A New Spin on Long-Range Interactions

Continuous Spin Particles and Predictions for their Interactions

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based on 2303.04816 (JHEP) with P. Schuster, **K. Zhou** and 2308.XXXXX with Schuster
(see also 1302.1198, 1302.1577, 1404.0675 with Schuster)

Outline

- Everything you learned about massless particles' spin in QFT is a special case. Helicities mix under Lorentz – controlled by ***spin scale*** ρ [Wigner 1939]
- **Coupling to matter particles is a predictive and (so far) well-behaved IR deformation of familiar theories** – we've had hints for a while [1302.1577], now have exact scheme to calculate both classical physics and amplitudes in putative theory
  - Punchline: Heuristic and example results
  - Superspace-like formalism for gauge theories of any massless particle [1404.0675]
  - Coupling matter particles to fields with nonzero ρ, aka "Continuous spin fields"
- (A few of the) open questions and future directions

Massless Spin, Covariantly

Spin state — Internal charges

Physical states take the form $|\,p^\mu, \sigma, n\rangle$

Spin σ characterizes state's transformation under **little group**.

Little group generators correspond with 3 components of $W^\mu = -\frac{1}{2}\epsilon^{\mu\nu\rho\sigma}J_{\nu\rho}p_\sigma$

For massive particle at rest, $W^\mu = (0, m\mathbf{J})$. (Spatial) components generate SO(3).

Natural relativistic invariant is $W^2 = -m^2\mathbf{J}^2 = -m^2 s(s+1)$

For massless particle:

$W^0, \mathbf{W}\cdot\hat{\mathbf{p}}$ are proportional to familiar helicity generator $R = \mathbf{J}\cdot\hat{\mathbf{p}}$. Transverse spatial components (W_x, W_y for $\mathbf{p} \propto \hat{\mathbf{z}}$) are less familiar:

$W_x \propto J_x + K_y$ and $W_y \propto -J_y + K_x$ generate transverse boost **and** rotation

These generators commute – they are the "translations" of ISO(2) little group

The natural relativistic invariant is $W^2 = -(W_x^2 + W_y^2)$ – independent of helicity R!

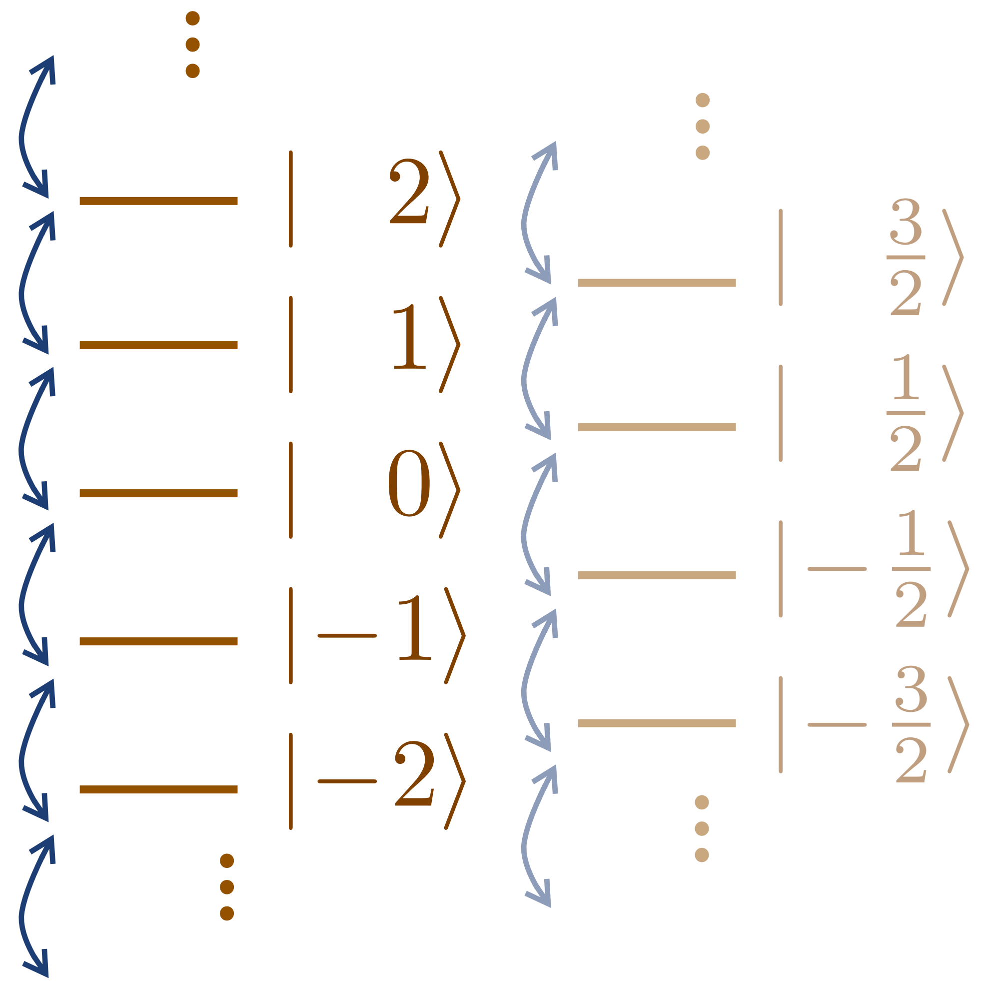
Massless Spin, Covariantly

It's convenient to work in a helicity eigenstate basis: $\mathbf{J}\cdot\hat{\mathbf{p}}\,|p,\sigma\rangle = \sigma\,|p,\sigma\rangle$,

Eigenvalues σ must be (half-)integer so that 4π rotation returns state to itself, since Lorentz group is doubly connected.

Build raising/lowering operators from "translations": $W_\pm = W_x \pm iW_y$, with $[R, W_\pm] = \pm W_\pm$
$W_\pm\,|p,\sigma\rangle = \rho\,|p,\sigma\pm 1\rangle$ where the invariant $W^2 = -\rho^2$ sets the **spin-scale** ρ.

σ-independent coefficient $\Rightarrow$ ***infinite*** ladder of states in one representation.



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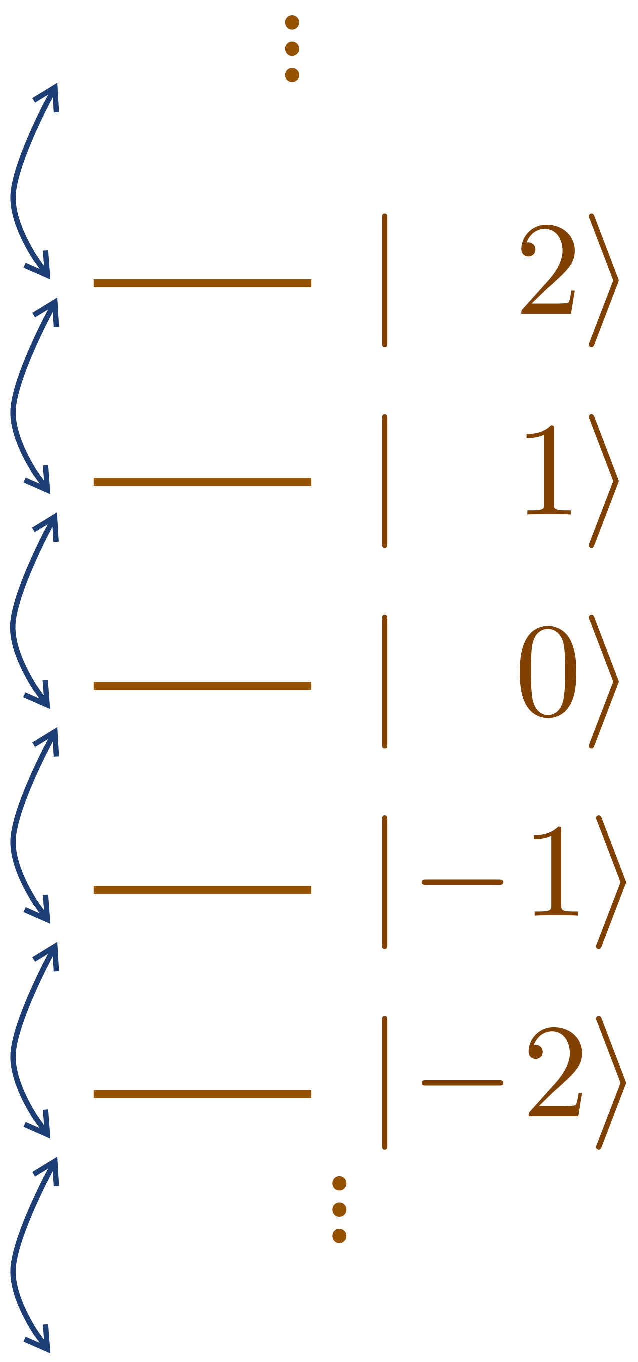


Exception: if $\rho = 0$ the states decouple. Each $|\,\sigma\rangle$ is a singlet representation, related only to $|-\sigma\rangle$ by CPT. **All known massless theories fall under this exception.**

The general case $\rho \neq 0$, where integer helicities mix under Lorentz – just as they do for massive particles – is known as "continuous" or "infinite" spin. (Every name is misleading. I invite you to find a better one)

Why has this possibility been ignored?

So far, quick counter-arguments that sound bad but don't survive close scrutiny.

Continuous spin includes high helicity states. Massless high spin is sick. Aren't these?

Robust constraints on high helicities (e.g. Weinberg-Witten, Weinberg soft theorems) all rely deeply on boost-invariance of helicity, so don't apply when $\rho \neq 0$.

Massive high spin is a somewhat better analogy, and can be consistent – *e.g.* nuclei and string theory

Incompatible with field theory?

