Simple model for the effect of detuning impedance on beam stability

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- Trying to answer to the question: Why is the horizontal plane more unstable than the vertical one in the presence of detuning impedance? – Preliminary!
Coasting-beam interacting with flat classical RW impedance

- Dispersion relation to solve

\[
1 = \Delta Q_{z,\gamma}^x(\Omega) \int \frac{\rho(\omega) \, d\omega}{Q_c - Q_{x,\gamma}(\omega)}
\]

\[
\int \rho(\omega) \, d\omega = 1 \quad Q_{x,y}(\omega) = Q_{x_0, y_0} + \Delta Q_{i,x,y}(\omega) \quad \Delta Q_{z}^x(\omega) = -\frac{A(1+j)}{\sqrt{\omega}} = -\Delta Q_{i}^x(\omega)
\]

\[
\Delta Q_{z}^y(\omega) = 2 \Delta Q_{z}^x(\omega)
\]

\[
\Delta Q_{i}^x(\omega) = \Delta Q_{i}^y(\omega)
\]
Coasting-beam interacting with flat classical RW impedance

- Dispersion relation to solve

\[ 1 = \Delta Q_{z}^{x,y} (\Omega) \int \frac{\rho(\omega) \, d\omega}{Q_c - Q_{x,y}(\omega)} \]

\[ \int \rho(\omega) \, d\omega = 1 \quad Q_{x,y}(\omega) = Q_{x_0,y_0} + \Delta Q_{i}^{x,y}(\omega) \quad \Delta Q_{z}^{x}(\omega) = -\frac{A(1 + j)}{\sqrt{\omega}} = -\Delta Q_{i}^{x}(\omega) \]

\[ \Delta Q_{z}^{y}(\omega) = 2 \Delta Q_{z}^{x}(\omega) \]

- This leads to

\[ -(1 + j) = \frac{1}{\int \frac{f(x) \, dx}{q - (1 + j)/\sqrt{x}}} \quad -2(1 + j) = \frac{1}{\int \frac{f(x) \, dx}{q + (1 + j)/\sqrt{x}}} \]

\[ q = \frac{Q_c - Q_{x_0,y_0}}{A/\sqrt{\Omega}} \quad x = \frac{\omega}{\Omega} \]
Stability limits with detuning impedance

assuming constant distribution \( f \) between 1/2 and 3/2 (similar results obtained with a Gaussian and also different spread’s widths)
Instability rise-times with detuning impedance

\[ \text{Re}(\Delta q_{\text{norm}}) - \text{Im}(\Delta q_{\text{norm}}) \]

- \text{Im}(q) = 1.65
- \text{Im}(q) = 0.3
Conclusion and next steps

- Instability rise-times WITHOUT detuning impedance
  - In x: $1/1 = 1$
  - In y: $\frac{1}{2} = 0.5 \Rightarrow y$-plane is more critical than $x$-plane by a factor 2

- Instability rise-times WITH detuning impedance
  - In x: $1/1.65 \approx 0.6$
  - In y: $1/0.3 \approx 3.3$

Next
- Check this simple model – Preliminary results!
- Develop more involved models
- Compare/benchmark/etc.
Conclusion and next steps

- Instability rise-times **WITHOUT** detuning impedance
  - In x: $1/1 = 1$
  - In y: $\frac{1}{2} = 0.5 \Rightarrow y$-plane is more critical than x-plane by a factor 2

- Instability rise-times **WITH** detuning impedance
  - In x: $1/1.65 \approx 0.6 \Rightarrow x$-plane is more critical than without detuning impedance by $\sim 70\%$ and more critical than y-plane by a factor $\sim 5$
  - In y: $1/0.3 \approx 3.3$

Next

- Check this simple model – Preliminary results!
- Develop more involved models
- Compare/benchmark/etc.
Conclusion and next steps

- Instability rise-times **WITHOUT** detuning impedance
  - $\ln x: 1/1 = 1$
  - $\ln y: \frac{1}{2} = 0.5 \Rightarrow y\text{-plane is more critical than } x\text{-plane by a factor } 2$

- Instability rise-times **WITH** detuning impedance
  - $\ln x: 1/1.65 \approx 0.6 \Rightarrow x\text{-plane is more critical than without detuning impedance by } \sim 70\% \text{ and more critical than } y\text{-plane by a factor } \sim 5$
  - $\ln y: 1/0.3 \approx 3.3$

- Next
  - Check this simple model – Preliminary results!
  - Develop more involved models
  - Compare/benchmark/etc.