PDFs from lattice data

Krzysztof Cichy, Luigi Del Debbio, Tommaso Giani

August 29

Background

PDFs, quasi-PDFs and pseudo-PDFs

Ioffe-time distribution

$$\mathcal{M}_\mu^{(0)}(n,P) = \langle P|\bar{\psi}_{\mathrm{q}}^{(0)}(n)\,\Gamma_\mu\,U(n,0)\,\psi_{\mathrm{q}}^{(0)}(0)\,|P\rangle$$

$$= 2P_\mu\,h_\Gamma\left(n\cdot P, n^2\right) + n_\mu\,\tilde{h}_\Gamma\left(n\cdot P, n^2\right)$$

Gamma structure: γ^+

Light-cone direction: $n = \left(0, n^-; \vec{0}\right), \quad n^2 = 0$

$\rightarrow \mathcal{M}_+^{(0)}(n,P) = 2P_+\,h_{\gamma^+}\left(n^-P^+, 0\right)$

$$\boxed{f_{\mathrm{q}}^{(0)}(x) = \left(2P^+\right)\int \frac{dn^-}{4\pi} e^{-ixP^+n^-}\,h_{\gamma^+}\left(n^-P^+, 0\right)}$$

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PDFs, quasi-PDFs and pseudo-PDFs

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PDF

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Pure-spatial case: $n = (0, 0, 0, z)\,, \quad n^2 = -z^2$

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quasi-PDF [X.Ji, 2013]

$$\hat{f}_{\mathsf{q}}^{(0)}(x) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{i\nu\,x} h_{\gamma^0}\left(\nu, -z^2\right)$$

pseudo-PDF [A. V. Radyushkin, 2017]

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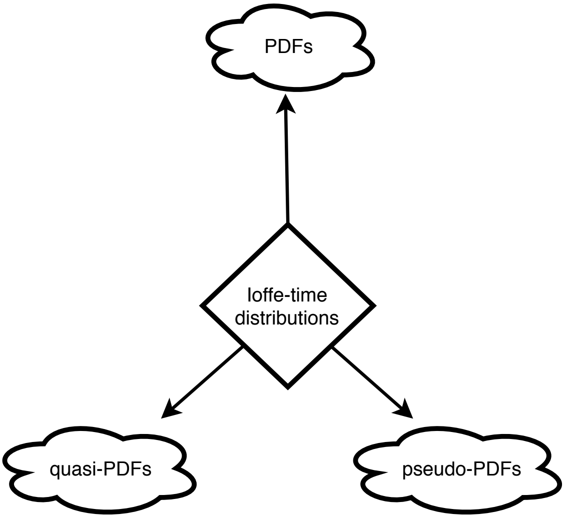
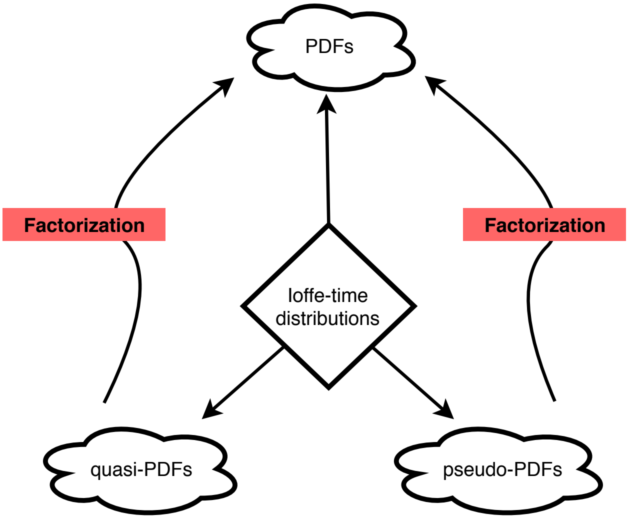

PDFs
Ioffe-time
distributions
quasi-PDFs
pseudo-PDFs


PDFs
Factorization
Factorization
Ioffe-time
distributions
quasi-PDFs
pseudo-PDFs

Factorization

$$\tilde{f}_A\left(x, P_z, \mu^2\right) = \int_{-1}^{1} \frac{dy}{|y|}\, C_A\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\mu}{\mu'}\right) f_A\left(y, {\mu'}^2\right) + \mathcal{O}\left(\frac{M^2}{P_z^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}\right)$$

Matching coefficient C_A :

