Simple model for the effect of detuning impedance on beam stability

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- Trying to answer to the question: Why is the horizontal plane more unstable than the vertical one in the presence of detuning impedance? – Preliminary!

Coasting-beam interacting with flat classical RW impedance

- Dispersion relation to solve

$$1 = \Delta Q_z^{x,y}(\Omega) \int \frac{\rho(\omega)\, d\omega}{Q_c - Q_{x,y}(\omega)}$$

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$$= \Delta Q_i^y(\omega)$$

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- This leads to

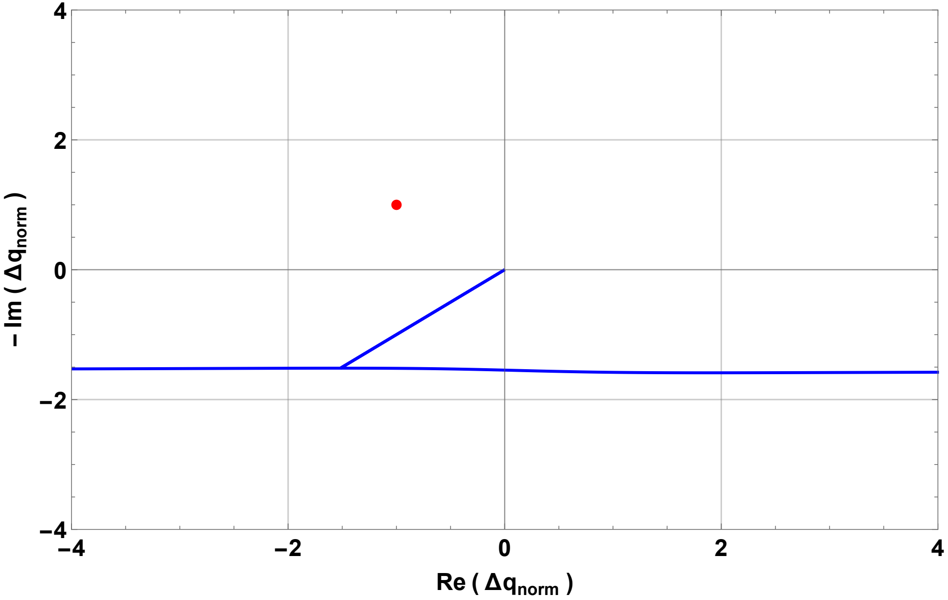
$$q = \frac{Q_c - Q_{x0,y0}}{A/\sqrt{\Omega}}$$

$$-(1+j) = \frac{1}{\int \frac{f(x)dx}{q-(1+j)/\sqrt{x}}} \qquad -2(1+j) = \frac{1}{\int \frac{f(x)dx}{q+(1+j)/\sqrt{x}}}$$

