Measuring the skew-sextupolar component of a crab-cavity from turn-by-turn observations

Michele Carlà, Lee Robert Carver, **Androula Alekou**, Hannes Bartosik
With the help of the SPS OP group

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From "LHC Crab Cavities Impedance and Multipole Update" J. A. Mitchell:

Multipole Components

J. Mitchell

- Impedance
- Power
- Multipoles
- Conclusions
- Appendix

- Last meetings: Questions about b_4 magnitude.
- Re-visited: Issues with CST field export and convergence

  Panofsky Wenzel method did not converge. Lorentz Force does.
- Solved. Benchmarked with K. Papke's code.

0.15 T/m @ 1MV

		b1	b2	b3	b4
SPS DQW (Dressed)					
LF	Re	33	6	1498	1026
	Im	0	-2	19	-383
LHC DQW (Dressed)					
LF	Re	33	6	1488	1048
	Im	0	-2	21	-292
LHC RFD (Dressed)					
LF	Re	34	0	-458	128
	Im	0	0	-74	55

Table 4: Evolution of b_n in units of $\mathrm{mT/m^{n-1}}$. Values correspond to a transverse deflecting voltage of 10 MV and are evaluated with 64 points around the azimuth at a radius of 30 mm.

- TDR: Limit of b_4 was 1000 units.
- TDR: Limits pending for higher components.

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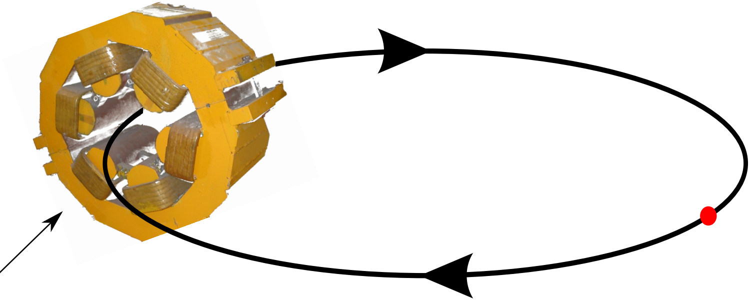
- b3 (sextupole) is the strongest multipole
- In the SPS crab cavities are installed rotated by 90°: b3 $\rightarrow$ a3 (skew sextupole)

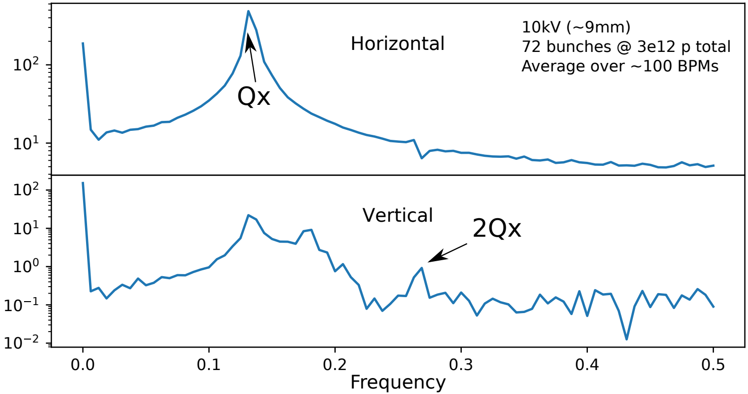


$$F_x \propto x \cdot y$$
$$F_y \propto x^2 + y^2$$

By exciting the horizontal betatron motion two vertical spectral lines are observed:

- V_{20}: spectral line with **frequency** $2Q_x$, V_{00}: **Static offset** of the orbit
- Both lines have **amplitude proportional to** a_3
- In SPS the strong vertical decoherence due to impedance does not allow to observe cleanly modes driven by the vertical betatron motion (H_{11}).

