Explorations in Hilbert space*

*danke Herr Lüthi

Brian Henning
École Polytechnique Fédérale de Lausanne

Based on work done in collaboration with
O. Delouche, T. Melia, H. Murayama, F. Riva, J. Thompson, M. Walters, ...

Beyond BSM 16.08.2023

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$S \sim M \sim 1/\epsilon$

$B \sim \epsilon$

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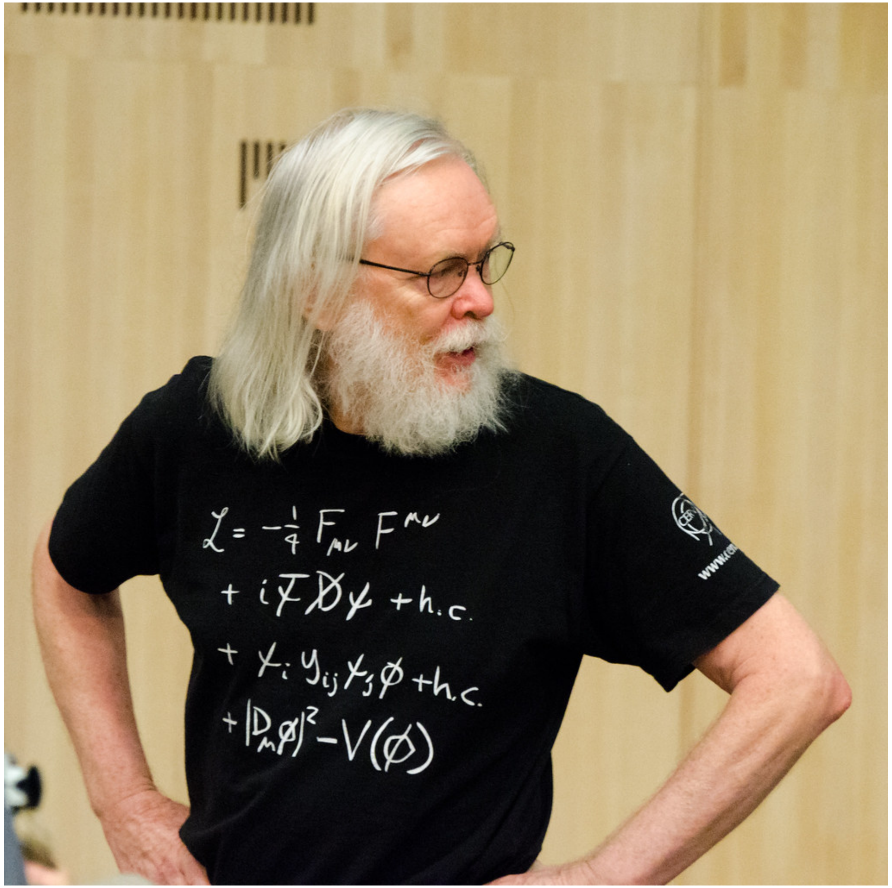
S~M~1/ϵ
B~ϵ

Bala Biött
Standard
Model

Beyond BSM 16.08.2023

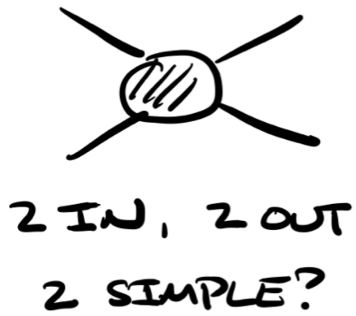
Our universe in 16 kB

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ &+ i\bar{\psi}\not{D}\psi + \text{h.c.} \\ &+ \psi_i y_{ij}\psi_j\phi + \text{h.c.} \\ &+ |D_\mu\phi|^2 - V(\phi)\end{aligned}$$



Scope of QFT

Particle physics deals with the **simplest** possible **systems**



⇒ The **scope** of questions **is** severely **narrow**

⇒ **"More is different" -P. Anderson**

⇒ **Abundance of "new physics"** lurking within theories we "know"

Opinion

What does a theorist do?

Considers dynamical systems, identifies the relevant dof, and finds a quantitative description for the dynamics

In this sense, we've barely scratched the surface

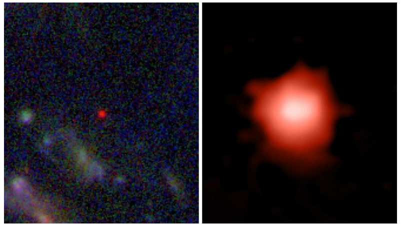
The universe at different scales

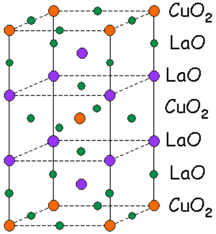
Standard Model

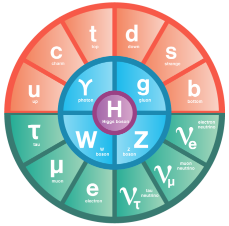
New Particles

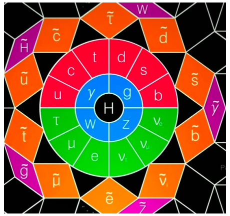
10^{27} 10^{20} 10^{10} 1m 10^{-10} 10^{-15} 10^{-35}

Very big → Very Small





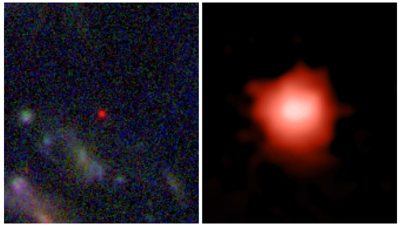


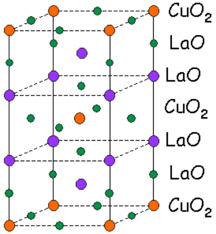


The universe at different scales

Very big

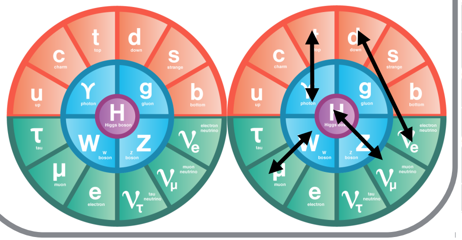
10^{27} 10^{20} 10^{10} 1m 10^{-10}





Standard Model

10^{-15}



New Particles

10^{-35}

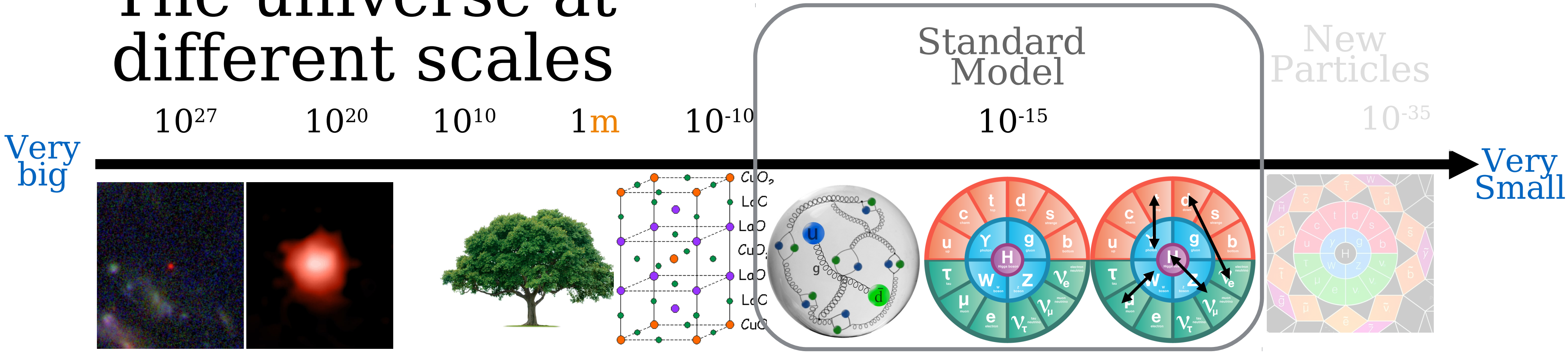
Very Small

Effective **F**ield **T**heory:

New Interactions

- Model independent
- Exhaustive
- Guide for experiments

The universe at different scales



New Physics under our nose: **QCD**

Effective **F**ield **T**heory:
New Interactions

- Model independent
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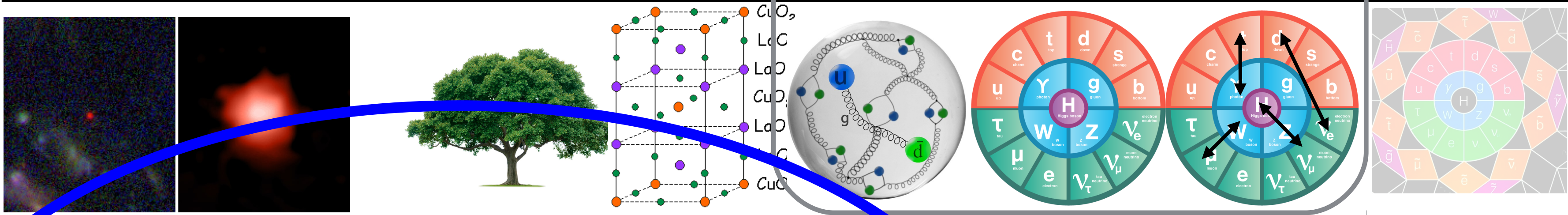
The universe at different scales

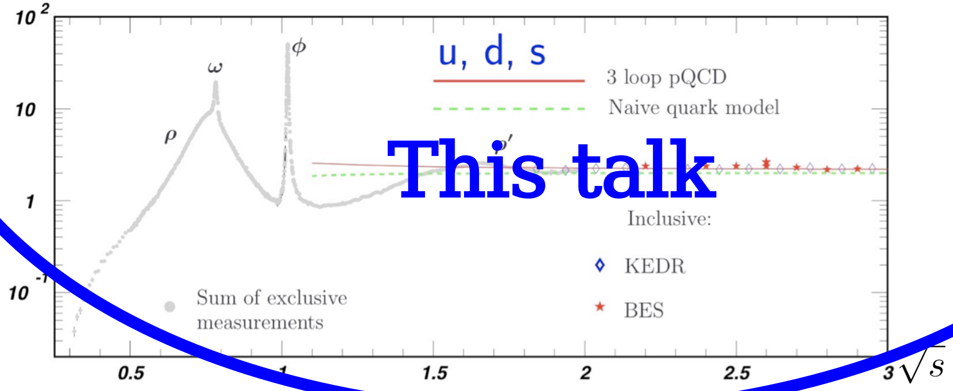
Very big — 10^{27} — 10^{20} — 10^{10} — 1m — 10^{-10} — Standard Model 10^{-15} — New Particles 10^{-35} — Very Small



New Physics under our nose: **QCD**

New numerical methods for strong coupling:

Hamiltonian truncation



Effective **F**ield **T**heory:

New Interactions

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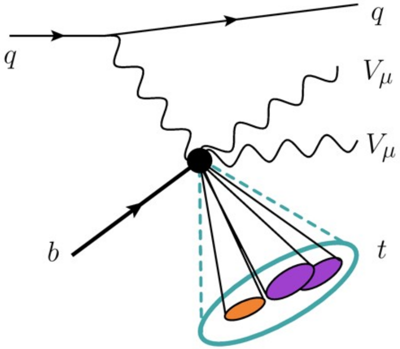
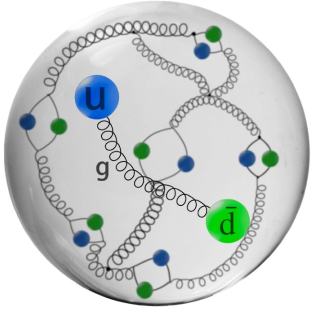


Hamiltonian truncation

a new tool for strong coupling

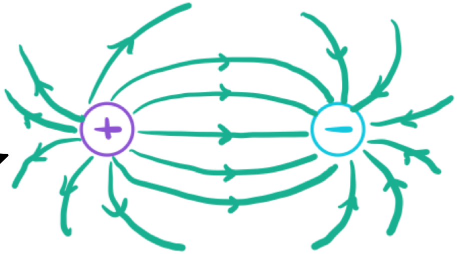
$$\mathcal{L}_{\rm QCD} = -\frac{1}{2}\mathrm{tr}(F^2) + \bar{\psi} i \not{D} \psi \xrightarrow{???}$$



Most striking feature of QCD is **confinement**

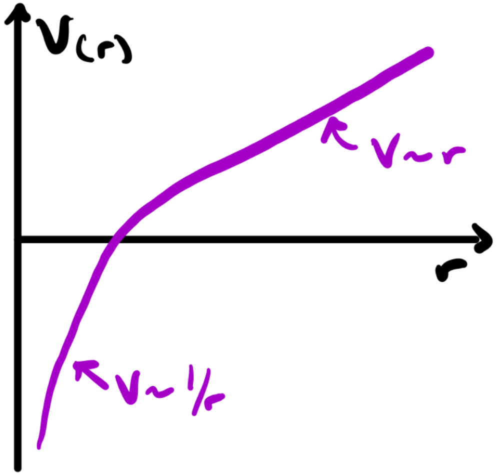
⇒ Inherently a **strongly coupled (nonperturbative)** phenomenon

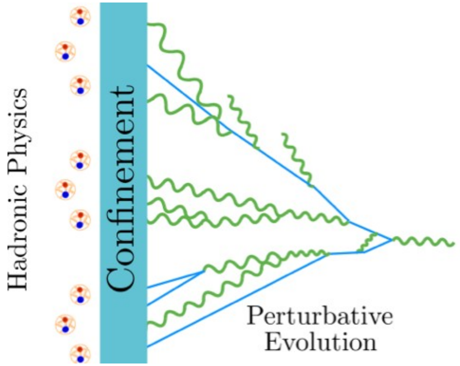
⇒ **A 50+ year old problem**



E&M

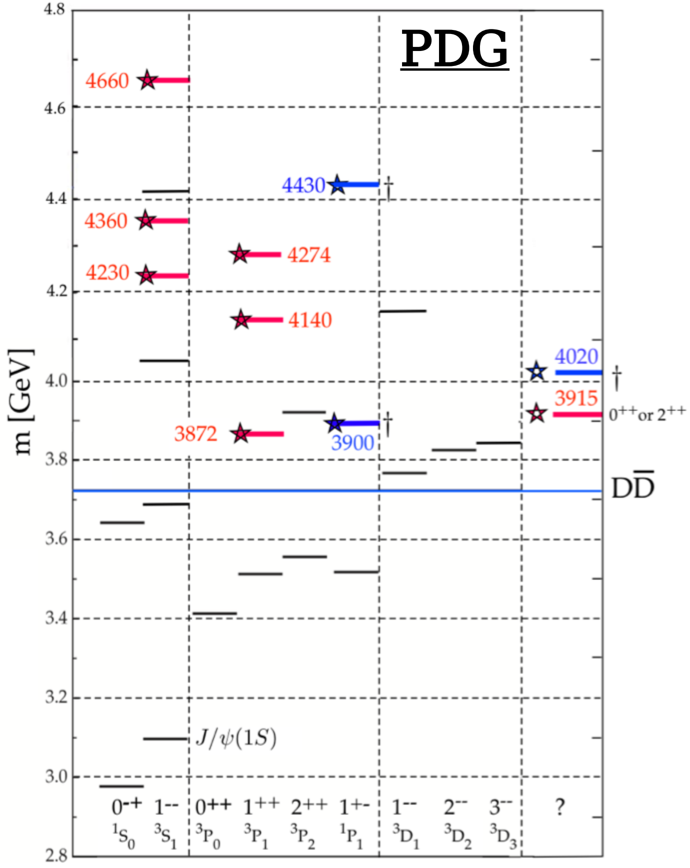
QCD



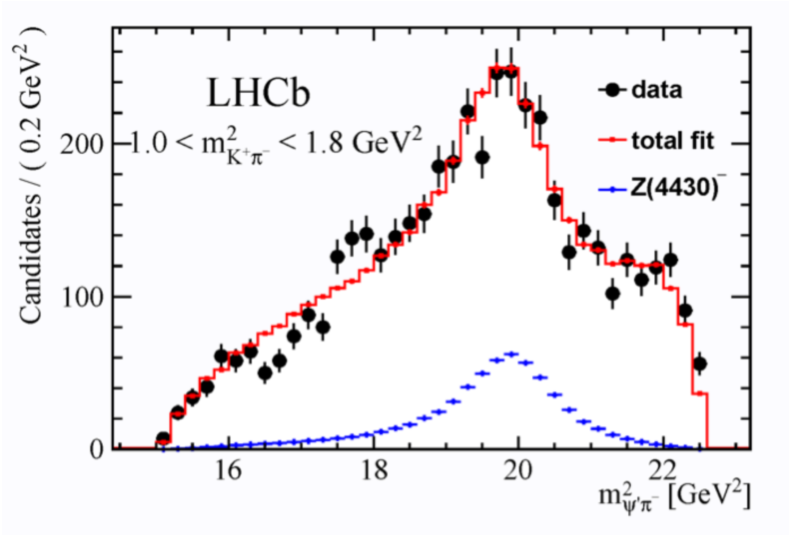


numerous open problems
BOTH qualitative AND quantitative

Charmonium spectrum
★ = exotics

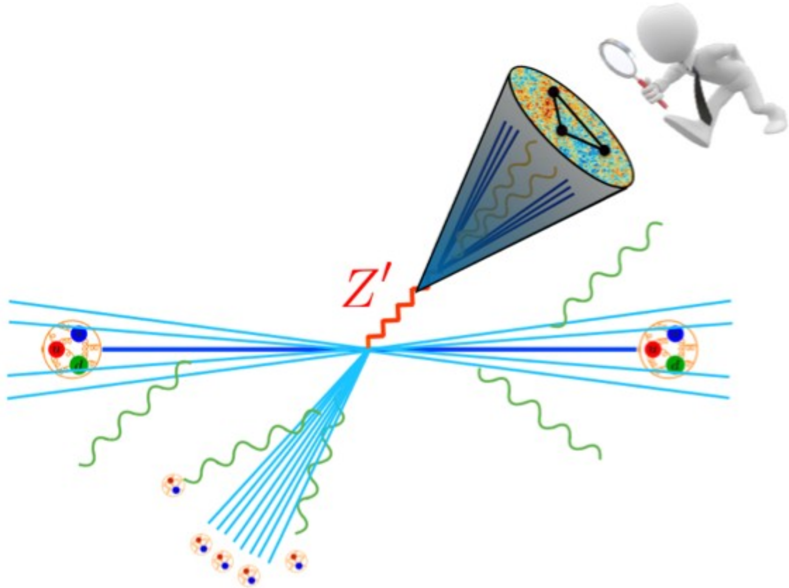


Tetraquark Z(4430)



