TEXAS A&M UNIVERSITY

BAYESIAN PRIORS, COPULAS, AND LARGE-DIMENSIONAL LIKELIHOODS

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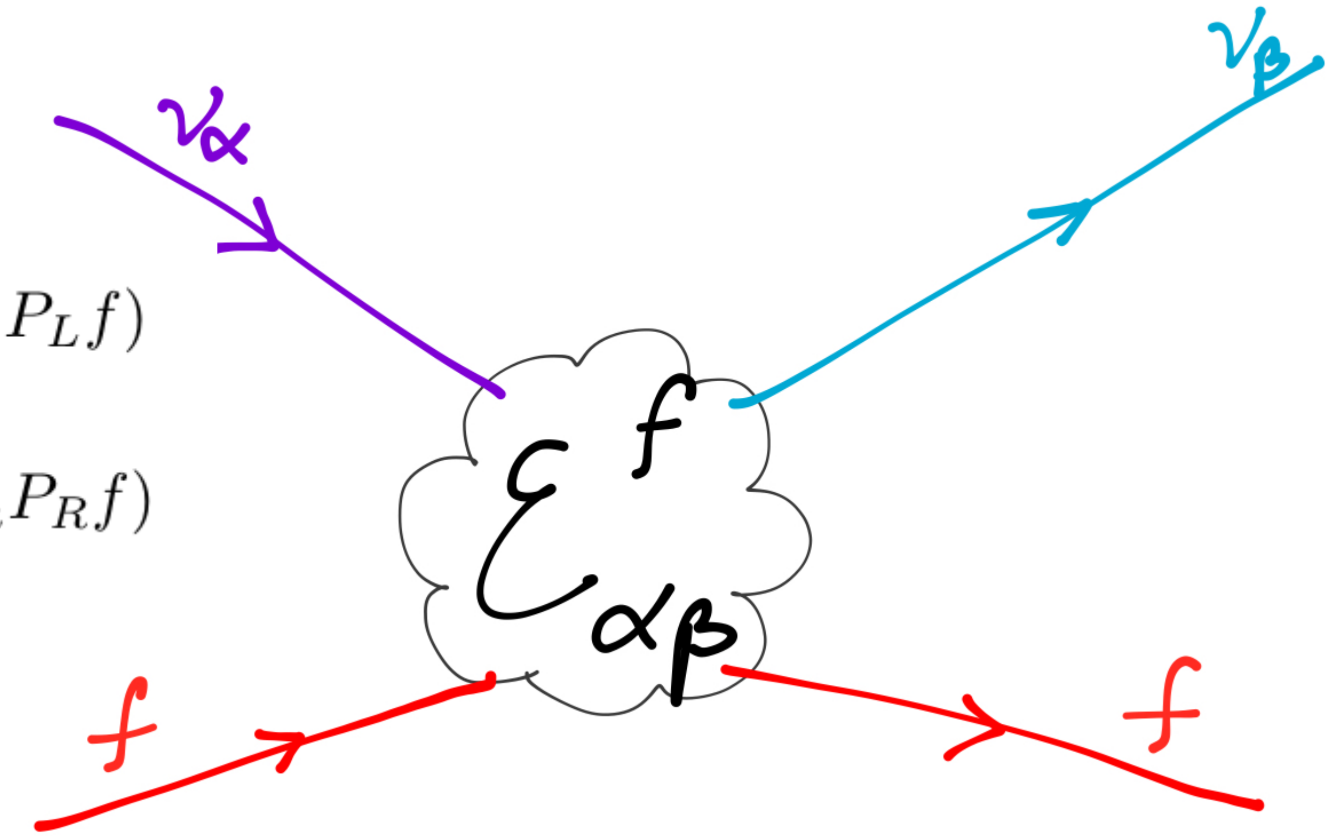
Dark Matter and Neutrino Computation Explored
(DANCE2019 @ Rice University)
October 29, 2019

Neutral Current Vector NSI

$$\mathscr{L}_{NSI} = -2\sqrt{2}G_F \sum_{f,\alpha,\beta} \epsilon_{\alpha\beta}^{f,L}(\bar{\nu}_\alpha \gamma^\mu \nu_\beta)(\bar{f}\gamma_\mu P_L f) + \epsilon_{\alpha\beta}^{f,R}(\bar{\nu}_\alpha \gamma^\mu \nu_\beta)(\bar{f}\gamma_\mu P_R f)$$



