Practical and Accurate Calculations of Radio Emission from Extensive Air Showers

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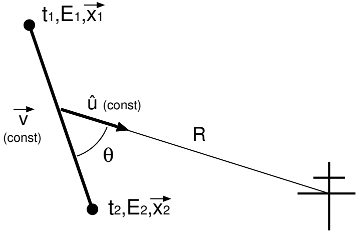
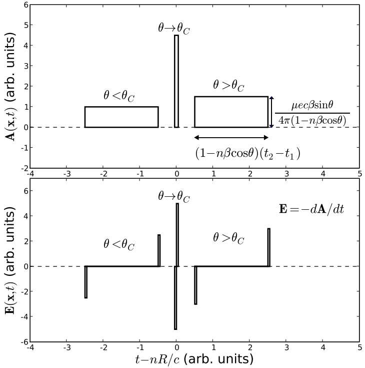
Today: e.g. ZHAireS in the time domain

- Contribution to $\vec{A}$ calculated separately for EVERY SINGLE TRACK!

$$\vec{A}(t,\hat{u}) = \frac{\mu e}{4\pi R c}\vec{\beta}_\perp \frac{\Theta(t-t_1^{det})-\Theta(t-t_2^{det})}{1-n\vec{\beta}\cdot\hat{u}}$$



(Phys.Rev.D81:123009,2010)



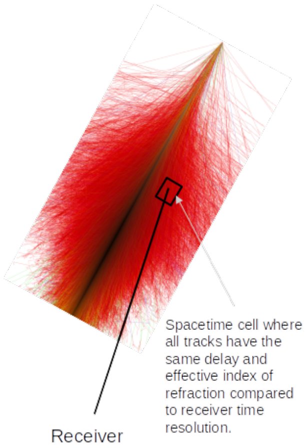
$t_{1,2}^{det}$ depend on $n_{eff} = \frac{1}{R}\int_0^R n(h)\,dl$ which is **VERY EXPENSIVE!**

Basic idea: Divide the shower in 4-D volumes

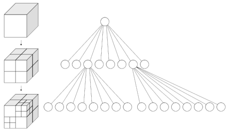
- Shower is divided into 4-D volumes of spatial sides L and time length L/c
- Volumes must obey Fraunhoffer condition for all observers:

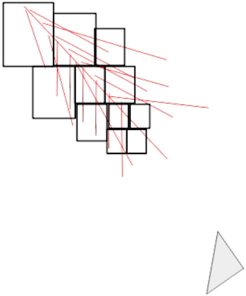
$$L < \frac{1}{\sin\theta}\sqrt{\frac{\lambda R}{2\pi}} \quad (\theta = 60^\circ)$$

- Cell is sufficiently small so that many terms of the $\vec{A}$ contribution are constant for the whole cell (depends on receiver time resolution)
- Contribution of all the tracks inside the cell is calculated only once, based on an average track for the cell
- Almost amounts to a macroscopic treatment of the shower, but retaining the microscopic precision



Cells have different sizes: Octree binning

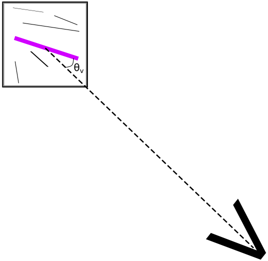




- Cell size optimization: Octree binning
- Start with spatial cells set to a large side length L
- Checks the Fraunhofer condition for that side L
- If side length does not meet the condition, L is divided by 2
  - We now have 8 cubes instead of the original
  - Far away from the observers, L is large
  - Close to the observers, L is small
- Calculate n_{eff} once per cell and store it
- Each cube is uniquely defined by its center point

Effective average track and $\vec{A}$ calculation

All tracks inside a volume are represented by a single "effective" track



- For a cell i and track j:
  (different notation but equivalent to the ZHS single track formula)

$$\vec{A}_i\left(\vec{x}, t\right) = \frac{\mu_0 (Q\vec{v})_{i\mathrm{eff}\perp}}{4\pi R_i} \left|\frac{dt'_{i\mathrm{eff}}}{dt}\right|_{t'=t'_{ret}} \Pi\left(t', t'_{1\mathrm{eff}}, t'_{2\mathrm{eff}}\right),$$

  where Π is a boxcar function,
  t' is source time, $\vec{x}$ and t are observer position and time,

