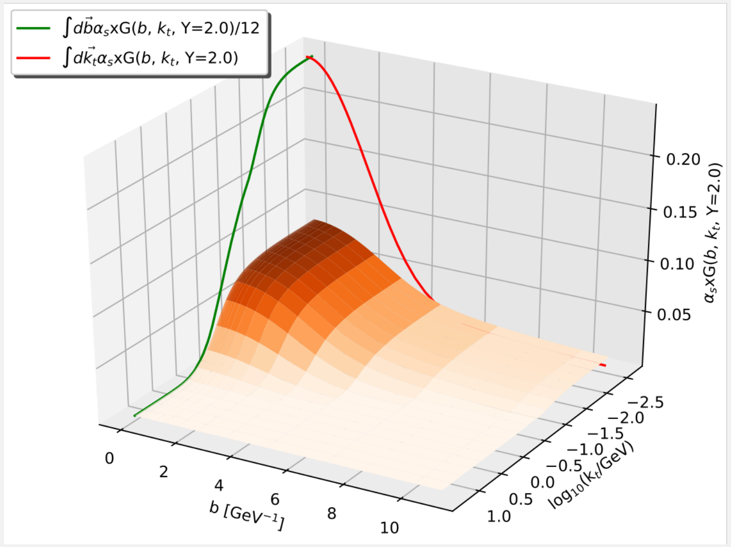
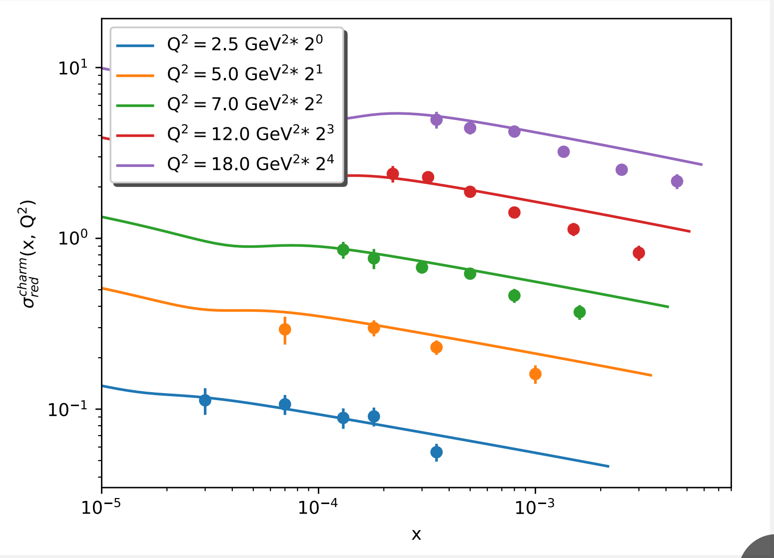
RESULTS









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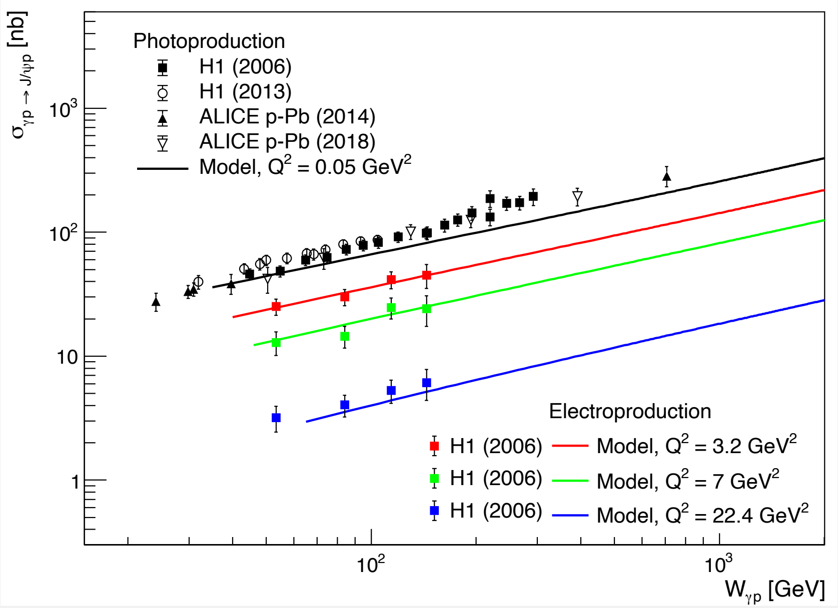

