

Production of Higgs bosons with large transverse momentum

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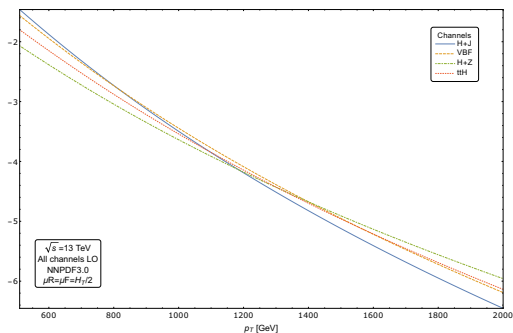
[ArXiv:1801.08226] and [ArXiv:1712.06549]

in collaboration with K. Melnikov(KIT), C. Wever(TUM) and J. Lindert(IPPP,UK)

N3PDF meeting

Higgs bosons with large transverse momentum

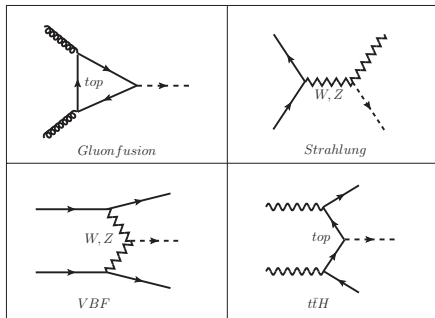
- Higgs transverse momentum distribution is used to constrain Higgs couplings
- Few channels for the top-Yukawa coupling
- Boosted Higgs $H \rightarrow b\bar{b}$ channel is competitive (at least complementary) to the $t\bar{t}H$ *C.Grojean, et al. 2013*
- Boosted Higgs provides an alternative approach to study the top Yukawa
- CMS probed already the high - $p_T > 450$ GeV region



LO rates for H + j (2j)

Higgs bosons with large transverse momentum

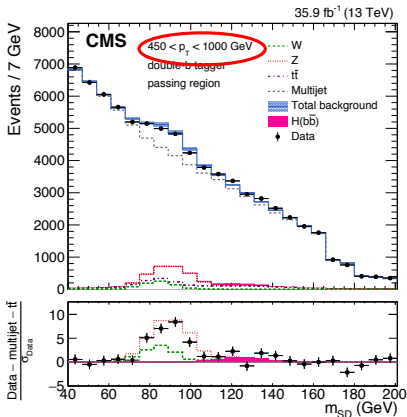
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Main channels

Higgs bosons with large transverse momentum

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CMS Physics Analysis Summary; ArXiv: 1709.05543

$$\sigma_{incl}^{CMS}(H \rightarrow b\bar{b}) = 74^{+51}_{-49} \text{ fb}$$

Since, only the LO cross-section was available (*R.K. Ellis, et al. ,1988; U. Baur, et al., 1990*) it led to huge theoretical uncertainties.

Table 1: Summary of the systematic uncertainties affecting the signal, W and Z+ jets processes. Instances where the uncertainty does not apply are indicated by “—”.

Systematic source	W/Z	H
Integrated luminosity	2.5%	2.5%
Trigger efficiency	4%	4%
Pileup	<1%	<1%
$N_2^{1,DDT}$ selection efficiency	4.3%	4.3%
Double-b tag	4% (Z)	4%
Jet energy scale / resolution	10/15%	10/15%
Jet mass scale (p_T)	0.4%/100 GeV (p_T)	0.4%/100 GeV (p_T)
Simulation sample size	2–25%	4–20% (ggF)
H p_T correction	—	30% (ggF)
NLO QCD corrections	10%	—
NLO EW corrections	15–35%	—
NLO EW W/Z decorrelation	5–15%	—

Anomalous couplings

- BSM physics \rightarrow deforming the top-Yukawa coupling
- Top-Yukawa coupling is known to about 20-30% (Observation of $t\bar{t}H$, CMS, 2018)

