RADIO PROPAGATION IN NON-UNIFORM MEDIA
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Dieder Van den Broeck*
Stijn Buitink
Krijn de Vries
Tim Huege
Uzair Latif

RADIO PROPAGATION IN NON UNIFORM MEDIA

WHY IS THIS IMPORTANT?

Understanding is needed on how radio propagates through natural material (polar ice, atmosphere…) => typically non uniform media

How to simulate propagation?

FDTD, typically computationally expensive

Parabolic equation solver, (https://arxiv.org/abs/2011.05997)

Raytracing, a high frequency limit

More specific:

1) How to describe Cerenkov emission in non uniform media?

2) How to simulate horizontal propagation in ice?

RAYTRACING WITH FERMAT'S PRINCIPLE

A good place to start

Observation of classically 'forbidden' electromagnetic wave propagation and implications for neutrino detection.

S. W. Barwick[a], E. C. Berg[a], D. Z. Besson[b,c], G. Gaswint[a], C. Glaser[a], A. Hallgren[d], J. C. Hanson[e], S. R. Klein[f], S. Kleinfelder[g], L. Köpke[h], I. Kravchenko[i], R. Lahmann[j], U. Latif[b], J. Nam[k], A. Nelles[l,m], C. Persichilli[a], P. Sandstrom[n], J. Tatar[a,o], E. Unger[d]

Describes raytracing with Fermat's principle. Overdensity => Guide the raypaths

FERMAT'S PRINCIPLE

HOW DOES IT WORK?

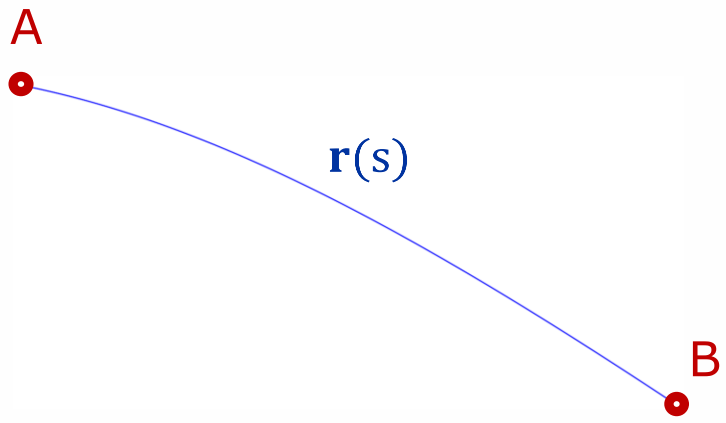
Minimize the variation in the optical path length for a path $\boldsymbol{r}$ between points A and B

$$OPL = \int_A^B n\big(\boldsymbol{r}(s)\big)\, ds$$

With $ds^2 = \boldsymbol{r}(s) \cdot \boldsymbol{r}(s)$

Leads to:

$$\frac{\partial n}{\partial \boldsymbol{r}} - \frac{d}{ds}\left(n(\boldsymbol{r}(s)\dot{\boldsymbol{r}}\right) = 0\,, \qquad \dot{\boldsymbol{r}} = \frac{d\boldsymbol{r}}{ds}$$



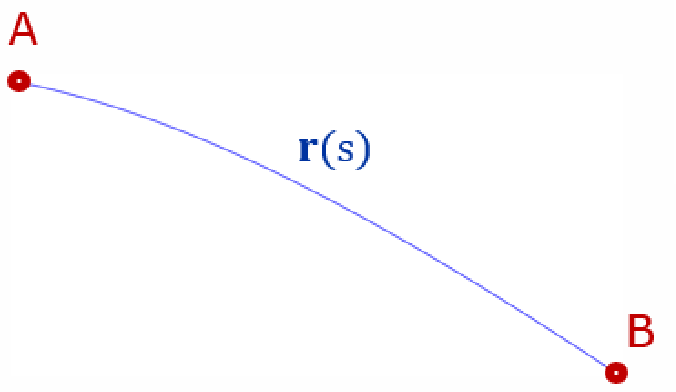
FERMAT'S PRINCIPLE

SOLVING FOR THE RAYPATH

Constrict the ray to be in the x,z plane, and define the optical momenta $p_{x_i} = n\,\dot{x}_i$ to get:

$$\dot{x} = \frac{p_x}{n} \qquad \dot{p}_x = \frac{\partial n}{\partial x}$$

$$\dot{z} = \frac{p_z}{n} \qquad \dot{p}_z = \frac{\partial n}{\partial z}$$



For the index of refraction profile an exponential profile is used

$$n(z) = A + Be^{-Cz}$$

To include perturbations: add a differentiable function to this over a range of z

RAYTRACING WITH FERMAT'S PRINCIPLE

WHAT INFORMATION DO WE WANT?

Assume a simple line model for the cascade, and define emission times:

$$t' = \frac{D}{c}$$

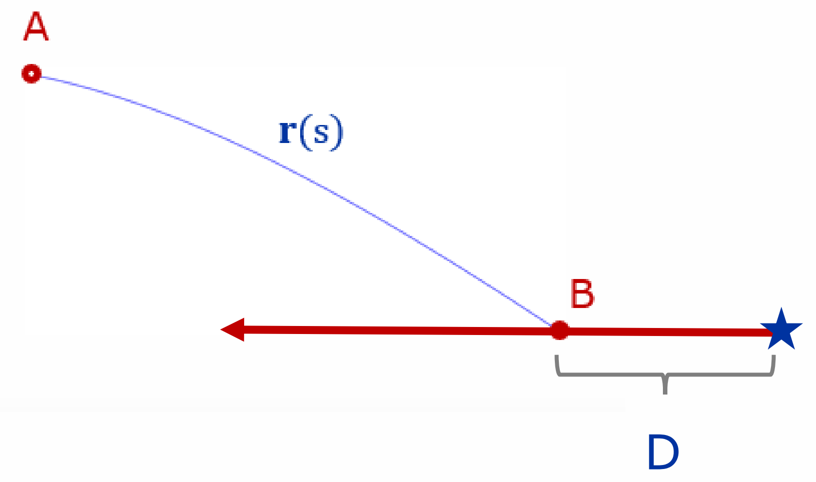
→ Distance from point $t' = 0$

For each ray, use the optical path length L to get travel time:

