ARENA 7th - 10th June 2022, Santiago de Compostela

Parameterization of the frequency spectrum of radio emission in the 30–80 MHz from inclined air showers

PIERRE AUGER OBSERVATORY

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Helmholtz International Research School for Astroparticle Physics

KIT
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UNSAM

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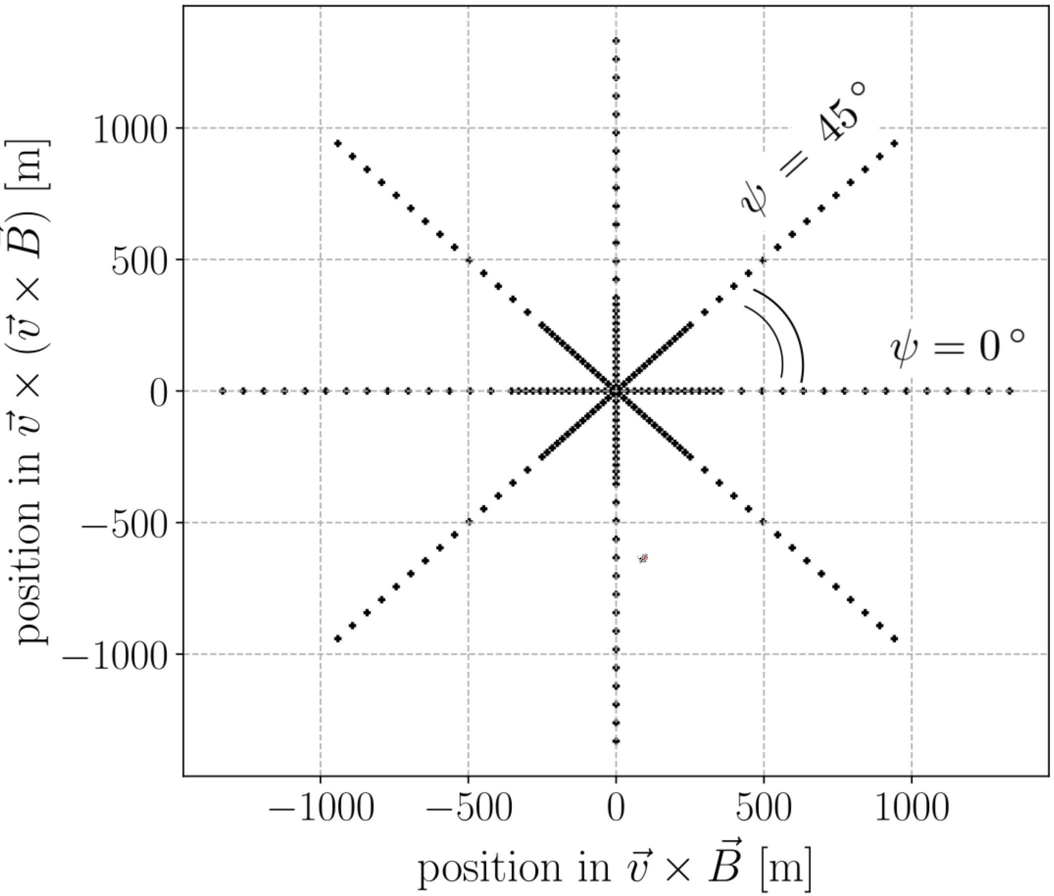
Simulation data-set



2160 CoREAS proton-simulations

Log(E) = [18.4, 18.6, ..., 20.0, 20.2] eV

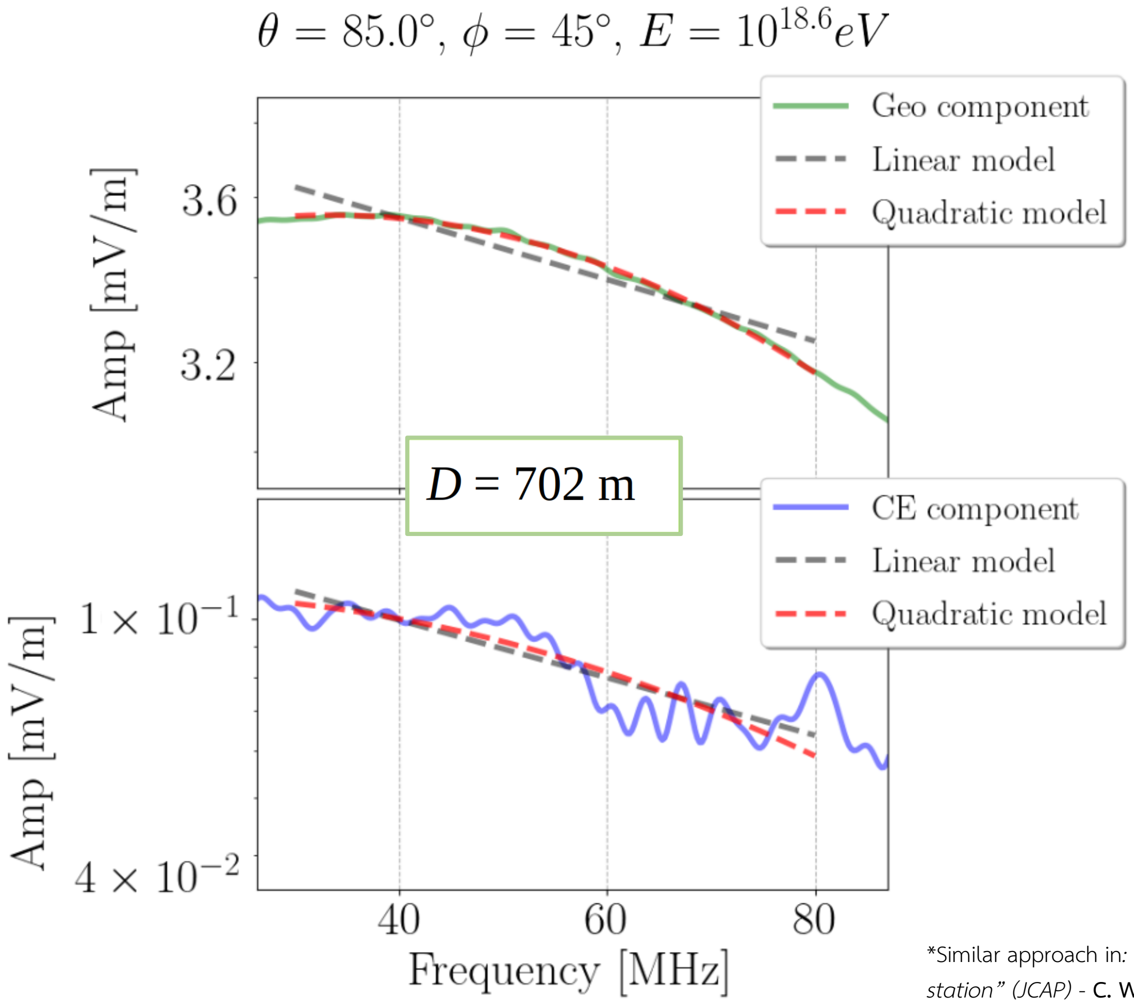
Azimuth = [0, 45, ..., 270, 315] deg

Zenith = [65.0, 67.5, ..., 82.5, 85.0] deg

Star-shaped grid

How does the spectrum of the pulses look like?

Frequency spectra and spectral fitting



Focus on the 30-80 MHz (often used, i.e. by the RD of AugerPrime)

Comparison of two fitting models*:

$$L(f) = A \cdot 10^{m_f \cdot (f - f_0)}$$

frequency slope(s)

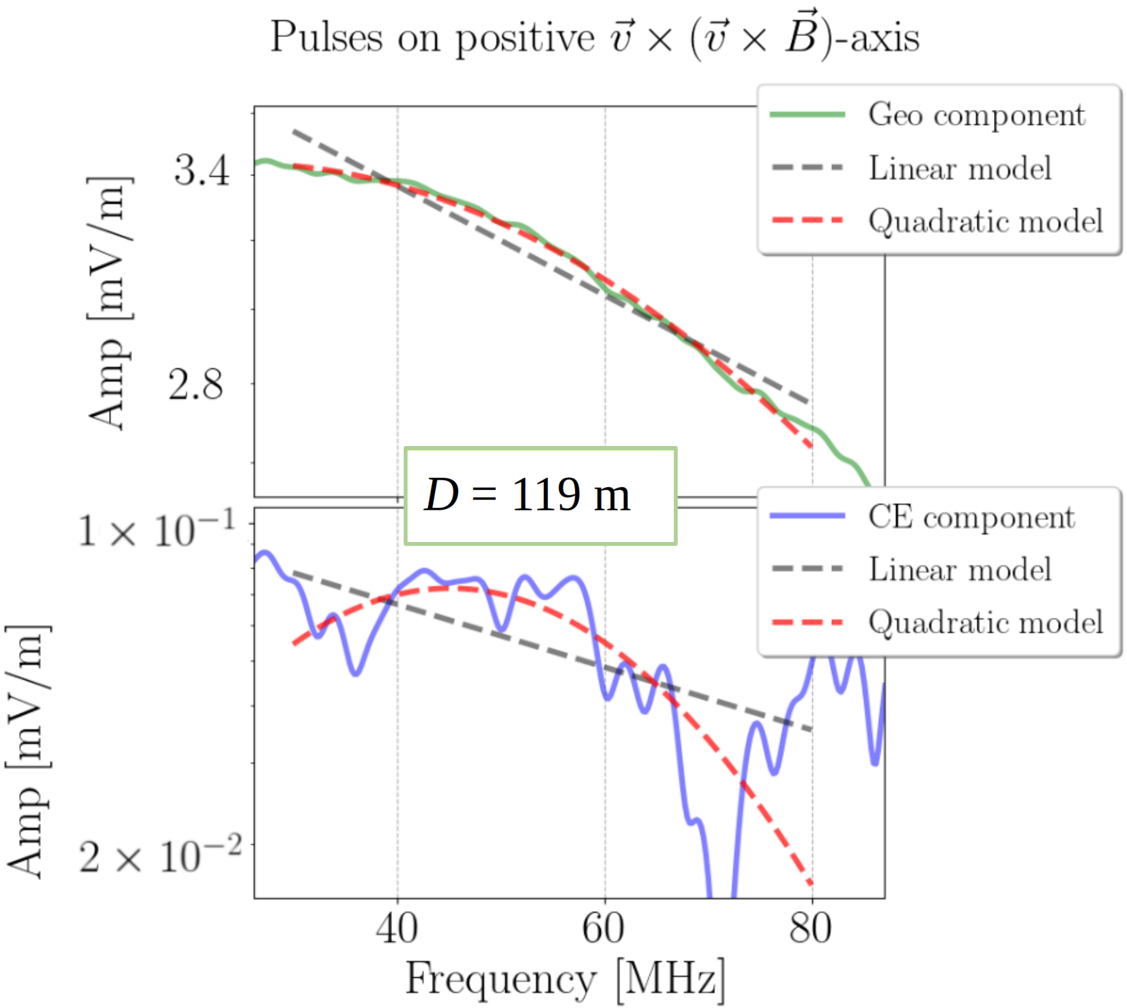
$$Q(f) = A \cdot 10^{m_f \cdot (f - f_0) + m_{f2} \cdot (f - f_0)^2}$$

quadratic term

Antennas position influences the spectral shape (see next slide's animation)

*Similar approach in: *"Reconstructing the cosmic-ray energy from the radio signal measured in one single station" (JCAP)* - C. Welling, C. Glaser, A. Nelles

Frequency spectra and spectral fitting



$\theta = 85.0°,\ \phi = 45°,\ E = 10^{18.6} eV$

r_{che} = 1406 m

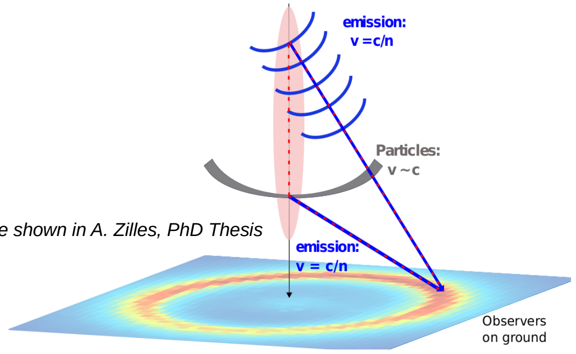
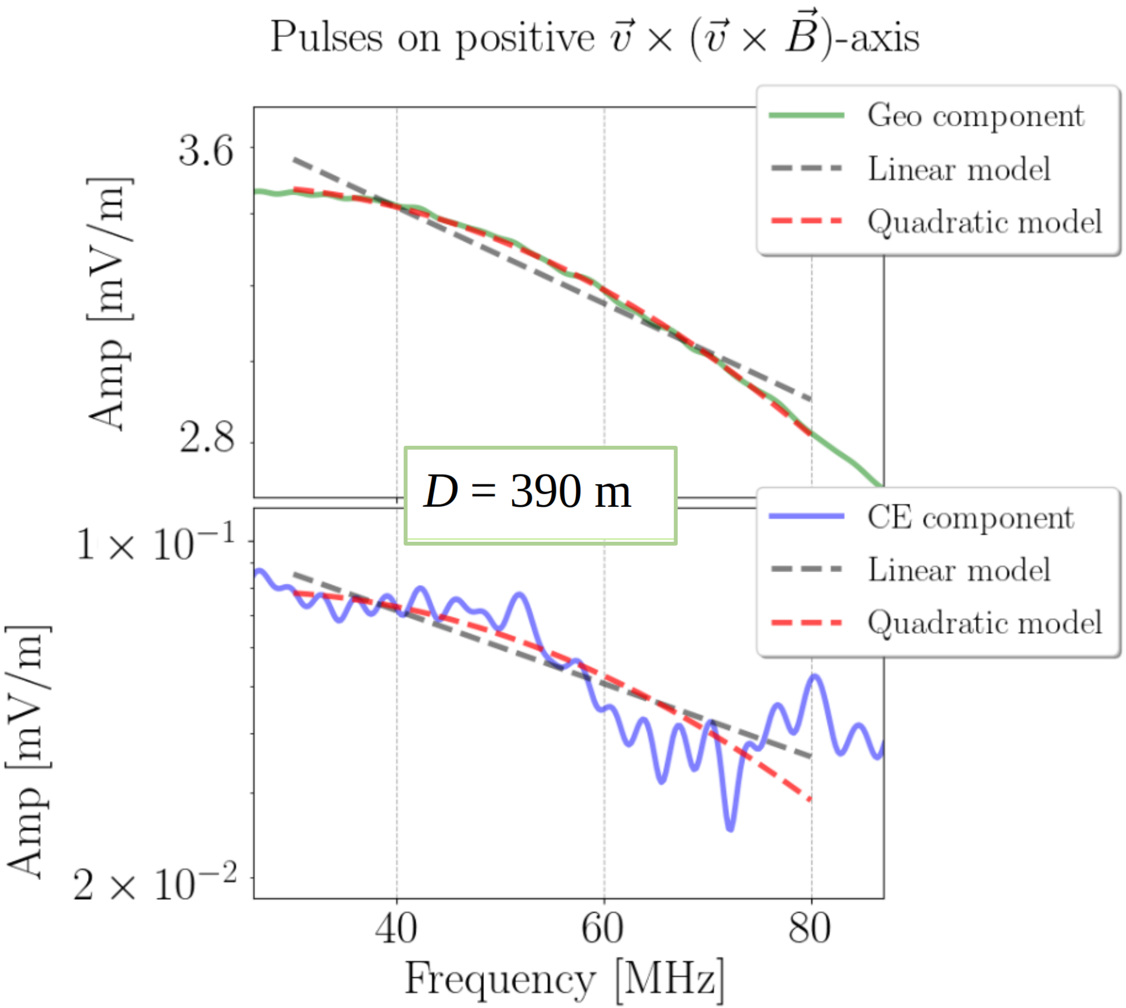


