Softening the UV without new particles

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$$B = 1 \longrightarrow SM$$

$$BBSM$$

$$SM = 1 \longrightarrow Beyond^2$$

w/ Riccardo Rattazzi 2306.12489

SM inspiration

My favorite part of physics is that is it possible to guess the answer

This talk : Naturalness

Naturalness : Using dimensional analysis to predict a new scale (sometimes this is the mass of a new particle)

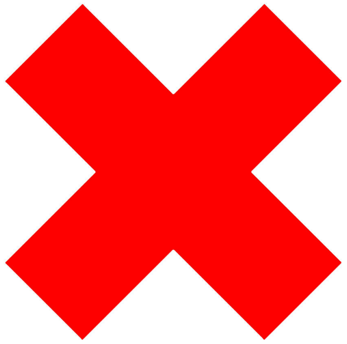
SM inspiration

Naturalness : Using dimensional analysis to predict a new scale (sometimes this is the mass of a new particle)

Believe : Experimental evidence indicates dimensional analysis always works

Just like Occam's Razor

Physics 

Math 

Familiar Examples

Simple Example

Drop a ball

Time when you hear a noise

$$d \sim g\, t^2$$

Predict distance to new physics

Familiar Examples

Simple Example

Guess the size of molecule

$$V = \frac{c_1}{r} + \frac{c_2}{r^2} + \frac{c_3}{r^3} + \frac{c_4}{r^4} + \cdots$$

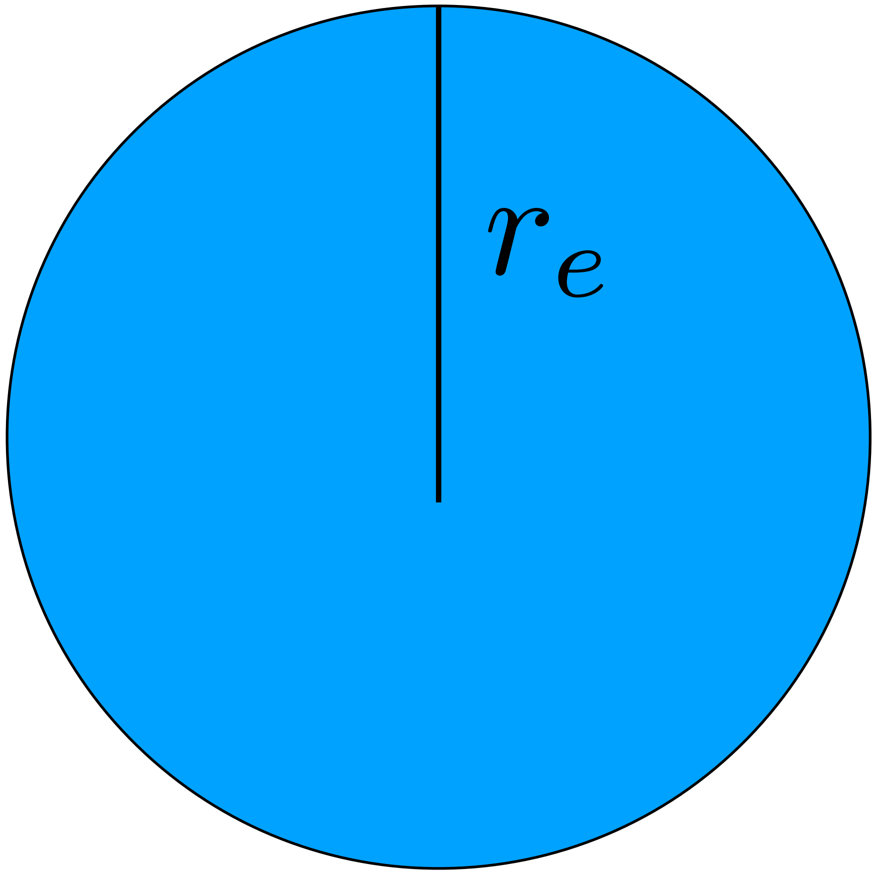
Use terms in the multipole expansion to guess size of object