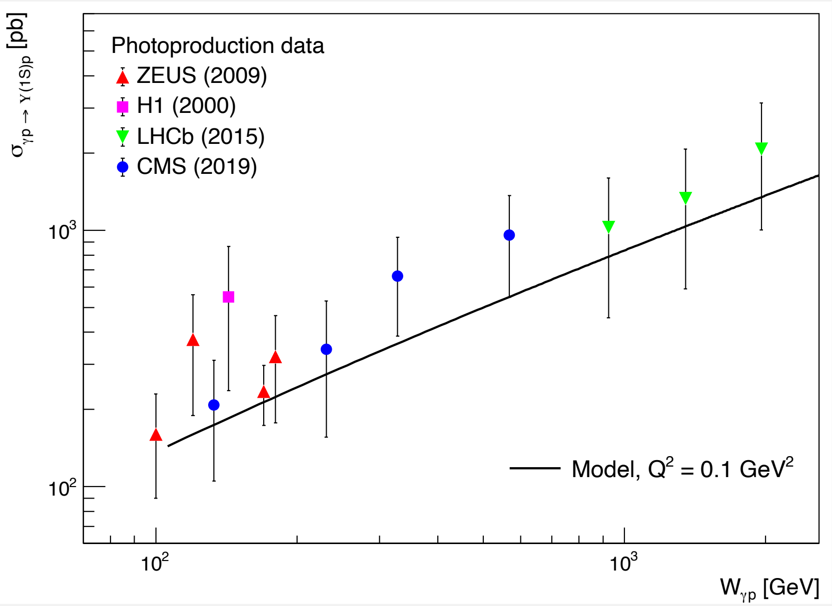

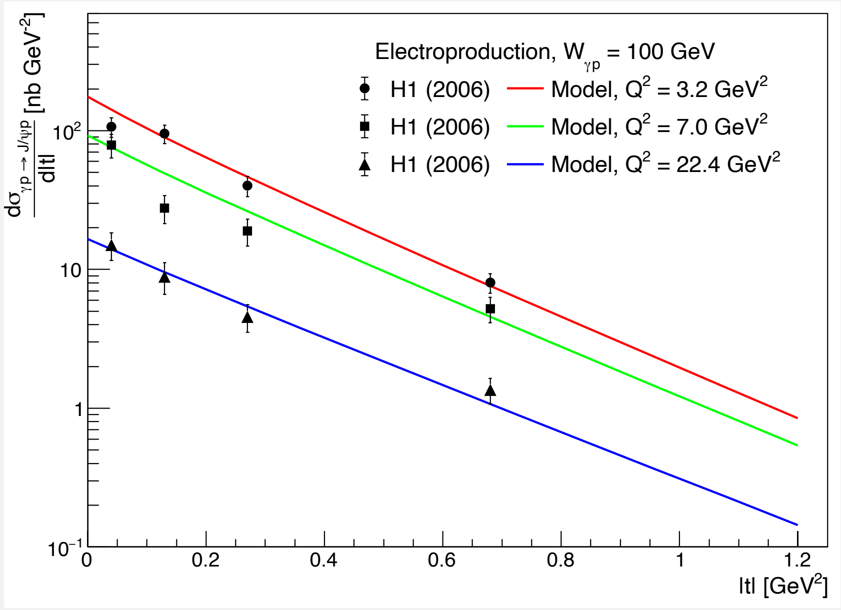

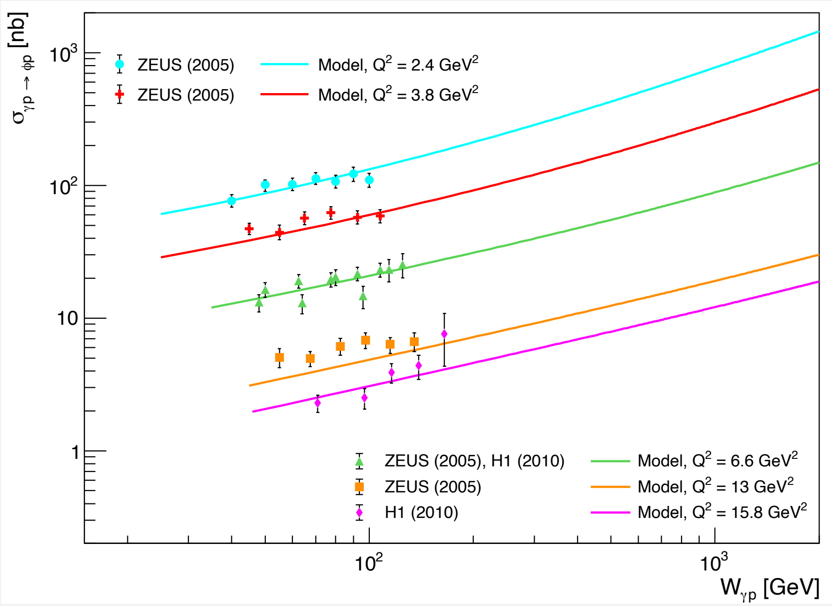

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SCATTERING AMPLITUDE DATASETS