- IR finite
- computable in perturbation theory

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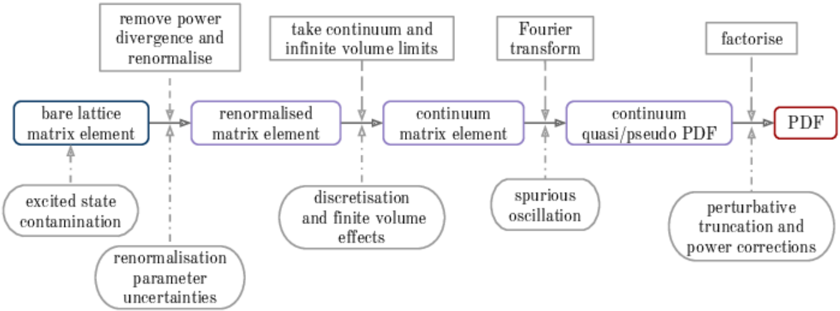
Lattice people approach

[arXiv:1811.00678, C.J. Monahan]



- $h\left(z,\mu^2\right)$, $\{\sigma_1^i,..,\sigma_k^i\}$
- $\tilde{f}\left(x,P_z,\mu^2\right) \sim \frac{2P_z}{4\pi}\sum_z e^{-ixP_z z} h\left(z,\mu^2\right)$
- $\tilde{f}\left(x,P_z,\mu^2\right) = \int_{-1}^{1}\frac{dy}{|y|}C\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\mu}{\mu'}\right) f\left(y,\mu'^2\right)$

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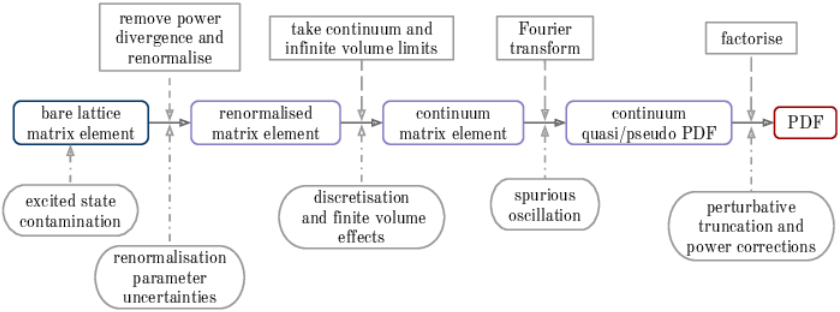

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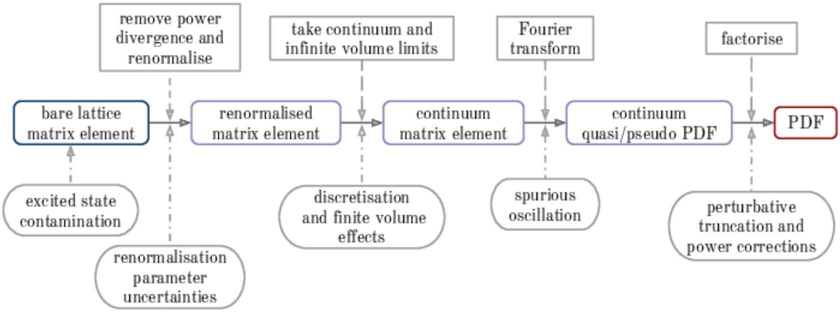

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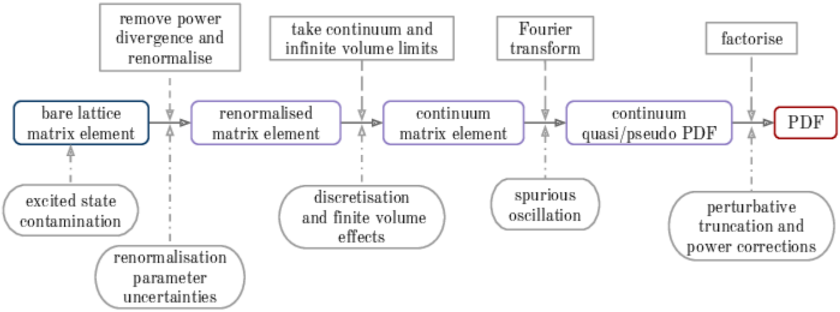

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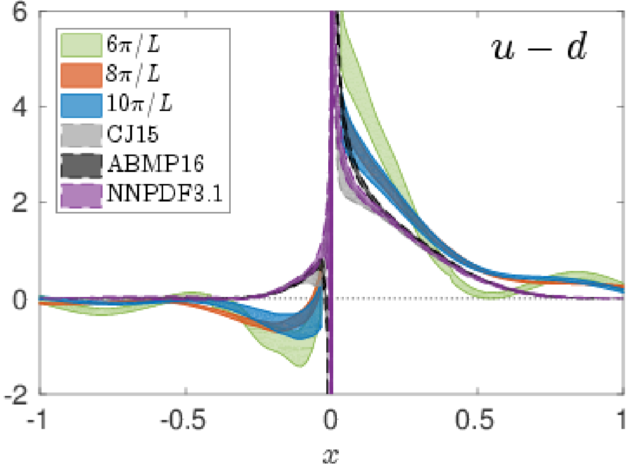
[arXiv:1803.02685, ETMC collaboration]



