# Non-additive distances and the relativity of the event

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2108.12362
2212.06156 (\w A. Tolley)
in progress (\w F. Nitti and A. Taskov)

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In this talk: Quantum gravity $\neq$ UV

still, $$g_{\mu\nu}(x) \longrightarrow \Psi[g_{\mu\nu}(x), \ldots]$$

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Good observables in QG:

Asymptotically flat: $\mathscr{A} \sim \langle \mathrm{out} \,|\, \mathrm{in} \rangle$

Asymptotically AdS: $\lim\limits_{r\to\infty} \langle \phi(x_1)\ldots\phi(x_n)\rangle$

In this talk: Quantum gravity $\neq$ UV

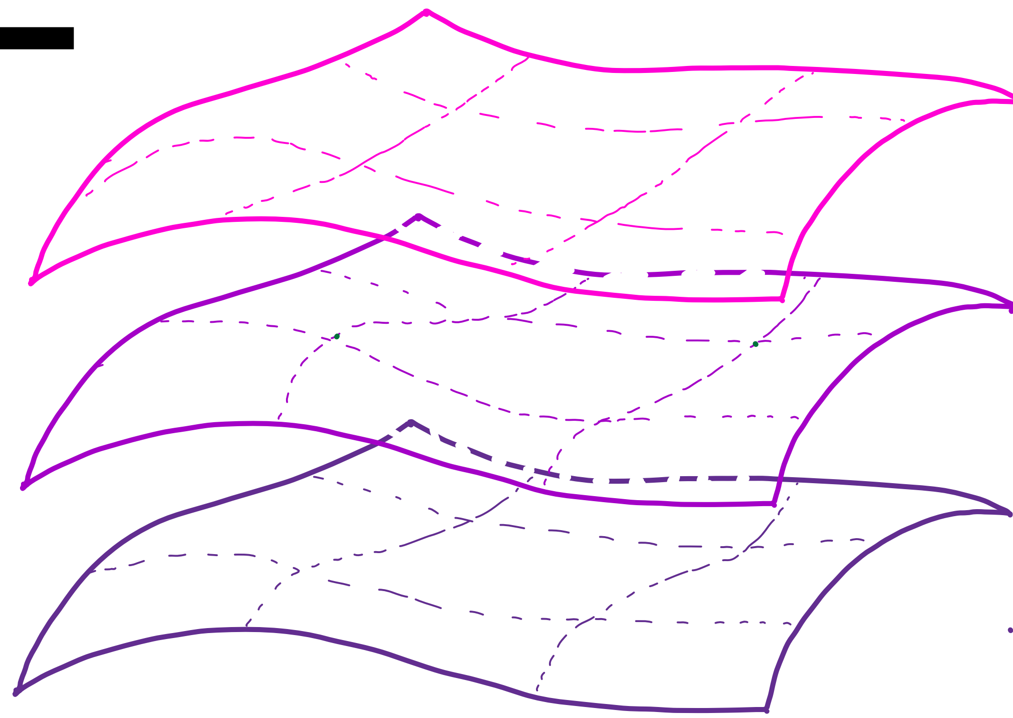
$$g_{\mu\nu}(x) \longrightarrow \Psi[g_{\mu\nu}(x), \ldots]$$

~~Good~~ observables in QG:

Anything more ``local”

?

One known difficulty (gauge invariance) is to identify an ``event” for each off-shell geometry: this is highly arbitrary

$\Psi\Big[$  $\Big]$

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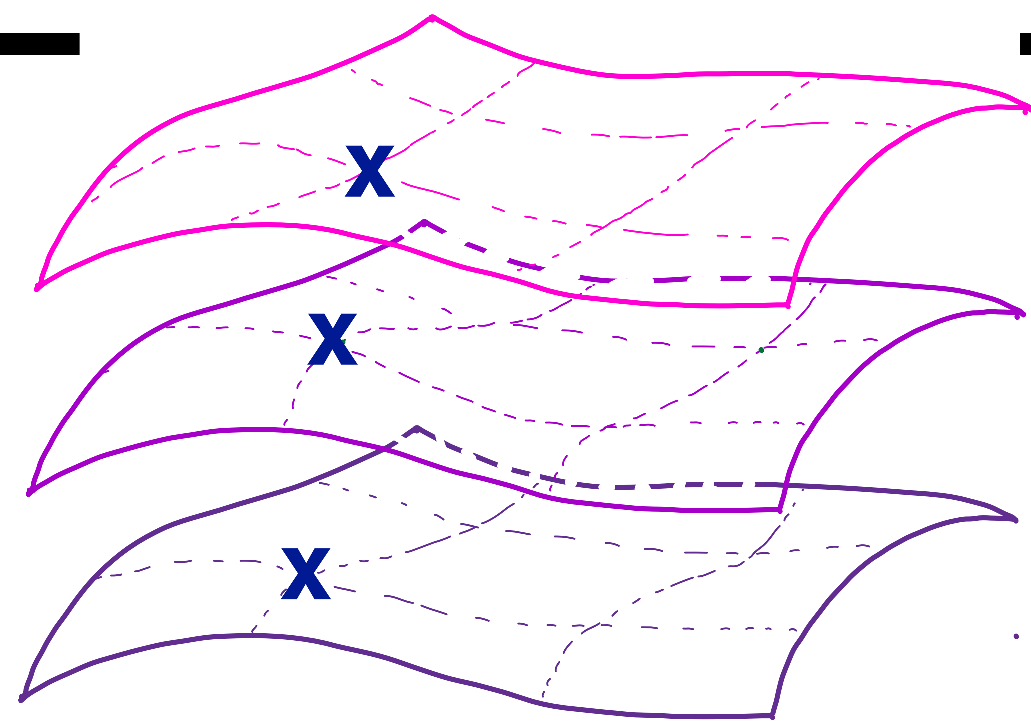
$\Psi\Big[$ 

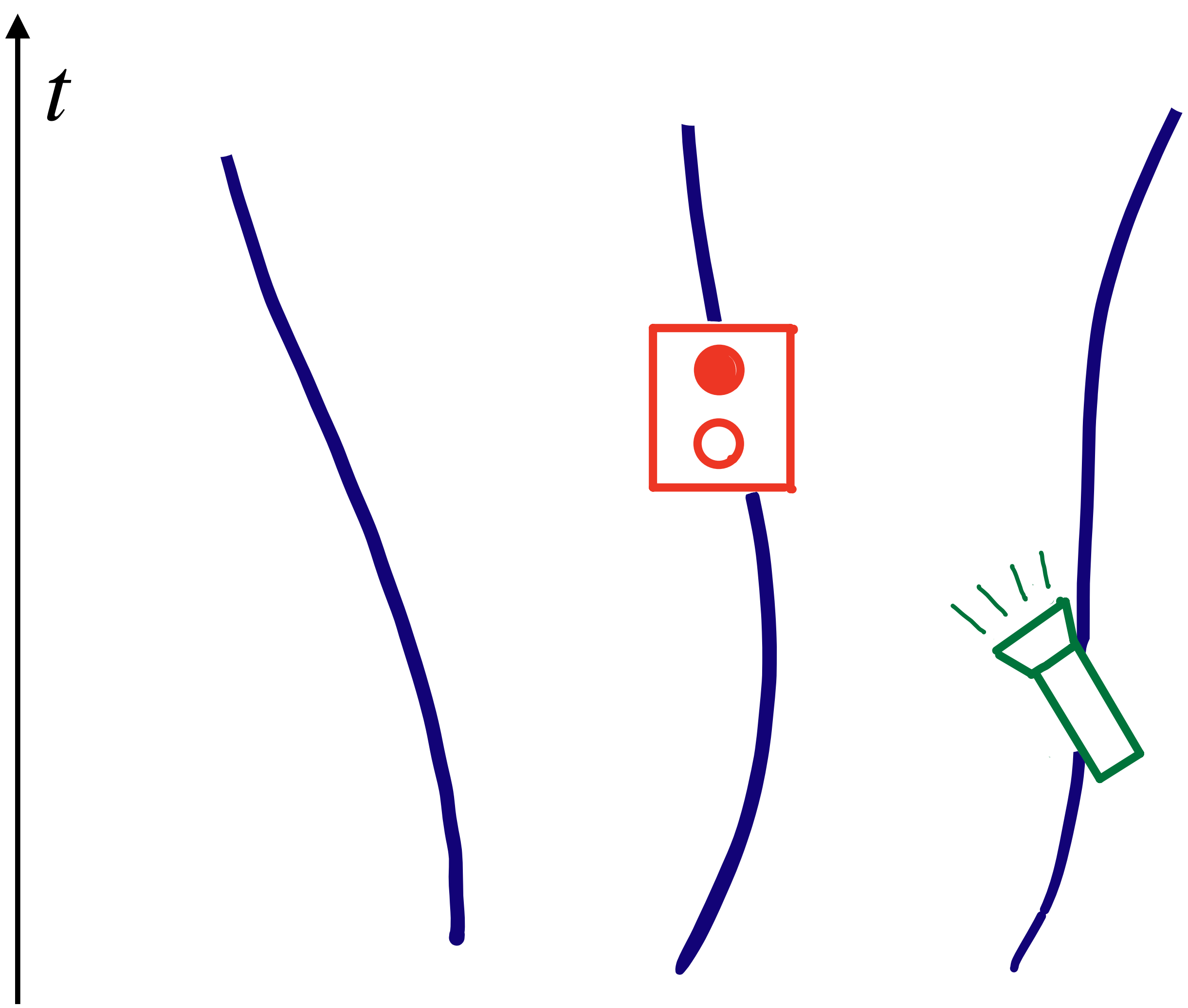
$\Big]$

## However, observers do not care

$$S \supset \sum_i \int dt_i \left[ -m_i + g\sigma_i \phi(X_i^\mu(t_i)) + j_i(t_i)\phi(X_i^\mu(t_i)) \right]$$

Unruh-DeWitt detector

Classical source




The detector clicking is an event!

$$(i^{th}, t_i^{click})$$

# A (non-relativistic) fluid of observers

Dubovsky, Gregoire, Nicolis, Rattazzi, 2006

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R - \mu^4\int d^4x\sqrt{-g}\sqrt{\det(g^{\mu\nu}\partial_\mu x^I\partial_\nu x^J)} + S_m[\Phi] + \ldots$$

– The three scalar fields $x^1, x^2, x^3$ label the observers.
– $x^I = $ const. is a geodesic on each classical solution
– $X^I = x^I$: unitary gauge. $\Psi[h_{ij}(X^i), x^i(X^k), \ldots] \to \Psi_U[h_{ij}(x^I), \ldots]$
– If no vorticity initially $\to N^i = 0$, $x^0$ proper time of the observers

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + \gamma_{ij}dx^i dx^j$$

In the observers' frame one can calculate e.g. $\langle G_{ret}(x,y)\rangle$ and study the causality relations among the observers

It is easier to use a proxy instead

$$\bar{d}(x,y) \;\equiv\; \sqrt{\langle d^2(x,y)\rangle}$$

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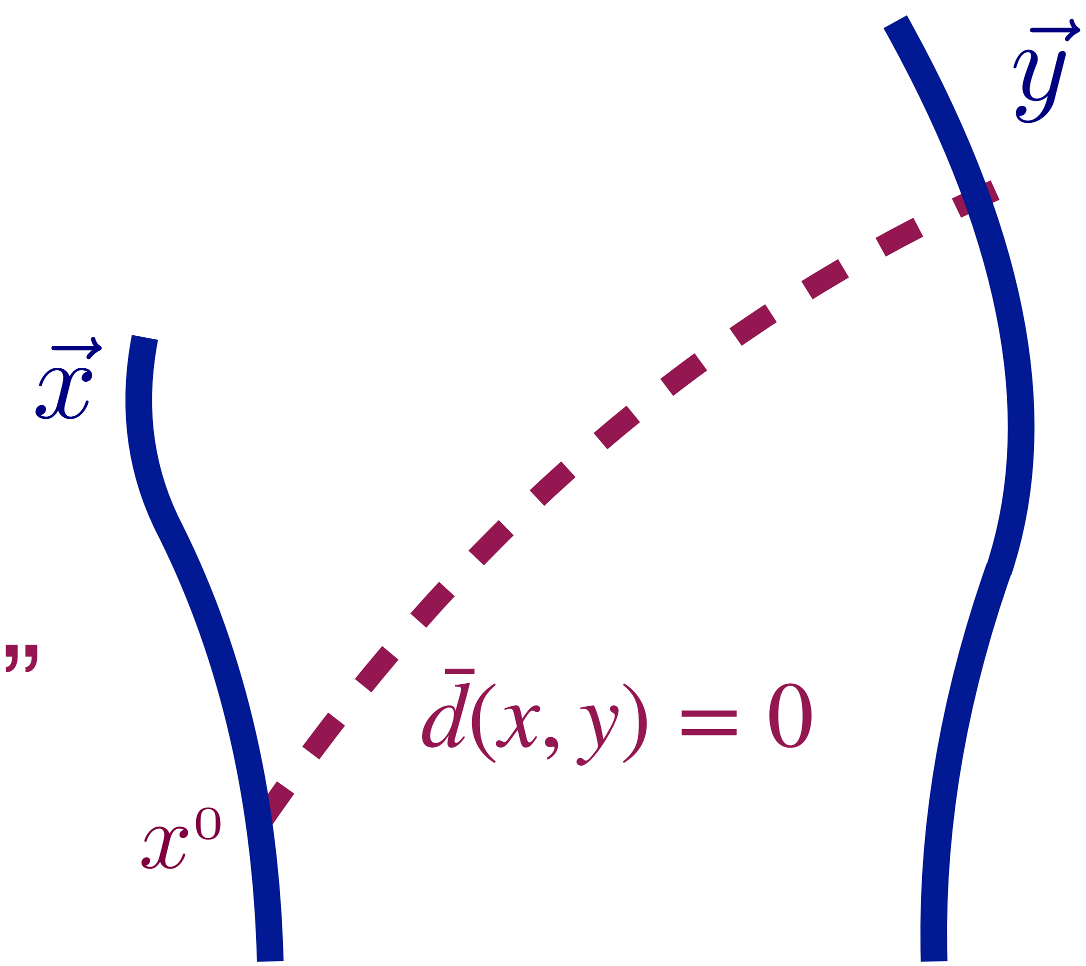
``Average light cone structure"