Todo : do this for methane

Familiar Examples

Classic Examples

Classical Radius of the electron



$$V_{E+M} \sim \frac{\alpha}{r_e} \sim m_e$$

“Anticipated” Quantum Mechanics

Familiar Examples

Classic Examples

Charged pion mass

$$m_{\pi^\pm}^2 - m_{\pi^0}^2 \sim \alpha\Lambda^2$$

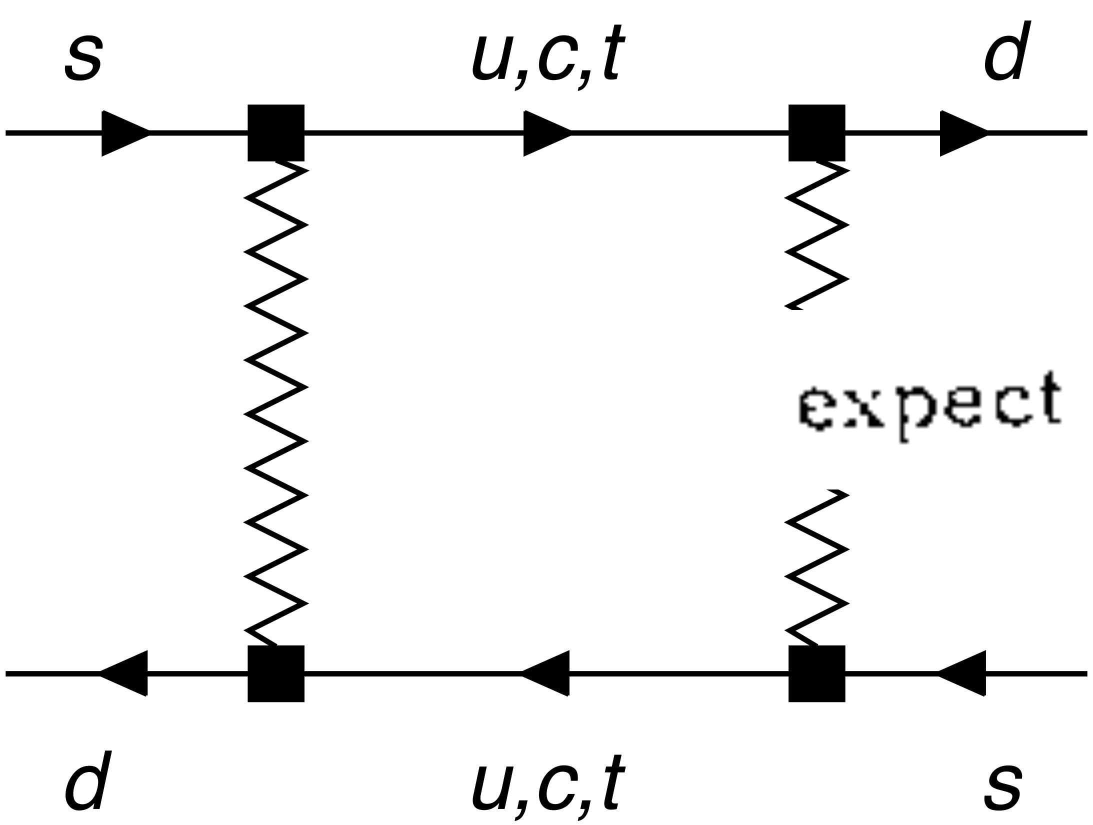
"Anticipated" Rho meson

Diagrams enforce dimensional analysis

Familiar Examples

Classic Examples

Charm quark



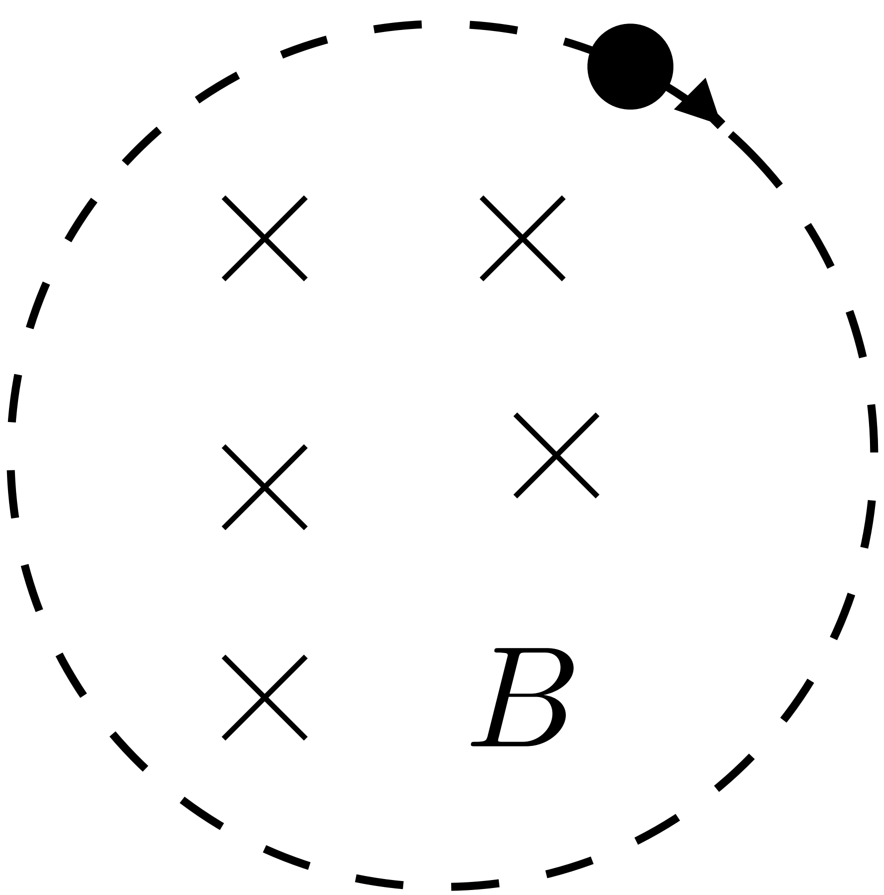
Prediction of charm quark

expect m_c to be less than, say, 10 GeV.

M.K. Gaillard and B. Lee, Phys.Rev.D 10 (1974) 897

Familiar Examples

Fermi in 1922 predicted (stimulated)
Schwinger pair-production



Larmor Radiation

$$\omega_\gamma \sim \gamma^2 \frac{eB}{m} \sim \gamma m$$

Something happens when

$$\gamma e B \sim m^2$$

Elephant in the Room

Guess the energy scale where something changes

$$\Lambda_{NP} \sim \frac{m_H}{y} \sim \mathcal{O}(1) \times 500\,\mathrm{GeV}$$

Tradition

What happens at this scale? New Particles!

1. Colored scalars
2. Colored particles
3. Particles

SUSY

Little Higgs

Extra dimensions

Twin Higgs

What’s going on?

Where are the 500 GeV new particles????

Dimensional Analysis does not predict O(1) numbers

To the extent that there is nothing there, we must be missing something important!

*Most Current Models are somewhat tuned

Twin Higgs
Axion/Relaxation solution
Cosmological Naturalness

Making Friends

Concrete example where ASSUMPTION that what appears is particles is INCORRECT

Dimensional analysis still correct, but instead new dynamics instead of new particles

Outline

- **IR EFT**
  - A Yukawa coupled fermion without quadratic divergences
- **UV completion**
  - Confirm we aren't tricking ourselves
- **Interpretation**
  - We still don't completely understand why our result is what it is
- **Conclusion**

IR EFT

$$\mathcal{L} = y\,\phi\,\overline{\psi}\psi$$

Quadratic divergence

$$\mathcal{L} = \sqrt{2}\,y\,f\,\sin\left(\frac{\phi}{2f}\right)\overline{\psi}\psi$$

No new particles
No y^2 divergence in effective potential

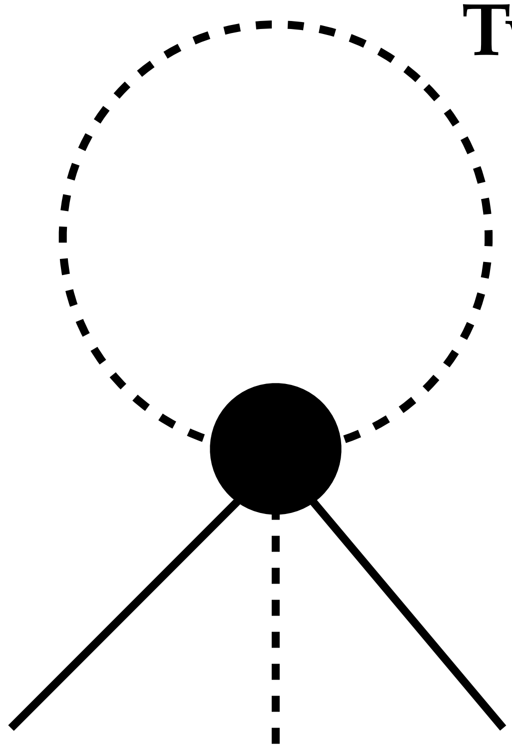
Warmup

Calculate the physical Yukawa coupling

$$\mathcal{L} = \sqrt{2}\, y\, f\, \sin\left(\frac{\phi}{2f}\right)\overline{\psi}\psi$$

Warmup

All corrections are multiplicative



Two derivatives on sine

Coupling Constant squared

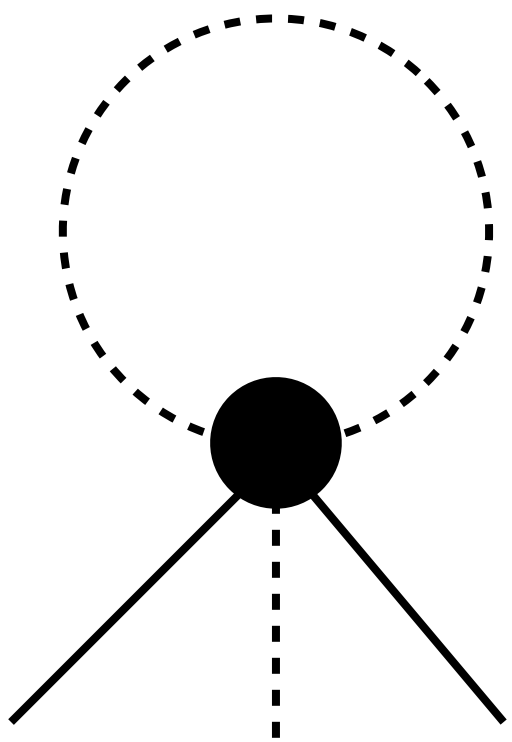
$$-\frac{1}{2}\left(\frac{1}{2f}\right)^2\int\frac{d^4p}{(2\pi)^4}\,\frac{i}{p^2}$$