PDFs from lattice data

Position space factorization theroem

$$\tilde{f}_A\left(x, P_z, \mu^2\right) = \int_{-1}^{+1} \frac{dy}{|y|}\, C_A\left(\frac{x}{y}, \frac{\mu}{|y| P_z}\right)\, f_A\left(y, \mu^2\right)$$

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$$\mathcal{O}^{\mathsf{Re}}_{\gamma^0}\left(z, \mu\right) \equiv \int_{-\infty}^{\infty} dx\, \cos\left(xP_z z\right) \int_{-1}^{+1} \frac{dy}{|y|}\, C_A\left(\frac{x}{y}, \frac{\mu}{|y| P_z}\right)\, f_A\left(y, \mu^2\right)$$

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Unpolarized isovector parton distribution

$$f_3\left(x,\mu^2\right) = \begin{cases} u\left(x,\mu^2\right) - d\left(x,\mu^2\right)\ , & \text{if} \quad x > 0 \\ -\bar{u}\left(-x,\mu^2\right) + \bar{d}\left(-x,\mu^2\right)\ , & \text{if} \quad x < 0 \end{cases}$$

$$\mathcal{O}^{\text{Re}}_{\gamma^0}\left(z,\mu\right) = \int_0^1 dx\ \mathcal{C}_3^{\text{Re}}\left(x,z,\frac{\mu}{P_z}\right) V_3\left(x,\mu\right)$$

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Lattice observables: can be simulated on the lattice and can be computed given a parametrization for the PDFs.

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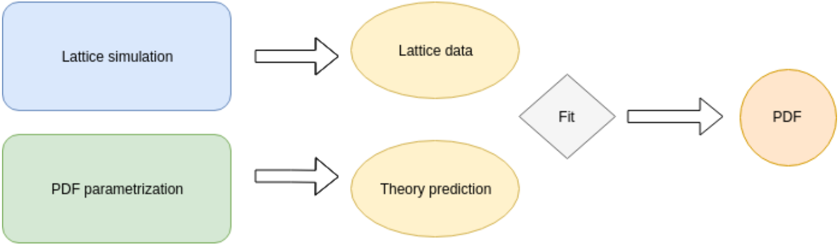
Lattice simulation
Lattice data
PDF parametrization
Theory prediction
Fit
PDF

FastKernel tables

$$\mathcal{O}^{\mathsf{Im}}_{\gamma^0}(z,\mu) = \int_0^1 dx\ \mathcal{C}^{\mathsf{Im}}_3\left(x, z, \frac{\mu}{P_z}\right) T_3\left(x,\mu^2\right)$$

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Interpolation of the PDF at a chosen scale (μ and μ_0)

$$T_3\left(x,\mu^2\right) = \sum_\beta T_3\left(x_\beta,\mu^2\right) \mathcal{I}^{(\beta)}(x) + \mathcal{O}\left[(x_{\beta+1} - x_\beta)^p\right]$$

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Observable in terms of FK table

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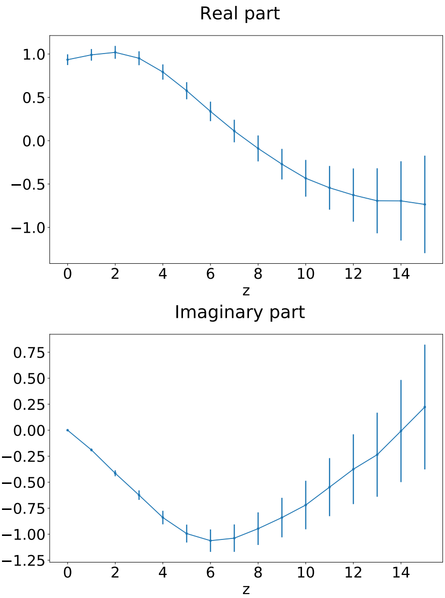
Lattice data



- data from ETMC collaboration
- γ_0 Dirac structure
- non-perturbative renormalization preocedure
- simulation at the physical pion mass
- data available for unpolarized, polarized and transversity cases
- $P_z = 10\pi/L$ (1.38GeV)

Lattice data systematics

- cut-off effects (finite value of lattice spacing a, UV regulator)
- finite volume effects (finite size of the box L, IR reguator)
- excited states contaminations
- truncation effects (coefficients to go from RI'-MOM to minimal subtraction)