$\varepsilon_{\alpha\beta}^{f}$

f: e, u, d → $\underline{3}$

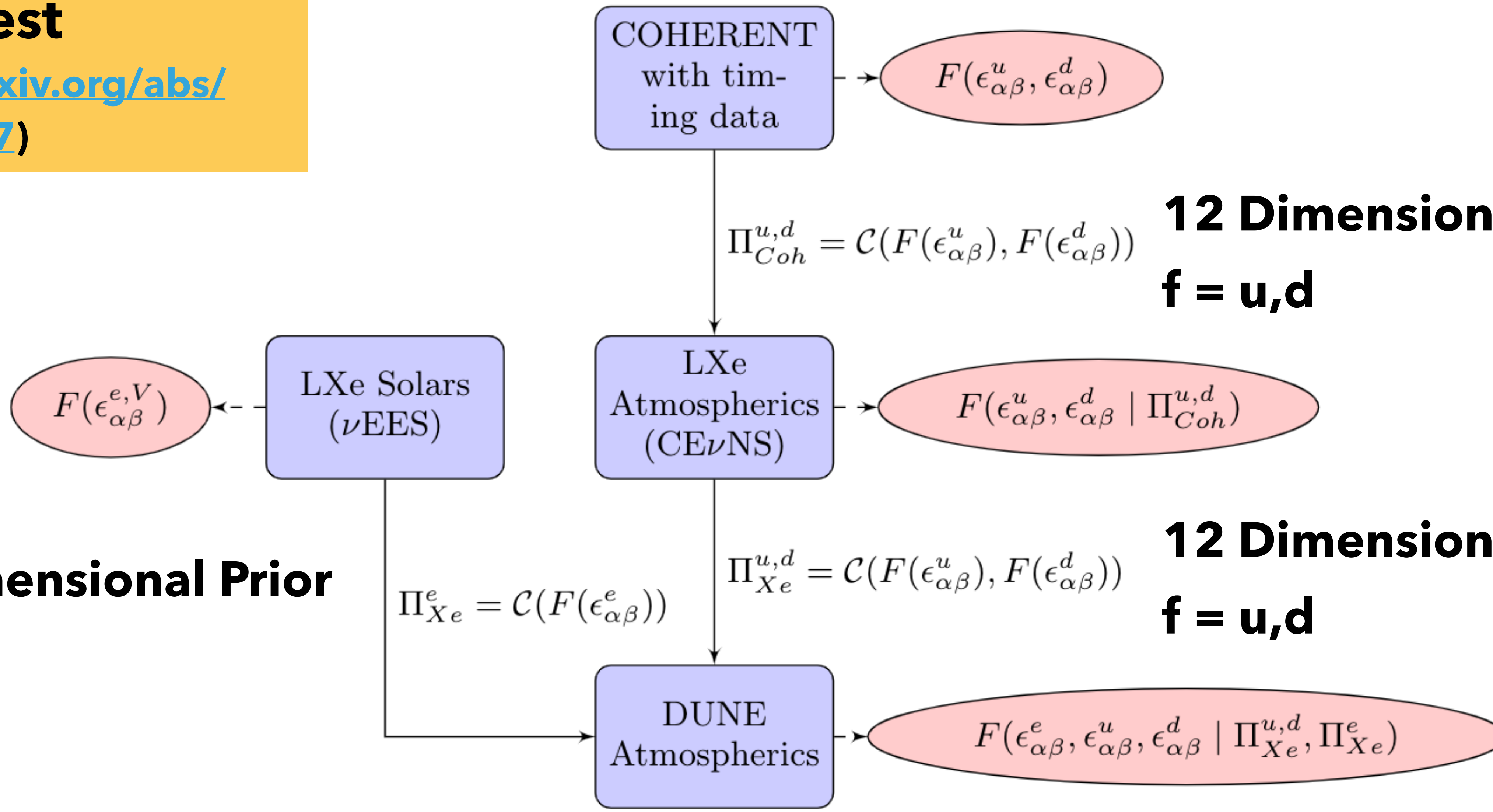
$\alpha\beta$: e, μ, τ → $\underline{6}$
symmetric

= 18 Real Parameters

(+3x3=9 complex phases)

(x2 for Chiral pieces!)

MultiNest
(https://arxiv.org/abs/0809.3437)



