

*HIGGS PAIR PRODUCTION :
NLO QCD AND UNCERTAINTIES*

Michael Spira (PSI)

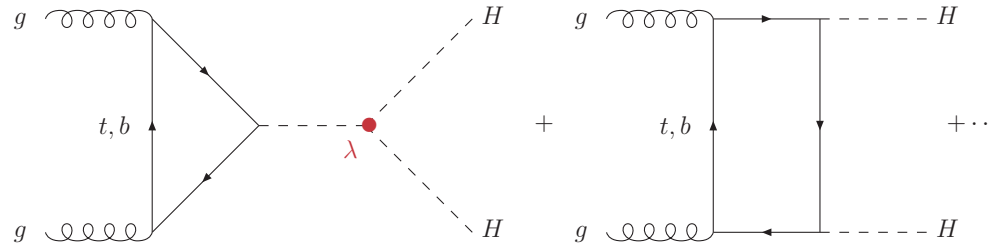
- I Introduction
- II Calculation
- III Conclusions

in collaboration with J. Baglio, F. Campanario, S. Glaus, M. Mühlleit-
ner, J. Streicher

I INTRODUCTION

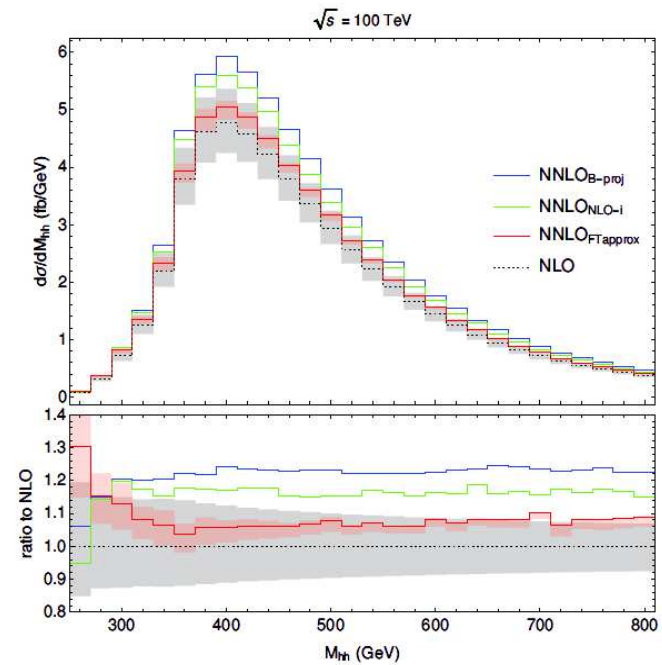
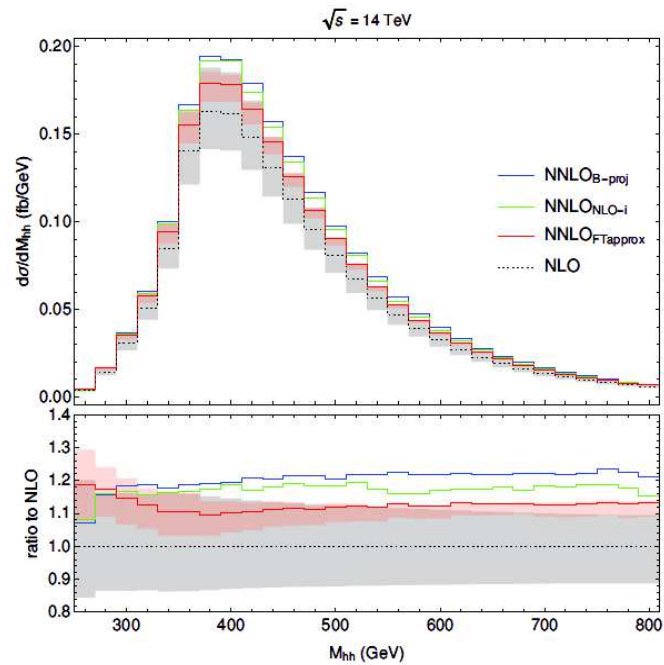
$gg \rightarrow HH$

SM



- third generation dominant: $t, b \rightarrow \frac{\Delta\sigma}{\sigma} \sim -\frac{\Delta\lambda}{\lambda}$
- 2-loop QCD corr.: $\lesssim 70\%$ [$M_H^2 \ll 4m_t^2, \mu = M_{HH}/2$] Dawson, Dittmaier, S.
- 2-loop QCD corr.: $\sigma = \sigma_0 + \frac{\sigma_1}{m_t^2} + \dots + \frac{\sigma_4}{m_t^8}$
[refinement: full LO at diff. level] Grigo, Hoff, Melnikov, Steinhauser
- NLO mass effects @ NLO in real corrections: $\sim -10\%$
Frederix, Frixione, Hirschi, Maltoni, Mattelaer, Torrielli, Vryonidou, Zaro
- NNLO QCD corrections: $\sim 20\%$ [$M_H^2 \ll 4m_t^2$] de Florian, Mazzitelli, Grigo, Melnikov, Steinhauser
- soft gluon resummation: $\sim 10\%$ [$M_H^2 \ll 4m_t^2$] Shao, Li, Li, Wang, de Florian, Mazzitelli
- NLO: small mass exp. [$Q^2 \gg m_t^2$] Davies, Mishima, Steinhauser, Wellmann