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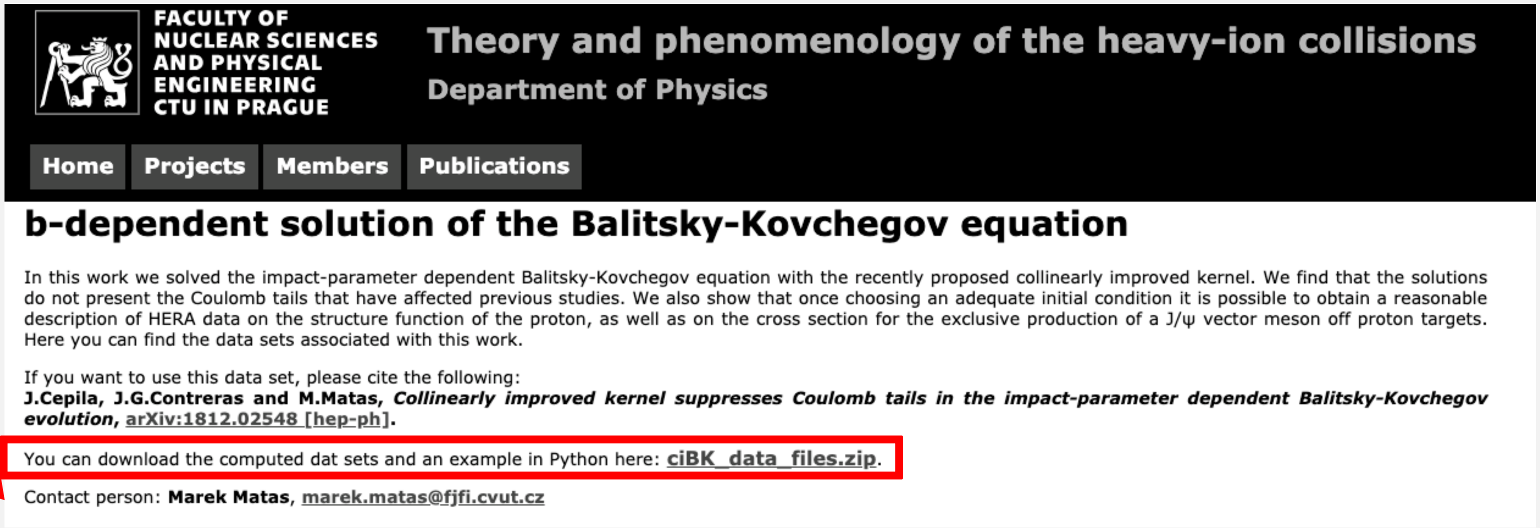

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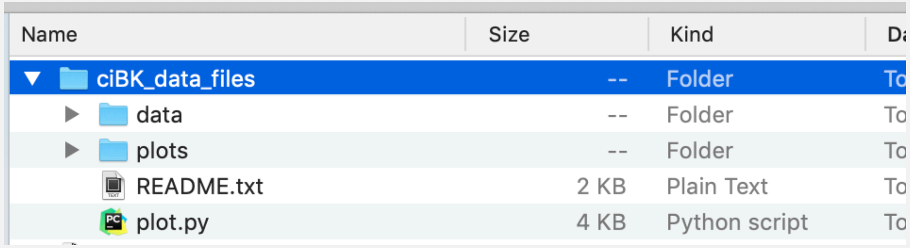
b-dependent solution of the Balitsky-Kovchegov equation

