Experimental fingerprints of shape coexistence

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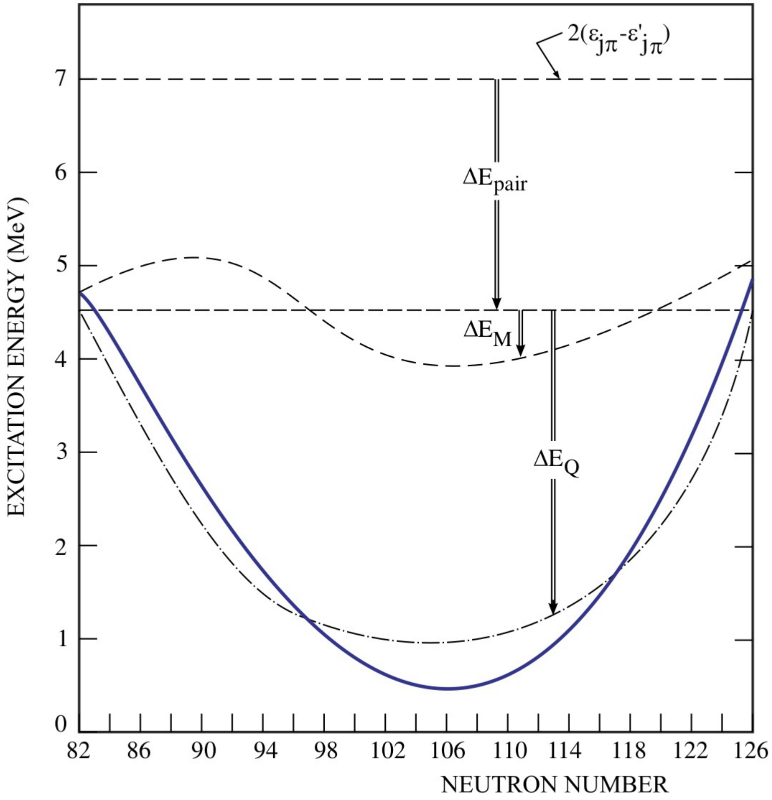
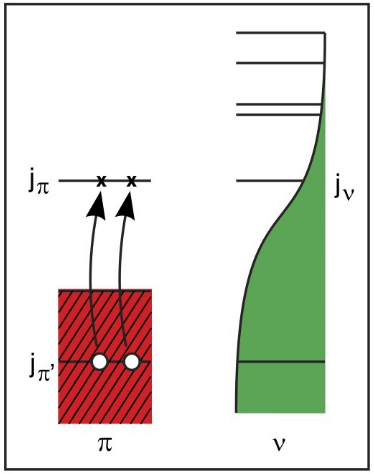
What observables can be used to conclude on shape coexistence?

- level energies
- transition probabilities
- transfer-reaction cross sections
- quadrupole moments: measure of the charge distribution in a given state
- charge radii
- complete sets of E2 matrix elements:
  possibility to determine quadrupole invariants and level mixing
- monopole transition strengths

Level energies

- can be used to conclude on shape coexistence if other data not availble (e.g. for very exotic nuclei)
- have to be put in some context – neighbouring isotopes, other states





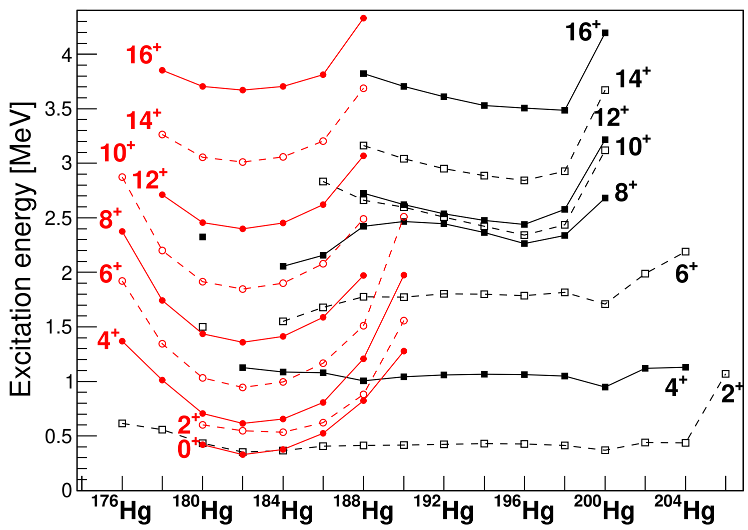
K. Heyde and J. Wood, Rev. Mod. Phys. 83, 1467 (2011)

- gain from correlation offsetting the shell gap increases towards mid shell
- characteristic parabolic behaviour of intruder states energies

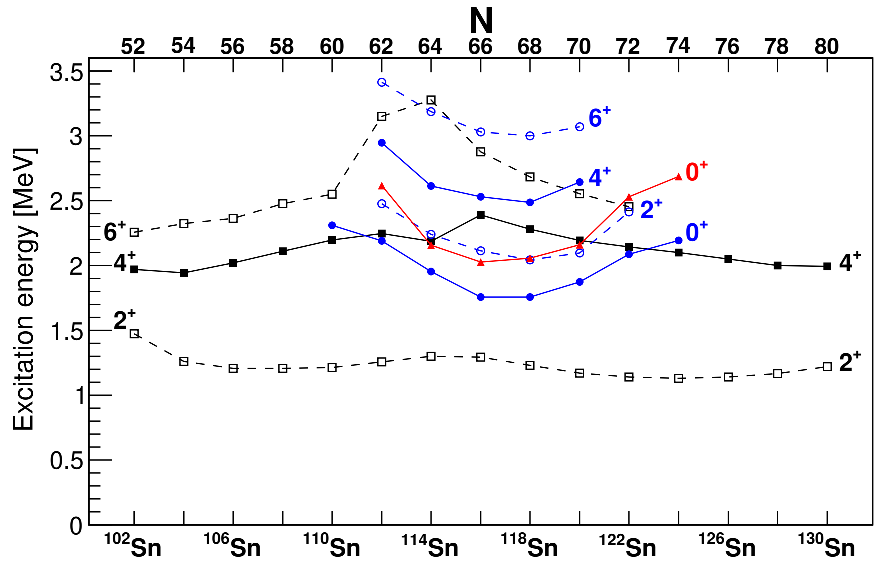
Level energies – systematics of isotopic chains

- parabolic behaviour experimentally observed for nuclei with A$>$100, less evident in lighter nuclei