Figure shown in A. Zilles, PhD Thesis

$$\theta_{\text{che}} = \text{arcos}\big(1/n(h)\big)$$

$$r_{\text{che}} = \tan(\theta_{\text{che}}) \cdot d_{\text{MAX}}$$

Frequency spectra and spectral fitting



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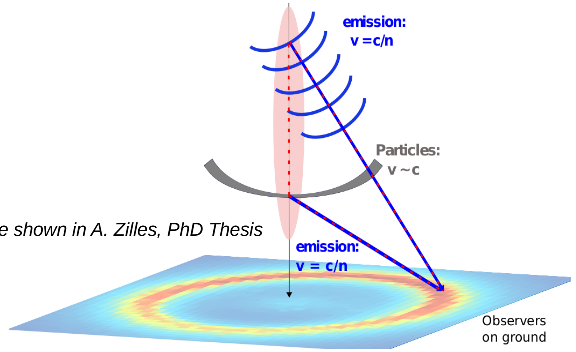

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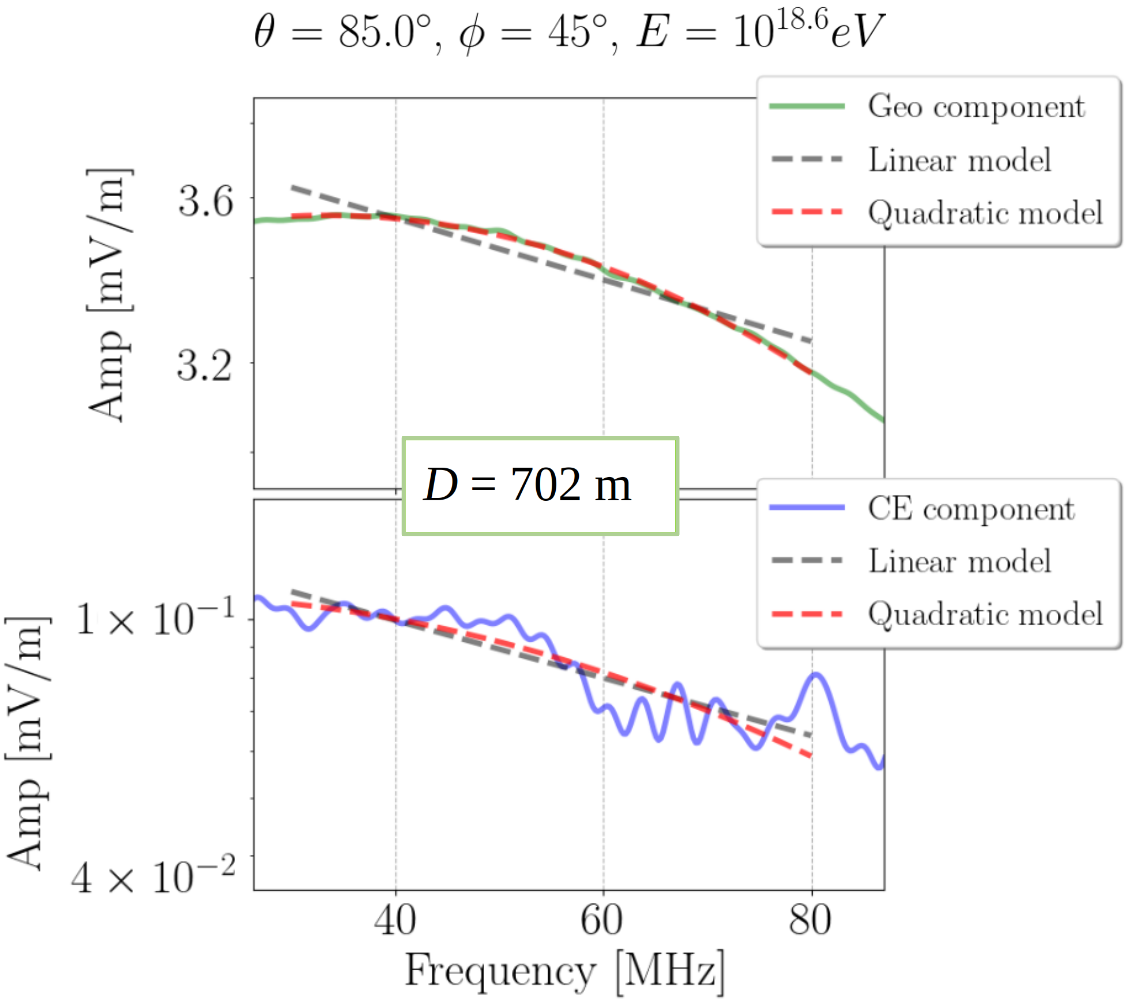
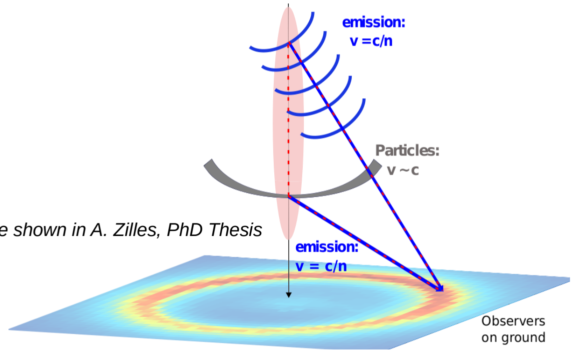
Frequency spectra and spectral fitting



$\theta = 85.0°, \phi = 45°, E = 10^{18.6} eV$

r_{che} = 1406 m



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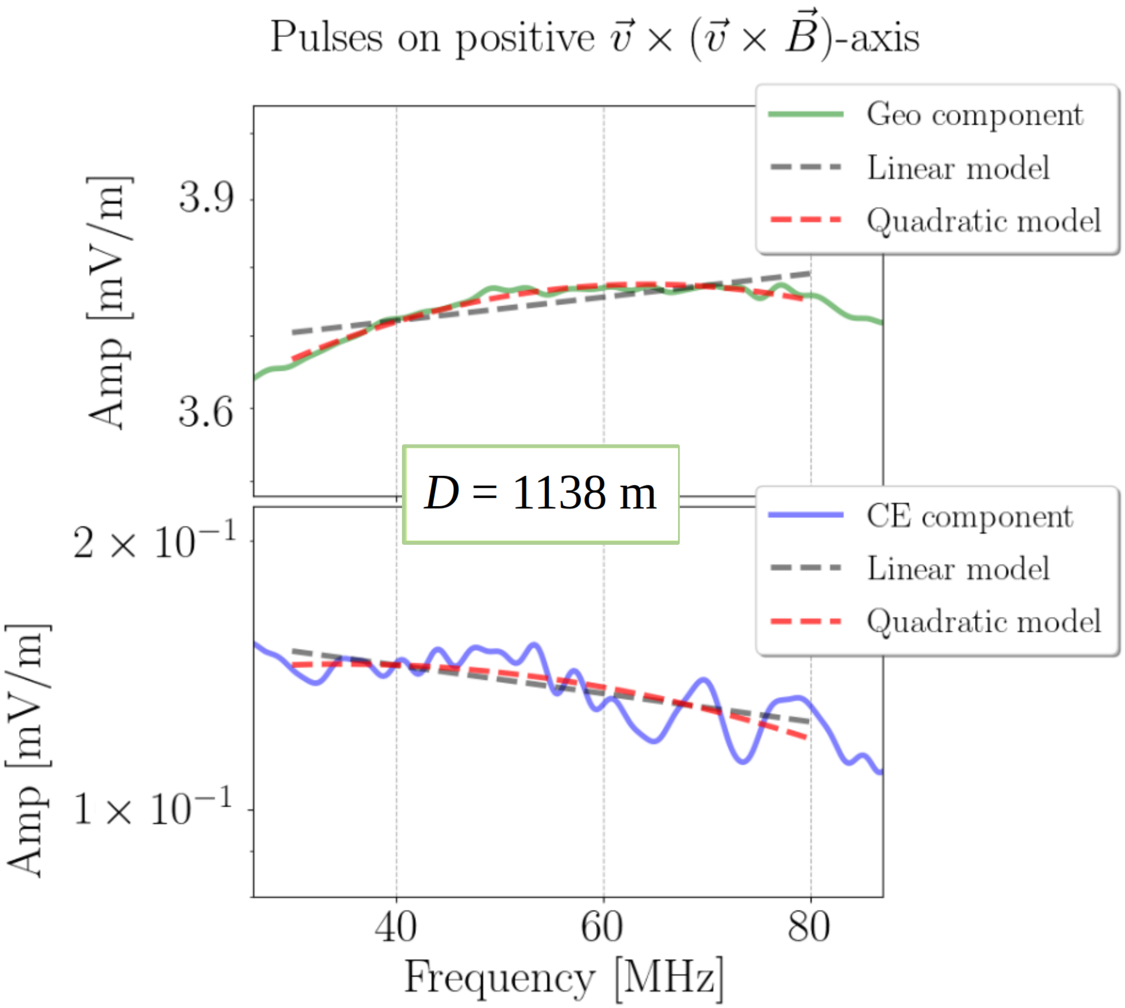
Frequency spectra and spectral fitting



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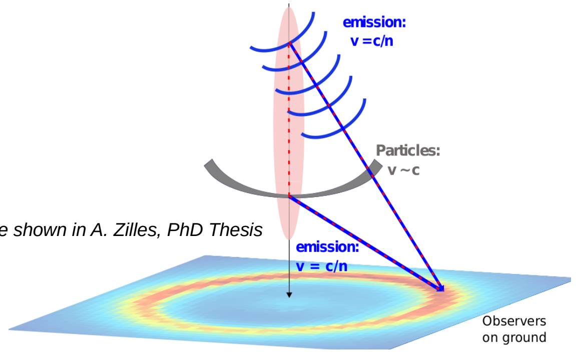

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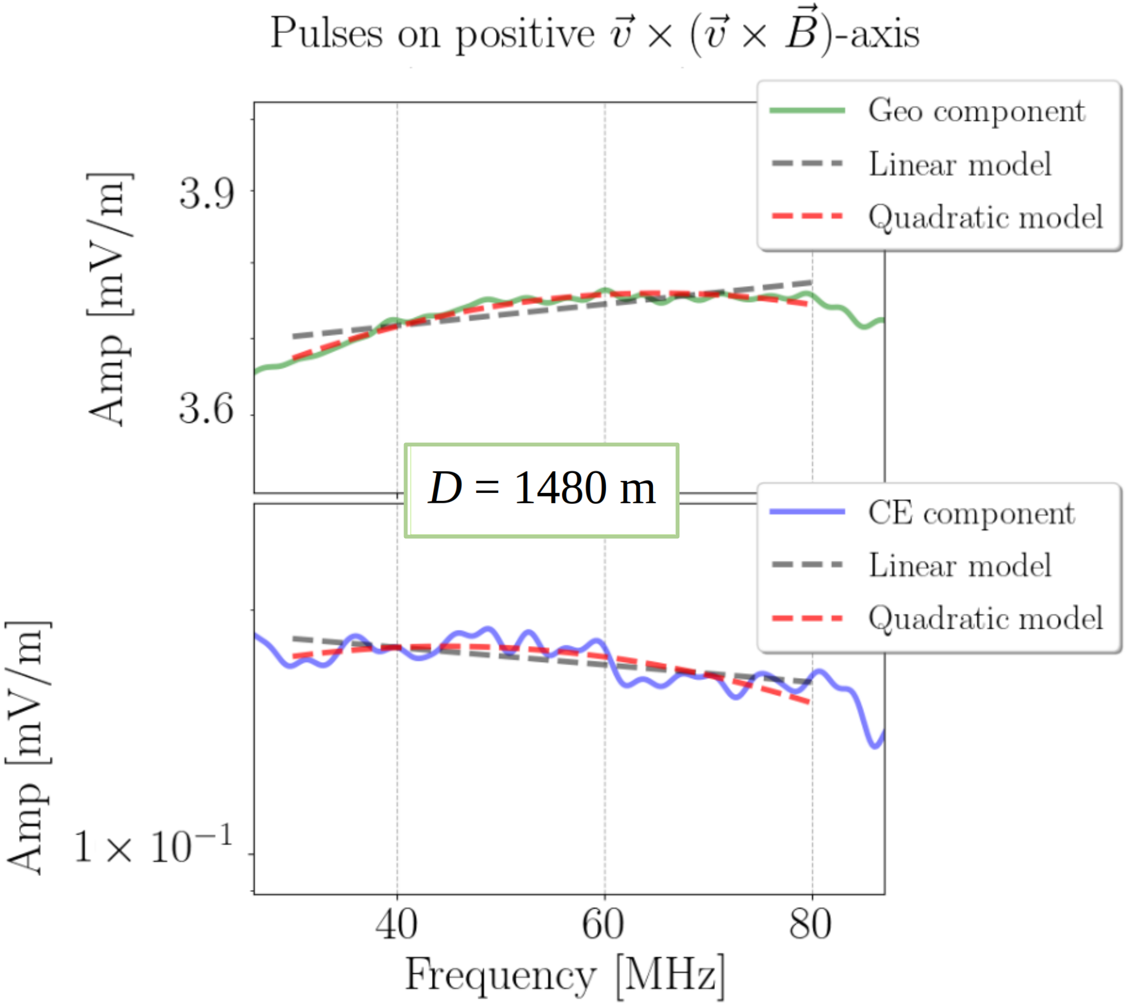
Frequency spectra and spectral fitting



