Using Woodcock viewpoint for weight calculation of occurrence biasing of charged particles

Jefferson Lab Collaboration Meeting
Parallel Session 2B
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Introduction

- Many material here presented at the last “GenProc” WG meeting
  - Sorry for repetition
  - Some new stuff though !
- Biasing interaction occurrence of charged particles = long pending development item
  - Because of the main difficulty being:
    - the variation of the cross-section over a step, because of energy loss
    - with the need to integrate this cross-section over the step for weight calculations
      - Integration which can not be “brute force” to not penalize the speed-up we try to get by biasing
- Laurent presented calculations he made in 2012:
  - That demonstrate the correctness of the so-called rejection technique
    - These require the track to do interactions
  - That propose a way to compute the weight in case of free-flights
    - To complement the case of tracks requested not to do interaction
- “Floating in the air” was also the so-called “Woodcock tracking” technique
  - which is based on the same principle than the rejection technique
  - but which has an elegant way to rephrase the problem
  - in my post-lunch nap on 30/07/2019 ;) after years of “digestion”, this triggered some reconsideration of the problem.

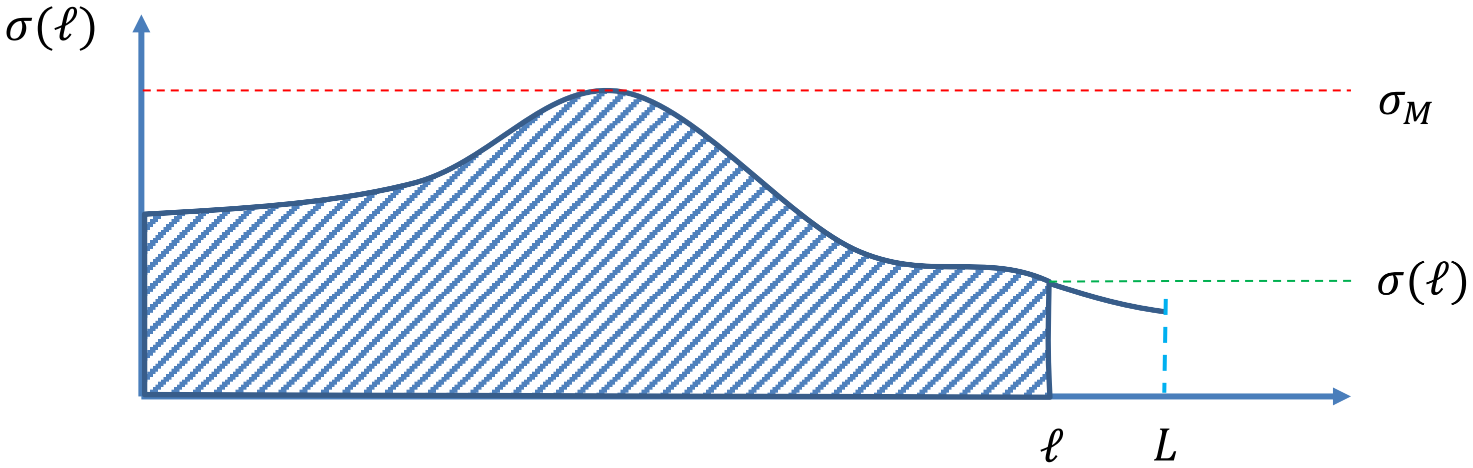
- **Main idea : how Woodcock viewpoint can provide a solution/an approach to the problem of taking into account the cross-section variation over steps in weight calculation**

Overview

- Rejection technique
- Woodcock tracking
- From Woodcock viewpoint to biasing
- Toy MC tests
  - Use of technique & improve first scheme
  - Testing varying biased cross-sections
  - Forced Free Flight
- Generalizing ?
- Conclusion

REJECTION TECHNIQUE

Rejection technique

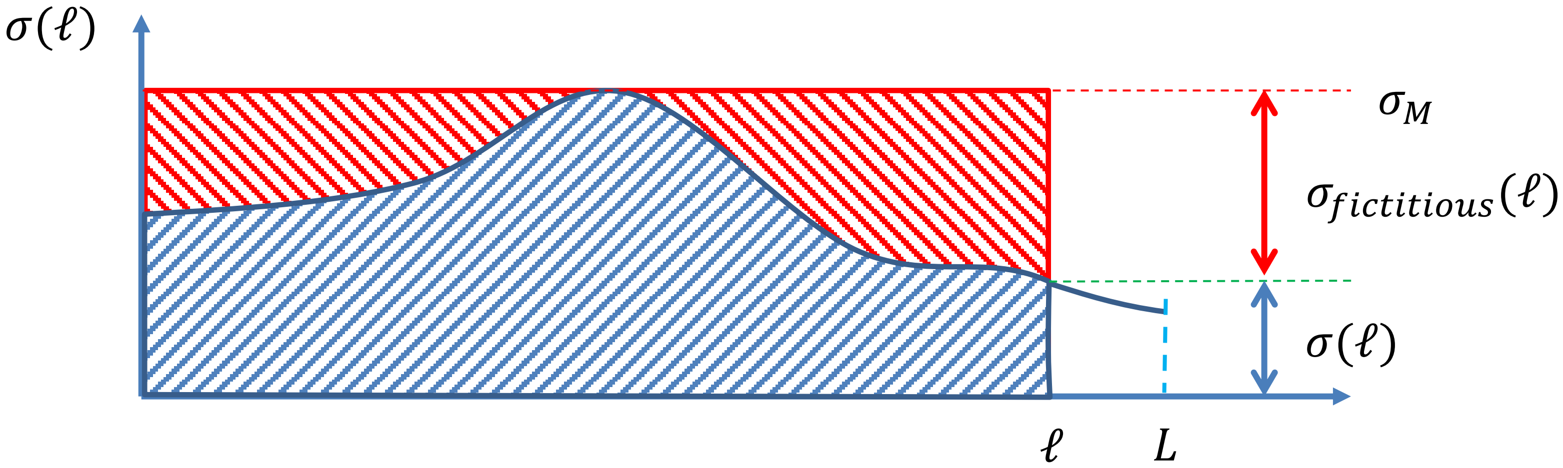


- Algorithm:
  - Determine σ_M on $[0, L]$
  - Sample ℓ using σ_M
  - Move track to ℓ and compute $\sigma(\ell)$
  - if $\mathrm{rand}(0,1) > \sigma(\ell)/\sigma_M$ reject interaction; else do it
- Laurent demonstrated the rejection provides the exact amount of interactions $\forall\ \ell$
  - Demonstration not straightforward
  - Based of the sum of number of interactions of tracks having suffered $0, 1, 2, \cdots, \infty$ rejected interactions before interacting at ℓ

- Rejection technique is used in the EM package, to account for variation of cross-section between the start and the end points of the step
  - Now used in the hadronics too.

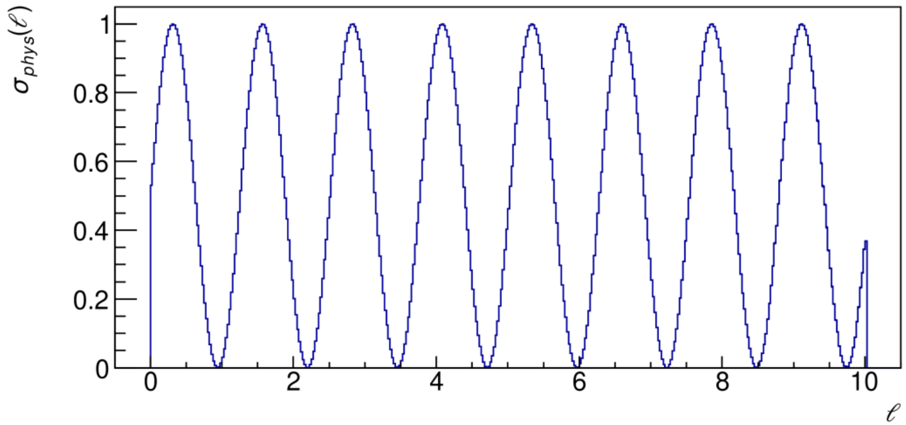
WOODCOCK VIEWPOINT

Woodcock viewpoint



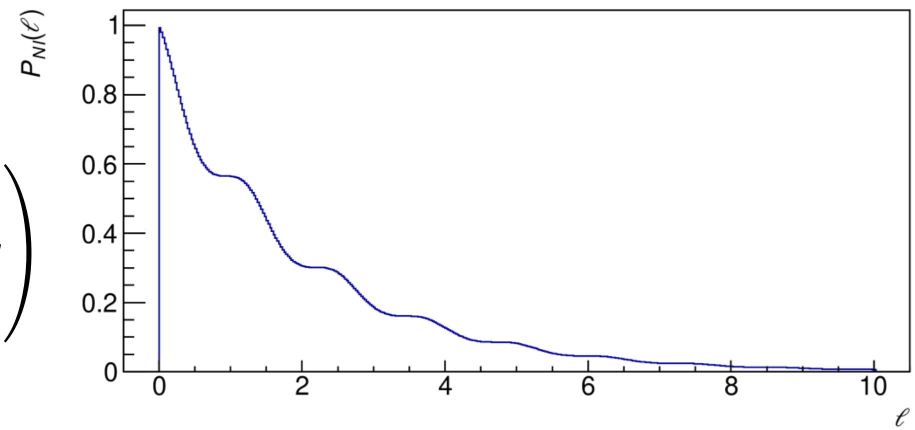
- The Woodcock tracking (invented for medical) rephrases the problem saying that:
  - We have a total (and constant) cross-section σ_M
  - This one is the sum of two physics process cross-sections:
    - The physical one $\sigma(\ell)$
    - And $\sigma_{fictitious}(\ell)$ which generates "fictitious interactions"
      - These are simply "void" interactions, from the fictitious process, which does nothing
- Algorithm is essentially the same as before
  - Determine σ_M on $[0, L]$; Sample ℓ according to total cross-section σ_M; Move track to ℓ
  - **And chose randomly between the two "processes"**
    - depending on their relative cross-sections at that point.
- This makes *de facto* the exact same sampling than the rejection one !
- But the Woodcock viewpoint –with the fictitious process- makes a tremendous change for what moving to biasing is concerned