Scenario	Cut-off	FVE	Excited states	Truncation
S1	10%	2.5%	5%	10%
S2	20%	5%	10%	20%
S3	30%	$e^{-3+0.062z/a}\%$	15%	30%
S4	0.1	0.025	0.05	0.1
S5	0.2	0.05	0.1	0.2
S6	0.3	$e^{-3+0.062z/a}$	0.15	0.3

Results

1. closure tests:
    - how good the convolution to get the lattice observables is in constraining PDFs
    - what we should expect given different systematics scenarios
2. fit:
    - fit results with different systematics scenarios
    - how lattice PDFs look like

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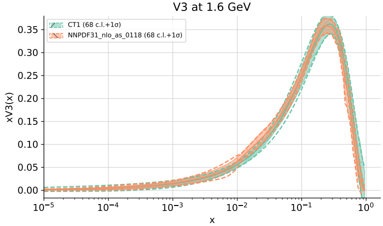
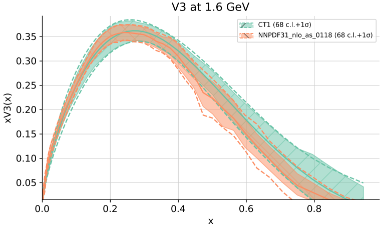
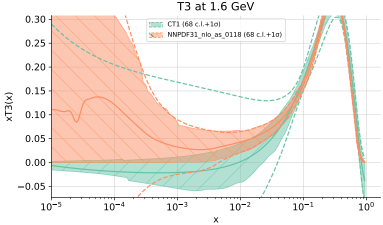
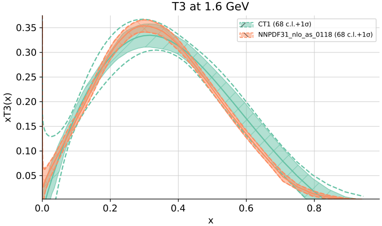
Closure tests

- small fake statistic
- no systematics



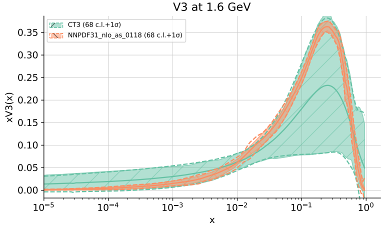


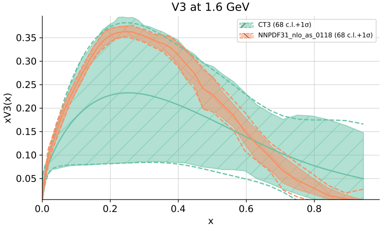


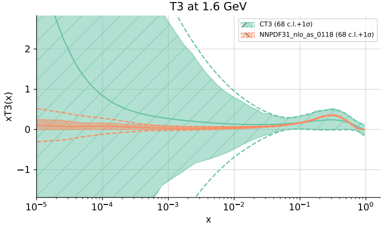


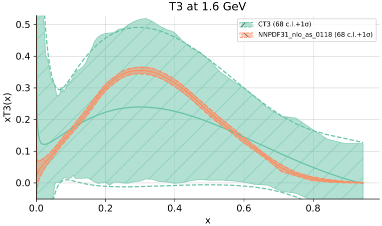
Closure tests

- real statistic
- real systematics scenario



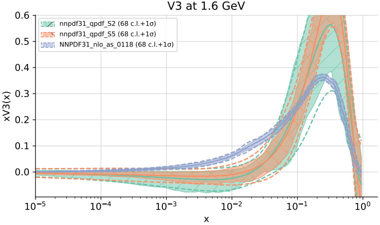


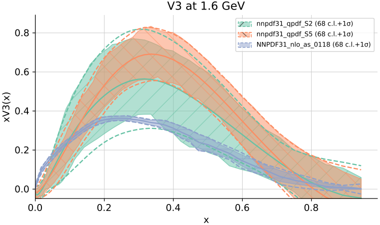


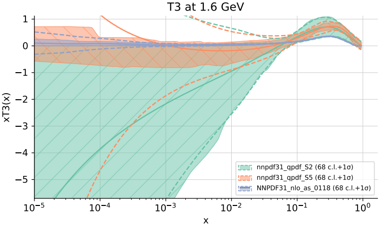


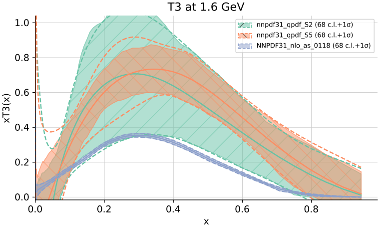
Fit results

S2 and S5 scenarios (more realistic ones)









Summary and future work

- increasing number of lattice data available
- we can treat them on the same footing as experimental data in a PDFs fit
- possible to include in the same fit lattice data coming from different lattice simulations/approaches
- polarized and trasnversity cases