Radiation of ionization electrons: the key-role of their 2-pt function of velocities

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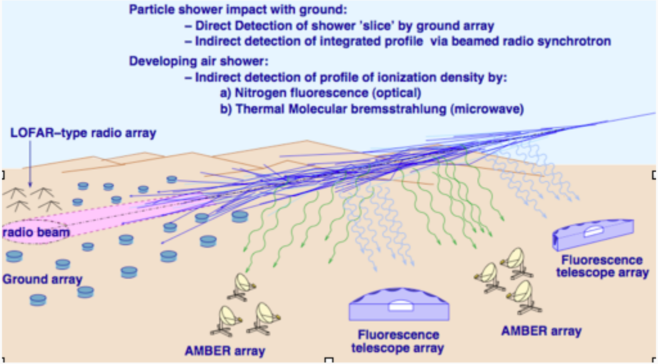
Plasma ionization electrons/neutral molecules

- Ionization electrons left as the cascade passes through the atmosphere
  - Profiling showers side-on (FD)
  - Up to 30-40 km
- Emission from the ionization electrons themselves?
  - MBR? – last-decade hopes triggered by SLAC measurements
  - Radar?



Emission from quasi-elastic collisions – I

- Radiated energy spectrum per solid unit angle:

$$\mathcal{E}(\omega,\Omega) = \frac{e^2}{16\pi^3\epsilon_0 c^3}\left\langle\left|\int \mathrm{d}t'\,(\mathbf{q}\times(\mathbf{q}\times\dot{\mathbf{v}}(t')))\exp(-i\omega t')\right|^2\right\rangle$$

- MBR modeled as a series of impulsive changes of velocities during τ:

$$\dot{\mathbf{v}}(t') = \sum_{k=1}^{N_{\mathrm{coll}}}\Delta\mathbf{v}_k\delta(t',t'_k)$$

- Radiated energy spectrum:

$$\mathcal{E}(\omega) = \frac{e^2}{6\pi^2\epsilon_0 c^3}\left\langle\left|\mathbf{v}_0+\sum_{k=1}^{N_{\mathrm{coll}}}\Delta\mathbf{v}_k\exp(-i\omega t'_k)-\mathbf{v}_{N_{\mathrm{coll}}}\exp(-i\omega\tau)\right|^2\right\rangle$$

- Two distinct regimes:
  - $t'_k \simeq \Gamma_{\mathrm{coll}}^{-1} \gg 2\pi/\omega \implies$ random phases $\implies$ incoherent regime $\implies \mathcal{E}(\omega) = N_{\mathrm{coll}}\mathcal{E}_1(\omega)$
  - $t'_k \simeq \Gamma_{\mathrm{coll}}^{-1} \ll 2\pi/\omega \implies$ phases close to 1 $\implies$ “1-collision regime”

Emission from quasi-elastic collisions – II

- Radiated energy spectrum per solid unit angle:

$$\mathcal{E}(\omega,\Omega) = \frac{e^2}{16\pi^3\epsilon_0 c^3}\left\langle\left|\int \mathrm{d}t'\,(\mathbf{q}\times(\mathbf{q}\times\dot{\mathbf{v}}(t')))\exp(-i\omega t')\right|^2\right\rangle$$

- Integration by parts and over all directions:

$$\mathcal{E}(\omega) = \frac{e^2\omega^2}{6\pi^2\epsilon_0 c^3}\iint \mathrm{d}t'\mathrm{d}t''\,\langle\mathbf{v}(t')\mathbf{v}(t'')\rangle\exp(-i\omega(t'-t''))$$

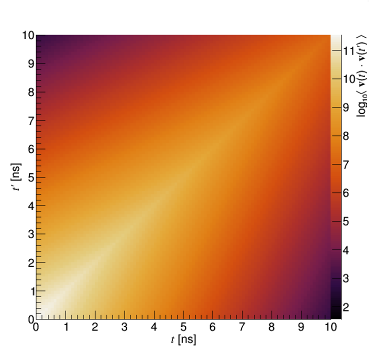
- Information encapsulated in the 2-pt function of the electron velocities:
    - Relativistic particles: small-angle scattering approximation, (Landau-Pomeranchuk approach)
    - Non-relativistic particles here, no such approximation possible