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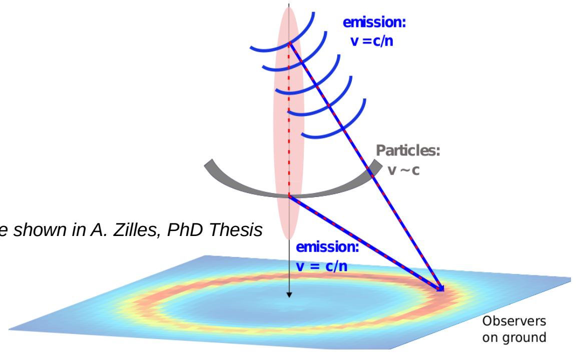
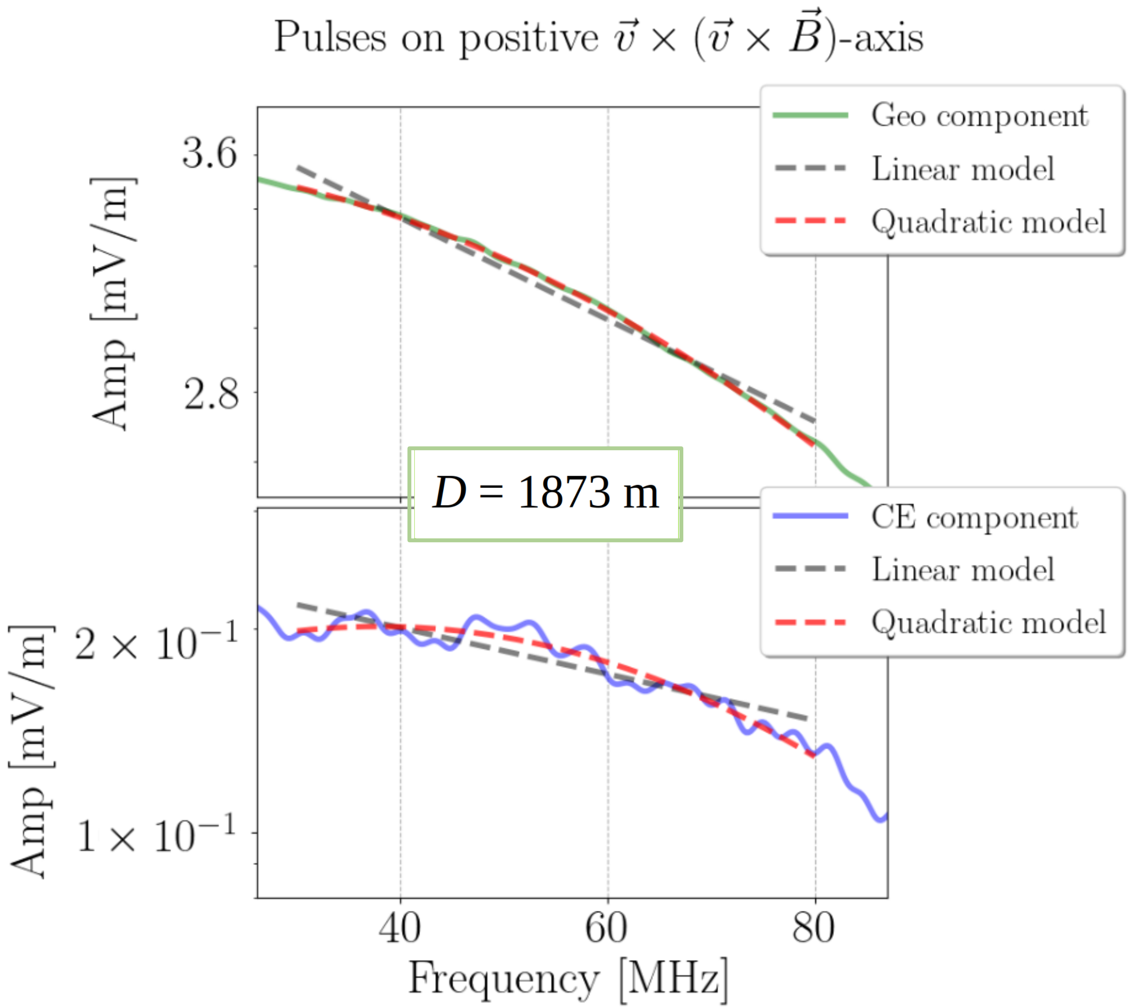


Figure shown in A. Zilles, PhD Thesis

$$\theta_{che} = \mathrm{arcos}\big(1/n(h)\big)$$
$$r_{che} = \tan(\theta_{che}) \cdot d_{MAX}$$

Frequency spectra and spectral fitting



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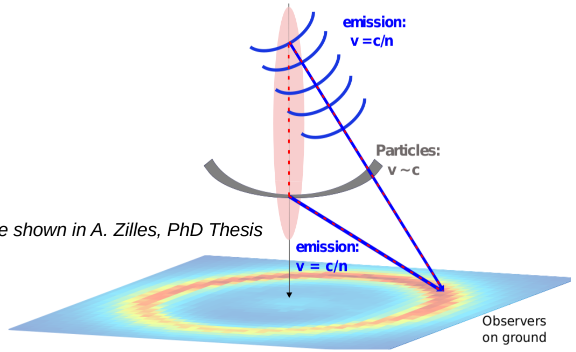

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$$\theta_{\text{che}} = \text{arcos}\big(1/n(h)\big)$$

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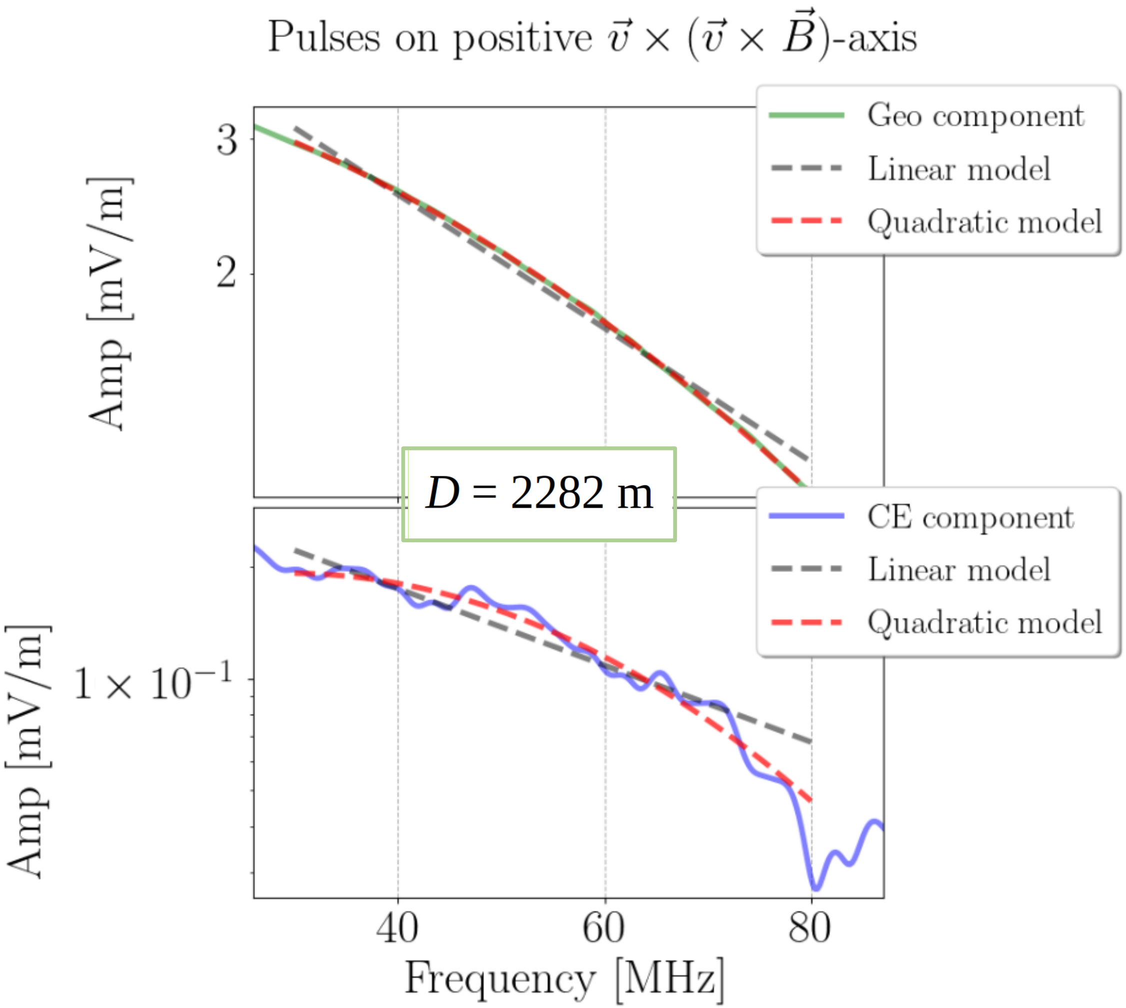
Frequency spectra and spectral fitting



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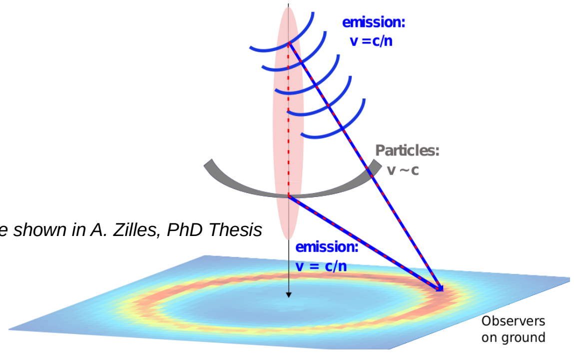

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$$\theta_{\text{che}} = \text{arcos}\big(1/n(h)\big)$$

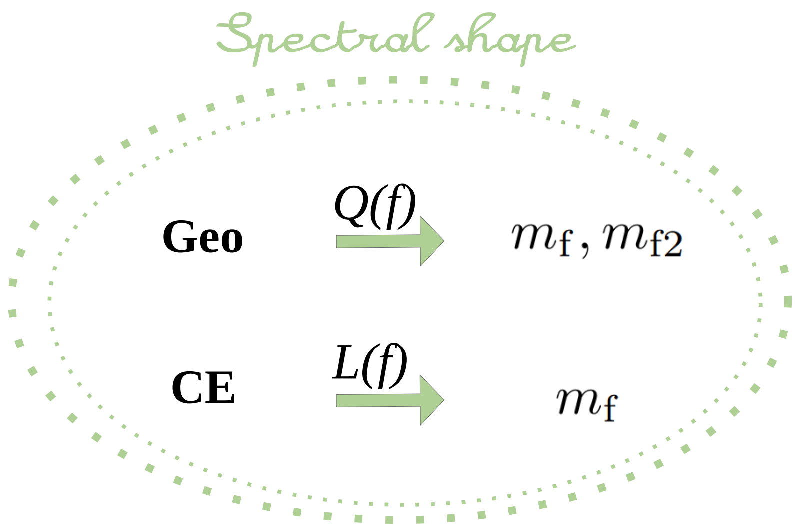
$$r_{\text{che}} = \tan(\theta_{\text{che}}) \cdot d_{\text{MAX}}$$

Study on the simulation data-set

(more in back-up slides)

1) To find a stable fitting procedure for *Q(f)* → frequency offset set to f_0 = 55 MHz

2) To determine which model describes better the signal components spectral shape



Goal and motivation

We want to extract additional information carried by the short transient radio pulses to be used in reconstruction tools to better constrain the geometry (e.g. core position)