In this work we solved the impact-parameter dependent Balitsky-Kovchegov equation with the recently proposed collinearly improved kernel. We find that the solutions do not present the Coulomb tails that have affected previous studies. We also show that once choosing an adequate initial condition it is possible to obtain a reasonable description of HERA data on the structure function of the proton, as well as on the cross section for the exclusive production of a J/ψ vector meson off proton targets. Here you can find the data sets associated with this work.

If you want to use this data set, please cite the following:
J.Cepila, J.G.Contreras and M.Matas, Collinearly improved kernel suppresses Coulomb tails in the impact-parameter dependent Balitsky-Kovchegov evolution, **arXiv:1812.02548 [hep-ph]**.

You can download the computed dat sets and an example in Python here: **ciBK_data_files.zip**.

Contact person: **Marek Matas**, **marek.matas@fjfi.cvut.cz**



All the datasets are publicly available at https://hep.fjfi.cvut.cz/bdepBK.php

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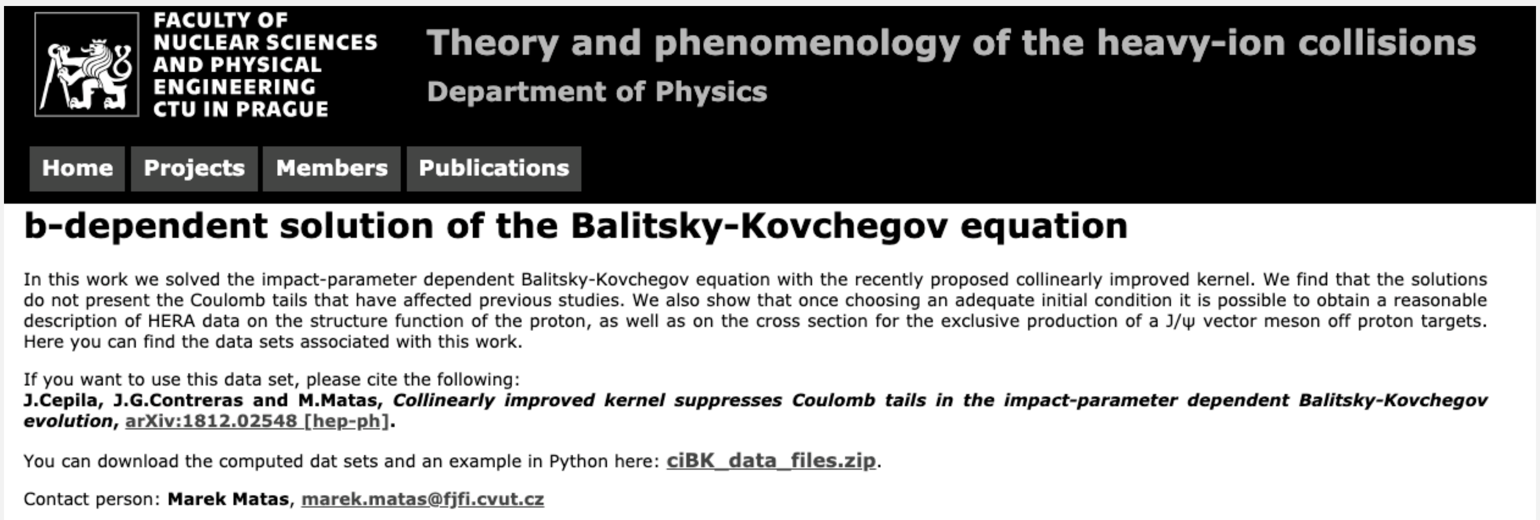