$$\Delta t = \frac{L}{c}$$

Look at the relation between emission and arrival times:

$$t = t' + \frac{L}{c}$$



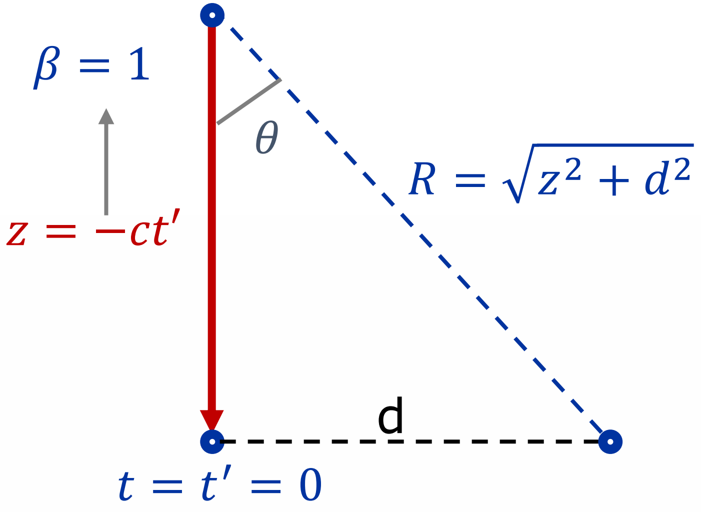
THE BOOSTFACTOR

The boostfactor $\frac{1}{\left(\frac{dt}{dt'}\right)}$ contains the information for geometric boosting.

Look at the uniform case (n constant):

$$t = t' + n\frac{\sqrt{(-ct')^2 + d^2}}{c}$$

$$\frac{dt}{dt'} = 1 - n \cdot \cos(\theta)$$



What if the derivative is zero?

$$\frac{dt}{dt'} = 0 \Rightarrow \cos(\theta) = \frac{1}{n} \Rightarrow \theta = \theta_{cerenkov}$$

A signal emitted over an interval $\Delta t'$ arrives in a shorter interval Δt

WHY DOES THE BOOSTFACTOR MATTER?

The end point formalism (arxiv.org/abs/1112.2126) :

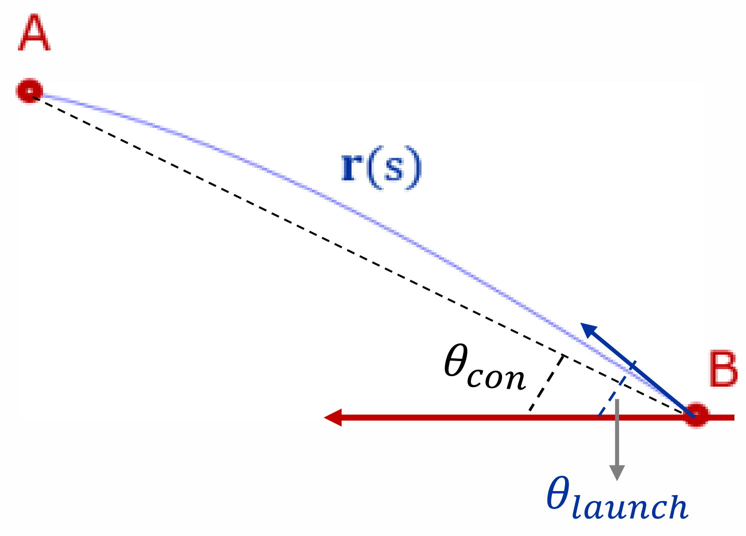
$$\vec{E}_{\pm}(\vec{x},t) = \pm \frac{1}{\Delta t}\frac{q}{c}\left(\frac{\hat{r}\times[\hat{r}\times\vec{\beta}^{*}]}{(1-n\vec{\beta}^{*}\cdot\hat{r})R}\right)$$

$(1-n\vec{\beta}^{*}\cdot\hat{r})$ → $Boostfactor^{-1}$

When calculating as $1 - n\beta\cos(\theta)$:

What n?

What θ?



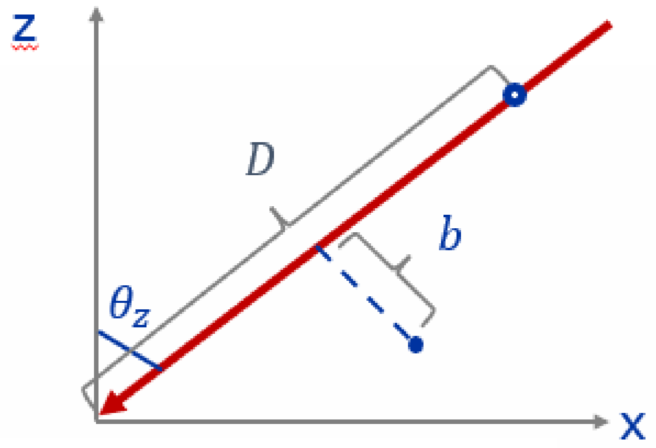
Previous studies (A. Timmermans, Ba. Thesis) show that a straight line approximation might not be valid for very inclined geometries in air