12 Dimensional Prior
f = u,d

12 Dimensional Prior
f = u,d

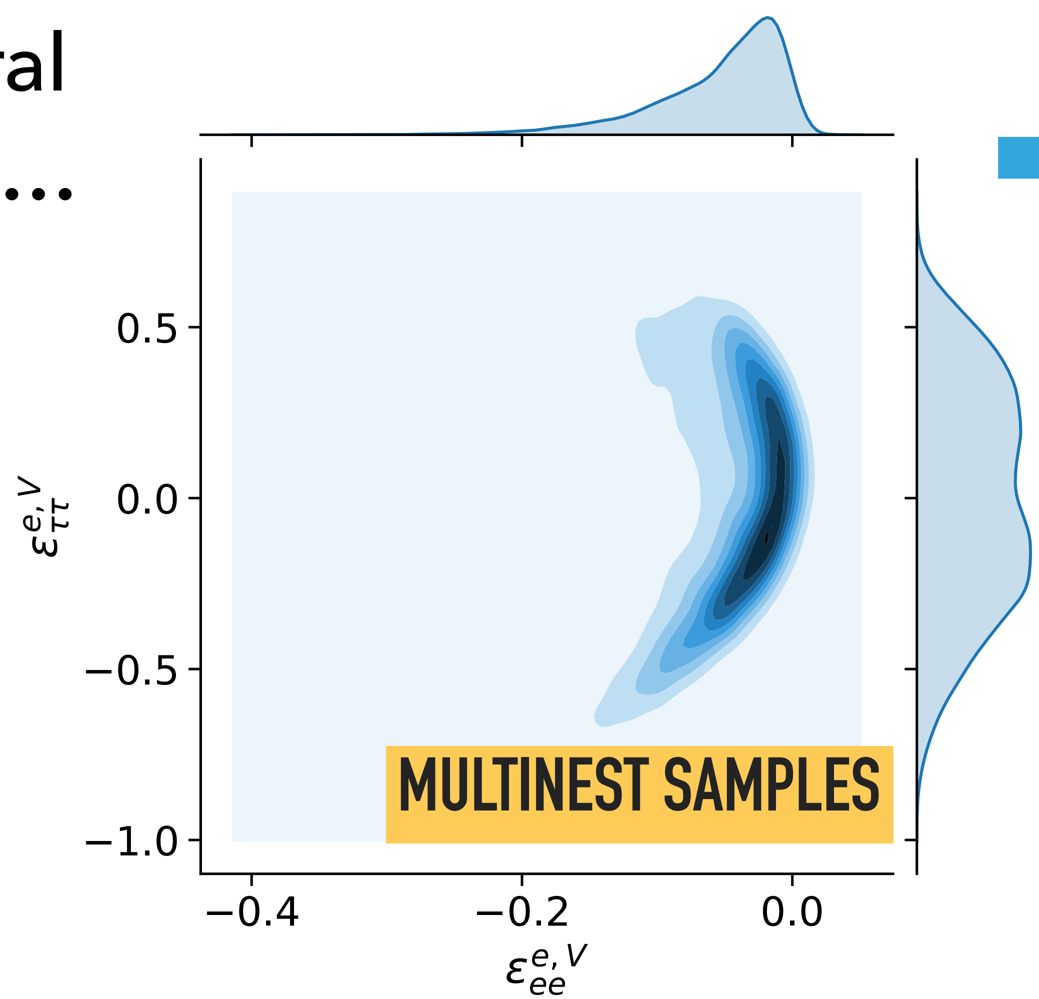
6 Dimensional Prior
f=e

- **We connect posteriors from one experiment to use as priors in complementary experiments**
- **We model the multivariate prior by sampling from a fitted/empirical Copula**

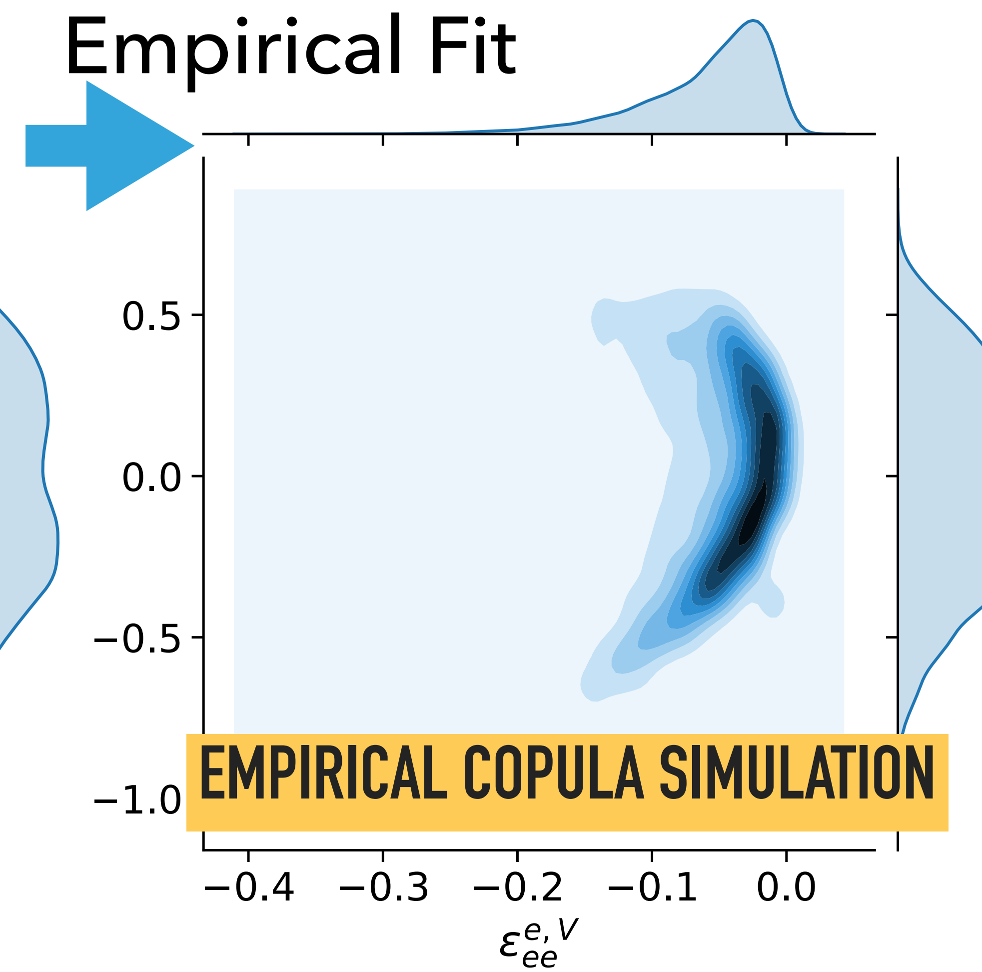
```python
# Simulate bivariate pairs empirically.
class EmpricalCopula:
  def __init__(self, datastr, i, j):
    # read in table
    robjects.r('data = read.table(file = "{0}", header=F)'.format(datastr))
    robjects.r('z = pobs(as.matrix(cbind(data[,{0}],data[,{1}])))'.format(i, j))
  def simulate(self, u, v):
    def ddv(v2):
      v2 = float(v2)
      robjects.r('u = matrix(c({0}, {1}), 1, 2)'.format(u, v2))
      return np.asarray(robjects.r('dCn(u, U = z, j.ind = 1)'))
    try:
      return float(pynv.inversefunc(ddv, y_values=v, domain=[0, 1], open_domain=[True, True]))
    except:
      print("passing...")
      return v
```

One can use several Copula libraries in...