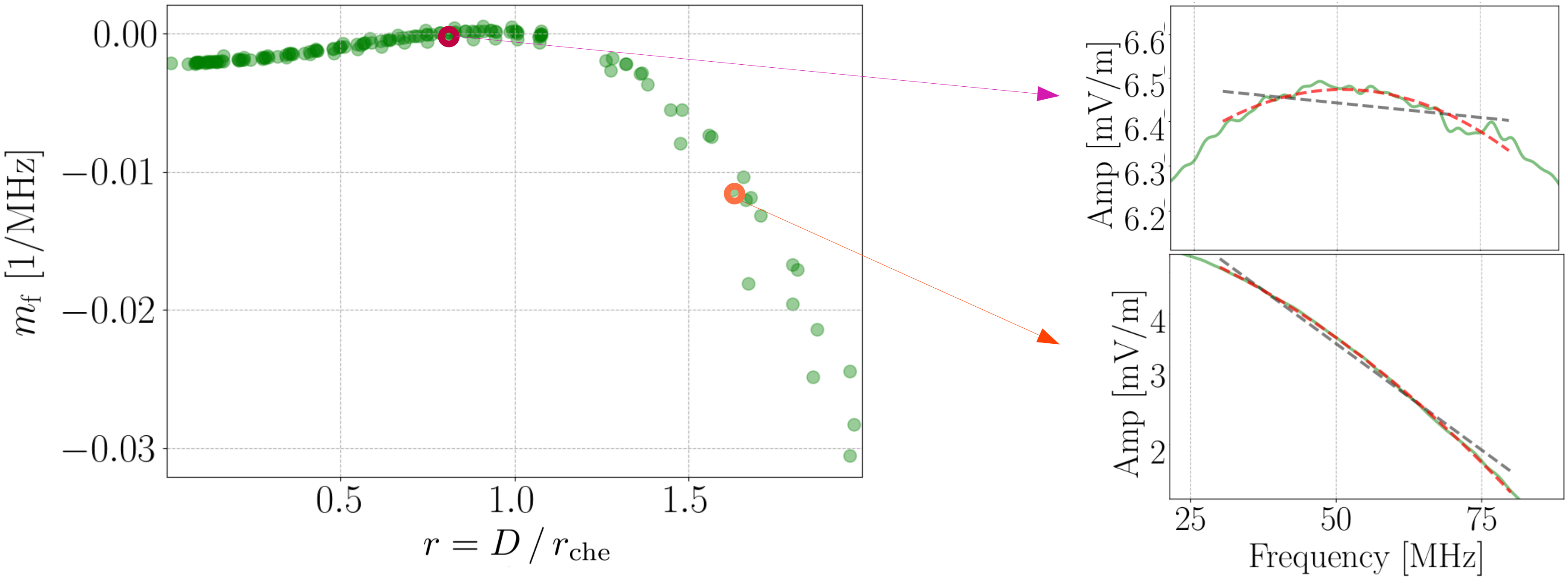
→ Parameterization of the spectral shape as a function of the lateral distance and d_{MAX}, the geometrical distance between core position and X_{MAX}

Previous works exploit dependence of the slope on X_{MAX} to achieve a reconstruction method (F. Canfora, S. Jansen)

Single-event lateral distribution

Frequency slopes of the Geo component from a simulation having $\theta = 85.0°,\ \phi = 45°,\ E = 10^{18.6}$ eV

Spectral fits at different r



Flattening of the spectrum → shorter pulses and coherence

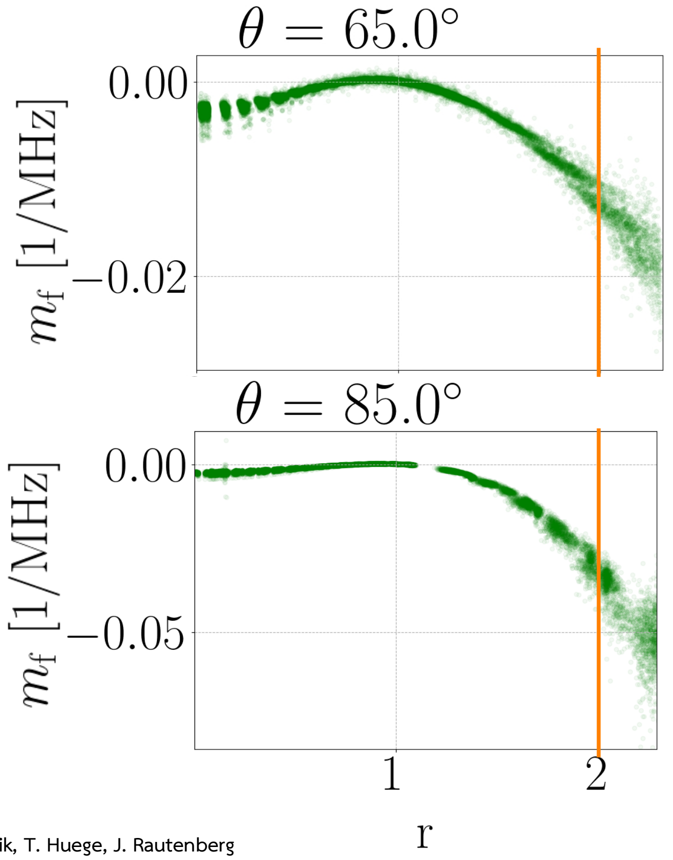
Parameterization of the spectral shape

Lateral distributions of simulations having the same **zenith angle** are studied separately.

Effect of noise introduced in the simulations by **thinning** increases with the lateral distance.

Stations above $r = 2$ are excluded from the analysis after applying:

1. *core refraction displacement correction**
2. *early-late correction of the distance***



* *"Refractive displacement of the radio-emission footprint of inclined air showers simulated with CoREAS"* - F. Schlüter, M. Gottowik, T. Huege, J. Rautenberg

** *"A Rotationally Symmetric Lateral Distribution Function for Radio Emission from Inclined Air Showers"* - T. Huege, L. Brenk, F. Schlüter

Parameterization of the Geo component

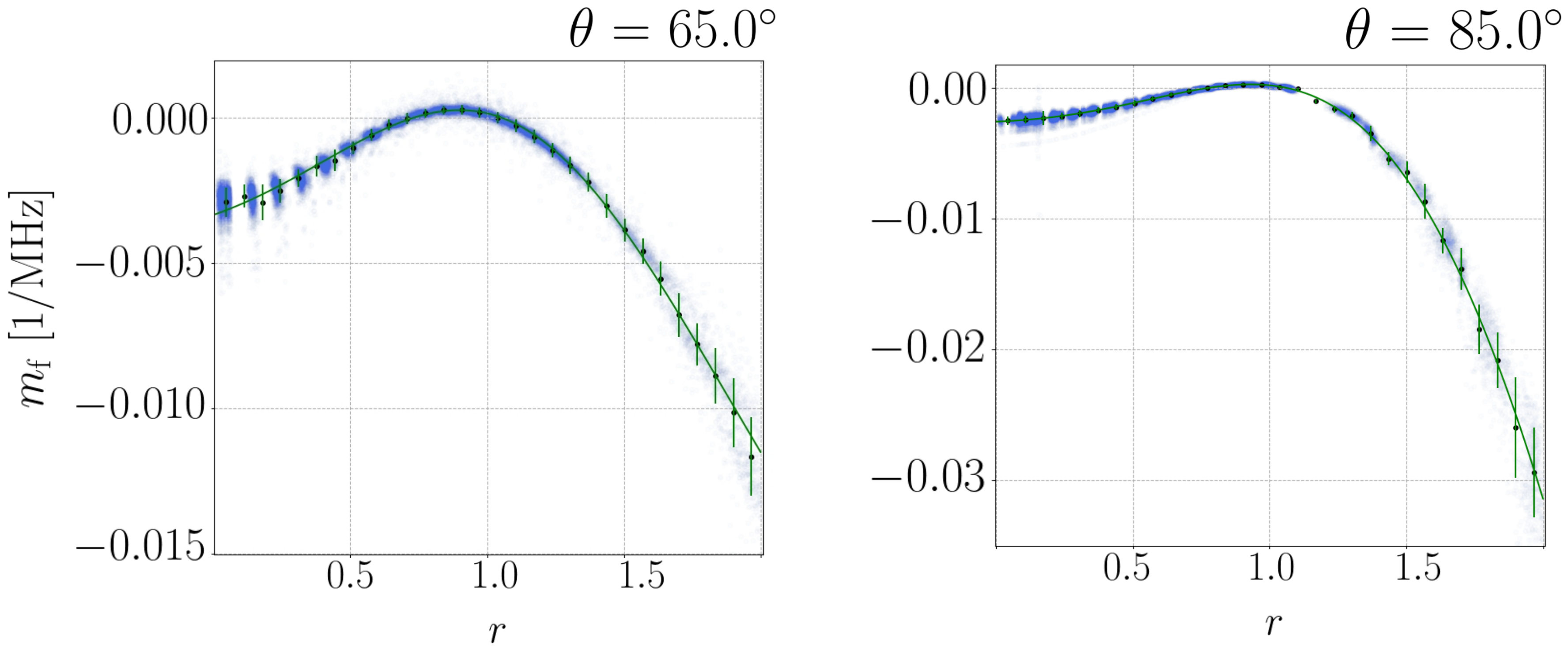
$$Q(f) = A \cdot 10^{m_{\mathrm{f}} \cdot (f - f_0) + m_{\mathrm{f2}} \cdot (f - f_0)^2}$$

frequency slope

quadratic term

Frequency slope: lateral distribution function

$$f(r) = A_1 \cdot \left[- e^{B \cdot (r - r_0)} + A_2 \cdot e^{-C \cdot (r - r_0)^2} \right]$$

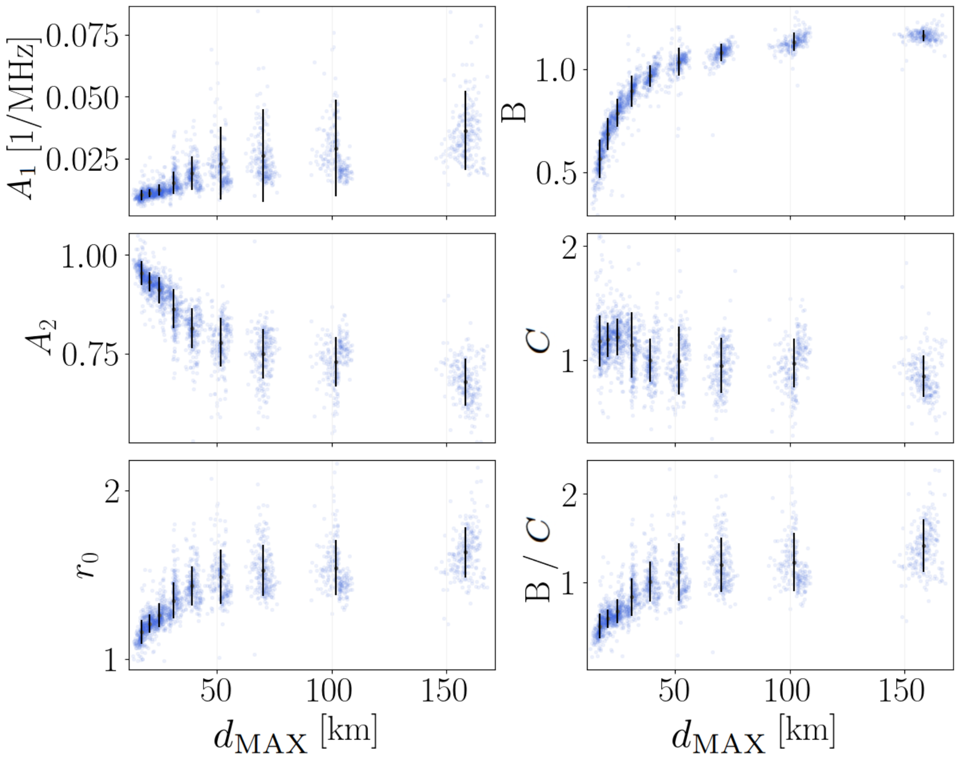


Free fit of mean values in lateral distance bins

Frequency slope

$$f(r) = A_1 \cdot \left[- e^{B\cdot(r-r_0)} + A_2 \cdot e^{-C\cdot(r-r_0)^2} \right]$$

Parameters distributions **(individual fits with 5 free parameters)**

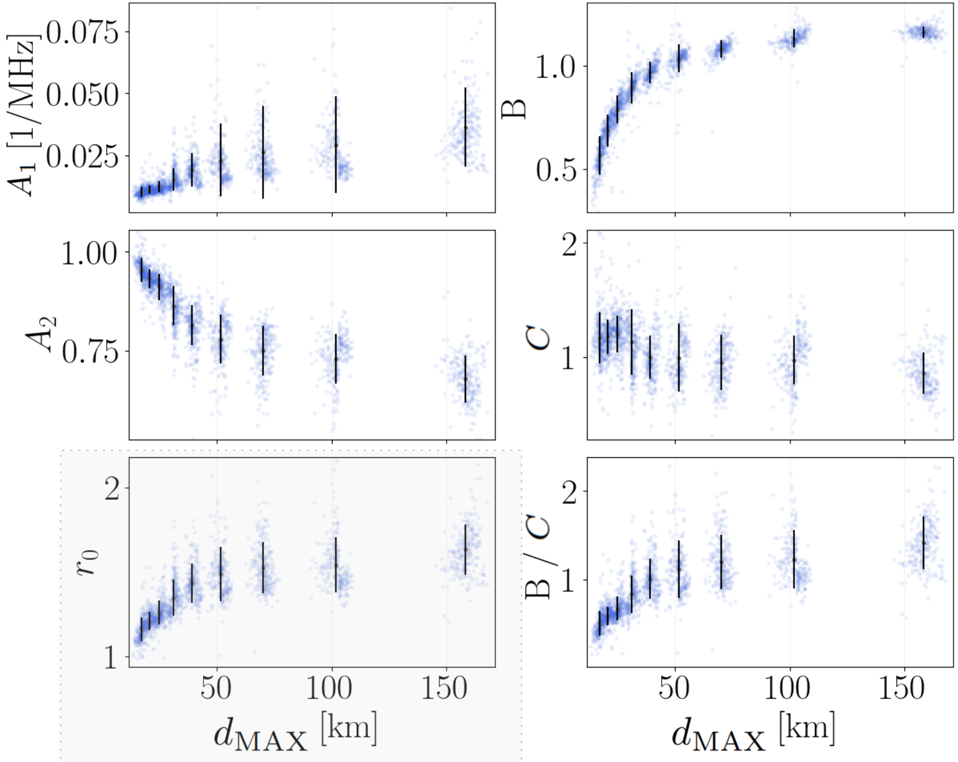


Fix one by one the 5 parameters by looking at their distributions expressed as a function of d_{MAX}