Discovery significance
Belle 2007: 5.2σ
LHCb 2014: 13.9σ

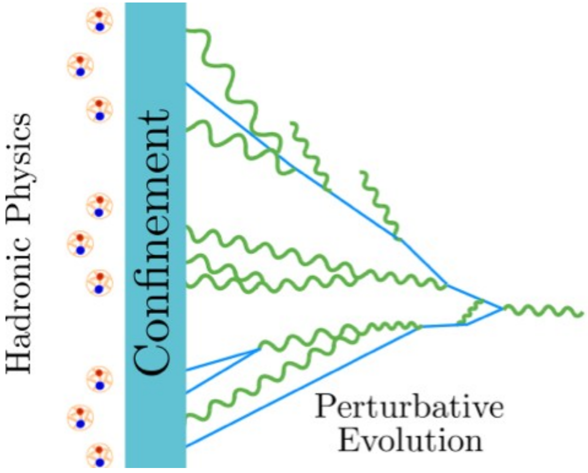
hadronization

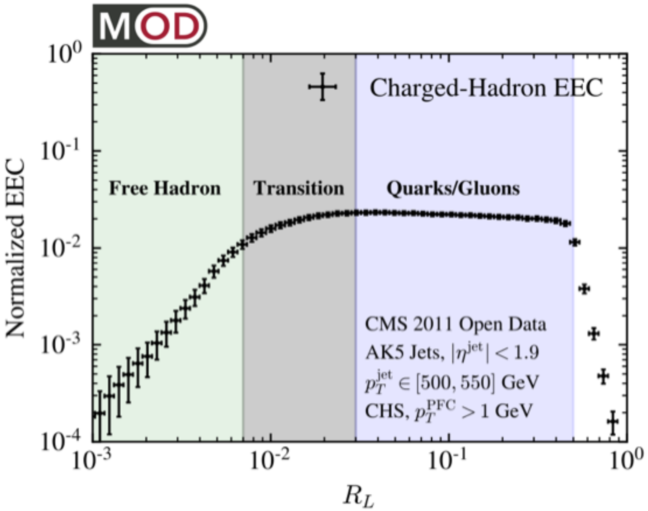


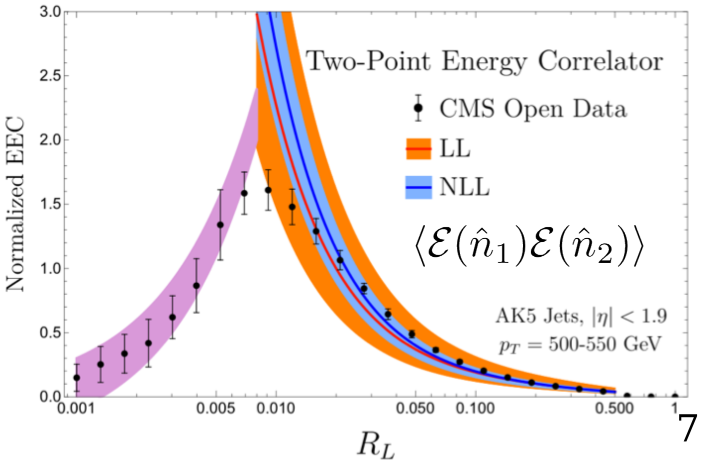
- Lots of **data** on jets
- **Clean** observables experimentally **and** theoretically
  - e.g. “energy correlators”

$$\mathcal{E}(\hat{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, n^i T_{0i}(t, r\hat{n})$$

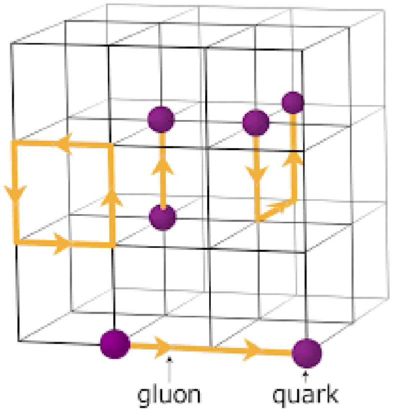
$$\langle\psi|\mathcal{E}(\hat{n}_1)\cdots\mathcal{E}(\hat{n}_k)|\psi\rangle$$



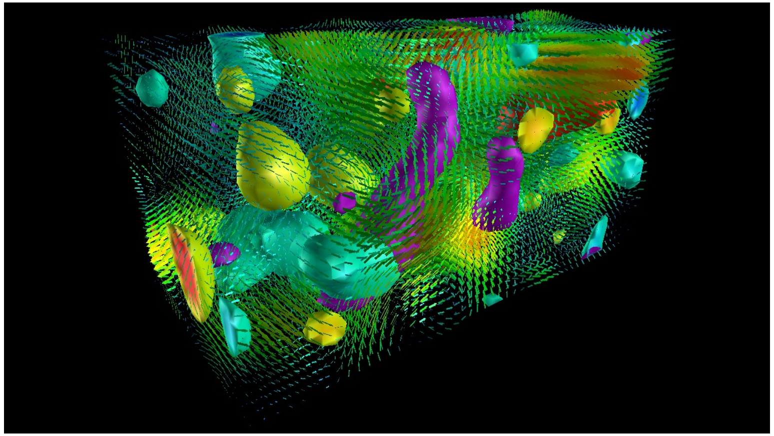




Current state-of-the-art: Lattice MC



$$\int D\phi\, e^{iS[\phi]} \xrightarrow{\text{imaginary time}} \Rightarrow \int D\phi\, e^{-S_E[\phi]}$$



✓ **General nonperturbative method**

✓ **Tremendously successful**

- ⇒ e.g. hadron spectroscopy
- ⇒ Absolutely crucial for experimental analyses

✗ **Inherently Euclidean**

- ⇒ No real time dynamics, e.g. scattering

✗ **No chiral fermions**

- ⇒ Can't put the SM on the lattice!

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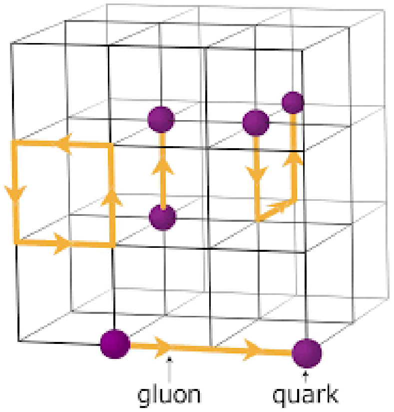

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NEED OTHER APPROACHES TO COMPLEMENT THE LATTICE!

$$P^\mu = \begin{pmatrix} H \\ \vec{P} \end{pmatrix} \xrightarrow{\text{Real time!}} H = i\partial_t$$

Will present another approach:

Hamiltonian truncation

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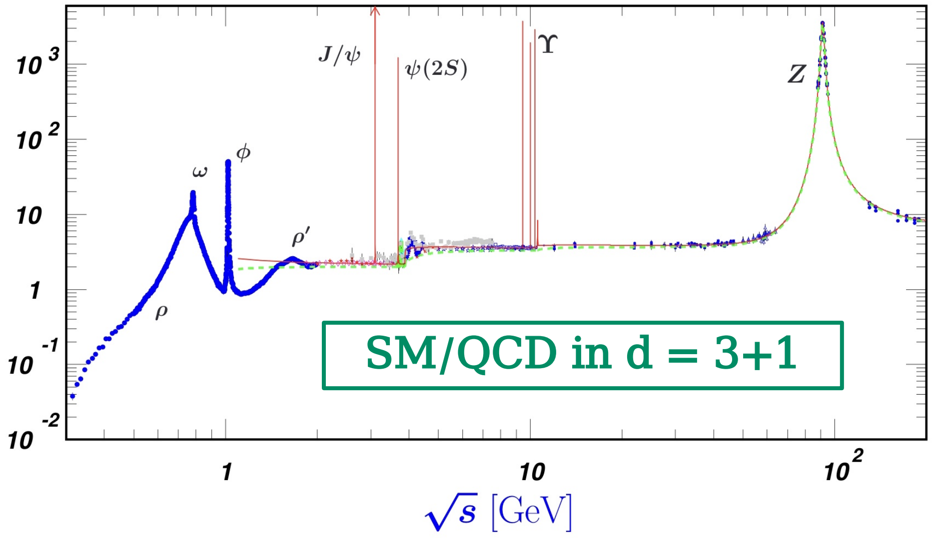
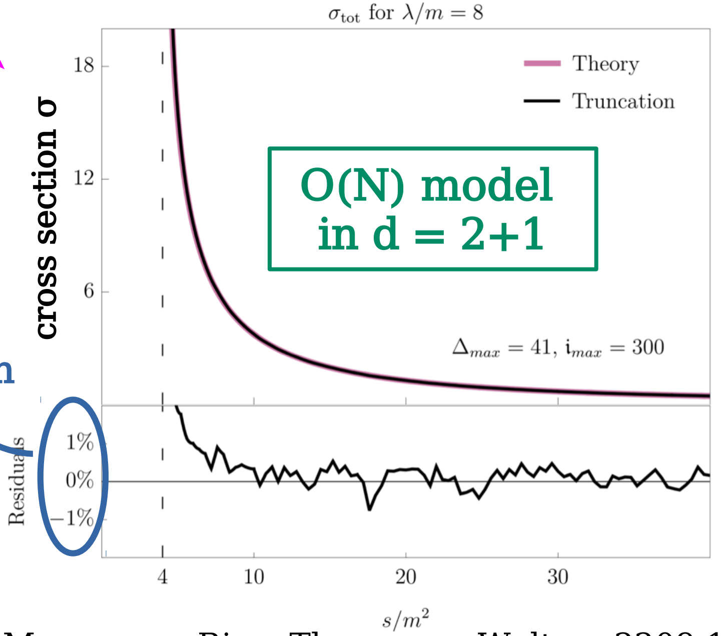
Where we want to go



Where we at

large coupling!

O(1%) precision



BH, Murayama, Riva, Thompson, Walters 2209.14306

Question

Can we make it our responsibility to make a theory collider at the same time as building the next collider(s)?

[in the spirit of brainstorming how to get the future we want, I recommend taking a hard look at messaging]

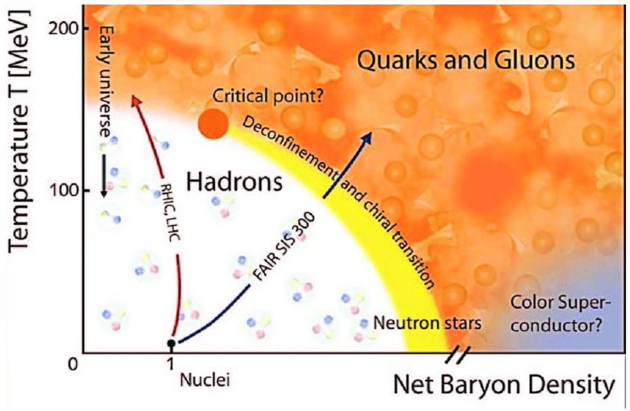
Observation/question

It appears (to me) that there is plenty of “new physics”

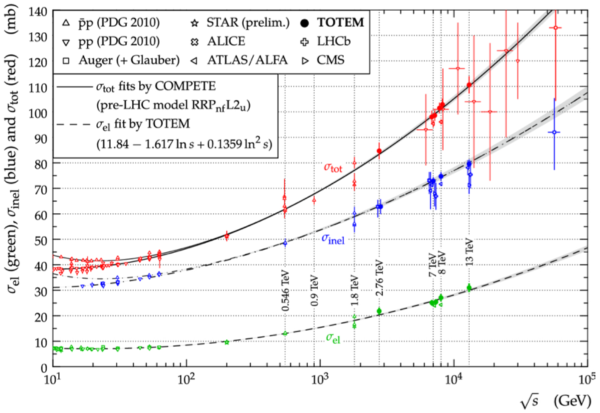
($\stackrel{\text{def}}{=}$ physics we don’t know how to describe)

being discovered at colliders

Why doesn’t this “count”?



[e.g.
Exotics,
diffractive scattering (Pomeron),
QGP,
hadronization (energy correlators),
...]



opinion

We need digestible, compact, and comprehensive materials clearly explaing what phenomenology we *could* be working on

(How else can we make informed decisions on our personal choices for research directions?)

Putting the quantum in QFT

QFT = QM on an infinite # of d.o.f.

$\Rightarrow$ States live in a Hilbert space $\longrightarrow$ $|\psi\rangle \in \mathcal{H}$

$\Rightarrow$ They obey Schrödinger eqn $\longrightarrow$ $H\,|\psi_\alpha\rangle = E_\alpha\,|\psi_\alpha\rangle$

$\Rightarrow$ Operators act on states $\longrightarrow$ $\mathcal{O}(\hat{\phi},\hat{\pi})\,,\ [\hat{\phi},\hat{\pi}] \sim i$

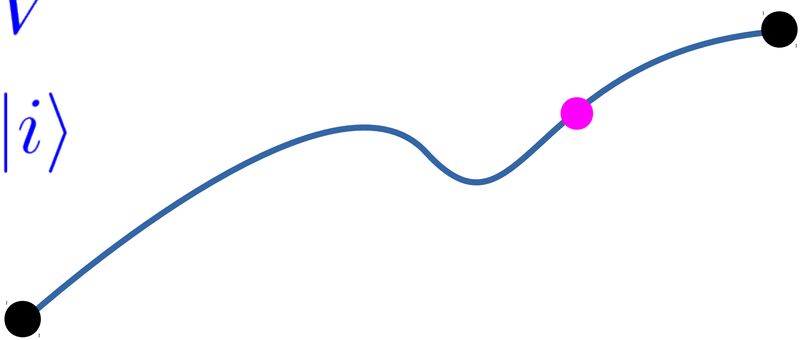
the dumbest idea which might actually work

start from known system

$$H = H_0 + V$$

$$H_0 \left| i \right\rangle = E_i \left| i \right\rangle$$

UV



deform with some relevant operator

$$V = \int d^{d-1}x \, \mathcal{O}_r(x)$$

IR

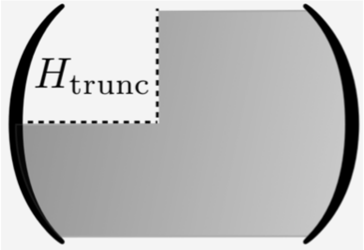
$$\langle i | H_0 + V | j \rangle$$

compute matrix elements

diagonalize

result approximates true spectrum

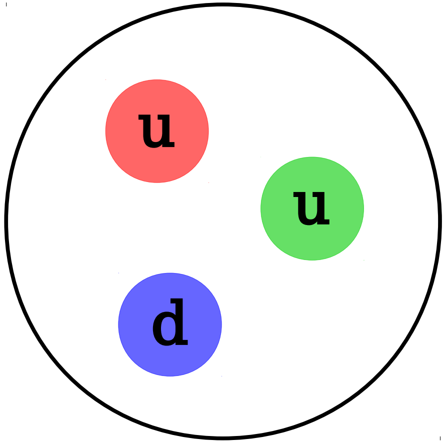
“Hamiltonian truncation”



HT output

$$|\psi\rangle = \sum_{\chi \in \mathcal{H}_{\mathrm{UV}}} c_\chi |\chi\rangle$$

e.g.



$$|p\rangle = c_{uud} |uud\rangle + \cdots$$

HT output

$$|\psi\rangle = \sum_{\chi \in \mathcal{H}_{\mathrm{UV}}} c_\chi \, |\chi\rangle$$

basis choice?