Early analyses didn't allow for gauge redundancy! No problem now (at least in free theory)

Are infinitely many states at fixed energy a problem? (Cross-sections, thermodynamics)

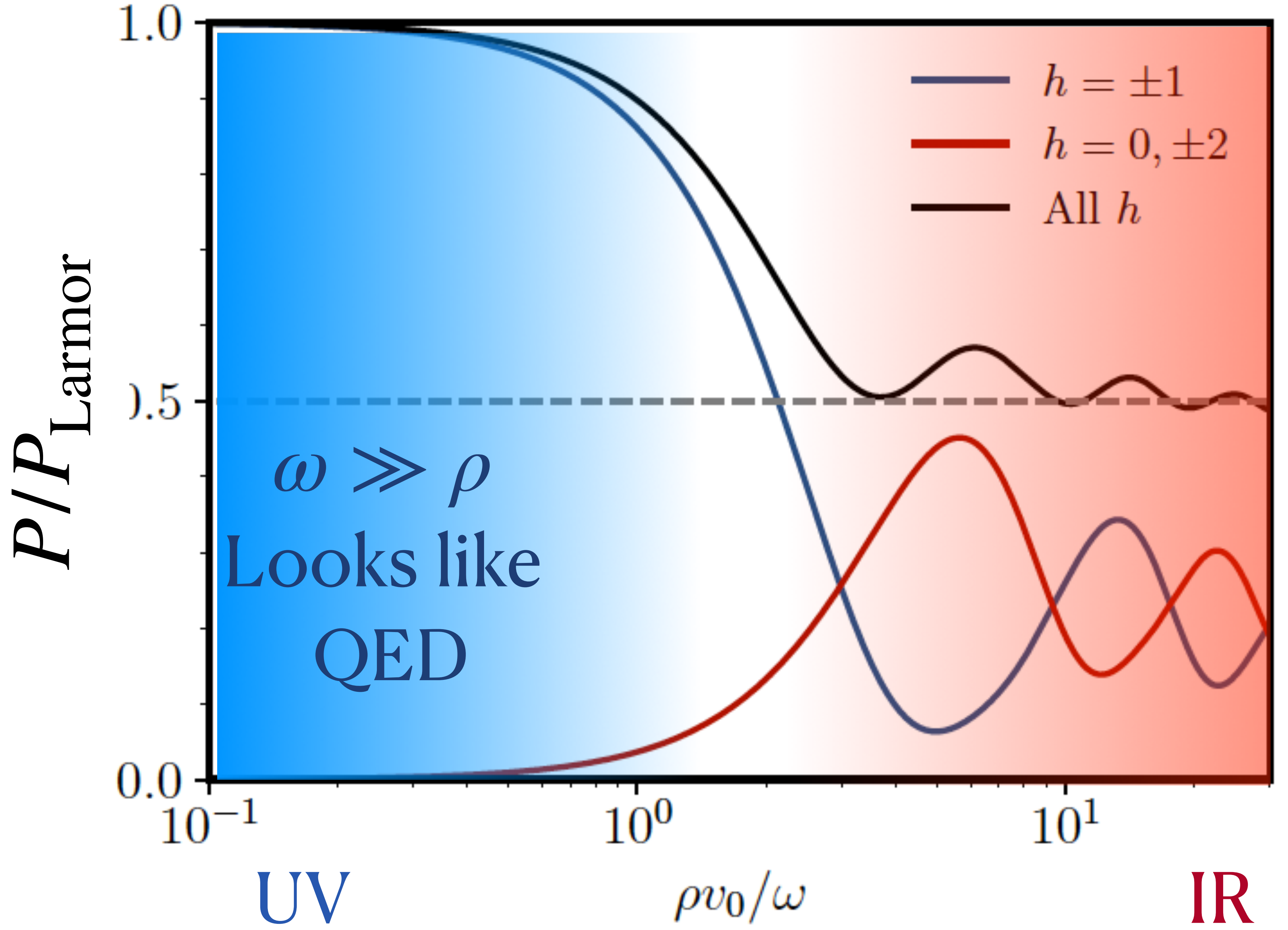
Very interesting resolution follows from Lorentz symmetry (at least for best-controlled calculations)

At frequencies $\gg \rho$, **all but one helicity have parametrically suppressed interactions.**
The dominant interaction can be "scalar-like", "vector-like", or "tensor-like"

Concrete Predictions: Vector-Like Coupling Class

Classical radiation from an oscillating particle:

$$P = \underbrace{\frac{e^2\omega^2 v_0^2}{12\pi}}_{\text{Larmor power}} \left(1 - \underbrace{\frac{9}{80}\frac{\rho^2 v_0^2}{\omega^2}}_{\text{Leading correction}} + \dots \right)$$

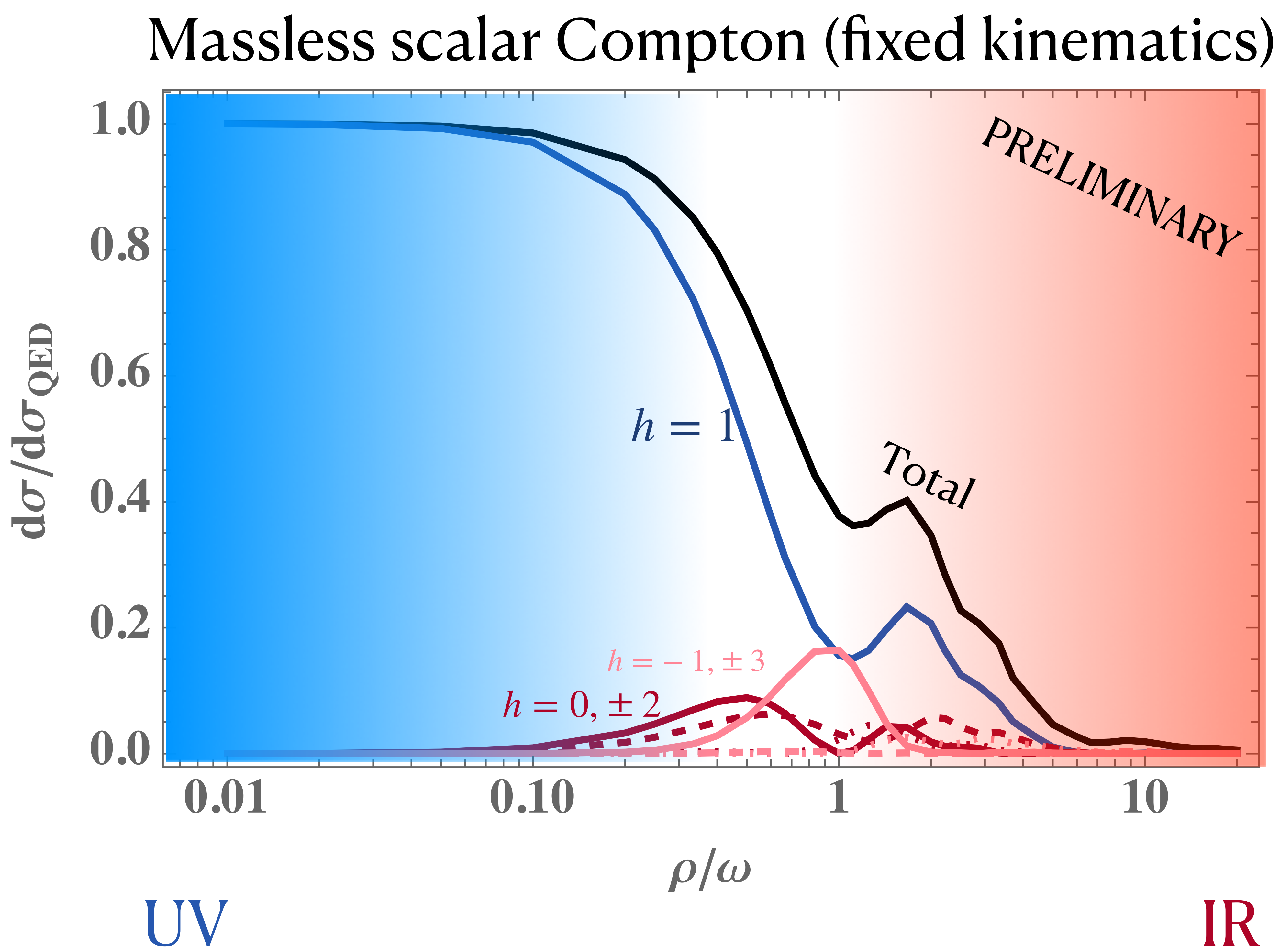


$\omega \ll \rho$

All $h \lesssim \rho/\omega$ relevant; finite limiting power

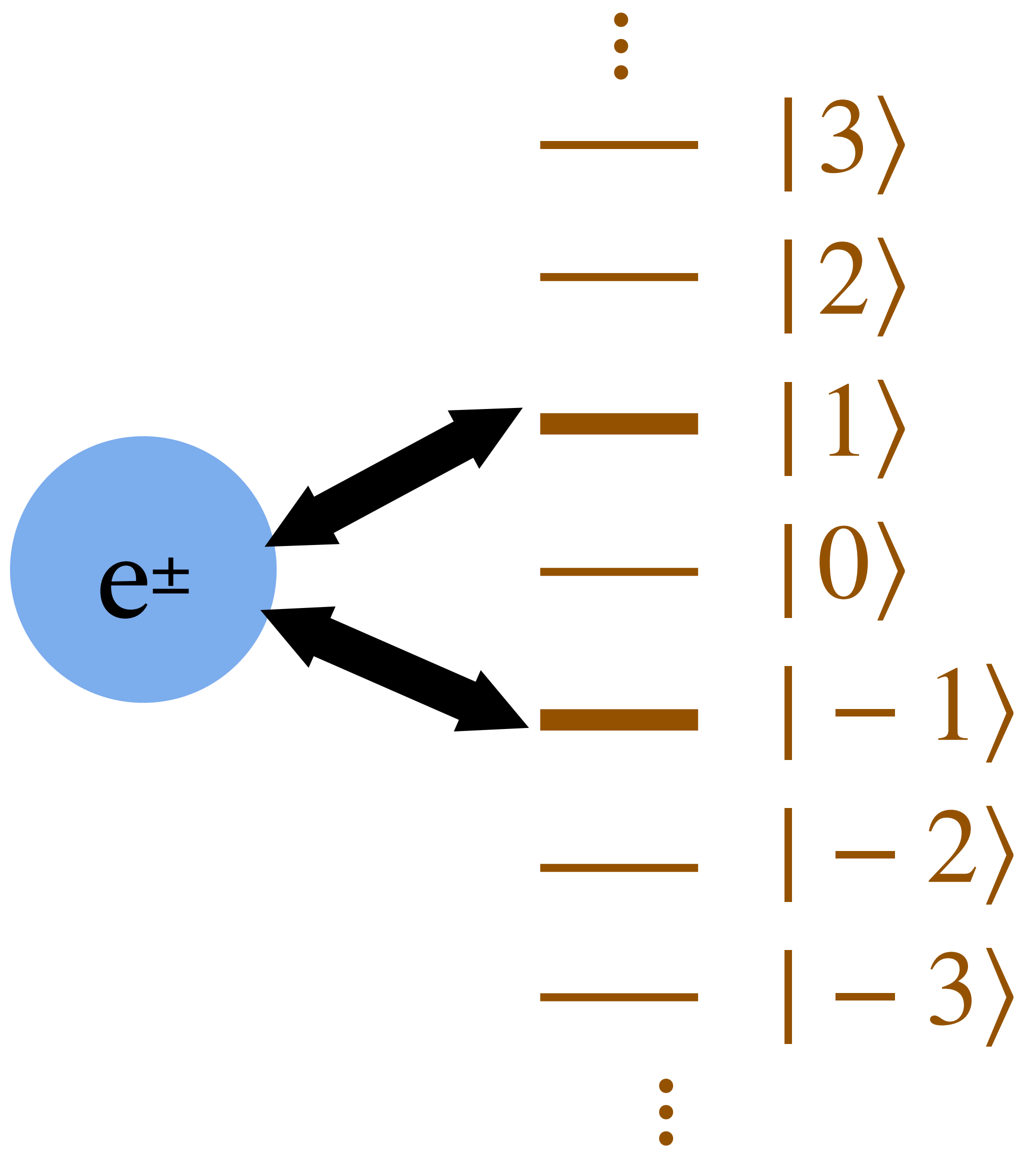
Scattering amplitudes computed using vertex operators

$$\hat{V}^{k,\eta}_{CSP}(t) \equiv ie^{ik\cdot z(t)}\, e^{-i\rho\frac{\eta\cdot \dot{z}(t)}{k\cdot \dot{z}(t)}} \left(\sqrt{2}ik\cdot\dot{z}(t)/\rho\right)$$