“Physical” test cross-section

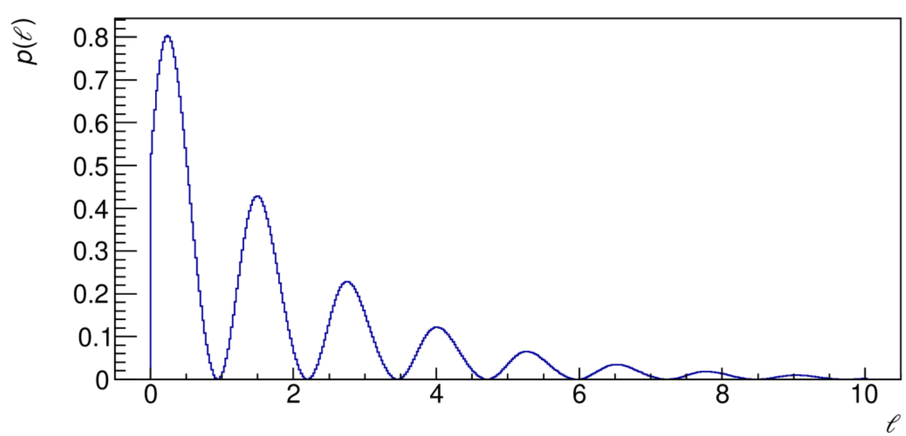


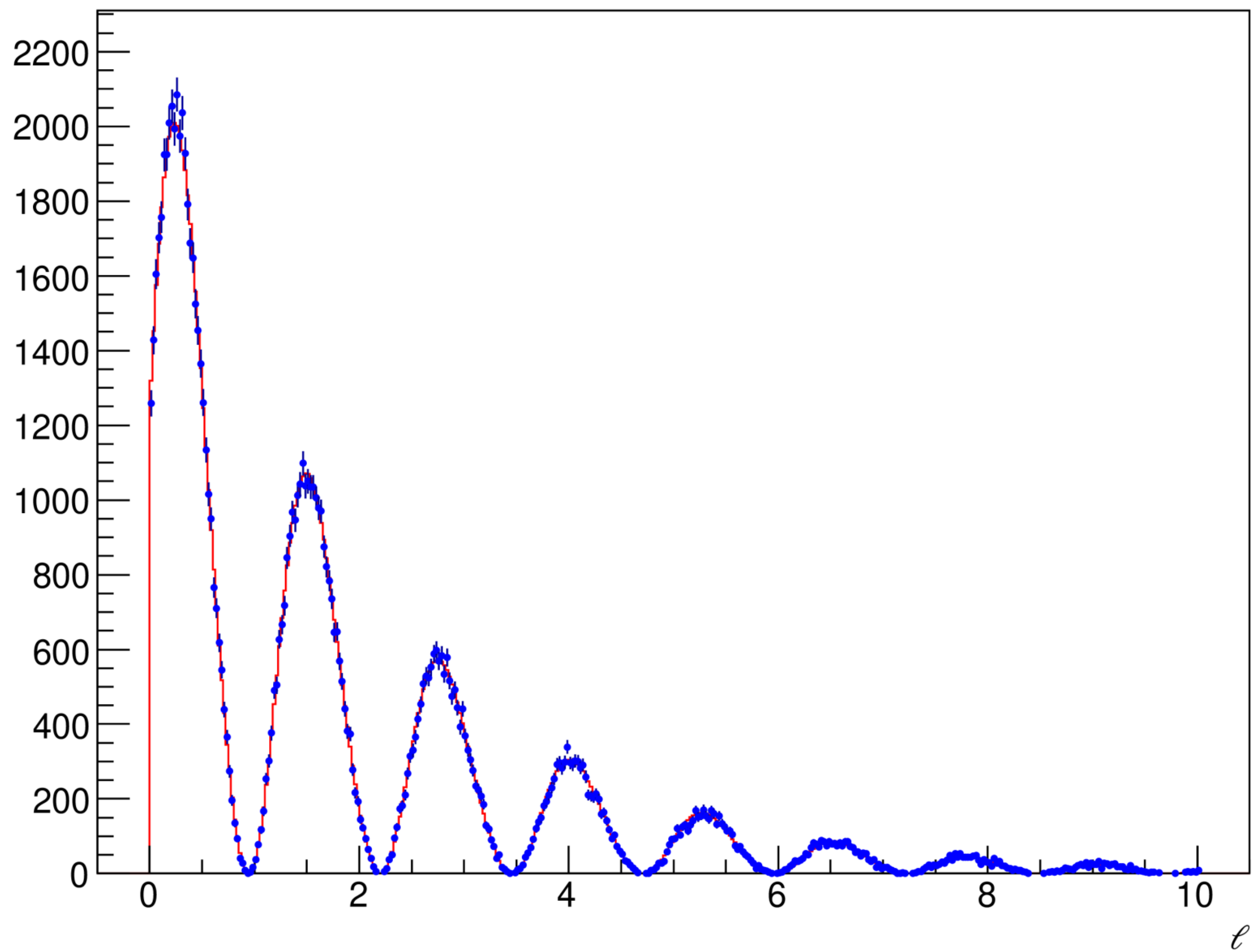
Related non-interaction probability over a path $0 \rightarrow \ell$

$$P_{NI}(0 \rightarrow \ell) = \exp\left(- \int_{0}^{\ell} \sigma_{phys}(s) \mathrm{d}s \right)$$



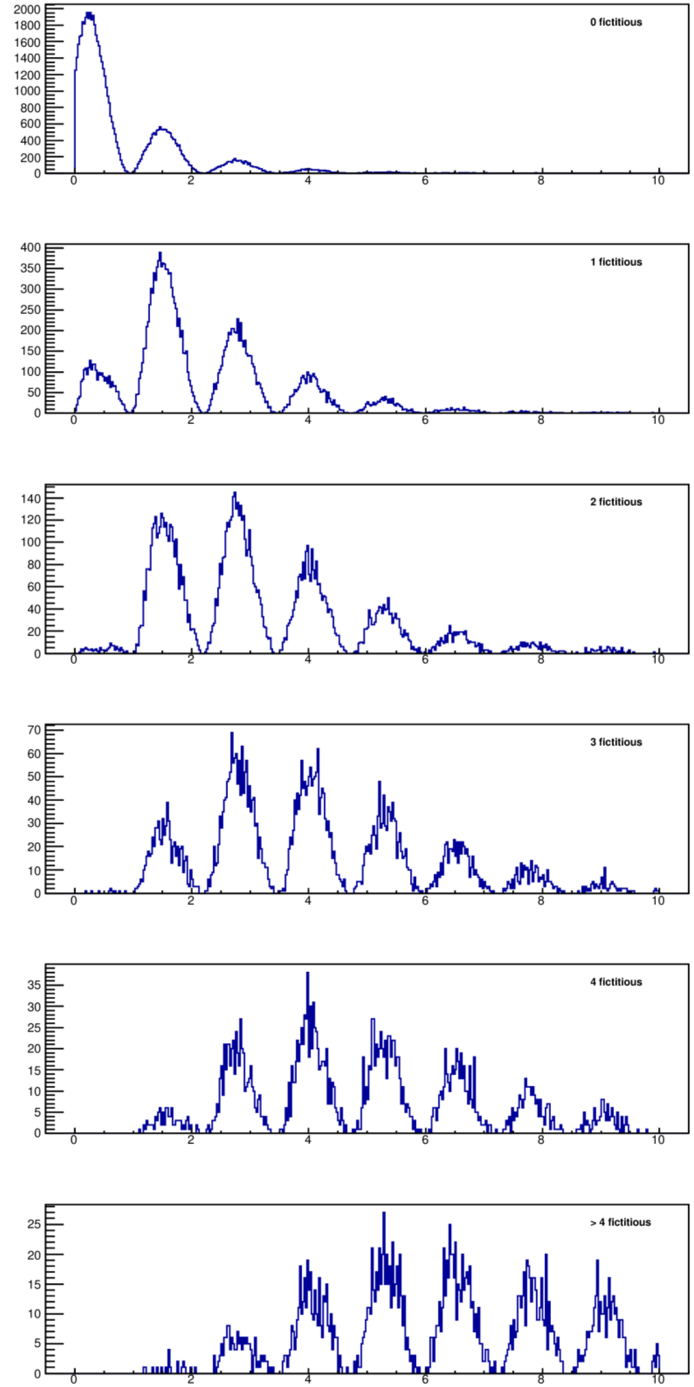
Related probability density function of interactions (product of the two above functions)





Distribution of interactions obtained by Woodcock sampling technique (100 k events)

Same events than previous page, but separated in samples with 0, 1, 2, 3, 4 and > 4 fictitious interactions before the physical one happens.