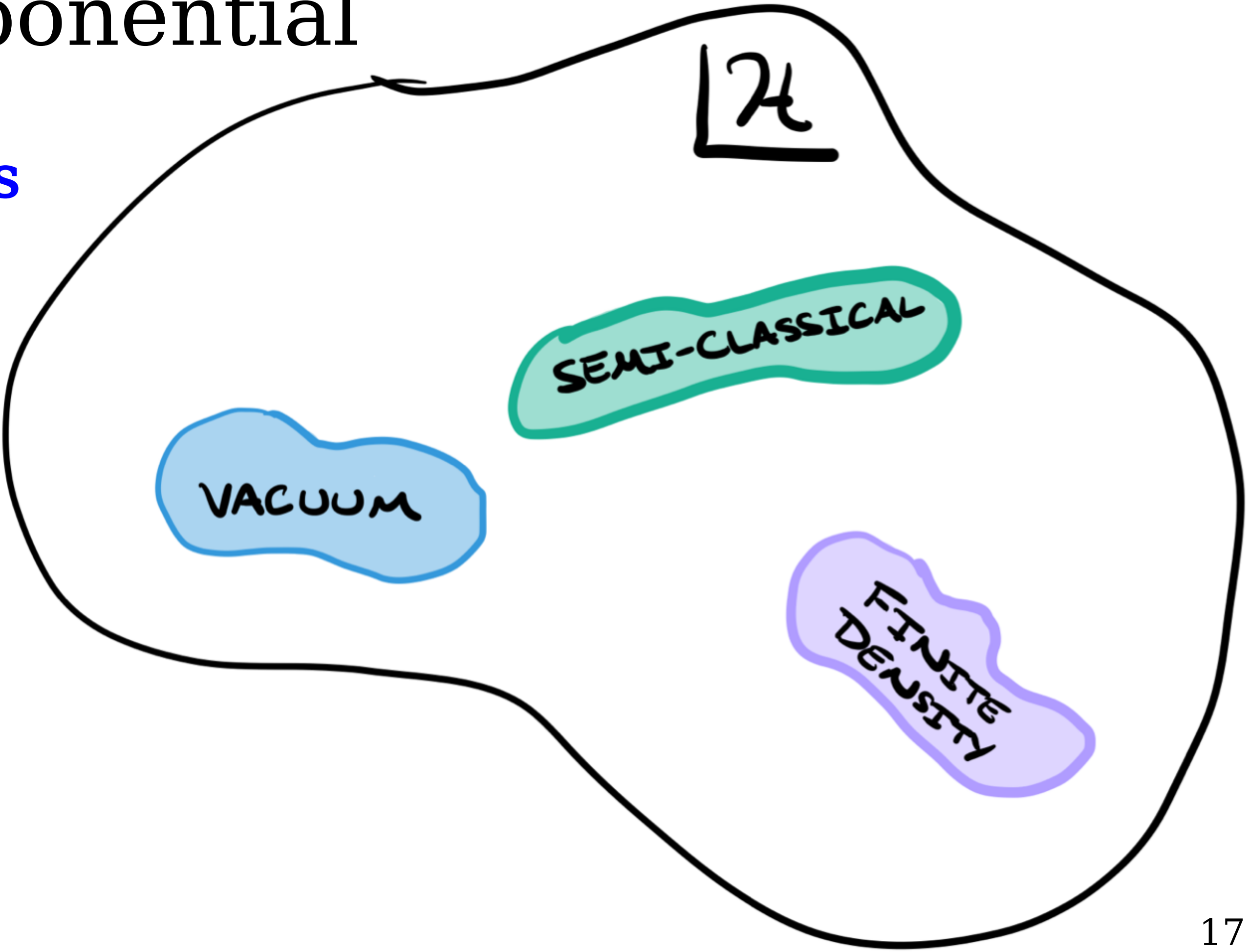
truncated sum

$\rightarrow$ convergence?

not independent questions

Basis choice: fighting the exponential

$\mathcal{H}_{IR} \subset \mathcal{H}_{UV}$

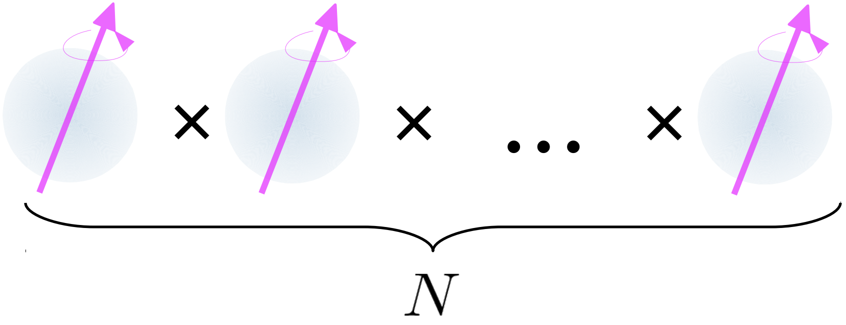


Quantum Hilbert spaces grow exponentially

⇒ How to isolate the relevant sector for desired physics?

basis choice?

- plane wave basis (e.g. DLCQ)
- tensor networks (MPS/PEPS)
  - organizing principle: information content
- partial waves (conformal basis)
  - organizing principle: spacetime symmetry



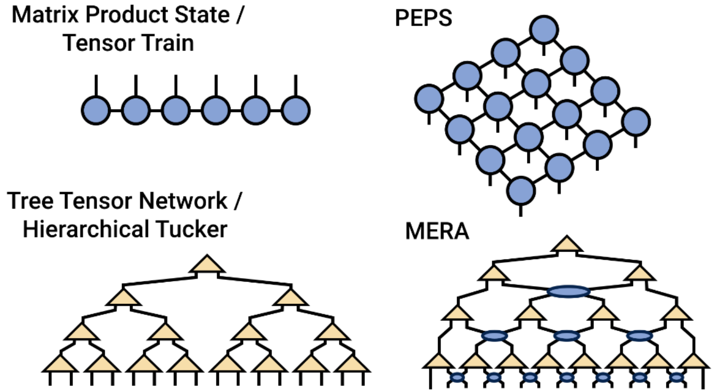
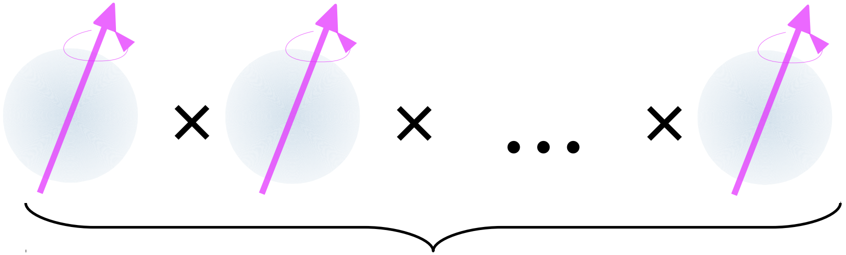


Fig: tensornetwork.org

Partial waves/phase space harmonics



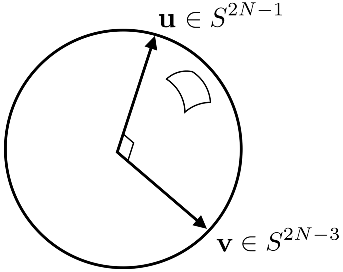
Don't treat independently—couple together and **ask properties about the collection of particles**

phase space measure

$$|\psi\rangle = \int d\Pi_n(\mathbf{p}_i)\ |\mathbf{p}_1 \ldots \mathbf{p}_n\rangle\ \underbrace{\langle \mathbf{p}_1 \ldots \mathbf{p}_n|\psi\rangle}_{\psi(\mathbf{p}_i)=\text{“wavefxn”}}$$

Free Hilbert space = wavefunctions on phase space

EFT amplitude bases (see later)



smart basis

$\Downarrow$

“spherical harmonics” on phase space

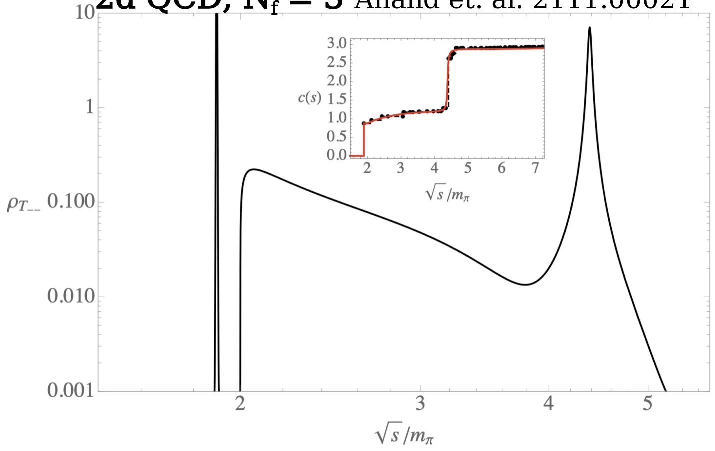
conformal basis

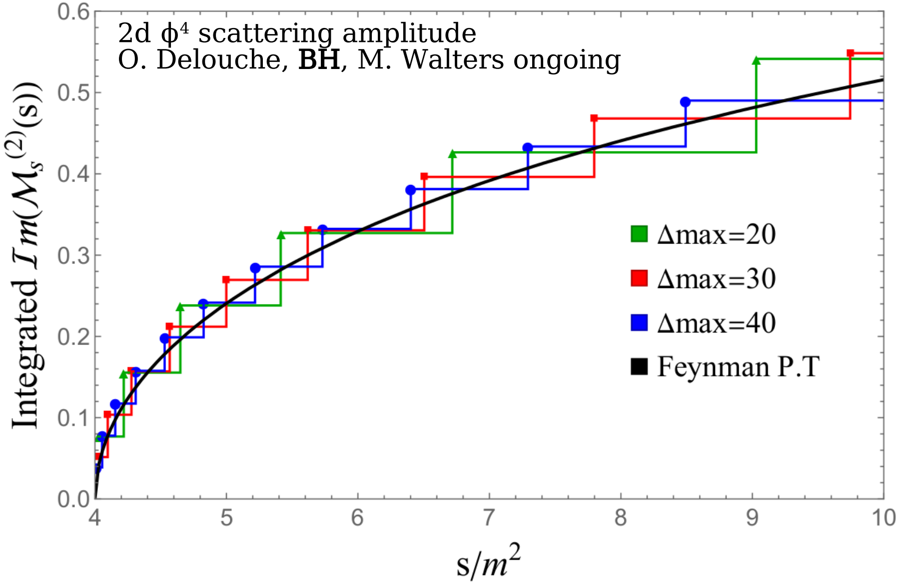
$$|\mathcal{O}(P)\rangle = \int dx\, e^{iPx}\mathcal{O}(x)\,|0\rangle$$

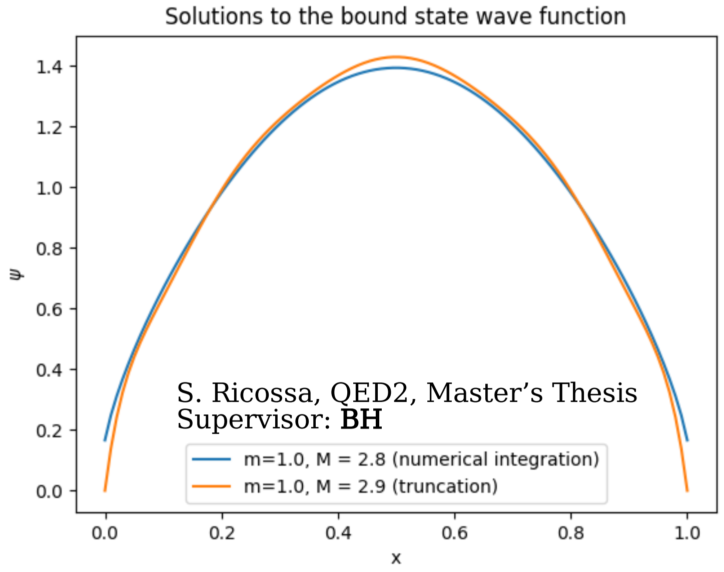
HT works splendidly in d = 1+1

- Exponential improvement over naïve Fock basis
  - # states = p(Δ_{max}) = # partitions of the integer Δ_{max}
- Laptop + Mathematica

2d QCD, N_f = 3 Anand et. al. 2111.00021







d>2: Harder...but worth it

→ Requires "bigger" basis

→ 2 truncation parameters

→ Lots of relevant couplings in d = 2+1

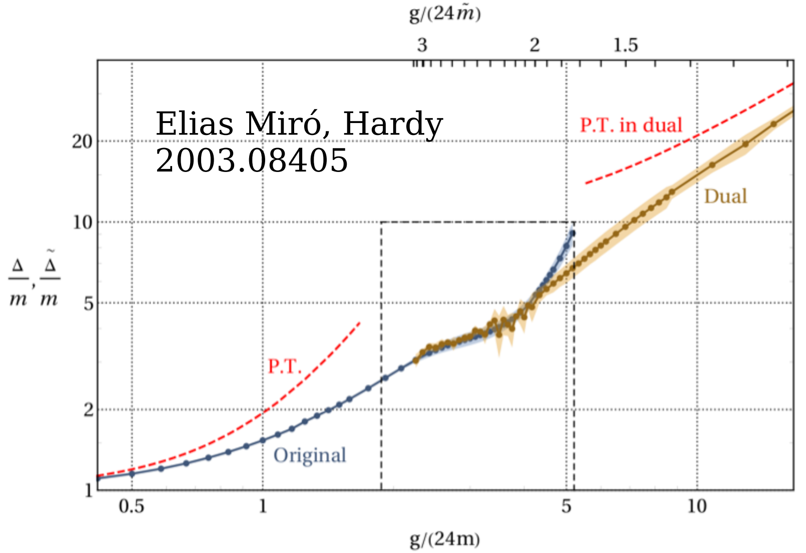
$$\lambda\phi^4 \;\; ; \;\; y\phi\bar{\psi}\psi \;\; ; \;\; \frac{1}{g^2}F^2\,,\, gA_\mu J^\mu$$

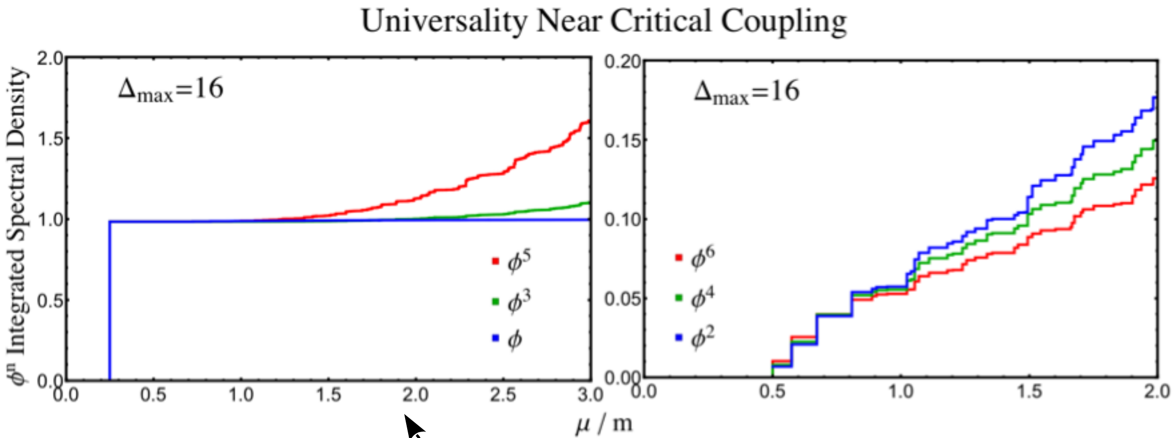
$$[\lambda] = 1 \;\; ; \;\; [y] = {}^1\!/_2 \;\; ; \;\; [g] = {}^1\!/_2$$