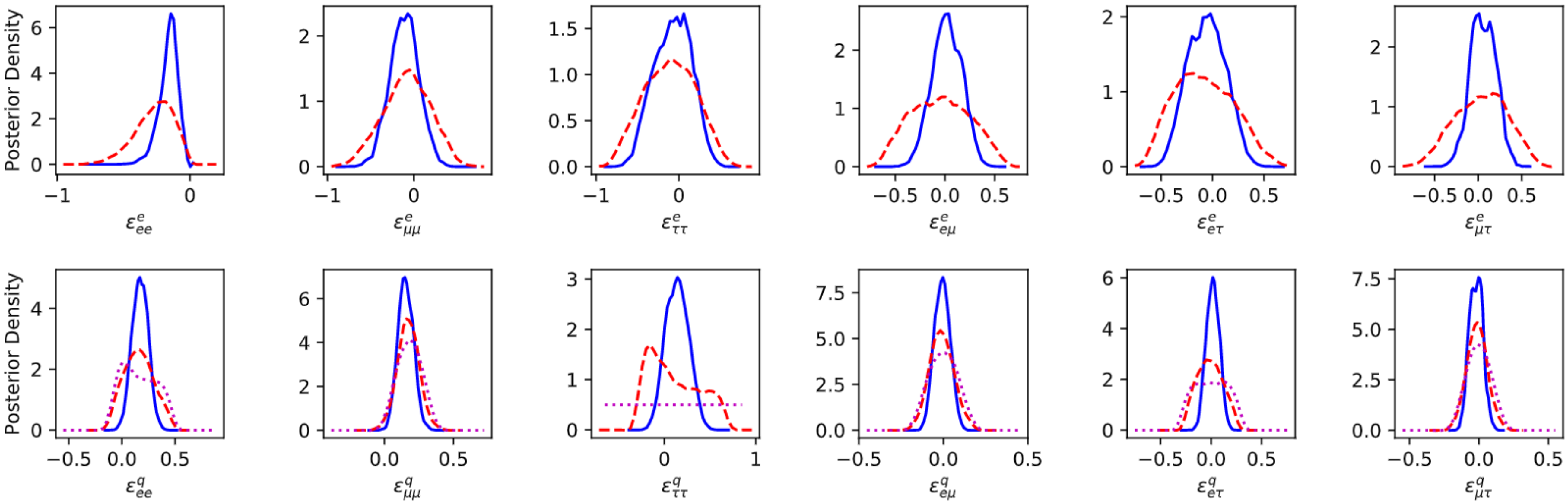
- R
- Python
- Matlab



Empirical Fit



PRELIMINARY



COHERENT (Prior 1)

LXe (Prior 2)

DUNE (Final Posterior)

Final stage of the prior flow:

d=18, MultiNest completes in ~7000 cpu-hours

Compare to single-likelihood global analysis: Fails to converge in under 21000 cpu-hours