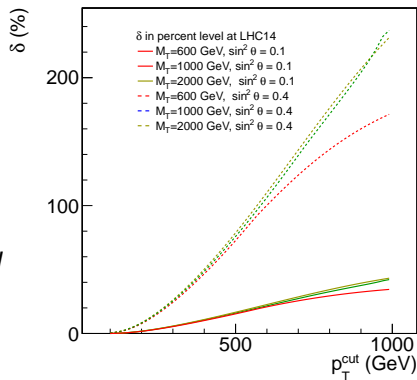
$$\frac{m_t}{v} \bar{t}tH \rightarrow -\kappa_g \frac{\alpha_s}{12\pi V} G_{\mu\nu}^a G^{\mu\nu,a} H + \kappa_t \frac{m_t}{v} \bar{t}tH$$

- **Problem:**

$$\sigma_{gg \rightarrow H} \sim \alpha_s^2 / v^2 (\kappa_g + \kappa_t)^2$$

- **Solution:** to go beyond inclusive cross-section

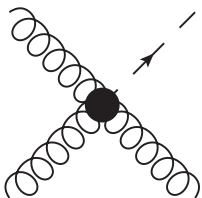
$$\sigma_{gg \rightarrow H+g} \sim \left(\kappa_g + \kappa_t \frac{4m_t^2}{p_{\perp}^2} \right)^2$$



Anrea Banfi, et al., 2013; Additional heavy fermion will increase the number of events relative to the SM

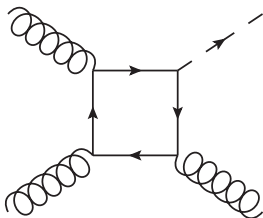
Low p_T vs High p_T

$$p_{T,H}^2 \ll 4m_t^2$$



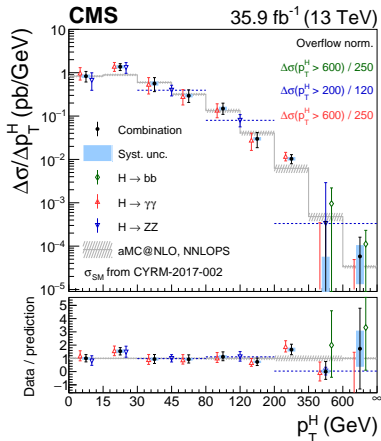
- $\{m_h, s, t, u\} \ll m_t$
- well established algorithmic approach to calculations (Large mass expansion)
- Higgs Effective Field Theory (HEFT) $m_t \rightarrow \infty$

$$p_{T,H}^2 \gg 4m_t^2$$

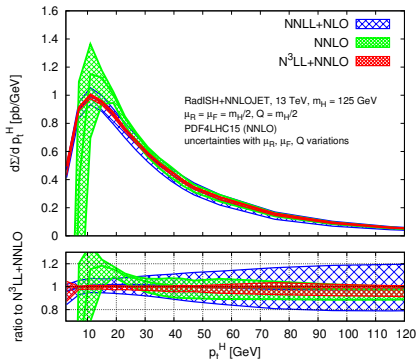


- $m_h < m_t \ll \{s, t, u\}$ hierarchy
- it is not clear what the degrees of freedom are here
- no robust algorithm *until recently*:
K. Melnikov, L. Tancredi, C. Wever, 2016

Low p_T



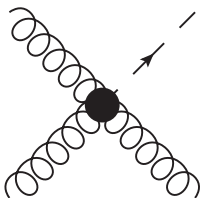
Phys. Lett. B 792 (2019) 369



Wojciech Bizo, Xuan Chen, Aude Gehrmann-De Ridder, Thomas Gehrmann, Nigel Glover, Alexander Huss, Pier Francesco Monni, Emanuele Re, Luca Rottoli, Paolo Torrielli → JHEP 1812 (2018) 132