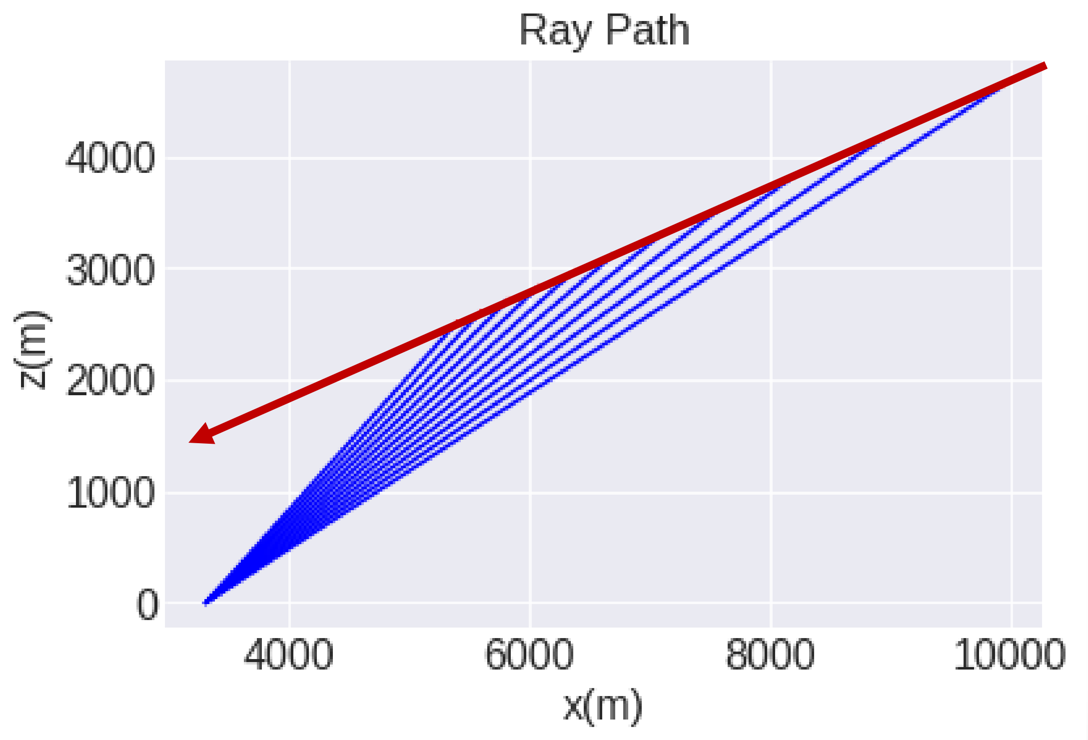
IN AIR BURSTS

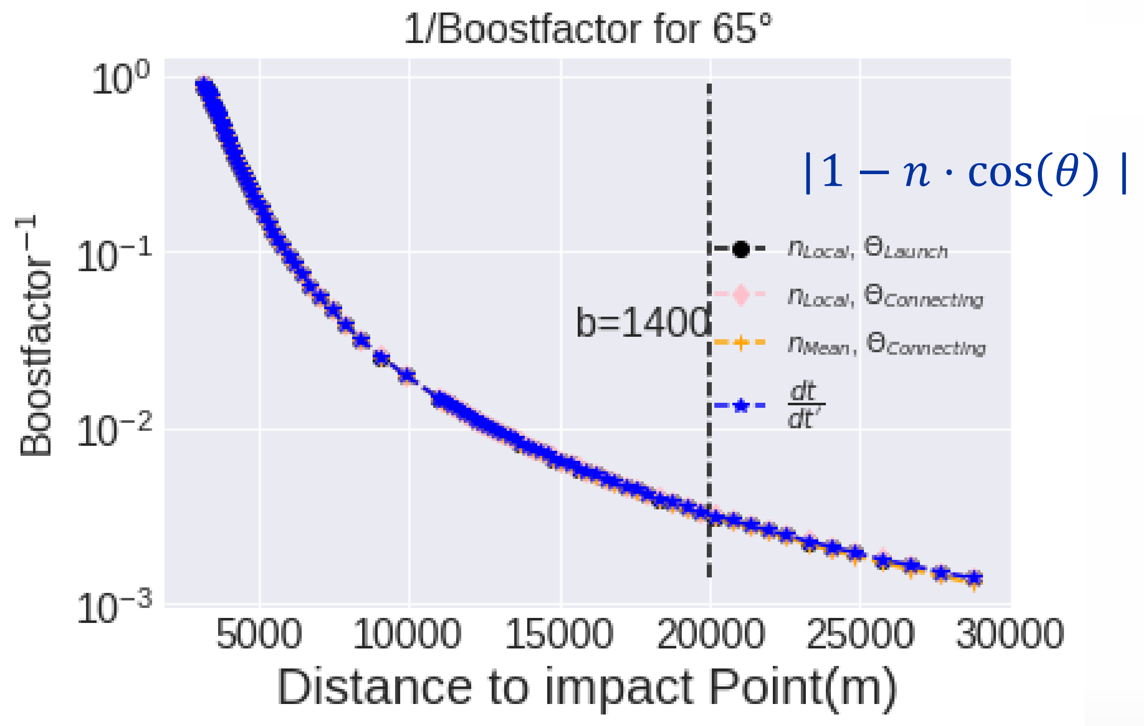
WHERE DOES THE APPROXIMATION CHANGE?