- NNLO Monte Carlo: inclusion of full top-mass effects @ NLO



Grazzini, Heinrich, Jones, Kallweit, Kerner, Lindert, Mazzitelli

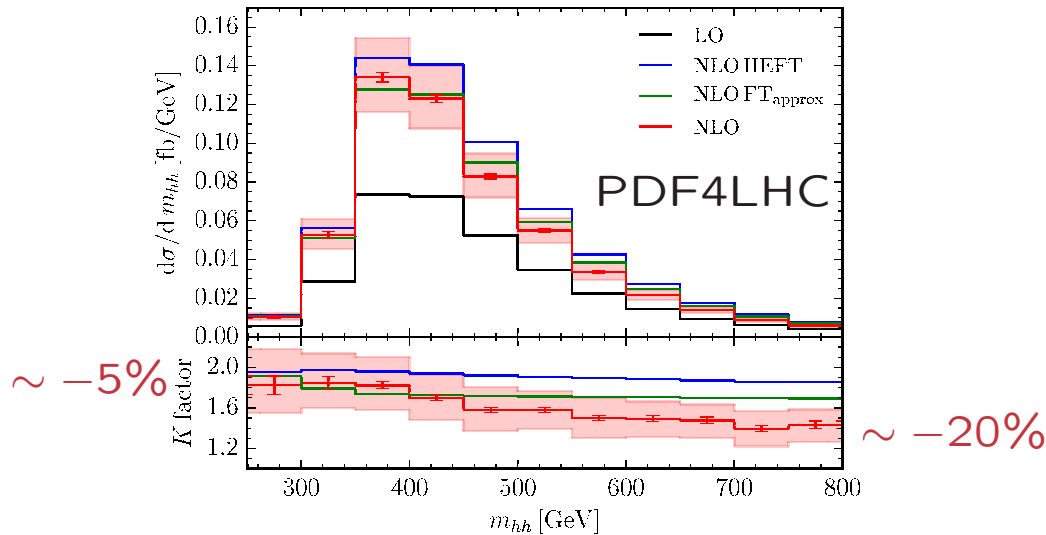
⇒ 20% effects beyond NLO

- NLO: matching to parton showers

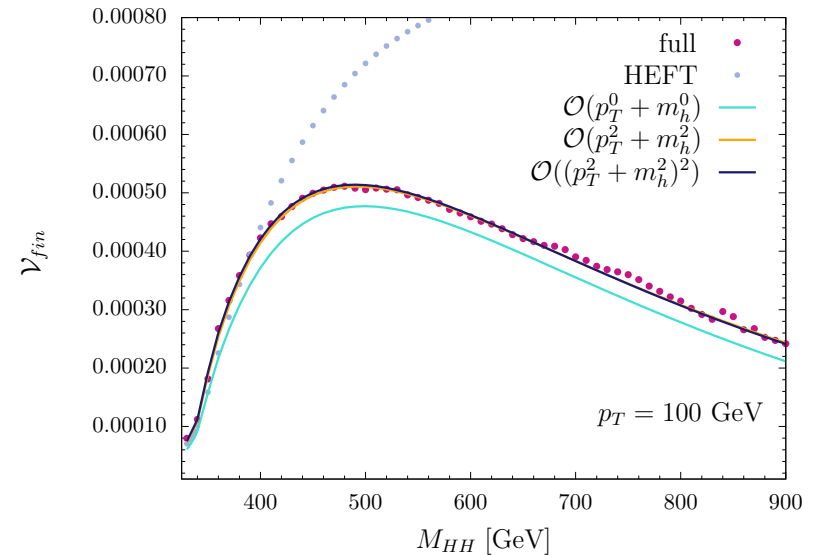
Heinrich, Jones, Kerner, Luisoni, Vryonidou

Full NLO calculation: top only

Numerical integration, sector decomposition, tensor reduction, contour deformation



Borowka, Greiner, Heinrich, Jones, Kerner
Schlenk, Schubert, Zirke



Boncianni, Degrassi, Giardino, Gröber

- 14 TeV: ($m_t = 173 \text{ GeV}$) $\sigma_{NLO} = 32.91(10)_{-12.8\%}^{+13.8\%} \text{ fb}$
- $\sigma_{NLO}^{HTL} = 38.75_{-15\%}^{+18\%} \text{ fb}$ (\leftarrow HPAIR)