Frequency slope

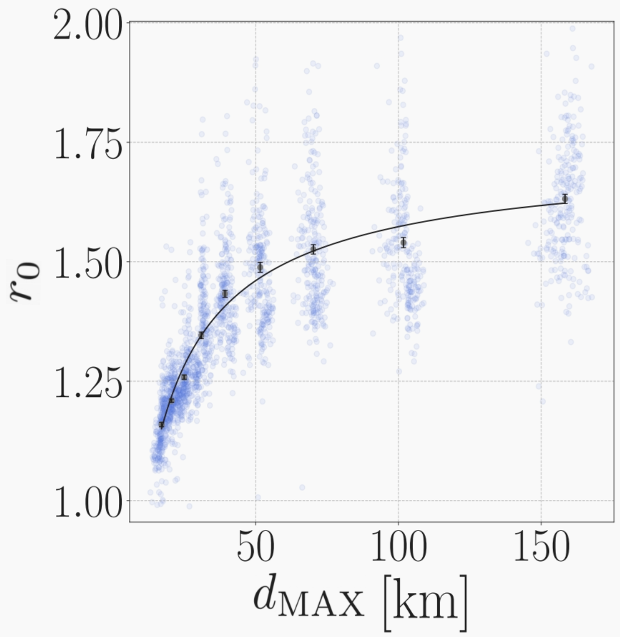
$$f(r) = A_1 \cdot \left[-e^{B\cdot(r-r_0)} + A_2 \cdot e^{-C\cdot(r-r_0)^2} \right]$$

Parameters distributions **(individual fits with 5 free parameters)**



1. $r_0\,(d_{\rm MAX}) = \dfrac{p_0}{d_{\rm MAX}} \cdot \log(d_{\rm MAX}) + p_1$

Fitting the mean values



Frequency slope

1. $$f(r) = A_1 \cdot \left[- e^{B\cdot(r-r_0)} + A_2 \cdot e^{-C\cdot(r-r_0)^2} \right]$$

2. $f(r, r_0\,(d_{\mathrm{MAX}}))$ **4 free parameters fits**

1. $$r_0\,(d_{\mathrm{MAX}}) = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \log(d_{\mathrm{MAX}}) + p_1$$

2. $$\frac{B(d_{\mathrm{MAX}})}{C\,(d_{\mathrm{MAX}})}$$

- Repeat the individual fits keeping free the remaining 4 parameters

- Look at the **new** parameters distributions expressed as a function of d_{MAX}

- Fix another parameter

Iterate until there are no parameters left

Frequency slope

1. $$f(r) = A_1 \cdot \left[- e^{B\cdot(r-r_0)} + A_2 \cdot e^{-C\cdot(r-r_0)^2} \right]$$

2. $f(r, r_0\,(d_{\mathrm{MAX}}))$ 4 free parameters fits

3. $f(r, r_0\,(d_{\mathrm{MAX}}), \frac{B(d_{\mathrm{MAX}})}{C\,(d_{\mathrm{MAX}})})$ 3 free parameters fits

4. $f(r, r_0\,(d_{\mathrm{MAX}}), \frac{B(d_{\mathrm{MAX}})}{C\,(d_{\mathrm{MAX}})}, A_2(d_{\mathrm{MAX}}))$ 2 free parameters fits

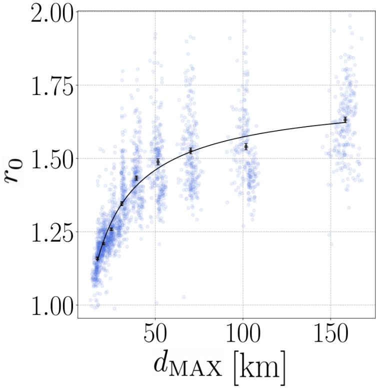
5. $f(r, r_0\,(d_{\mathrm{MAX}}), \frac{B(d_{\mathrm{MAX}})}{C\,(d_{\mathrm{MAX}})}, A_2(d_{\mathrm{MAX}}), C(d_{\mathrm{MAX}}))$ 1 free parameter fits

1. $r_0\,(d_{\mathrm{MAX}}) = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \log(d_{\mathrm{MAX}}) + p_1$
2. $\frac{B(d_{\mathrm{MAX}})}{C\,(d_{\mathrm{MAX}})}$
3. $A_2(d_{\mathrm{MAX}})$
4. $C\,(d_{\mathrm{MAX}})$
5. $A_1(d_{\mathrm{MAX}})$

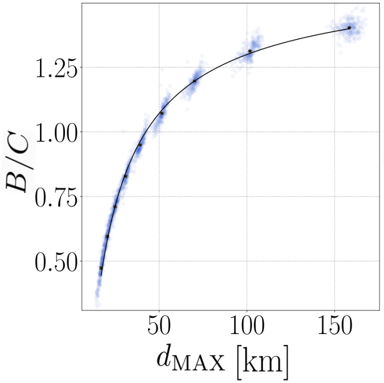
$$\rightarrow \quad f(r, r_0\,(d_{\mathrm{MAX}}), \frac{B(d_{\mathrm{MAX}})}{C\,(d_{\mathrm{MAX}})}, A_2(d_{\mathrm{MAX}}), C(d_{\mathrm{MAX}}), A_1(d_{\mathrm{MAX}}))$$

Frequency slope

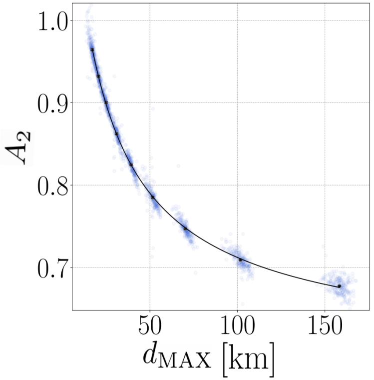
1.



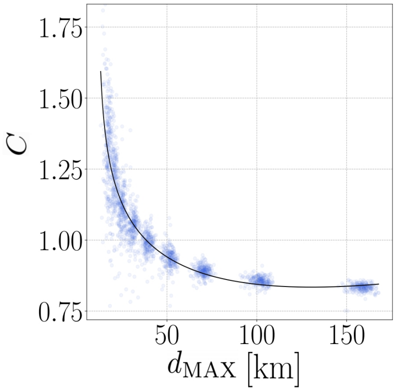
2.



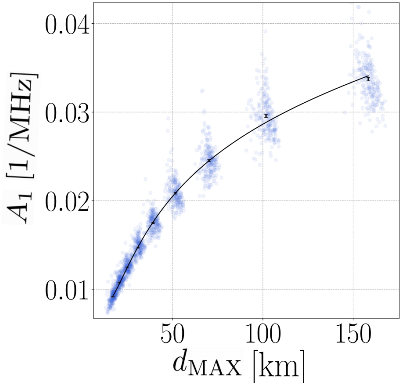
3.



4.



5.



1. $r_0\,(d_{\mathrm{MAX}}) = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \log(d_{\mathrm{MAX}}) + p_1$

2. $\frac{B(d_{\mathrm{MAX}})}{C\,(d_{\mathrm{MAX}})} = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \log(d_{\mathrm{MAX}}) + p_1$

3. $A_2(d_{\mathrm{MAX}}) = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \left[\log(p_1 \cdot d_{\mathrm{MAX}}) - p_2\right] + p_3$

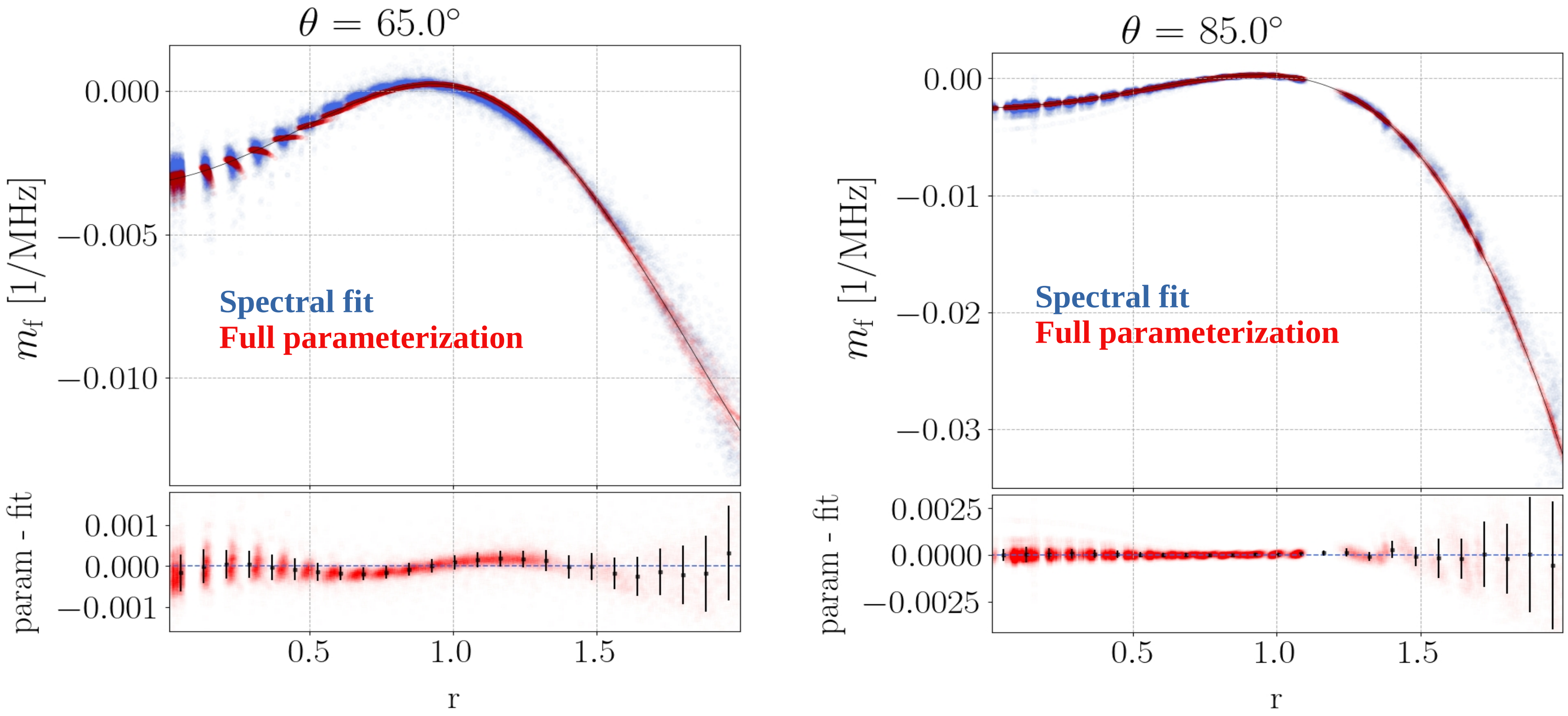
4. $C\,(d_{\mathrm{MAX}}) = p_0^{-d_{\mathrm{MAX}}} - p_1 \cdot \log(d_{\mathrm{MAX}} - p_2) + p_3$

5. $A_1(d_{\mathrm{MAX}}) = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \log(p_1 \cdot d_{\mathrm{MAX}}) + p_2 \cdot d_{\mathrm{MAX}} + p_3$

In total 16 constant values

(see back-up slides)

Frequency slope

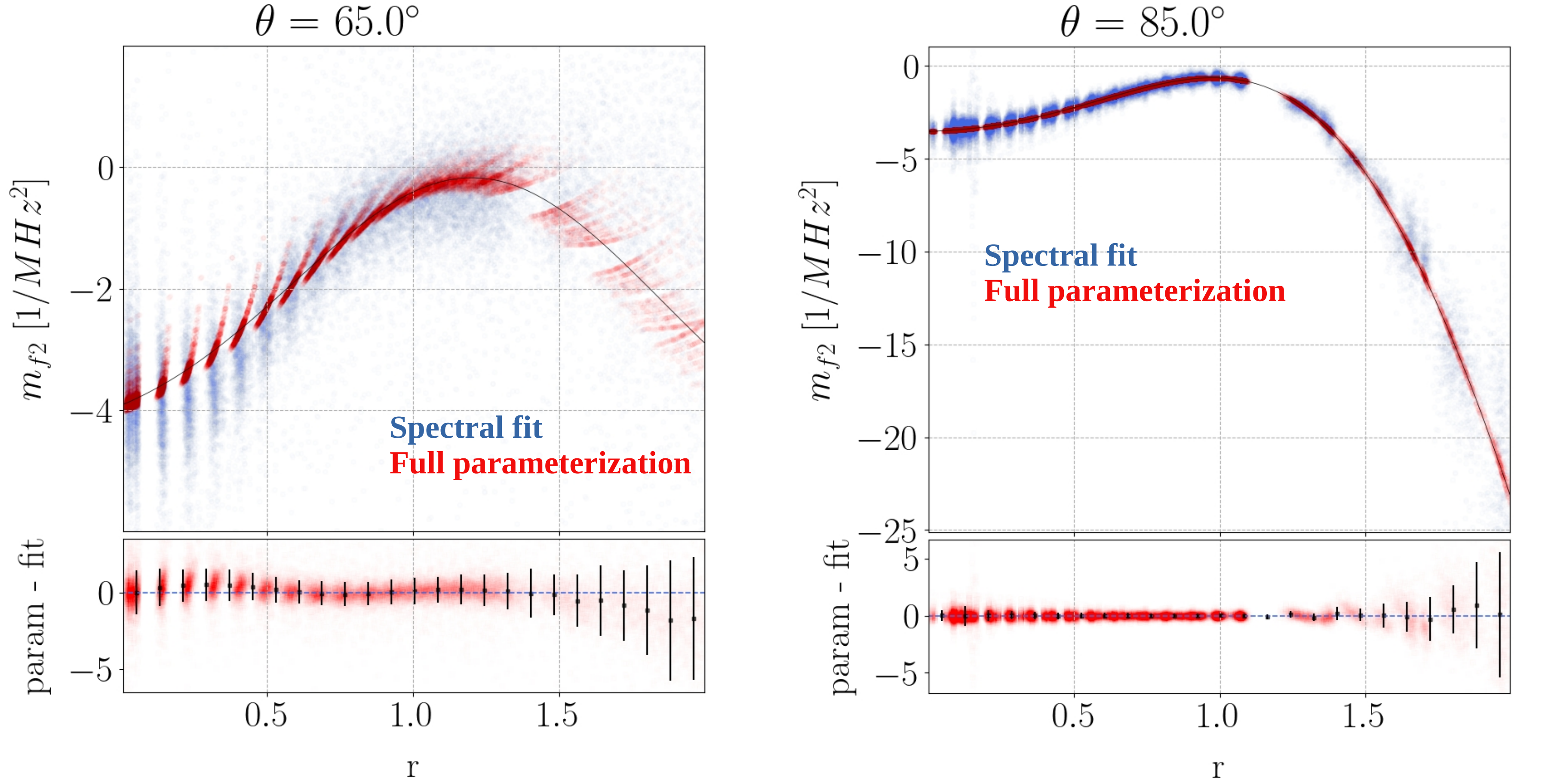


Quadratic term

(more in back-up slides)