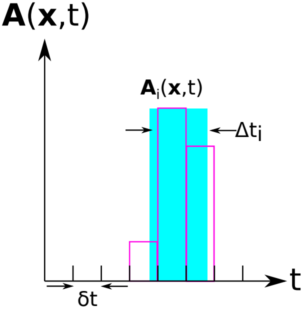
$$(dt')_{i\mathrm{eff}} = \left|t'_{2\mathrm{eff}} - t'_{1\mathrm{eff}}\right| = \left|\frac{\sum_j w_{ij}\, q_{ij}\, v_{ij}\, dt'_{ij}}{\sum_j w_{ij}\, q_{ij}\, v_{ij}}\right| \text{ and}$$

$$(Q\vec{v})_{i\mathrm{eff}} = \frac{\sum_j w_{ij}\, q_{ij}\, \vec{v}_{ij}\, dt'_{ij}}{\sum_j dt'_{ij}}$$

- Average track inside cell i is completely defined by $(dt')_{i\mathrm{eff}}$ and $(Q\vec{v})_{i\mathrm{eff}}$

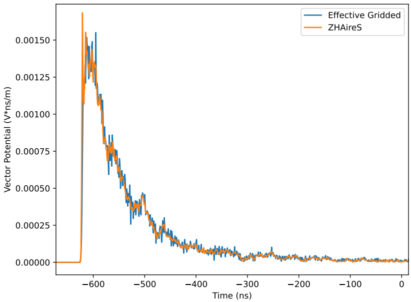
Time binning the Radiation at the Observer

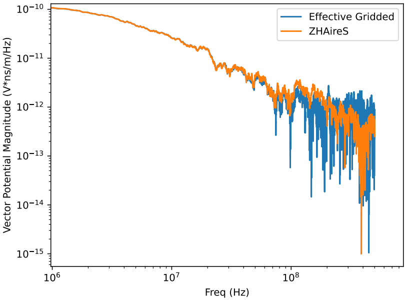
- Add the contribution of all cells to a resulting $\vec{A}$ at the observer
- Similar to ZHAIRES
- Boxcar function $\Pi\left(t', t'_{1\text{eff}}, t'_{2\text{eff}}\right)$ gives rise to time window Δt_i
- δt is a fixed time bin width set by the receiver
- Bins that fully overlap with Δt_i filled with the full value of $\vec{A}_i\left(\vec{x}, t\right)$
- Contribution of partially overlapping bins scaled by the fraction of bin overlap



Comparison with full ZHAireS simulations

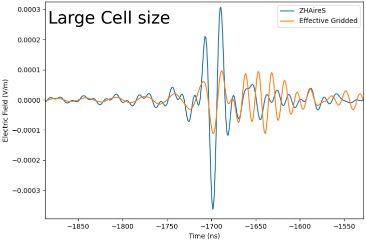
- Input: particle tracks obtained from ZHAireS
- Calculations using the ZHS algorithm compared to this work
- Thinning of 10^{-3} and $\theta = 70^\circ$