BACKUP

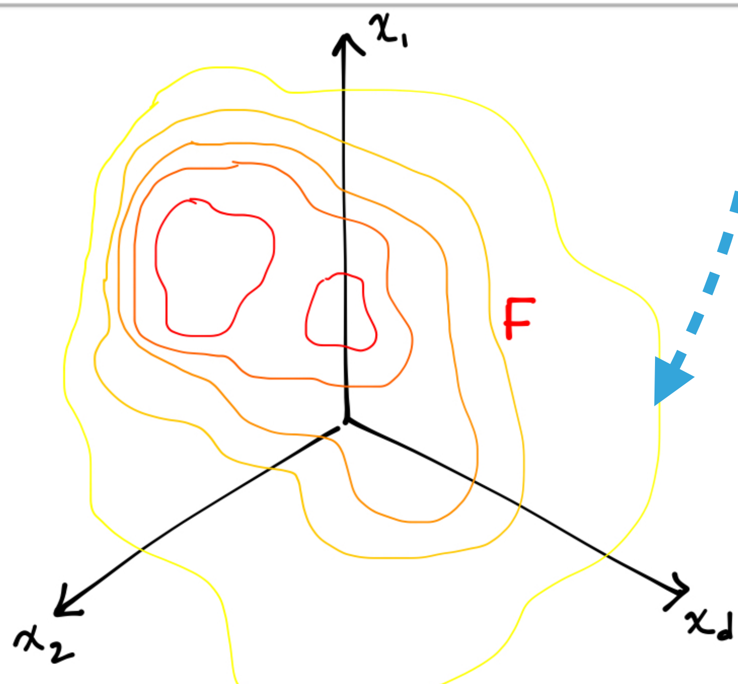
THE COPULA AND SKLAR'S THEOREM

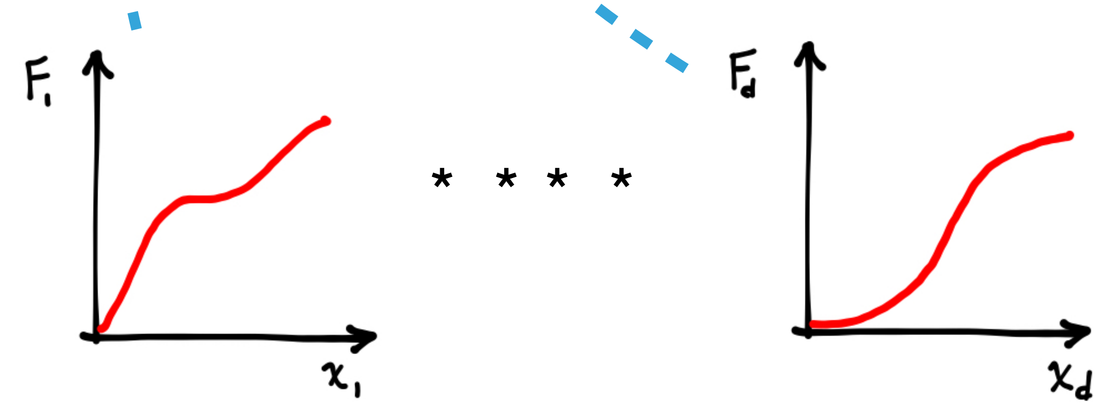
Definition 1 *A d-dimensional copula, $C : [0,1]^d \; : \to \; [0,1]$ is a cumulative distribution function (CDF) with uniform marginals.*

Theorem 2 (Sklar's Theorem 1959) *Consider a d-dimensional CDF, F, with marginals $F_1, \ldots, F_d$. Then there exists a copula, C, such that*

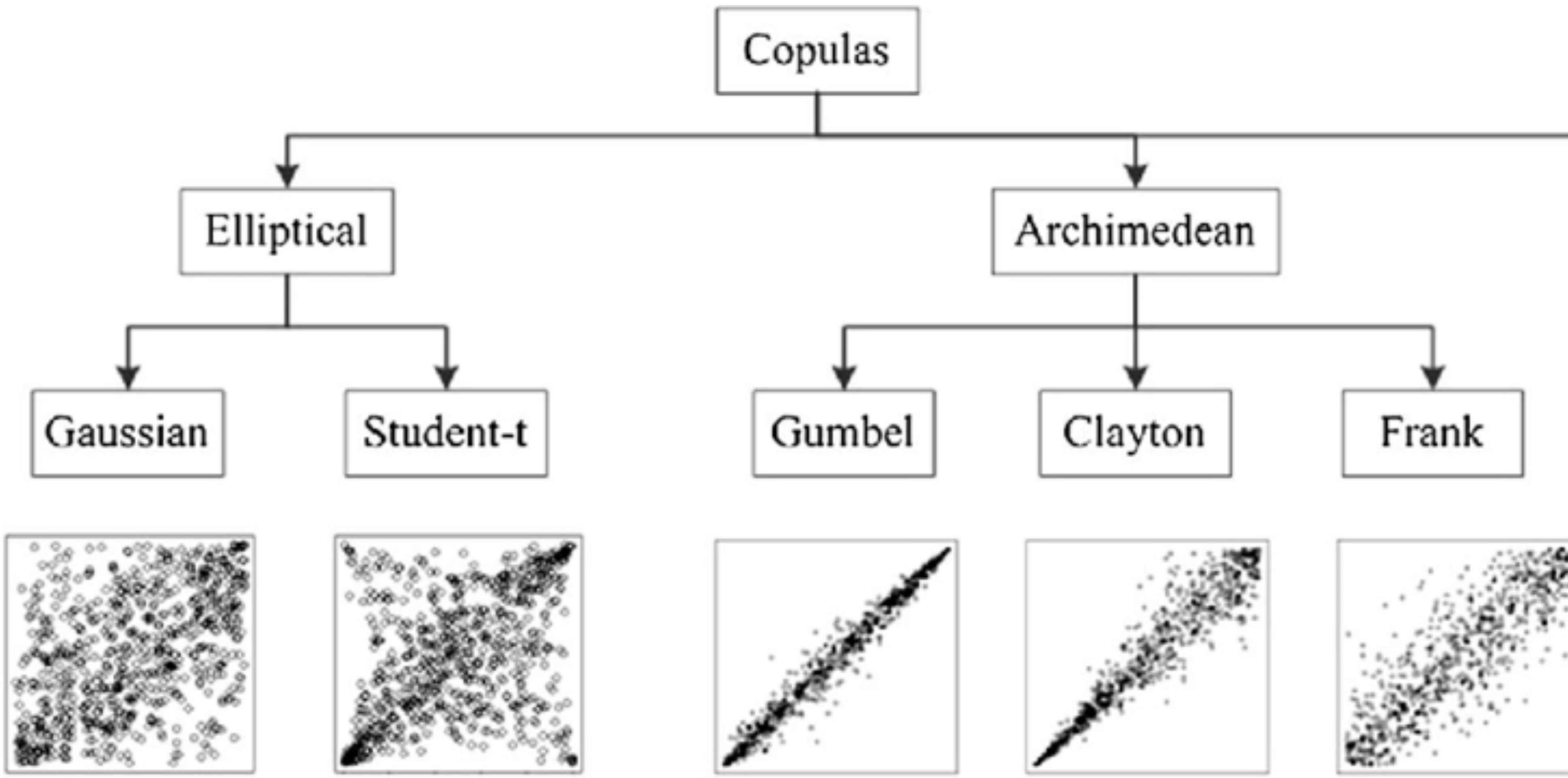
$$F(x_1, \ldots, x_d) \;=\; C\left(F_1(x_1), \ldots, F_d(x_d)\right) \tag{2}$$

for all $x_i \in [-\infty, \infty]$ and $i = 1, \ldots, d$.