FROM WOODCOCK VIEWPOINT TO BIASING

Woodcock & Biasing

Invention ?
Re-invention ?
Re-phrasing of an existing technique ?

- The Woodcock viewpoint makes it easy the move to biasing:
  - In the analog world we have total physical and fictitious cross-sections:

$$\sigma_M^a = \sigma_{phys}^a(\ell) + \sigma_{fictitious}^a(\ell)$$

  - That we replace by their biased version in the biased world:

$$\sigma_M^b = \sigma_{phys}^b(\ell) + \sigma_{fictitious}^b(\ell)$$

- From there, we apply the formalism we already know (see last general paper):
  - For a step ending with no interaction (eg : geometry), we multiply the track weight by the non-interaction weight, ratio of the non-interaction probabilities $P_{NI}^{a(b)}(0 \to \ell)$:

$$w_{NI}(0 \to \ell) = \frac{P_{NI}^a(0 \to \ell)}{P_{NI}^b(0 \to \ell)}$$

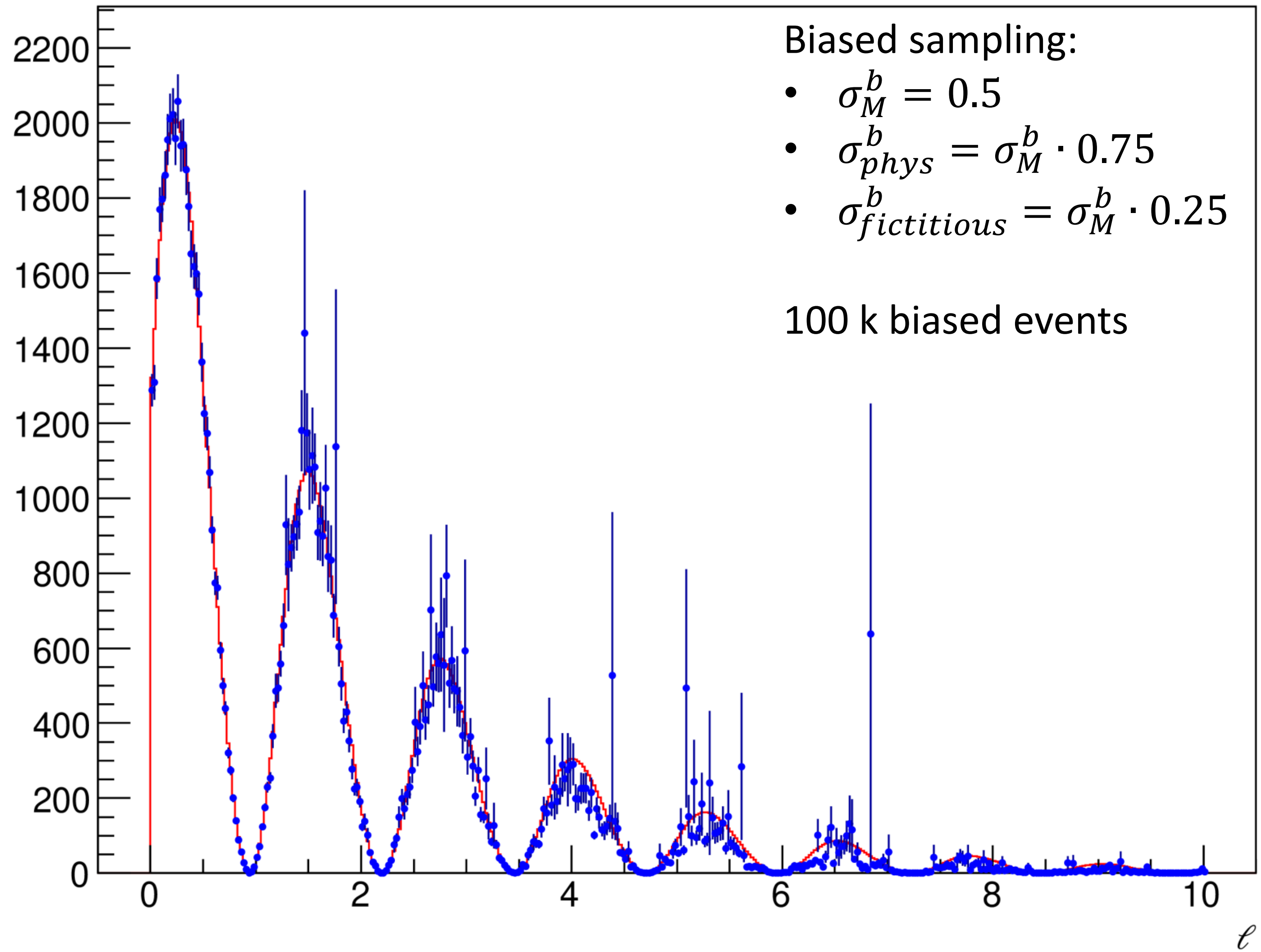
$$P_{NI}^{a(b)}(0 \to \ell) = \exp\left(-\int_0^\ell \sigma_M^{a(b)} \cdot ds\right) = \exp\left(-\sigma_M^{a(b)} \cdot \ell\right)$$

  - For a step ending with an interaction by process i, i = "physical" of "fictitious", we multiply the track weight by the interaction weight:

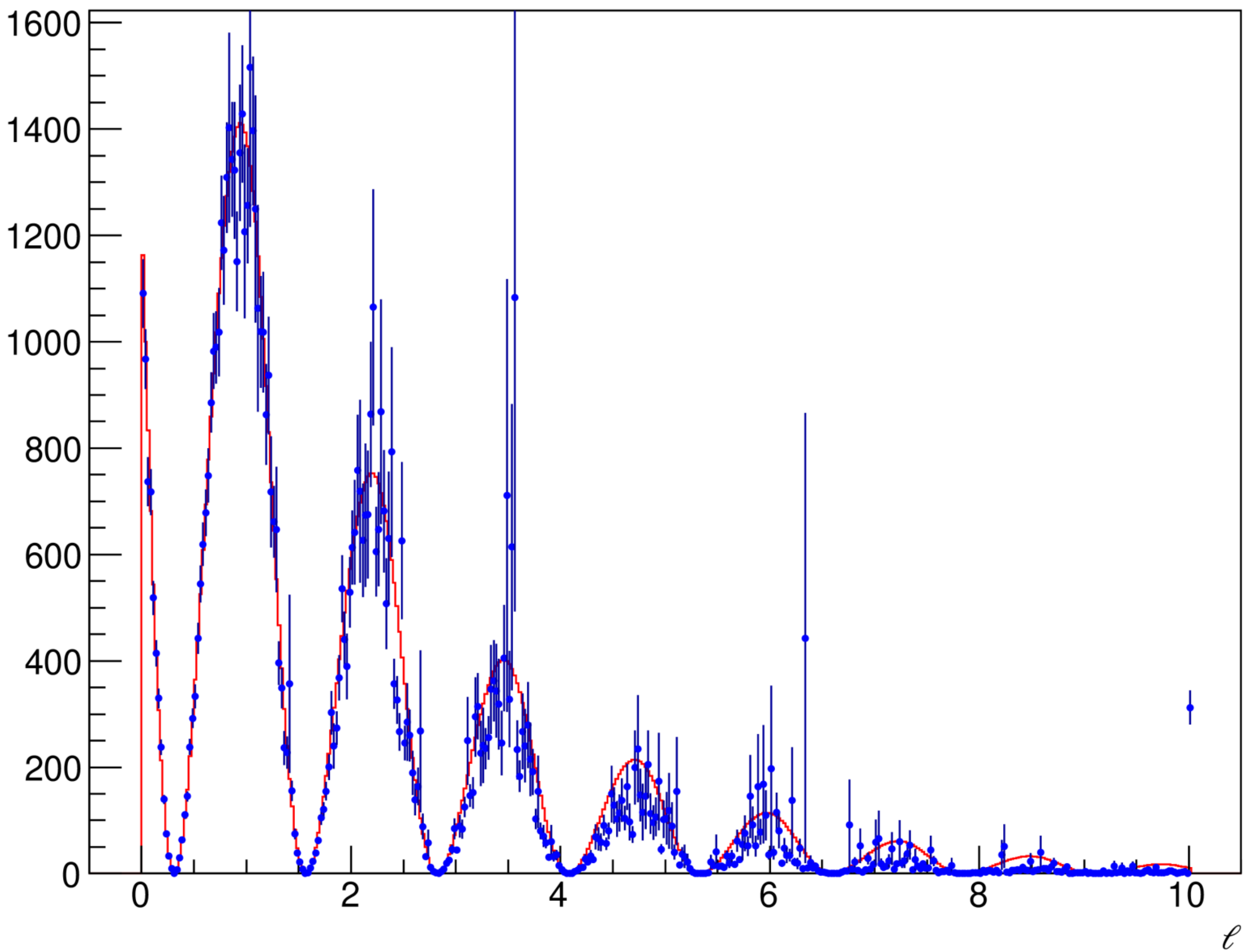
$$w_I(\ell) = w_{NI}(0 \to \ell) \cdot \frac{\sigma_i^a(\ell)}{\sigma_i^b(\ell)}$$

- And we're done !