Look at estimators and compare to numerically computed derivative

All estimators follow the derivative





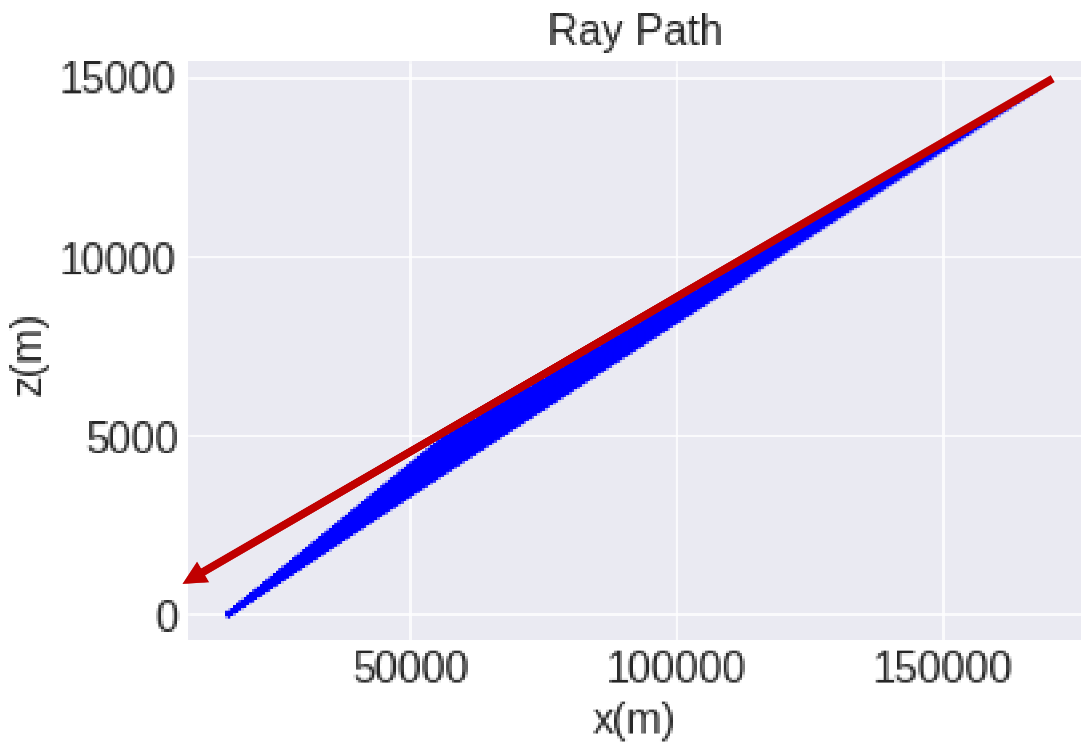


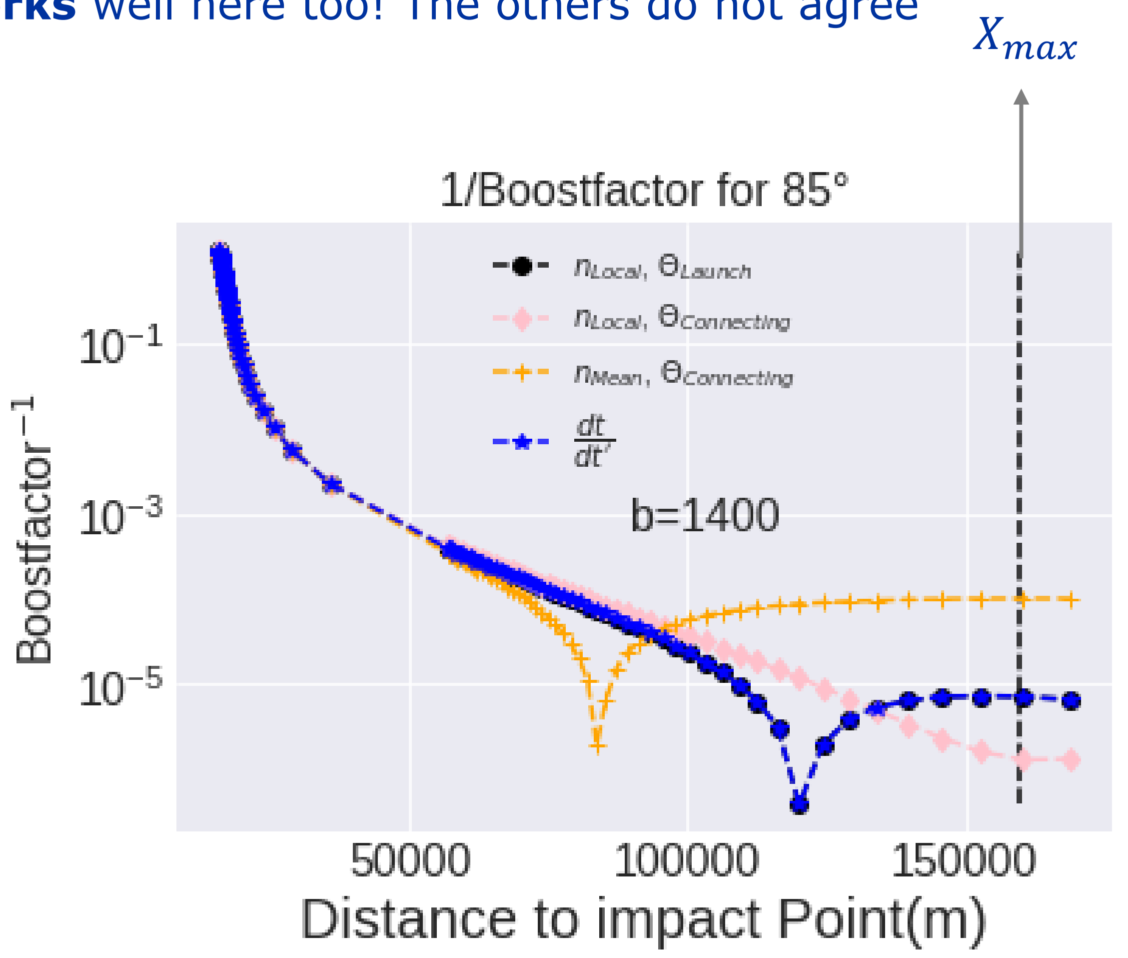
IN AIR BURSTS

WHAT ABOUT INCLINED SHOWERS?