$\Rightarrow -15\%$ mass effects on top of LO

• new expansion/extrapolation methods:

(i) $1/m_t^2$ expansion + conformal mapping + Padé approximants

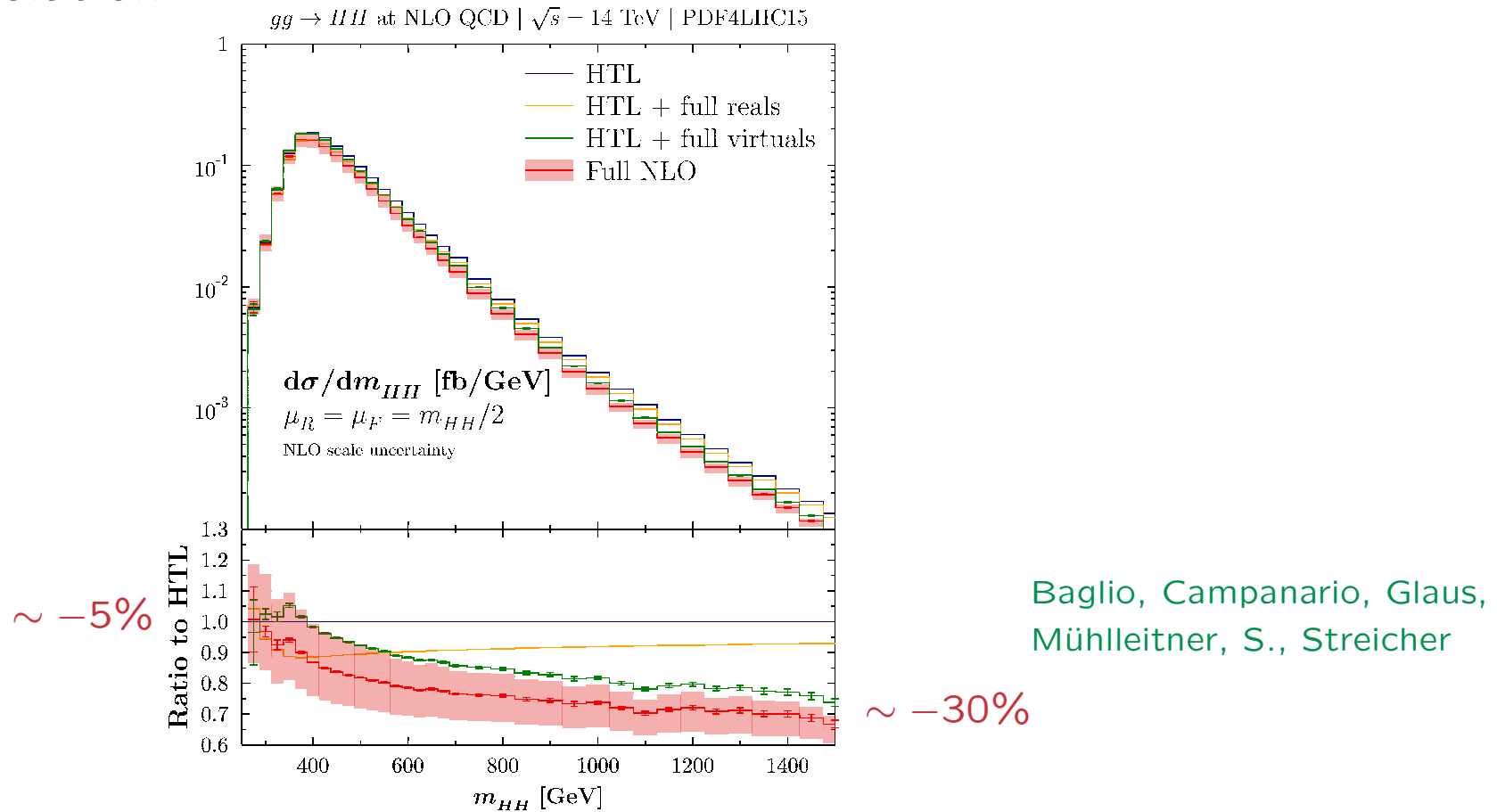
Gröber, Maier, Rauh

(ii) p_T^2 expansion

Boncianni, Degrassi, Giardino, Gröber

Full NLO calculation: top only

Numerical integration, IR subtraction, no tensor reduction, Richardson extrapolation



- 14 TeV: ($m_t = 172.5$ GeV) $\sigma_{NLO} = 32.78(7)_{-12.5\%}^{+13.5\%}$ fb
 $\sigma_{NLO}^{HTL} = 38.66_{-15\%}^{+18\%}$ fb (← HPAIR)