Symmetry factor

Loop

Warmup

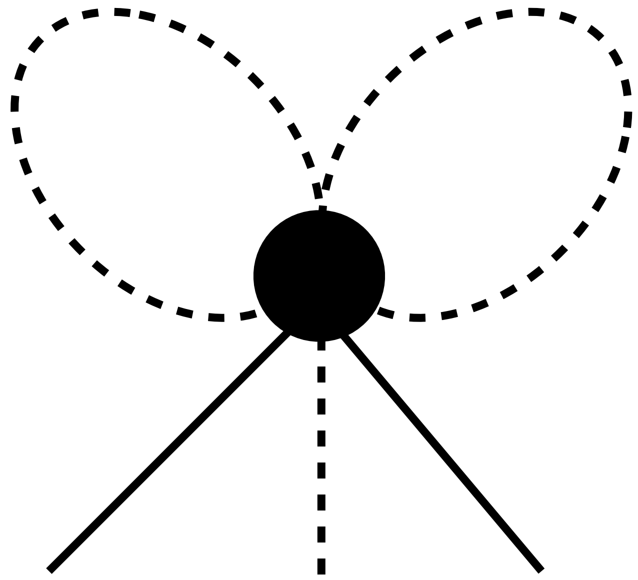
Later calculations are more easily done in position space



$$-\frac{1}{2}\left(\frac{1}{2f}\right)^2 D_\phi(0)$$

Warmup

2 loop



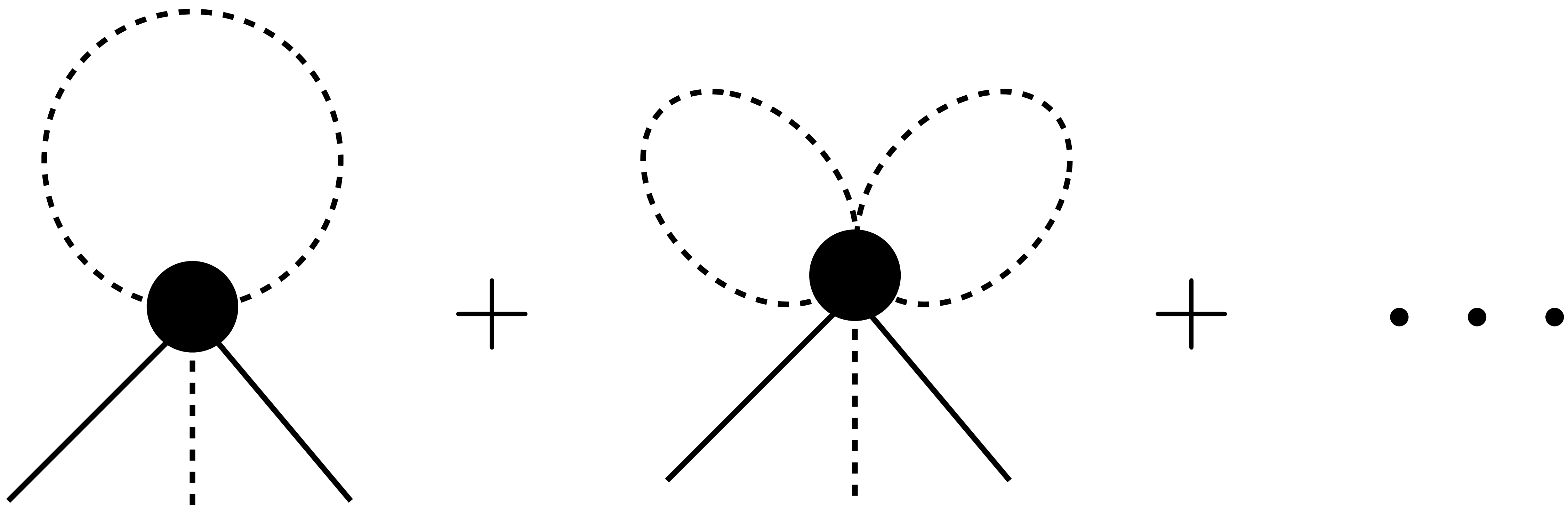
Previous result squared

$$\frac{1}{2}\left(-\frac{D_\phi(0)}{8f^2}\right)^2$$

Additional cyclic symmetry factor

Warmup

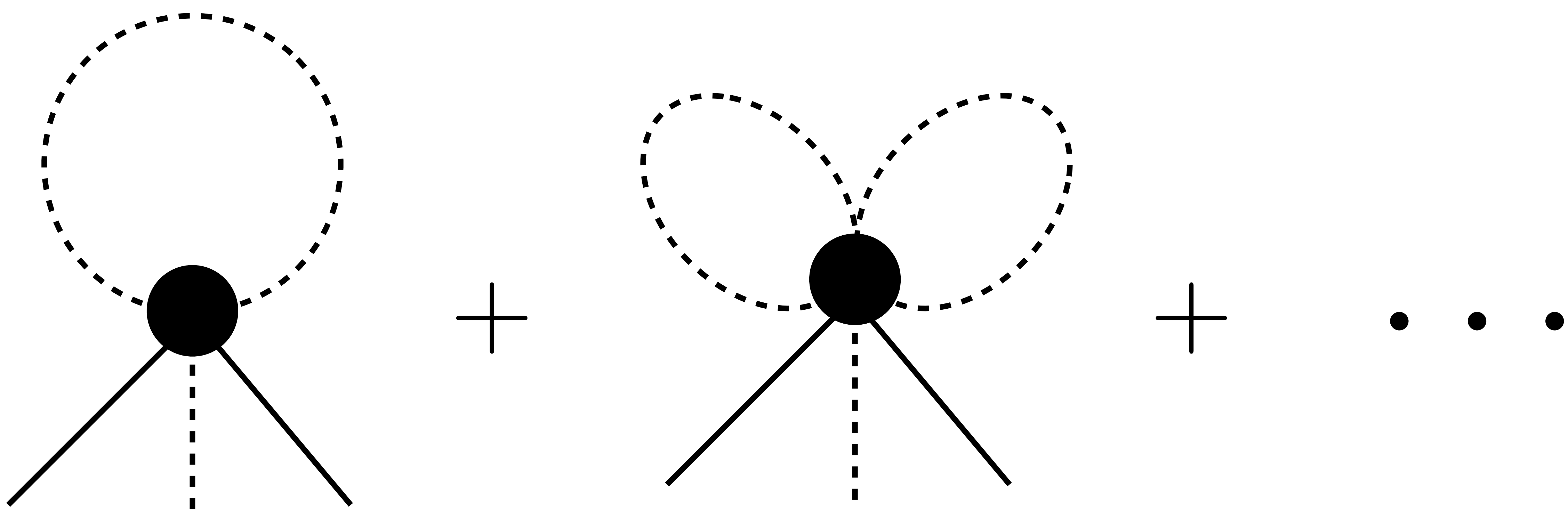
All corrections are multiplicative



$$\sum_n \frac{1}{n!}\left(-\frac{D_\phi(0)}{8f^2}\right)^n$$

Warmup

All corrections are multiplicative



$$\mathcal{L} = \sqrt{2}\, y\, f\, \sin\left(\frac{\phi}{2f}\right) \overline{\psi}\psi\, e^{-D_\phi(0)/8f^2}$$

Warmup

End Result

$$y_{\rm obs} = y\, e^{-D_\phi(0)/8f^2}$$
$$= y\, e^{-\Lambda^2/32\pi^2 f^2}$$

Divergences resum into an exponential suppression instead of enhancement!