TOY MC TEST 1
Use of technique
& improve first scheme



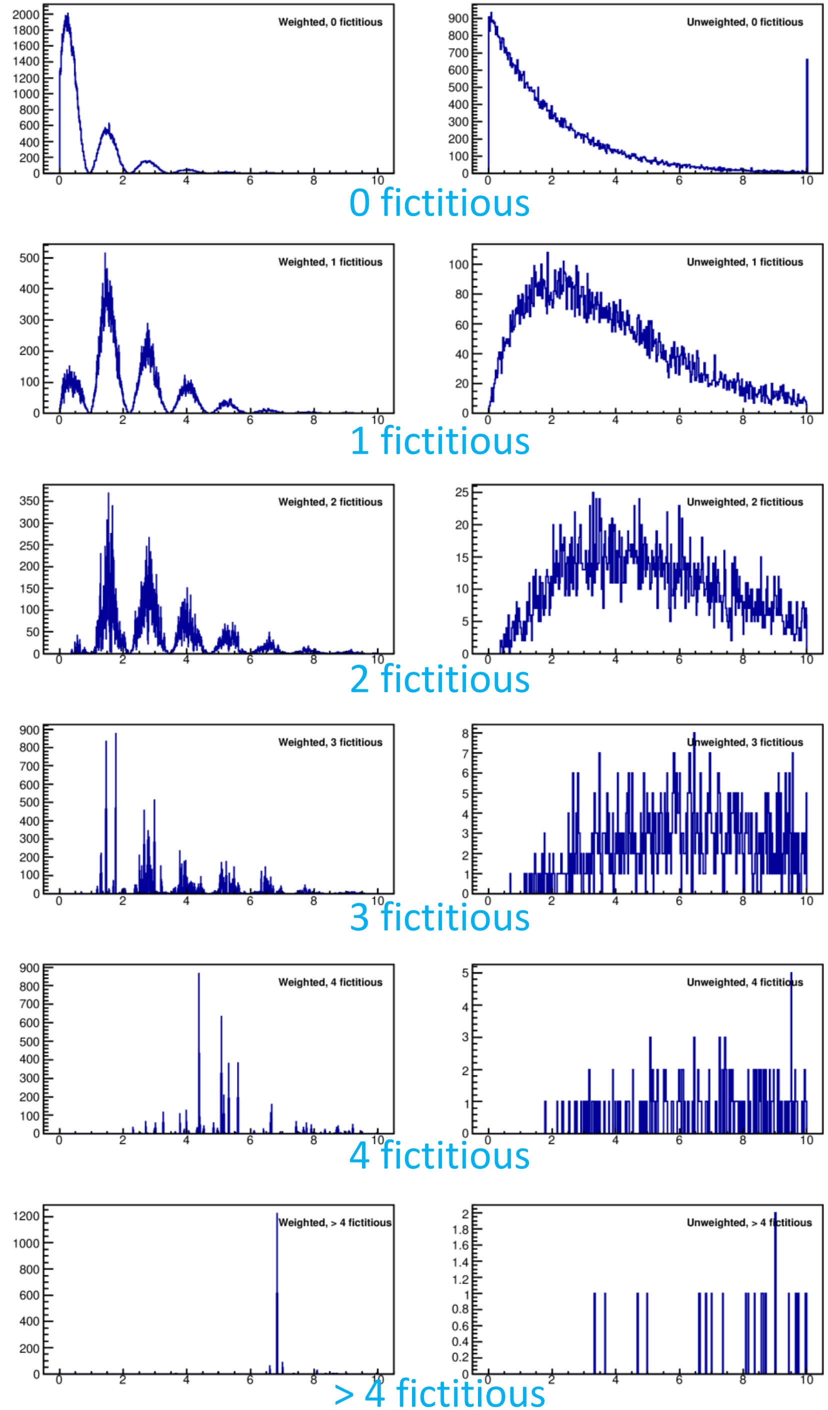
Reconstructed distribution of physical interactions using biasing