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$$d^2(0,x) = g_{\mu\nu}x^\mu x^\nu + \frac{1}{2}g_{\mu\nu,\rho}\,x^\mu x^\nu x^\rho - \frac{1}{12}\left(g_{\alpha\beta}\Gamma^\alpha_{\mu\nu}\Gamma^\beta_{\rho\sigma} - 2g_{\mu\nu,\rho\sigma}\right)x^\mu x^\nu x^\rho x^\sigma + \mathcal{O}(x^5)$$

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This is not the geodesic distance of any metric

# Non-additive distances (Euclidean signature)

Problem:

given $d(x, z)$ and $0 < R < d(x, z)$: Find $y$ s.t.

$$d(x, y) = R, \qquad d(y, z) = d(x, z) - R$$

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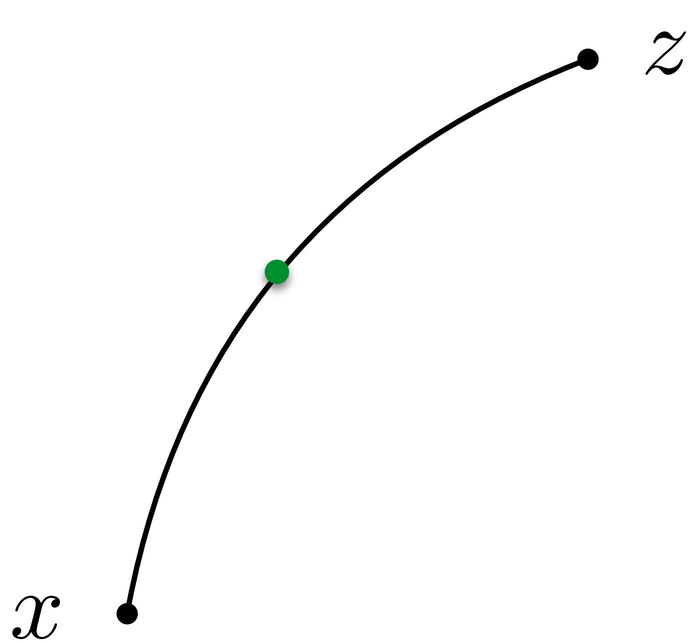
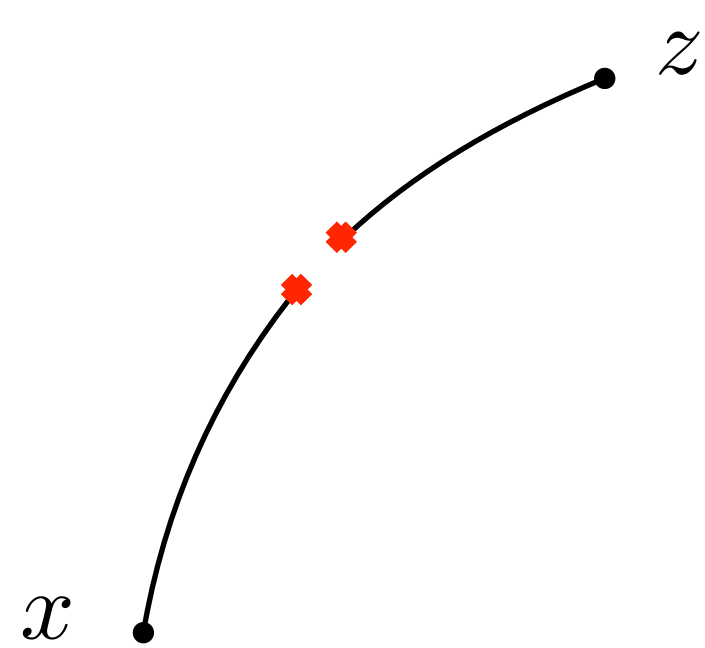
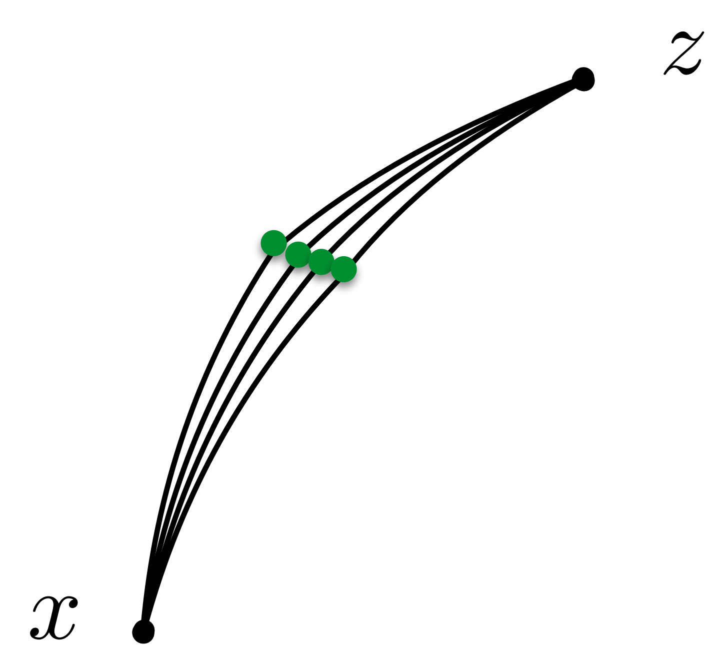



Additive:
only one solution




Subadditive:
no solution




Superadditive:
infinite solutions

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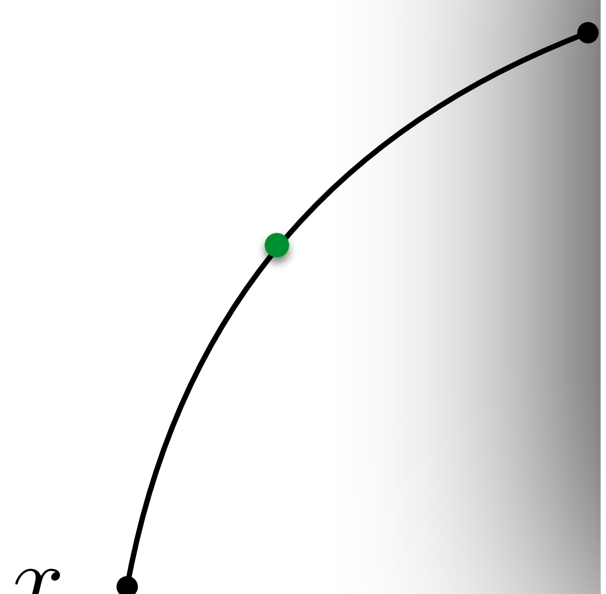
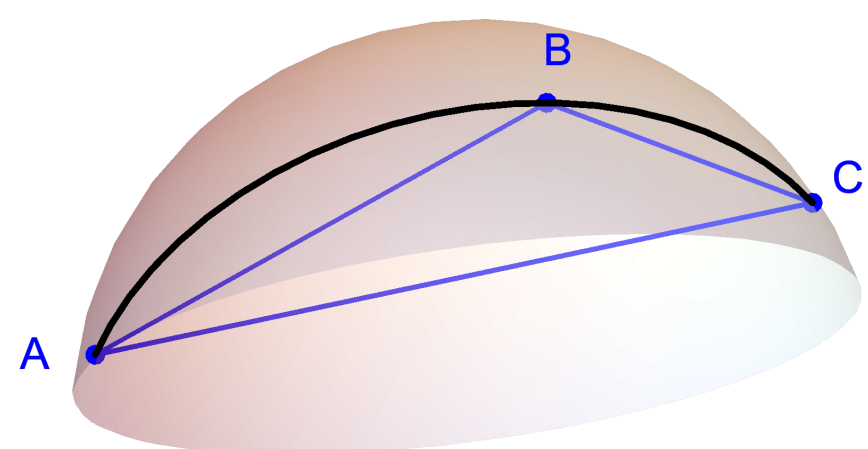
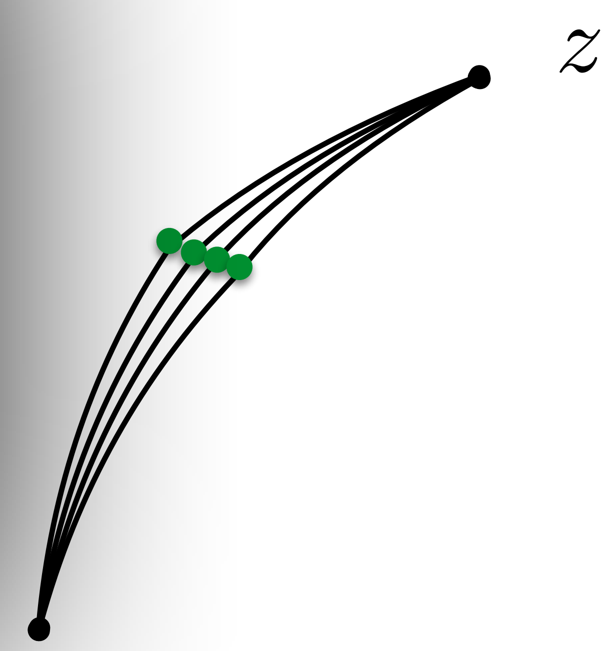



Similar to chordal distances




Additive:
only one solution

Subadditive:
~~no solution~~

Superadditive:
infinite solutions

# Non-additive distances

$d(x, y) \to g_{\mu\nu}(x)$ Always possible

$$\boxed{g_{\mu\nu}(x) \equiv -\frac{1}{2} \lim_{y \to x} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu} d^2(x, y)}$$

$g_{\mu\nu}(x) \to d(x, y)$ Only if $d(x, y)$ is additive

## Chordal distance analogy

You need e.g. extrinsic curvature to calculate $d(x, y)$

# Measuring non-additivity

If additive, $d(x, y)$ has unit gradient

(

Hamilton-Jacobi equation for a particle $S = -m \int d\tau \sqrt{-g^{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}}$

$$g^{\mu\nu}\partial_\mu S \partial_\nu S + m^2 = 0$$

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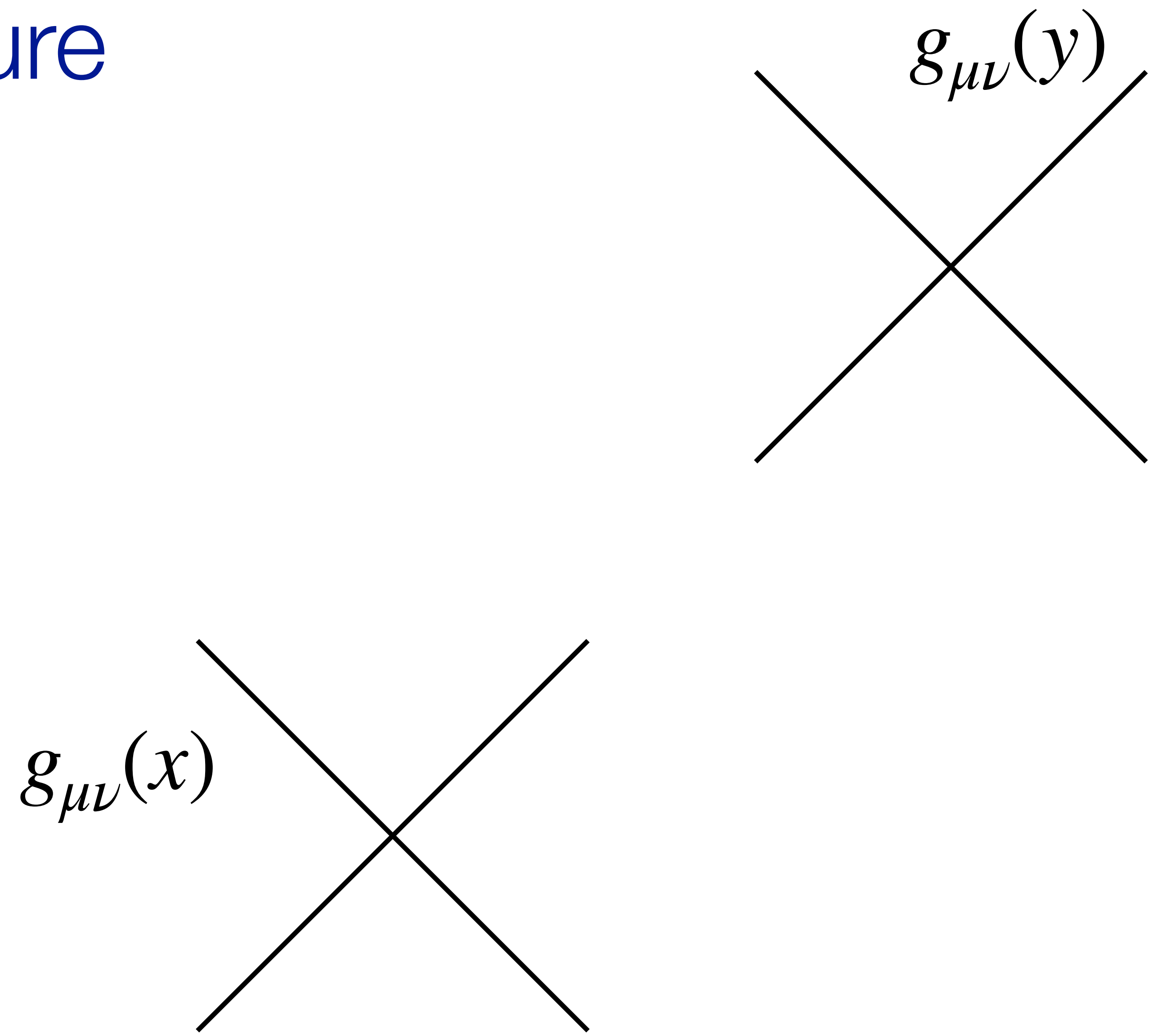
$$C(x,y) \equiv \frac{1}{4} \frac{\partial\, d^2(x,y)}{\partial y^\mu} \frac{\partial\, d^2(x,y)}{\partial y^\nu} \; g^{\mu\nu}(y) - d^2(x,y)$$