FAMILIES OF COPULAS



On the aggregation of credit, market and operational risks - Li, *et al* (2015)

EXAMPLE: SIMULATING A GAUSSIAN COPULA

$$C_P^{Gauss}(\mathbf{u}) \;:=\; \mathbf{\Phi}_{\boldsymbol{P}}\left(\Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_d)\right)$$

Let P be a correlation matrix and compute the *Cholesky* decomposition $P = A^{T}A$

1. Generate $(Z_1,\ldots,Z_d) \sim (N(0,1), \ldots, N(0,1))$

   Normal variates

2. $\mathbf{X} = A^{T}\mathbf{Z}$ Correlate the variates!

3. $\mathbf{Y} = (\Phi(X_1),\ldots,\Phi(X_d))$ Return them to [0,1]

4. Return $(u_1 = F_1^{-1}(Y_1),\ldots,u_d = F_d^{-1}(Y_d))$

   Match them up with our marginals

EXAMPLE: GAUSSIAN COPULA

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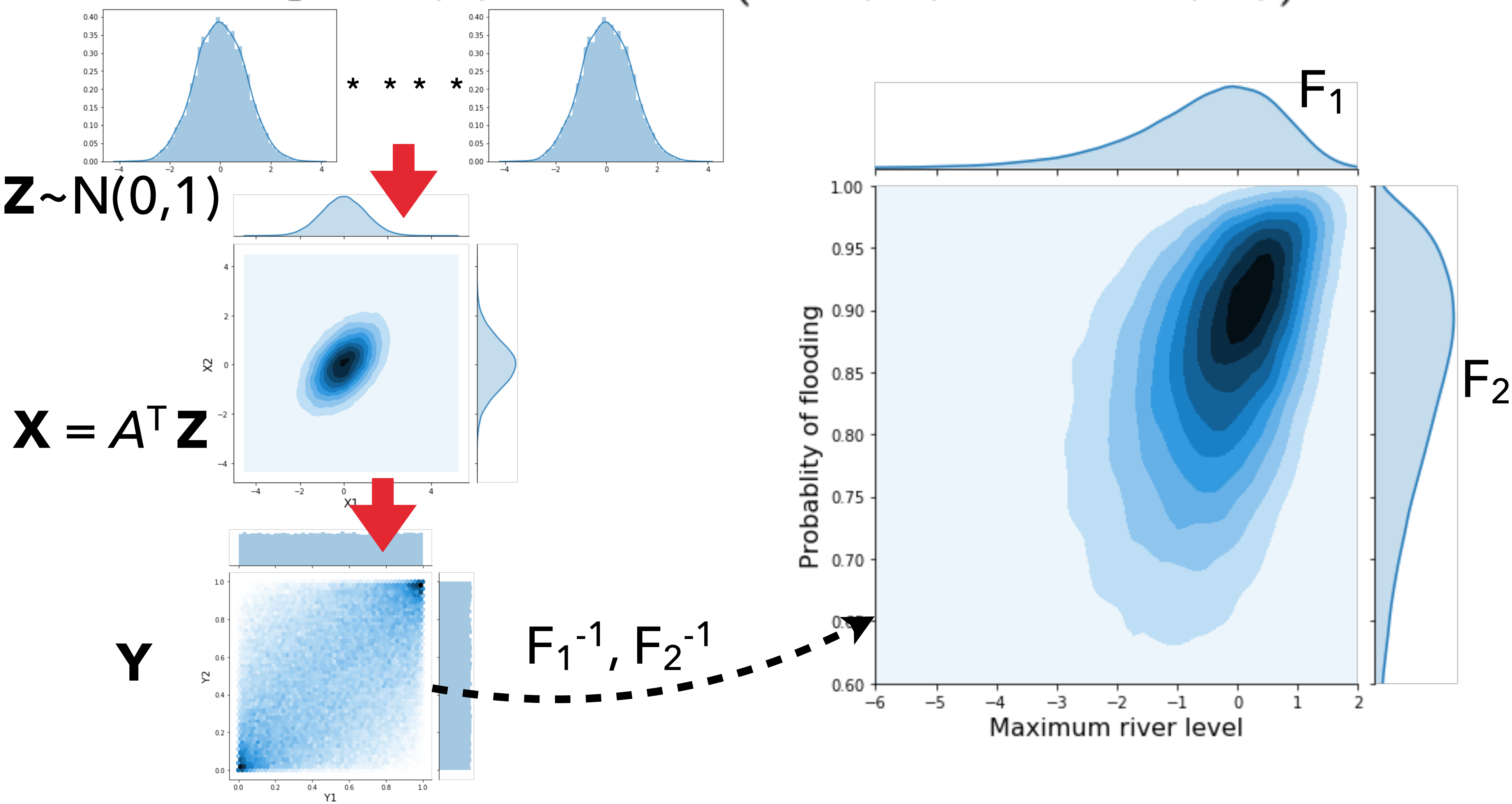