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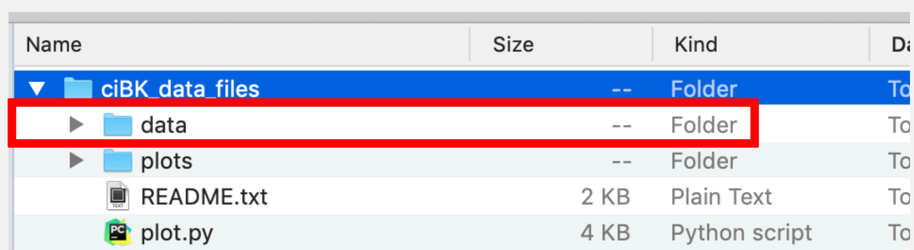

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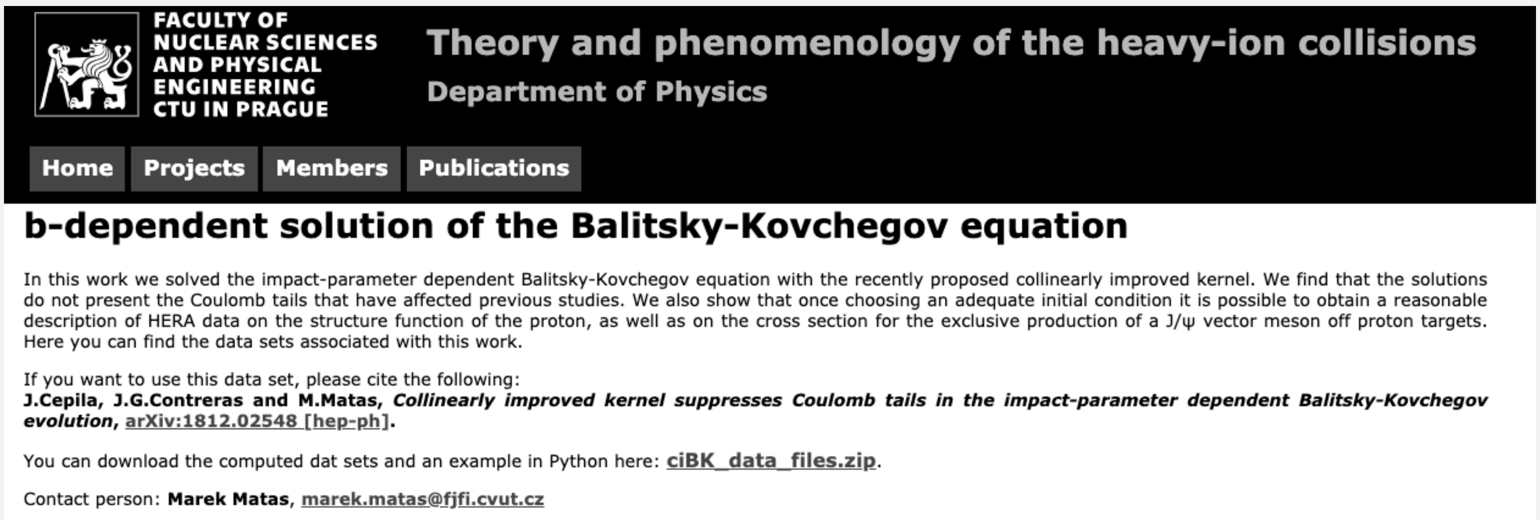

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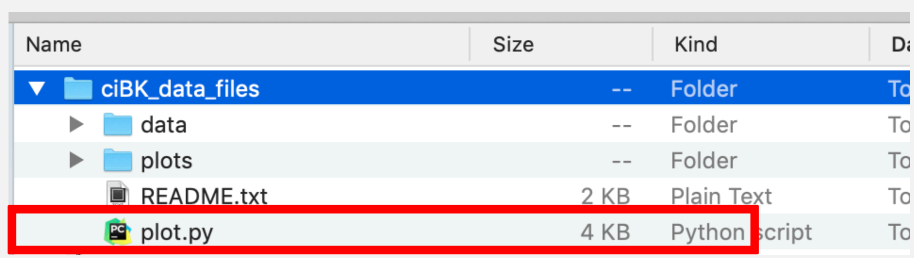