Hg isotopes, Z=80



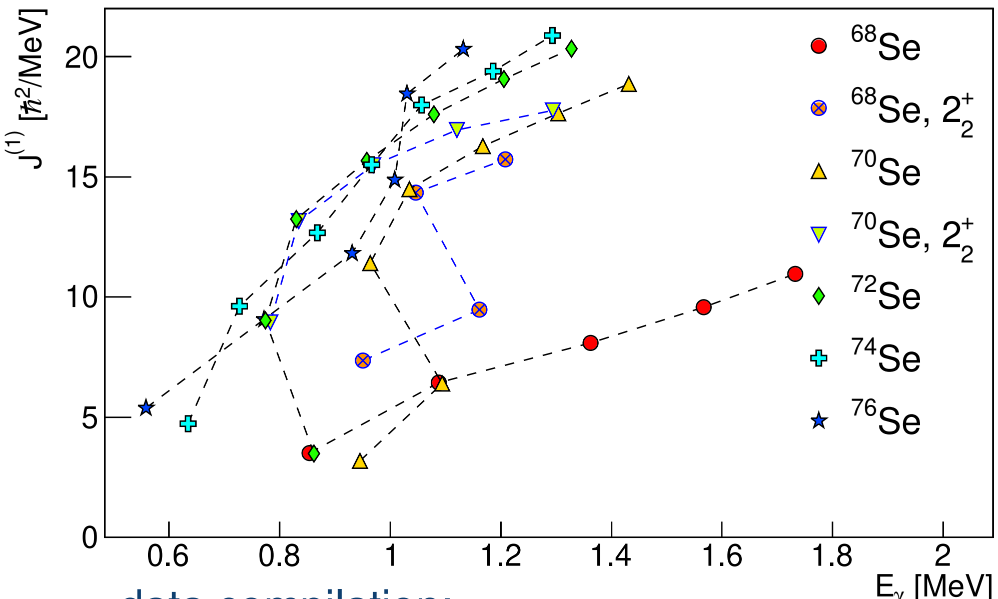
Sn isotopes, Z=50



data compilation: P. Garrett, MZ, E. Clément, Prog. Part. Nucl. Phys. 124, 103931 (2022)

Level energies – moments of inertia

- ^{72,74,76}Se: presence of bands built on low-lying 0^+ states
- ^{76}Se: different transition strengths in the gsb and the band built on the 0_2^+ state: $B(E2; 2_3^+ \rightarrow 0_2^+)$ = 31(5) W.u. versus $B(E2; 2_1^+ \rightarrow 0_1^+)$) = 44(1) W.u.; (S. Mukhopadhyay *et al.*, PRC 99, 014313 (2019))
- ^{72,76}Se: negative quadrupole moments of 2_1^+ states (J. Henderson *et al.*, PRL 121, 082502 (2018); A.E. Kavka, NPA 593, 177 (1995))



data compilation:

P. Garrett, MZ, E. Clément,

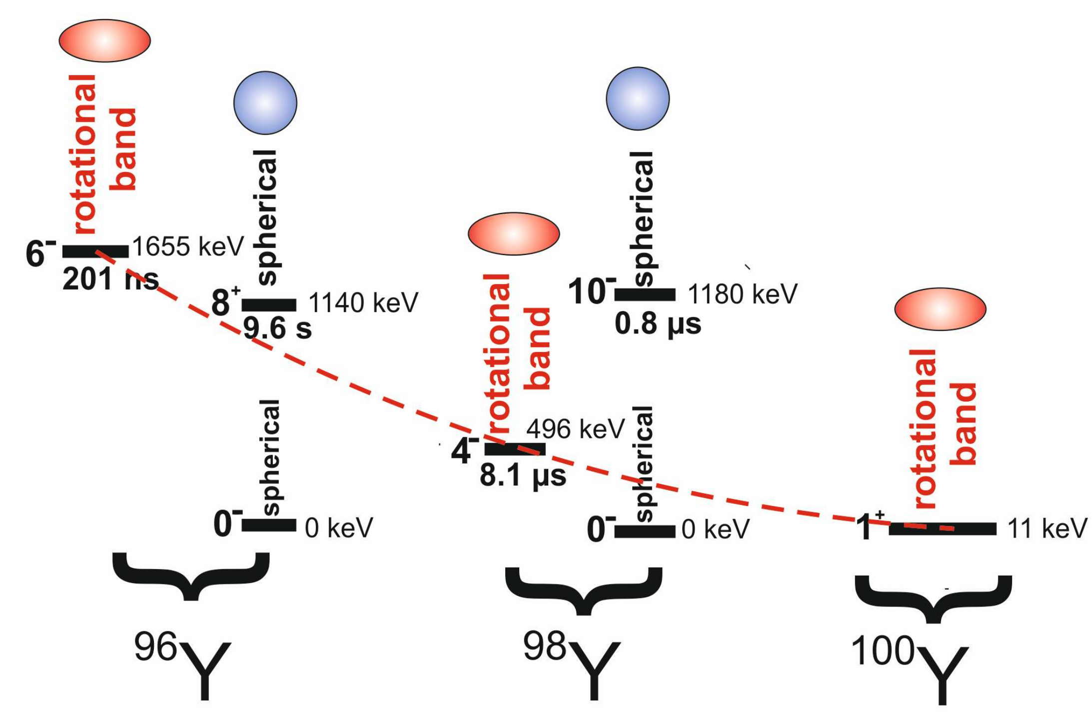
Prog. Part. Nucl. Phys. 124, 103931 (2022)

- ^{68,70}Se: no excited 0^+ states known, but in particular for ^{68}Se very different moment of inertia in the ground state band (S.M. Fischer *et al.*, PRL 84, 4064 (2000))

$\rightarrow$ conclusion on shape coexistence in ^{68,70}Se and different shapes of their ground states with respect to heavier Se

Level energies – rotational bands in less deformed nuclei

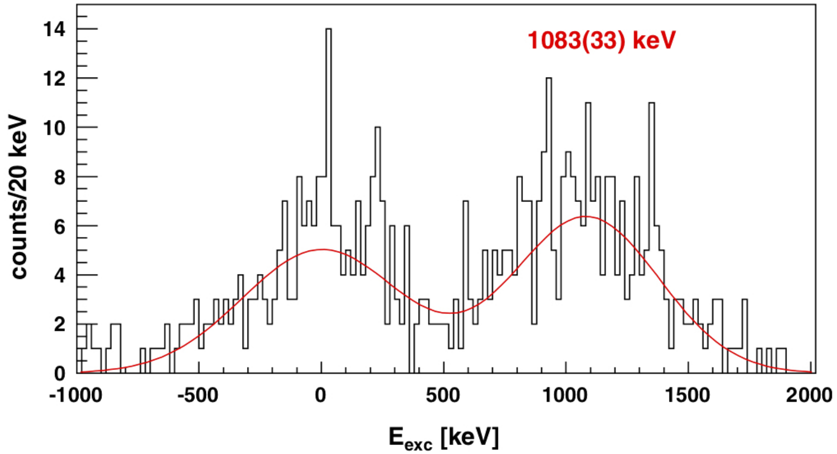
• it is much easier to find deformed configurations in nuclei with nearly spherical ground states, than vice versa!