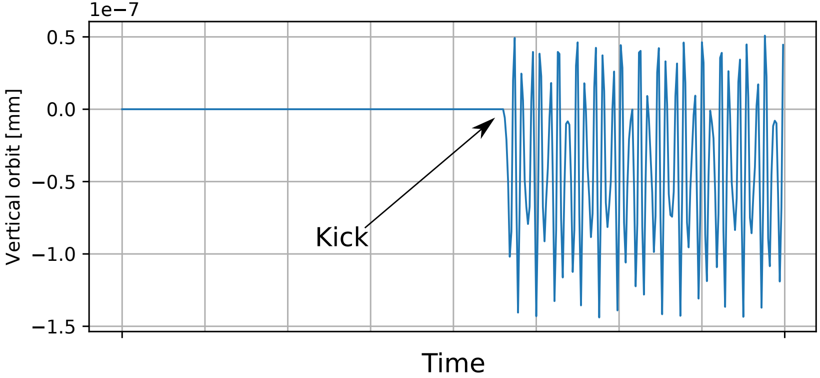
Spectral analysis of V_{20}



For each acquisition:

1. **Q_x**: average over each horizontal BPM (à la Laskar)
2. **H_{10} amplitude, phase and damping** (**damping**: average over each horizontal BPM)
3. **Undamp** the **vertical** signal
4. Evaluate **amplitude and phase** of **V_{20}** for each vertical BPM

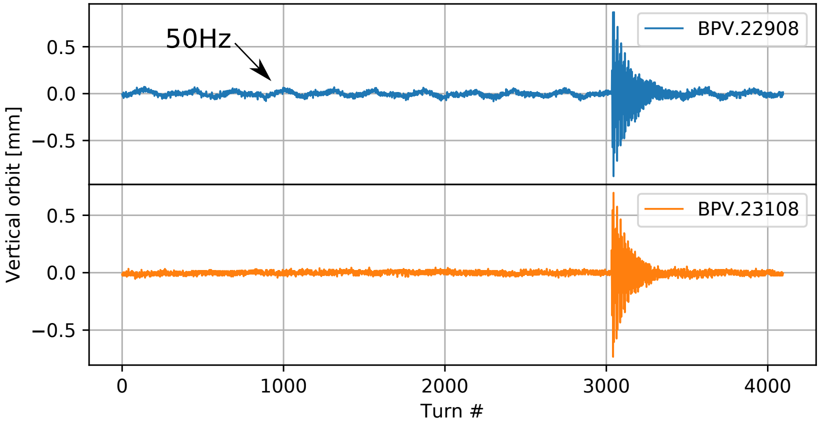
Analysis of V_{00}



For each acquisition:

1. **Orbit** is obtained from the average of ∼**1000 turns before the kick**
2. $\mathbf{V_{00}}$ is the difference of the orbit and the average of ∼**100 turns after the kick**

Typical BPM signal... no averaging, no filtering



- **Amplitude and phase of 50Hz** is evaluated using **3000 turns** before the kick
- **50Hz is purged** from the signal

Measurements/Experimental results

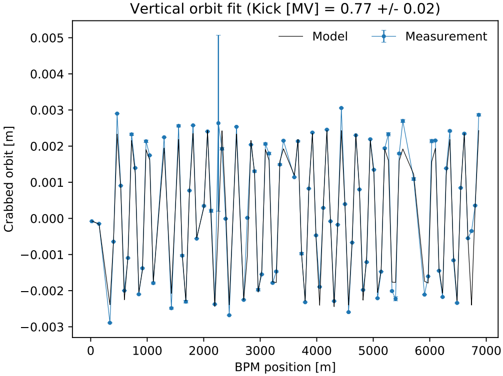
- **20/10/2017: positive test** with a **static skew-sextupole**
  - No skew sextupoles is present in SPS, a 5 mm **vertical bump in an octupole** (LOE.33002) was used to produce a **feed-down**.
  - The measurement was repeated for an **octupole strength of K3 = $\pm$2, $\pm$5** and a **vertical bump of $\pm$5mm**
  - **Q20 optics** was used.

- **10/10/2018:** measurement with the **crab cavity**
  - 2 acquisitions for a crab cavity **voltage of: 0.1 and $\pm$1 MV**
  - **Q26 optics** was used.