Additive:
$C = 0$

Subadditive:
$C < 0$

Superadditive:
$C > 0$

# Lorentz signature

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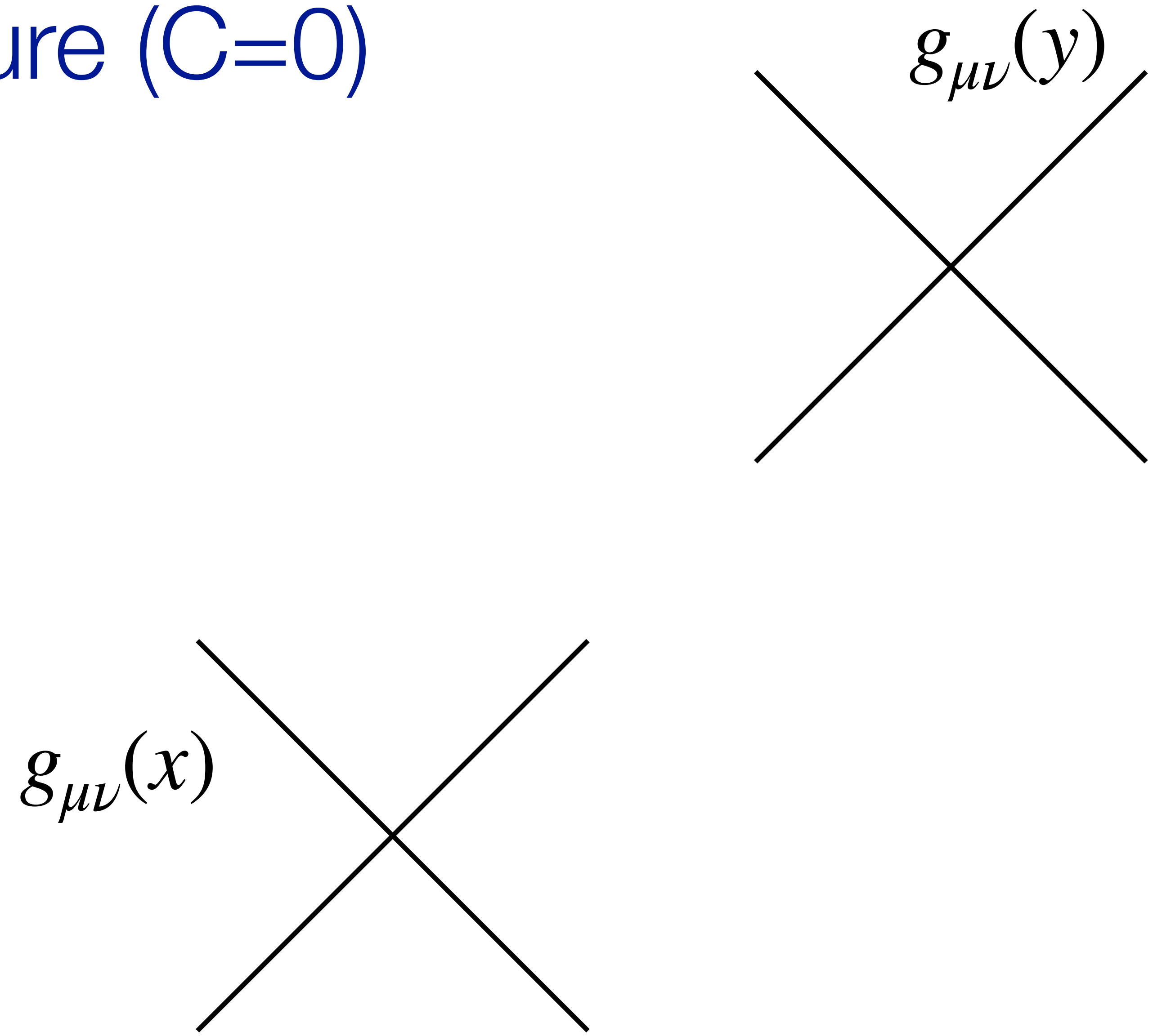
# Lorentz signature (C=0)

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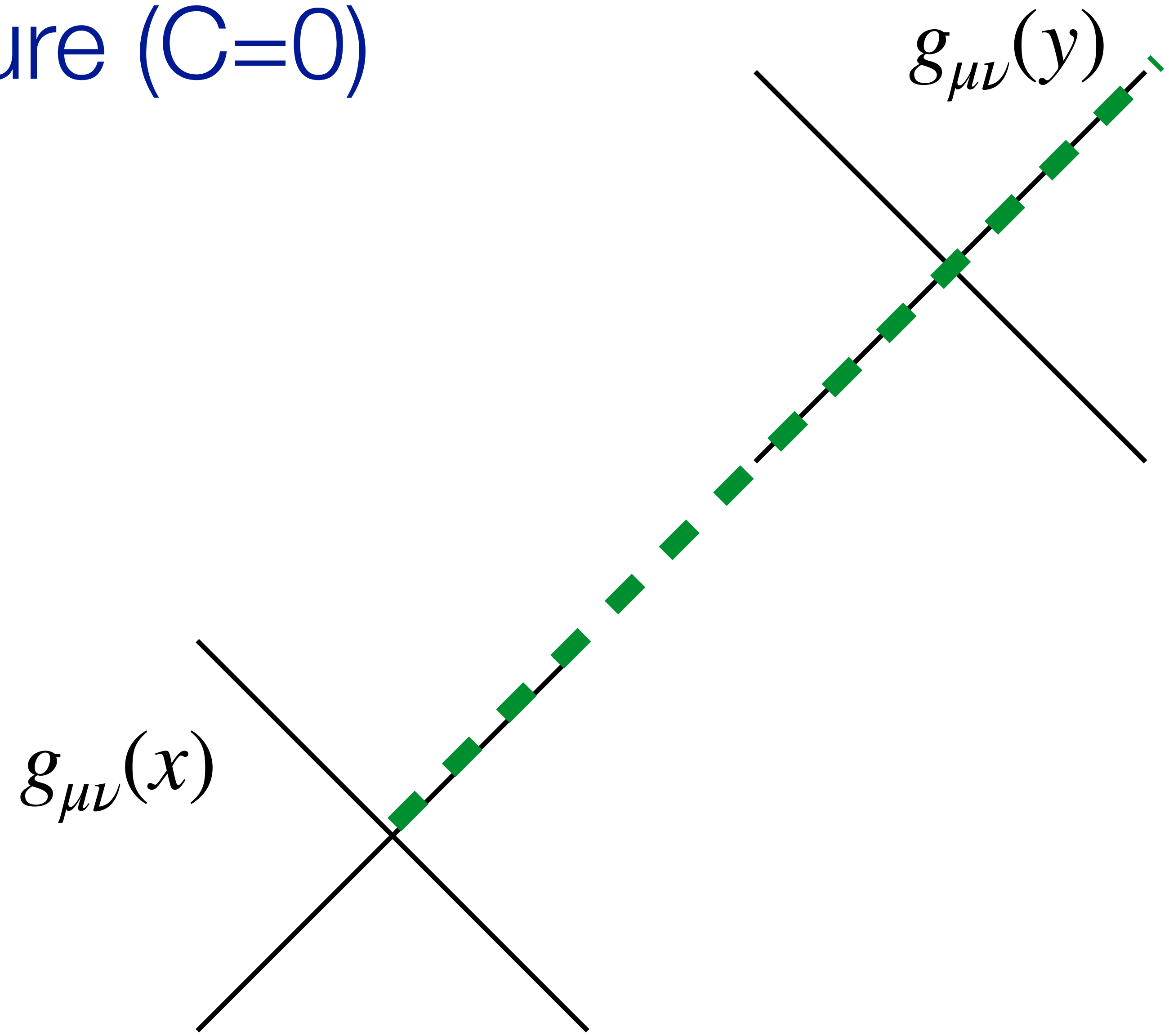


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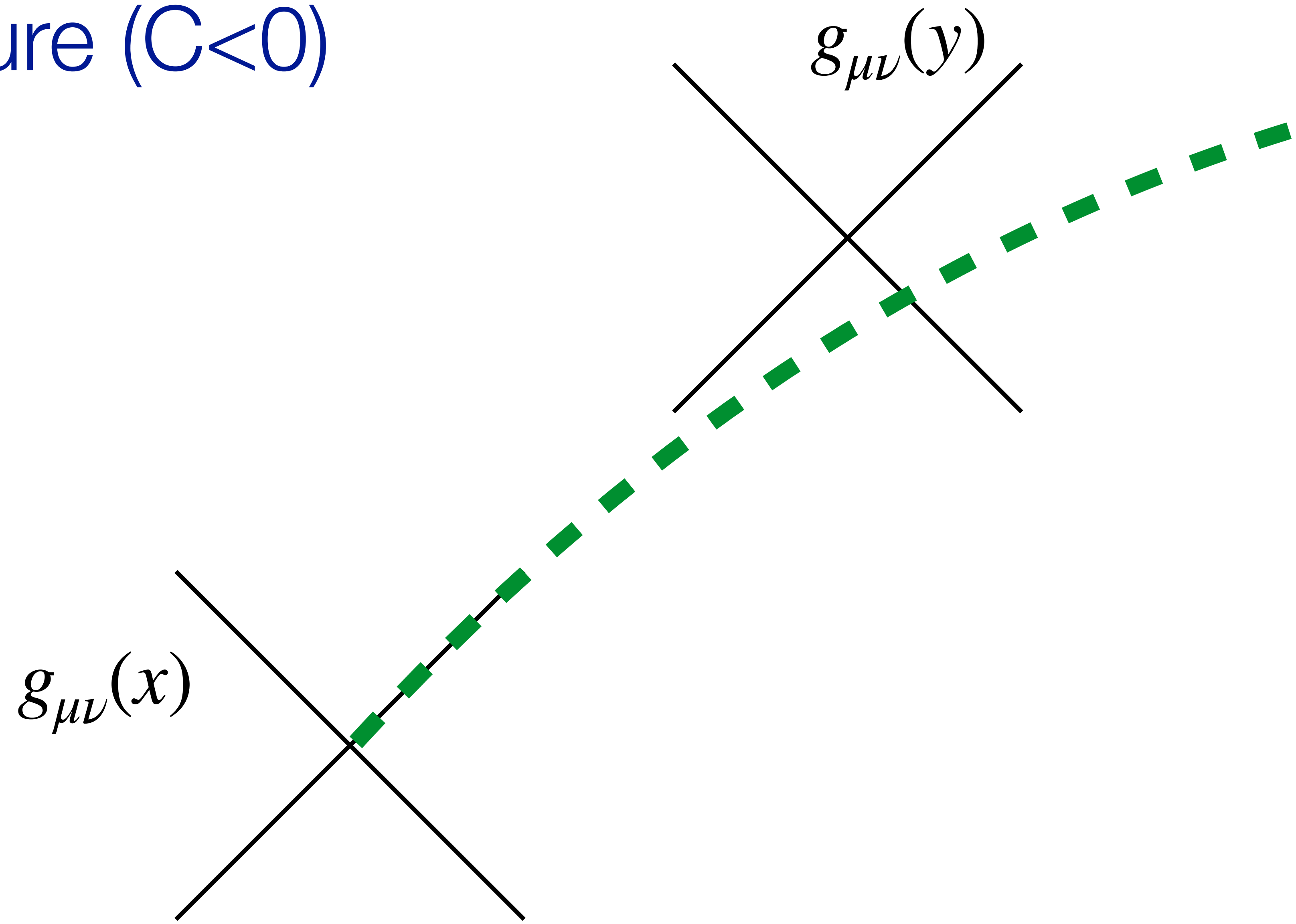
# Lorentz signature (C<0)

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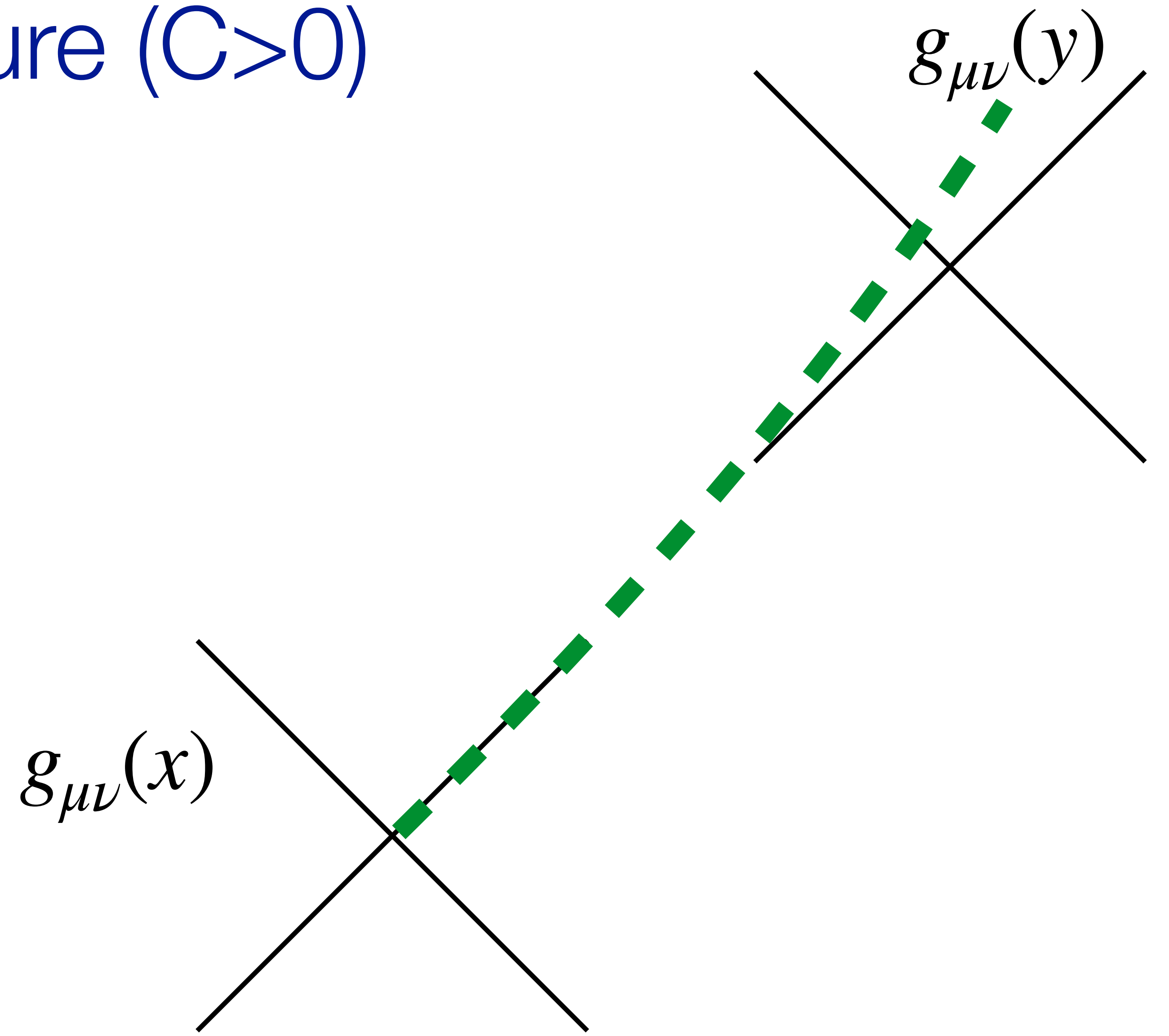
# Lorentz signature (C>0)