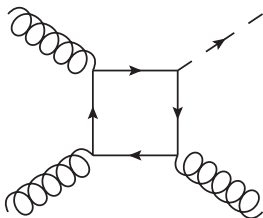
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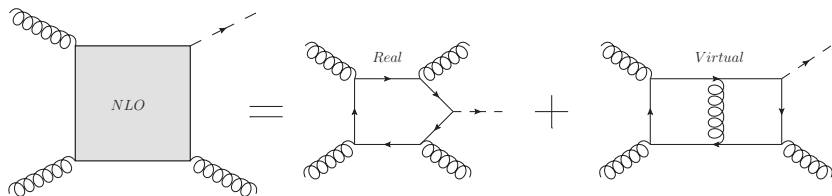
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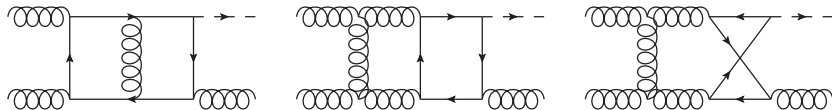
NLO corrections to Higgs + Jet



NLO corrections consist of two parts

- Real corrections are known analytically (*V. Del Duca, et al., 2001*)
- We used *OpenLoops* implementation of real corrections (*F. Cascioli, et al., 2012*)
- It is well understood how to combine these two pieces together (*S. Frixione, Z. Kunstz, A. Signer, 1995; S. Catani, M.H. Seymour, 1996*)
- Missing analytical results for the virtual amplitude

Virtual corrections



Examples of Feynman diagrams for H + Jet

- A four-scale problem: three external (s, p_T, m_h) and one internal (m_t)
- 264 Feynman integrals
- ~~Complicated reduction~~ (*KIRA, FIRE6* and other private codes)
- No complete analytic result with the full top mass dependence (*R. Bonciani, et al., 2016 and 2019*)
- Only numerical results with the full top mass available (*S. P. Jones, et al., 2018*)

Master integrals

Computing master integrals for the Higgs boson + jet \rightarrow bottleneck

- 6 top-sectors computed (cf. arXiv:1907.13156 & arXiv:1609.06685)



- One is still missing



- Integrals contain elliptic kernels \rightarrow had to come up with a particular expansion along a contour (arXiv:1907.13234)



no recipe to treat integrals on elliptic curves in full generality (except, if there is only one elliptic curve arXiv:1712.07089)

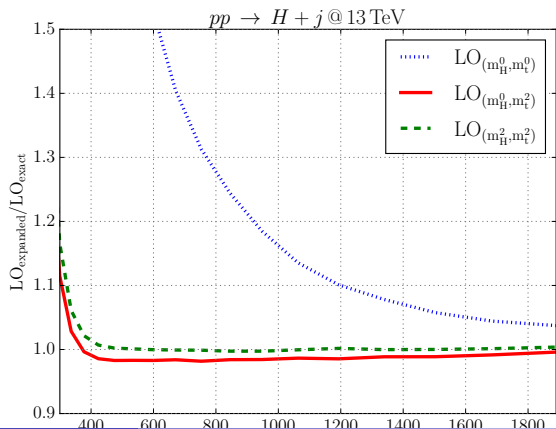
- many different entries “letters” at GPLs (49 for planar integrals and 69 for non-planar)

Virtual corrections → Different approach instead of exact results

Hierarchy $m_h < m_t \ll \{s, t, u\}$ suggests → Expansion in small parameters $(-\frac{m_h^2}{4m_t^2}, -\frac{m_t^2}{s})$ using Differential Equation approach (DEQ). It allows to calculate the virtual amplitude for H + jet production.