The estimator with **local n and launch angle works** well here too! The others do not agree

Similar results found by A.Timmermans





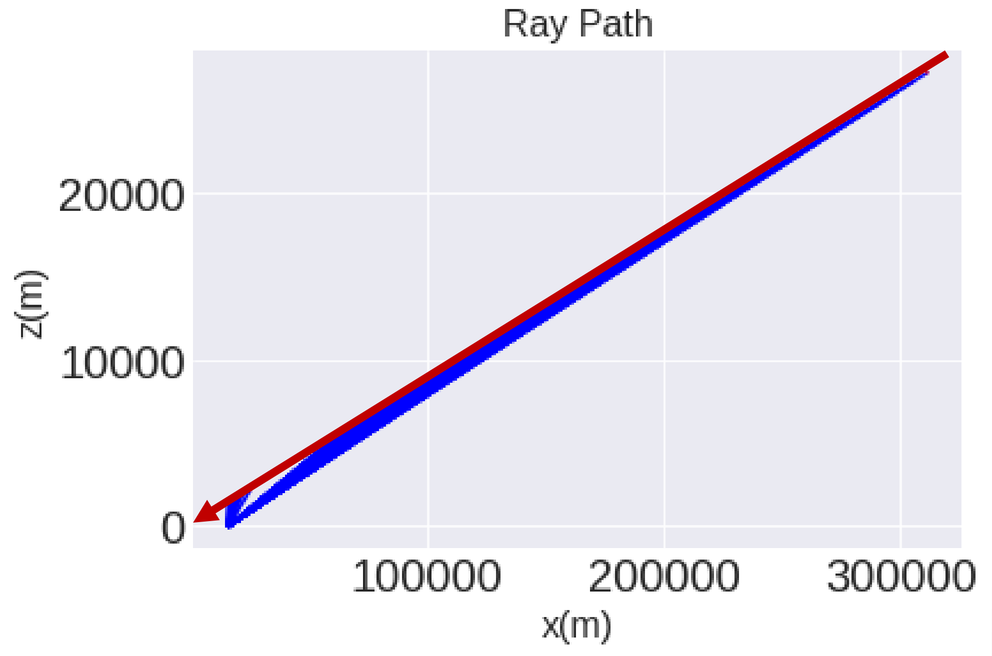
IN AIR BURSTS

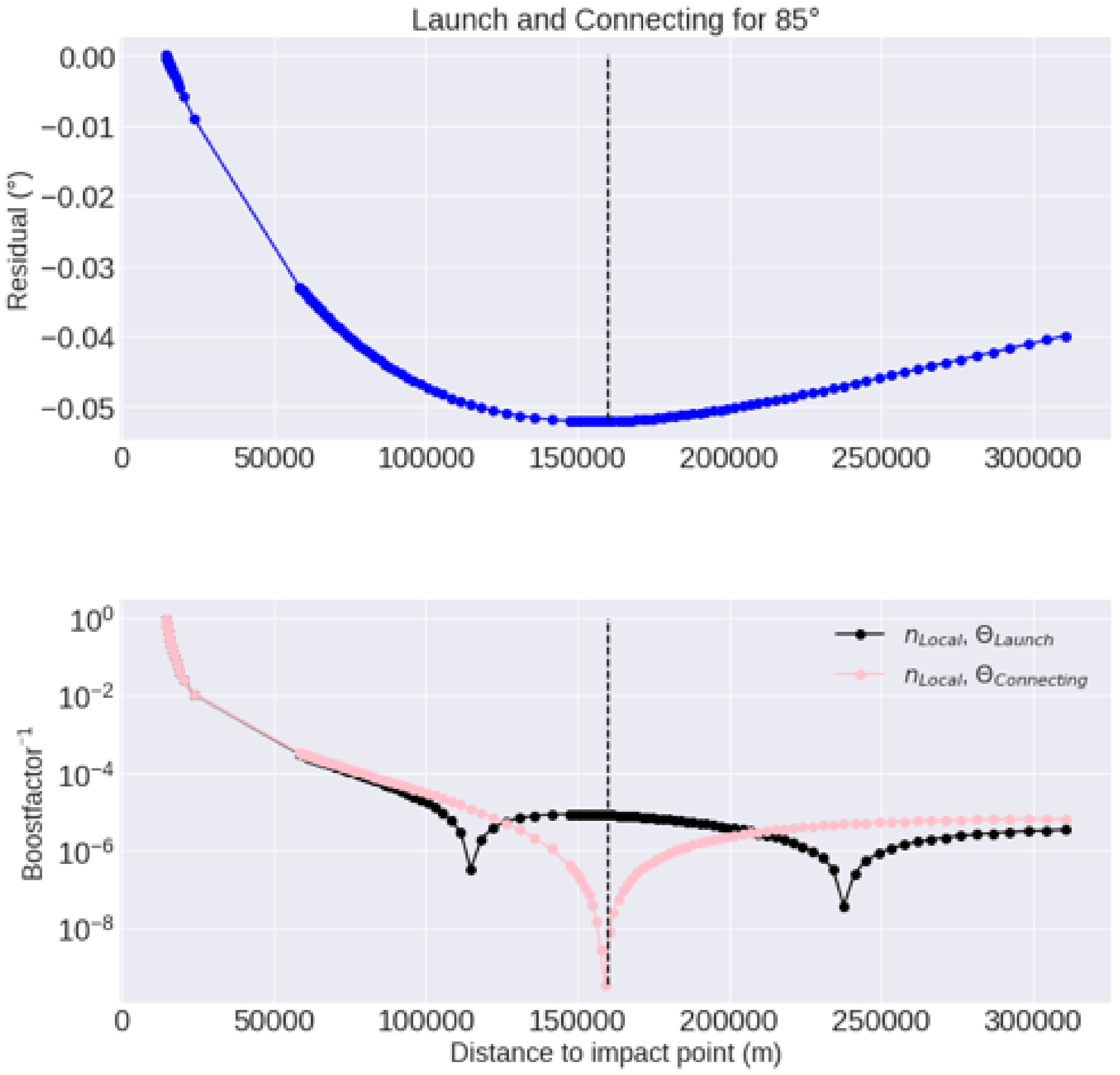
HOW BAD IS IT REALLY?