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# Calculating non-additivity

Coordinate expansion:

$$C(0,x) = \frac{1}{4}\left(\bar{g}^{\alpha\beta}\langle\Gamma_{\alpha\mu\nu}\rangle\langle\Gamma_{\beta\rho\sigma}\rangle - \langle g_{\alpha\beta}\Gamma^{\alpha}_{\mu\nu}\Gamma^{\beta}_{\rho\sigma}\rangle\right) x^{\mu}x^{\nu}x^{\rho}x^{\sigma} + \mathcal{O}(x^5)$$

Non-additivity builds up at large separation

# Calculating non-additivity

Observers' frame: $$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + \gamma_{ij}dx^i dx^j$$

$$\begin{aligned} C = \frac{1}{4}\Bigg[&\frac{1}{4}\left(\langle\dot\gamma_{ij}\dot\gamma_{lk}\rangle - \langle\dot\gamma_{ij}\rangle\langle\dot\gamma_{lk}\rangle\right)x^i x^j x^k x^l \\ &- \left(\langle\gamma^{pq}\dot\gamma_{pi}\dot\gamma_{qj}\rangle - \bar\gamma^{pq}\langle\dot\gamma_{pi}\rangle\langle\dot\gamma_{qj}\rangle\right)t^2 x^i x^j \\ &- 2\left(\langle\Gamma^p_{ij}\dot\gamma_{pk}\rangle - \bar\gamma^{pq}\langle\Gamma_{pij}\rangle\langle\dot\gamma_{qk}\rangle\right)t\, x^i x^j x^k \\ &- \left(\langle\gamma^{pq}\Gamma_{pij}\Gamma_{qkl}\rangle - \bar\gamma^{pq}\langle\Gamma_{pij}\rangle\langle\Gamma_{qkl}\rangle\right)x^i x^j x^k x^l\Bigg]\,, \end{aligned}$$

← positive definite

← negative definite

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E.g. Thermal state of gravitons: $$C(0,x) \simeq -\frac{T^4}{M_P^2}x^4$$

effect important at $\ell \sim \dfrac{M_P}{T^2}$

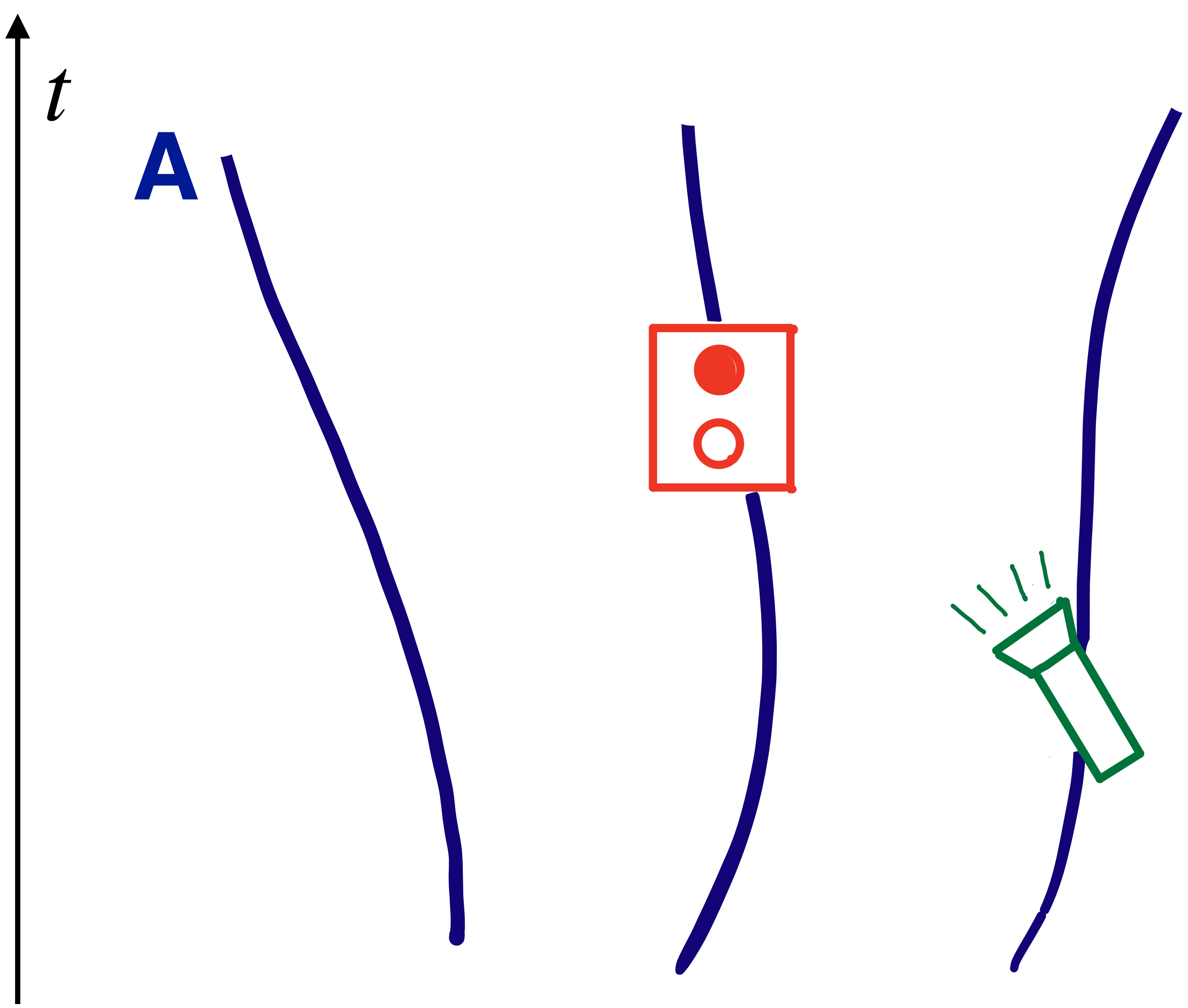
→ $\sim H^{-1}$ IF gravitons were in eq.

→ $\sim t_{page}/\sqrt{S}$

(With F. Nitti and A. Taskov, to appear)

# The relativity of the event

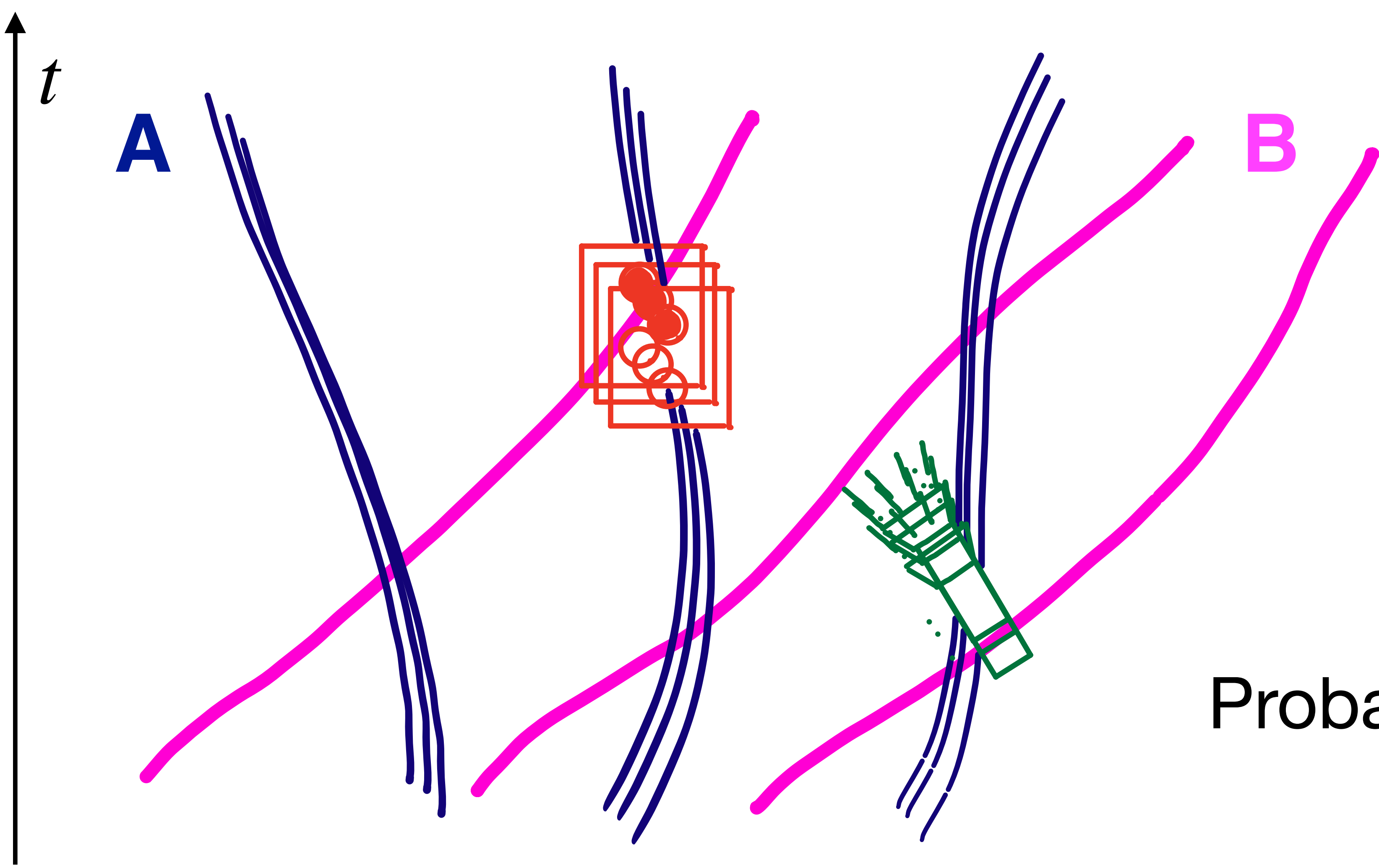
The clicking has definite coordinates $x_A^{click}$ in the A-frame



(With  F. Nitti and A. Taskov, to appear)

## The relativity of the event

In any other frame (e.g. that of a boosted set of observers B) it is has indefinite coordinates




$$P(x_B | x_A^{click}; \Psi)$$

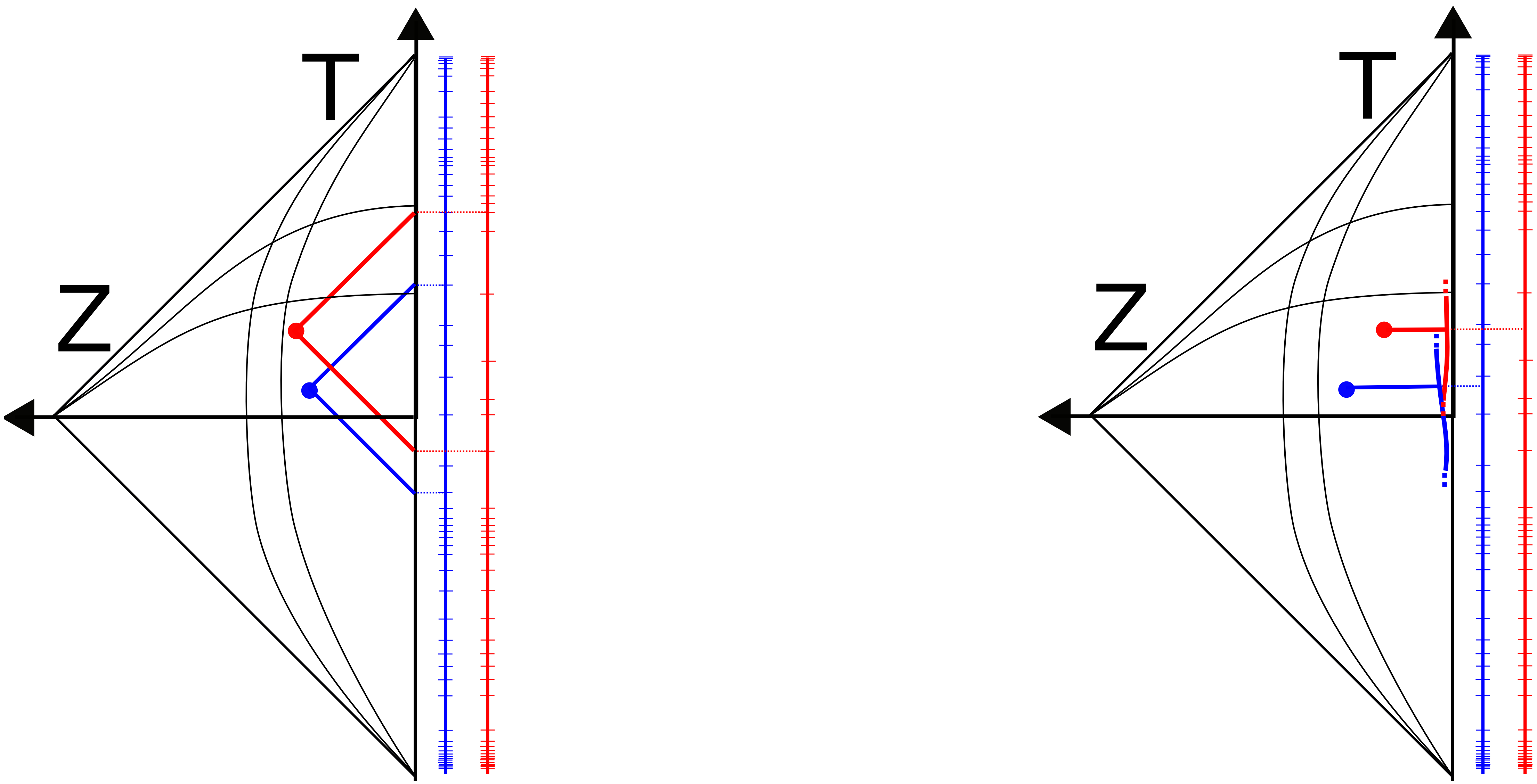
Probabilistic coordinate transf.