Spectral fit

$$\sigma(f) = a \cdot Amp^{\mathrm{MAX}} + b \cdot Amp(f)$$



Parameterization of the CE component

$$L(f) = A \cdot 10^{m_{\mathrm{f}} \cdot (f - f_0)}$$

frequency slope

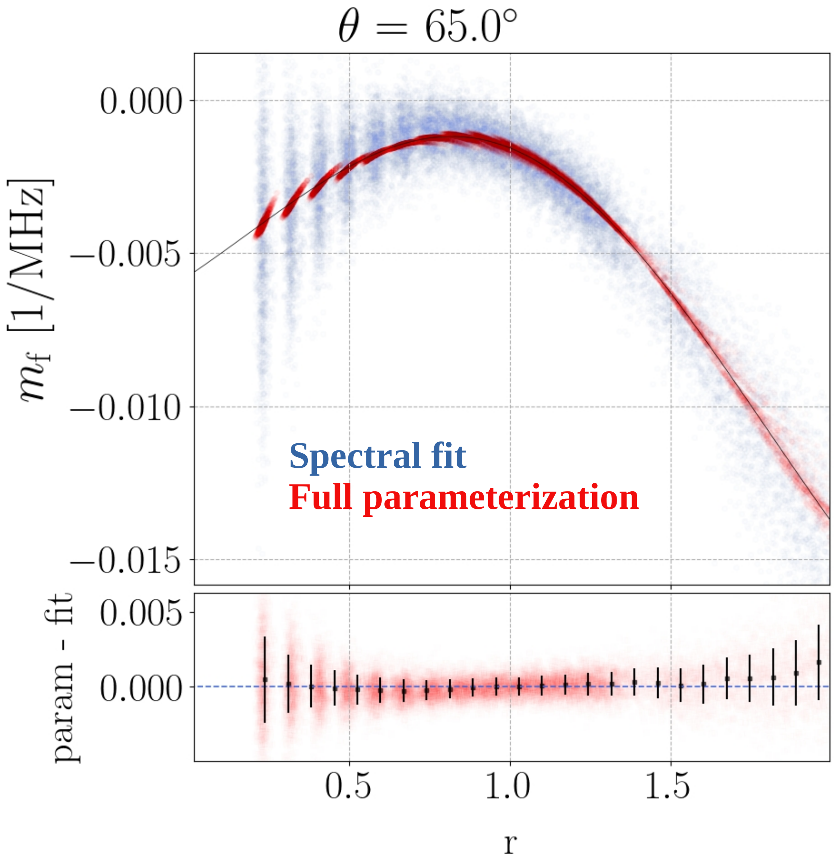
Frequency slope

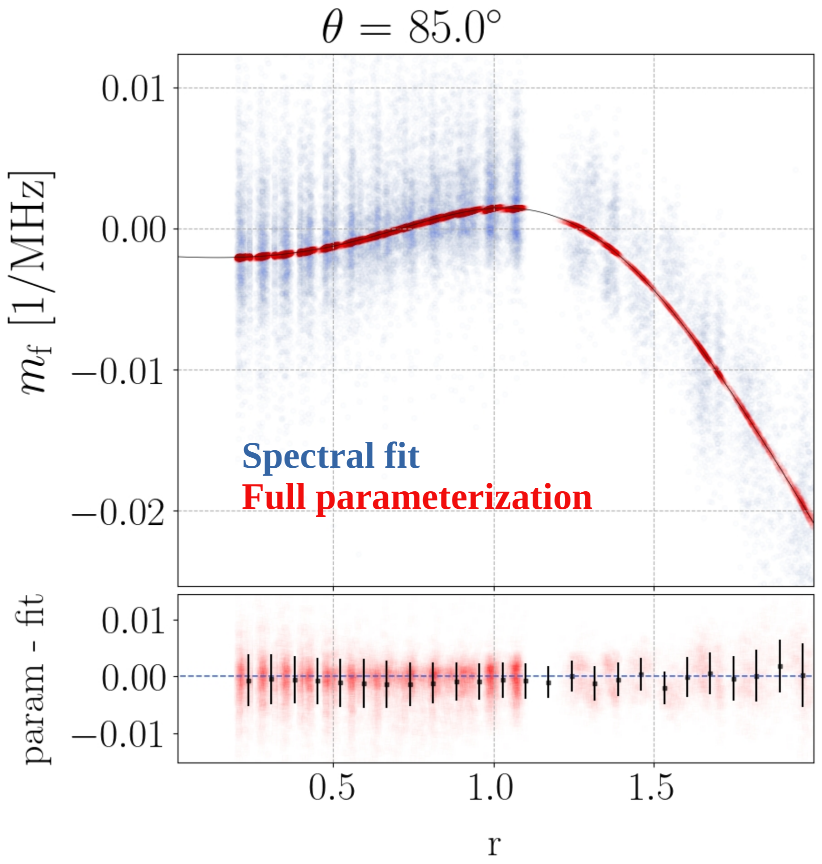
(more in back-up slides)

Spectral fit

Stations selected: 0.2 < r < 2.0

$$\sigma(f) = a \cdot Amp^{\mathrm{MAX}} + b \cdot Amp(f)$$





Conclusions

The **Charge-excess spectrum** can be described by a linear model, while the quadratic model describes better the **Geomagnetic spectrum**.

For both components, a parameterization of the spectral shape expressed as a function of r and d_{MAX} was derived.

The parameterization is valid in the 30-80 MHz bandwidth and for observer positions at $r < 2$.

Given zenith angle, X_{MAX} and antenna position, the slope and quadratic term can be **analytically calculated** and exploited in reconstruction algorithm to better constrain the geometry.

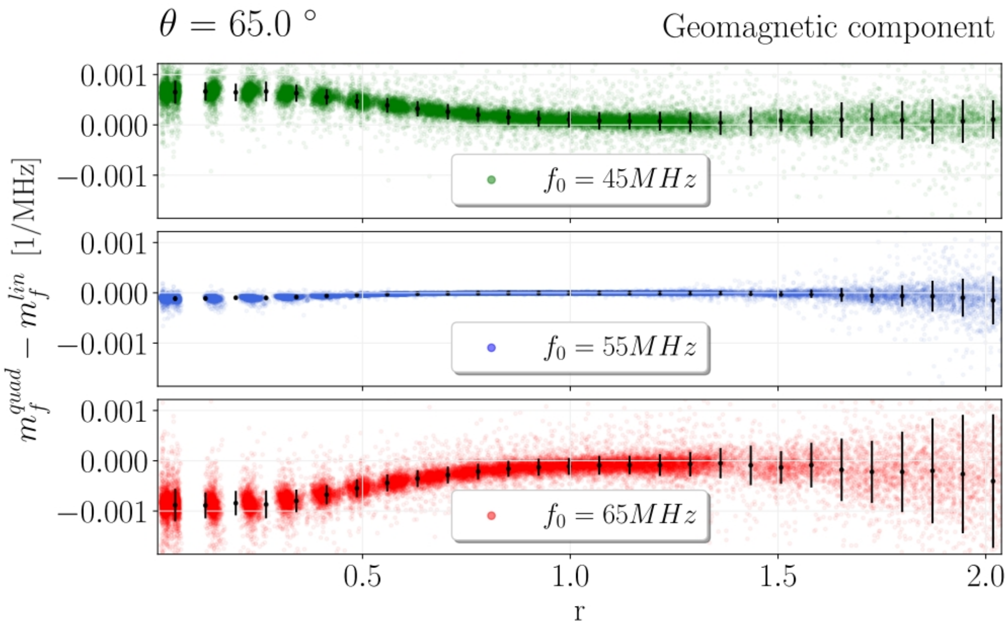
Back-up

Quadratic model fitting-procedure

1) Linear model fit with meaningful starting values for A and slope

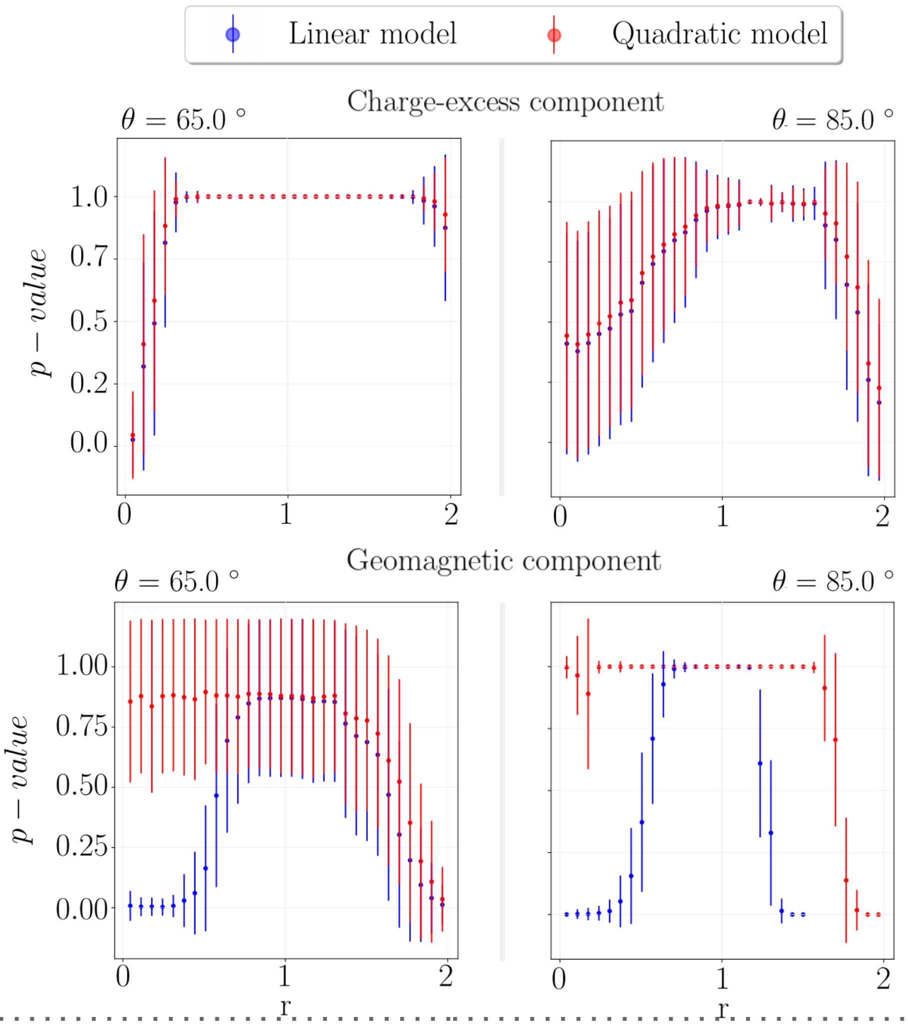
2) Quadratic model fit fixing the slope

3) Quadratic model fit with starting values from previous steps



Minimize flip of the sign of frequency slope → $f_0 = 55\,\mathrm{MHz}$

P-values distributions



Parameterization: Geomagnetic frequency slope

$$f(r) = A_1 \cdot \left[- e^{B\cdot(r-r_0)} + A_2 \cdot e^{-C\cdot(r-r_0)^2} \right]$$

1. $r_0\left(d_{\mathrm{MAX}}\right) = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \log(d_{\mathrm{MAX}}) + p_1$

2. $\frac{B(d_{\mathrm{MAX}})}{C\left(d_{\mathrm{MAX}}\right)} = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \log(d_{\mathrm{MAX}}) + p_1$

3. $A_2(d_{\mathrm{MAX}}) = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \left[\log(p_1 \cdot d_{\mathrm{MAX}}) - p_2\right] + p_3$

4. $C\left(d_{\mathrm{MAX}}\right) = p_0^{-d_{\mathrm{MAX}}} - p_1 \cdot \log(d_{\mathrm{MAX}} - p_2) + p_3$

5. $A_1(d_{\mathrm{MAX}}) = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \log(p_1 \cdot d_{\mathrm{MAX}}) + p_2 \cdot d_{\mathrm{MAX}} + p_3$

Constant values				
Parameter	p_0	p_1	p_2	p_3
r_0	-3.558	1.738		
B/C	-7.078	1.625		
A_2	3.468	0.2335	-0.4804	0.5863
C	0.9985	0.2155	11.69	0.6492
A_1 [MHz^{-1}]	-0.3792	0.2008	4.056	0.0359

Parameterization: Geomagnetic quadratic term

$$f(r) = A_1 \cdot \left[-e^{B\cdot(r-r_0)} + A_2 \cdot e^{-C\cdot(r-r_0)^2} \right]$$