⇒ -15% mass effects on top of LO

II CALCULATION

$$\sigma_{\text{NLO}}(pp \rightarrow HH + X) = \sigma_{\text{LO}} + \Delta\sigma_{\text{virt}} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}$$

$$\sigma_{\text{LO}} = \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{LO}}(Q^2 = \tau s)$$

$$\Delta\sigma_{\text{virt}} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \hat{\sigma}_{\text{LO}}(Q^2 = \tau s) \quad C$$

$$\Delta\sigma_{gg} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \frac{d\mathcal{L}^{gg}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -z P_{gg}(z) \log \frac{M^2}{\tau s} \right. \\ \left. + d_{gg}(z) + 6[1 + z^4 + (1 - z)^4] \left(\frac{\log(1 - z)}{1 - z} \right)_+ \right\}$$

$$\Delta\sigma_{gq} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_{q, \bar{q}} \frac{d\mathcal{L}^{gq}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \left\{ -\frac{z}{2} P_{gq}(z) \log \frac{M^2}{\tau s(1 - z)^2} + d_{gq}(z) \right\}$$

$$\Delta\sigma_{q\bar{q}} = \frac{\alpha_s(\mu)}{\pi} \int_{\tau_0}^1 d\tau \sum_q \frac{d\mathcal{L}^{q\bar{q}}}{d\tau} \int_{\tau_0/\tau}^1 \frac{dz}{z} \hat{\sigma}_{\text{LO}}(Q^2 = z\tau s) \quad d_{q\bar{q}}(z)$$

$$C \rightarrow \pi^2 + \frac{11}{2} + C_{\Delta\Delta}, \quad d_{gg} \rightarrow -\frac{11}{2}(1 - z)^3, \quad d_{gq} \rightarrow \frac{2}{3}z^2 - (1 - z)^2, \quad d_{q\bar{q}} \rightarrow \frac{32}{27}(1 - z)^3$$

uncertainties due to m_t

- transform $m_t \rightarrow \overline{m}_t(\mu)$ ($\overline{\text{MS}}$)

→ modification of mass CT

- use $m_t, \overline{m}_t(\overline{m}_t)$ and scan $Q/4 < \mu < Q \rightarrow$ uncertainty = envelope:

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=300 \text{ GeV}} = 0.0298(7)_{-34\%}^{+6\%} \text{ fb/GeV},$$

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=400 \text{ GeV}} = 0.1609(4)_{-13\%}^{+0\%} \text{ fb/GeV},$$

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=600 \text{ GeV}} = 0.03204(9)_{-30\%}^{+0\%} \text{ fb/GeV},$$

$$\left. \frac{d\sigma(gg \rightarrow HH)}{dQ} \right|_{Q=1200 \text{ GeV}} = 0.000435(4)_{-35\%}^{+0\%} \text{ fb/GeV}$$

- **preliminary** interpolation:

$$\sigma(gg \rightarrow HH) = 32.78_{-17\%}^{+4\%} \text{ fb} \quad \text{(preliminary)}$$

uncertainties due to m_t for single Higgs

- transform $m_t \rightarrow \overline{m}_t(\mu)$ ($\overline{\text{MS}}$)

→ modification of mass CT

- use $m_t, \overline{m}_t(\overline{m}_t)$ and scan $Q/4 < \mu < Q \rightarrow$ uncertainty = envelope:

$$\sigma(gg \rightarrow H)|_{M_H=125 \text{ GeV}} = 42.17^{+0.4\%}_{-0.5\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=300 \text{ GeV}} = 9.85^{+7.5\%}_{-0.3\%} \text{ pb}$$

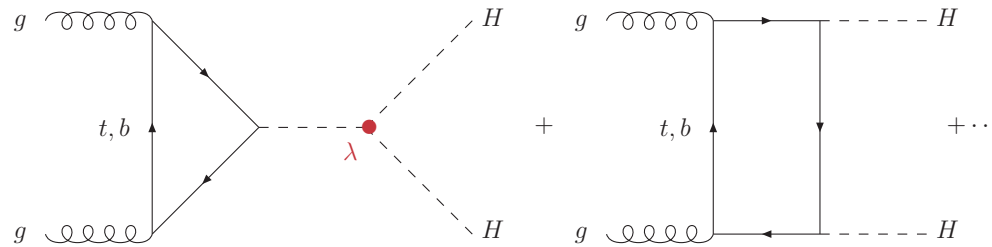
$$\sigma(gg \rightarrow H)|_{M_H=400 \text{ GeV}} = 9.43^{+0.1\%}_{-0.9\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=600 \text{ GeV}} = 1.97^{+0.0\%}_{-15.9\%} \text{ pb}$$