(With F. Nitti and A. Taskov, to appear)

# Application: the event horizon in JT gravity

Different ``frames" discussed in

Blommaert, Mertens, Verschelde ``Rods and Clocks in JTgravity" 1902.11194



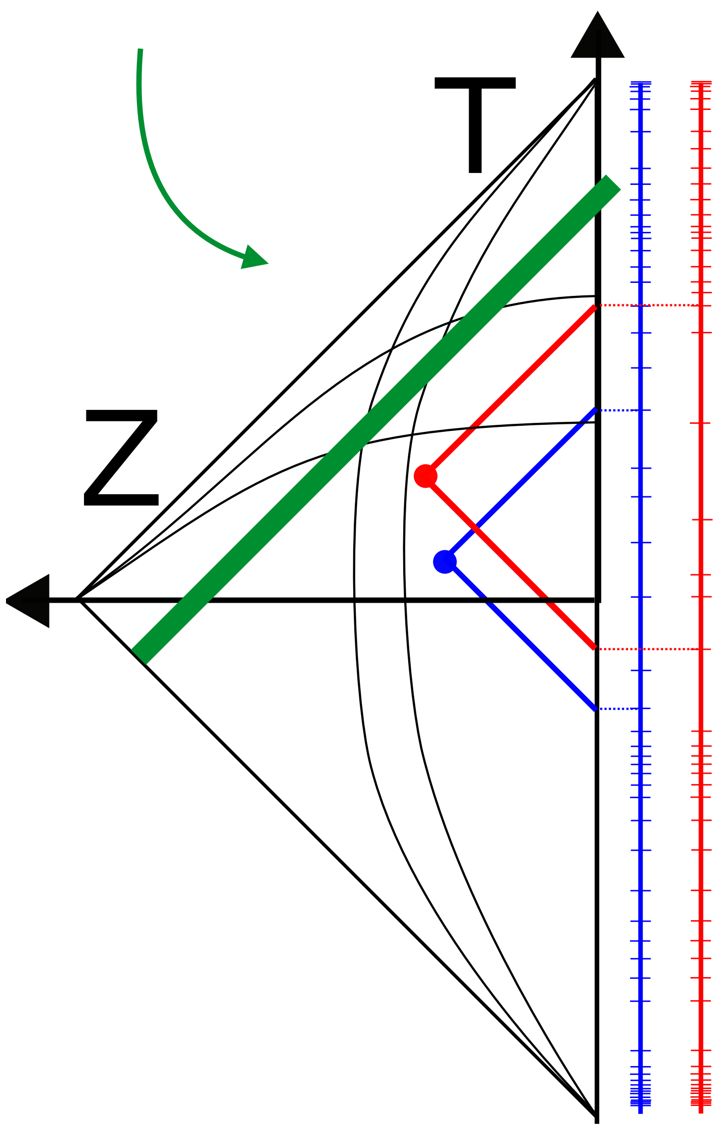
(With F. Nitti and A. Taskov, to appear)

# Application: the event horizon in JT gravity

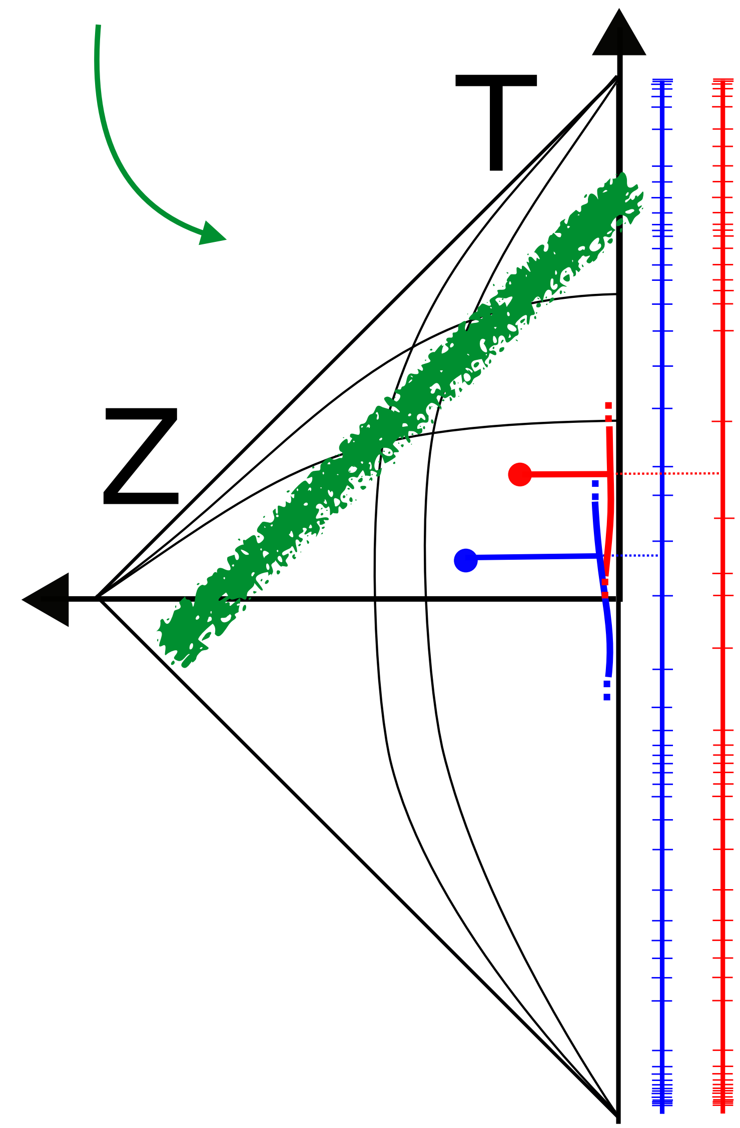
Different ``frames” discussed in

Blommaert, Mertens, Verschelde ``Rods and Clocks in JTgravity” 1902.11194

definite surface in the ``lightlike frame”




fuzzy in the ``spacelike geodesics frame”

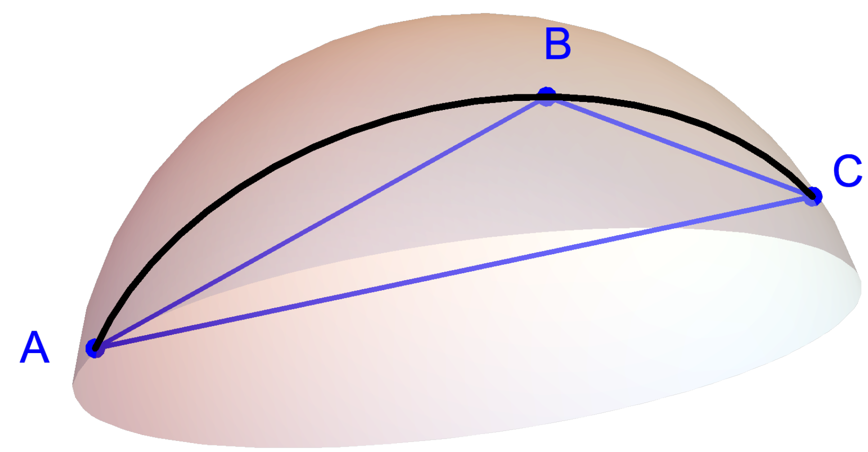


# BACKUP SLIDES

Result in Euclidean signature:

Average distances always *subadditive*

Similar to chordal distances



$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + \gamma_{ij}dx^i dx^j$$

# Lorentz signature (unitary gauge)

$$\begin{aligned} C = \frac{1}{4}\Big[&\frac{1}{4}\left(\langle\dot\gamma_{ij}\dot\gamma_{lk}\rangle - \langle\dot\gamma_{ij}\rangle\langle\dot\gamma_{lk}\rangle\right)x^i x^j x^k x^l \\ &- \left(\langle\gamma^{pq}\dot\gamma_{pi}\dot\gamma_{qj}\rangle - \bar\gamma^{pq}\langle\dot\gamma_{pi}\rangle\langle\dot\gamma_{qj}\rangle\right)t^2 x^i x^j \\ &- 2\left(\langle\Gamma^p_{ij}\dot\gamma_{pk}\rangle - \bar\gamma^{pq}\langle\Gamma_{pij}\rangle\langle\dot\gamma_{qk}\rangle\right)t\, x^i x^j x^k \\ &- \left(\langle\gamma^{pq}\Gamma_{pij}\Gamma_{qkl}\rangle - \bar\gamma^{pq}\langle\Gamma_{pij}\rangle\langle\Gamma_{qkl}\rangle\right)x^i x^j x^k x^l\Big]\ , \end{aligned}$$

— No non-additivity along time ($\overrightarrow{x} = 0$).

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— No non-additivity along time ($\overrightarrow{x} = 0$).

— Negative definite pieces

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + \gamma_{ij}dx^i dx^j$$

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— No non-additivity along time ($\vec{x} = 0$).

— Negative definite pieces

— Positive definite

# Examples:

– Superposition of plane waves: $C < 0$

– Fluctuations around homogeneous background: $C < 0$

Thermal state of gravitons: $$C(0,x) \simeq \frac{T^4}{M_P^2}\Delta x^4$$

effect important at $\ell \sim \frac{M_P}{T^2}$

– FRW: $C < 0$ if $w > -\frac{1}{3}$

# Causality

Given $\langle d^2(x,y)\rangle$ one can define a metric tensor $\langle g_{\mu\nu}\rangle = \bar{g}_{\mu\nu}$.

$$\bar{g}_{\mu\nu}(x) \;\equiv\; -\frac{1}{2}\lim_{y\to x}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\langle d^2(x,y)\rangle$$

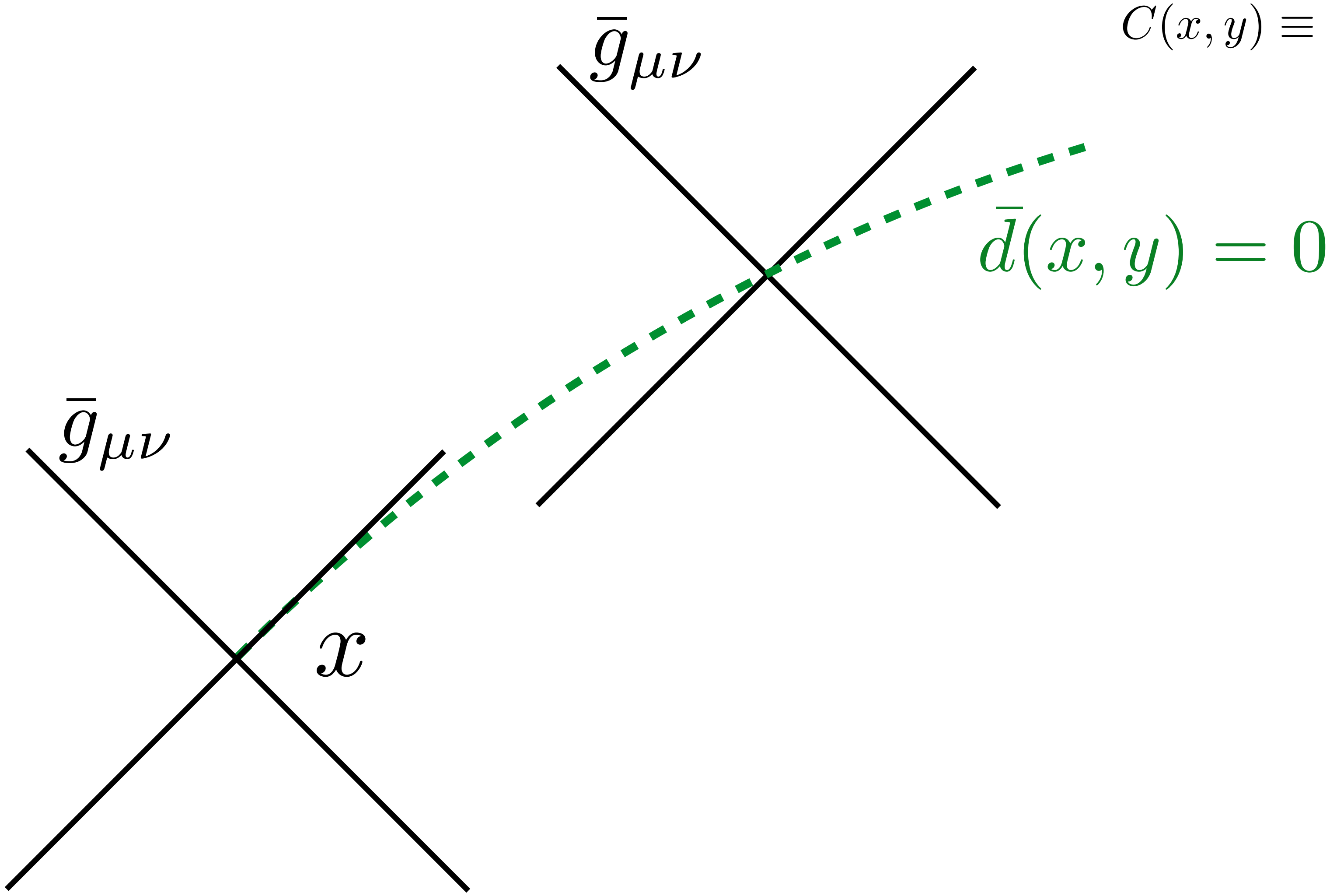
But there is more to $\langle d^2(x,y)\rangle$ than $\langle g_{\mu\nu}\rangle$ !