1. $$r_0(d_{\text{MAX}}) = -p_0 + p_1 \cdot d_{\text{MAX}} - \frac{p_2}{d_{\text{MAX}}^2} \cdot \left[\log(p_3 \cdot d_{\text{MAX}}) - d_{\text{MAX}}\right]$$
2. $$\frac{B(d_{\text{MAX}})}{C(d_{\text{MAX}})} = -p_0^{-d_{\text{MAX}}} - p_1^{d_{\text{MAX}}^2} + p_2$$
3. $$A_2(d_{\text{MAX}}) = -p_0 + p_1 \cdot d_{\text{MAX}} + \frac{p_2}{d_{\text{MAX}}^2} \cdot \left[\log(p_3 \cdot d_{\text{MAX}}) - d_{\text{MAX}}^2\right]$$
4. $$C\left(d_{\text{MAX}}\right) = \frac{p_0}{d_{\text{MAX}}} \cdot \log(p_1 \cdot d_{\text{MAX}}) - p_2 \cdot d_{\text{MAX}} + p_3$$
5. $$A_1(d_{\text{MAX}}) = p_0 \cdot d_{\text{MAX}} + p_1$$

Constant values				
Parameter	p_0	p_1	p_2	p_3
r_0	-1.219	0.0019	-8.768	558448
B/C	1.014	1.00001	1.824	
A_2	73.28	-0.0009	-74.21	0.0482
C	59.77	0.0898	-0.0018	-0.2631
A_1 [MHz^{-2}]	$1.169 \cdot 10^{-6}$	$2.957 \cdot 10^{-5}$		

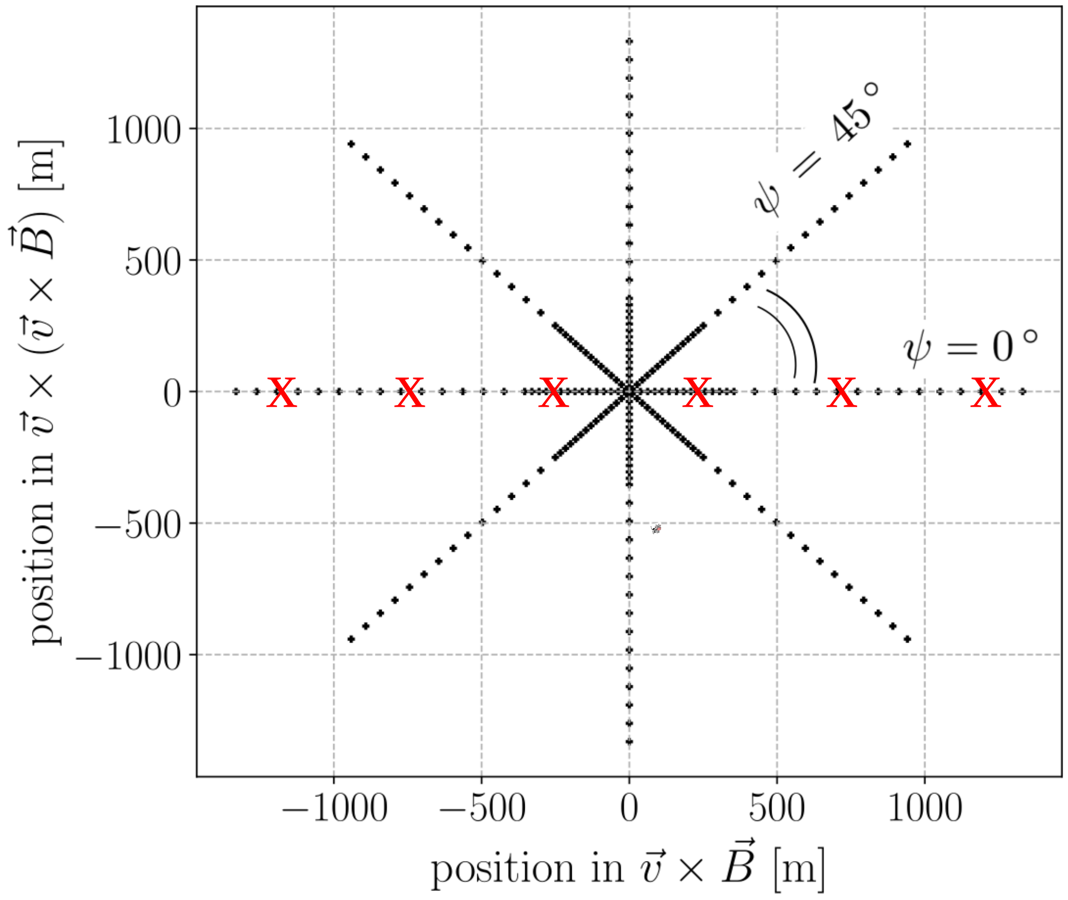
Parameterization: Charge-excess frequency slope

$$f(r) = A_1 \cdot \left[- e^{B\cdot(r-r_0)} + A_2 \cdot e^{-C\cdot(r-r_0)^2} \right]$$

1. $$r_0(d_{\mathrm{MAX}}) = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \log(d_{\mathrm{MAX}}) + p_1$$
2. $$\frac{B(d_{\mathrm{MAX}})}{C(d_{\mathrm{MAX}})} = p_0 \cdot d_{\mathrm{MAX}} + p_1$$
3. $$A_2(d_{\mathrm{MAX}}) = p_0 \cdot d_{\mathrm{MAX}} + p_1$$
4. $$C\left(d_{\mathrm{MAX}}\right) = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \log(d_{\mathrm{MAX}}) + p_1$$
5. $$A_1(d_{\mathrm{MAX}}) = \frac{p_0}{d_{\mathrm{MAX}}} \cdot \left[\log(p_1 \cdot d_{\mathrm{MAX}}) - p_2\right] + p_3$$

Constant values				
Parameter	p_0	p_1	p_2	p_3
r_0	-2.021	1.320		
B/C	0.0016	0.2764		
A_2	0.0009	0.8870		
C	-10.54	2.570		
A_1 [MHz^{-2}]	0.0270	388.0	4.748	0.0089

Field decomposition



Geomagnetic and Charge-excess field decomposition*

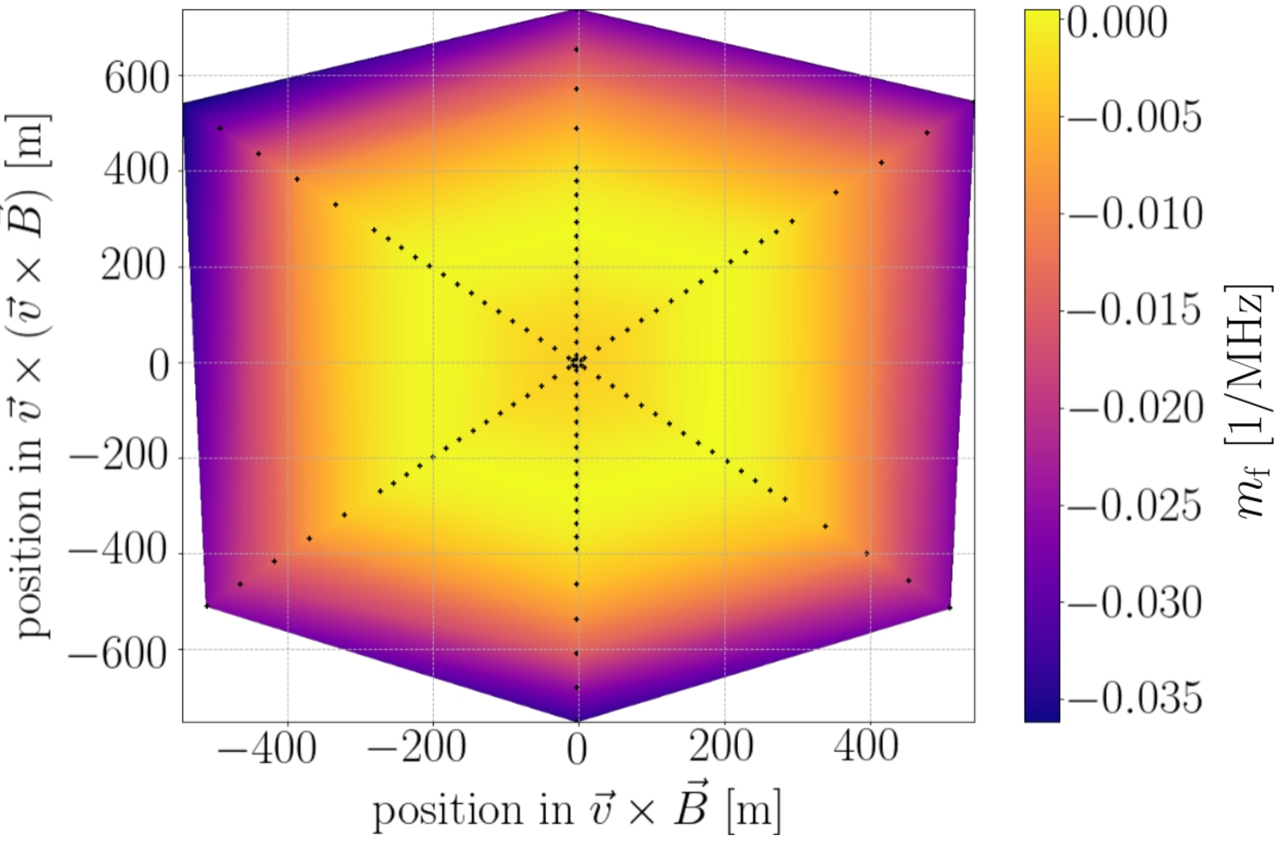
$$E_{\vec{v}\times\vec{B}}(\vec{r},t) = E_{geo}(\vec{r},t) + cos\,\psi\, E_{ce}(\vec{r},t)$$
$$E_{\vec{v}\times(\vec{v}\times\vec{B})}(\vec{r},t) = sin\,\psi\, E_{ce}(\vec{r},t)$$

Positions on the $\vec{v} \times \vec{B}$- axis are excluded from the analysis

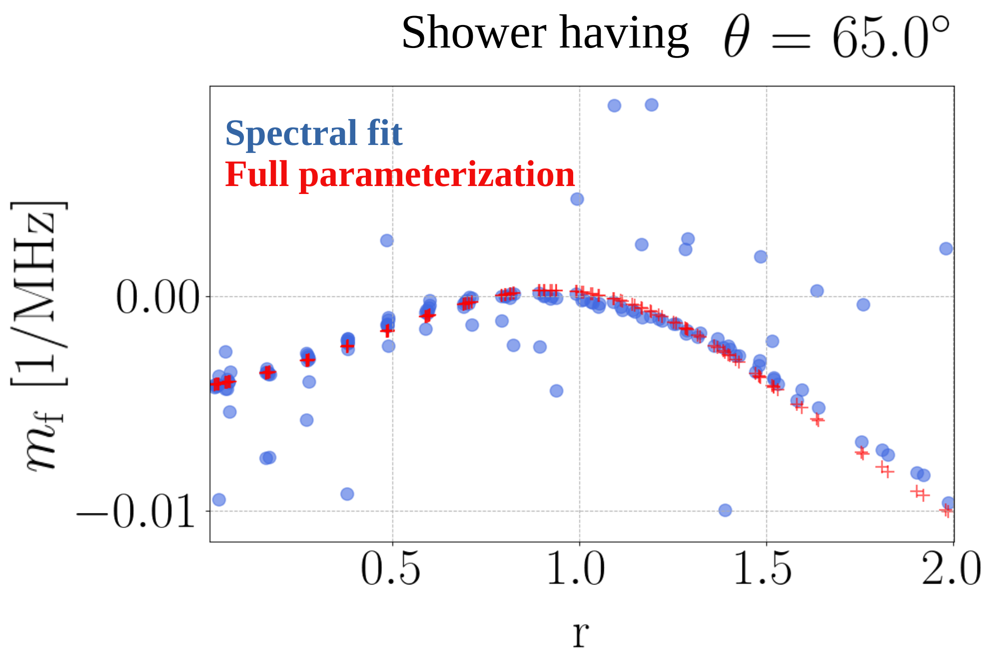
*As in *"Simulation of radiation energy release in air showers", JCAP,* C. Glaser, M. Erdmann, J. R. Hörandel, T. Huege, J. Schulz