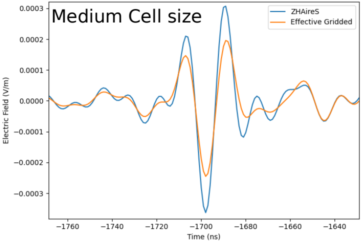


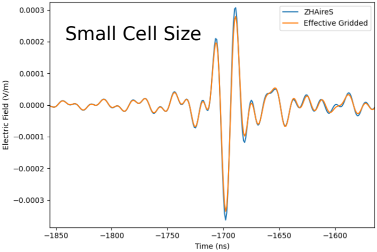


Importance of correct cell sizing

- Bandpass filtered pulse 30-80 MHz, thinning 10^{-3} and $\theta = 70^{\circ}$

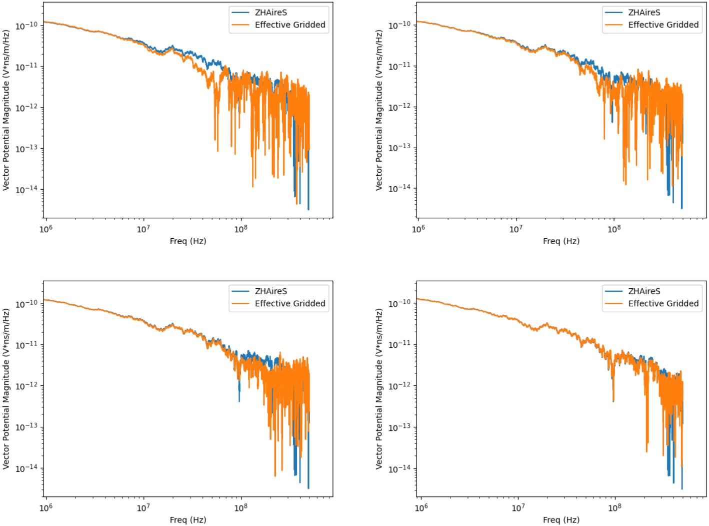






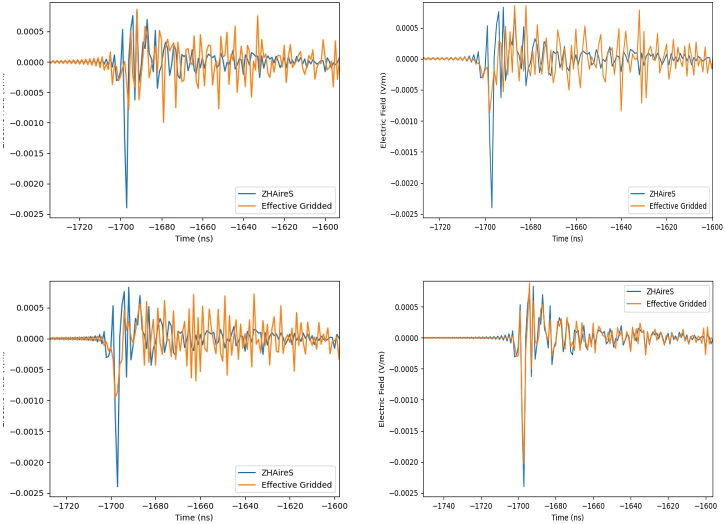
Importance of correct cell sizing

- Spectra, thinning 10^{-3} and $\theta = 70^{\circ}$



Importance of correct cell sizing

- Full bandwidth pulse, thinning 10^{-3} and $\theta = 70^\circ$ (cell size depends on λ)



Advantages

- Only a single calculation per 4-D volume
- Theoretically much faster than track-by-track calculations
- Almost a macroscopic approach, but retaining the precision of the successful microscopic formalism.
  - Maybe parametrizations of average tracks could be devised
- More detailed calculations that are too expensive on a track-by-track basis can be performed
  - These can be calculated in advance for each cell
  - Atmospheric effects: e.g. curved propagation
    - Important effect specially for upgoing horizontal showers
    - Very relevant for satellite and balloon experiments
  - Antenna patterns could also be taken into account in the simulation

Questions?

Other applications of Radio...

BACKUP

Math

$\vec{A}_i(\vec{x},t) = \frac{\mu_0}{4\pi}\iiint_V d^3\vec{x}\sum_i \frac{\vec{J}_{i\perp}(\vec{x}_i',t_i')}{|\vec{x}-\vec{x}_i'|}\left|\frac{dt'}{dt}\right|_{t'=t_i'}$ In the case of a single track, we can use:
$\vec{J}(\vec{x}',t') = \rho(\vec{x}',t')\,\vec{v}$, where $\rho(\vec{x}',t') = q\delta(\vec{x}'-\vec{v}t')\,\Pi(t',t_1',t_2')$

The δ function models the track as an infinitely thin linear charge density with particle velocity $\vec{v}$. We then obtain:

$$\vec{A}(\vec{x},t) = \frac{\mu_0 q}{4\pi R}\vec{v}_\perp\left|\frac{dt'}{dt}\right|_{t'=t'_{ret}}\Pi(t',t_1',t_2')$$

If, instead of a single track, we have a charged current density vector $\vec{J}_i$ at a location $\vec{x}_i$ that is approximately constant over a volume ΔV_i:

$$\vec{A}_i(\vec{x},t) = \frac{\mu_0}{4\pi}\Delta V_i\frac{\vec{J}_{i\perp}(\vec{x}_i',t'_{ret})}{|\vec{x}-\vec{x}_i'|}\left|\frac{dt'}{dt}\right|_{t'=t'_{ret}}$$

In this case, our $\vec{J}_\perp$ is no longer a single track, but a collection of tracks within a cell. $\Delta V_i\vec{J}_{i\perp} = Q_i\vec{v}_{i\perp}$, where Q_i is charge within the cell:

$$\vec{A}_i(\vec{x},t) = \frac{\mu_0}{4\pi}\frac{Q_i\vec{v}_{i\perp}(\vec{x}_i',t'_{ret})}{|\vec{x}-\vec{x}_i'|}\left|\frac{dt'}{dt}\right|_{t'=t'_{ret}}$$