Reconstructed distribution of fictitious interactions using biasing

Weighted (reconstructed) distributions

Unweighted (generated) distributions

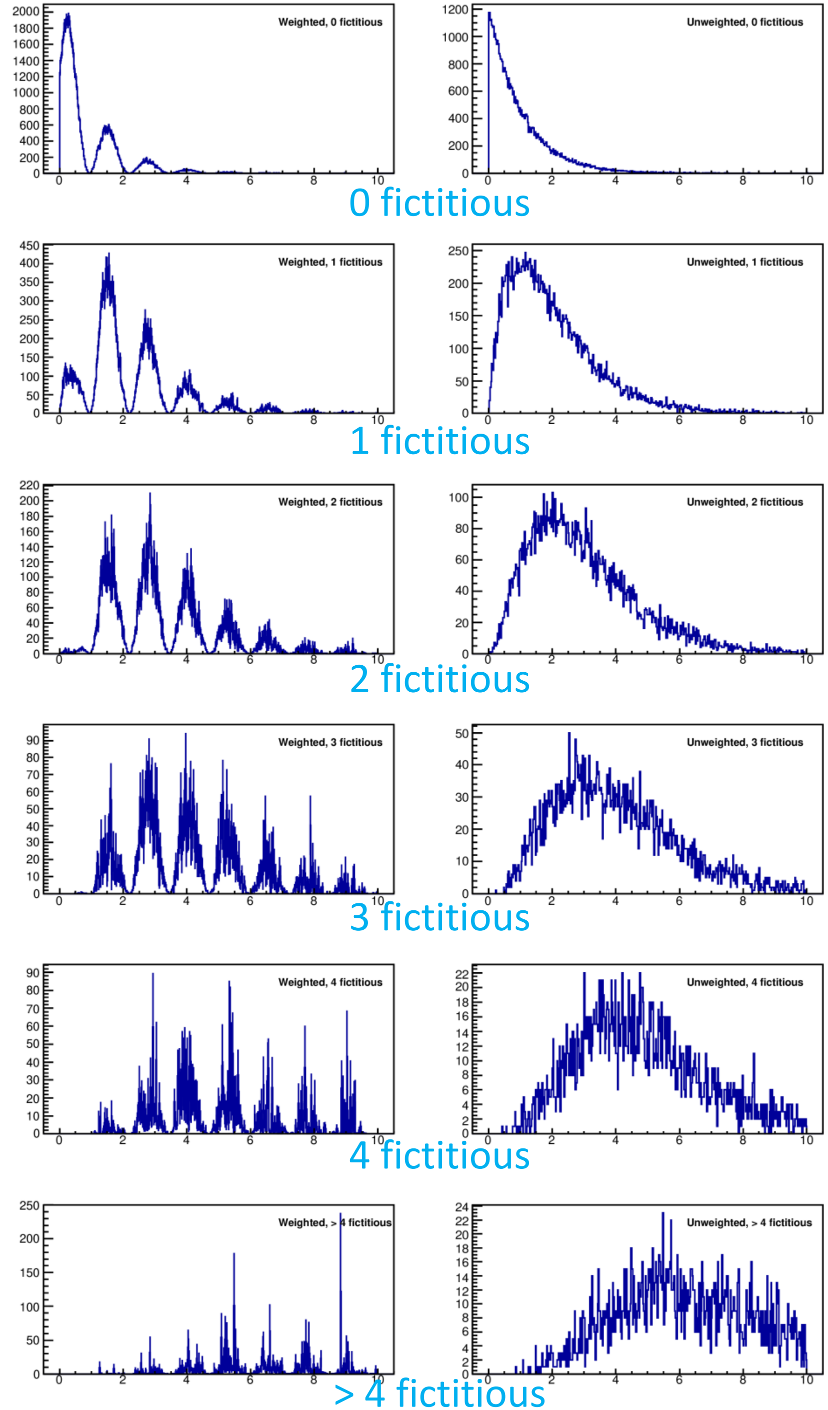


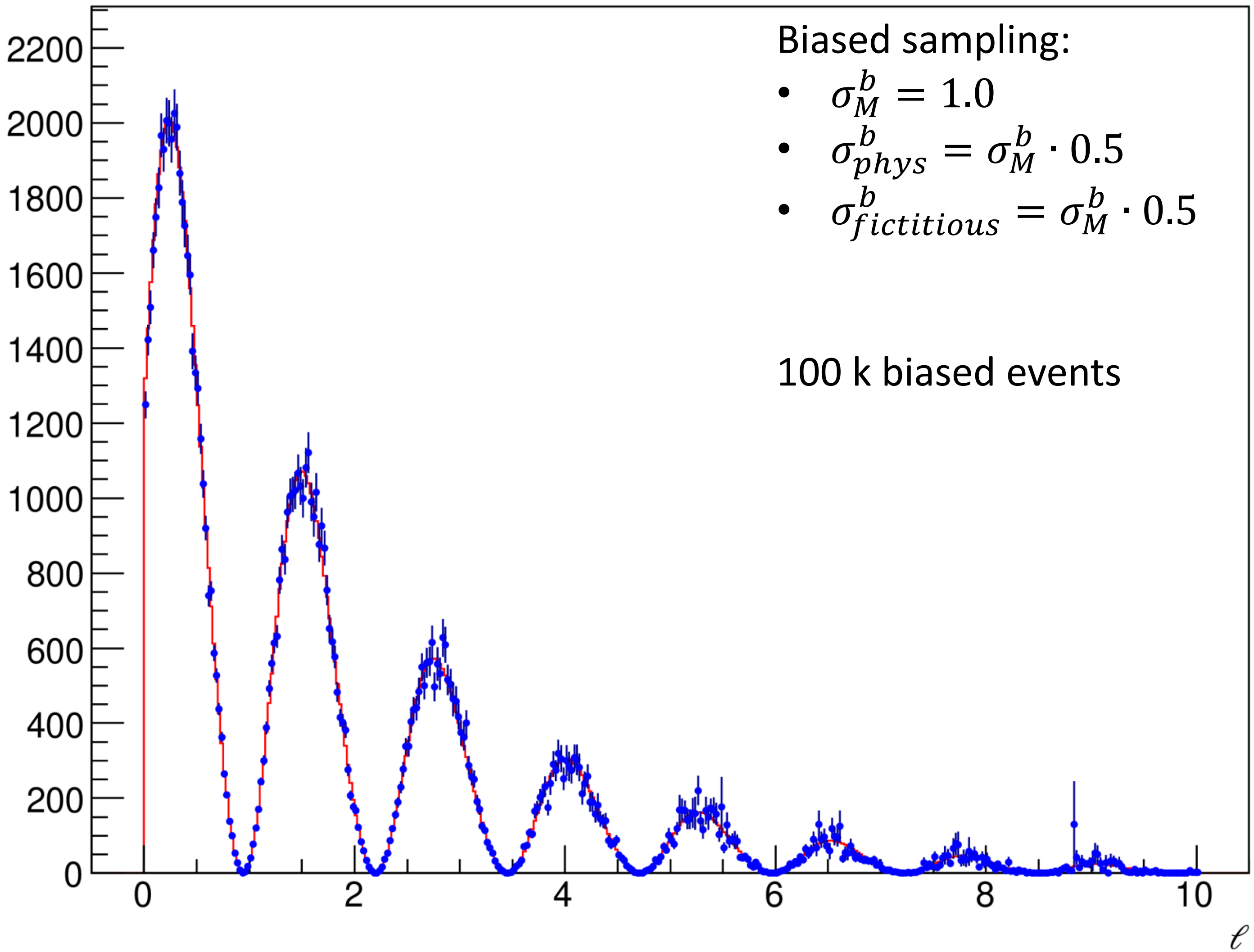
Weighted (reconstructed) distributions

Unweighted (generated) distributions

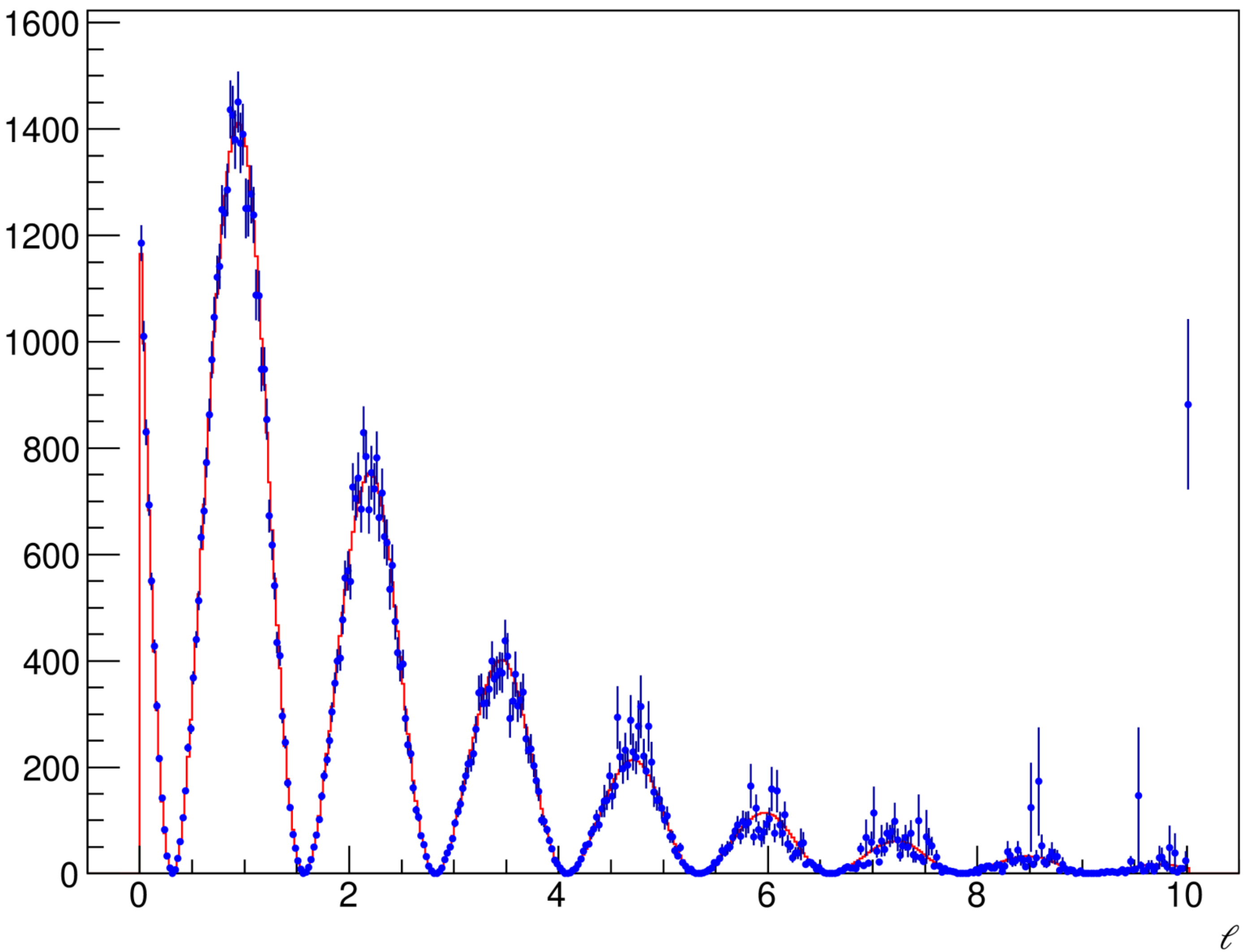
Redo with more adapted biased sampling to avoid depletions observed in previous page:

- $\sigma_M^b = 1.0$
- $\sigma_{phys}^b = \sigma_M^b \cdot 0.5$
- $\sigma_{fictitious}^b = \sigma_M^b \cdot 0.5$





Reconstructed distribution of physical interactions using biasing



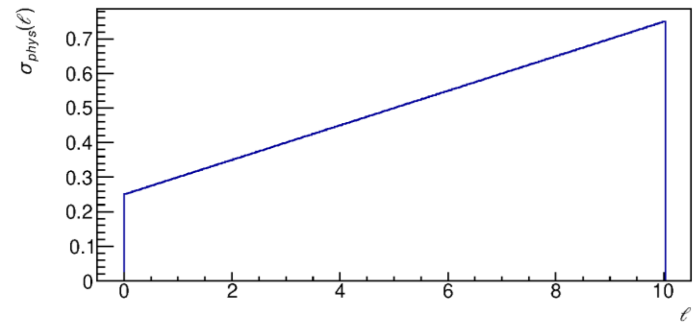
Reconstructed distribution of fictitious interactions using biasing