This boostfactor is for particles
with momentum along the cascade axis

Momentum in a real cascade is distributed

⇒ Non trivial result for realistic radio simulation

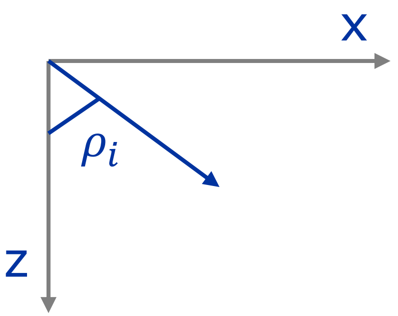


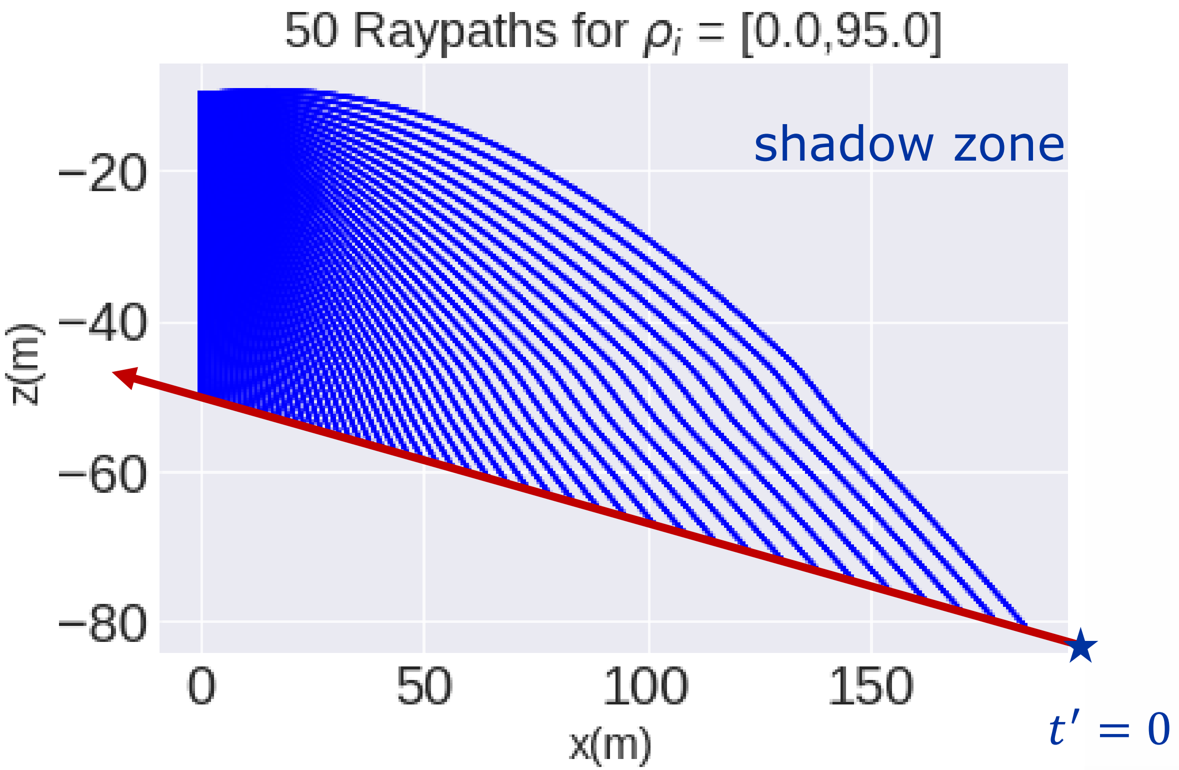


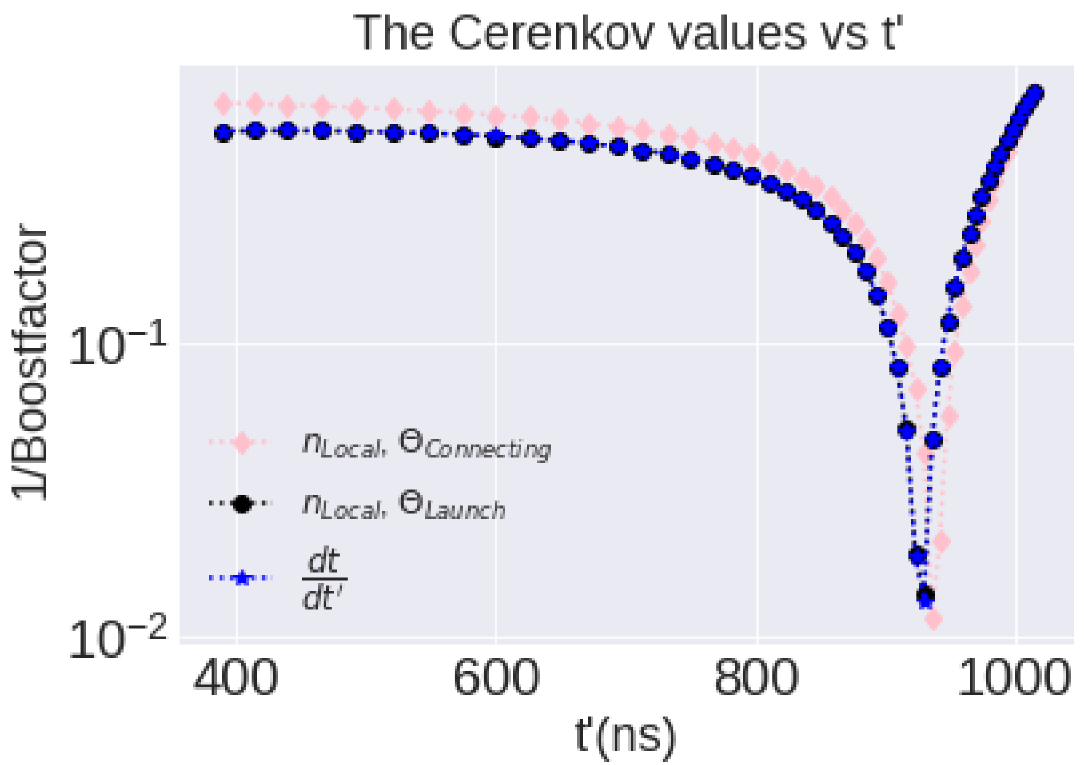
HOW ABOUT IN ICE?