$\langle g_{\mu\nu}\rangle \Delta x^\mu \Delta x^\nu = 0$: where we expect the photon to be detected in the immediate vicinity of the emission.

Further away: see where $\langle d^2(x,y)\rangle = 0$

# Subadditive causality ($C < 0$)

$$C(x,y) \equiv \frac{1}{4}\,\frac{\partial\, d^2(x,y)}{\partial y^\mu}\,\frac{\partial\, d^2(x,y)}{\partial y^\nu}\; g^{\mu\nu}(y) - d^2(x,y)$$



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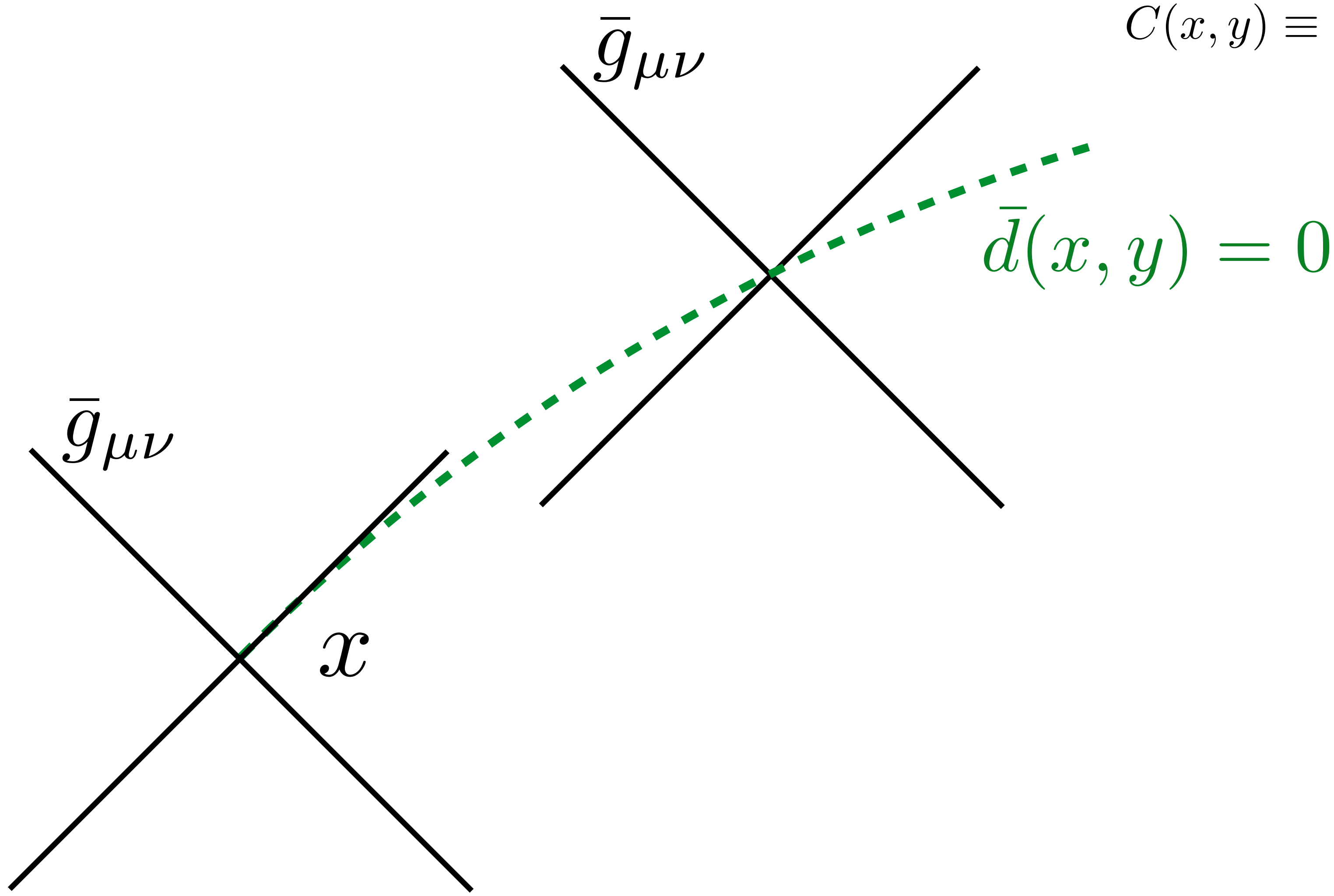


Two causal structures at play. One *rigid* defined at each point. One dependent on the two extremes $x$ and $y$.

Photons are ``prompt" wrt the rigid structure given by $\bar g_{\mu\nu}$

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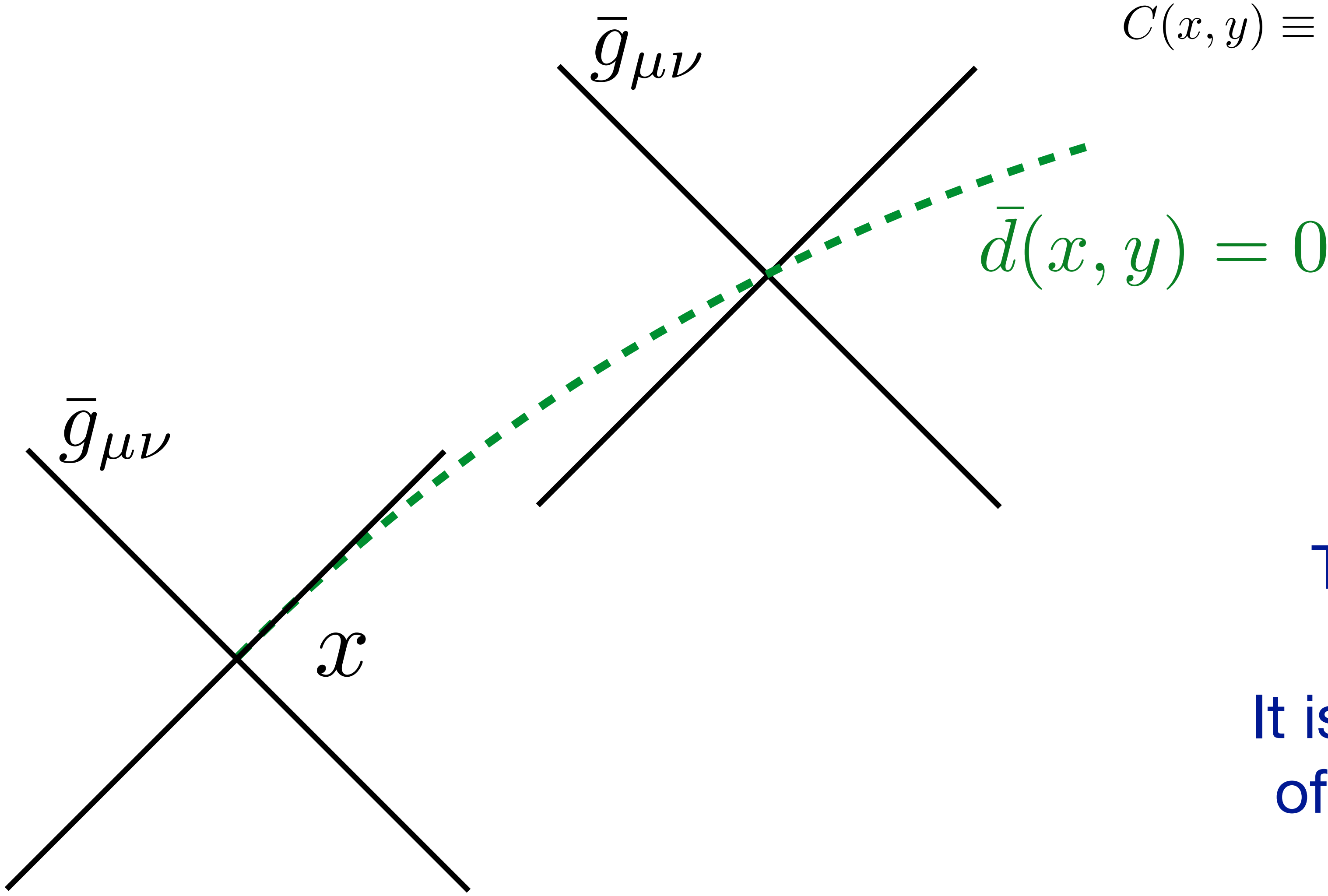


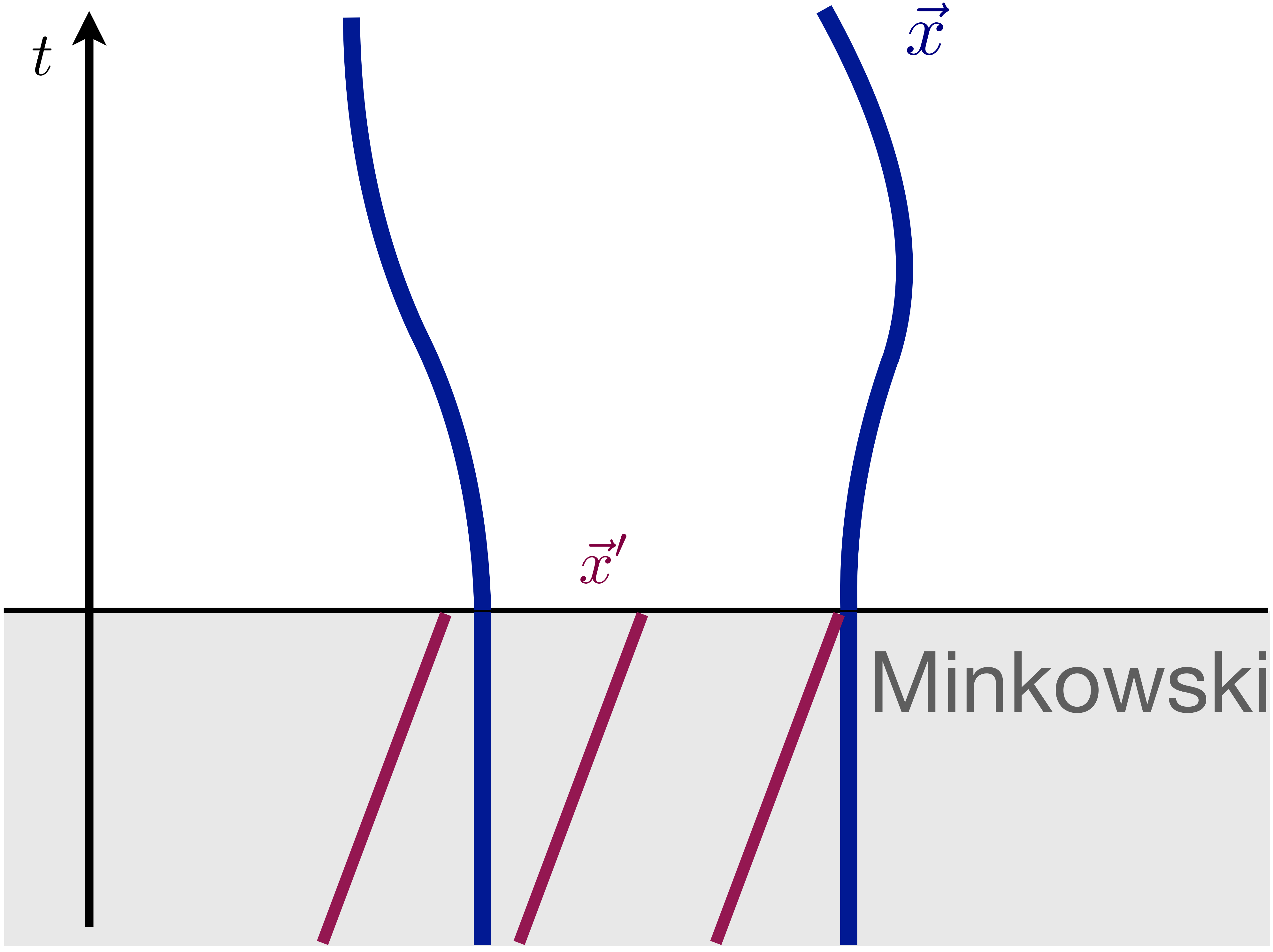

This is not the trajectory
of any light ray.
It is just where the ensemble
of events where we expect
to receive it

Two causal structures at play. One *rigid* defined at each point. One dependent on the two extremes $x$ and $y$.