Also true for mass term!

Calculate faster

Perturbation theory is a way of doing path integral

$$y e^{i\phi/2f} \psi\psi^c \int D\delta\phi e^{iS} e^{i\delta\phi/2f} + h.c.$$

Just do path integral

$$e^{iS + \int d^4x \delta^4(x - x_0) i\delta\phi(x)/2f}$$

Delta function source terms

Calculate faster

Complete the square

$$S = -\frac{1}{2}\int d^4x\, d^4y J(x) D_\phi(x-y) J(y)$$

$$J = \frac{1}{2f}\delta^4(x-x_0)$$

Calculate faster

Complete the square to get

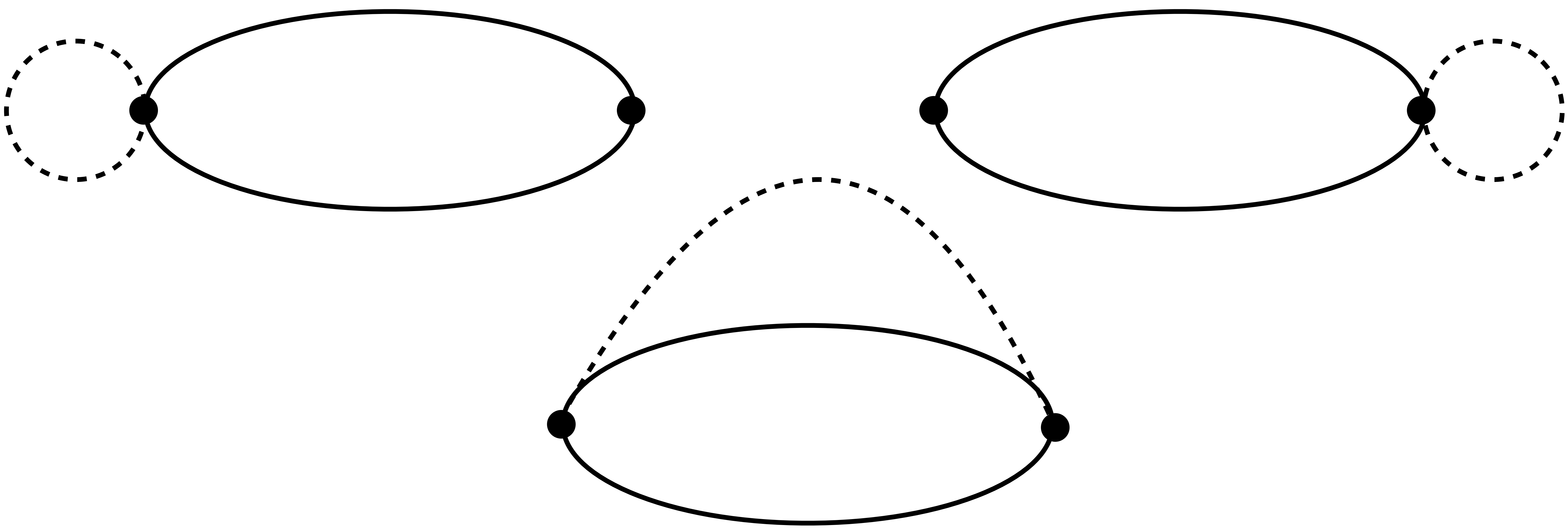
$$\mathcal{L} = \sqrt{2}\, y\, f\ \sin\left(\frac{\phi}{2f}\right) \overline{\psi}\psi\ e^{-D_\phi(0)/8f^2}$$

Same as Feynman Diagrams

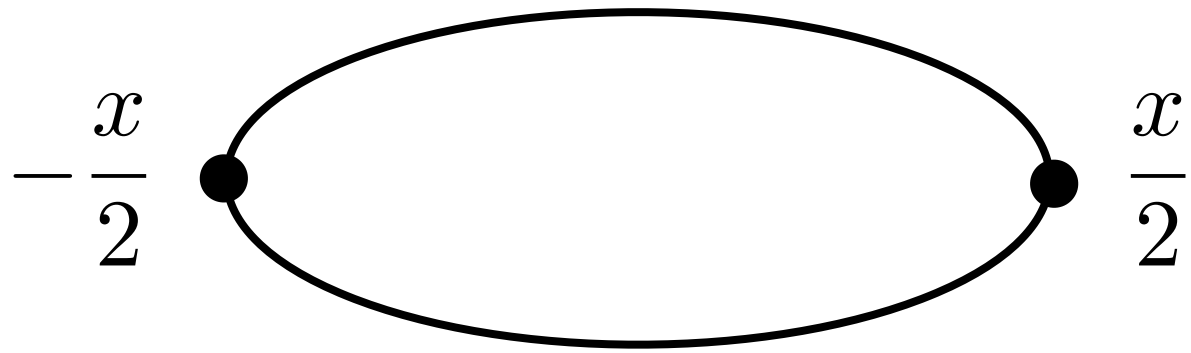
Took you long enough

Finally calculate the fermion loop

Scalar loops can hit either Yukawa or reach between them



Fermion Loop



$$\delta V_{\mathrm{eff}} = -\frac{y^2 f^2}{2} \cos\left(\frac{\phi_0}{f}\right) \int d^4x \, \mathrm{Tr}\left(D_\psi(x) D_\psi(-x)\right)$$

Fermion loop in position space

$$\int D\delta\phi \, e^{i(\delta\phi(-x/2)+\delta\phi(x/2))/2f} \, e^{-S}$$

Fluctuations

Fermion Loop

Complete the square

$$\delta V_{\text{eff}} = \frac{y^2 f^2}{2\pi^2} \int \frac{dx^2}{x^4} \int D\delta\phi \, e^{-\frac{1}{8f^2}(D_\phi(x)+D_\phi(-x)+2D_\phi(0))}$$

Fermion Loop

Complete the square

$$\delta V_{\text{eff}} = \frac{y^2 f^2}{2\pi^2} \int \frac{dx^2}{x^4} \int D\delta\phi \, e^{-\frac{1}{8f^2}(D_\phi(x)+D_\phi(-x)+2D_\phi(0))}$$

Takes Yukawa couplings to IR value

$$y_{\text{obs}} = y(\Lambda) e^{-D_\phi(0)/8f^2}$$

Fermion Loop

Complete the square

$$\delta V_{\text{eff}} = \frac{y^2 f^2}{2\pi^2} \int \frac{dx^2}{x^4} \int D\delta\phi \, e^{-\frac{1}{8f^2}(D_\phi(x)+D_\phi(-x)+2D_\phi(0))}$$

Divergent integral made finite

$$-\frac{1}{8f^2}(D_\phi(x) + D_\phi(-x)) = -\frac{1}{16\pi^2 f^2 x^2}$$

Fermion Loop

Final Result

$$\delta V_{\text{eff}} = \frac{y_{\text{obs}}^2 f^2}{2\pi^2} \int \frac{dx^2}{x^4}\, e^{-\frac{1}{8f^2}\left(D_\phi(x)+D_\phi(-x)\right)} \cos\left(\frac{\phi_0}{f}\right)$$

$$= 8 y_{\text{obs}}^2 f^4 \cos\left(\frac{\phi_0}{f}\right)$$

Finite correction to the potential

No quadratic divergence after resuming diagrams

Fermion Loop

$$\delta V_{\mathrm{eff}} = 8y_{\mathrm{obs}}^2 f^4 \cos\left(\frac{\phi_0}{f}\right)$$

Main result of this talk!