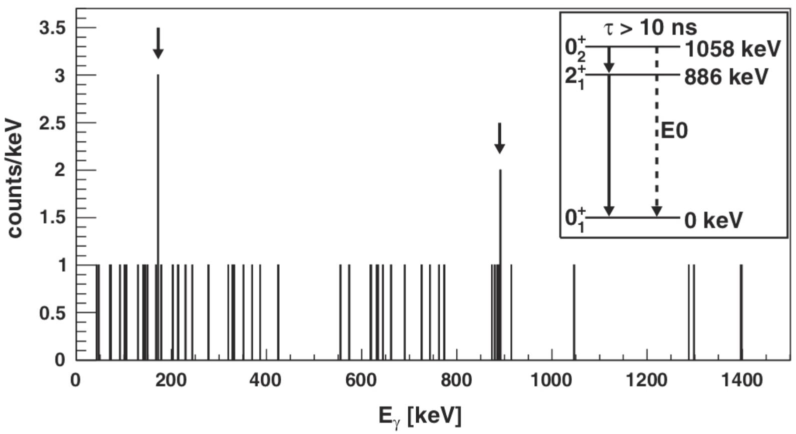
L. Iskra *et al.*, EPL 117, 12001 (2017)

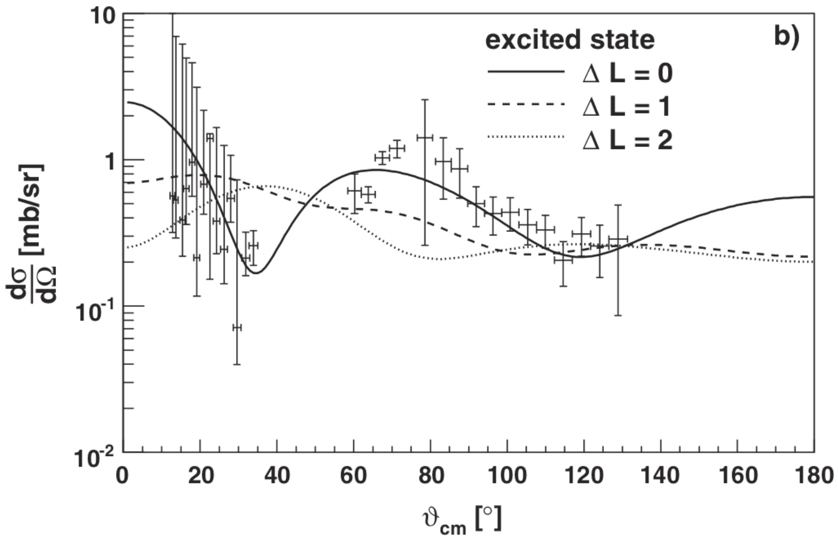
Finding spherical states in deformed nuclei – example of ^{32}Mg

K. Wimmer *et al.*, PRL 105, 252501 (2010)



- $2n$ transfer from spherical normal-order configuration in ^{30}Mg g.s. populates preferentially the spherical normal-order excited state in ^{32}Mg



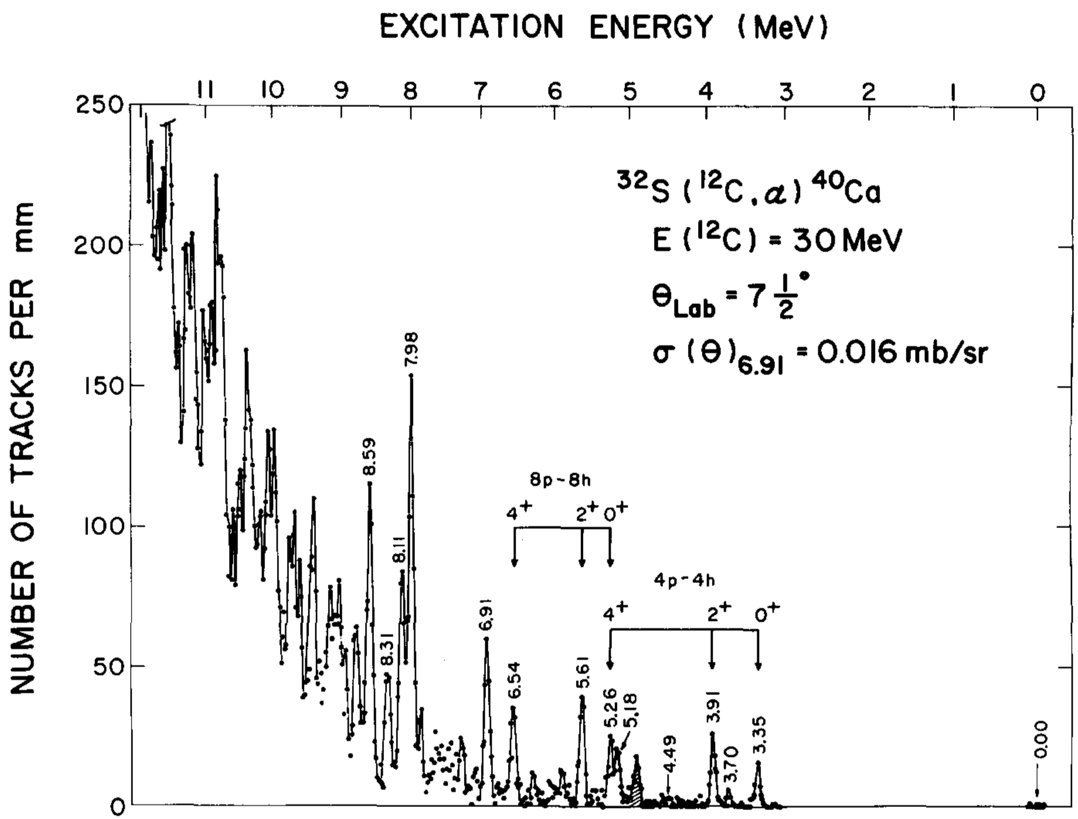


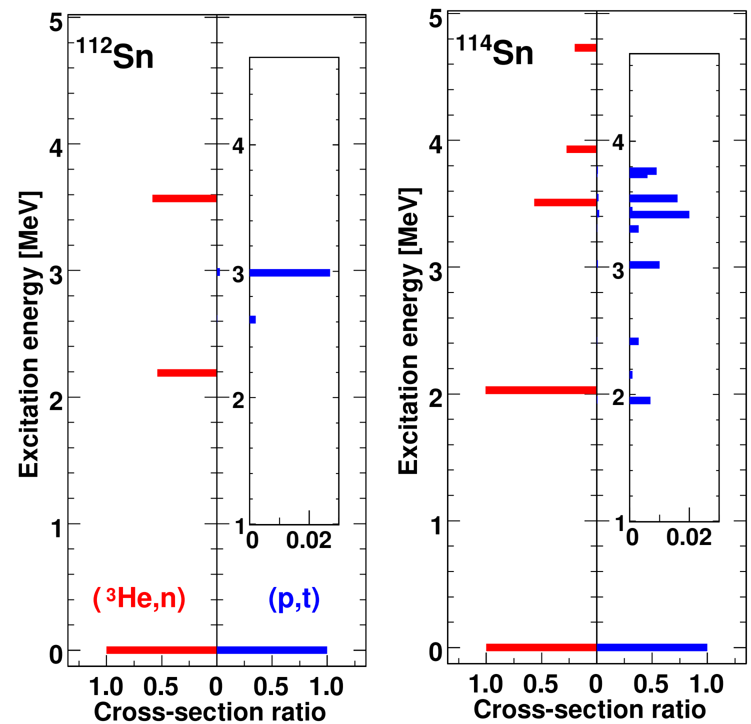
- level spin confirmed by proton angular distributions
- excitation energy precisely measured from γ-ray decay in coincidence with protons