Photons are ``prompt” wrt the rigid structure given by $\bar g_{\mu\nu}$

(w/ F. Nitti, A. Taskov,
A. Tolley, to appear)
see also 1902.11194

# Relativity 2.0: the relativity of the event



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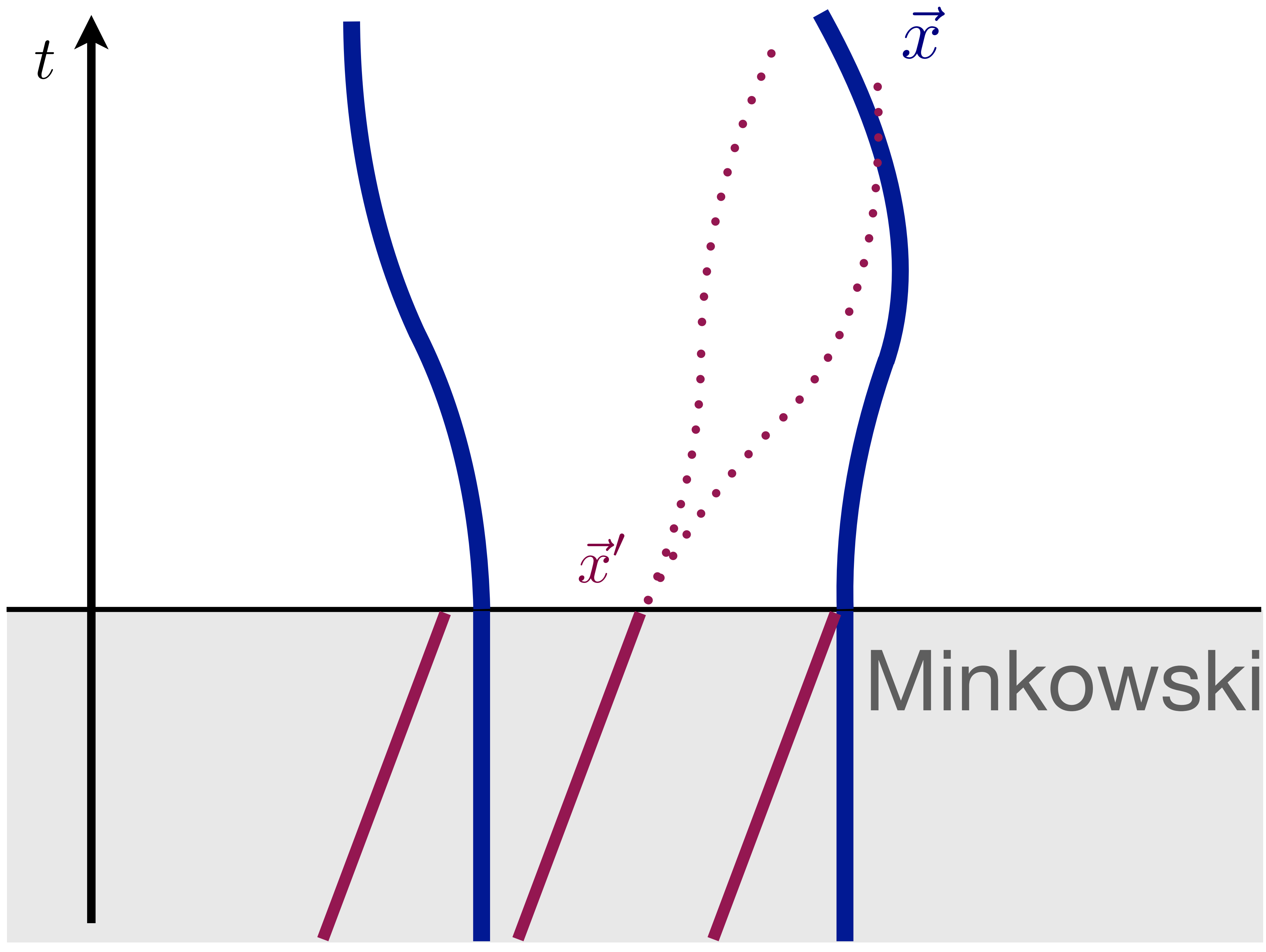

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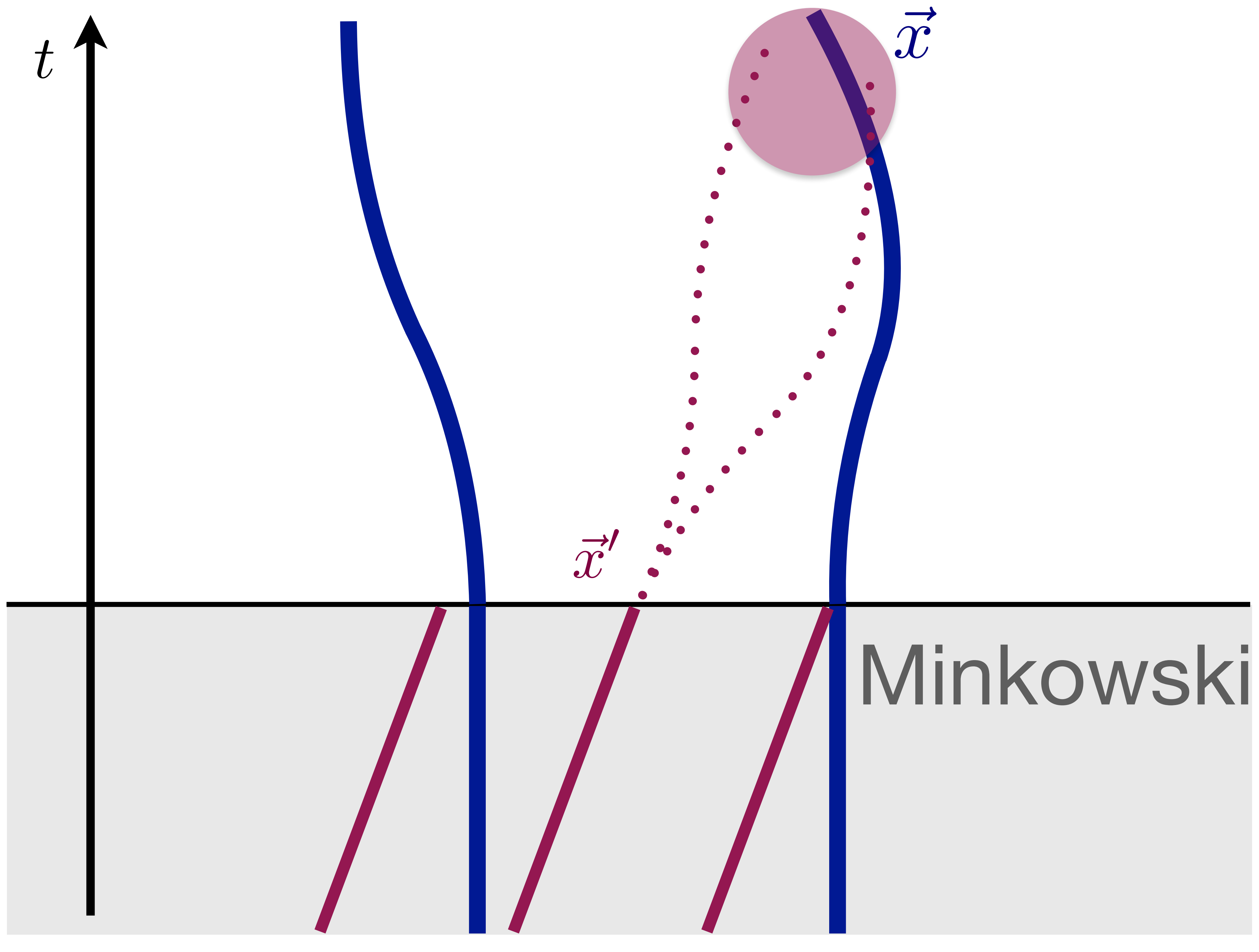

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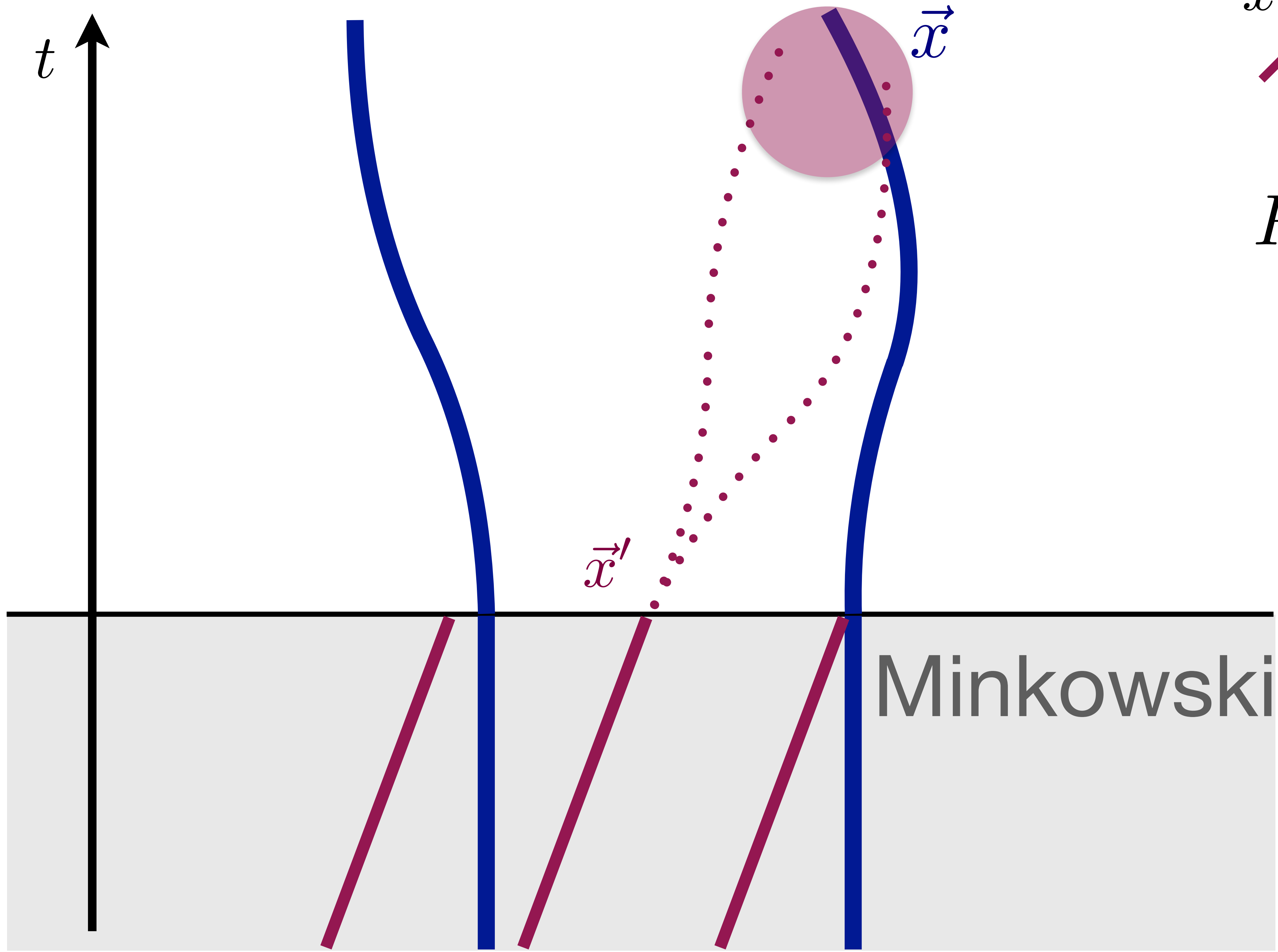
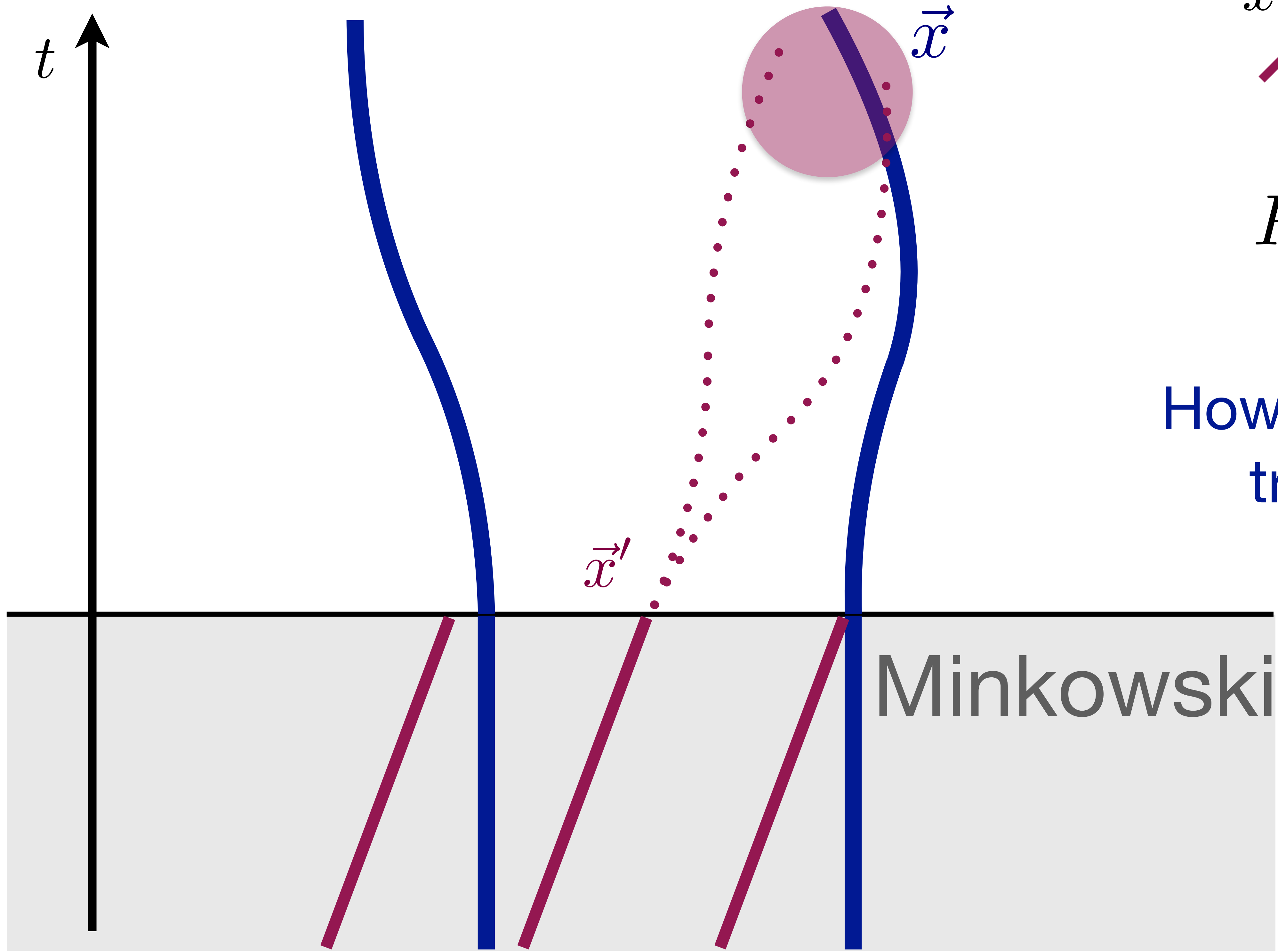



$$\cancel{x' = x'(x)}$$

$$P(x'|x)$$

(w/ F. Nitti, A. Taskov,
A. Tolley, to appear)
see also 1902.11194

# Relativity 2.0: the relativity of the event




$$\cancel{x' = x'(x)}$$

$$P(x'|x)$$

How does $\bar{d}(x, y)$ transform?