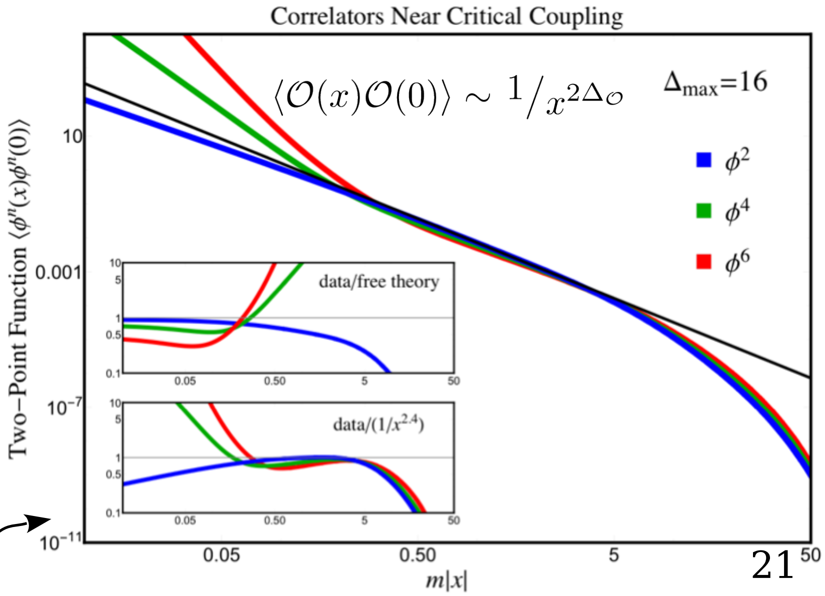
⇒ **lots of strong coupling!**

→ Fewer exact results

⇒ **uncharted territory!**







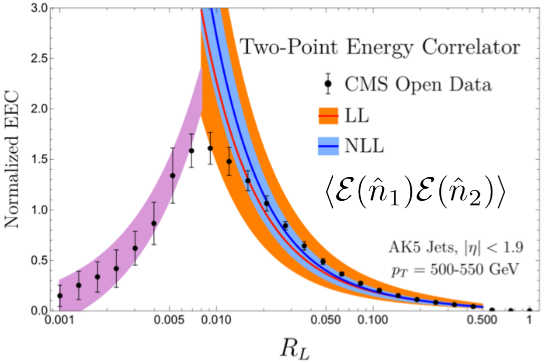
Anand, Katz, Khandker, Walters 2010.09730

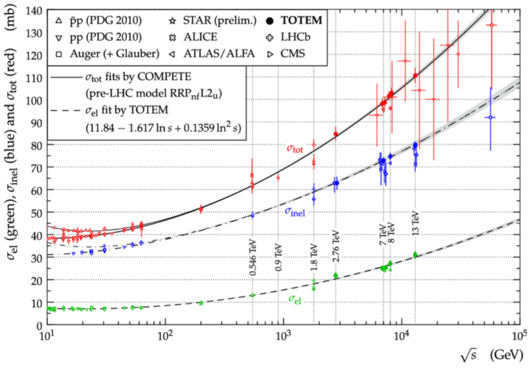
Truncation philosophy

1) Pick an observable

$$1 = \sum_i |\psi_i\rangle\langle\psi_i|$$

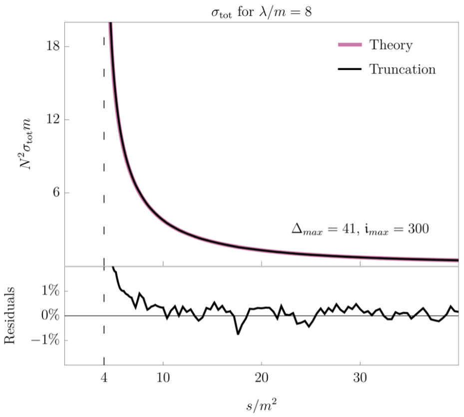
$$\langle T\{\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\mathcal{O}_k(x_k)\}\rangle$$

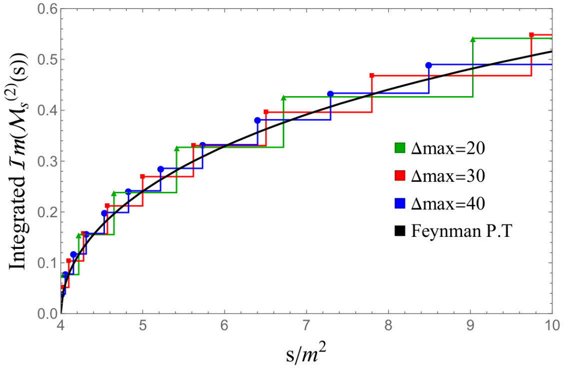




2) Learn to compute with Hamiltonian

3) Apply truncation





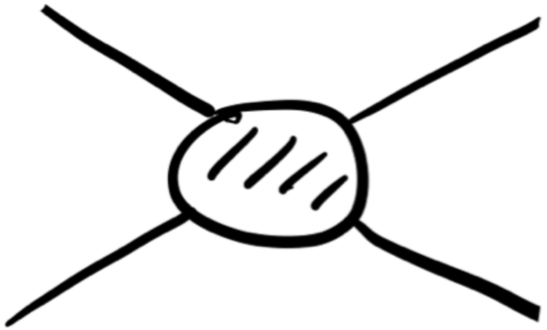
$$* \langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \sum_n \langle 0|\mathcal{O}(x)|n\rangle \langle n|\mathcal{O}(y)|0\rangle$$

$$H|n\rangle = E_n|n\rangle\,,\ \mathcal{O}(x) = e^{iPx}\mathcal{O}(0)e^{-iPx}$$

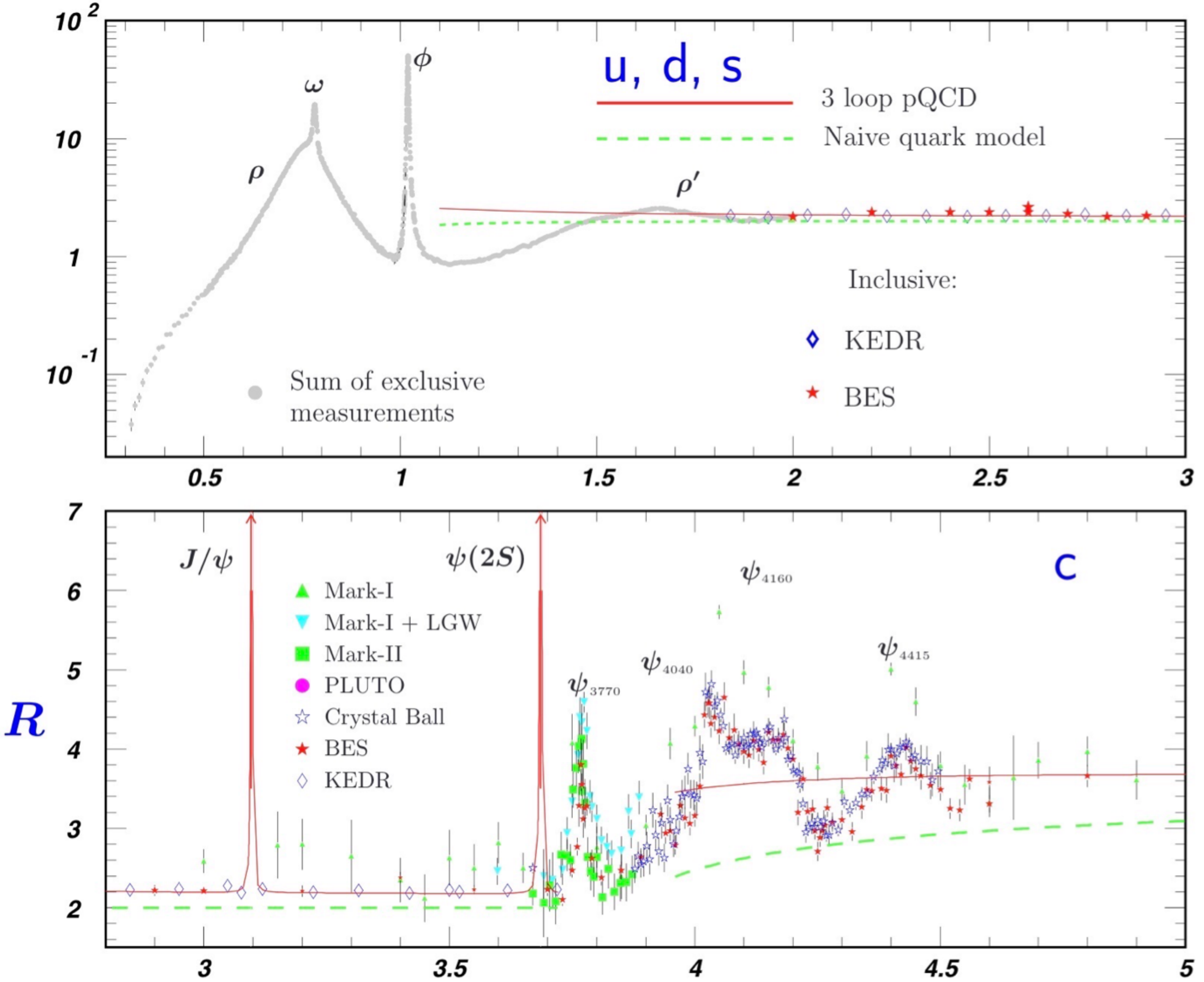
things like

$$\left.\begin{array}{l}\text{SPECTRAL INFO}\\ \text{2-POINT FUNCTIONS}^*\end{array}\right\}\ \text{super cool!}$$

TIME TO GO AFTER THE
FUNDAMENTAL OBSERVABLE
IN RELATIVISTIC FIELD THEORY



The dream



u, d, s
3 loop pQCD
Naive quark model
Inclusive:
KEDR
BES
Sum of exclusive measurements
ω
ϕ
ρ
ρ'
10^2
10
1
10^{-1}
0.5
1
1.5
2
2.5
3
R
c
J/ψ
$\psi(2S)$
ψ_{3770}
ψ_{4040}
ψ_{4160}
ψ_{4415}
Mark-I
Mark-I + LGW
Mark-II
PLUTO
Crystal Ball
BES
KEDR
7
6
5
4
3
2
3
3.5
4
4.5
5

Truncation output:

(approximate) spectrum $\Leftrightarrow \left\{ E_i \, , \, |\psi_i\rangle \right\} , \ \hat{H} \, |\psi_i\rangle = E_i \, |\psi_i\rangle$

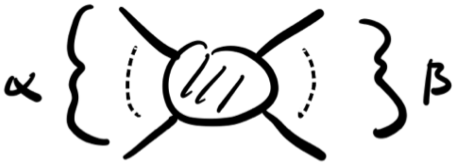
$\Rightarrow$ gives (approximate) resolution of identity: $1 \approx \sum_{i=1}^{N} |\psi_i\rangle \langle\psi_i|$

Fundamental question:

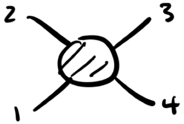
GIVEN THE ENERGY EIGENSTATES,

HOW DO YOU COMPUTE THE S-MATRIX?

How to compute $\mathcal{M}$ from $|\psi\rangle$?



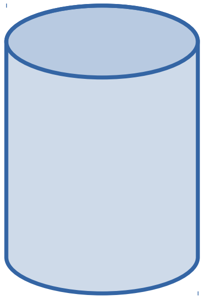
$S_{\beta\alpha} = \langle \psi_\beta^- | \psi_\alpha^+ \rangle$



$= \langle p_3 p_4 ; \text{out} | p_1 p_2 ; \text{in} \rangle$

PROBLEM: How are truncation states related to in/out-states?

Think finite volume e.g. $\mathbb{R} \times S^{d-1}$



DISCRETIZING continuum $\Rightarrow$ IR cutoff = finite "box"

Prevents formal identification

of asymptotic states

$\mathcal{M}$ from $|\psi\rangle$?

Lippmann-Schwinger equation

$$(E_\alpha - H_0)|\psi_\alpha\rangle = V|\psi_\alpha\rangle$$

continuum consequence : H_0 guaranteed to have E_α eigenvalue $\exists\ |\phi_\alpha\rangle$ s.t. $H_0|\phi_\alpha\rangle = E_\alpha|\phi_\alpha\rangle$

$\mathcal{M}$ from $|\psi\rangle$?

Lippmann-Schwinger equation

$$\left(E_\alpha - H_0\right)|\psi_\alpha\rangle = V|\psi_\alpha\rangle$$

continuum consequence : H_0 guaranteed to have E_α eigenvalue $\exists\ |\phi_\alpha\rangle$ s.t. $H_0|\phi_\alpha\rangle = E_\alpha|\phi_\alpha\rangle$

$$|\psi_\alpha^\pm\rangle = |\phi_\alpha\rangle + \frac{1}{E_\alpha - H_0 \pm i\epsilon}|\psi_\alpha^\pm\rangle$$

$\pm i\epsilon$ physically specifies a boundary condition

Truncation: H_0, H finite dim matrices

Discrete spectra for H_0,H generically differ

No need for iϵ!

BH, Murayama, Riva, Thompson, Walters
arXiv:2209.14306

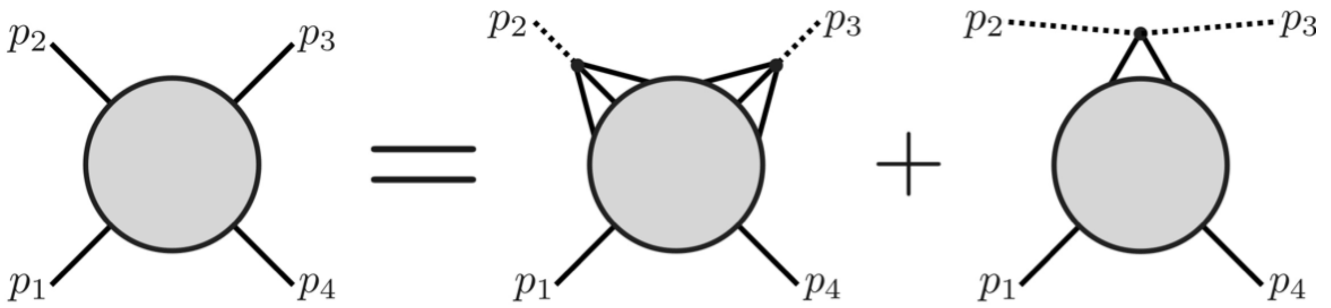
$\mathcal{M}$ from $|\psi\rangle$

scattering amplitude $\xrightarrow{\text{LSZ}}$ **correlation function**

$$\mathcal{M}(p_i) \sim (\Box_1 + m^2)\cdots(\Box_4 + m^2)\langle T\,\phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$$

evaluate by inserting the identity

$$1 \approx \sum_{i=1}^{N} |\psi_i\rangle\langle\psi_i|$$



$\mathcal{M}$ from $|\psi\rangle$

$$\underbrace{\mathcal{M}(p_i) = \langle \mathbf{p}_4\mathbf{p}_3, \text{out}|\mathbf{p}_2\mathbf{p}_1, \text{in}\rangle}_{\text{on-shell, } p_i^2 = m^2} \sim (\Box_2 + m^2)(\Box_3 + m^2)\, \langle \mathbf{p}_4 | T\phi(x_3)\phi(x_2)|\mathbf{p}_1\rangle$$

$$\sim \underbrace{(p_3^2 - m^2)(p_2^2 - m^2)}_{\text{multiplying by zero}}\, \underbrace{\langle \mathbf{p}_4 | T\phi_3\phi_2|\mathbf{p}_1\rangle}_{\text{develops poles which cancel zeroes}}$$

on-shell, $p_i^2 = m^2$ ⇛ multiplying by zero ⇛ develops poles which cancel zeroes

Issue: resolution of identity is approximate

$$1 \approx \sum_{i=1}^{N} |\psi_i\rangle\langle\psi_i|$$

Multiplying exact zero by approximate pole

→ delicate numerical game ⇒ want to avoid!