Before trying to interpret this result, a few is to dot and a few ts to cross

- **Unitarity**
- **Does anything happens at the scale 4πf ?**
- **UV completion**

Unitarity

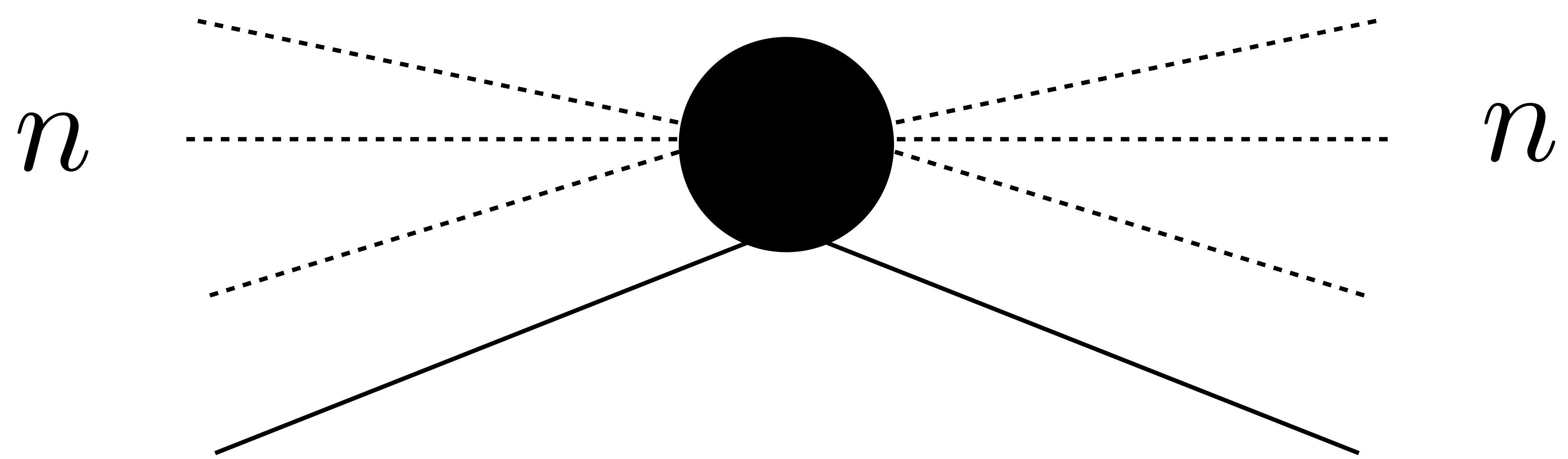
Unitarity Limit - Result only matters if UV cutoff can be taken large

$$\sqrt{2} y f \sin\left(\frac{\phi}{2f}\right) \psi\psi^c \qquad\qquad \Lambda > 4\pi f$$

No Yukawa means no UV cutoff so it depends on Yukawa coupling

Unitarity

Unitarity Limit on scalar + fermion matrix element



$$M_{n\to n} \sim y \left(\frac{E}{4\pi f n^{3/2}}\right)^{2n-4} \lesssim 1 \qquad \frac{\sqrt{s}}{n} \lesssim 4\pi f \log^{1/2} y$$

Unitarity

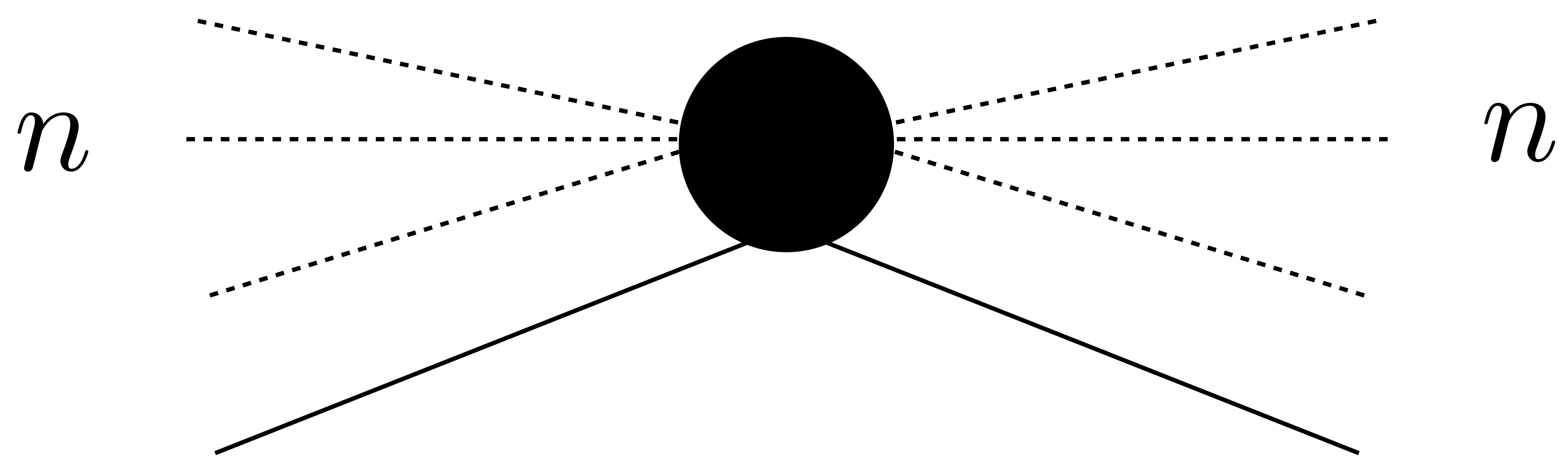
$$\frac{\sqrt{s}}{n} \lesssim 4\pi f \log^{1/2} y$$

Large UV cutoff requires small Yukawa

Had a hint of this from the exponential nature of the divergent corrections to the Yukawa coupling

Many Legs + Final States

What physically happens at this new scale?



$$M_{n\to n} \sim y\left(\frac{E}{4\pi f n^{3/2}}\right)^{2n-4} \lesssim 1 \qquad\qquad n_{\max} \sim \left(\frac{E}{f}\right)^{2/3}$$