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Amplitudes

Amplitudes for $H \rightarrow ggg$ & $H \rightarrow q\bar{q}g$

$$\begin{aligned}\mathcal{A}_{H \rightarrow ggg}(p_1^{a_1}, p_2^{a_2}, p_3^{a_3}) &= f^{a_1 a_2 a_3} \epsilon_1^\mu \epsilon_2^\nu \epsilon_3^\rho \mathcal{A}_{\mu\nu\rho}^g(s, t, u, m_t), \\ \mathcal{A}_{H \rightarrow q\bar{q}g}(p_1^j, p_2^k, p_3^a) &= i T_{jk}^a \epsilon_3^\mu(p_3) \bar{u}(p_1) \mathcal{A}_\mu^q(s, t, u, m_t) v(p_2).\end{aligned}$$

Tensor decomposition (*T. Gehrmann, et al., 2011*)

$$\begin{aligned}\mathcal{A}_{\mu\nu\rho}^g(s, t, u, m_t) &= F_1^g g_{\mu\nu} p_{2\rho} + F_2^g g_{\mu\rho} p_{1\nu} + F_3^g g_{\nu\rho} p_{3\mu} + F_4^g p_{3\mu} p_{1\nu} p_{2\rho}, \\ \mathcal{A}_\mu^q &= F_1^q (\not{p}_3 p_{2\mu} - p_2 \cdot p_3 \gamma^\mu) + F_2^q (\not{p}_3 p_{1\mu} - p_1 \cdot p_3 \gamma^\mu).\end{aligned}$$

where $F_j^i = \sum_k R_{jk}^i(s, t, u, m_t; \epsilon) \mathcal{I}_k$ are form factors; a linear combination of rational arguments and **scalar integrals**

Reduction of scalar integrals

Scalar integrals

$$\mathcal{I}_{\text{top}}(a_1, a_2, \dots, a_8, a_9) = \int \frac{\mathcal{D}^d k \mathcal{D}^d l}{[1]^{a_1} [2]^{a_2} [3]^{a_3} [4]^{a_4} [5]^{a_5} [6]^{a_6} [7]^{a_7} [8]^{a_8} [9]^{a_9}}$$

Integration-by-parts (IBP; *K. G. Chetyrkin and F. V. Tkachov, 1981*)

$$\int \frac{\partial}{\partial k^\mu} \left(q^\mu \prod_{j=1}^J \frac{1}{[N]^{a_j}} \right) \mathcal{D}k \mathcal{D}l = 0 \quad q^\mu = \{k^\mu, p^\mu\}$$

via IBP the following mapping is done

$$\{\mathcal{I}_1, \mathcal{I}_2, \dots, \mathcal{I}_N\} \rightarrow \{\mathfrak{I}_1, \mathfrak{I}_2, \dots, \mathfrak{I}_{\tilde{N}}\}$$

\mathfrak{I}_i are Master Integrals(MI); 458 Master integrals to compute (crossings included)

Differential equations

DEQs

Taking derivatives w.r.t m_t, s, t, u and applying IBPs

$$\partial_k \mathfrak{I}_i(\kappa, \eta, z, \epsilon) = \sum_j A_{ij}^k(\kappa, \eta, z, \epsilon) \mathfrak{I}_j(\kappa, \eta, z, \epsilon), \quad k \in \{\kappa, \eta, z\}.$$

"Normalized" variables

$$\kappa = -\frac{m_h}{4m_t}, \eta = -\frac{m_t}{s}, z = \frac{u}{s}, \quad 0 < \kappa, \eta \ll 1, z > 0, s < 0$$

Note that both κ, η are numerically small!

Ansatz

Once constructed, DEQs are analyzed. DEQs admit the following solutions

$$\mathfrak{I}_i(\kappa, \eta, z, \epsilon) = \sum_{j,k,l,m \in \mathbb{Z}, n \in \mathbb{N}} c_{i,j,k,l,m,n}(z, \epsilon) \eta^{j-k\epsilon} \kappa^{l/2-m\epsilon} \log^n(\kappa).$$

Solving DEQs

Inserting Ansatz

By inserting the ansatz into DEQs we simplify the problem significantly. Namely, we mapped DEQs onto algebraic equations

$$\mathfrak{J} \rightarrow c_{i,j,k,l,m,n}(z, \epsilon)$$

Combining this with $\kappa, \eta \ll 1$ we get a finite system of linear equations after truncating linear series; in practice it is a very sparse system. **This takes care of κ, η DEQs**.