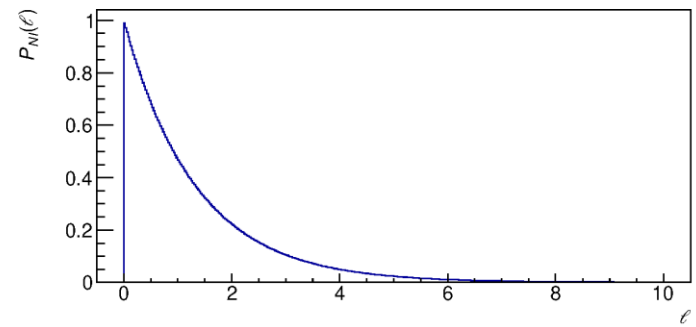
TOY MC TEST 2

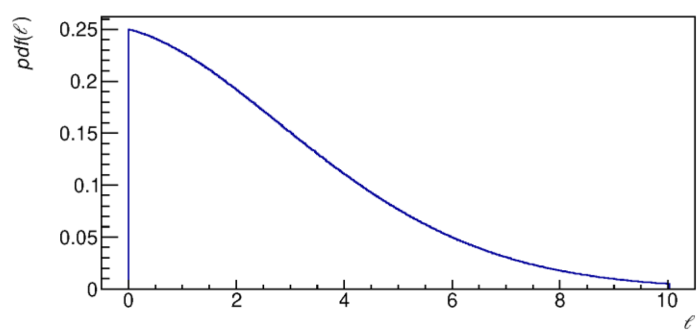
Testing varying biased cross-sections

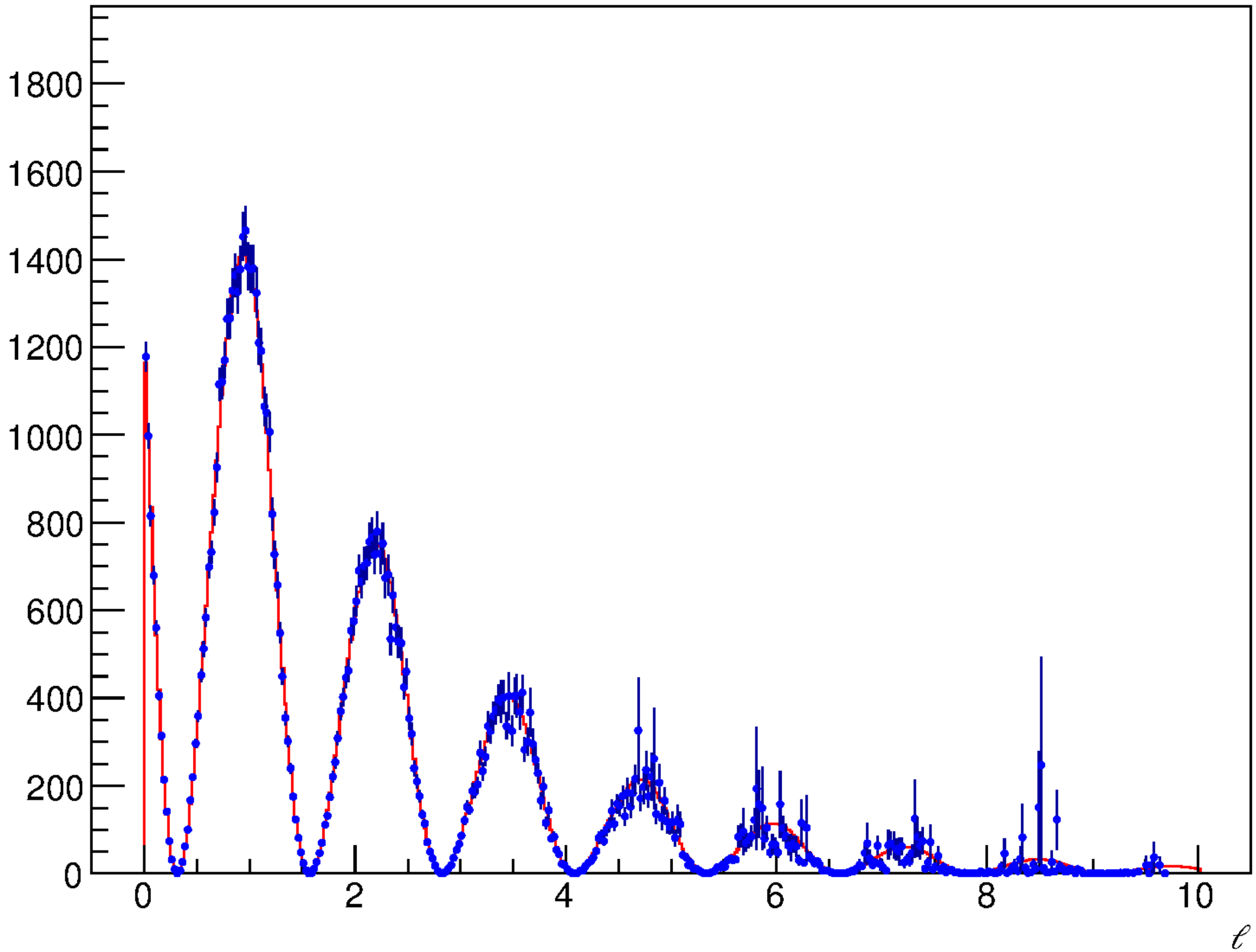
Testing varying biased cross-sections

- In previous test, biased cross-sections (physical and fictitious) were constant
- The Woodcock-based formalism supports however varying biased cross-sections
- Try with linearly varying biased cross-sections:
  - $\sigma^{b}_{phys}(\ell) = 0.25 + 0.05 \cdot \ell$
  - Then
    - $\sigma^{b}_{M} = 0.75$
    - $\sigma^{b}_{fictitious}(\ell) = \sigma^{b}_{M} - \sigma^{b}_{phys}(\ell)$
- (Note that we can anticipate some problems near $\ell \sim 10$ as the fictitious cross-section goes to zero, and the rareness of such events may lead to large weights.)