DOES THE ESTIMATION HOLD HERE?

$1 - n_{local} \cdot (\cos(\theta_{launch})) = \frac{dt}{dt'}$ seems to work here too, even with perturbation

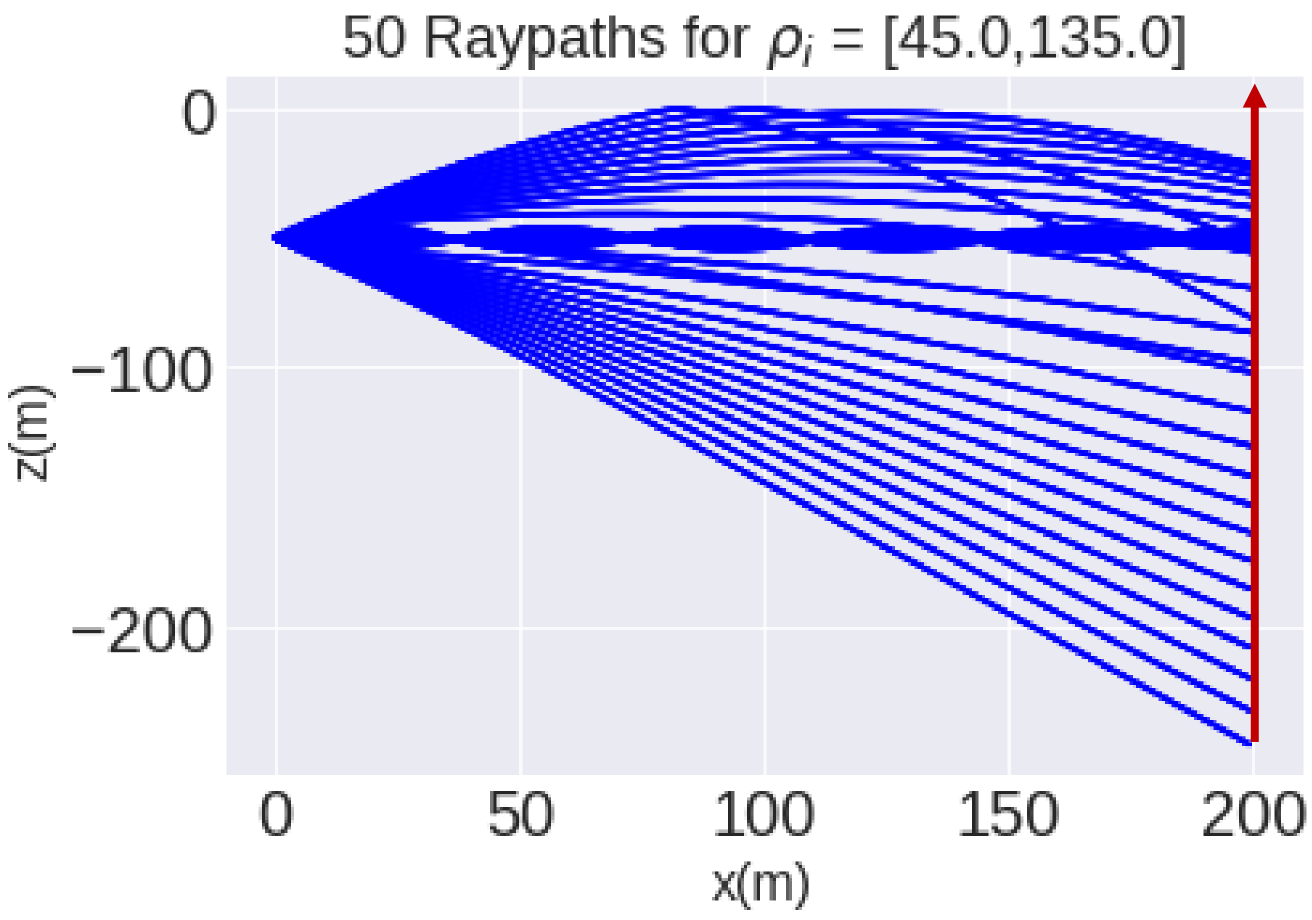






HOW ABOUT IN ICE?

CAN AN OVERDENSITY GUIDE THE RAYPATHS?