Information from transfer reactions

- identfication of 4p-4h and 8p-8h structures in ^{40}Ca (α-particle transfer); admixture of the 4p-4h configuration to 8p-8h states

- proton domination in the wave functions of the excited 0^+ states in ^{112,114,116,118}Sn



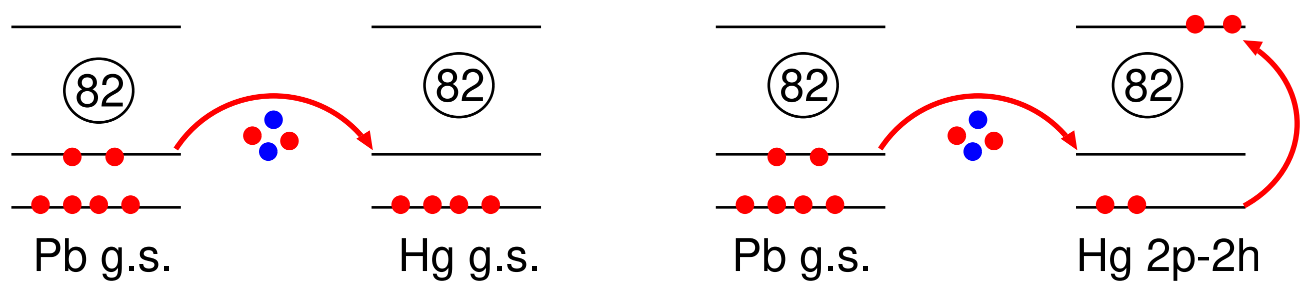


R. Middleton *et al.*, Phys. Lett. 39B, 339 (1972)

P. Guazzoni *et al.*, PRC 85, 054609 (2012), PRC 69, 024619 (2004)

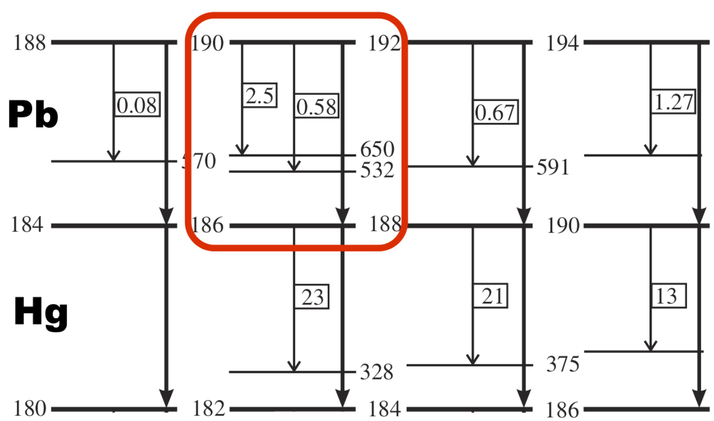
Alpha-decay hindrance

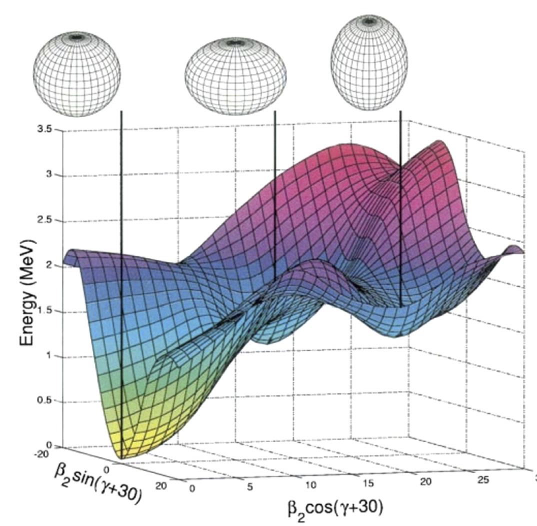
- similar type of information as from transfer reactions in lighter nuclei



- ground states of Pb nuclei are spherical (experimentally confirmed by charge radii measurements) $\rightarrow$ the same is true for ground states of Hg nuclei, while their excited states are dominated by the 2p-2h configuration

- triple shape coexistence in ^{186}Pb was deduced using this method





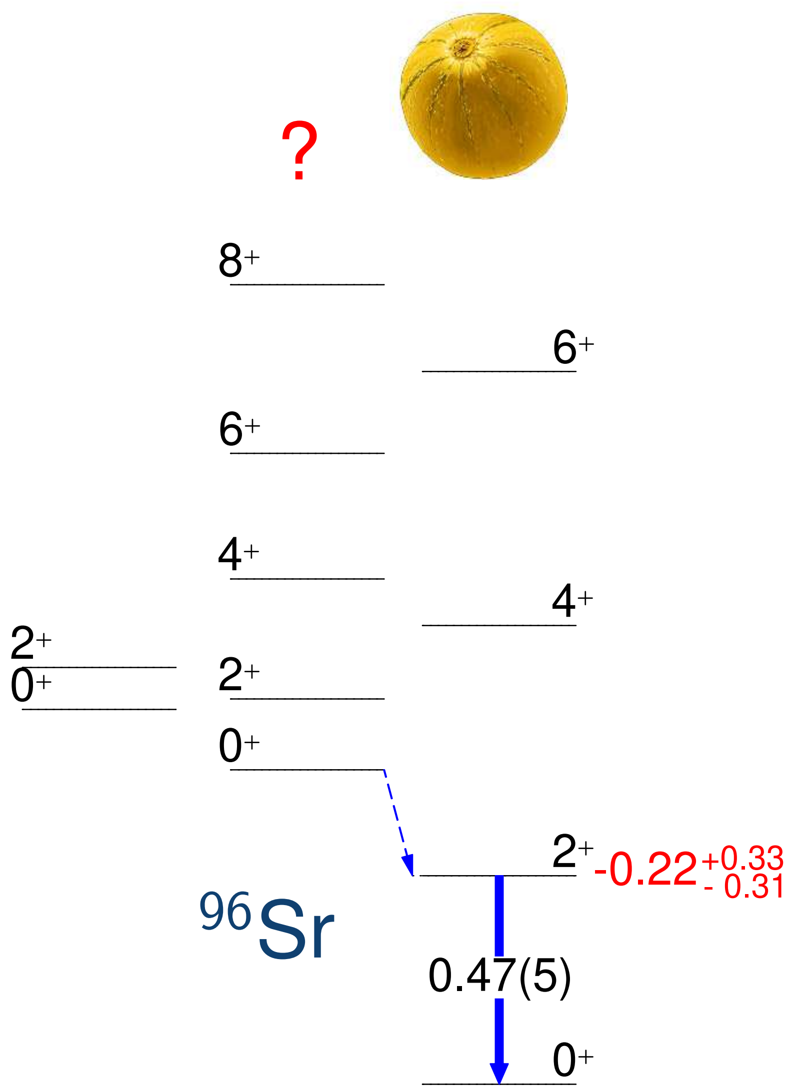
compilation: K. Heyde and J.L. Wood, RMP 83, 1467 (2011)