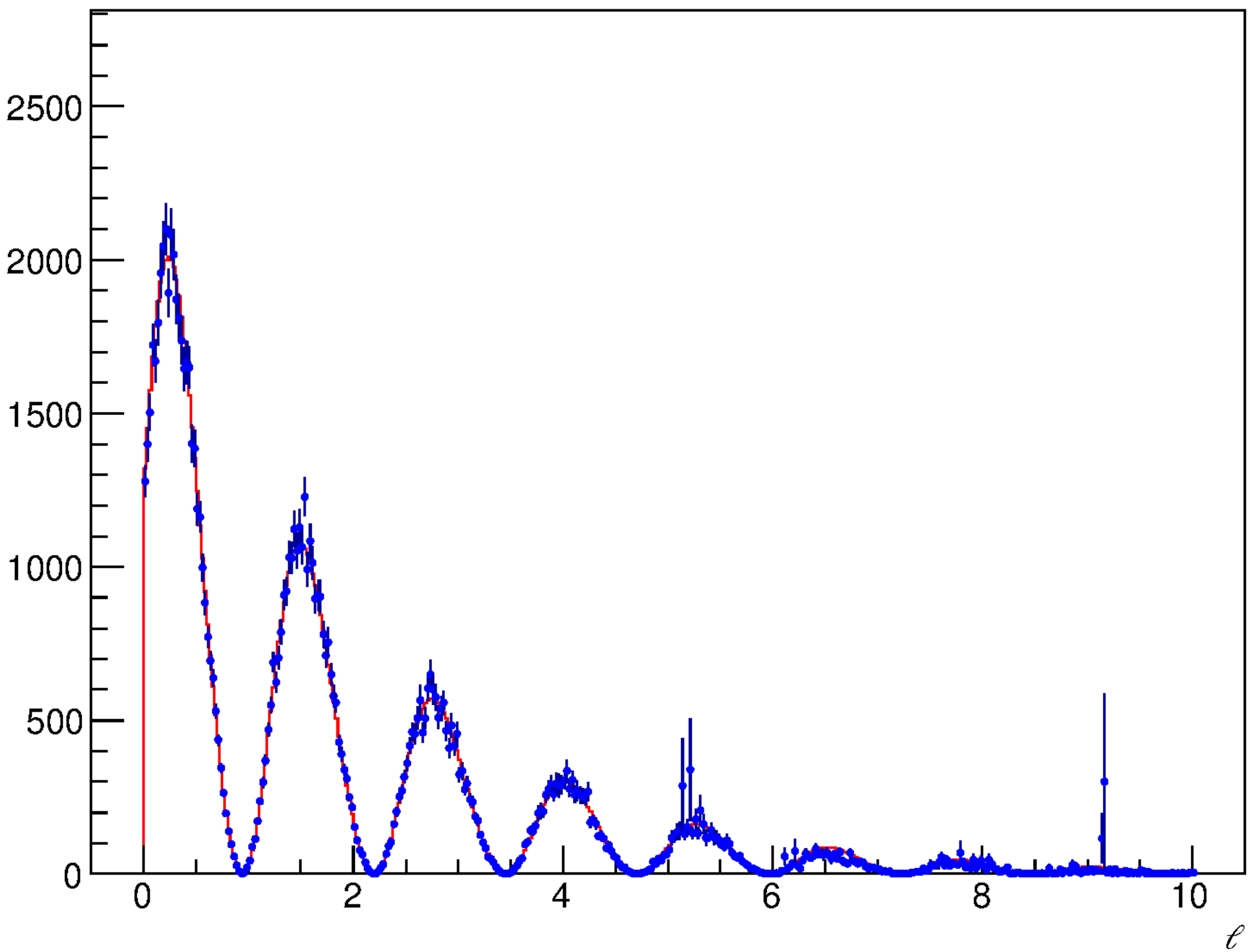








Reconstructed distribution of fictitious interactions using biasing



Reconstructed distribution of physical interactions using biasing

TOY MC TEST 3
Forced Free Flight

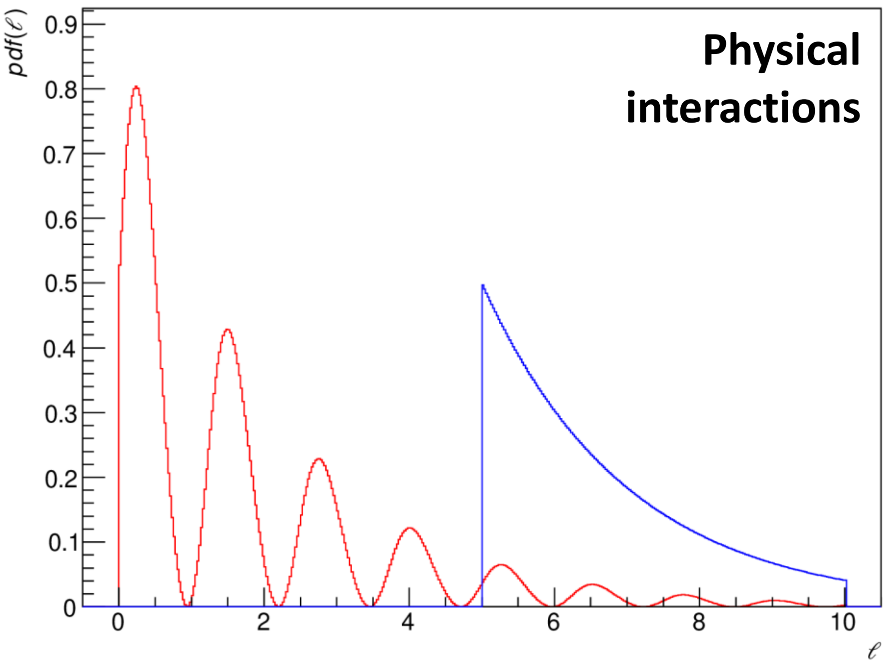
Forced Free Flight

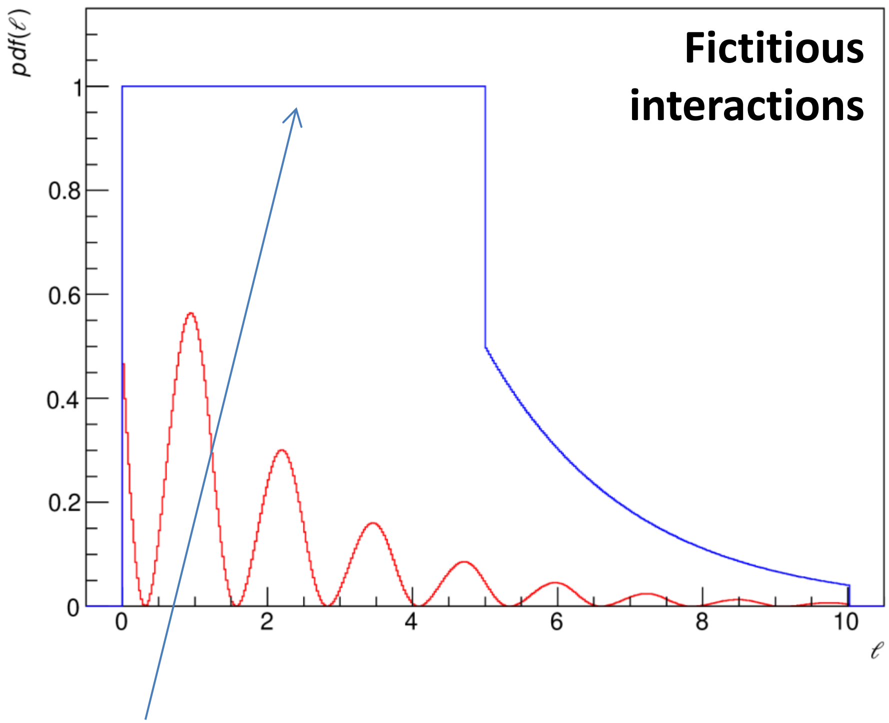
- Forced free flight intervenes for neutrals in the “forced interaction scheme” (in MCNP terminology)
  1. Split the track in two copies at the volume entrance
  2. One copy is forced to have an interaction (elastic or not) inside some volume
  3. The other copy being the opposite : no interaction, it does a forced free flight
     - It represents the unscattered flux

- For charged particles, we consider having a similar use-case
- We hence need a forced free flight for charged particles
  – With no interactions → no discrete interactions
  – But still continuous interactions
    - So that the track has realistic energy loss at volume exit
  – Note that the track is not guaranteed to exit the volume
    - As it may exhaust its energy in the volume
- We try a scheme in which a domain has only fictitious interactions
  – So to make the amount of particles preserved
-

	$\ell < 5$	$\ell > 5$
σ^b_{phys}	0.0	0.5
$\sigma^b_{fictitious}$	1.0	0.5

Distributions of physical and fictitious interactions in the analog and biased schemes