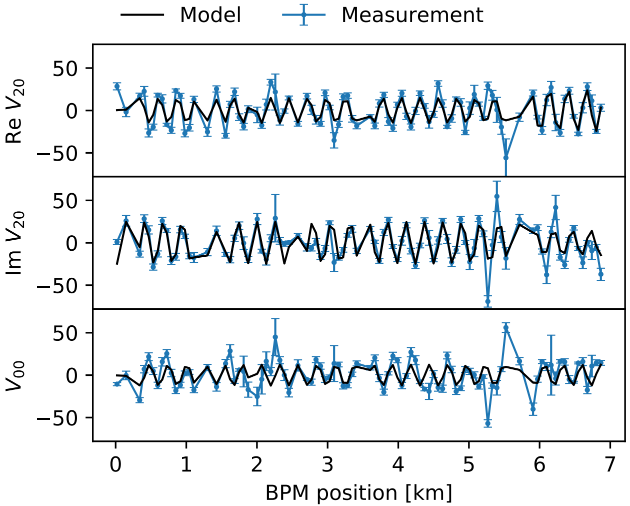
  - **An a3 several times above the expected value was observed!**

Part of the disagreement comes from the BPM frequency response

- SPS BPMs have a narrow pass-band filtere centered around 200 MHz
- → BPM does not measure the *'center of mass'* of the bunch
- **It measures the** 200 MHz **component of the 'center of mass'**



To work around the problem the crab-cavity voltage is determined from the vertical orbit and the measured a_3 is normalized by this value.



Two independent fits: $V_{20} \to a_3 = 0.99\,\mathrm{T/m}$, ψ: $-76°$,
$V_{00} \to a_3 = 0.90\,\mathrm{T/m}$ (expected a_3: $0.15\,\mathrm{T/m}$)

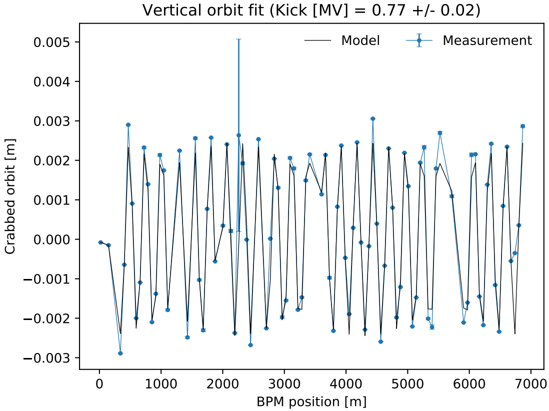
Are there other sources of V_{00} and V_{20}?

- **Vertical orbit + octupoles** $\Rightarrow$ **feed-down** to skew-sextupole!

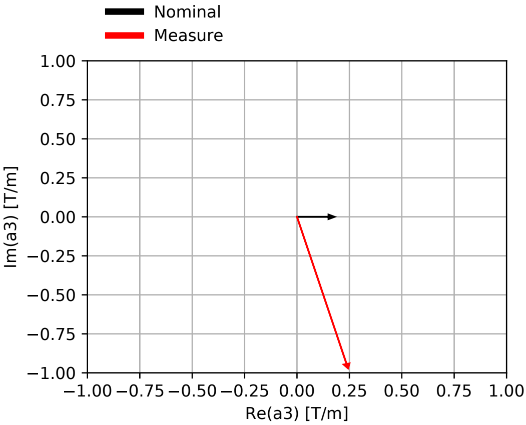
  Octupoles were off during the measurement ...residual field?