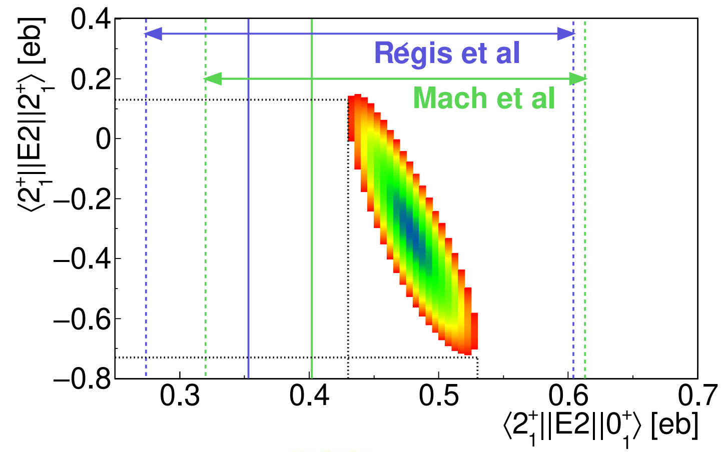
A. Andreyev *et al.*, Nature 405, 431 (2000)

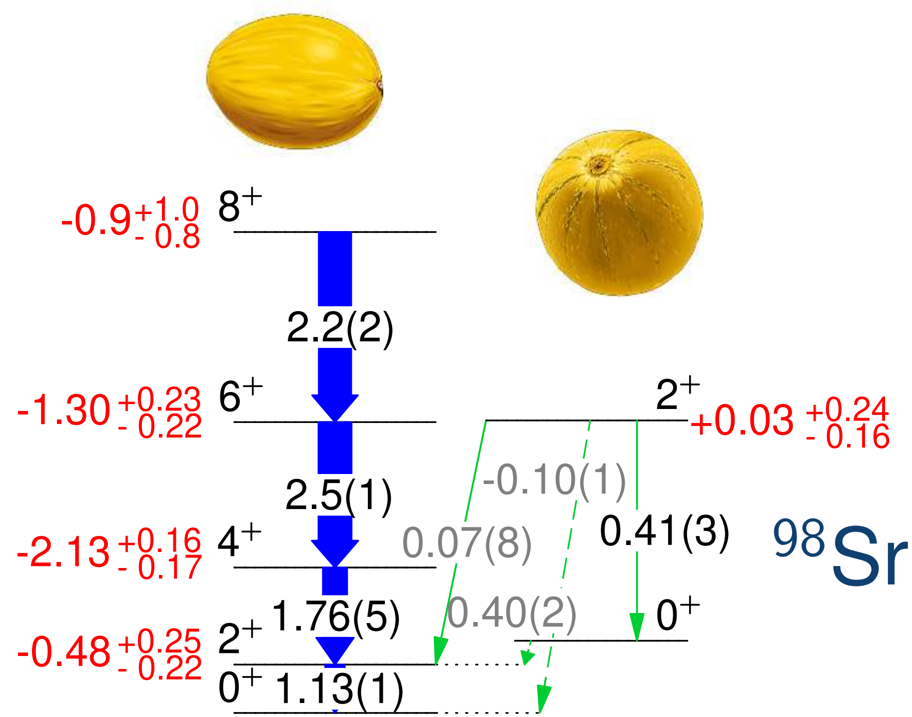
Transition probabilities in ^{96,98}Sr

E. Clément, MZ et al, PRL 116, 022701 (2016)

Coulomb excitation at REX-ISOLDE: ^{96}Sr on ^{109}Ag, ^{120}Sn, ^{98}Sr on ^{60}Ni, ^{208}Pb



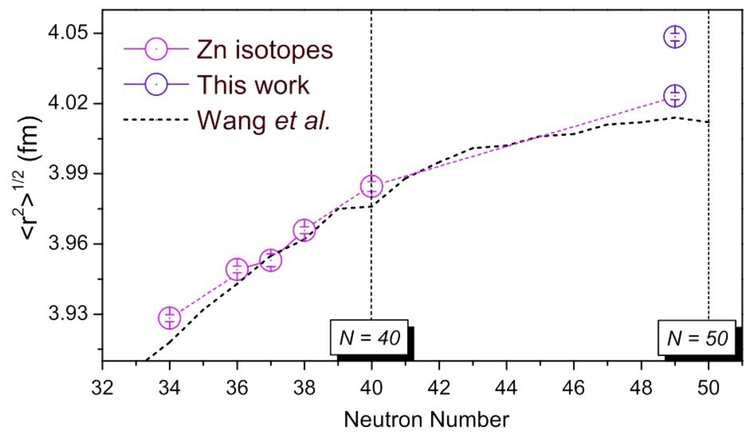




Laser spectroscopy data

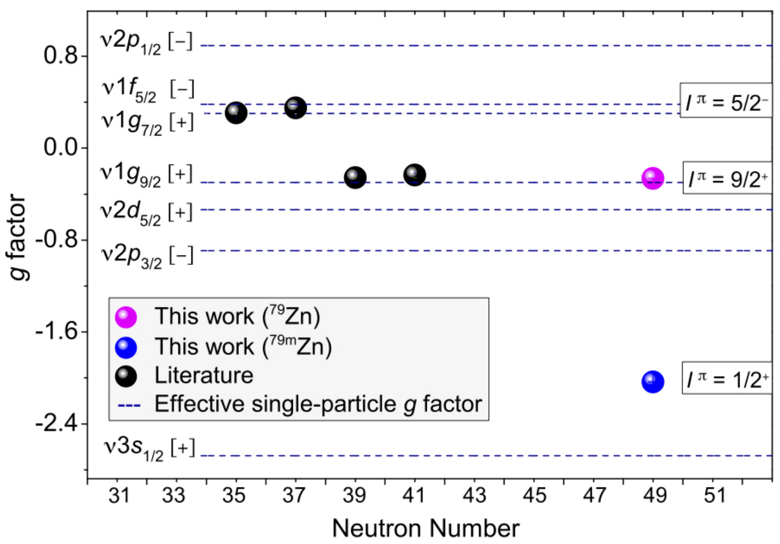
- precise measurements of charge radii, spectroscopic quadrupole moments, g factors for long-lived states

Example of ^{79}Zn:



- large isomer shift for the $1/2^+$, 1-MeV isomer in ^{79}Zn

- combined with $\beta_2 \approx 0.14$ deduced from B(E2) values in ^{78,80}Zn, results in $\beta_2 \approx 0.22$ for the isomer



- 1p-2h neutron configuration determined from the measured g factor

- first evidence for shape coexistence in the immediate vicinity of ^{78}Ni

X.F. Yang *et al.*, PRL 116, 182502 (2016)