Exact zeros and approximate poles

Analytically: $\mathbf{0} \times \frac{\mathbf{1}}{\mathbf{0}} = \mathcal{M}$ **Numerically:** $\mathbf{0} \times \frac{\mathbf{1}}{\text{small}} = \mathbf{0}$

Resolution:
USE EOM!

$$\frac{\delta S}{\delta \phi_x} = 0 \quad \Rightarrow \quad \underbrace{\left(\Box_x + m^2\right)}_{D_x \equiv \Box_x + m^2} \phi_x \equiv \underbrace{J_x}$$

Source, e.g.
$\frac{\lambda}{3!}\phi^3$, $(m^2 - m_0^2)\phi$

$$D_3 D_2 \langle p_4 | T \phi_3 \phi_2 | p_1 \rangle = \langle p_4 | T J_3 J_2 | p_1 \rangle - i \langle p_4 | \frac{\partial J}{\partial \phi}(x_2) | p_1 \rangle \, \delta^d(x_3 - x_2)$$

Understanding ingredients

$$D_3 D_2 \langle p_4 | T\, \phi_3 \phi_2 | p_1 \rangle = \langle p_4 | T\, J_3 J_2 | p_1 \rangle - i \langle p_4 | J_2' | p_1 \rangle \, \delta^d(x_3 - x_2)$$

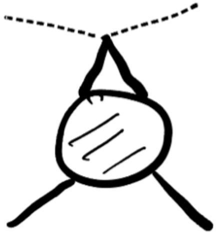
$\langle p_4 | J \, J \, | p_1 \rangle$

$1 = \Sigma |\psi\rangle\langle\psi|$

TRUNCATION CALC MIMICS DISPERSION RELATION

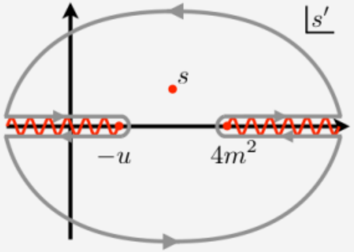
$$\mathcal{M}(s) \sim \int ds' \, \frac{\mathrm{Im}(\mathcal{M}(s'))}{s - s'}$$

$\underline{+ \text{ subtractions}}$



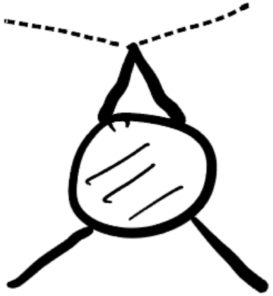
$= \langle p_4 | J' | p_1 \rangle = \lambda \langle p_4 | \phi^2 | p_1 \rangle \sim \overline{\lambda} + \cdots$

Reproduces fixed-u **dispersion relation:**

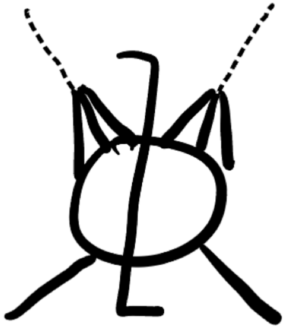


$$\boxed{\mathcal{M}(s,t) = \frac{1}{\pi}\int ds'\,\frac{\mathrm{Im}[\mathcal{M}(s',t')]}{s'-s-i\epsilon} + \frac{1}{\pi}\int dt'\,\frac{\mathrm{Im}[\mathcal{M}(s',t')]}{t'-t-i\epsilon} + \text{subtraction terms}}$$

$$\boxed{\begin{aligned}\mathcal{M}(s,t) = \frac{1}{Z}\Bigg[\sum_\alpha \Bigg(&\frac{\langle \mathbf{p}_4|J(0)|M_\alpha^2;\mathbf{p}_1+\mathbf{p}_2\rangle\langle M_\alpha^2;\mathbf{p}_1+\mathbf{p}_2|J(0)|\mathbf{p}_1\rangle}{M_\alpha^2-s-i\epsilon} \\ &+ \frac{\langle \mathbf{p}_4|J(0)|M_\alpha^2;\mathbf{p}_1-\mathbf{p}_3\rangle\langle M_\alpha^2;\mathbf{p}_1-\mathbf{p}_3|J(0)|\mathbf{p}_1\rangle}{M_\alpha^2-t-i\epsilon}\Bigg) + \langle \mathbf{p}_4|J'(0)|\mathbf{p}_1\rangle\Bigg]\end{aligned}}$$



$$\langle p_4|J'|p_1\rangle$$



$$\langle p_4|\underbrace{J\,J}_{1=\sum|\psi\rangle\langle\psi|}|p_1\rangle$$

Summary of recipe

$$D_3 D_2 \langle p_4 | T\, \phi_3 \phi_2 | p_1 \rangle = \langle p_4 | T\, J_3 J_2 | p_1 \rangle - i \, \langle p_4 | J_2' | p_1 \rangle \, \delta^d (x_3 - x_2)$$

$$\langle p_4 | J J | p_1 \rangle = \sum_{i=1}^{N} \langle p_4 | \, J | \psi_i \rangle \, \langle \psi_i | J \, | p_1 \rangle$$

can easily read
off stable states*
from output

*below continuum

matrix elements
straightforward to compute
using truncation data

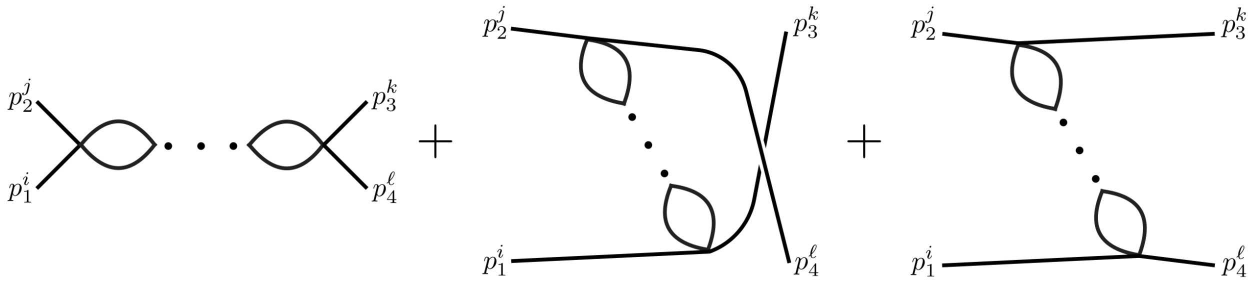
$$\langle p_i | J(x) | \psi \rangle = e^{i(p_i - p_\psi)x} \, \langle p_i | J(0) | \psi_i \rangle$$

Implementing on a strongly coupled theory

$$d = 2+1 : \; O(N) \text{ model}, \; N \to \infty$$

$$V = \tfrac{1}{2} m^2 \vec{\phi}^2 + \tfrac{1}{N} \tfrac{\lambda}{4} \left(\vec{\phi}^2\right)^2; \text{ fixed } \lambda, m^2, \; \frac{\lambda}{m} = \begin{array}{c}\text{dimensionless} \\ \text{parameter}\end{array}$$

At large N: particle changing processes suppressed



$$\mathcal{M}^{ijkl}(s,t,u) = \frac{1}{N}\left(\mathcal{M}(s)\delta^{ij}\delta^{kl} + \mathcal{M}(t)\delta^{ik}\delta^{jl} + \mathcal{M}(u)\delta^{il}\delta^{jk}\right) + O\left(\frac{1}{N^2}\right)$$

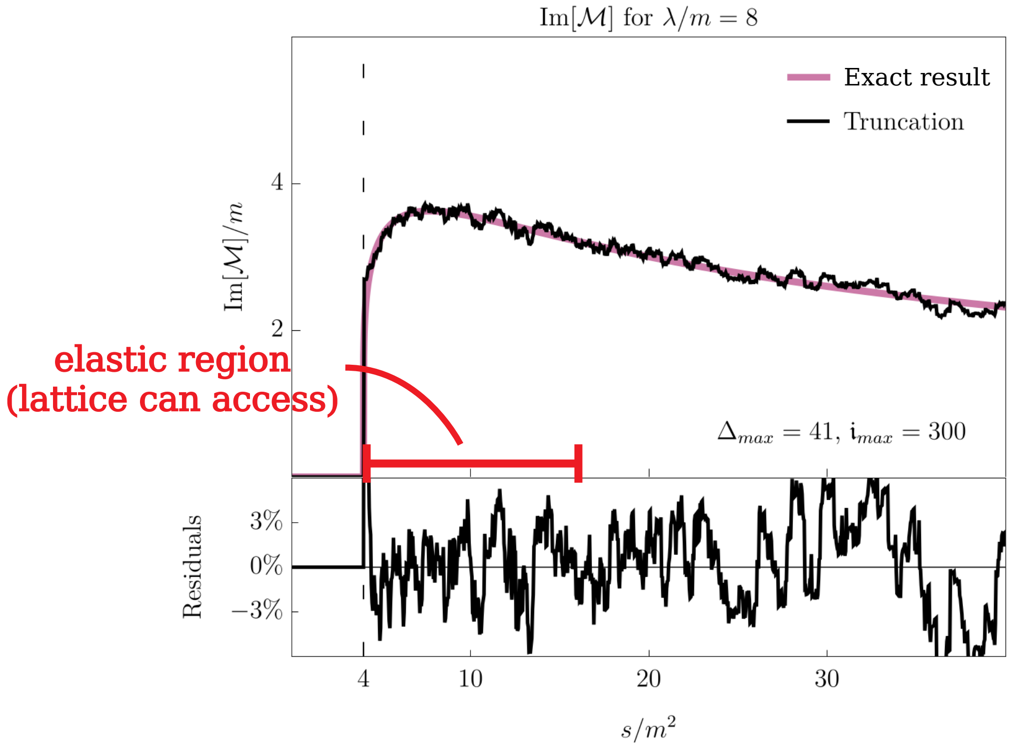
$$\mathcal{M}(s) = -\frac{2\lambda}{1 + \frac{\lambda}{8\pi\sqrt{s}}}\left[\log\left(\frac{\sqrt{s}+2m}{\sqrt{s}-2m}\right) + i\pi\right]$$

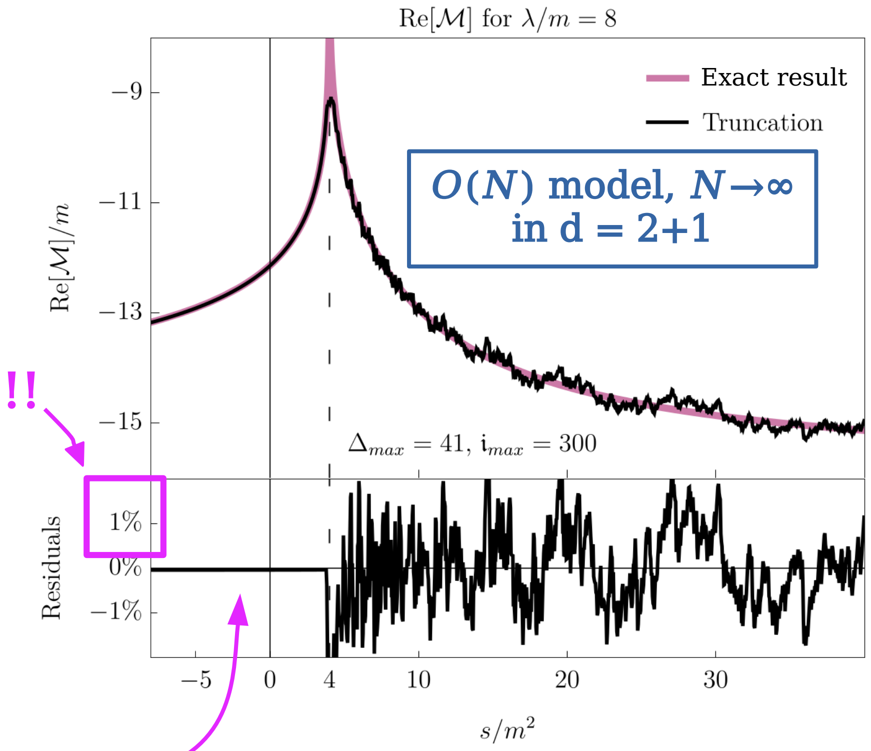
BH, Murayama, Riva, Thompson, Walters
arXiv:2209.14306

results

$O(N)$ model: repulsive interaction
⇒ no bound states

large coupling!



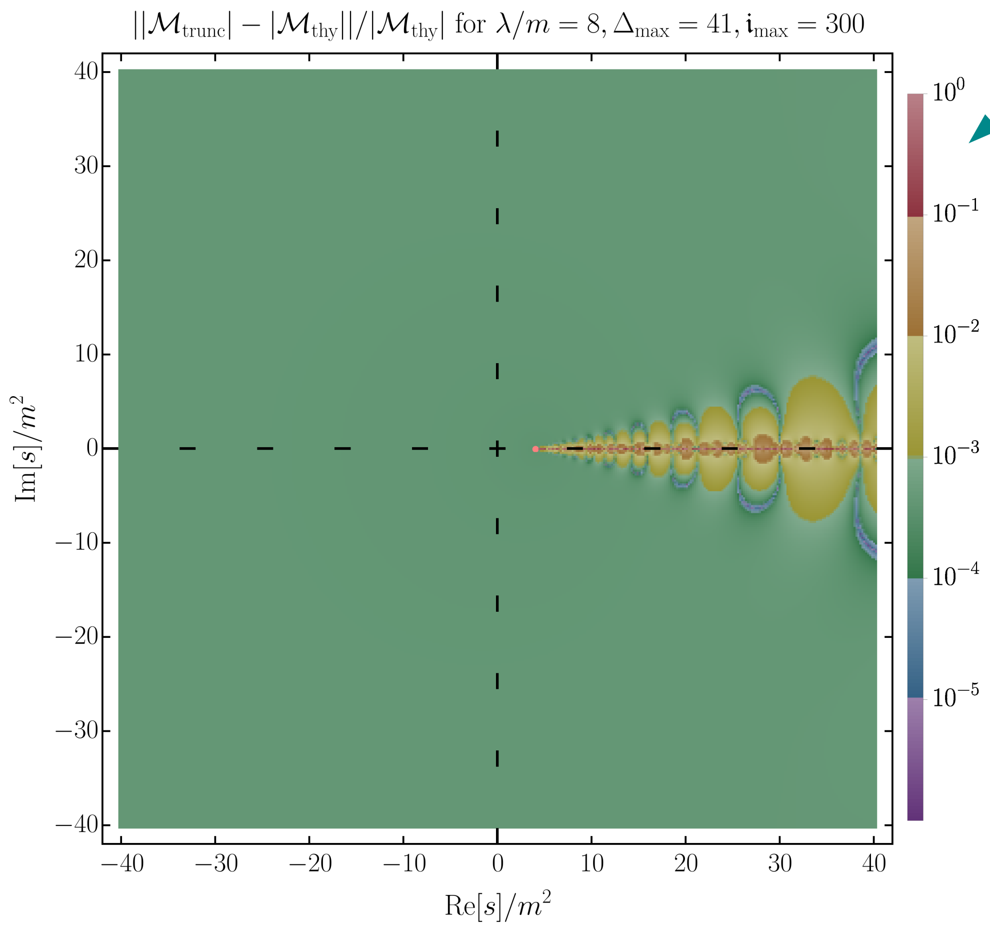


Clear appearance of threshold @ $s = 4m^2$

Best convergence outside physical regime

At high-E, perturbative regime: $\text{Re}(\mathcal{M}) \sim \lambda = \text{const}$ $\quad \text{Im}(\mathcal{M}) \sim \lambda^2 / s$