Continuous Spin Particles are like familiar massless particles with an associated dark sector

Covariant interactions single out **one** helicity with unsuppressed coupling (e.g. $|h|=1$)



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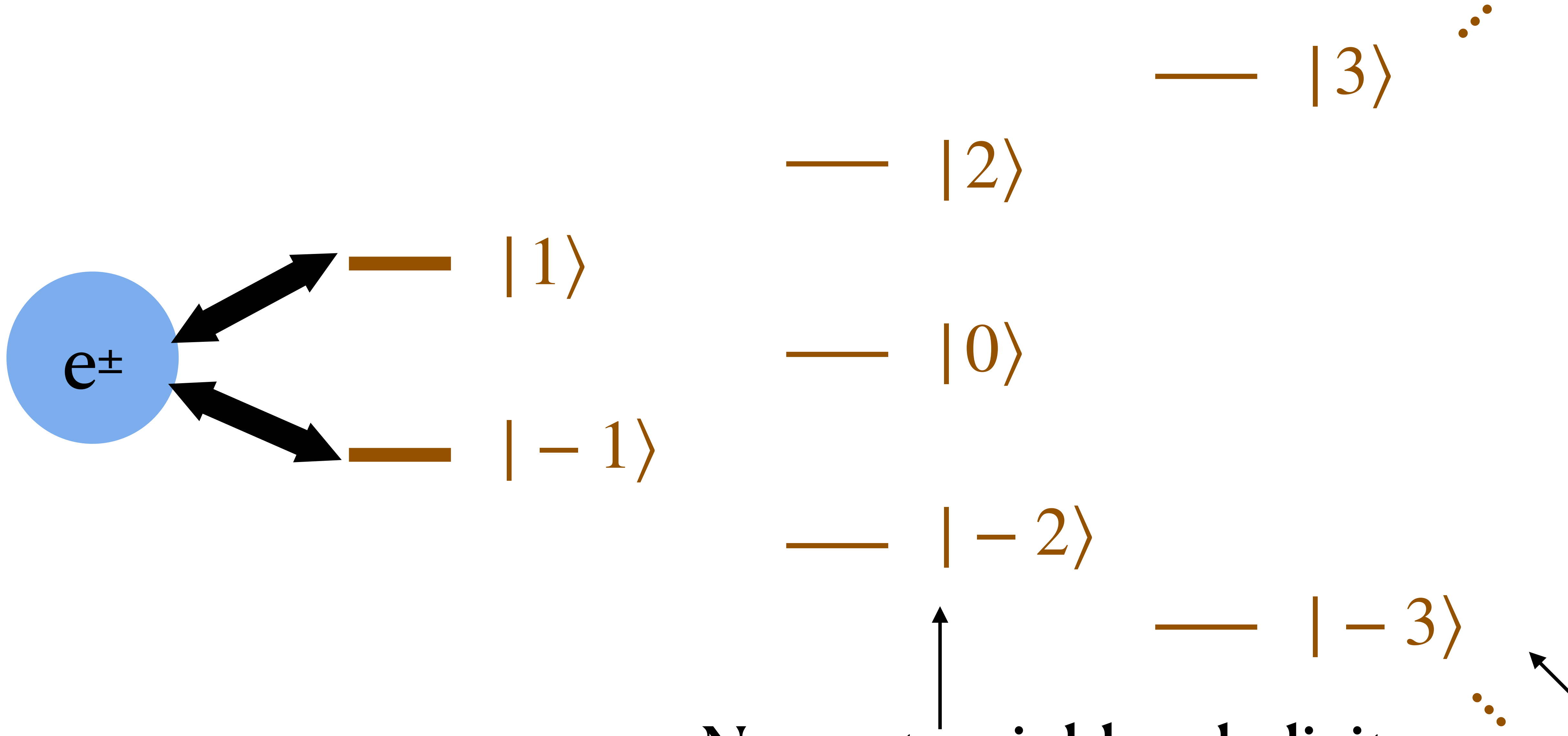

Nearest-neighbor helicity states' interactions suppressed by ρ/ω

Higher helicities' interactions suppressed by more powers of ρ/ω

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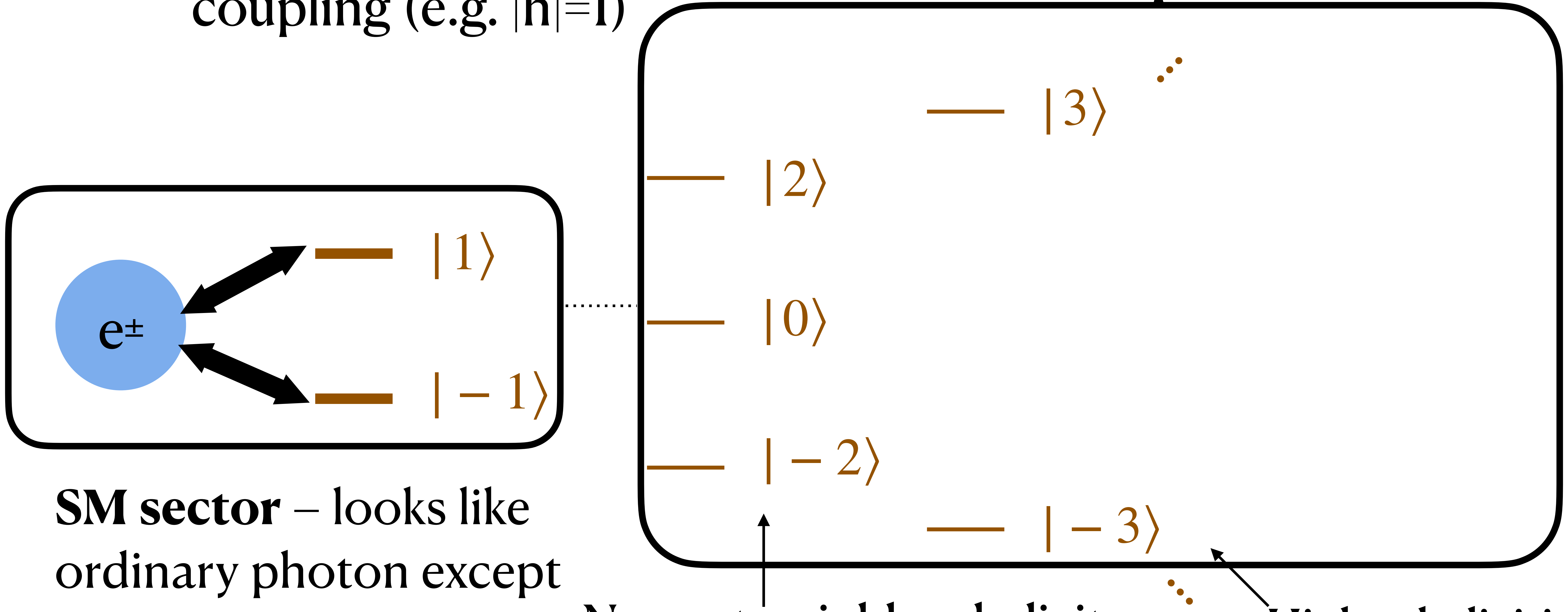
Continuous spin "dark" sector



SM sector – looks like ordinary photon except in deep IR $\omega \lesssim \rho$

Nearest-neighbor helicity states' interactions suppressed by ρ/ω

Higher helicities' interactions suppressed by more powers of ρ/ω

$\Rightarrow$ Only lowest-energy phase space of partner modes thermalizes, with finite (polynomial $\rightarrow$ logarithmically growing)

$$\sum_h \delta n(h)$$

Why should we care?

- Theorist: “Because it’s there”

  Falls out simply from postulates of relativity and quantum mechanics $\Rightarrow$ worth understanding!

- Phenomenologist: “Because it might **really** be there”

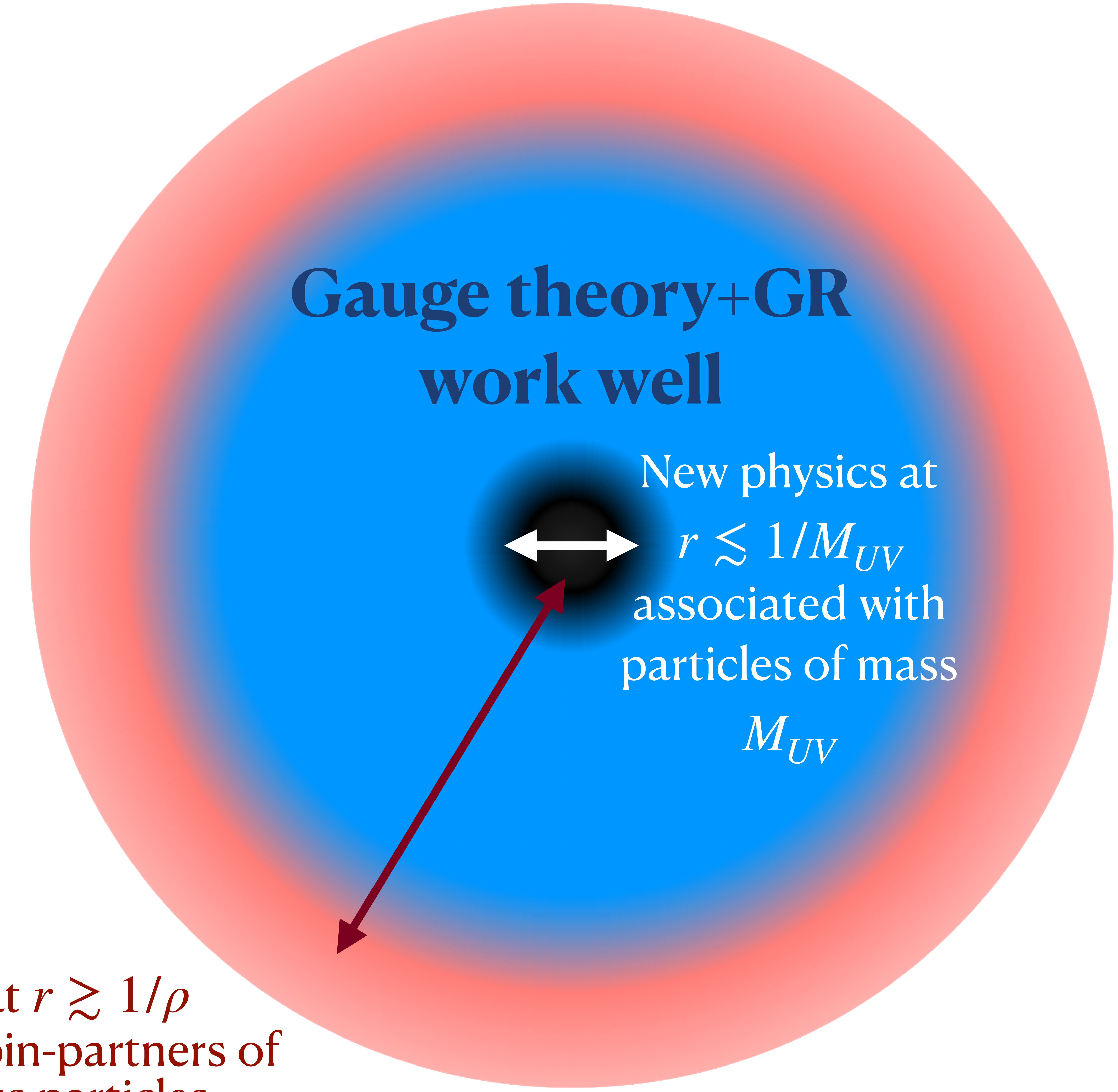
  Can think about experimental measurements/constraints on the spin-scale of photons and gravitons

  All SM fields are either fundamentally massless (before EWSB) or unnaturally light.