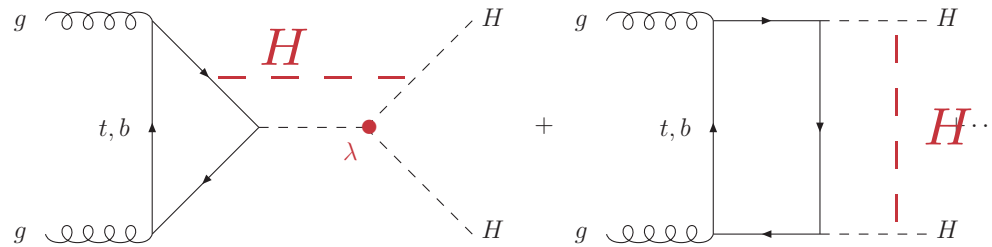
$$\sigma(gg \rightarrow H)|_{M_H=900 \text{ GeV}} = 0.230^{+0.0\%}_{-22.3\%} \text{ pb}$$

$$\sigma(gg \rightarrow H)|_{M_H=1200 \text{ GeV}} = 0.0402^{+0.0\%}_{-26.0\%} \text{ pb}$$

- different scales for y_t in triangle (Q) and box (M_H) diagrams?
 → has to hold at all orders



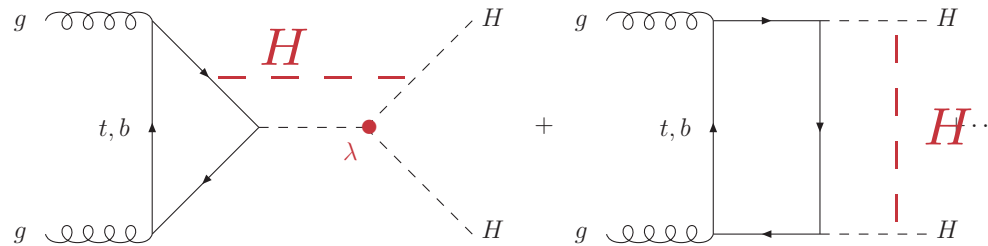
- different scales for y_t in triangle (Q) and box (M_H) diagrams?
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elw. corrections

⇒ same scales in all diagrams

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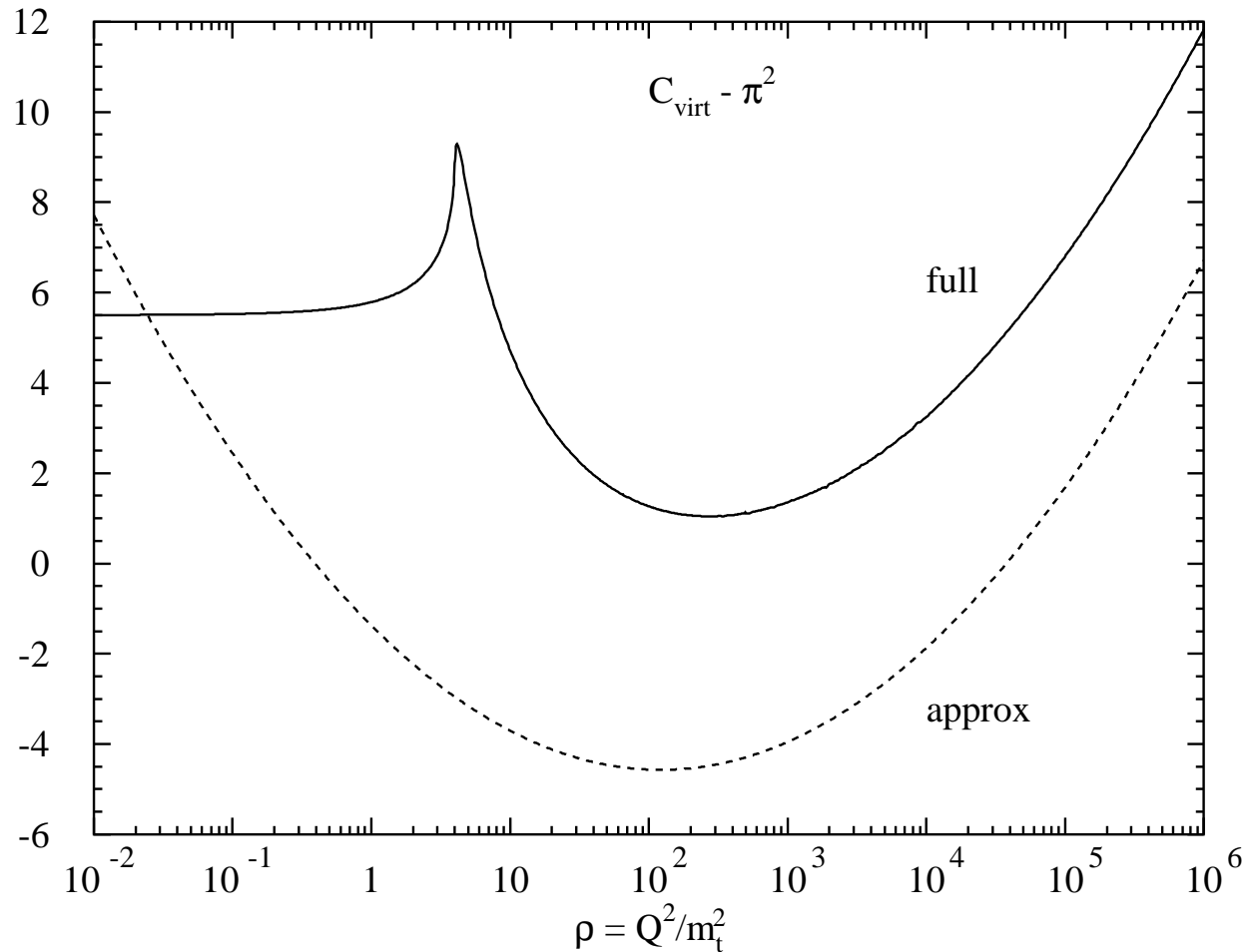
elw. corrections