results

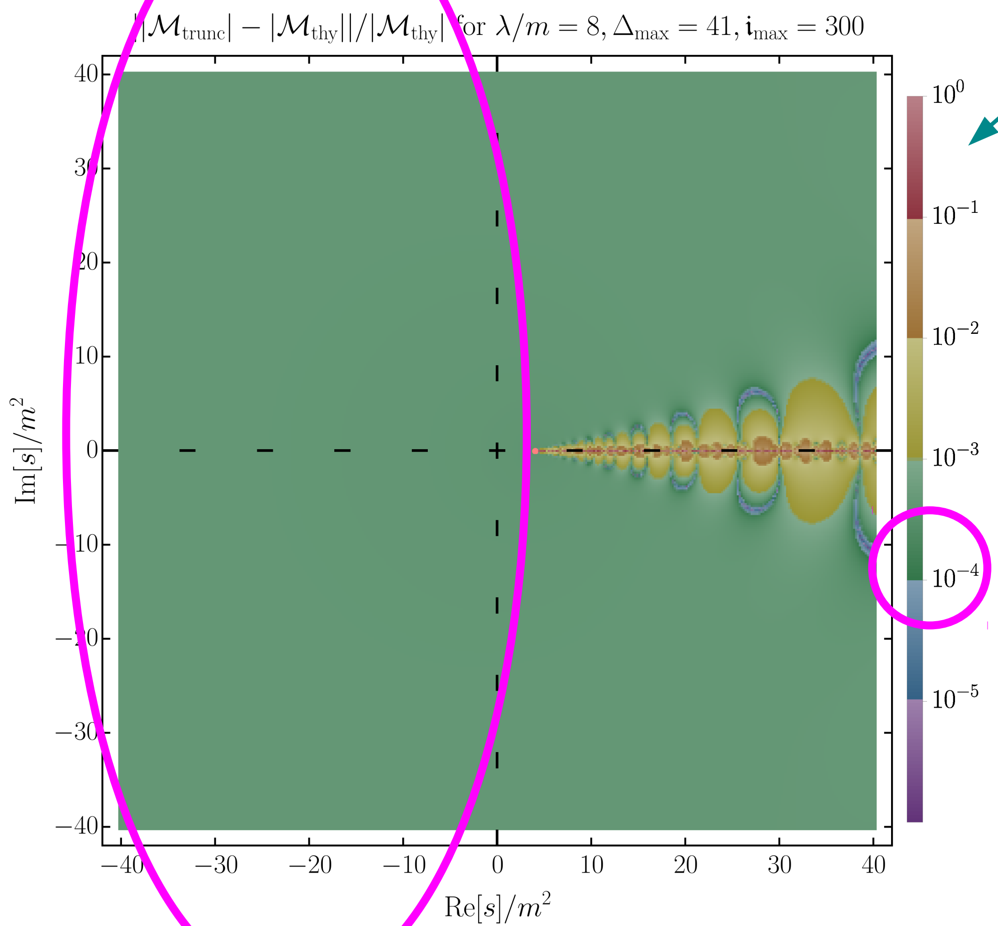


complex s-plane

- Can explore analytic behavior

BH, Murayama, Riva, Thompson, Walters
arXiv:2209.14306

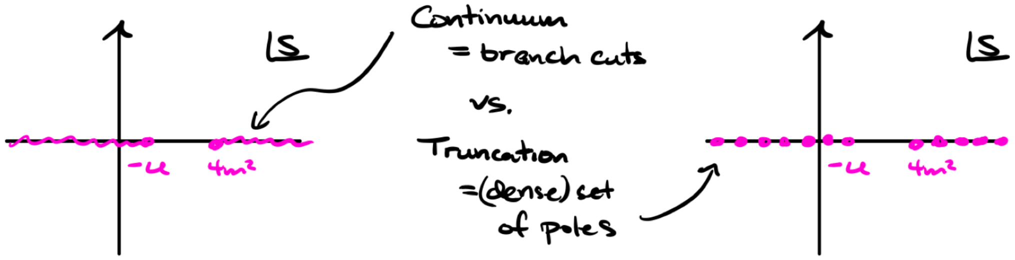
results



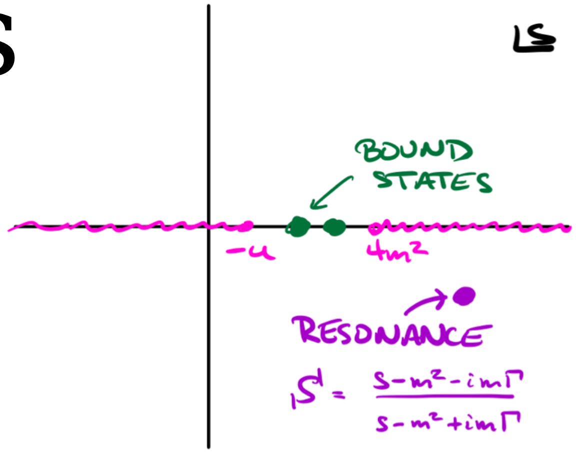
BH, Murayama, Riva, Thompson, Walters
arXiv:2209.14306

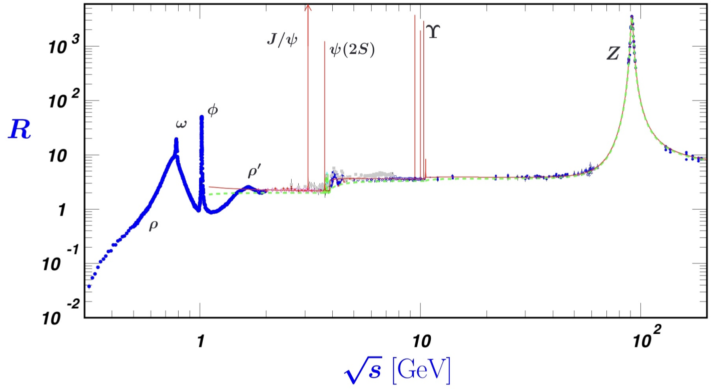
- Can explore analytic behavior
- Rapid convergence throughout complex plane

Scattering goals



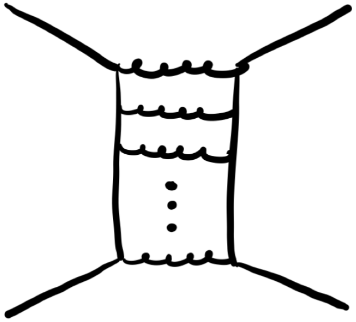
Probe analytic structure

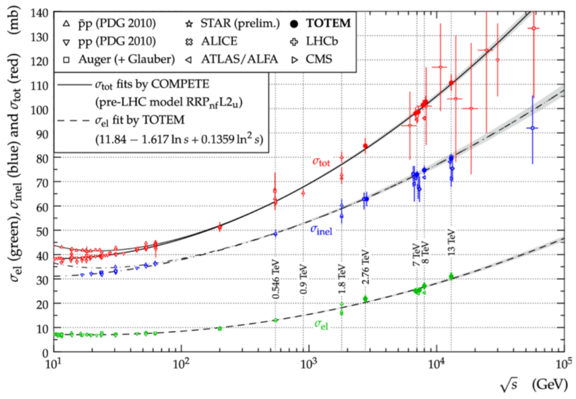




Bound state scattering from first principles

Forward scattering/ Regge physics

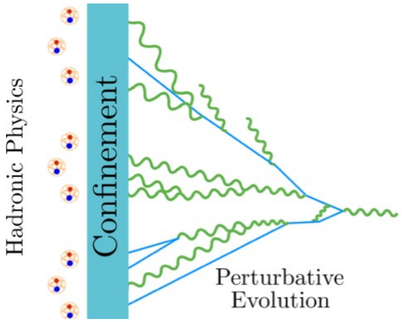


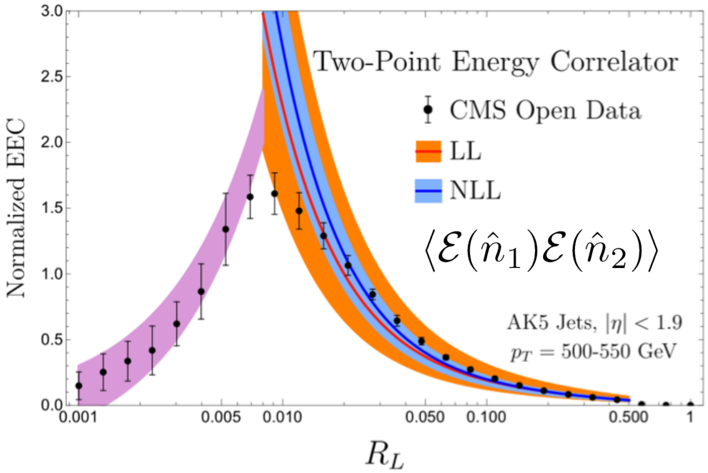


Gauge theories

QCD in d = 3+1

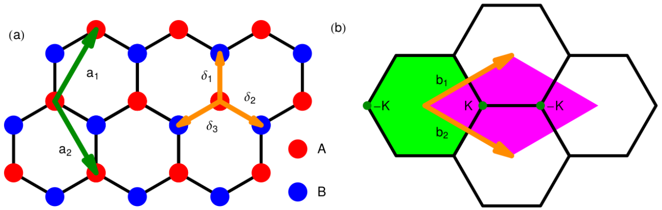
✓ Confining (for small N_f)





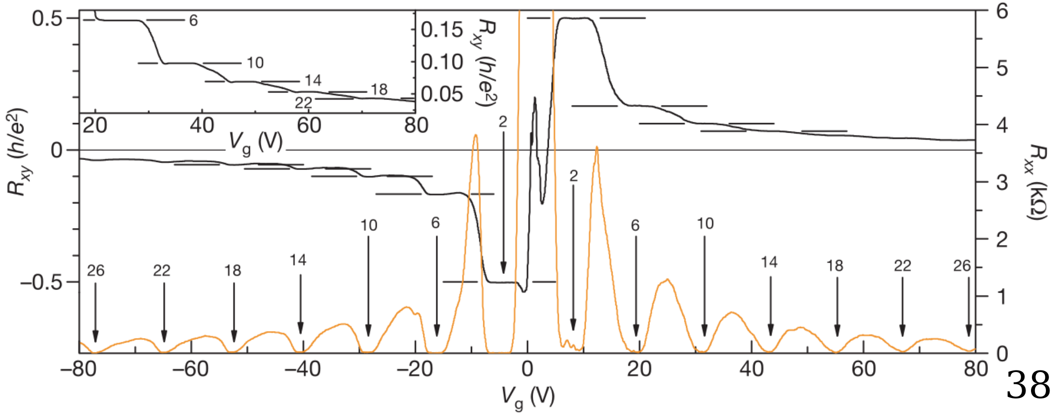
QED in d = 2+1

✓ Confining (for small N_f)



Graphene honeycomb lattice
Unconventional QHE
from: arXiv:0706.3016

QHE in graphene
Zhang et. al., Nature 438, 201-205 (2005)



GAUGE THEORIES

TWO POSSIBLE APPROACHES

$H_0 =$ FREE QUARKS & GLUONS

$$V \sim g A_\mu J^\mu + g A^2 \partial A$$

$H_0 =$ "SOLVABLE" INTERACTING UV FIXED POINT
(e.g. Banks-Zaks, SU(3) $N_f = 16$)

$$V \sim m \bar{q} q$$

QCD

+: FAMILIAR

−: MARGINAL ($d = 3+1$)

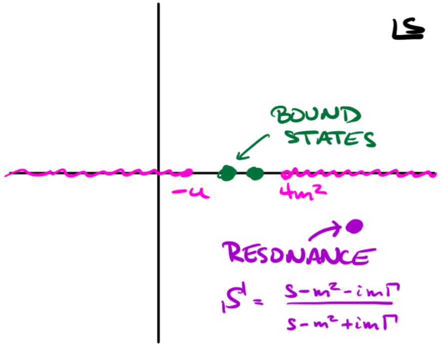
−: NOT GAUGE INVARIANT

+: RELEVANT

+: GAUGE INVARIANT

−: NEED TO "SOLVE" H_0

Future directions: HT



resonances

d = 1+1

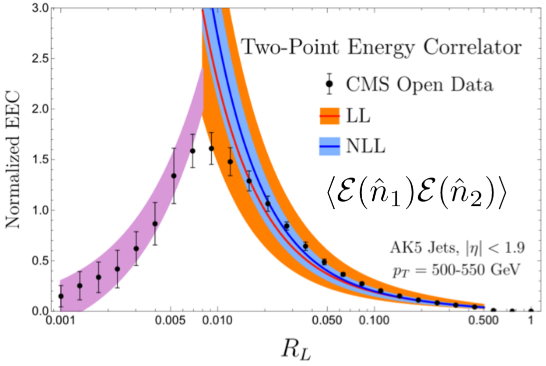
– check against integrable systems

– complement S-matrix bootstrap

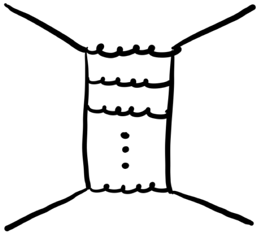
– ongoing with O. Delouche, M. Walters

scattering

energy correlators



$$\mathcal{E}(\hat{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, n^i T_{0i}(t, r\hat{n})$$



forward/Regge physics

– e.g. ϕ^3 or Yukawa in d=2+1

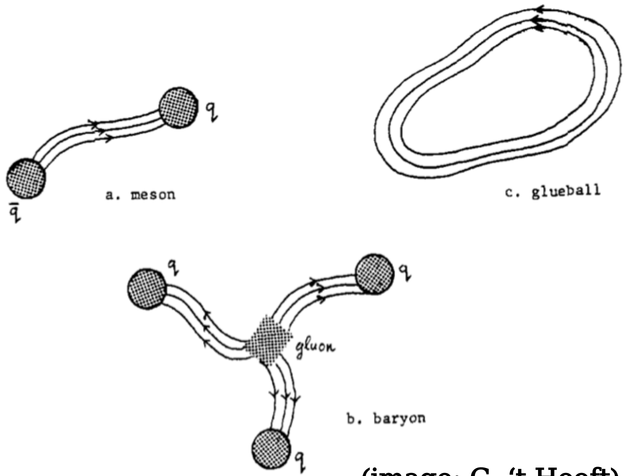
generalize our prescription to QM?

– ongoing with S. Monin, M. Walters

Future directions: HT

Banks-Zaks ⇒ QCD

⇒ Banks-Zaks data (ongoing)
with Karateev, Kosmopoulos, Ricossa, Riembau, Riva, Walters



(image: G. 't Hooft)

gauge theories

QED3

concrete mysteries; tension between methods; relevance to cond-mat

Two approaches:

1) Start from free theory
2) Start from interacting fixed point

– ongoing with J. Thompson, M. Walters, ...

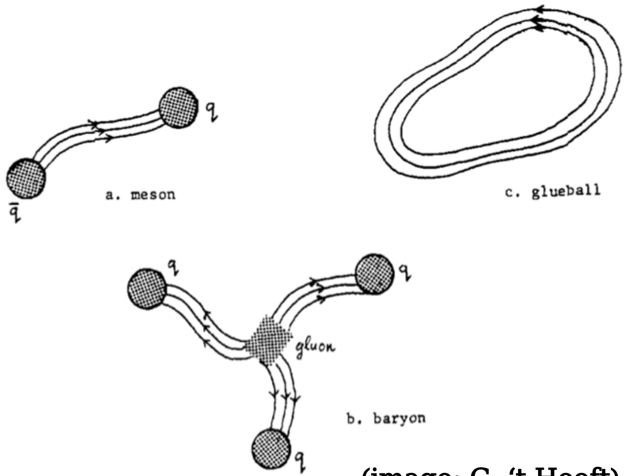
gauge theories in d = 1+1

QED2
→screening vs confinement
– ongoing with K. Farnsworth , S. Ricossa

Future directions: HT

Banks-Zaks ⇒ QCD

⇒ Banks-Zaks data (ongoing)
with Karateev, Kosmopoulos, Ricossa, Riembau, Riva, Walters



(image: G. 't Hooft)

gauge theories

QED3

concrete mysteries; tension between methods; relevance to cond-mat

Two approaches:
1) Start from free theory
2) Start from interacting fixed point

– ongoing with J. Thompson, M. Walters, ...

gauge theories in d = 1+1

QED2
→screening vs confinement
– ongoing with K. Farnsworth , S. Ricossa

PLENTY of projects, ranging from pheno, to formal, to numerical
⇒ something for everyone!