  Thinking about models with non-zero spin scales may illuminate new approaches to many SM problems.

Worldview

- Lorentz invariance → massless particles have a spin-scale. **Is it zero or non-zero?**
- The non-zero option makes more sense than previously thought, and looks like IR modification of familiar theories with testable consequences
- If viable, we should think of the Standard Model as an effective theory with both UV and IR completions.
- **If inconsistent, deserves a proper burial**



New physics at $r \gtrsim 1/\rho$ associated with spin-partners of known massless particles

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A Field Theory for All Helicities

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- Continuous spin particle has modes of every helicity, that separate into singleton representations as $\rho \to 0$
- Helicity $\pm h$ modes typically described by gauge theory of rank-h tensor fields
  - Notably subtle – many components: 2 physical, rest are pure gauge
- Continuous spin field should, in $\rho \to 0$ limit, decompose into similar modes.
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$$\Psi(\eta, x) \equiv \phi^{(0)}(x) + \eta^\mu \phi^{(1)}_\mu(x) + \eta^\mu \eta^\nu \phi^{(2)}_{\mu\nu}(x) + \ldots$$

Lorentz acts as $x \to \Lambda x, \eta \to \Lambda \eta$

introduced in 1404.0675 – complementary pedagogical discussion in 2303.04816

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- Action:

$$S = \frac{1}{2}\int_{\eta,x} \delta'(\eta^2+1)(\partial_x \Psi)^2 + \frac{1}{2}\delta(\eta^2+1)(\Delta\Psi)^2 \quad \text{with } \Delta\Psi \equiv \partial_\eta \cdot \partial_x + \rho$$

What is Vector Superspace?

- Naively divergent integral – can regulate by analytic continuation of η^0, or just divide by "volume" $\int_\eta \delta(\eta^2+1)$. Symmetry relates integral to differential operator $J_0(\sqrt{\partial_\eta^2})$, e.g.

$$\int_\eta \delta(\eta^2+1)\eta^\mu\eta^\nu = \frac{1}{4}g^{\mu\nu}\int_\eta \delta(\eta^2+1)\eta^2 = -\frac{1}{4}g^{\mu\nu}$$

- Basic job: relating off-shell Lorentz transformation to little group action on-shell

Basis states' orthonormality, tree unitarity of CSP exchange, and little group covariance of matrix elements all follow from one identity: Whenever $\delta(\eta^2+1)k\cdot\partial_\eta F(\eta)=0$,

$$\int_\eta \delta'(\eta^2+1)F(\eta) = \int_C F(\eta)$$

C is unit circle of $\vec{\eta}$'s orthogonal to $\vec{k}$ (in any frame) ~ unit circle in "little group E_2 plane"

- Enables an enlarged spacetime symmetry that mixes spins

Free action is invariant under a "bosonic supertranslation" $\delta x^\mu = \omega^{\mu\nu}\eta_\nu$ [Rivelles '14].

A Field Theory for All Helicities: A Bit of Intuition

When $\rho = 0$, action encodes familiar actions for tensor components, e.g.

$$\mathscr{L}[\Psi \to \phi(x)] = \frac{1}{2}\int_\eta \underbrace{\delta'(\eta^2+1)}_{\text{gives } 1}\underbrace{(\partial_x\Psi)^2}_{\partial_x\phi} + \frac{1}{2}\,\delta(\eta^2+1)(\partial_x\cdot\underbrace{\partial_\eta}_{=0}\Psi)^2 \quad = \frac{1}{2}\,(\partial_x\phi)^2$$

$$\mathscr{L}[\Psi \to \sqrt{2}\eta^\mu A_\mu] = \frac{1}{2}\int_\eta \delta'(\eta^2+1)\underbrace{(\partial_x\Psi)^2}_{2\,(\eta_\mu\partial_x A^\mu)^2} + \frac{1}{2}\,\delta(\eta^2+1)\underbrace{(\partial_x\cdot\partial_\eta\Psi)^2}_{2\,(\partial_\mu A^\mu)^2} = -\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}(\partial_\mu A^\mu)^2$$

But working in η-space directly is simple and powerful.

A Field Theory for All Helicities

Analogy with Maxwell Action

Action	$\int_x -\frac{1}{2}(\partial_\mu A_\nu)^2 + \frac{1}{2}(\partial \cdot A)^2$	$\frac{1}{2}\int_{x,\eta} \delta'(\eta^2+1)(\partial_x \Psi)^2 + \frac{1}{2}\,\delta(\eta^2+1)(\Delta\Psi)^2$
Equation of Motion	$\Box A_\mu - \partial_\mu \partial \cdot A = 0$	$\delta'(\eta^2+1)\,\Box\,\Psi(\eta,x) - \frac{1}{2}\Delta(\delta(\eta^2+1)\Delta\Psi) = 0$
Gauge Invariance	$A_\mu \simeq A_\mu + \partial_\mu \epsilon(x)$	$\Psi(\eta,x) \simeq \Psi(\eta,x) + \overbrace{\left(\eta\cdot\partial_x - \frac{1}{2}(\eta^2+1)\Delta\right)}^{D}\epsilon(\eta,x) + (\eta^2+1)^2\chi(\eta,x)$

For $\rho = 0$, a nice re-packaging of Fronsdal actions for all rank-h tensor fields.

For $\rho \neq 0$, Δ mixes tensor components of $\Psi \Rightarrow$ **much** simpler in η-space

Each result has 1- or 2-line proof, using standard IBP and two δ-fn identities:

$$\delta'(\eta^2+1)Df = \frac{1}{2}\Delta\left(\delta(\eta^2+1)f\right),\quad \delta(\eta^2+1)\Delta D\epsilon = \Box\,\epsilon$$

A Field Theory for All Helicities

Gauge fixing and physical states

Covariant Gauge Fixing	$\partial \cdot A = 0$	$\Delta\Psi(\eta, x) = 0$
Gauge-Fixed EOM	$\Box A_\mu = 0$	$\Box \Psi = 0$
Residual Gauge Freedom	$\Box \epsilon = 0$	$\Box \epsilon = \Delta\epsilon = 0$
Physical states (helicity basis)	$\psi_{\pm,k}(x) = e^{-ik\cdot x}\,\epsilon_\pm^\mu$	$\psi_{h,k}(\eta, x) = e^{-ik\cdot x}\,(\pm i\eta\cdot\epsilon_\pm)^{\lvert h\rvert}\; e^{-i\rho\eta\cdot q}$
	$\epsilon_- = \epsilon_+^*, \epsilon_\pm \cdot k = 0, \epsilon_+ \cdot \epsilon_- = -2$	$q\,.\,k = 1, q\,.\,\epsilon_\pm = 0$

Helicity basis states with $\rho \neq 0$ are simple functions of η, but **not** tensors!
Another reason to work in vector superspace

Perspective on Free Field Theory

Good, so we can describe a continuous spin particle as a free particle.

Old no-go's were wrong, but so what? We know there are physical obstructions to helicity-3 interactions, but they also admit a free field theory.

From S-matrix perspective, can see this via Weinberg soft theorems

From field theory perspective, the problem is that you can't build a conserved current from matter.

Existence of free theory says nothing about consistency of interactions, but gives us a framework to start looking.

A Field Theory for All Helicities

Coupling to currents

Current Term in Action	$\delta S = -\int_x A^\mu(x) J_\mu(x)$	$\delta S = \int_{x,\eta} \delta'(\eta^2+1)\Psi(\eta,x)J(\eta,x)$
Continuity condition from gauge-invariance	$\partial_\mu J^\mu = 0$	$\delta(\eta^2+1)\Delta J(\eta,x) = 0$
EOM in suitable gauge	$\Box A^\mu = J^\mu$	$\Box \Psi(\eta,x) = J(\eta,x)$

Once we have found a suitable current, can use familiar machinery to compute physical quantities, e.g.

- Classical radiation and CSP-exchange forces [2303.04816]
- Scattering amplitudes [2308.xxxxx]

Currents from Worldlines

Ordinary EM Example

For technical reasons, we've worked with matter **particles** and their **worldlines** rather than matter **fields**.

Well-established but less familiar, so as a refresher let's look at a few E&M examples

$$S_{free}[z(\tau), E(\tau)] = \int d\tau \frac{\dot{z}^2}{2E} + \frac{1}{2} m^2 E$$

Einbein $E(\tau) \Rightarrow$ reparametrization invariant
$E(\tau) = 1/m$ gives proper time parametrization.