z - integration

Ansatz helps, but we still need to integrate z-DEQs

$$\frac{\partial}{\partial z} c(z, \epsilon) = \epsilon M(z) c(z, \epsilon)$$

Integrated in terms of Goncharov's polylogarithms

$$G(\underbrace{l_1, \dots, l_n}_{\text{weight } n}; z) := \int_0^z dz' \frac{G(l_2, \dots, l_n; z')}{z' - l_1},$$

$$G(; z) = 1, \quad G(\underbrace{0, \dots, 0}_{n \text{ times}}; z) = \frac{1}{n!} \log^n(z).$$

An example

Consider a simple diagram: a bubble.

$$\frac{\partial}{\partial \mu} \text{---} \circ \text{---} = -\frac{2\epsilon - 1}{2(\mu + 1)} \cdot \text{---} \circ \text{---} + \frac{2(\epsilon - 1)}{\mu(\mu + 1)} \cdot \text{---} \circ \text{---}$$

with $\mu = 4m^2/s$. Expanding in μ

$$\frac{\partial}{\partial \mu} \text{---} \circ \text{---} = \frac{1}{2}(\mu - 1)(2\epsilon - 1) \text{---} \circ \text{---} - 2(\mu - 1)(\epsilon - 1)\mu^{-\epsilon} + o(\mu^2).$$

We “guess” a solution

$$\text{---} \circ \text{---} = \text{---} \circ \text{---} + \frac{\mu(1 - 2\epsilon)}{2} \cdot \text{---} \circ \text{---} + 2 \cdot \text{---} \circ \text{---} + o(\mu^2).$$

Comparison with full result

Simplifications

- 49 letters \rightarrow $\{-1, 0, 1\}$ letters (correspond to HPLs)
- Elliptic sectors “disappear”

Complications

- it is a different system \rightarrow
- not all mathematical limits are accessible

Determining boundary conditions is difficult.

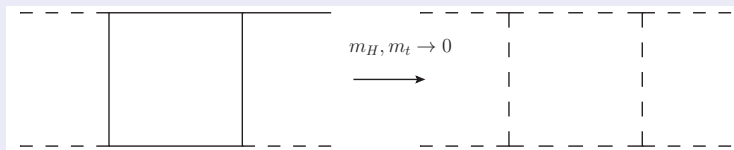
Last step is always the hardest one

Massless two-loop master diagrams

$$\mathfrak{I} = M0 + M1 \cdot \eta + \dots$$

$$\frac{\partial \mathfrak{I}}{\partial \eta} = (A + B \cdot \eta + \dots) \mathfrak{I} + \text{subtopologies}$$

Convenient to solve for $M0$, since they are known (T. Gehrmann, E. Remiddi, 2000 and 2001). In other words, master integrals for Higgs + Jet are master integrals in massless limit (*not always*)

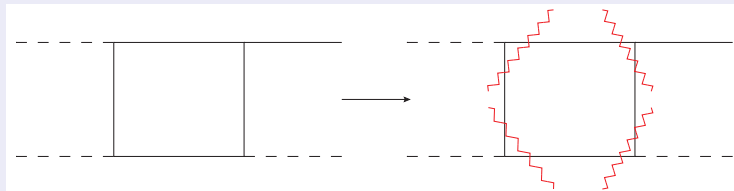


Massless limit

Last step is always the hardest one

Singularities

Master-integrals admits certain singularities



cut for a triangle integral

At equation level, we can check that is satisfied

$$\frac{\partial \mathcal{J}}{\partial \eta} = \dots + \frac{\sum b_j M_j}{p_1 \cdot p_4} + \dots$$

Hence, we should $\sum b_j M_j \rightarrow 0$

Last step is always the hardest one

Mellin-Barns transformation (M. Czakon and A. Smirnov;
V. Smirnov, 1999; J.B Tausk, 1999)