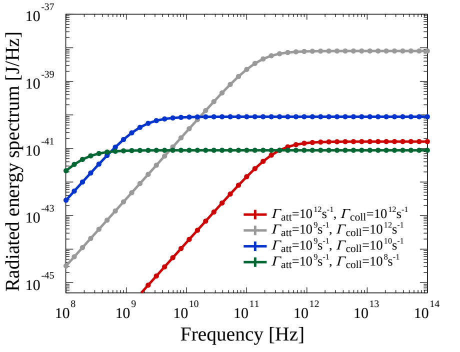
Example 1

- Elastic collisions at a rate Γ_{coll} during $\tau = \Gamma^{-1}$

$$\langle \mathbf{v}(t') \cdot \mathbf{v}(t'') \rangle = v_0^2 \times \begin{cases} \exp\left(-\Gamma t' - \Gamma_{\text{coll}}\left(t' - t''\right)\right) & \text{if } t' \geq t'', \\ \exp\left(-\Gamma t'' - \Gamma_{\text{coll}}\left(t'' - t'\right)\right) & \text{otherwise.} \end{cases}$$

$$\mathcal{E}(\omega) = \frac{e^2 v_0^2}{3\pi^2 \epsilon_0 c^3} \frac{(\Gamma + \Gamma_{\text{coll}})\omega^2}{\Gamma\left((\Gamma + \Gamma_{\text{coll}})^2 + \omega^2\right)}$$



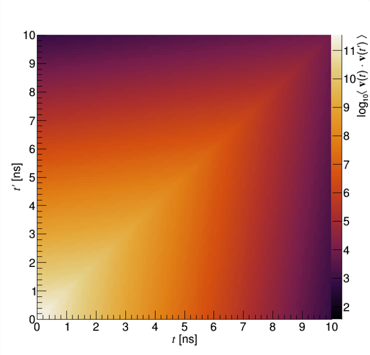


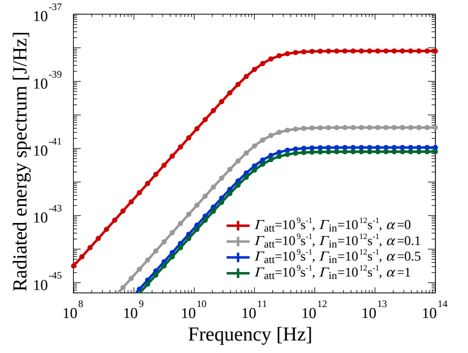
Example 2

- ☛ Adding inelastic collisions with velocity loss α

$$\langle \mathbf{v}(t') \cdot \mathbf{v}(t'') \rangle = v_0^2 \times \begin{cases} \exp\left(-\Gamma t' - \left(\Gamma_{\mathrm{el}} + \Gamma_{\mathrm{in}}\right)\left(t' - t''\right) - \alpha(2-\alpha)\Gamma_{\mathrm{in}} t''\right) & \text{if } t' \geq t'', \\ \exp\left(-\Gamma t'' - \left(\Gamma_{\mathrm{el}} + \Gamma_{\mathrm{in}}\right)\left(t'' - t'\right) - \alpha(2-\alpha)\Gamma_{\mathrm{in}} t'\right) & \text{otherwise.} \end{cases}$$

$$\mathcal{E}(\omega) = \frac{e^2 v_0^2}{3\pi^2 \epsilon_0 c^3} \frac{(\Gamma + \Gamma_{\mathrm{el}} + \Gamma_{\mathrm{in}})\omega^2}{\left(\Gamma + \alpha(2-\alpha)\Gamma_{\mathrm{in}}\right)\left((\Gamma + \Gamma_{\mathrm{el}} + \Gamma_{\mathrm{in}})^2 + \omega^2\right)}$$





Example 3: air-shower plasma

- GH longitudinal profile, NKG lateral profile
- Distribution in position, velocity, production time: $n' \equiv n(\mathbf{x}', \mathbf{v}'_0, t'_0)$
- Radiated energy:

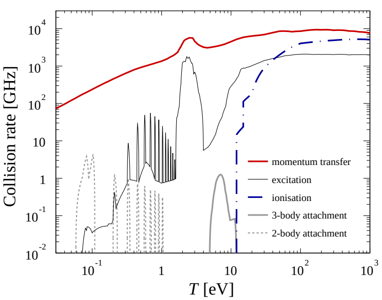
$$\begin{aligned} \mathcal{E}(\omega) &= \frac{e^2\omega^2}{6\pi^2\epsilon_0 c^3} \iint \mathrm{d}\mathbf{x}'\mathrm{d}\mathbf{x}'' \iint \mathrm{d}\mathbf{v}'_0\mathrm{d}\mathbf{v}''_0 \iint \mathrm{d}t'_0\mathrm{d}t''_0 \int_{t'_0}^{\infty} \int_{t''_0}^{\infty} \mathrm{d}t'\mathrm{d}t'' \\ &\times \left\langle (n'\mathbf{v}(t')) \cdot (n''\mathbf{v}(t''))\right\rangle e^{-i\omega(t'-t'')} \end{aligned}$$