⇒ same scales in all diagrams

- scales for small Q : look at $1/m_t^2$ expansion
 [top mass effects small]
 ⇒ NNLO expansion available

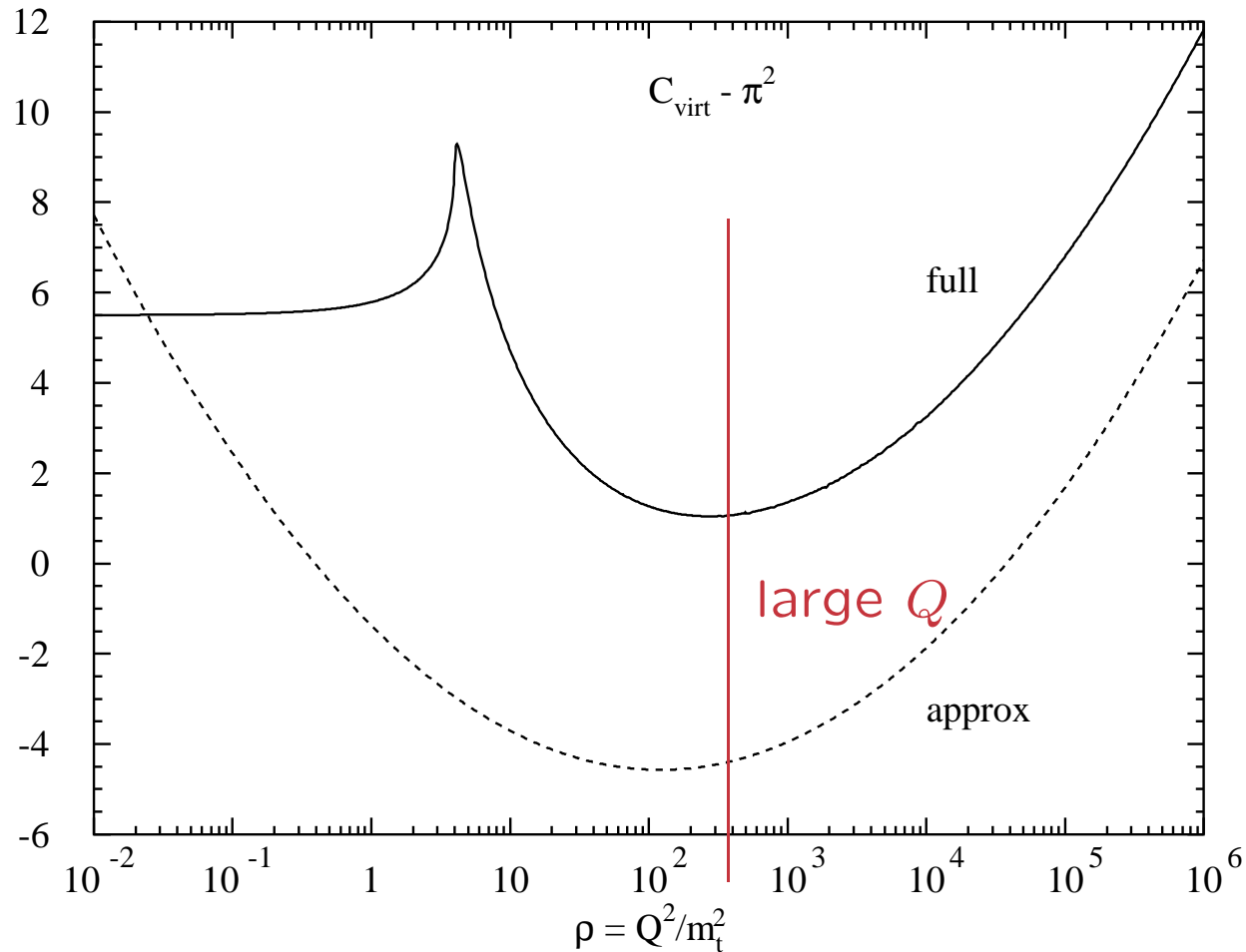
- triangles for $Q^2 \gg 4M_t^2$:

$$C \rightarrow \frac{C_A - C_F}{12} \left[\log \frac{Q^2}{m_t^2} - i\pi \right]^2 - C_F \left[\log \frac{Q^2}{m_t^2} - i\pi \right] + 3C_F \log \frac{\mu_t^2}{m_t^2} + \mathcal{O}(1)$$



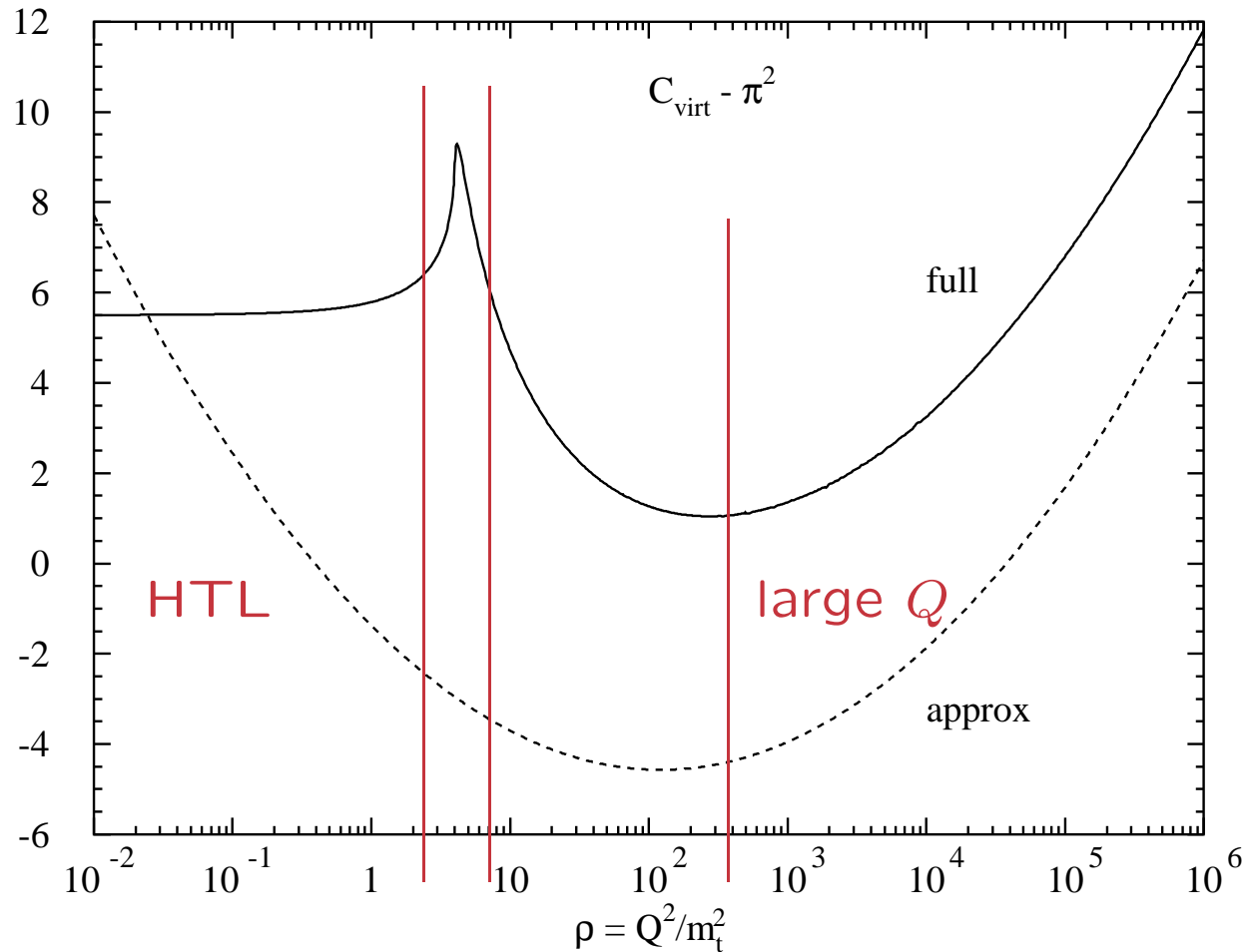
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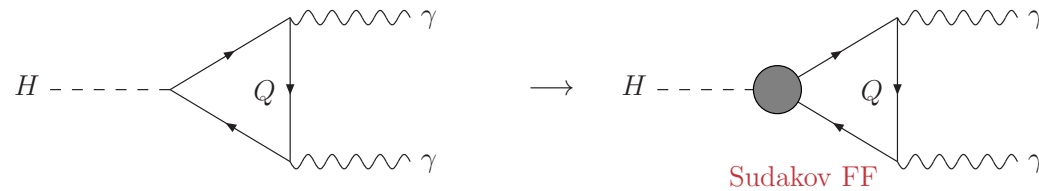