- **Vertical orbit + normal sextupoles** $\Rightarrow$ **second order** excite V_{00}/V_{20}

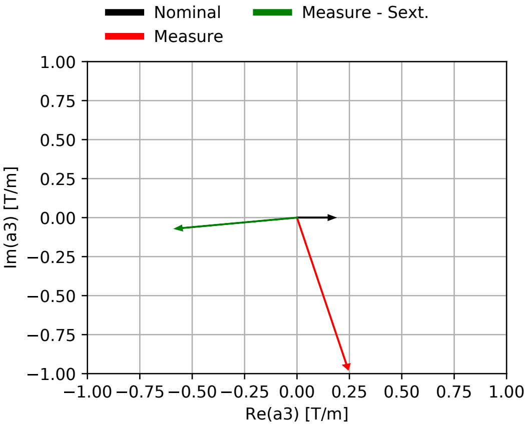
  Is a second order effect but there are plenty of sextupoles



Real and Imaginary part of a_3 from V_{20} (and simulation)

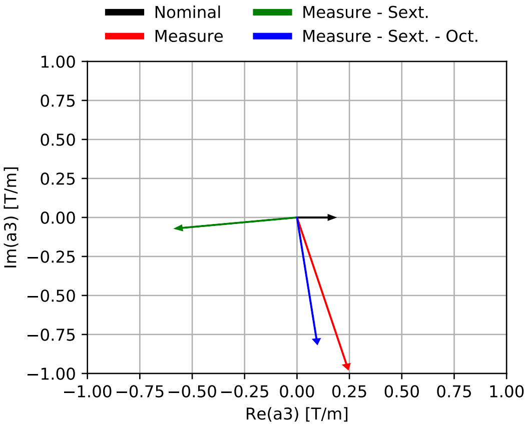


Now the **sextupoles** contribution is **removed**:



- Second order effect from the **sextupoles** is ...**huge**

also **octupoles** contribution is **removed too**:

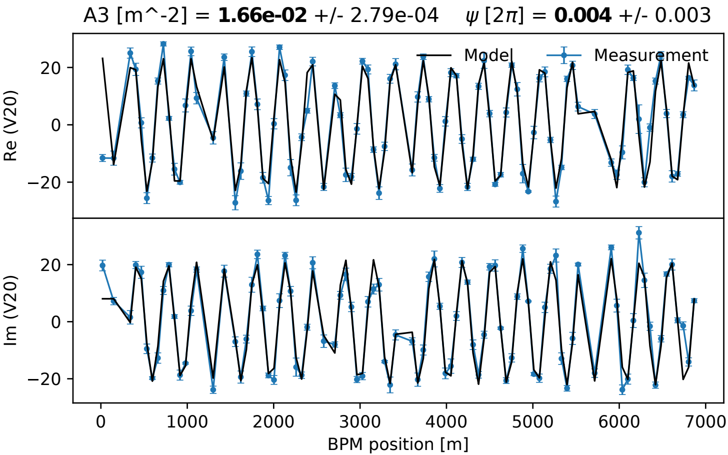


- Second order effect from the **sextupoles** is ...**huge**
- Also **Octupoles** have a **strong impact**

Summary

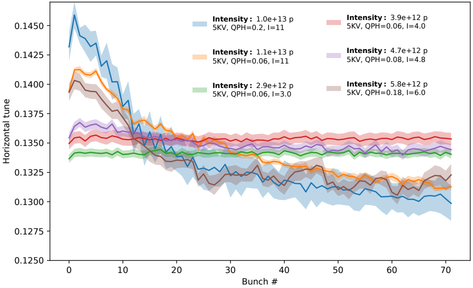
- Measurements of V_{00} and V_{20} with a static skew sextupole (vertical bump in an octupole) shows agreement with theory
- **Crab-cavity** measurement instead **is far from expectations**
- It was found that **sextupoles and octupoles**, due to the large **vertical orbit** induced by the crab-cavity, play an important **role in the analysis**.
- Including sextupoles and octupoles in the analysis requires a **good understanding** of the SPS **non-linear model**
- While the **sextupole** model is "quite" **robust** the octupole one is questionable...
- **Work to understand the octupole model is still undergoing!**

Skew sextupole strength from V_{20} (Octupole)

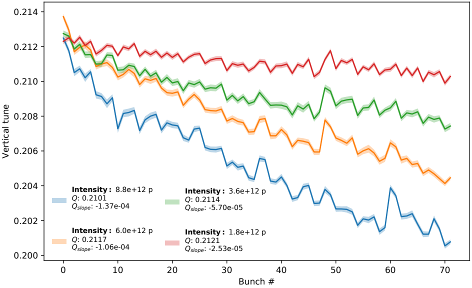


SPS multibunch detuning (what lucky coincidence!)