Quadrupole moments of excited states

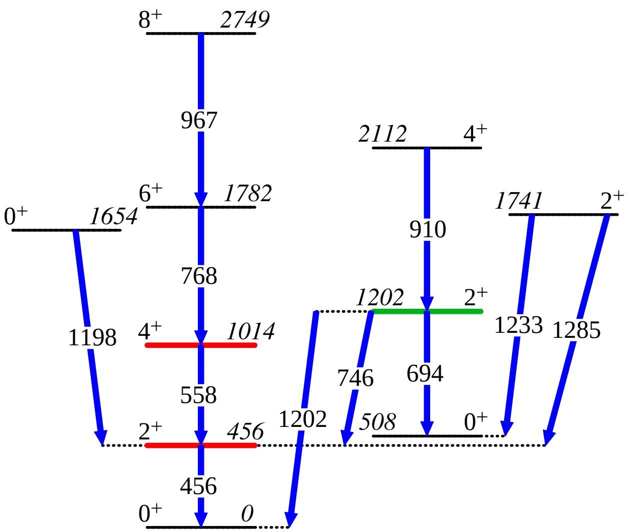
E. Clément *et al.*
Phys. Rev. C75, 054313 (2007)

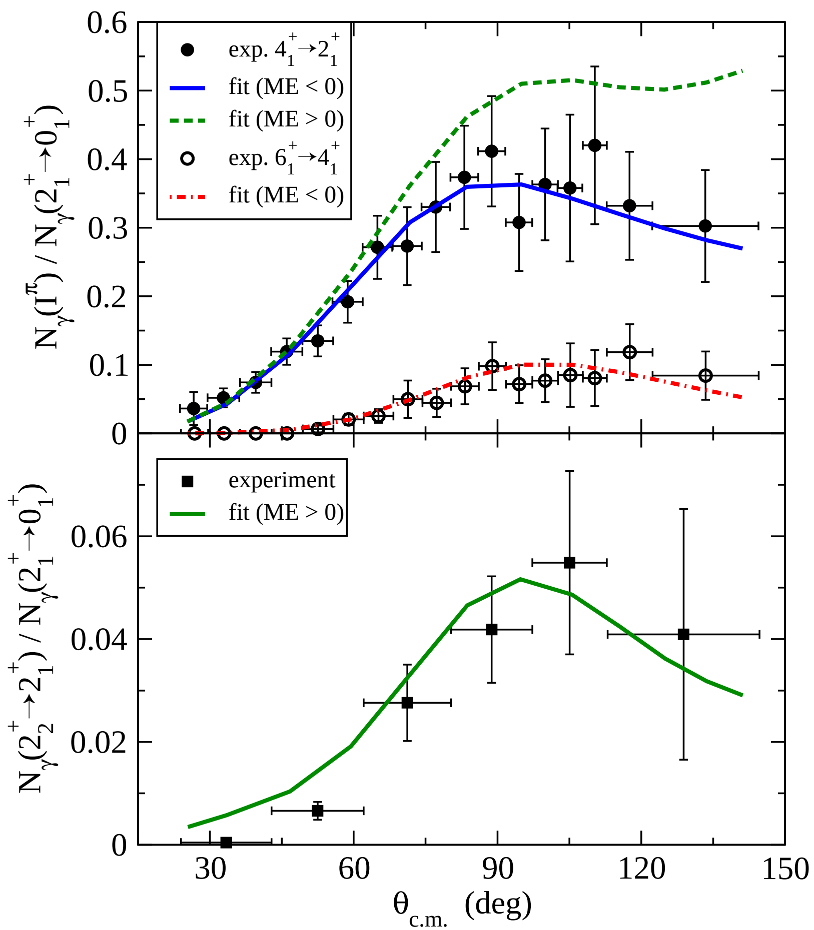
- prolate-oblate shape coexistence in ^{74,76}Kr

- first Coulomb-excitation measurement of spectroscopic quadrupole moments using a radioactive beam

$$\langle 2_1^+ \| E2 \| 2_1^+ \rangle = -\, 0.70^{-0.33}_{-0.30}$$
$$\langle 4_1^+ \| E2 \| 4_1^+ \rangle = -\, 1.02^{+0.59}_{-0.21}$$

$$\langle 2_2^+ \| E2 \| 2_2^+ \rangle = +\, 0.33^{+0.28}_{-0.23}$$





- spectroscopic quadrupole moments are zero for J=0,1/2 – complication for even-even nuclei

Quadrupole sum rules

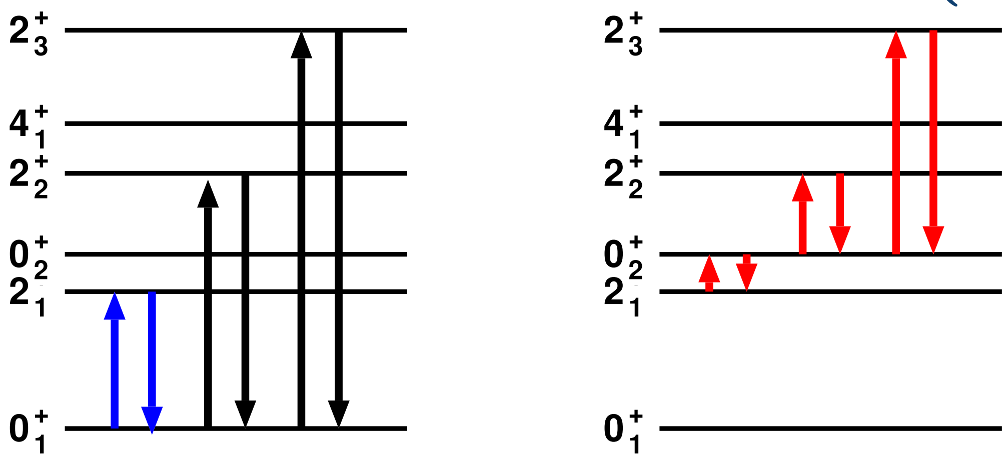
D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683
K. Kumar, PRL 28 (1972) 249

- electromagnetic multipole operators are spherical tensors – products of such operators coupled to angular momentum 0 are rotationally invariant

- in the intrinsic frame of the nucleus, the E2 operator may be expressed using two parameters Q and δ related to charge distribution:

$$\begin{aligned} E(2,0) &= Q\cos\delta \\ E(2,2) = E(2,-2) &= \frac{Q}{\sqrt{2}}\sin\delta \\ E(2,1) = E(2,-1) &= 0 \end{aligned}$$

$$\frac{\langle Q^2\rangle}{\sqrt{5}} = \langle i|\,[E2\times E2]^0\,|i\rangle = \frac{1}{\sqrt{(2I_i+1)}}\sum_t \langle i||E2||t\rangle\langle t||E2||i\rangle \left\{\begin{array}{ccc} 2 & 2 & 0 \\ I_i & I_i & I_t \end{array}\right\}$$



$\langle Q^2\rangle$: measure of the overall deformation;
for the ground state – extension of B(E2; $0^+ \rightarrow 2^+$) = $((3/4\pi)eZR_0^2)^2\ \beta_2{}^2$