- we can express a sum as a contour integral (*MBtools*)

$$(A_1 + A_2)^{-\nu} = \frac{1}{2\pi i} \int_{-i\infty}^{i\infty} d\sigma A_1^\sigma A_2^{-\nu-\sigma} \frac{\Gamma(-\sigma)\Gamma(\nu + \sigma)}{\Gamma(\nu)}$$

- we want a particular branch $((m_t^2)^{-\epsilon}, (m_t^2)^{-2\epsilon}, \dots)$
- The main difficulty \rightarrow Mellin-Barns representation
- Advantage \rightarrow many constants could be found simultaneously by pinching

$$\int \frac{\mathcal{D}^d k \mathcal{D}^d l}{[1]^{\# + a_1} [2]^{\# + a_2} [3]^{\# + a_3} \dots}$$

that is putting $a_i \rightarrow 0$ we can find boundaries for subtopologies.

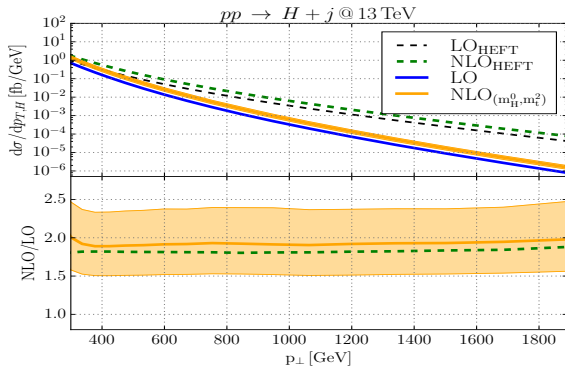
Last step is always the hardest one

Computing Feynman integral in a kinematic point

It is last resort method. Just few suggestions

- We have to extract a particular branch $((m_t^2)^{-\epsilon}, (m_t^2)^{-2\epsilon}, \dots)$
- Change of variables
- Bring integrand to a such a form where one can integrate it to hypergeometric functions
- Hypergeometric functions are well understood.
- Look for asymptotics of hypergeometric functions to extract the branch you need

Higgs + Jet at NLO



$p_{H,T}$ distribution above the top mass threshold

PDF:NNPDF3.0

$m_t = 173.2\text{GeV}$

$m_h = 125\text{GeV}$

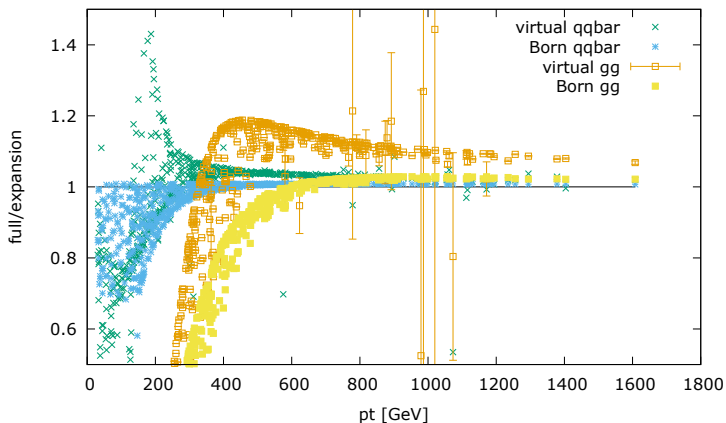
$\mu = \{1/2, 2\}\mu_0$

the top mass
effect

4 – 6%

	LO _{HEFT} [fb]	NLO _{HEFT} [fb]	K	LO [fb]	NLO [fb]	K
$p_{\perp} > 400$ GeV	33.8 ^{+44%} _{-29%}	61.4 ^{+20%} _{-19%}	1.82	12.4 ^{+44%} _{-29%}	23.6 ^{+24%} _{-21%}	1.90
$p_{\perp} > 450$ GeV	22.0 ^{+45%} _{-29%}	39.9 ^{+20%} _{-19%}	1.81	6.75 ^{+45%} _{-29%}	12.9 ^{+24%} _{-21%}	1.91
$p_{\perp} > 500$ GeV	14.7 ^{+44%} _{-28%}	26.7 ^{+20%} _{-19%}	1.81	3.80 ^{+45%} _{-29%}	7.28 ^{+24%} _{-21%}	1.91
$p_{\perp} > 1000$ GeV	0.628 ^{+46%} _{-30%}	1.14 ^{+21%} _{-19%}	1.81	0.0417 ^{+47%} _{-30%}	0.0797 ^{+24%} _{-21%}	1.91