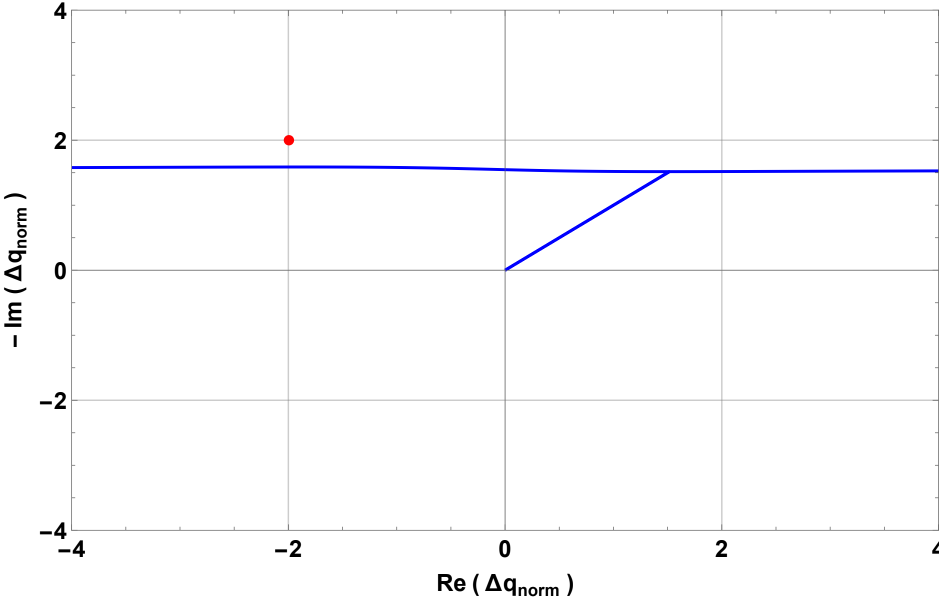
$$x = \frac{\omega}{\Omega}$$

Stability limits with detuning impedance

In x

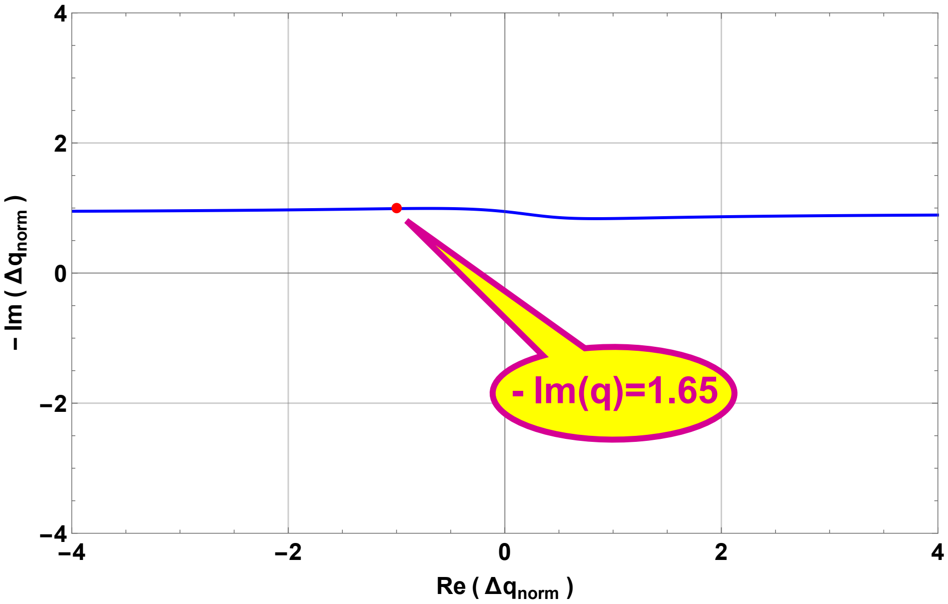


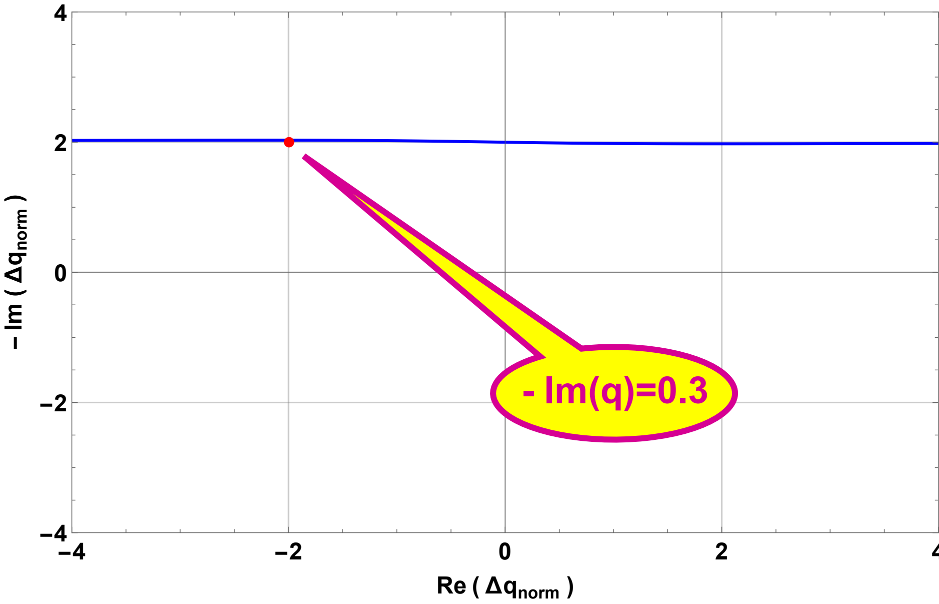
In y



Assuming constant distribution f between 1/2 and 3/2 (similar results obtained with a Gaussian and also different spread's widths)

Instability rise-times with detuning impedance





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  - In x: 1/1.65 ≈ 0.6 => x-plane is more critical than without detuning impedance by ~ 70% and more critical than y-plane by a factor ~ 5
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- Next
  - Check this simple model – Preliminary results!
  - Develop more involved models
  - Compare/benchmark/etc.