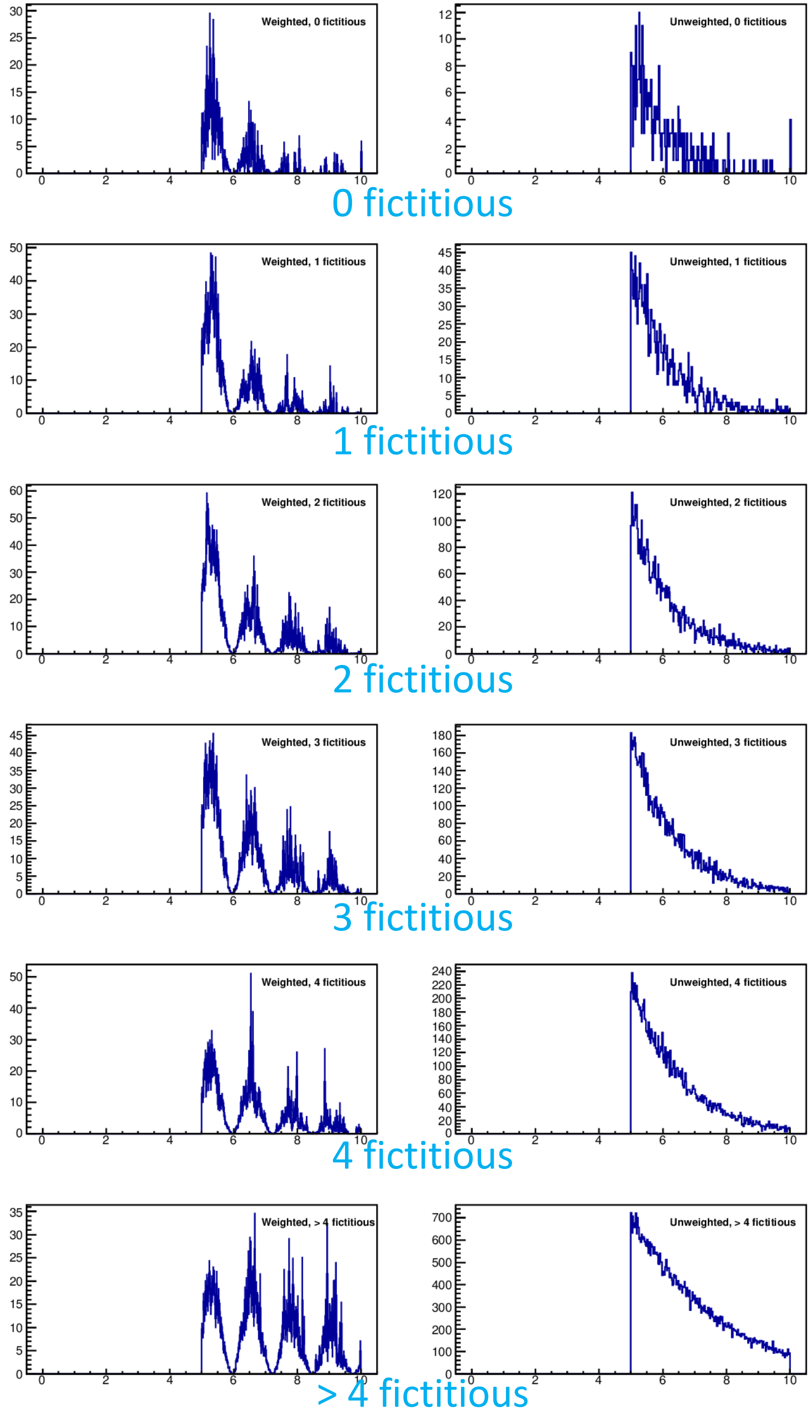


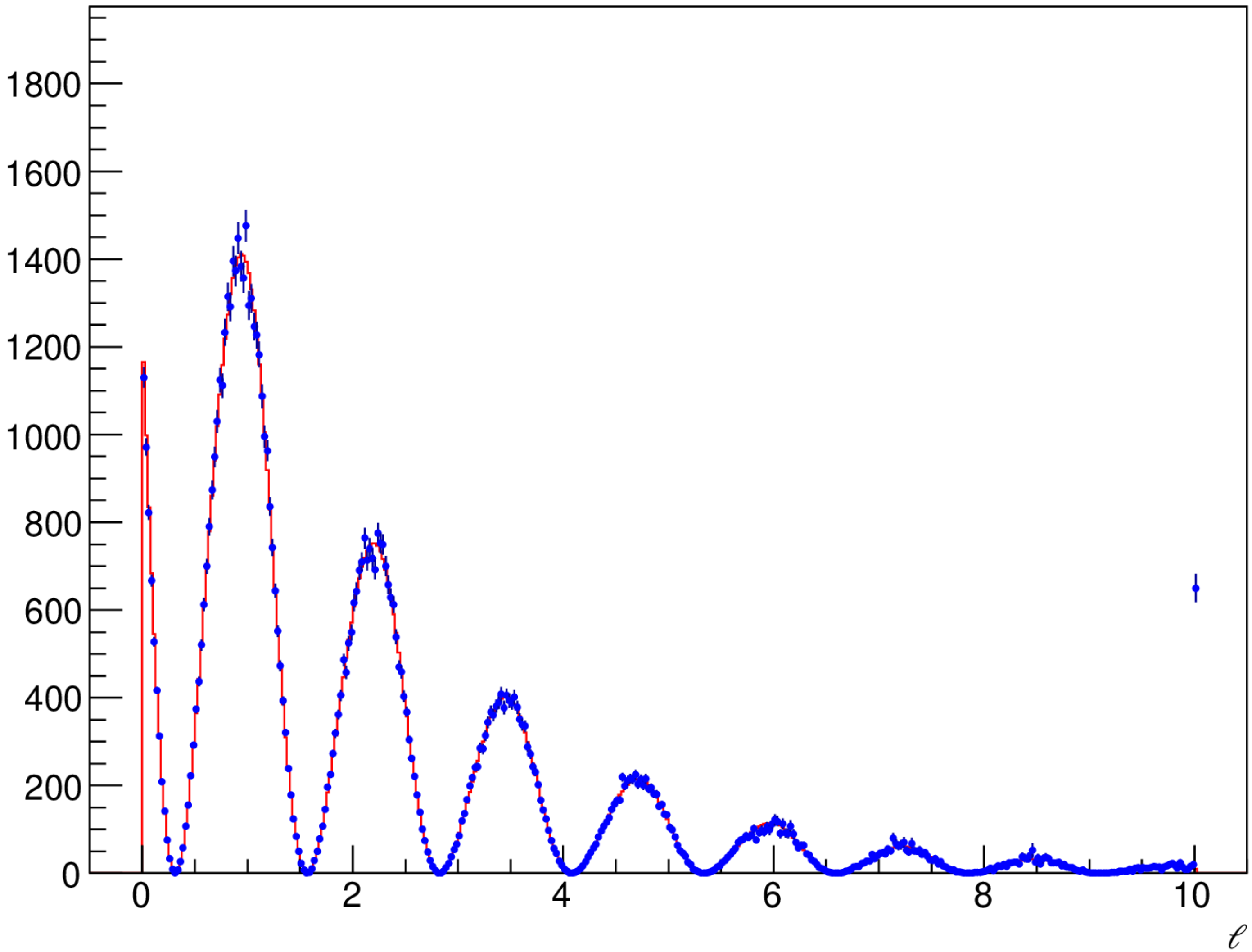


Note the particular feature with fictitious interactions : we have interactions ($pdf \neq 0$) but they don't consume particles, hence the possibility for a plateau !

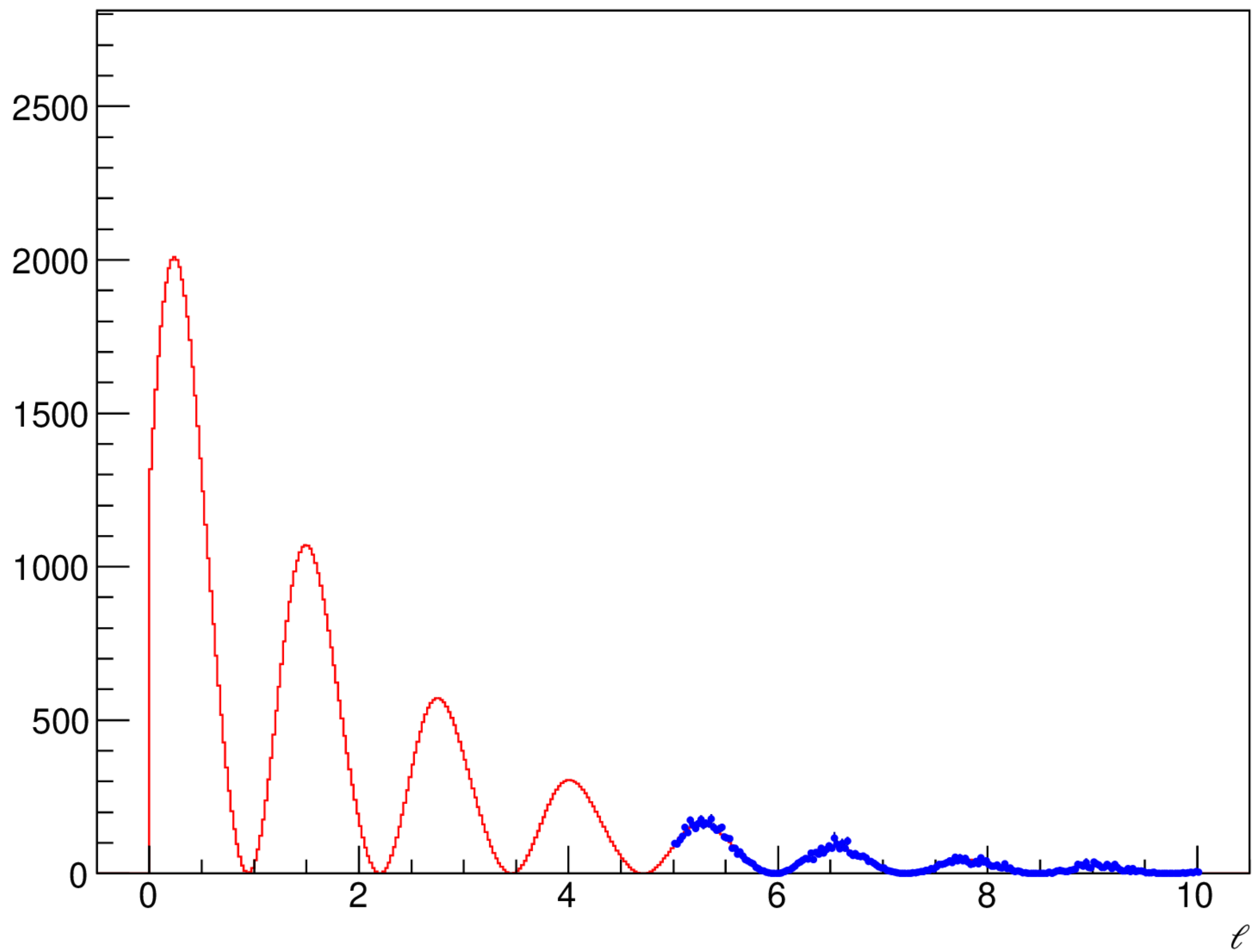
Weighted (reconstructed) distributions

Unweighted (generated) distributions





Reconstructed distribution of fictitious interactions :
they are all present, as expected



Reconstructed distribution of physical interactions : they are only present at $\ell > 5$, and with the proper normalization, hence correct weight, as desired !

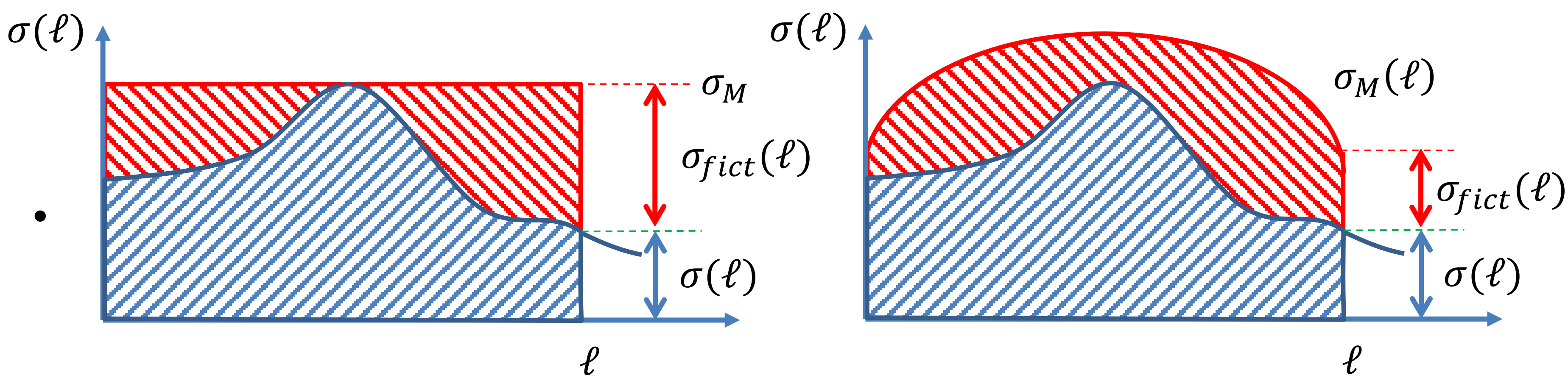
GENERALIZING ?

Considering varying maximum cross-section

Generalizing ?

Invention ?
Re-invention ?
Re-phrasing of an existing technique ?

- Woodcock approach needs a maximum of the XS in the segment $[0, L]$
  - If the maximum is far above the average, many fictitious will happen
    - This can be a penalty
  - **Fatal case** : it can happen maximum == ∞, specially on the biasing side
    - The important "forced interaction" use-case has such a divergence of the cross-section
    - In such case, the technique can't be applied
- But using a (maximum and) constant cross-section is for convenience
- Instead we could:
  - Maximum → "majorant" $\sigma_M(\ell) > \sigma_{phys}(\ell);\ \forall\ell$
    - So that we still have an amount of fictitious interaction everywhere
  - Constant → Choose $\sigma_M(\ell)$ convenient enough so that the related pdf can be sampled easily
  - **Next test to be done in this toy MC study**



Conclusion

- The smart Woodcock viewpoint of interpreting a track which makes a step with no interaction as a “fictitious interaction” is very convenient to allow to take into account the variation of cross-sections
- And it allows an easy application to biasing !

- On Toy MC samples, the biasing technique looks working well
  - Tested with constant and varying biased cross-sections
    - To reconstruct analog distributions from varying cross-sections
  - Tested with a free-flight scheme
  - It looks to have interesting observables to help diagnosing poor biased sampling:
    - Eg : comparison of weighted and unweighted distributions as function of the number of fictitious interaction
  - It looks to have interesting handle(s) to improve poor sampling:
    - Eg : amount of fictitious interactions
- Generalization ?
  - Going from constant maximum to varying upper bound cross-section
  - Needed for the forced interaction scheme for example

- Moving to G4 implementation will require technical help:
  - how to know or calculate the maximum cross-section over a step

- (And of course and as usual : manpower remains a issue…)