$$J^\mu(x) = \int d\tau\, q\, \dot{z}^\mu(\tau)\, \delta^{(4)}(x - z(\tau))$$

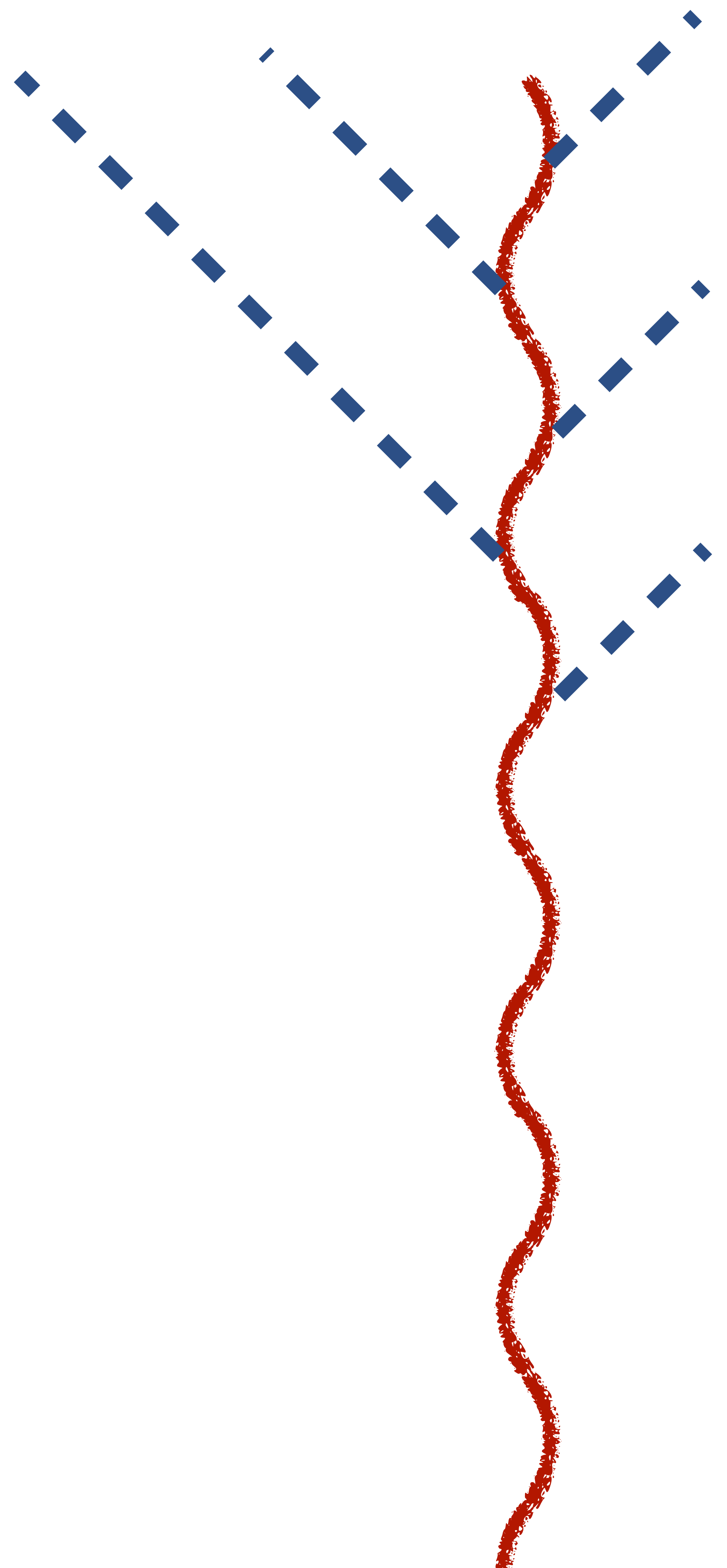
$$\partial \cdot J(x) = -\int d\tau\, \partial_\tau \left[q\, \delta^{(4)}(x - z(\tau)) \right]$$

Conserved as long as worldlines only begin and end at charge-conserving vertices.

Currents from Worldlines

EM: Classical Radiation from a Moving Particle

$$\frac{dP_h}{d\omega d\hat{\mathbf{r}}} = \frac{\omega^2}{8\pi^2} |\epsilon^*_{h,k} J(\eta, k)|^2 \quad \text{with k} = (\omega, \omega\hat{\mathbf{r}})$$



For simple harmonic motion, power

$$P_{Larmor} = \frac{e^2\omega^2 v_0^2}{12\pi}$$

Currents from Worldlines

EM: Amplitudes

Compute amplitudes from path integral for worldline in EM field (Feynman 1950)

$$A(p, p', k_i, \epsilon_i) = \int_{\mathscr{P}[x,x']} Dz(\tau)\, e^{-S_{free}[z]}\, e^{-i\,p\cdot x}\, e^{i\,p'\cdot x'} \prod \int \underbrace{dt_i \left(\epsilon_i \cdot \dot{z}(t_i) e^{-ik_i\cdot z(t_i)}\right)}_{A^{\mu}_{ext} J_{\mu}} \Bigg|_{LSZ}$$

Current and Maxwell field theory are all you need to know to build amplitudes!

(More pieces needed for YM or GR theories with self-interacting fields)

Currents from Worldlines

EM: Amplitudes

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Modern "string-inspired" approach to evaluation (Strassler, Schubert, ...): matter Fourier phases and photon-current couplings → vertex operators; solving Gaussian path-integral exactly leaves integral over the insertion points t_i.

Very different organization from Feynman diagrams but identical result.

Fully general treatment of loops, multiple worldlines, etc.

Matter Currents: The Key Ingredient

To couple a particle's worldline to CSP field, need to find current from worldline data satisfying continuity condition.

$$J(\eta, x) = \int d\tau\, f(\underbrace{x - z(\tau),}_{\text{Can't have just } \delta\text{-function support}} \dot{z}(\tau), \eta)$$

<— Worldline-local ansatz

$$= \int d\tau\, d^4k\ e^{ik\cdot(z(\tau)-x)} f(k, \dot{z}, \eta),\ \text{continuity condition}\ \ (-ik\cdot\partial_\eta + \rho)f = 0$$

General Solutions

Most general solution to continuity condition (up to total derivative terms) can be written as

$$f(k,\dot{z},\eta) = e^{-i\rho\frac{\eta\cdot\dot{z}}{k\cdot\dot{z}}}\hat{g}(k\cdot\dot{z}) + \underline{\mathcal{O}X(k,\dot{z},\eta)}$$

"Shape" terms

where free eom is $\delta'(\eta^2+1)\mathcal{O}\Psi = 0$

Analogous to charge radius etc. operators in E&M

$\Rightarrow$ As in E&M, shape terms do not couple to continuous spin radiation

$\Rightarrow$ Worldline interactions with radiation **fully** determined by $\hat{g}$.

But shape terms can qualitatively change the space-time support of the current, as well as couplings to off-shell CSPs.

Currents in Space-Time

Although shape terms don't couple to continuous spin radiation, they **can** change the spacetime localization of the current – e.g. family of currents

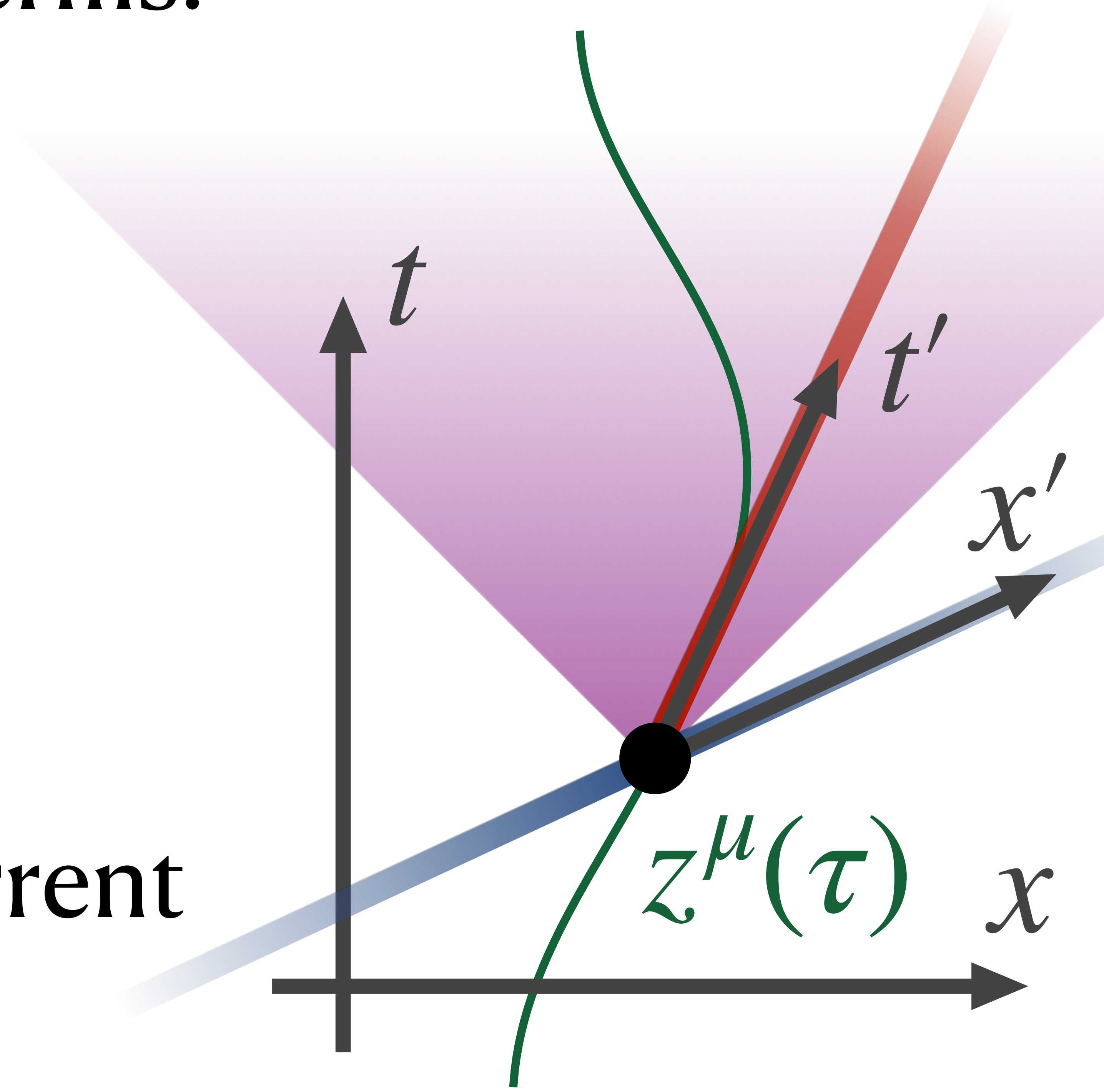
$$f(k, \dot{z}, \eta) \propto e^{-i\rho \frac{\eta \cdot V(\dot{z}, k)}{k \cdot V(\dot{z}, k)}}$$

Satisfy continuity condition for any V – differ only by shape terms.
But they have different localization properties:

$V = \dot{z}$ "temporal"

$V = k + \beta\rho\dot{z}$ "inhomogeneous"

$V = k - (k \cdot \dot{z})\, \dot{z}$ "spatial"



Parametrization of general solution in terms of "temporal" current was arbitrary – not physically privileged.