MultiNest
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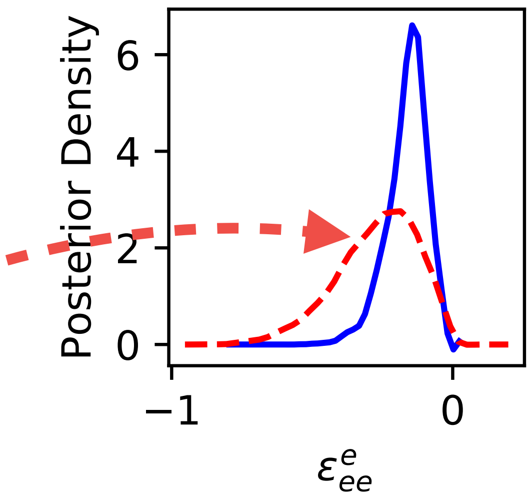
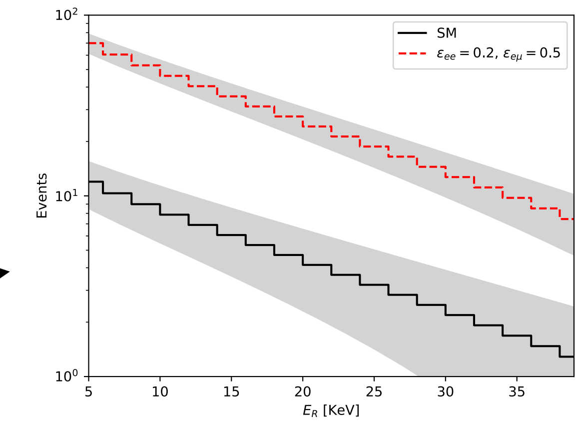


Draw from Prior Distribution on ε

$$dN = \sum_{\alpha,\beta} \frac{\mathcal{E}}{M_T} P_{\beta\alpha} \frac{d\Phi_\alpha}{dE_\nu} \frac{d\sigma_{\beta\alpha}(E_r, E_\nu)}{dE_r} dE_r dE_\nu$$

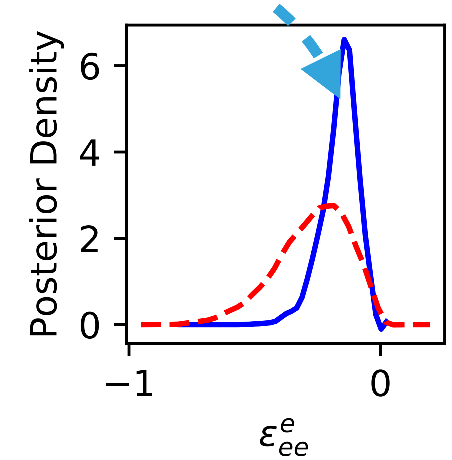
Computed with [pyCEvNS]()