Comparison with numerical results



Courtesy of C. Wever and M. Kerner

Summary

- The two loop amplitude has been computed up to the subleading order in the top mass squared
- We combined existing real amplitude with our result, to produce the higgs p_T distribution
- We found that the NLO QCD corrections increase the LO prediction by 100 % and the ration of NLO over LO is of $O(2)$ and stable at large values of the transverse momentum
- Scale uncertainty $\pm 20\%$

Outlook

- This method can be used in different calculations related to (HL)-LHC physics (for instance, we work now on mixed QCD&EW for Z boson + jet production)
- Constraining anomalous couplings $\mathcal{L} \sim c_t t\bar{t}h + c_g G_{\mu\nu}^a G_a^{\mu\nu}$
- Taking seriously a method of a small parameter \rightarrow develop various instruments to compute various asymptotic expansions in high energy limits (reductions, solving differential equations etc.)

Thank you!

Backup slides

Real corrections with open loops

- Partonic channels contributing to the $H + j$ @NLO

$$q\bar{q} \rightarrow Hgg, gg \rightarrow Hgg, qg \rightarrow Hqg, \dots$$

- Receives contributions from kinematical regions where one parton become soft or collinear to another parton
- This requires a delicate approach of these regions in phase space integral
- *Openloops* algorithm is publicly available program which is capable of dealing with these singular regions in a numerically stable way
- Crucial ingredient is tensor integral reduction performed via expansions in small Gram determinants (*Cascioli et al., 2012* and *Denner et al., 2003 - 2017*)
- Exact top mass dependence kept throughout for one-loop computations

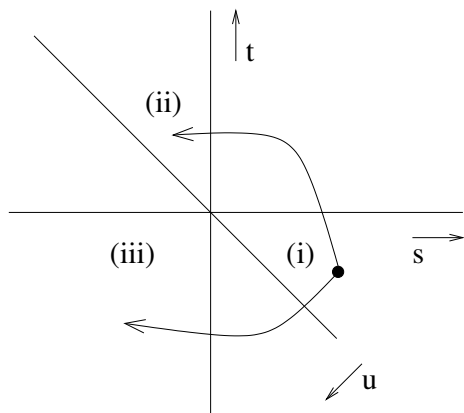
Pole substructure

S. Catani, M.H. Seymour, 1996

$$(F^i)_j^{(1),UV} = (F^i)_j^{(1),\text{fin}}, \quad (F^i)_j^{(2),UV} = l_1^i(\epsilon)(F^i)_j^{(1),UV} + (F^i)_j^{(2),\text{fin}}$$

Keep in mind that $(F^i)_j^{(1),UV}$ kept exact through out the calculations. In the end the whole expression is expanded in the Higgs and top mass.

Analytic continuation



C. Anastasiou et al., 2016

$$(i) : s \rightarrow s + i0,$$

$$(ii) : t \rightarrow t + i0,$$

$$(iii) : u \rightarrow u + i0.$$

The analytic continuation is done in two steps;

Consider analytic continuation from (i) \rightarrow (ii). First we cross $t = 0$, which would require to do a transformation of the kind: $x \rightarrow 1/x$. $t \rightarrow t + i0$.

Next, we cross $s = 0$, hence a value of $s \rightarrow s + i0$. Apply complex analysis, we can get the following formulas for transformation:

$$\log(-t/s) \rightarrow \log(-t/s) - 2i\pi$$

$$\log(-u/s) \rightarrow \log(u/s) - i\pi$$