Currents in Space-Time: Causality

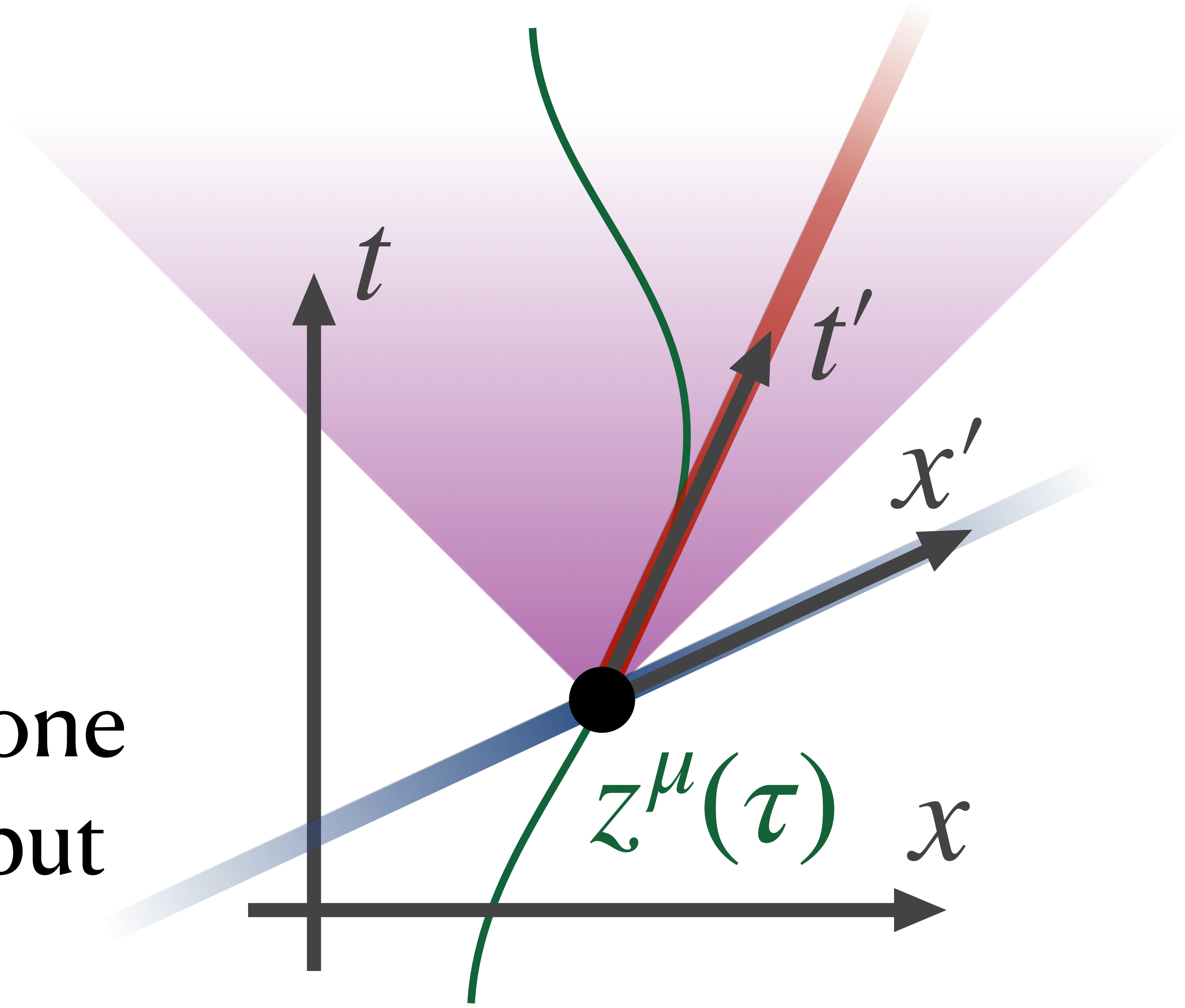
Some ansatz currents **admit retarded/advanced forms supported in source's forward/ backward lightcone → manifestly causal equations of motion**

$$\partial_x^2 \Psi(x) \propto \int d\tau \, j_R(x - z(\tau))$$

Field at point depends on particle trajectories in past causal cone

$$m\ddot{z}^\mu(\tau) \propto \int \Psi(\eta, x) j_A(x - z(\tau))$$

Particle's acceleration at point depends only on fields in its causal past



This feature, and detailed non-local structure, suggestive of integrating out intermediate fields. We suspect this can be done at Lagrangian level to yield local & manifestly causal action, but no concrete realization yet.

(Could Rivelles' supertranslation-like symmetry be a hint?)

Even equal-time interactions (c.f. Coulomb-gauge QED) can yield causal dynamics – could this also happen for less causal-looking continuous spin currents?

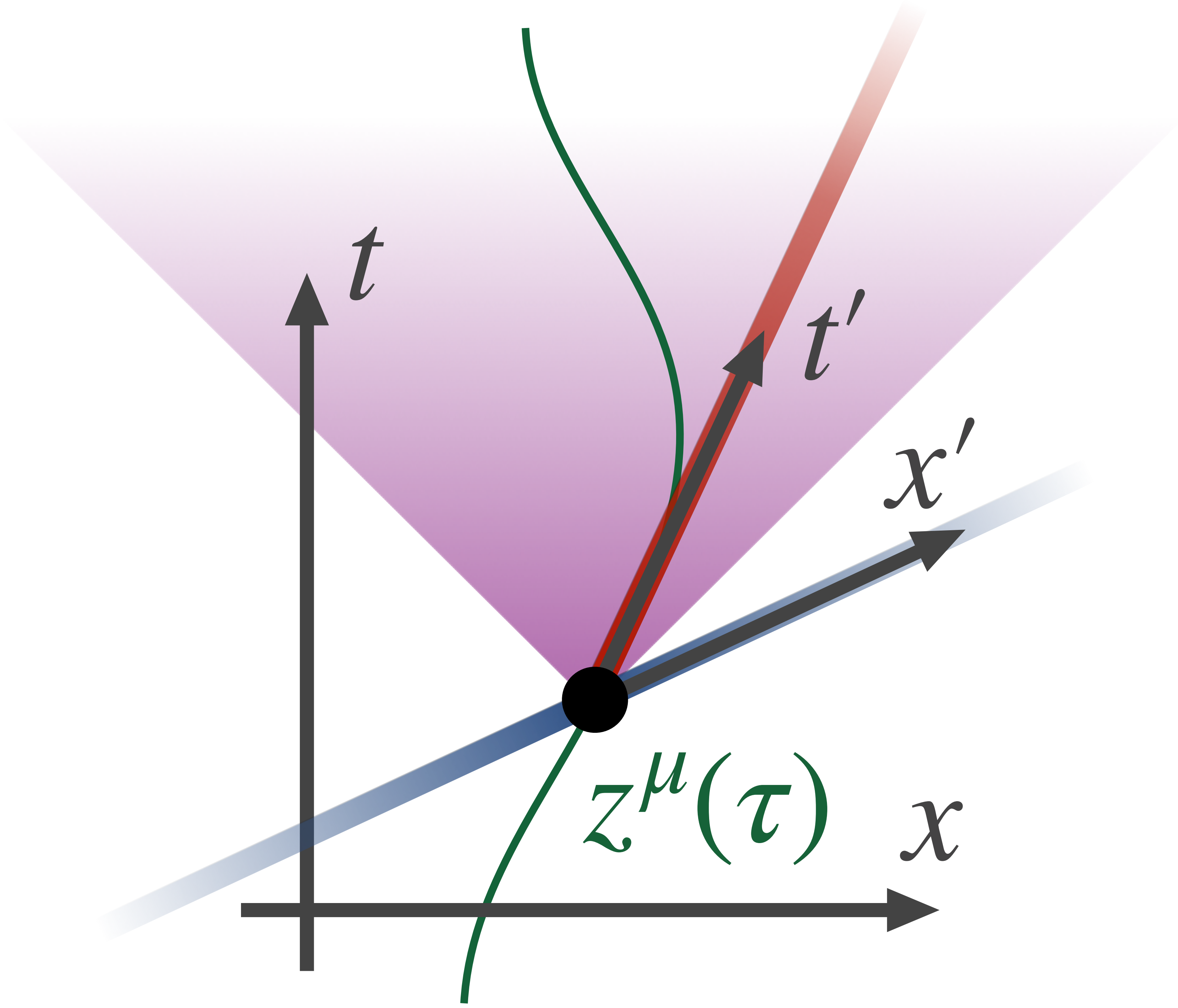
Shape Questions and Off-Shell Physics

Different "current shapes" do have different physics – whenever off shell continuous spin fields are involved, e.g.

— classical static force law (1/r for spatial and temporal, with ρ-corrections for inhomogeneous) and velocity-dependent corrections

— tree-level matter-matter scattering via CSP exchange

— **renormalized CSP and matter propagators**



Exploring consistency properties of these less universal amplitudes will likely help to understand which current structures are consistent, physically "minimal".

The path(s) forward

- Goal: Kill or define theory more sharply/completely
- Classification of appropriately conserved, WL-local worldline currents [2303.04816]

→"**Shut up and calculate**" focused on the wide variety of "universal" reactions (classical and amplitudes) that involve only on-shell CSP radiation, and therefore are unaffected by shape.

→Currents are suggestive of an extended object with more d.o.f., or additional spacetime fields. Looking for **inspired guess** for the structure of the source currents, i.e. "**CSP string**" could be fruitful, but not a prerequisite to progress.

→ Already have **definite rules** for constructing general amplitudes from given current. Demanding sensible properties for off-shell CSP amplitudes (e.g. 2-point functions) will likely single out correct current within each universality class, i.e. "**CSP bootstrap**"

General Solutions

Most general solution to continuity condition (up to total derivative terms) can be written as

$$f(k, \dot{z}, \eta) = e^{-i\rho \frac{\eta \cdot \dot{z}}{k \cdot \dot{z}}} \hat{g}(k \cdot \dot{z}) + \mathcal{O}X(k, \dot{z}, \eta)$$

Worldline interactions with on-shell radiation **fully** determined by $\hat{g}$.

Expanding $\hat{g}$ in Taylor series gives "universality classes" of currents:

$$\hat{g} = \begin{cases} g & \text{scalar-like current} \\ \frac{e}{\rho} k \cdot \dot{z} & \text{vector-like current} \\ (k \cdot \dot{z})^n / \Lambda^n & \text{non-minimal currents*} \end{cases}$$

Classical results in these cases are main focus of 2303.04816 (scalar-like and vector-like currents)

GR-like special case – more later (non-minimal currents*)

Limiting Behavior: $k^0 \gg \rho$

Look at small-ρ behavior of current:

$$f(k,\eta,\dot{z}) = -\frac{e}{\rho}k\cdot\dot{z}(\tau)\,e^{-i\rho\frac{\eta\cdot\dot{z}}{k\cdot\dot{z}}}$$

$$= \boxed{-\frac{e}{\rho}k\cdot\dot{z}(\tau)} + \boxed{ie\,\eta\cdot\dot{z}(t)} + \mathcal{O}(\rho)$$

Physically irrelevant (changes J by total τ-derivative)