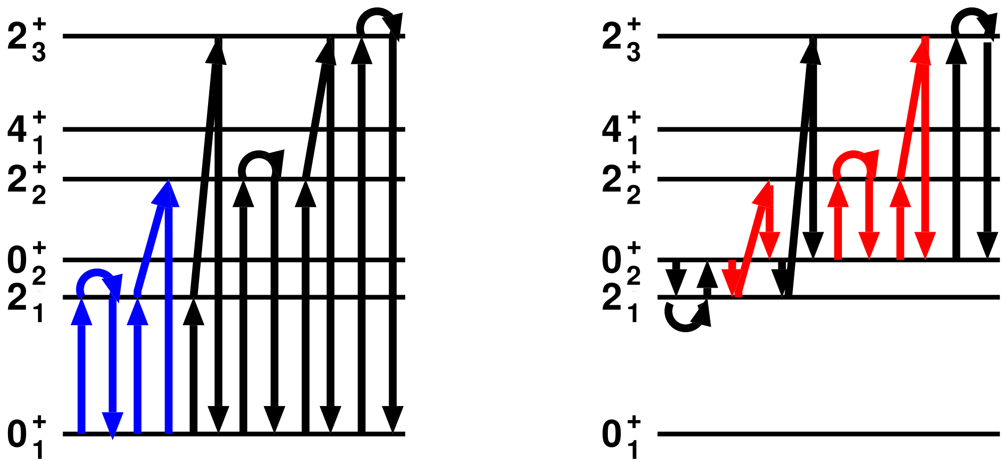
Contributions to $\langle Q^2\rangle$ in ^{100}Mo: K. Wrzosek-Lipska *et al.*, PRC 86 (2012) 064305

Quadrupole sum rules: triaxiality

D. Cline, Ann. Rev. Nucl. Part. Sci. 36 (1986) 683

K. Kumar, PRL 28 (1972) 249

$$\sqrt{\frac{2}{35}}\langle Q^3 \cos 3\delta\rangle = \langle i|\{[E2\times E2]^2\times E2\}^0|i\rangle$$
$$= \frac{1}{(2I_i+1)}\sum_{t,u}\langle i\|E2\|u\rangle\langle u\|E2\|t\rangle\langle t\|E2\|i\rangle \left\{\begin{array}{ccc} 2 & 2 & 2 \\ I_i & I_t & I_u \end{array}\right\}$$



$\langle \cos 3\delta\rangle$: measure of triaxiality

- relative signs of E2 matrix elements are needed: can we get them experimentally?

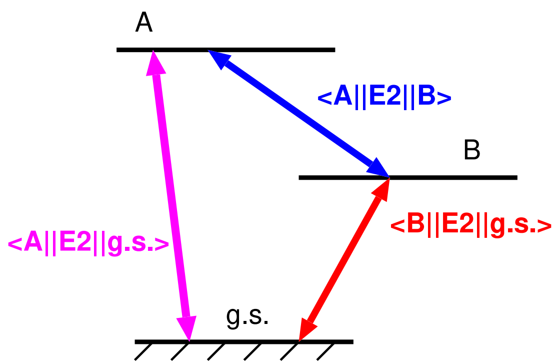
Contributions to $\langle Q^3 \cos 3\delta\rangle$ in ^{100}Mo: K. Wrzosek-Lipska *et al.*, PRC 86 (2012) 064305

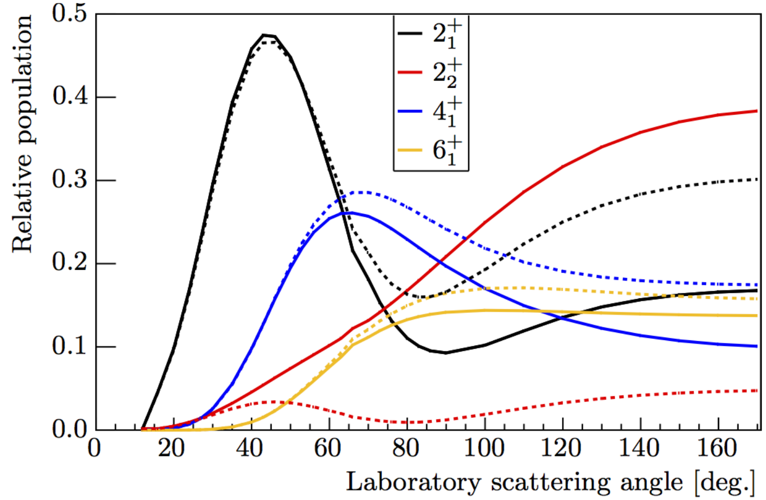
Relative signs of E2 matrix elements

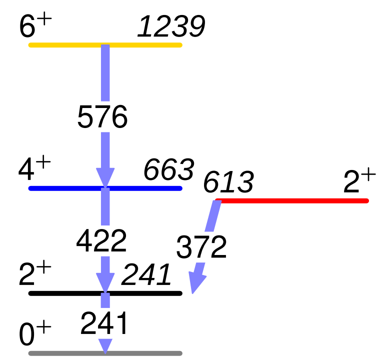
- Coulomb-excitation cross section are sensitive to relative signs of MEs: result of interference between single-step and multi-step amplitudes

- excitation amplitude of state A: $a_A \sim \langle A||E2||g.s.\rangle + \langle B||E2||g.s.\rangle\langle A||E2||B\rangle$

- excitation probability ($\sim a_A^2$) contains interference terms $\langle A||E2||g.s.\rangle\langle B||E2||g.s.\rangle\langle A||E2||B\rangle$







^{110}Ru

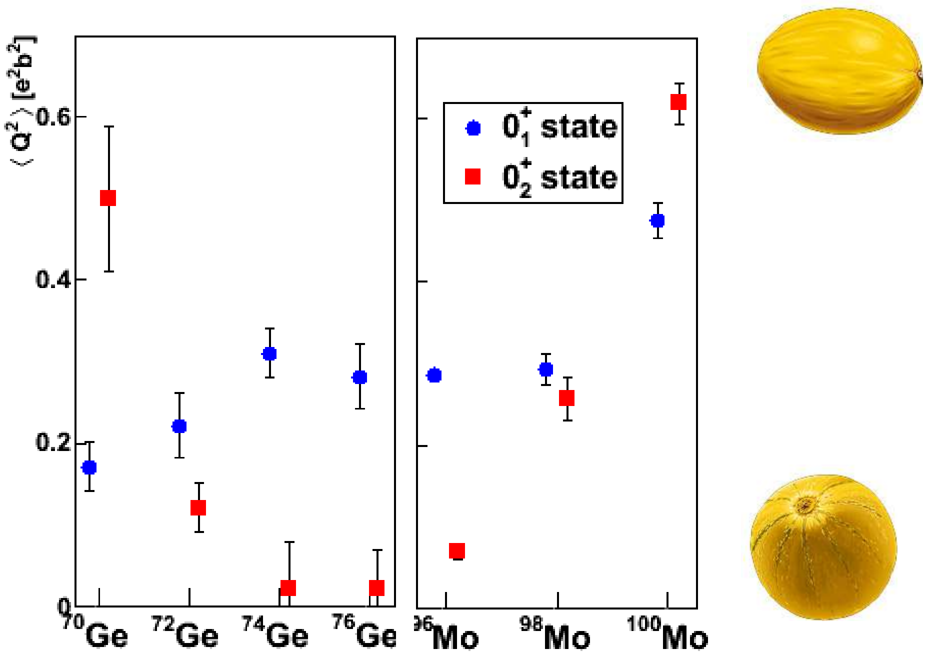
- negative $\langle 2_1^+||E2||2_2^+\rangle$ (solid lines): much higher population of 2_2^+ at high CM angles