threshold

S., Djouadi, Graudenz, Zerwas

resummation for large Q

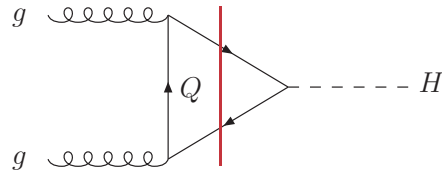
- Abelian logs (C_F): $H \rightarrow \gamma\gamma$



Kotsky, Yakovlev, PLB 418 (1998) 335 (LL)
Akhoury, Wang, Yakovlev, PRD 64 (2001) 113008 (NLL)

- non-Abelian logs (C_A): LL related to IR singularities \rightarrow exponentiate
Liu, Penin, PRL 119 (2017) no.26, 262001; JHEP 1811 (2018) 158
- non-Abelian NLL?
- remainder (NNLL)?
- boxes? (more scales)

- threshold: \mathcal{P} -wave QCD potential \rightarrow Coulomb singularities



- matrix element $\propto \beta^2$, phase space $\propto \beta$
imaginary part $\propto \beta^3$ @ LO

- Coulomb singularity at each order in imaginary part:

$$C_{Coul} = \frac{Z}{1 - e^{-Z}} = 1 + \frac{Z}{2} + \dots \text{ with } Z = C_F \frac{\pi \alpha_s}{\beta}$$

\Rightarrow step in imaginary part @ N³LO

\Rightarrow log. sing. in real part @ N³LO (\leftarrow dispersion integral)

- solution: non-relativistic Green-function in threshold range
[real part renormalized, finite top width]

Melnikov, S., Yakovlev, ZPC 64 (1994) 401

- remainder?

III CONCLUSIONS

- Higgs pair production at full NLO for variable top/Higgs masses [top loops]
- top mass effects on top of LO up to 20–30%
- factorization/renormalization scale uncertainties $\sim 15\%$
- uncertainties due to scale/scheme choice of m_t sizeable $\lesssim 30\%$
→ reduction unclear

BACKUP SLIDES

- pole mass \leftrightarrow $\overline{\text{MS}}$ mass:

$$\overline{m}_t(M_t) = \frac{M_t}{1 + \frac{4\alpha_s(M_t)}{3\pi} + 10.9 \left(\frac{\alpha_s(M_t)}{\pi}\right)^2}$$

$$\overline{m}_t(\mu) = \overline{m}_t(M_t) \frac{c[\alpha_s(\mu)/\pi]}{c[\alpha_s(M_t)/\pi]}$$

$$c(x) = \left(\frac{7}{2}x\right)^{\frac{4}{7}} [1 + 1.398x + 1.793x^2 - 0.6834x^3]$$

$$M_t = 172.5 \text{ GeV}$$

$$\overline{m}_t(\overline{m}_t) = 163.015161017019 \text{ GeV}$$

M_HH	mt (M_HH/4)	mt (M_HH/2)	mt (M_HH)
125	189.209370262526	176.772460597358	166.501914700149
260	176.139964023672	165.972836934324	156.889554725476
275	175.247098219568	165.224863654266	156.188624671063
300	173.888433241807	164.084218616097	155.118481503625
350	171.556916171559	162.101622772544	153.272150436136
375	170.543285547792	161.158290295641	152.465560631846
400	169.611142167793	160.289697463114	151.721739637882
500	166.501914700149	157.384965182267	149.226383426185
600	164.084218616097	155.118481503625	147.270941230420
700	162.101622772544	153.272150436136	145.672596390682
800	160.289697463114	151.721739637882	144.326704798025
900	158.737886290123	150.390138497802	143.168060367441
1000	157.384965182267	149.226383426185	142.153427561240
1100	156.188624671063	148.195135247933	141.252743160739
1200	155.118481503625	147.270941230420	140.444302478362
1300	154.152026867353	146.434896300904	139.711950260189
1400	153.272150436136	145.672596390682	139.043354388391
1500	152.465560631846	144.972828986822	138.428898934501