## Conclusions:

- The metric is not enough!
- Effect generically small in perturbative situations
- A lot of potential applications
- New mathematical structures…?

# Distance within a normal neighborhood

Geodesic distance can be expressed in a coordinate expansion

$$\left\langle d^2(0,x)\right\rangle = \left\langle g_{\mu\nu}\right\rangle x^\mu x^\nu + \frac{1}{2}\left\langle g_{\mu\nu,\rho}\right\rangle x^\mu x^\nu x^\rho - \frac{1}{12}\left(\left\langle g_{\alpha\beta}\Gamma^\alpha_{\mu\nu}\Gamma^\beta_{\rho\sigma}\right\rangle - 2\left\langle g_{\mu\nu,\rho\sigma}\right\rangle\right) x^\mu x^\nu x^\rho x^\sigma + \mathcal{O}(x^5)$$

We want to evaluate $\quad \bar{d}(x,y) \equiv \sqrt{\langle d^2(x,y)\rangle}$

The unitary gauge coordinates $x$ drop from averages

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The unitary gauge coordinates $x$ drop from averages

$$\langle d^2(0,x)\rangle = \langle g_{\mu\nu}(0)\rangle x^\mu x^\nu + \ldots$$

$$\bar{g}_{\mu\nu}(x) \equiv -\frac{1}{2}\lim_{y\to x}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}\bar{d}^2(x,y)$$

The metric tensor defined locally with $\bar{d}(x,y)$ is nothing else than $\langle g\rangle$ !

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Terms higher than linear cannot be reproduced by an average metric

$$C(0,x) = \frac{1}{4}\left(\bar{g}^{\alpha\beta}\langle\Gamma_{\alpha\mu\nu}\rangle\langle\Gamma_{\beta\rho\sigma}\rangle - \langle g_{\alpha\beta}\Gamma^\alpha_{\mu\nu}\Gamma^\beta_{\rho\sigma}\rangle\right) x^\mu x^\nu x^\rho x^\sigma + \mathcal{O}(x^5)$$

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Non-additivity builds up at large separation. Can we infer about the sign?

$$C(0,x) = -\frac{1}{4}\langle Q_a \eta^{ab} Q_b\rangle\,,$$

$$Q_a = \left(e^\alpha_a\,\Gamma_{\alpha\mu\nu} - e_{\beta a}\,\bar{g}^{\alpha\beta}\langle\Gamma_{\alpha\mu\nu}\rangle\right) x^\mu x^\nu\,,$$

# We can actually calculate it!

Example: thermal state of gravitons at temperature $T$

$$C(0,x) \simeq \frac{T^4}{M_P^2}\Delta x^4$$

$\Leftarrow$ effect important at $\ell \sim \dfrac{M_P}{T^2}$

Conjecture: Average distances are generally subadditive in QG

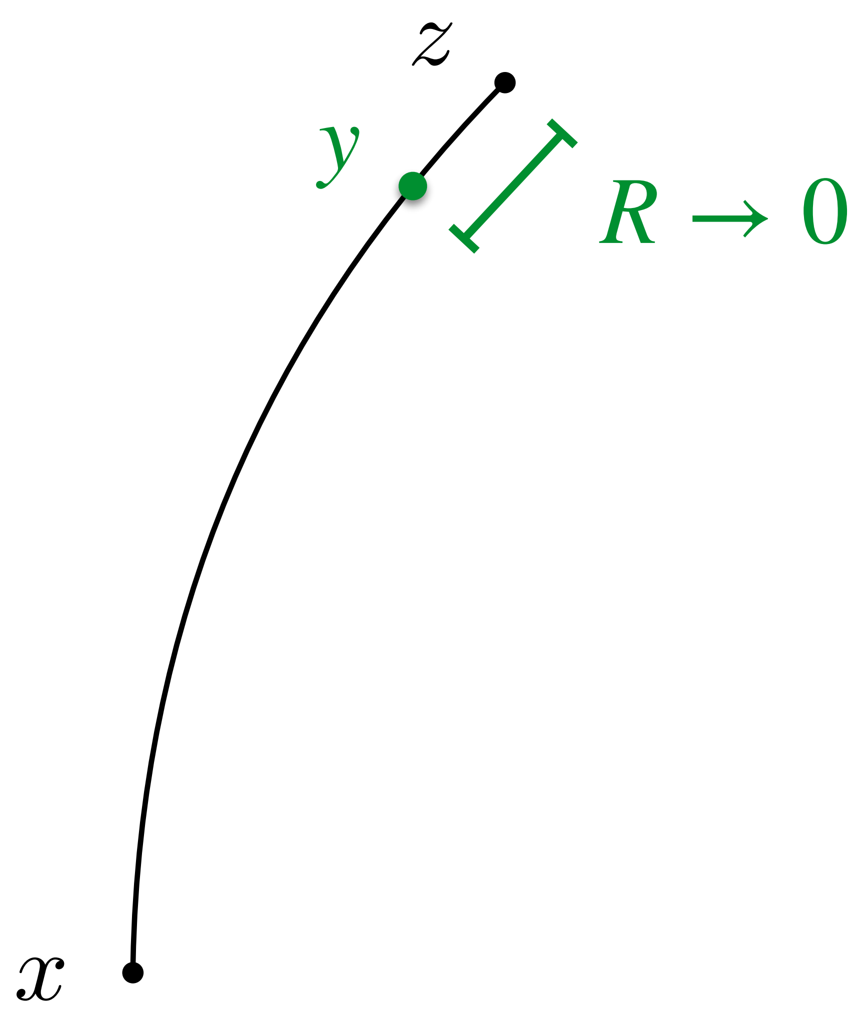
# Superadditive causality ($C > 0$)

$$x \prec y \ \wedge \ y \prec z \quad \not\Rightarrow \quad x \prec z$$

$$\langle[\mathcal{A}(x), \mathcal{A}(y)]\rangle \neq 0, \quad \langle[\mathcal{A}(y), \mathcal{A}(z)]\rangle \neq 0, \quad \langle[\mathcal{A}(x), \mathcal{A}(z)]\rangle \approx 0$$

Conjecture: Subadditivity the outcome of evolution from relatively ``standard” initial conditions

# The Third-Point-Problem: differential version




$$d(y,z) = R$$

$$d(x,y) = d(y,z) + \frac{\partial d(x,z)}{\partial z^i} n^i R$$

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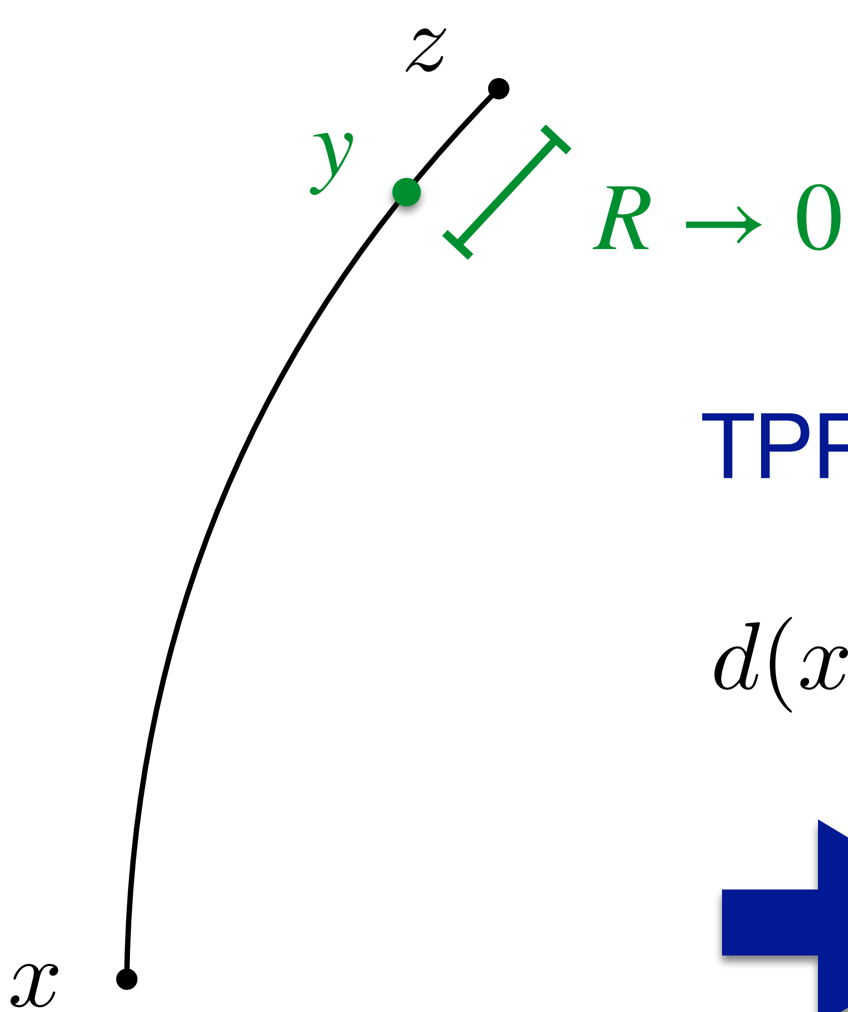


$$d(y,z) = R$$

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TPP:

$$d(x,y) + d(y,z) = d(x,z)$$

$$\boxed{\frac{\partial d(x,z)}{\partial z^i} n^i = -1}$$

The size of the gradient of $d(x,z)$ in $z$ determines how many solutions to the TPP: the character of $d(x,z)$

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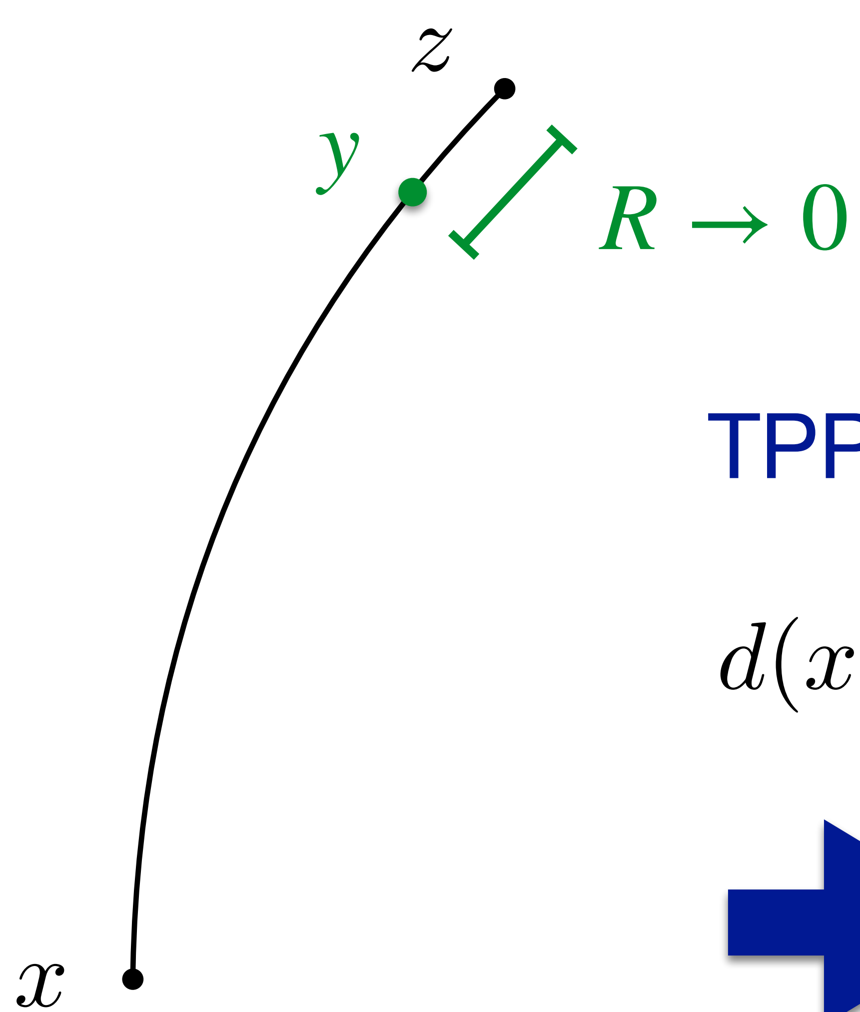


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$$\Rightarrow \quad \boxed{\frac{\partial d(x,z)}{\partial z^i} n^i = -1}$$

The size of the gradient of $d(x,z)$ in $z$ determines how many solutions to the TPP: the character of $d(x,z)$

Additivity = the gradient of $d(x,z)$ in $z$ has unit norm