Many Legs + Final States

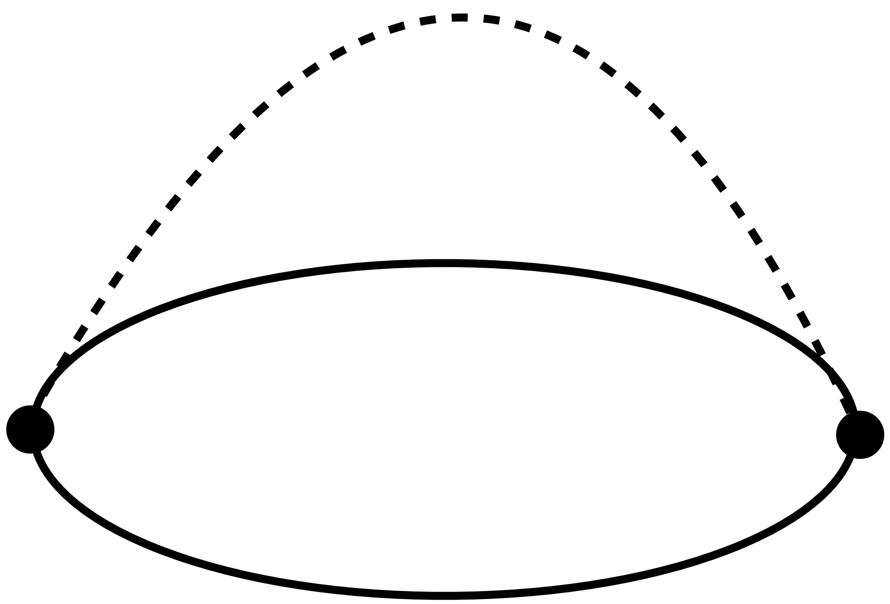
What physically happens at this new scale?

Multiple scalar final states become important

$$n_{\max} \sim \left(\frac{E}{f}\right)^{2/3}$$

Many Legs + Final States

Many loops become more important than few loops



Surprisingly, by making diagrams more divergent, the loop becomes convergent

$$e^{-\frac{1}{16\pi^2 f^2 y^2}}$$

$$n_{\text{loops}} \sim \frac{1}{16\pi^2 f^2 y^2}$$

Outline

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- **Conclusion**

A simple starting point

Our starting point is everyone's favorite toy theory

$$\mathcal{L} = |\partial\Phi|^2 + m_\Phi^2 \Phi\Phi^\dagger - \frac{\lambda_\Phi}{4}(\Phi\Phi^\dagger)^2$$

Spontaneous Symmetry breaking with a Higgs mode and a Goldstone

A simple starting point

Break U(1) but preserve a Z_N

$$\delta \mathcal{L} = Y(\Phi^{N/2} - \Phi^{\dagger, N/2})\psi\psi^c$$

We will be interested in the loop of fermions

$$\delta \mathcal{L} = \lambda_N \Phi^N$$

High dimensional operator is why multi-particle final states become important

Limit under consideration

$F, N \to \infty$	**Symmetry breaking scale and classical dimension large. Many legs**
$\lambda N \to 0$	**Loop expansion small**
$\lambda N^2 = \text{const.}$	**1-Loop effects are O(1) important**
$m_\rho = \text{const.}$	**Radial mode has a fixed mass**
$F/N = f = \text{const.}$	**New scale**

Nice Simplification

Useful Simplification

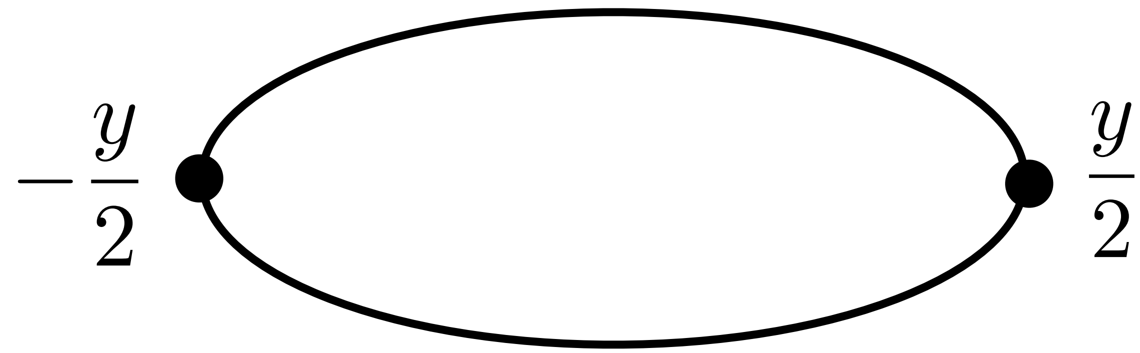
$$\Phi^{N/2} = (\frac{F+\rho}{\sqrt{2}})^{N/2} e^{i\phi/2f} = \frac{F^{N/2}}{2^{N/4}}(1+\frac{\rho}{Nf})^{N/2} e^{i\phi/2f}$$

$$\approx \frac{F^{N/2}}{2^{N/4}} e^{(\rho+i\phi)/2f}$$

Radial Mode also exponentiates

Radial mode and pNGB appear the same up to an factor of i

Fermion Loop



$$\delta\lambda_N = -\frac{iY(\Lambda)^2}{8}\int d^4y\,\mathrm{Tr}\left(D_\psi(y)D_\psi(-y)\right)$$

Fermion loop in position space

$$\int D\delta\rho\, D\delta\phi\, D\psi\, e^{\left(\delta\rho(\frac{y}{2})+i\delta\phi(\frac{y}{2})+\delta\rho(-\frac{y}{2})+i\delta\phi(-\frac{y}{2})\right)/2f}\, e^{iS}$$

Fluctuations

Fermion Loop

Complete the square

$$\delta\lambda_N = \frac{Y(\Lambda)^2}{8\pi^2}\int \frac{dy_E^2}{y_E^4} e^{\frac{1}{8f^2}\left(2D_\rho(0)-2D_\phi(0)+D_\rho(y)-D_\phi(y)+D_\rho(-y)-D_\phi(-y)\right)}$$

Fermion Loop

Complete the square

$$\delta\lambda_N = \frac{Y(\Lambda)^2}{8\pi^2}\int \frac{dy_E^2}{y_E^4} e^{\frac{1}{8f^2}\left(2D_\rho(0)-2D_\phi(0)+D_\rho(y)-D_\phi(y)+D_\rho(-y)-D_\phi(-y)\right)}$$

Takes Yukawa couplings to IR value

$$Y(0) = Y(\Lambda)e^{(D_\rho(0)-D_\phi(0))/8f^2}$$

We will see that from the UV perspective all we are doing is the RG resummation

Fermion Loop

Complete the square

$$\delta\lambda_N = \frac{Y(\Lambda)^2}{8\pi^2}\int \frac{dy_E^2}{y_E^4} e^{\frac{1}{8f^2}\left(2D_\rho(0)-2D_\phi(0)+D_\rho(y)-D_\phi(y)+D_\rho(-y)-D_\phi(-y)\right)}$$

Divergent integral made finite

$$\frac{1}{8f^2}\left(D_\rho(y) - D_\phi(y) + D_\rho(-y) - D_\phi(y)\right)$$

$$= -\gamma_{\Phi^N}\frac{1 - m_\rho\, y\, K_1(m_\rho\, y)}{m_\rho^2\, y^2}$$

Fermion Loop

Final Result

$$\delta\lambda_N = \frac{Y(0)^2}{8\pi^2}\int \frac{dy_E^2}{y_E^4} e^{-\gamma_{\Phi N}\frac{1-m_\rho y_E K_1(m_\rho y_E)}{m_\rho^2 y_E^2}}$$