η-space form of usual vector current

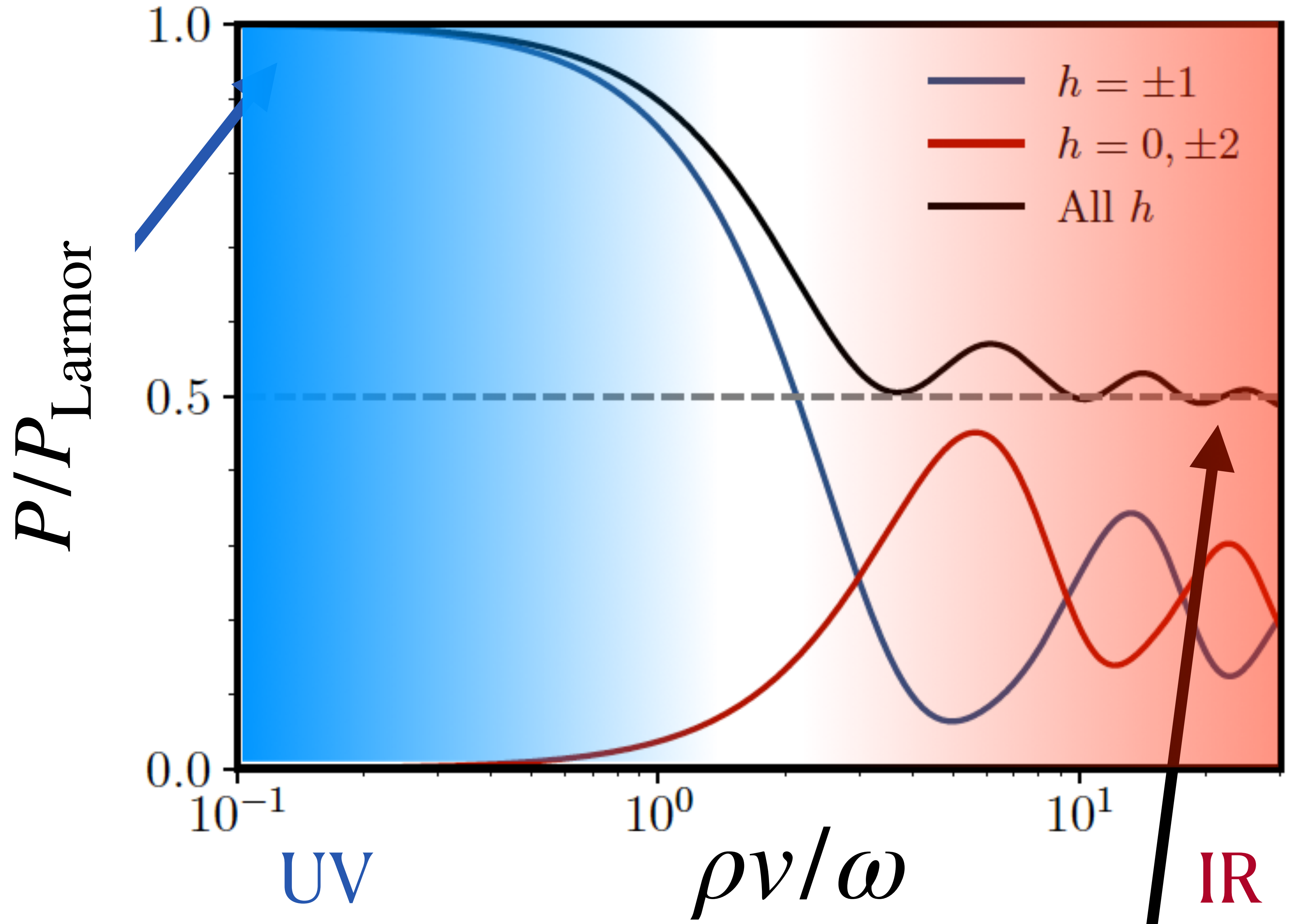
⇒ Leading physical effects should be QED-like!

$$J(\eta,x) = \int d\tau\, d^4k\; e^{ik\cdot(z(\tau)-x)} f(k,\dot{z},\eta)$$

Radiation from a Moving Particle

Indeed, for vector-like currents,

$$P = \underbrace{\frac{e^2\omega^2 v_0^2}{12\pi}}_{\text{Standard Larmor power}}\left(1 - \frac{9}{80}\frac{\rho^2 v_0^2}{\omega^2} + \dots\right)$$



For small $\rho v/\omega$, power **matches Larmor** and **dominated by h=±1 modes**

(Physical manifestation of formal correspondence noted earlier)

At large $\rho v/\omega$, power spread among many modes, harmonics **but total power emitted has finite limit.**

Compton-Like Amplitudes

Structure of the calculation is identical to QED – η-dependent vertex operator yields "unconstracted be contracted with basis wave-functions to get polarization amplitudes.

$$M(p_0, p_3, \{k_1,\eta_1\}, \{k_2,\eta_2\}) = 2\int_{-1}^{1} dx \left(\eta_1 - \frac{\eta_1\cdot P_1(x)}{k_1\cdot P_1(x)}k_1\right)\cdot\left(\eta_2 - \frac{\eta_2\cdot P_2(x)}{k_2\cdot P_2(x)}k_2\right) e^{-i\rho\frac{\eta_1\cdot P_1(x)}{k_1\cdot P_1(x)} - i\rho\frac{\eta_2\cdot P_2(x)}{k_2\cdot P_2(x)}.}$$

$$P_{1,2}(x) = p_3 - p_0 \pm x\,k_{2,1}$$

→ at endpoints $x = \pm 1$, these are momenta appearing in $s(u)$-channel photon vertex

Polarization amplitudes are Fourier transforms of this expression,

$$A(p_0, p_3, \{k_i, h_i\}) = \int \frac{d\phi_i}{2\pi} e^{ih_i\phi_i} M(p_0, p_3, \{k_i, \eta_i(\phi_i)\})$$

η(ϕ) lies on unit circle orthogonal to k, e.g. $(0, \cos\phi, i\sin\phi, 0)$ for $k = (k,0,0,k)$

(1) no unphysical singularities, (2) sensible at physical singularities, (3) finite angle-differential cross-section at all energies.

High Energy Limit

1

$$M(p_0, p_3, \{k_1, \eta_1\}, \{k_2, \eta_2\}) = \; 2\int_{-1}^{1} dx \left(\eta_1 - \frac{\eta_1 \cdot P_1(x)}{k_1 \cdot P_1(x)} k_1\right) \cdot \left(\eta_2 - \frac{\eta_2 \cdot P_2(x)}{k_2 \cdot P_2(x)} k_2\right) \boxed{e^{-i\rho\frac{\eta_1 \cdot P_1(x)}{k_1 \cdot P_1(x)} - i\rho\frac{\eta_2 \cdot P_2(x)}{k_2 \cdot P_2(x).}}}$$

This clearly has smooth $\rho \to 0$ limit (just drop phase).

Linear in η_1 and η_2 implies only Fourier modes $h = \pm 1$ survive. [In this case, $\eta_i \sim \epsilon_i$]

The integral above is simply a Feyman parametrization of the standard scalar-QED result! (This form makes gauge invariance and permutation symmetry manifest, but obscures both locality and vanishing of “both +” amplitudes)

For high-energy scaling, simply Taylor-expand in ρ.

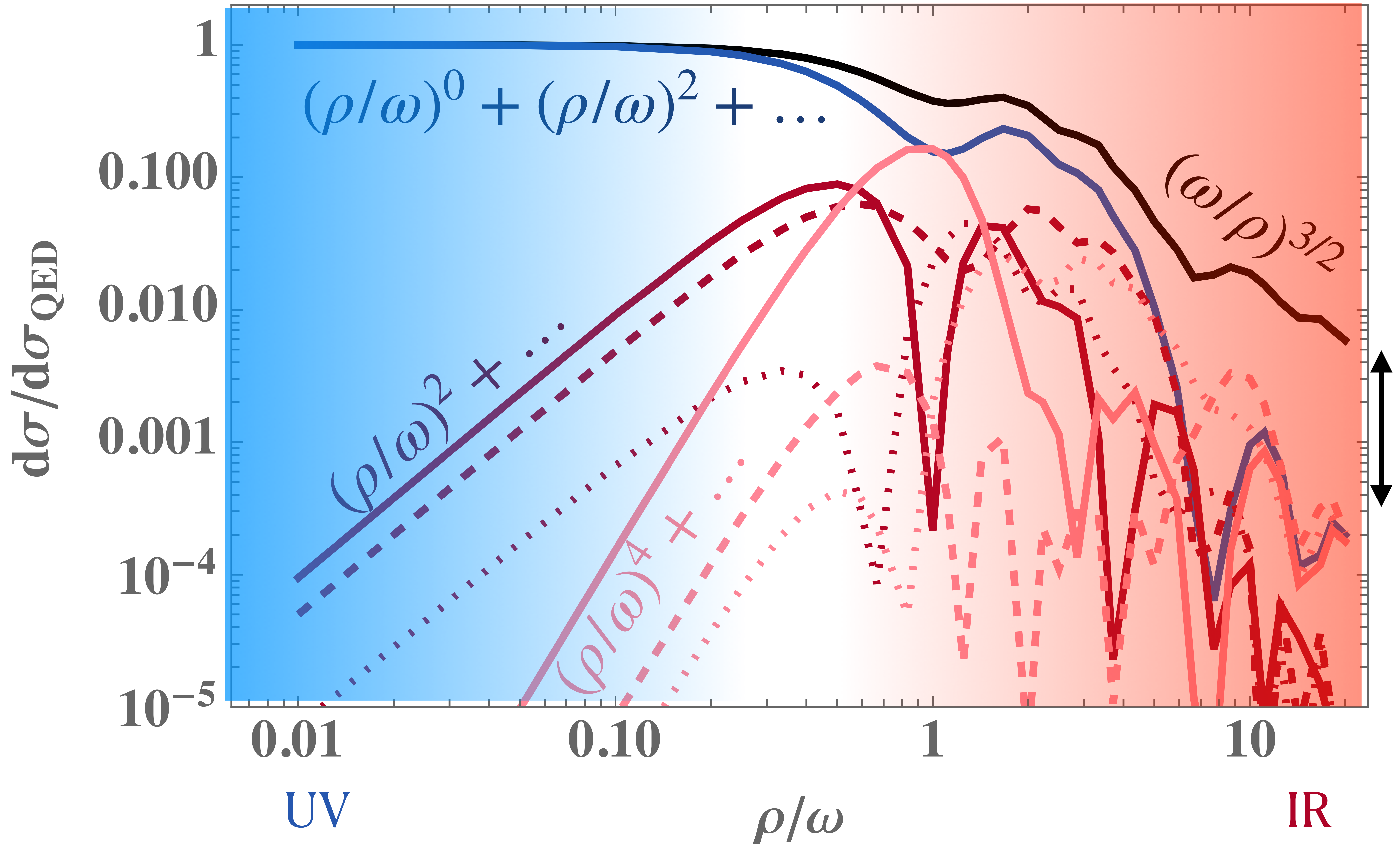
Compton-Like Amplitudes

Amplitude scaling at high energies:

		h_1=-2	h_1=-1	**h_1=0**	h_1=1	h_1=2	
h_2=-2		$O(\rho^2/E^2)$	$O(\rho/E)$	$O(\rho^2/E^2)$	$O(\rho/E)$	$O(\rho^2/E^2)$	
h_2=-1		$O(\rho/E)$	$\boxed{O(\rho^2/E^2)}$	$O(\rho/E)$	$O(1)$	$O(\rho/E)$	
h_2=0		$O(\rho^2/E^2)$	$O(\rho/E)$	$O(\rho^2/E^2)$	$O(\rho/E)$	$O(\rho^2/E^2)$	
h_2=1		$O(\rho/E)$	$O(1)$	$O(\rho/E)$	$\boxed{O(\rho^2/E^2)}$	$O(\rho/E)$	
h_2=2		$O(\rho^2/E^2)$	$O(\rho/E)$	$O(\rho^2/E^2)$	$O(\rho/E)$	$O(\rho^2/E^2)$	

Like classical radiation, becomes democratic **but with finite sum** at lower energies.