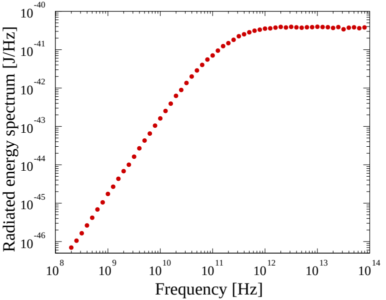
- Absence of coherence between particles:

$$\left\langle (n'\mathbf{v}(t')) \cdot (n''\mathbf{v}(t''))\right\rangle = n'\delta(\mathbf{x}', \mathbf{x}'')\delta(\mathbf{v}'_0, \mathbf{v}''_0)\delta(t'_0, t''_0)\ \left\langle \mathbf{v}(t') \cdot \mathbf{v}(t'')\right\rangle .$$

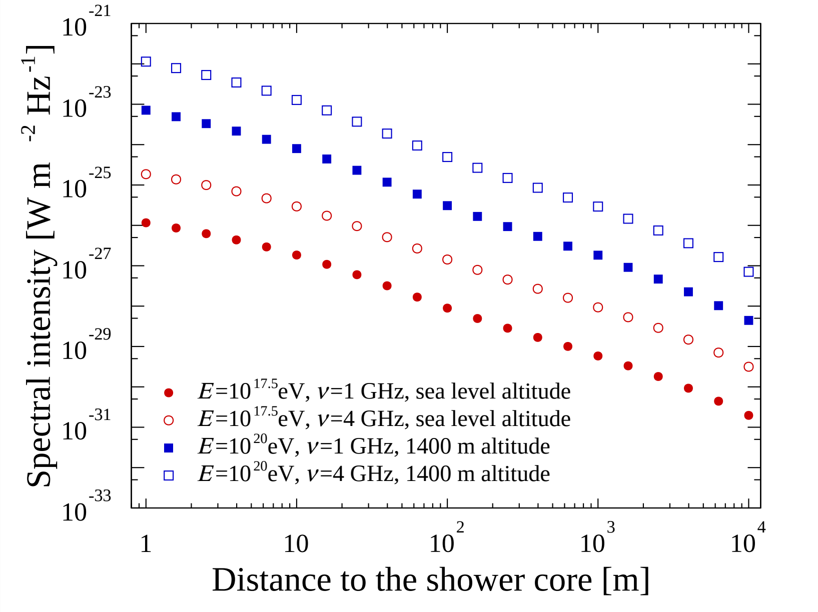
Example 3: air-shower plasma

- Collision term: atmospheric "chemistry" [Rosado et al., Astropart. Phys. 02 (2014) 003]
- MC integration



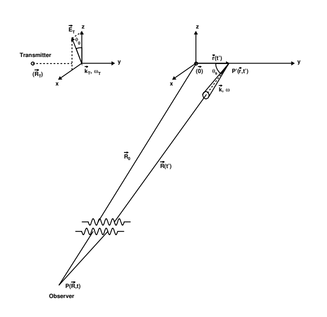


Example 3: air-shower plasma



Radar echoes of extensive air showers?

- ☛ SLAC measurement in ice [Allison et al., PRL 124 091101 (2020)]
- ☛ EAS as a hard target: old idea from the 40's
  - Idea revived in the early 2000s [Gorham, Astropart. Phys. 15 177 (2001)]
  - Reflection coefficients destroyed by the high collision rate [Stasielak et al., Astropart. Phys. 73 14 (2016)]
  - Excluded by TARA [Abbasi et al., Astropart. Phys. 87 1 (2017)]
- ☛ EAS as a soft target?
  - SLAC measurement: large number of pulses averaged ($\sim$ average small-scale shower)
  - Re-radiation of the incident wave [NB: $\propto \langle \mathbf{v}(t')\mathbf{v}(t'')\rangle$, **not** $\langle \dot{\mathbf{v}}(t)\rangle^2$]
  - Thomson cross section extremely small
  - Coherence effects to "inflate" the Thomson power?