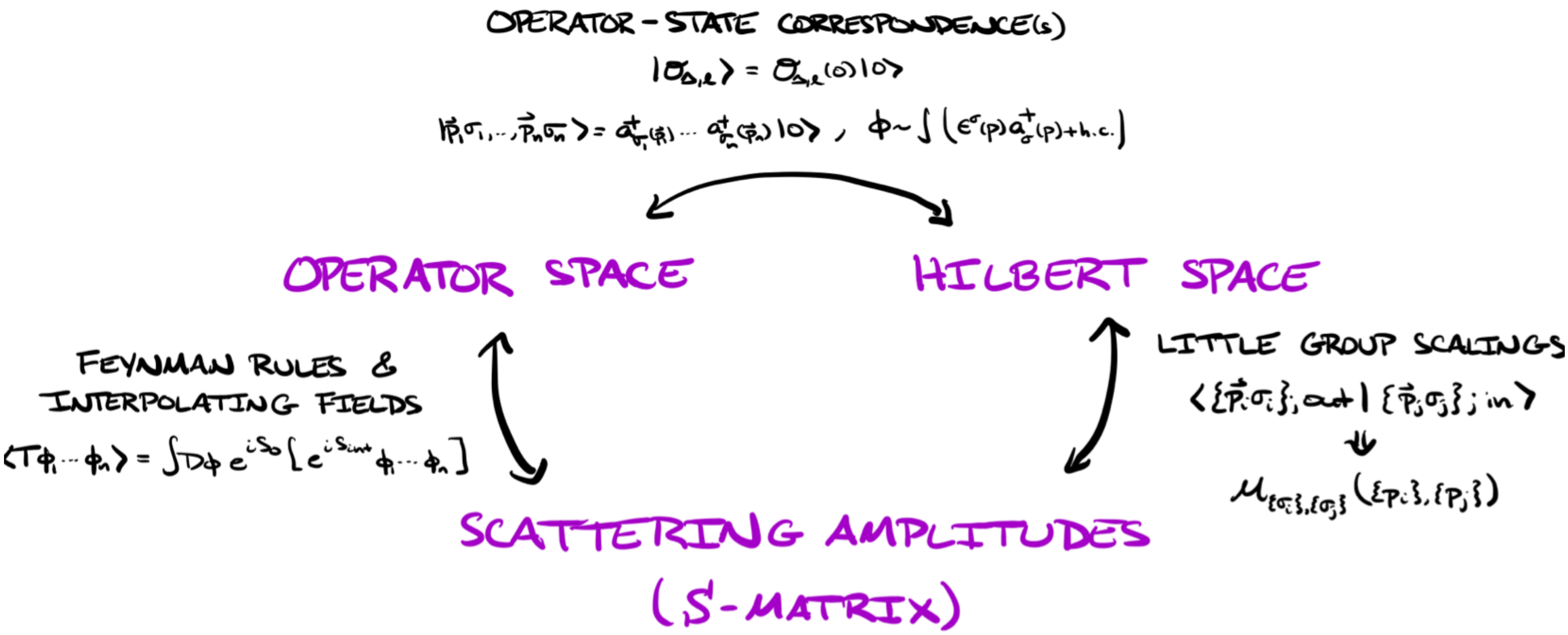
Observation

“becoming a better physicist” and “career advancement” are not always the same path

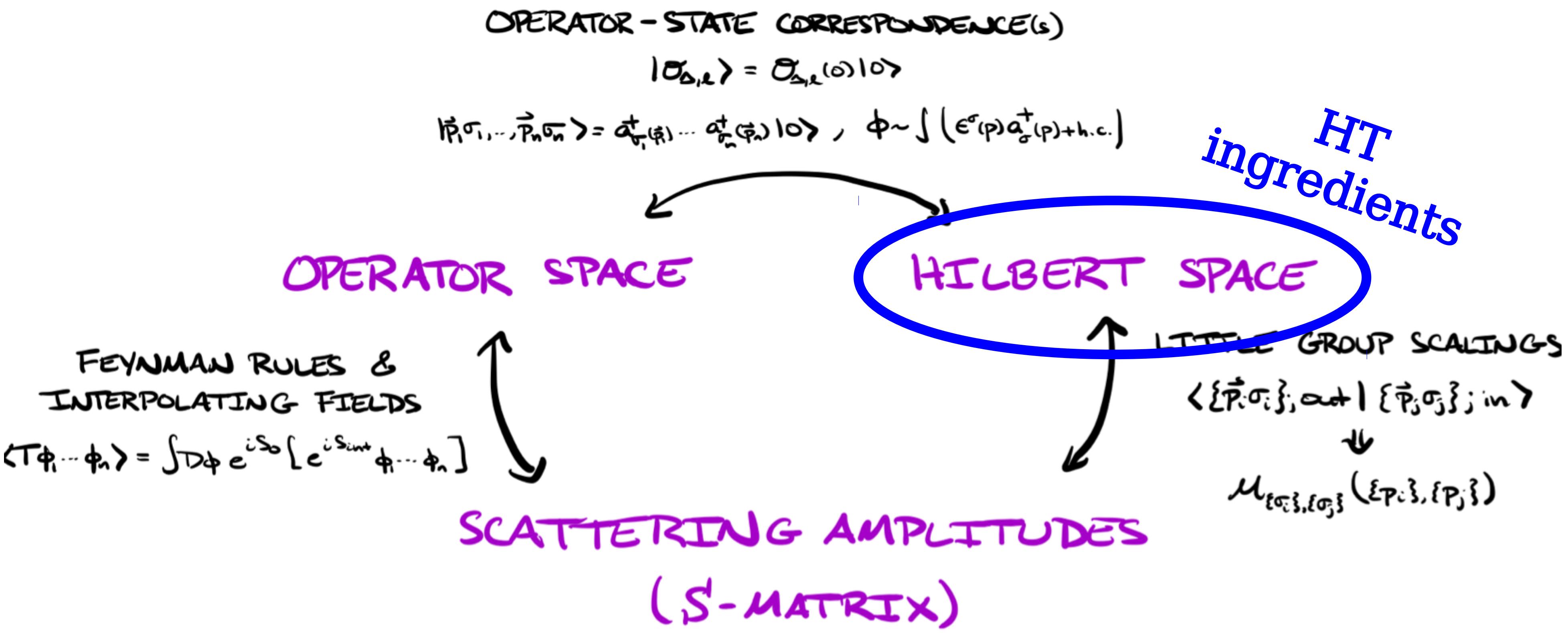
I worry these paths are diverging

I think “how do we work on the longstanding, big questions” plays a role here into (some set of) the weeds

isomorphic problems



isomorphic problems



Reminder: two input ingredients

$$\text{STATES} \;\Rightarrow\; |\psi\rangle\,,\;\; \langle\psi|\psi\rangle < \infty$$

$$\underbrace{\langle\psi|H|\psi'\rangle}_{\text{Born level}} \;\Leftarrow\; \text{MATRIX ELEMENTS}$$

$\Rightarrow$ **Ingredients recyclable for many different theories**

Computational dream

$$H \sim \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ k \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & \cdots & k \end{array} \\ \begin{pmatrix} c_{1H1} & c_{1H2} & \cdots & c_{1Hk} \\ & c_{2H2} & \cdots & c_{2Hk} \\ & & \ddots & \\ & & & c_{kHk} \end{pmatrix} \end{array}$$

$c_{iHj} =$ OPE coeff

$$\langle \mathcal{O}_i | H | \mathcal{O}_j \rangle \simeq \langle \mathcal{O}_i H \mathcal{O}_j \rangle \propto \underbrace{c_{iHj}}$$

Deforming from free theories

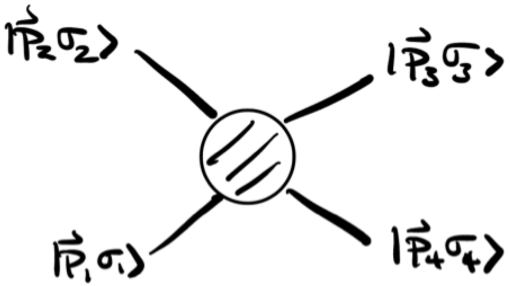
$$H = \underbrace{H_0}_{\text{free = LOTS of structure}} + V$$

efficient ways to determine??

Said another way, can we <u>actually solve</u>

a CFT $\equiv \{\{(\Delta, l)\}, \{c_{ijk}\}\}$??

Building Fock spaces



QFT with S-matrix

$\Leftrightarrow$ $\exists$ scattering states

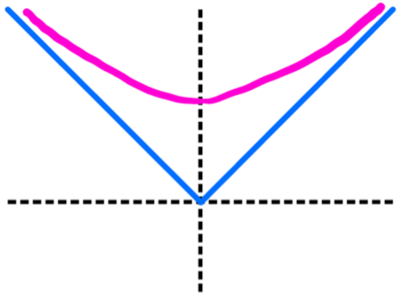
$\Rightarrow$ Fock space at t→−∞ or +∞

$\Rightarrow$ Furnishes unitary rep of ISO(d-1,1)

$\Rightarrow$ Single particle: $\mathcal{H}_1 = \{\, |\vec{p}, \sigma\rangle \,\}$

FOCK: $\mathcal{H} = \bigoplus_n \text{(a)sym}^n(\mathcal{H}_1)$

$\vec{p} \in$ $SO(d-1,1)\big/SO(d-1)$ OR $SO(d-1,1)\big/ISO(d-2)$ $=$



Scalar Fock space

$$\mathcal{H}_1 = \{\, |\mathbf{p}\rangle \,\} \equiv \Pi_1(p) = \begin{array}{c}\text{single particle}\\ \text{phase space}\end{array}$$

$$\langle \mathbf{p}|\mathbf{q}\rangle = \eth(\mathbf{p}-\mathbf{q}) \quad \Leftrightarrow \quad \mathbf{1} = \int \text{đ}p \; |\mathbf{p}\rangle \, \langle \mathbf{p}|$$

Scalar Fock space

$$\mathcal{H}_1 = \{\, |\mathbf{p}\rangle \,\} \equiv \Pi_1(p) = \text{ single particle phase space}$$

$$\langle \mathbf{p}|\mathbf{q}\rangle = \not{\delta}(\mathbf{p}-\mathbf{q}) \quad \Leftrightarrow \quad \mathbf{1} = \int đp\ |\mathbf{p}\rangle\langle\mathbf{p}|$$

Arbitrary state $|\psi\rangle \in \mathcal{H}_1$

$$|\psi\rangle = \int đp\ |\mathbf{p}\rangle\langle\mathbf{p}|\psi\rangle = \int đp\, \psi(\mathbf{p})\, |\mathbf{p}\rangle = \text{“wave packet”}$$

Scalar Fock space

$$\mathcal{H}_1 = \{ \, |\mathbf{p}\rangle \, \} \equiv \Pi_1(p) = \text{ single particle phase space}$$

$$\langle \mathbf{p}|\mathbf{q}\rangle = \delta\!\!\!{}^-(\mathbf{p}-\mathbf{q}) \quad \Leftrightarrow \quad \mathbf{1} = \int đp \; |\mathbf{p}\rangle \langle \mathbf{p}|$$

$$\text{Arbitrary state } \; |\psi\rangle \in \mathcal{H}_1$$

$$|\psi\rangle = \int đp \; |\mathbf{p}\rangle \langle \mathbf{p}|\psi\rangle = \int đp \, \psi(\mathbf{p}) \, |\mathbf{p}\rangle = \text{“wave packet”}$$

$$\langle \psi|\psi\rangle = \int đp \; |\psi(\mathbf{p})|^2$$

$$\Rightarrow \; \boxed{\mathcal{H}_1 = L^2(\Pi_1)}$$

Scalar Fock space

$$\mathcal{H} = \bigoplus_n S^n(\mathcal{H}_1) \equiv \bigoplus_n \Pi_n(\mathbf{p}_1, \ldots, \mathbf{p}_n)$$

$$\begin{aligned}
\int d\Pi_n(\mathbf{p}_i) &= \int đ\mathbf{p}_1 \cdots đ\mathbf{p}_n \\
&= \int d^d P\, \delta^d(P - \sum_i p_i) đ\mathbf{p}_1 \cdots đ\mathbf{p}_n \\
&\equiv \int d^d P\, d\Pi_n^P(\mathbf{p}_i)
\end{aligned}$$

$\Pi_n^P \equiv$ n-particle phase space with total momentum P^μ

COMPACT

Scalar Fock space

Free "Hilbert space"
$=$ Fock space

$$|n(\mathbf{p}_i)\rangle \equiv |\mathbf{p}_1 \cdots \mathbf{p}_n\rangle = a^\dagger_{\mathbf{p}_1} \cdots a^\dagger_{\mathbf{p}_2} |0\rangle$$

$$\mathbf{1} = \sum_{n,p} |n(\mathbf{p}_i)\rangle \langle n(\mathbf{p}_i)|$$

completeness

arbitrary state

$$|\psi\rangle = \sum_{n,p} \psi^{(n)}(\mathbf{p}_i)\, |n\rangle \qquad |\psi^{(n)}(P)\rangle = \int d\Pi_n(\mathbf{p}_i)\delta^d(P - p_1 - \cdots - p_n)\, \psi^{(n)}(\mathbf{p}_i)\, |n\rangle$$

$$\langle \psi'^{(n')}(P')|\psi^{(n)}(P)\rangle = \delta^d(P-P')\delta_{nn'} \underbrace{\int d\Pi_n(\mathbf{p}_i)\, \psi'^{*}(\mathbf{p}_i)\psi(\mathbf{p}_i)\delta^d(P - \sum_i p_i)}$$

Hilbert space $=$square-integrable functions

on phase space: $L^2(\Pi_n^P)$

Massless phase space

BH, T. Melia
1902.06747
1902.06754

- $\Rightarrow$ momentum conservation
- $\Rightarrow$ on-shell
- $\Rightarrow$ Lorentz invariance

constraints define a manifold in phase space

$$\delta(p_1^2)\cdots\delta(p_n^2)\times\delta^4\big(P^\mu-(p_1^\mu+\cdots+p_n^\mu)\big)$$

use spinors

$$\delta^4\Big(P_{\alpha\dot\alpha}-(\lambda^1\widetilde{\lambda}^1+\cdots+\lambda^n\widetilde{\lambda}^n)_{\alpha\dot\alpha}\Big)$$

Massless phase space

BH, T. Melia
1902.06747
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⇒ momentum conservation

⇒ on-shell

⇒ Lorentz invariance

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Want a set of class functions on the manifold

generalized spherical harmonics

Massless phase space

BH, T. Melia
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- $\Rightarrow$ **momentum conservation**
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use spinors

$$\delta^4\Big(P_{\alpha\dot\alpha}-(\lambda^1\widetilde{\lambda}^1+\cdots+\lambda^n\widetilde{\lambda}^n)_{\alpha\dot\alpha}\Big)$$

$$\lambda=\{\lambda_\alpha{}^i\}=\begin{pmatrix}\lambda_1{}^1 & \cdots & \lambda_1{}^N\\ \lambda_2{}^1 & \cdots & \lambda_2{}^N\end{pmatrix}$$

Want a set of class functions on the manifold

$\hookrightarrow$ **generalized spherical harmonics**

$$\lambda\to g\lambda U^T\ \left(\lambda_\alpha{}^i\to g_\alpha{}^\beta U^i{}_j\lambda_\beta{}^j\right)\quad\Rightarrow\quad P=\lambda\lambda^\dagger$$

$$g\in SL(2,\mathbb{C})\,,\ U\in U(N)\supset U(1)^N$$

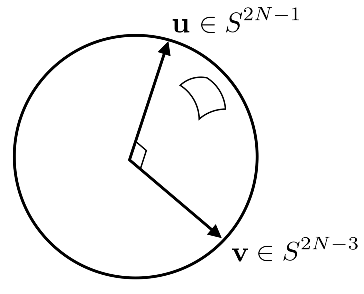
U(N) invariant!

$$\int d\Pi_n^P\ \Rightarrow\ \int d\lambda d\lambda^\dagger\delta(P-\lambda\lambda^\dagger)$$

BH, T. Melia
1902.06747
1902.06754

geometry of phase space

$$\delta^4(P - \lambda\lambda^\dagger) \xrightarrow{\text{c.o.m.}} P_{\alpha\dot\alpha} = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix} = \begin{pmatrix} \left|\vec\lambda_1\right|^2 & \vec\lambda_1 \cdot \vec\lambda_2^* \\ \vec\lambda_2 \cdot \vec\lambda_1^* & \left|\vec\lambda_2\right|^2 \end{pmatrix}$$



$\Longleftrightarrow$

$$\vec v^2 = r^2$$
$$\vec u^2 = r^2$$
$$\vec v \cdot \vec u = 0$$

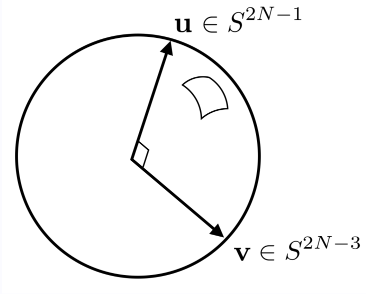
$\Longleftrightarrow$

geometry basically complex version of two orthogonal spheres

BH, T. Melia
1902.06747
1902.06754

geometry of phase space

$$\delta^4\left(P - \lambda\lambda^\dagger\right) \xrightarrow{\text{c.o.m.}} P_{\alpha\dot\alpha} = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix} = \begin{pmatrix} \left|\vec\lambda_1\right|^2 & \vec\lambda_1\cdot\vec\lambda_2^* \\ \vec\lambda_2\cdot\vec\lambda_1^* & \left|\vec\lambda_2\right|^2 \end{pmatrix}$$



$$\Longleftrightarrow \quad \begin{aligned} \vec v^2 &= r^2 \\ \vec u^2 &= r^2 \\ \vec v\cdot\vec u &= 0 \end{aligned} \quad \Longleftrightarrow$$

geometry basically complex version of two orthogonal spheres

$$G/H = U(N)\big/U(N-2) \qquad \text{“Stiefel manifold”}\ V_2(\mathbb{C}^N)$$

Grassmannian ⊂ Stiefel $\quad G_2(\mathbb{C}^N) = U(N)\big/U(N-2)\times U(2)$

$\boxed{\textbf{states} \Leftrightarrow \textbf{harmonics on phase space}}$

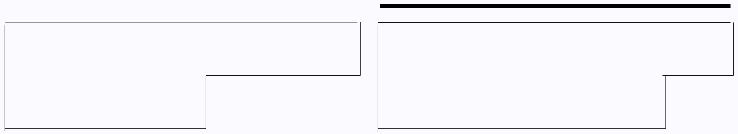
“conformal – helicity duality”

$$4d:\ SU(2,2)\times U(N)$$
$$3d:\ Sp(4,\mathbb{R})\times O(N)$$
$$2d:\ SL(2,\mathbb{R})\times O(N)$$

(math world: reductive dual pairs/Howe duality/oscillator representation)

upshot on Stiefel harmonics

harmonics labeled by Young diagrams
(with at most two rows)



these dictate specific polynomials in the spinors

comments:

1) each shape corresponds to operators

2) multiple operators belong to same shape

  a) these involve particles with different spin

3) these operators are conformal primaries

Construct states algebraically

e.g.