High energy behavior

$$m_\rho y \ll 1$$

Anomalous Dimension $\gamma_{\Phi^N} = \frac{\lambda N^2}{32\pi^2}$

At high energies $\frac{\gamma_{\Phi^N}}{2}\log(m_\rho y)$

Exactly the differential RG running expected from Conformal Perturbation Theory

$$\frac{\gamma_{\Phi^N}}{2}\log(m_\rho y) = (\gamma_{\Phi^N} - 2\gamma_{\Phi^{N/2}})\log(m_\rho y)$$

Low energy behavior

$$m_\rho y \gg 1$$

Exponential Suppression $e^{-\gamma_\Phi N/(m_\rho^2 y^2)}$

Low energy behavior

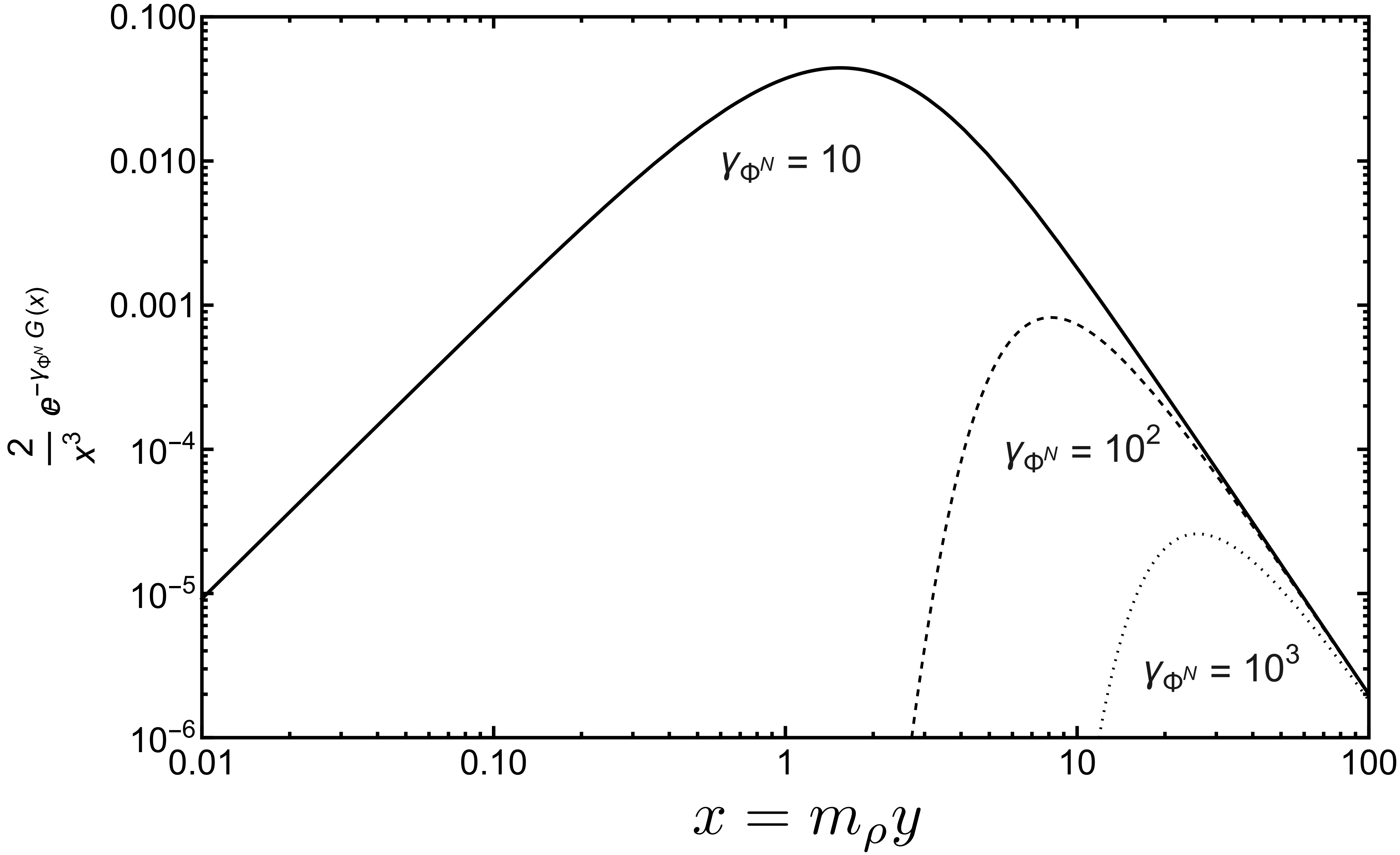
$$m_\rho y \gg 1$$

Exponential Suppression $\quad e^{-\gamma_{\Phi^N}/(m_\rho^2 y^2)}$

Energy scale introduced by exponential $\quad m_\rho/\sqrt{\gamma_{\Phi^N}} \sim F/N = f$