- Incident wave:

$$\mathbf{E}_{\mathrm{T}} \simeq \frac{U_{\mathrm{T}}}{R_{\mathrm{T}}} \exp\left(i\omega_{\mathrm{T}}\left(t - \mathbf{q}_{\mathrm{T}} \cdot \int_{\mathbf{r}}^{\mathbf{R}_T} n(\mathbf{r}')\mathrm{d}\mathbf{r}'/c\right)\right)$$

- Spectral energy density re-radiated:

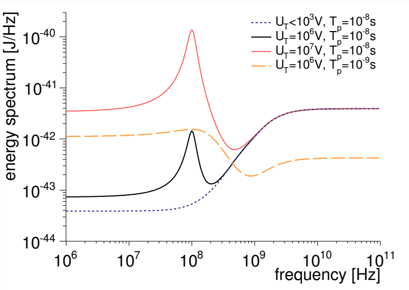
$$\mathcal{E}_{\nu}(\Omega) = \frac{e^2}{16\pi^3 \epsilon_0 c^3} \left\langle \left| \sum_{p=1}^{N} \left(\frac{2eU_{\mathrm{T}}\mathbf{n}_{\Omega}}{mR_{\mathrm{T}}} \sum_{j=0}^{N_{\mathrm{coll}}^p} \frac{\sin\left(\left(2\pi\Delta\nu - \Delta\mathbf{k}\cdot\mathbf{v}_{jp}\right)\Delta t_{jp}/2\right)}{2\pi\Delta\nu - \Delta\mathbf{k}\cdot\mathbf{v}_{jp}} + \sum_{j=0}^{N_{\mathrm{coll}}^p} \Delta\mathbf{v}_{jp} e^{-i2\pi\nu t_{jp} - i\mathbf{k}\cdot(\mathbf{R}_{0p} - \mathbf{r}_p(t_{jp}))} - \mathbf{v}_{N_{p\mathrm{coll}}} e^{-i2\pi\nu T_p} + v_{0p} \right) \right|^2 \right\rangle$$

Case of a plasma of uniform density

$$\mathcal{E}_\nu(\Omega) = \frac{e^2}{16\pi^3\epsilon_0 c^3}\left\langle\left|\sum_{p=1}^{N}\left(\frac{2eU_\mathrm{T}\mathbf{n}_\Omega}{mR_\mathrm{T}}\sum_{j=0}^{N^p_\mathrm{coll}}\frac{\sin\left(\left(2\pi\Delta\nu-\Delta\mathbf{k}\cdot\mathbf{v}_{jp}\right)\Delta t_{jp}/2\right)}{2\pi\Delta\nu-\Delta\mathbf{k}\cdot\mathbf{v}_{jp}}+\sum_{j=0}^{N^p_\mathrm{coll}}\Delta\mathbf{v}_{jp}e^{-i2\pi\nu t_{jp}-i\mathbf{k}\cdot(\mathbf{R}_{0p}-\mathbf{r}_p(t_{jp}))}-\mathbf{v}_{N_{p\mathrm{coll}}}e^{-i2\pi\nu T_p}+v_{0p}\right)\right|^2\right\rangle$$

- First term: Thomson scattering in the presence of collisions, smoothly peaked for frequencies around the incident one with a width proportional to $\langle T_p^{-2}\rangle$
- For a volume large enough compared to $(2\pi/\nu_\mathrm{T})^3$, $\Delta\mathbf{k}\cdot\mathbf{R}_{0p}$ in the phase implies destructive interferences between electrons located in $\mathbf{R}_0$ and in $\mathbf{R}'_0=\mathbf{R}_0+\pi/\Delta\mathbf{k}$
- Plasma with uniform density: energy effectively radiated from this scattering entirely due to density fluctuations that produce an unbalanced number of electrons at different positions
- Amplitude of such fluctuations proportional to $\sqrt{N} \implies$ Radiated energy proportional to N (so-called Incoherent Scatter Radar)

Case of an EAS



- Plasma highly non-uniform $\Longrightarrow$ Possibility to have amplitudes scaling as $\sim N$
- Inflate signals with clever interferences: low frequency incident wave
- Radiated energy frequency spectrum?

$$\mathcal{E}(\omega) = \frac{e^2\omega^2}{6\pi^2\epsilon_0 c^3} \iint \mathrm{d}\mathbf{x}'\mathrm{d}\mathbf{x}'' \iint \mathrm{d}\mathbf{v}_0'\mathrm{d}\mathbf{v}_0'' \iint \mathrm{d}t_0'\mathrm{d}t_0'' \int_{t_0'}^{\infty} \int_{t_0''}^{\infty} \mathrm{d}t'\mathrm{d}t''$$
$$\times \quad \langle (n'\mathbf{v}(t')) \cdot (n''\mathbf{v}(t'')) \rangle \, e^{-i\omega(t'-t'')}$$

☛ $\langle (n'\mathbf{v}(t')) \cdot (n''\mathbf{v}(t'')) \rangle = ???$