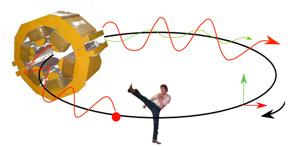
Horizontal plane



Vertical plane

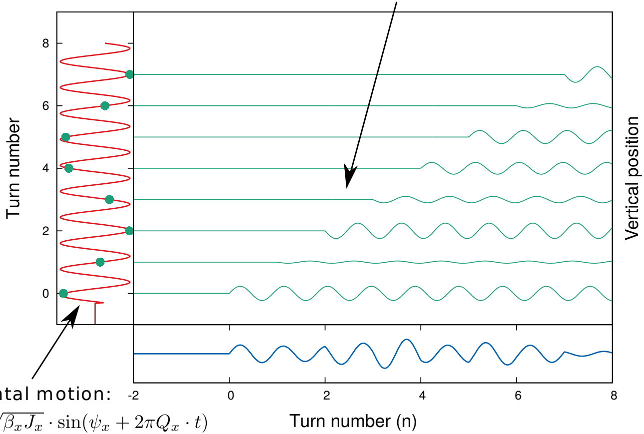


- 72 bunches in the ring
- **Horizontal** plane is **ok** up to $\sim 4 \cdot 10^{12}$
- **Vertical** plane exhibits a strong **tuneshift**
- **No excitation on the vertical plane allowed!** ...but we don't need it



Induced vertical motion at turn 'n' induced by kick 't':

$$A_3 \cdot x^2(t) \cdot \theta(s_{\mathrm{bpm}} - s_{\mathrm{skew}} + (n-t)C) \cdot \sqrt{\beta_y^{\mathrm{bpm}} \beta_y^{\mathrm{skew}}} \sin(\Delta\psi_y + 2\pi(n-t)Q_y)$$



Horizontal motion:

$$x(t) = \sqrt{\beta_x J_x} \cdot \sin(\psi_x + 2\pi Q_x \cdot t)$$

$$y(n) = \sum_{t=0}^{n} A_3 \sqrt{\beta_y^{\mathrm{bpm}} \beta_y^{\mathrm{skew}}} \sin(\Delta\psi_y + 2\pi(n-t)Q_y) \times \left[\sqrt{\beta_x^{\mathrm{skew}} J_x} \sin(\psi_x^{\mathrm{skew}} + 2\pi t Q_x)\right]^2$$

$$y(n) = \sum_{t=0}^{n} A_3 \sqrt{\beta_y^{\text{bpm}} \beta_y^{\text{skew}}} \sin(\Delta\psi_y + (n-t)\nu_y) \times \left[\sqrt{\beta_x^{\text{skew}} J_x} \sin(\psi_x^{\text{skew}} + t\nu_x) \right]^2$$

$$\Downarrow \qquad \sum_{t=0}^{n} e^{it\nu} = \frac{1 - e^{i(n+1)\nu}}{1 - e^{i\nu}}$$