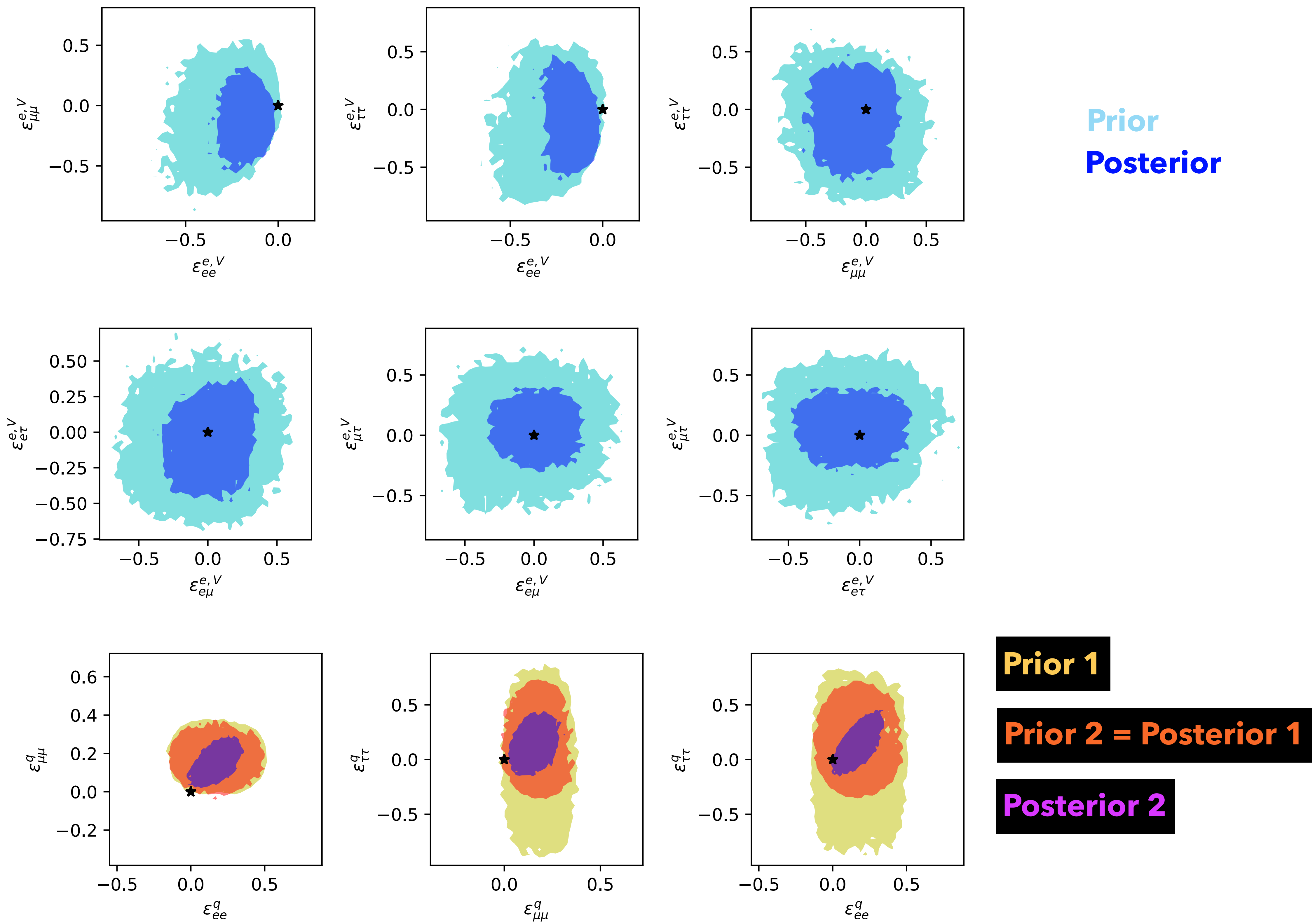
Simulate model prediction due to ε



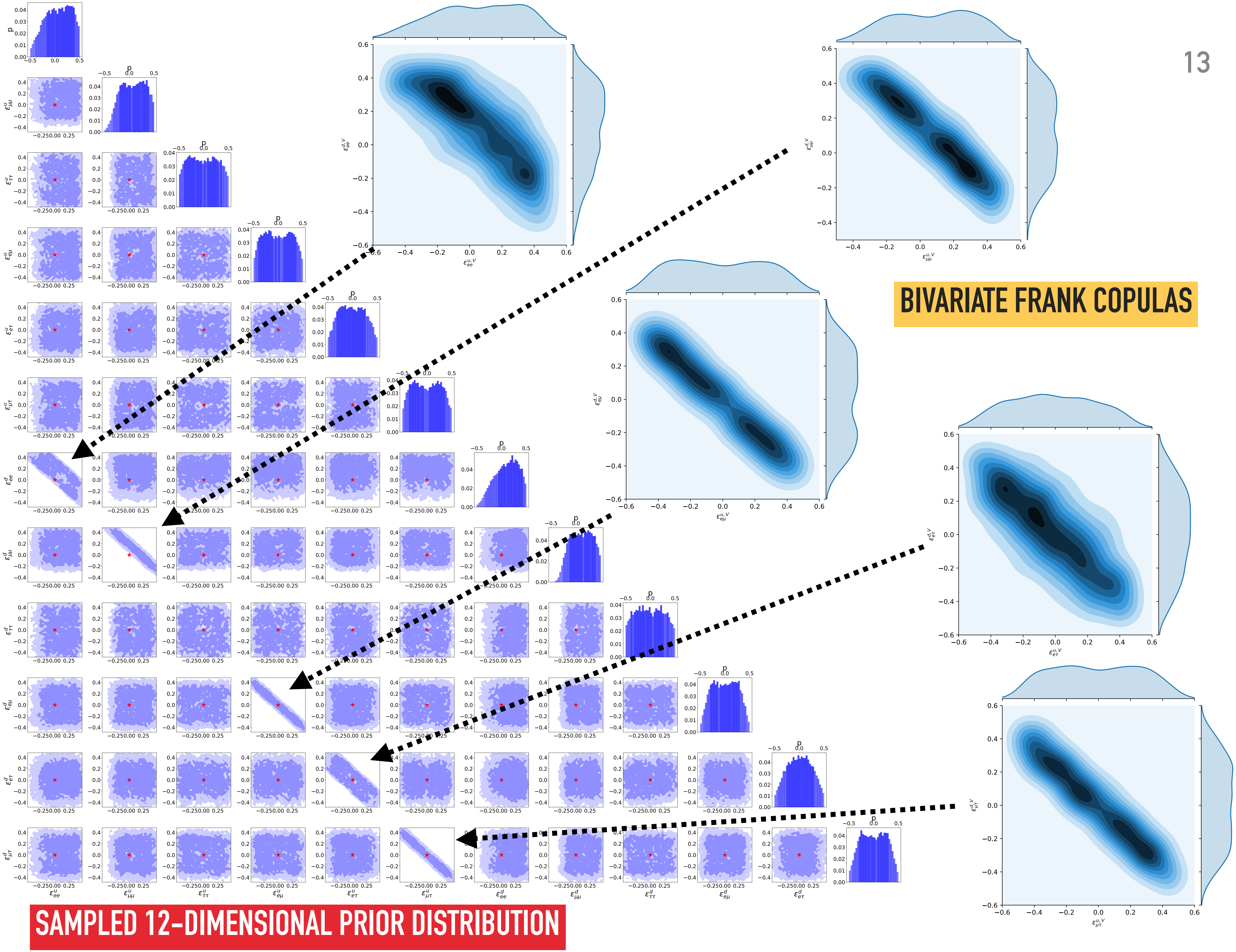
Calculate Probability that ε explains the data

$$f(\vec{\epsilon}) = \frac{\mathcal{L}(\mathcal{D} \mid \vec{\epsilon}; H)\Pi(\vec{\epsilon})}{Z}$$





PRELIMINARY RESULTS


BIVARIATE FRANK COPULAS
SAMPLED 12-DIMENSIONAL PRIOR DISTRIBUTION

Copula References:

- *An intuitive, visual guide to copulas*: https://twiecki.io/blog/2018/05/03/copulas/
- *An Introduction to Copulas*, Martin Haugh http://www.columbia.edu/~mh2078/QRM/Copulas.pdf
- *MULTINEST: an efficient and robust Bayesian inference tool for cosmology and particle physics* *https://arxiv.org/pdf/0809.3437.pdf*
- *Li, 1999: On Default Correlation: A Copula Function Approach*
- *Somnath Chatterjee, Modelling credit risk* - Bank of England 2015