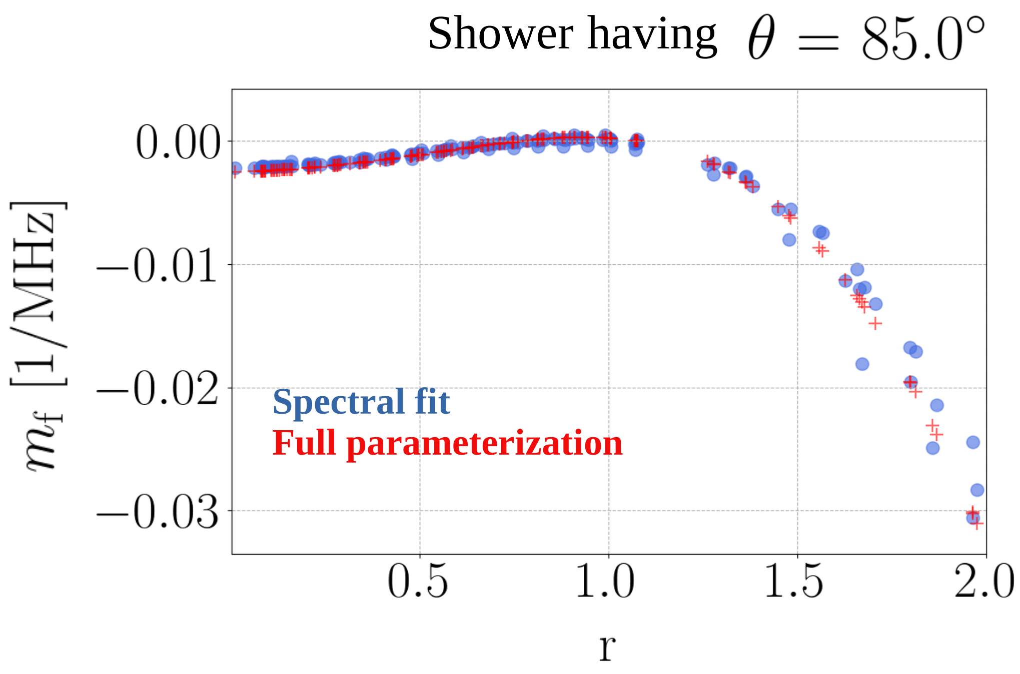
Single-event slope footprint

Geomagnetic component - $\theta = 85.0°,\ \phi = 45°,\ E = 10^{18.6}$ eV

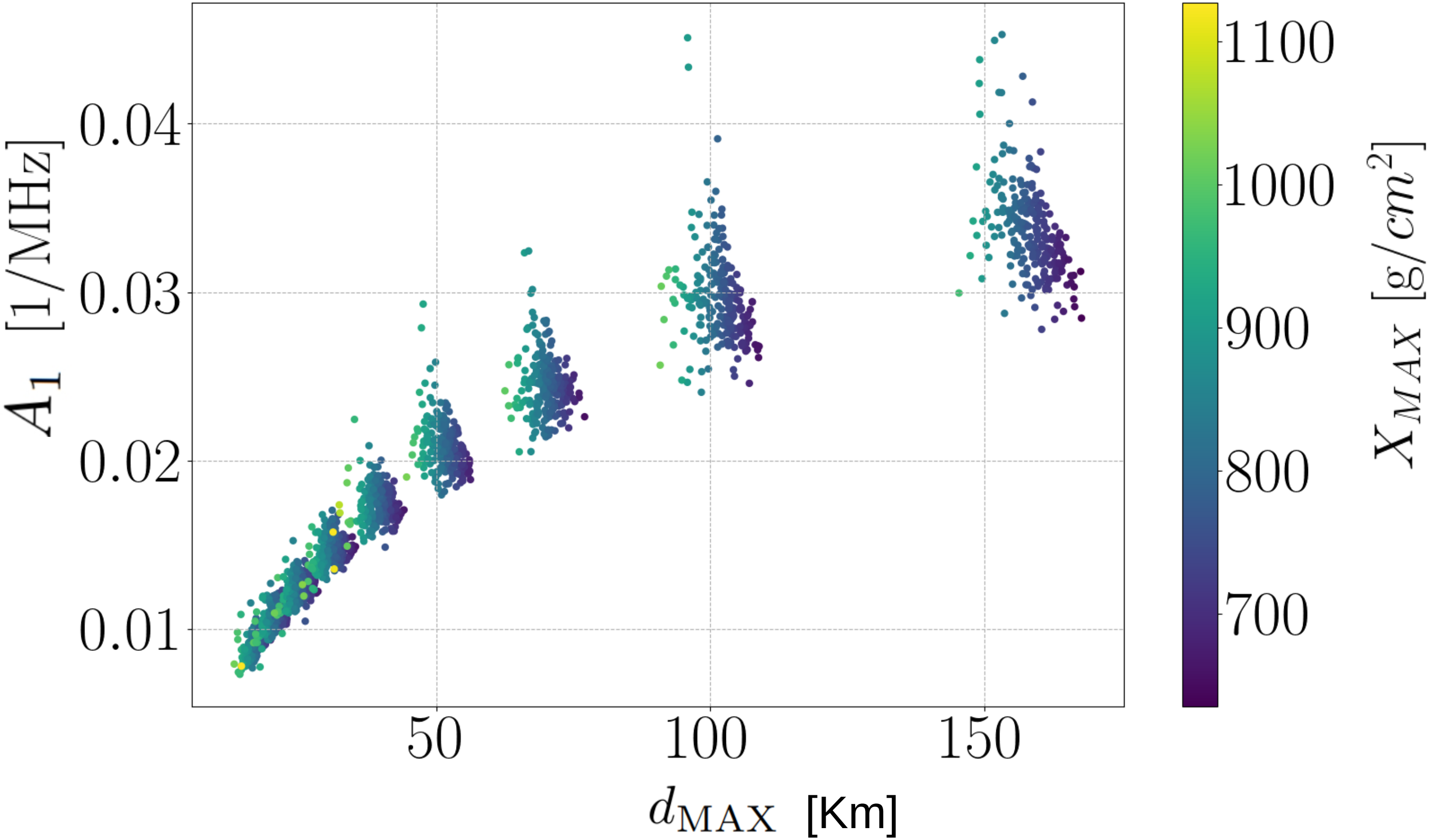


Slope lateral distribution: fit vs parameterized values



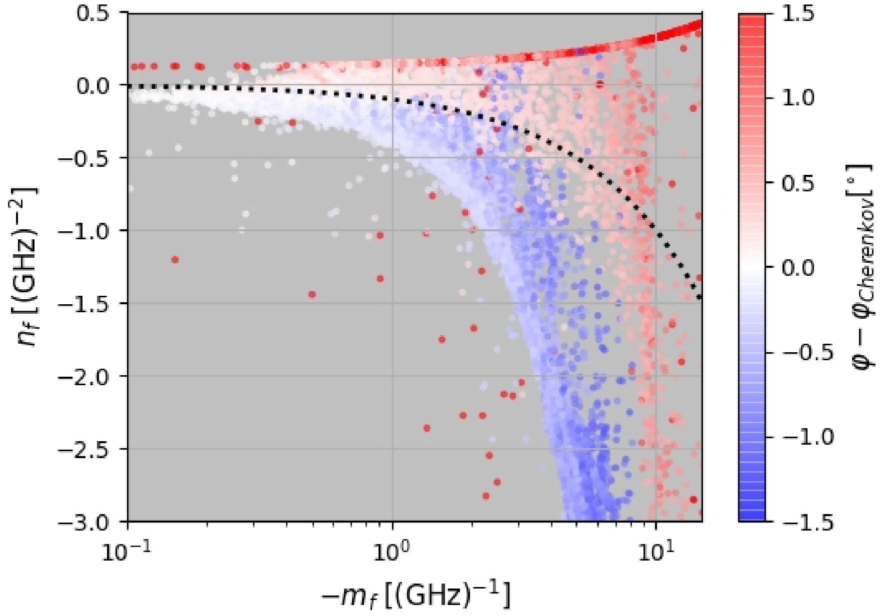


Xmax dependence



Broader frequency bandwidth

Quadratic term of a broader frequency bandwidth (80 – 300 MHz, ARIANNA) shows interesting features

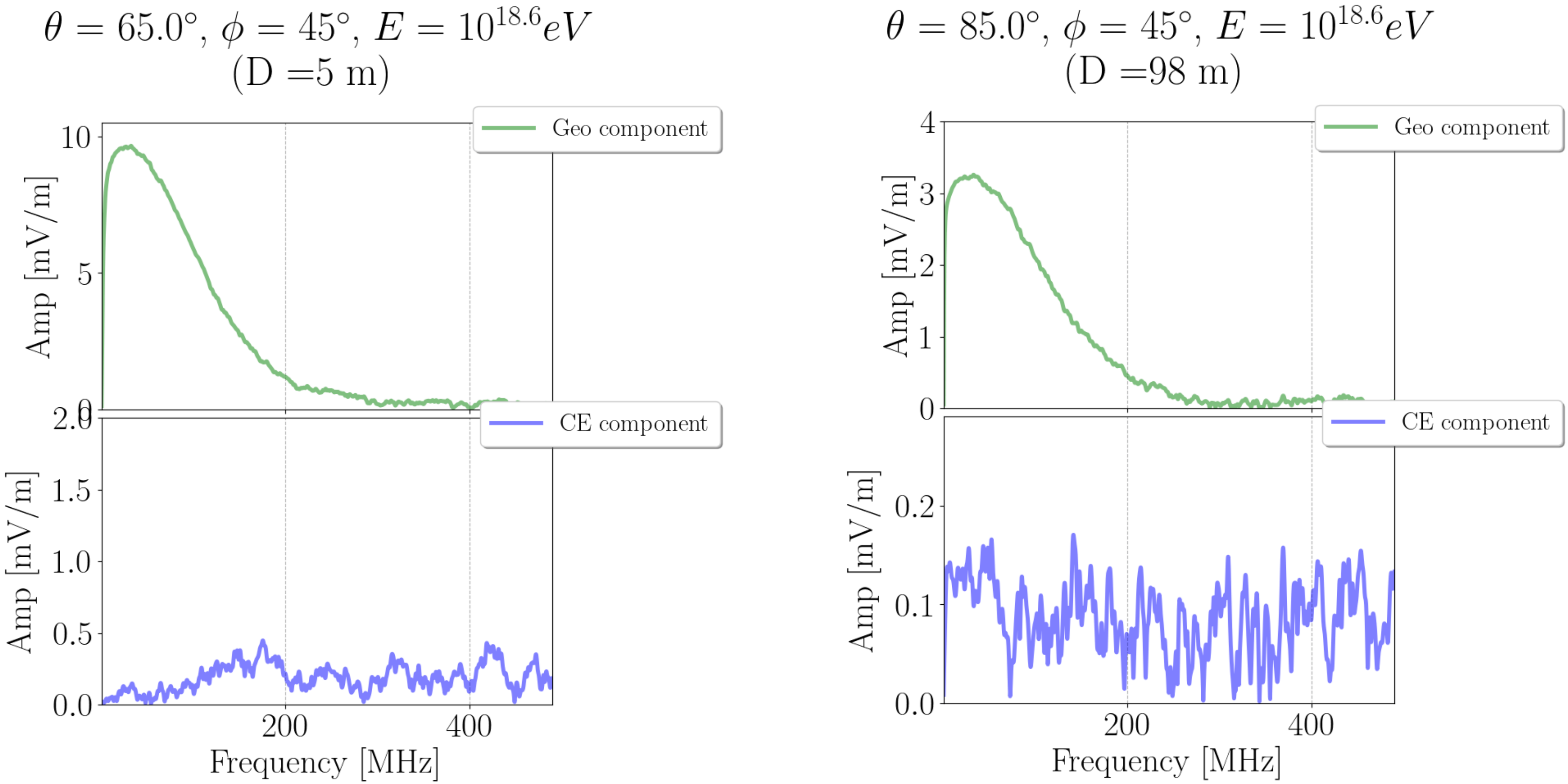


From *"Reconstructing the cosmic-ray energy from the radio signal measured in one single station"*, C. Welling, C. Glaser, A. Nelles

$$\begin{pmatrix}\mathcal{E}_\theta \\ \mathcal{E}_\phi\end{pmatrix} = \begin{pmatrix}A_\theta \\ A_\phi\end{pmatrix} 10^{f\cdot m_f + (f-80\mathrm{MHz})^2\cdot n_f}\ \exp(\Delta\, j)$$

Positive quadratic correction identifies signals measured outside the Cherenkov ring, negative otherwise

Broader frequency bandwidth

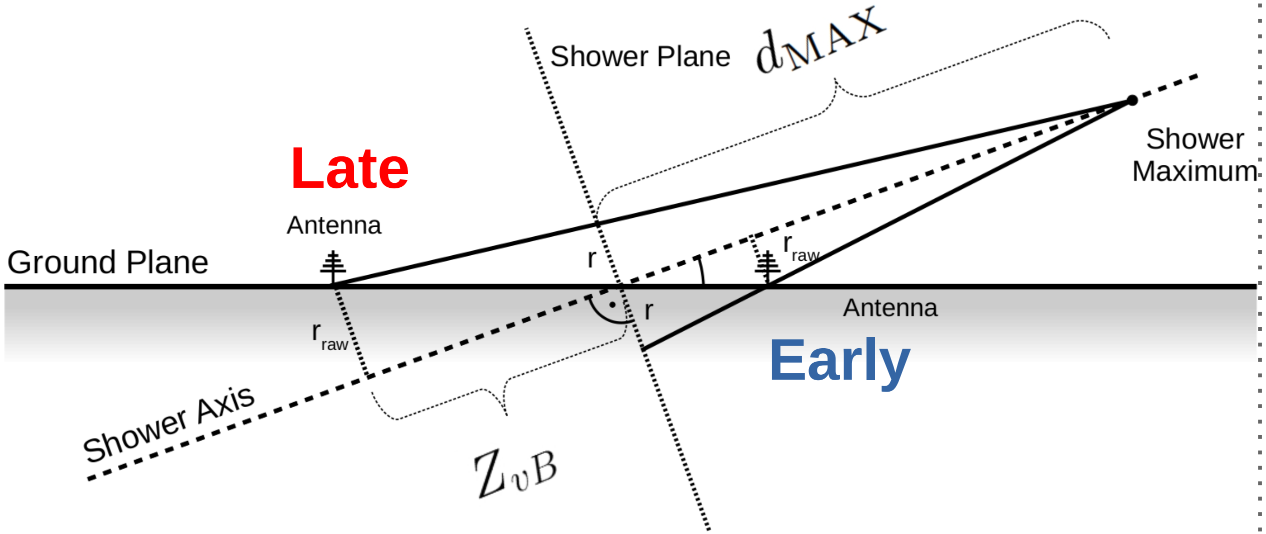


For wider frequency regions, an additional correction, not only quadratic, could be needed

"Splitting" feature

$c = 1 + (Z_{vB} \,/\, d_{MAX})$
$r = c * r_{raw}$

From "A Rotationally Symmetric Lateral Distribution Function for Radio Emission from Inclined Air Showers" - T. Huege, L. Brenk, F. Schlüter



Positive vxvxB: upstream (early)
Negative vxvxB: downstream (late)

$Z_{vB} \,/\, d_{MAX}$

Early-late "splitting" effect

Positive vxvxB:
upstream (early)
Negative vxvxB:
downstream (late)