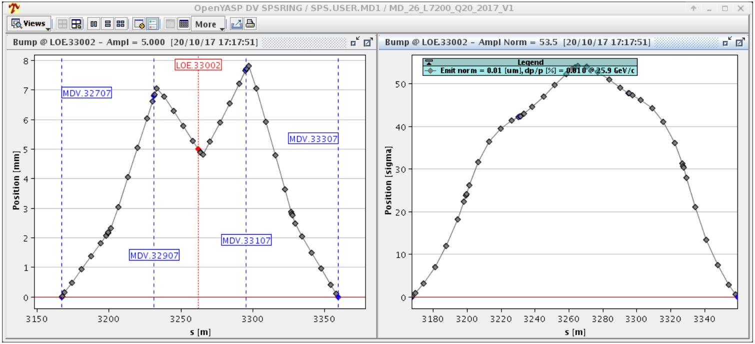
$$y(n) = \mathbf{V_{20}} + V_{02} + V_{01} + V_{00}$$

$$V_{20} = A_3 J_x \frac{\beta_x^{\text{p}} \sqrt{\beta_y^{\text{o}} \beta_y^{\text{p}}}}{8i} \cdot \left[\frac{e^{i(2\nu_x + \nu_y - \Delta\psi_y + 2\psi_x^{\text{p}})}}{e^{i(2\nu_x+\nu_y)} - 1} - \frac{e^{i(2\nu_x - \nu_y + \Delta\psi_y + 2\psi_x^{\text{p}})}}{e^{i(2\nu_x - \nu_y)} - 1} \right] e^{2i\nu_x n} - \text{c.c.}$$

The skew-sextupole **drives** an oscillation with frequency $\mathbf{2}Q_x$ on the **vertical plane** $\propto A_3 \cdot J_x$

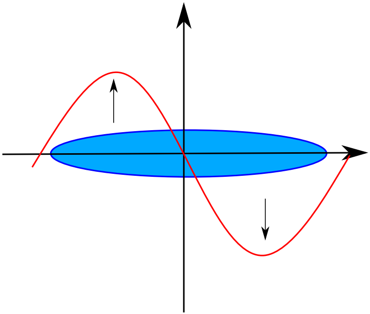
No skew-sextupoles in SPS: Octupole + vertical bump

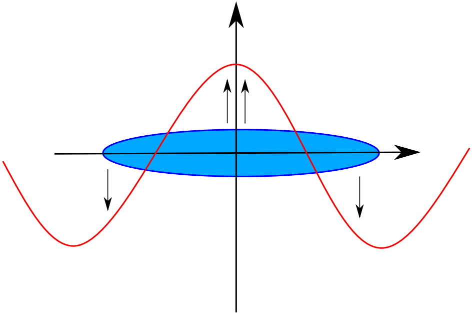
- LOE.33002 was used
- $\pm$5mm vertical bump
- $K_3 = \pm\ 5\ \&\ \pm 2\ \mathrm{m}^{-4}$ ($K_3 = \pm 2$ produces an A_3 very close to the C.C. one $\simeq 0.013\mathrm{m}^{-3}$)



Time dependent A_3 + longitudinal beam emittance

- Standard operation: head and tail of the bunch see opposite $A_3 \rightarrow$ average to 0
- Running the crab-cavity on-crest $\rightarrow A_3$ does not average to zero





- Bunch length(4σ): 3ns
- Energy spread(1σ): 1.5‰

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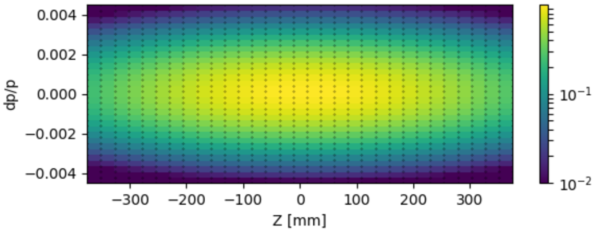

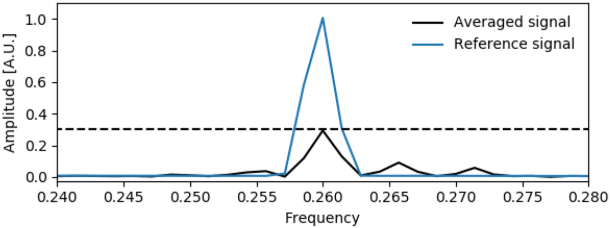

Bunch length(4σ): 3ns Energy spread(1σ): 1.5‰ C.C.: 680kV