Exact behavior

Position Space Integrand



A new scale appears

E

Conformal Perturbation Theory

Differential running

m_ρ

Multi Particle Final state important

$F/N = f$

Exponential Unimportant - Usual Quadratic Divergence

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Dimensional Analysis

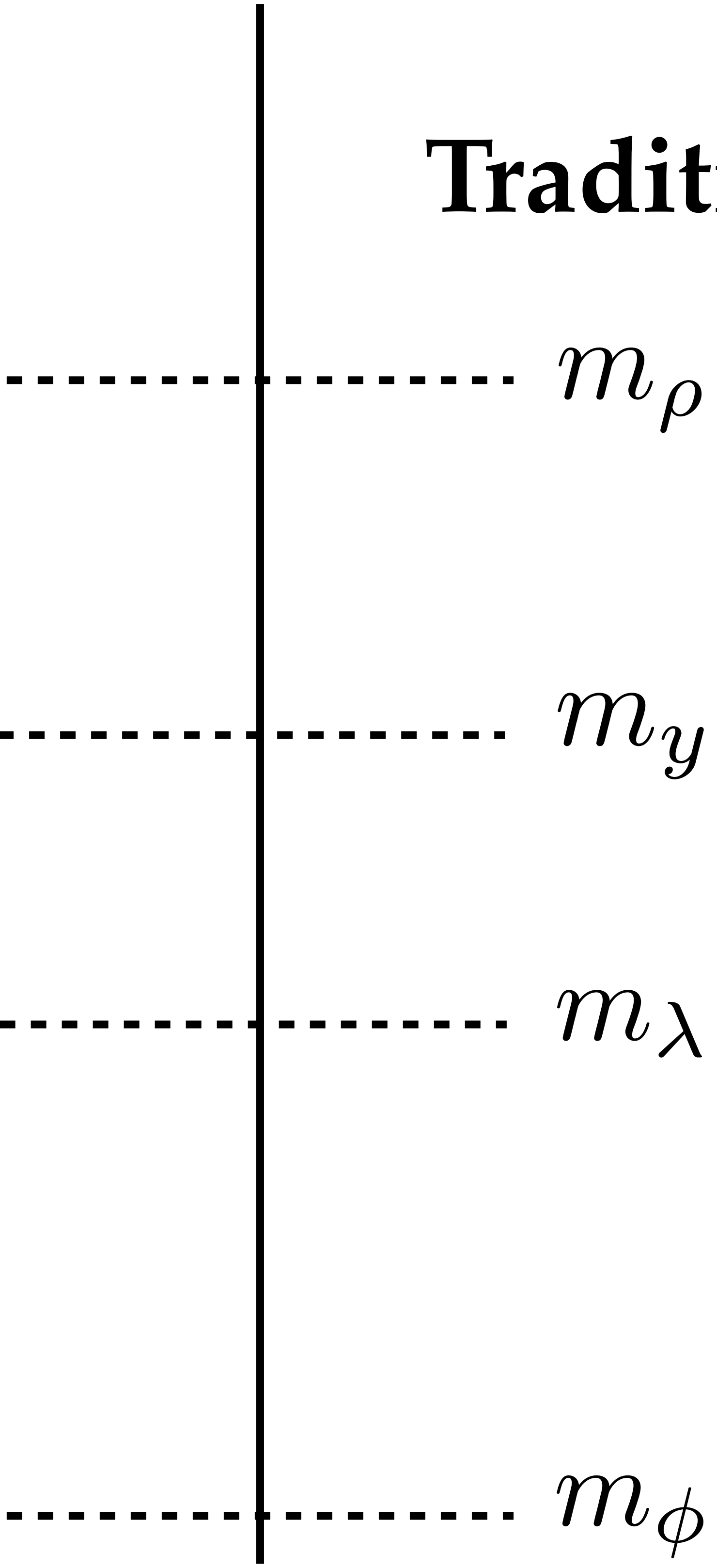
Traditional Statements : New Particles

$$m_\lambda^2 \sim \frac{32\pi^2 m_\phi^2}{\lambda_\phi}$$ **Quartic Coupling**

$$m_y^2 \sim \frac{8\pi^2 m_\phi^2}{y^2}$$ **Yukawa Coupling**

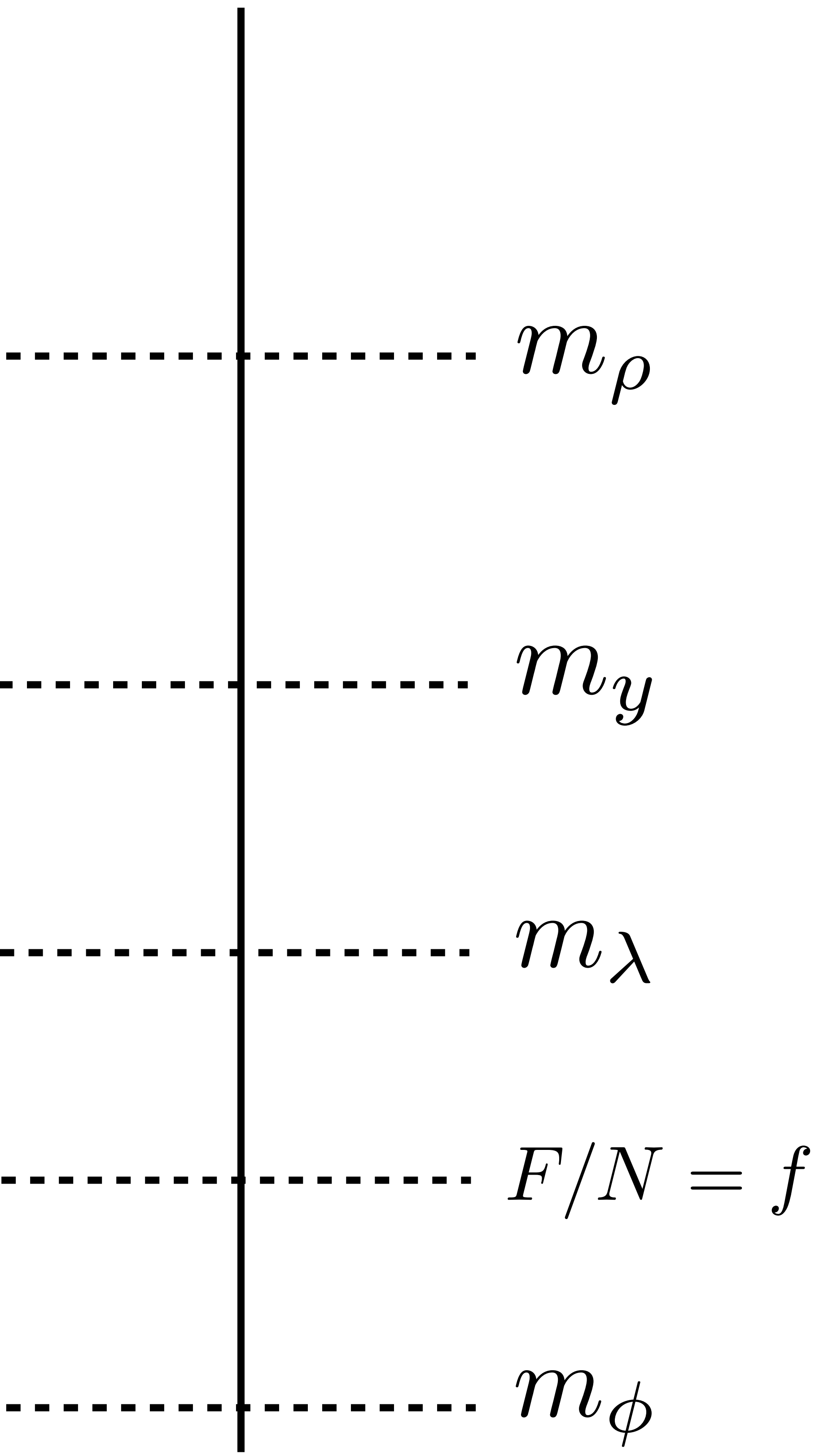
Dimensional Analysis

Traditional Statements : New Particles



Standard estimates of mass of new particles parametrically incorrect

Dimensional Analysis



Dimensional analysis does not say what has to happen

Successfully predicts when the many legged phenomenon becomes important

Dimensional Analysis

Dimensional analysis doesn’t fail

People have mis-interpret what needs to happen at the scale

Why?

Why does this happen?

Who knows

$$\mathcal{L} = \sqrt{2}\, y\, f\, \sin\left(\frac{\phi}{2f}\right)\overline{\psi}\psi$$

Essential Singularity as $f \to 0$

Essential Singularity in the UV $e^{\pm\#/f^2x^2}$

Dimensional Analysis

Something odd with dimensional analysis

Probably I’m just too stupid to realize what’s going on yet

Dim. Analysis	**Result**
$m_\phi \sim y\, E$	$m_\phi \sim y f$

What is the spurion argument that forces the energy scale to be f and not the UV cutoff?

Conclusion

Standard expectations of "naturalness" are a bit too simple

New physics is not always new particles

Extend to non-abelian, gauge interactions, …

What about the Higgs boson?

Perhaps new particles are not what occurs at the TeV scale

Multiple Higgs final state in conjunction with top quarks?

Large Yukawas?

$$m_h \approx m_t/\sqrt{2}$$