Add a cosine window at -50 m depth (peak strength 0.15)

=> Raypaths get trapped in the perturbation

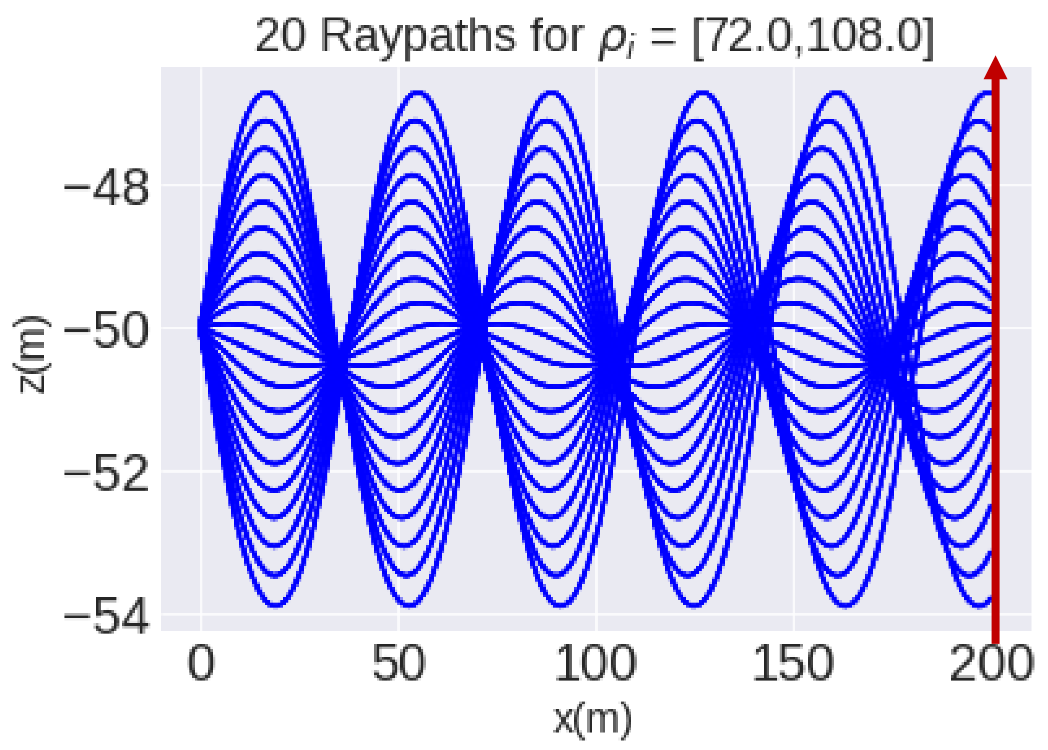
What does this mean for the shadow zone?

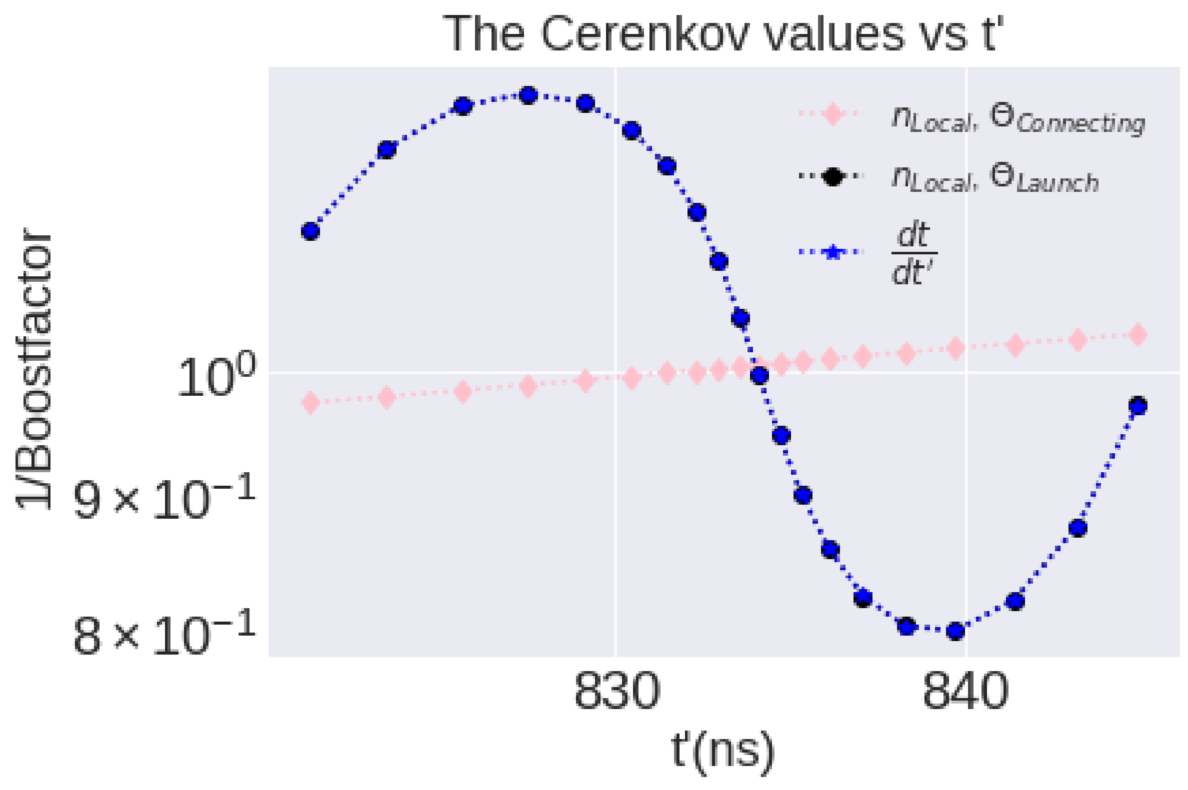
HOW ABOUT IN ICE?

IS THE BOOSTFACTOR STILL RELEVANT?

Look at the rays that remain trapped in the overdensity, can we use the same estimator here?

$$1 - n_{local} \cdot (cos(\theta_{launch}) = \frac{dt}{dt'}$$ **holds**





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WHAT ARE THE MAIN RESULTS? HOW TO PROCEED?

When describing the boostfactor, using a Cerenkov **estimator with local n and launch angle works (for now)**

The raytracer finds **waveguide like behavior for overdensities**

Waveguide propagation is typically frequency dependent => needs to be taken into account

What effect does launch vs connecting have? => Simulate worst case scenarios