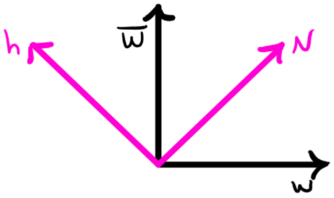
$$|l, \mu = (\mu_1, \ldots, \mu_3)\rangle \simeq F^3$$

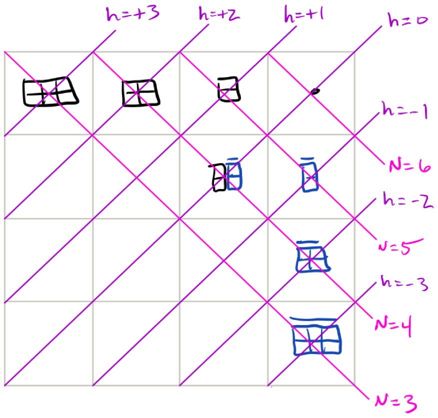
now apply U(N) lowering op:

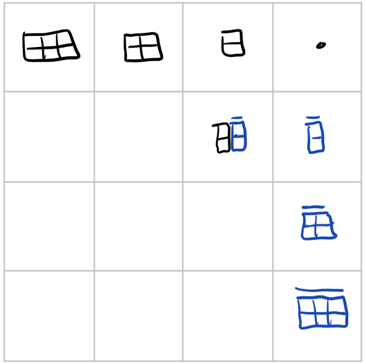
$$L_- |l, \mu\rangle \sim |l, \mu'\rangle \simeq \widetilde{\psi} F \psi$$

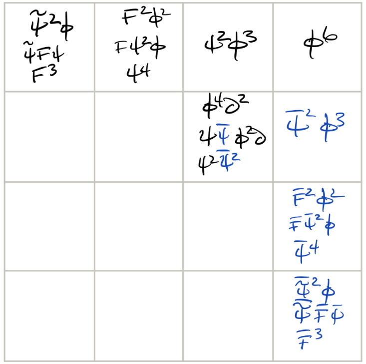
Phase space harmonics

The families of operators belong to the same Grassmann harmonic!









Explains structure of EFT non-renormalization/helicity selection rules

Cheung & Shen 1505.01844
Azatov, Contino, Machado, Riva 1607.05236
Further extensions in recent years...

Method used to construct dim-8 ops in SMEFT

Li, Shu, Xiao, Yu 2005.00008, 2012.11615
Dong, Ma, Shu, Zheng 2202.08350

2- and 3-pt functions

Treating λ as a Fock operator, with deformed commutation relations $[\lambda_a, \lambda_b^\dagger] = z_{ab}$, gives a very efficient computation of 2-pt functions

2- and 3-pt functions

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4 Correlation functions

4.1 Higher spin currents two-point fxns: $\langle J_l J_{l'} \rangle$

Focus on the currents for a single scalar field, so $l \in 2\mathbb{Z}$:

$$J^\phi_{2l} = N_{2l}\frac{1}{2}\left[(\hat\lambda + i\hat\eta)^{2l} + (\hat\lambda - i\hat\eta)^{2l}\right] \tag{31}$$

with N_{2l} a normalization factor. Binomial expanding the terms we arrive at

$$J_{2l} = N_{2l}\sum_{k=0}^{l}\binom{2l}{2k}(-1)^k\hat\lambda^{2(l-k)}\hat\eta^{2k}. \tag{32}$$

The two point function is

$$\langle J_{2l}(x)J_{2l'}(y)\rangle = 2!N_lN_{l'}\int d^2\widetilde\lambda d^2\widetilde\eta\, e^{-\frac{i}{2}(\lambda^2+\eta^2)z}\sum_{k=0}^{l}\sum_{k'=0}^{l'}(-1)^{k+k'}\binom{2l}{2k}\binom{2l'}{2k'}\hat\lambda^{2(l-k)}\hat\eta^{2k}\,\hat\lambda^{2(l'-k')}\hat\eta^{2k'} \tag{33}$$

Let's evaluate this using the Fock space method, instead of evaluating all the gaussian integrals. Here, the basic ingredient is $[\lambda_a, \lambda_b^\dagger] = \bar z_{ab}$ (deforming the canonical commutation relations from $[\lambda_a, \lambda_b^\dagger] = \delta_{ab}$).

$$\begin{aligned}
\langle\lambda^m|\lambda^n\rangle &= \langle 0|\lambda_{a_1}\cdots\lambda_{a_m}\lambda^\dagger_{b_1}\cdots\lambda^\dagger_{b_n}|0\rangle && (34)\\
&= \delta_{mn}\,\langle 0|\lambda_{a_1}\cdots\lambda_{a_n}\lambda^\dagger_{b_1}\cdots\lambda^\dagger_{b_n}|0\rangle && (35)\\
&= \delta_{mn}\,\langle 0|\lambda_{a_2}\cdots\lambda_{a_n}\left[\left([a_1,b_1]\lambda^\dagger_{b_2}\cdots\lambda^\dagger_{b_n}\right) + \left(\lambda^\dagger_{b_1}[a_1,b_2]\lambda^\dagger_{b_3}\cdots\lambda^\dagger_{b_n}\right) + \cdots\right]|0\rangle && (36)\\
&\;\vdots\\
&= \delta_{mn}\sum_{\text{Wick}}[\lambda_{a_1},\lambda^\dagger_{b_1}]\cdots[\lambda_{a_n},\lambda^\dagger_{b_n}]\,\langle 0|0\rangle && (37)\\
&= \delta_{mn}\,n!\,\bar z_{a_1}^{(b_1}\cdots\bar z_{a_n}^{b_n)}\,\langle 0|0\rangle && (38)
\end{aligned}$$

Now

$$\begin{aligned}
\langle\hat\lambda^{2(l-k)}\hat\eta^{2k}|\hat\lambda^{2(l'-k')}\hat\eta^{2k'}\rangle &= \delta_{l-k,l'-k'}\delta_{kk'}(2k)!(2(l-k))!\\
&\quad\times\left[\bar z_{a_1}^{(b_1}\cdots\bar z_{a_{2k}}^{b_{2k})}\cdot\bar z_{a_{2k+1}}^{(b_{2k+1}}\cdots\bar z_{a_{2l}}^{b_{2l})}\right]^{\text{sym } b_j}_{\text{sym } a_i}\langle 0|0\rangle_\lambda\,\langle 0|0\rangle_\eta && (39)\\
&= \delta_{ll'}\delta_{kk'}\frac{(2k)!(2(l-k))!}{(2l)!}(2l)!\,\bar z_{a_1}^{(b_1}\cdots\bar z_{a_{2l}}^{b_{2l})}\,\langle 0|0\rangle^2 && (40)
\end{aligned}$$

whence

$$\langle\mathrm{Re}[(\hat\lambda + i\hat\eta)^{2l}]|\mathrm{Re}[(\hat\lambda + i\hat\eta)^{2l'}]\rangle = \sum_{k=0}^{l}\sum_{k'=0}^{l'}(-1)^{k+k'}\binom{2l}{2k}\binom{2l'}{2k'}\cdot\frac{\delta_{ll'}\delta_{kk'}}{\binom{2l}{2k}}(2l)!\,\bar z_{a_1}^{(b_1}\cdots\bar z_{a_{2l}}^{b_{2l})}\,\langle 0|0\rangle^2 \tag{41}$$

$$= \delta_{ll'}\left[\sum_{k=0}^{l}\binom{2l}{2k}\right](2l)!\,\bar z_{a_1}^{(b_1}\cdots\bar z_{a_{2l}}^{b_{2l})}\,\langle 0|0\rangle^2 \tag{42}$$

so that we arrive at

$$\boxed{\langle\,\mathrm{Re}[(\hat\lambda + i\hat\eta)^{2l}]\,|\,\mathrm{Re}[(\hat\lambda + i\hat\eta)^{2l'}]\,\rangle = \delta_{ll'}\,2^{2l-1}\,(2l)!\,\bar z_{a_1}^{(b_1}\cdots\bar z_{a_{2l}}^{b_{2l})}\,\langle 0|0\rangle_\lambda\,\langle 0|0\rangle_\eta} \tag{43}$$

Taking $\langle 0|0\rangle = 1/(4\pi\sqrt{-z^2})$ we arrive at

$$\boxed{\langle J_{a_1\cdots a_{2l}}(x)\,J_{b_1\cdots b_{2l'}}(y)\rangle = 2!\,\delta_{ll'}\,N_{2l}^2\,2^{2l-1}\,(-i)^{2l}\left(\frac{1}{4\pi\sqrt{-z^2}}\right)^2(2l)!\,\bar z_{a_1}^{(b_1}\cdots\bar z_{a_{2l}}^{b_{2l})}} \tag{44}$$

2- and 3-pt functions

Treating λ as a Fock operator, with deformed commutation relations $[\lambda_a, \lambda_b^\dagger] = z_{ab}$, gives a very efficient computation of 2-pt functions

Can such a method be extended to 3-pt functions/matrix elements???

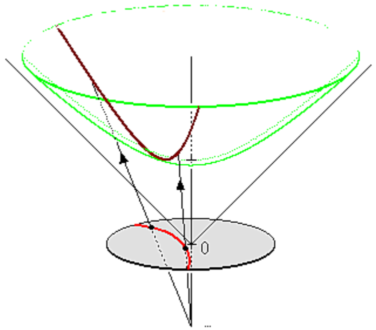
$$\langle \mathcal{O}_f(x_1)\mathcal{O}_H(x_2)\mathcal{O}_i(x_3)\rangle \propto c_{fHi}$$

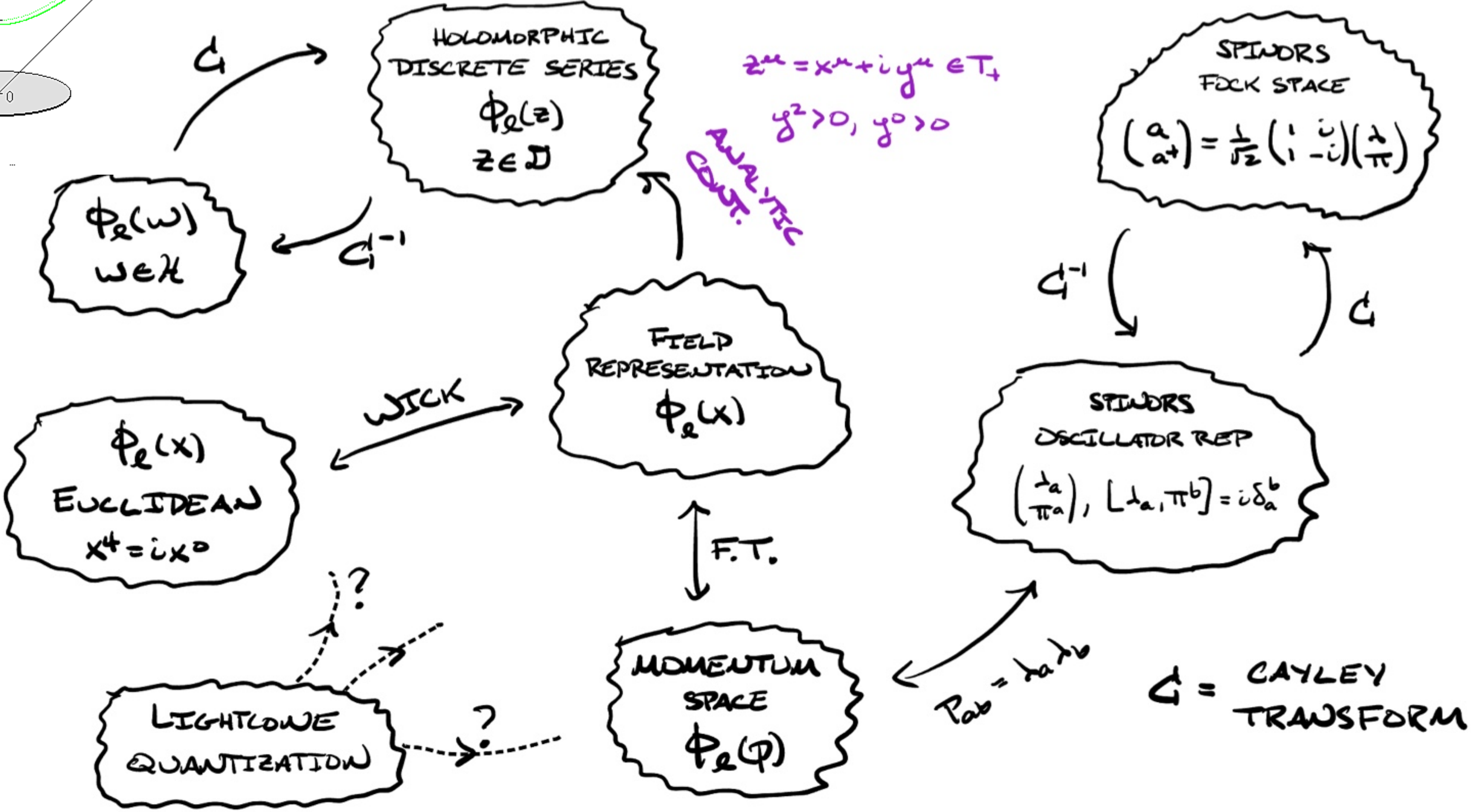
$$\Rightarrow \quad \langle l_f \mu_f | H | l_i \mu_i \rangle \propto c_{\mu_f H \mu_i}$$

$$\langle l_f \mu'_f | H | l_i \mu'_i \rangle = \langle l_f \mu_f | L_+ H L_- | l_i \mu_i \rangle \propto c_{\mu'_f H \mu'_i}$$

Related by group theory??

A variety or realizations



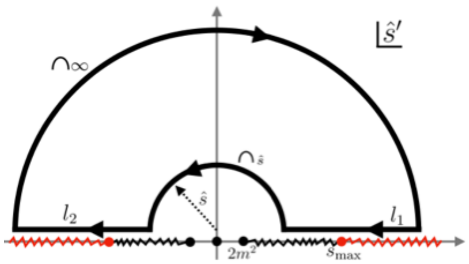


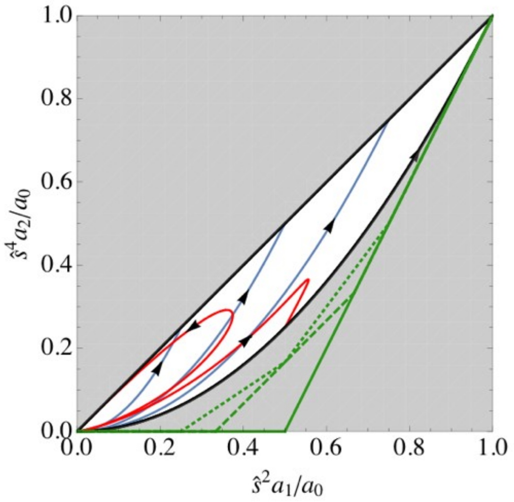
Is there a "best" picture?

Other applications: EFT

operators/EFT amplitudes

phase space (Grassmannian) harmonics and EFT positivity





generalize to massive particles (hard, but useful!)

$$\underbrace{\delta(p_1^2 - m_1^2)\cdots\delta(p_k^2 - m_k^2)\,\delta^4(P^\mu - \sum_i p_i^\mu)}$$

Massive phase space manifold: Is there a “nice” geometric formulation?

A bunch of other questions: identical particles (symmeterization); non-renormalization thms; efficient construction algorithms; amplitudes in d = 2+1; ...

Observation

we have significant representation and environment issues (to put it mildly, IMO)

physics, and theoretical physics in particular, do not have a good reputation

what does this mean for our future?

THANK YOU!