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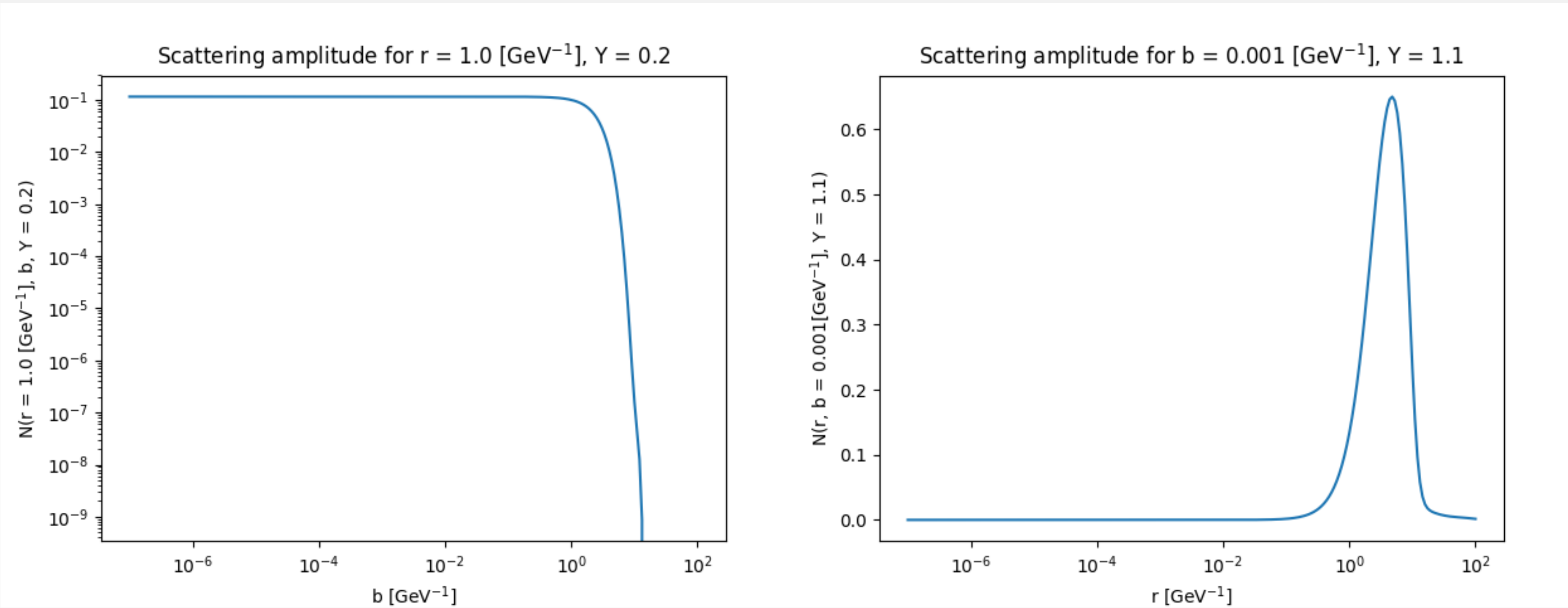

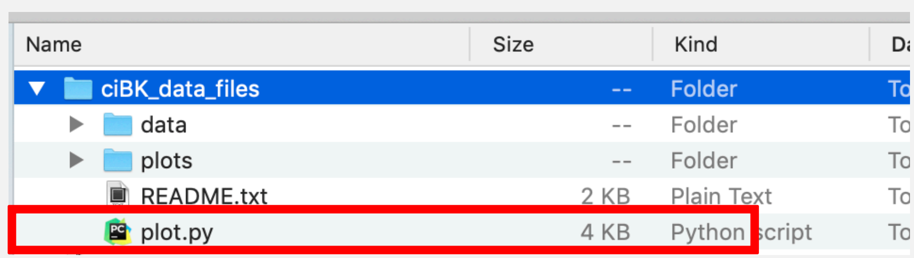

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CONCLUSIONS

- The predictive power of the the impact-parameter dependent BK equation can be spoiled by the unphysical growth of the so-called Coulomb tails.
- These can be suppressed by suppressing the evolution for large daughter dipoles r_1 and r_2.
- The collinearly improved kernel suppresses the Coulomb tails so that the b-dependent BK equation describes data over a large phase-space and various processes.
- We have currently published a paper with all details

  Phys. Rev. D 100, 054015.

THANK YOU FOR YOUR ATTENTION

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D. Bendova, J. Cepila, J. G. Contreras, M. Matas; arXiv:1907.12123