Compton-Like Cross-Section: UV to IR



Modes with $|h| \lesssim \rho/\omega$ are relevant

Scalar-Like Radiation

Analogous results for $\hat{g} = g$

Power radiated in minimally coupled scalar

$$P = \frac{g^2\omega^2}{24\pi} \times \begin{cases} \boxed{v_0^2 \left(1\right.} - \boxed{\frac{(\rho v/\omega)^2}{20}} \left. + \ldots \right) & h = 0 \\ \boxed{(\rho v/\omega)^2/2} + \ldots & h = \pm 1 \\ \mathcal{O}(\rho v/\omega)^{2|h|} & |h| \geq 2 \end{cases}$$

Subleading in $\rho v/\omega$

$$\bar{P} = \begin{cases} g^2 a_0^2/24\pi & a_0 \gg \rho v_0^2 \\ g^2 \rho a_0/8\pi^2 & a_0 \ll \rho v_0^2 \end{cases}$$

Power falls off at low acceleration, but ***slower*** than for ordinary scalar

Compton-Like Amplitudes

Messy expression, but (1) no unphysical singularities, (2) sensible factorization limit at physical singularities, (3) at energies ≫ ρ, recover scalar amplitudes,

	h_1=-2	h_1=-1	**h_1=0**	h_1=1	h_1=2
h_2=-2		$O(\rho^3/E^3)$	$O(\rho^2/E^2)$	$O(\rho^3/E^3)$	
h_2=-1	$O(\rho^3/E^3)$	$O(\rho^2/E^2)$	$O(\rho/E)$	$O(\rho^2/E^2)$	$O(\rho^3/E^3)$
h_2=0	$O(\rho^2/E^2)$	$O(\rho/E)$	$\boxed{\frac{1}{s}+\frac{1}{u}-\frac{\rho^2}{8}\frac{p_\perp^2(s^3+u^3)}{s^3u^3}+\ldots}$	$O(\rho/E)$	$O(\rho^2/E^2)$
h_2=1	$O(\rho^3/E^3)$	$O(\rho^2/E^2)$	$O(\rho/E)$	$O(\rho^2/E^2)$	$O(\rho^3/E^3)$
h_2=2		$O(\rho^3/E^3)$	$O(\rho^2/E^2)$	$O(\rho^3/E^3)$	

Compton-Like Amplitudes

Messy expression, but (1) no unphysical singularities, (2) sensible factorization limit at physical singularities, (3) at energies $\gg \rho$, recover scalar amplitudes,

	h_1=-2	h_1=-1	**h_1=0**	h_1=1	h_1=2
h_2=-2		$O(\rho^3/E^3)$	$O(\rho^2/E^2)$	$O(\rho^3/E^3)$	
h_2=-1	$O(\rho^3/E^3)$	$O(\rho^2/E^2)$	$O(\rho/E)$	$O(\rho^2/E^2)$	$O(\rho^3/E^3)$
h_2=0	$O(\rho^2/E^2)$	$O(\rho/E)$	$\frac{1}{s}+\frac{1}{u}-\frac{\rho^2}{8}\frac{p_\perp^2(s^3+u^3)}{s^3u^3}+\ldots$	$O(\rho/E)$	$O(\rho^2/E^2)$
h_2=1	$O(\rho^3/E^3)$	$O(\rho^2/E^2)$	$O(\rho/E)$	$O(\rho^2/E^2)$	$O(\rho^3/E^3)$
h_2=2		$O(\rho^3/E^3)$	$O(\rho^2/E^2)$	$O(\rho^3/E^3)$	

(4) at energies $\ll \rho$, angle-differential cross-section is finite

Model-Building Opportunities

One tantalizing (but very speculative and premature!) potential application:

Hierarchy Problem ≃ “scalars that generate long-range 1/r potentials are unnatural”

Scalar with non-derivative couplings to heavy matter receives radiative mass corrections

Goldstones have shift symmetry – protects mass terms, but also forbids non-derivative couplings

?

Continuous spin field admits non-derivative scalar-like interactions, **and** Lorentz requires it to stay massless to avoid d.o.f. discontinuity (aka gauge “symmetry” protects mass)

If these models are renormalizable, they admit something close to a radiatively stable massless scalar.

Perhaps related mechanism can stabilize massive but light Higgs?

Graviton-Like CSPs

Linearized graviton couples to worldline current $m\int d\tau\, \dot{z}^{\mu}\dot{z}^{\nu}\, \delta^{(4)}(x - z(\tau))$.

This is not physically conserved for an accelerating worldline – must sum over $T^{\mu\nu}$ for all worldlines, and fields through which they interact.

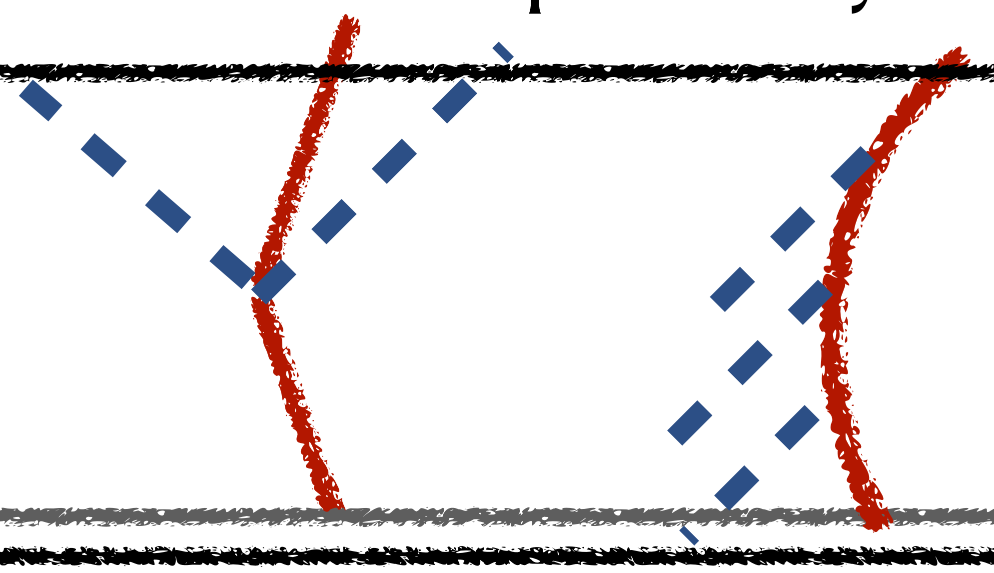
But there are limits where the non-conservation is "small" and linearized treatment is useful.

This "linearized GR current" has a natural generalization to $\rho \neq 0$. Graviton soft factor arguments do too.

But, like $\rho = 0$ perturbative GR, need universal couplings (including self-interactions) for full consistency.

What else can one compute?

Universal:
depend only on $\hat{g}$



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- Radiation from worldline undergoing a single instantaneous kick –reproduces soft factor results found in 2013.
- Force on a particle induced by plane-wave radiation background
- Continuous spin field $\Psi(\eta, x)$ sourced by a particle at rest or in motion
- Inter-particle force laws (static force-law is just 1/r for our simple ansatz – phases cancel out – but other ansatz currents can give $O(\rho r)$ corrections)

Depend on "shape terms"– we have only looked at simplest example currents

- QM in path integral
  - Continuous spin radiation/absorption amplitudes for free scalar matter
  - CSP-mediated atomic transitions (toy bosonic atoms)
  - Tree amplitudes with intermediate CSPs
  - Wavefunction renormalization of continuous spin field
  - Continuous spin fields in loops
- Classical response of macroscopic detectors to continuous spin radiation

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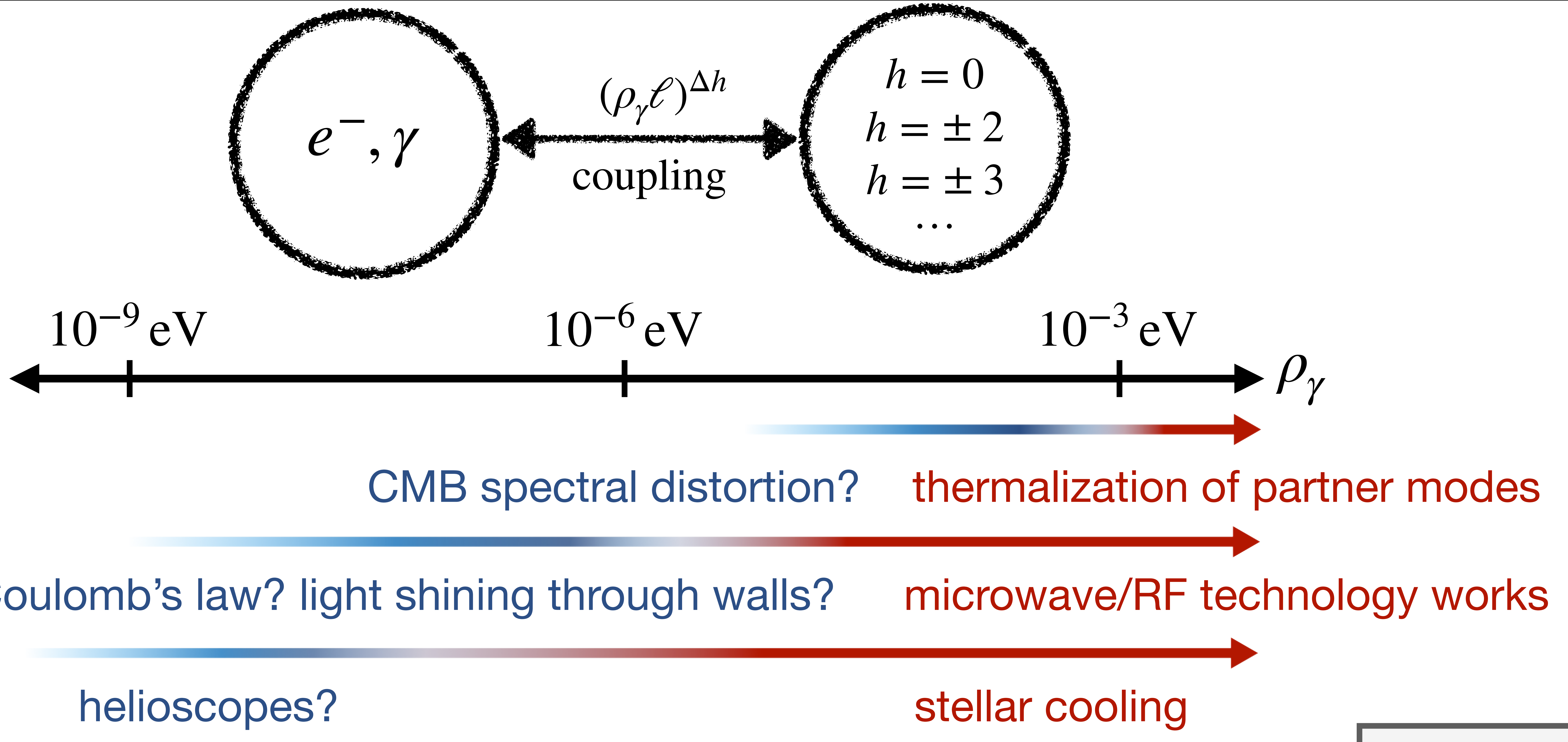
+ currents from fermionic matter worldlines probably similar – but not yet studied

Experimental Opportunities

Continuous spin field with vector-like coupling looks like photon + a dark sector.
Could our photon have non-zero ρ?



Very rough, preliminary estimates!

Can also study graviton's spin scale !

Conclusions

- Lorntz invariance → massless particles have a spin-scale. **Is it zero or non-zero?**
- The non-zero option makes more sense than previously thought, and has testable consequences
- If inconsistent, deserves a proper burial
- If viable, we should think of the Standard Model as an effective theory with both UV and IR completions.