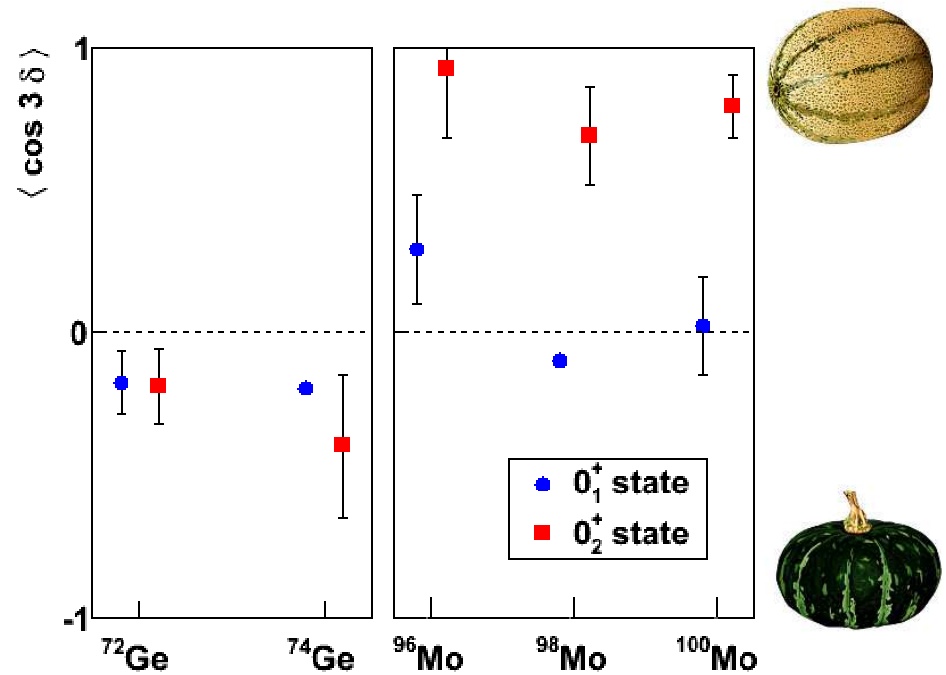
- sign of a product of matrix elements is an observable

Shape evolution of $^{96-100}$Mo

MZ *et al.*, Nucl. Phys. A 712 (2002) 3

K. Wrzosek-Lipska *et al.*, PRC 86 (2012) 064305

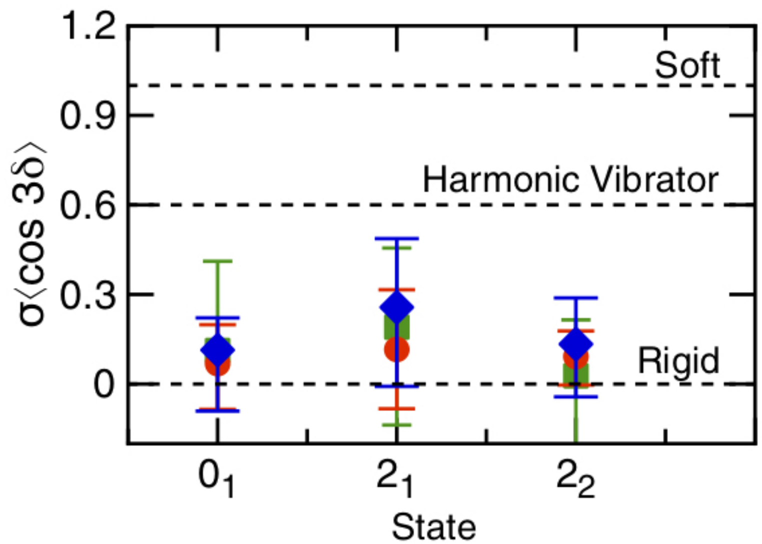


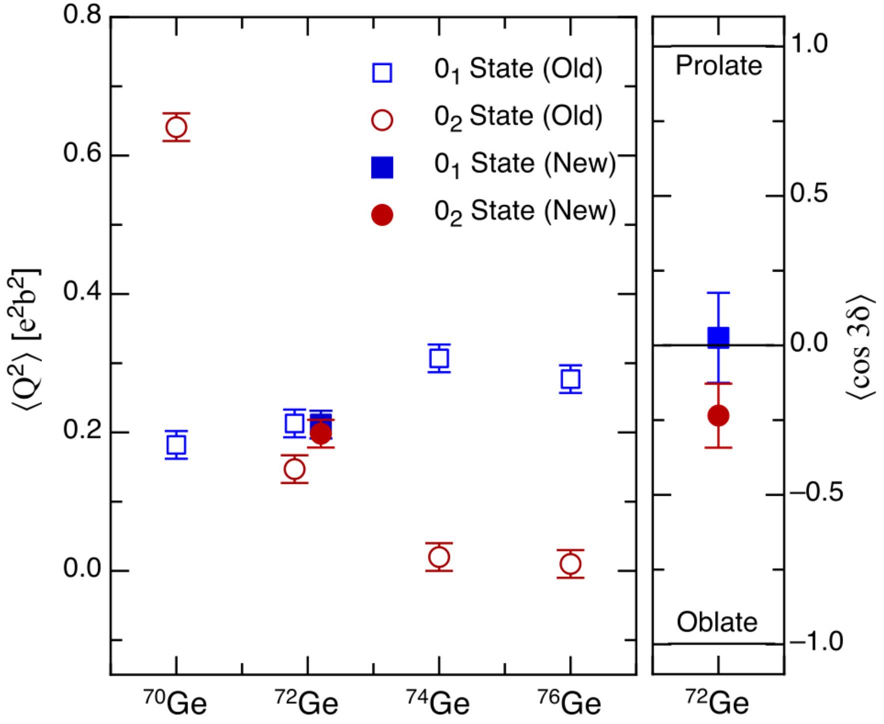


- ^{72,74,76}Ge, ^{96}Mo: coexistence of the deformed ground state with a spherical 0_2^+
- ground states of the Mo isotopes triaxial, deformation of 0_2^+ increasing with N
- shape coexistence in ^{98}Mo manifested in a different triaxiality of 0_1^+ and 0_2^+

Quadrupole invariants – example of ^{72,76}Ge

A.D. Ayangeakaa *et al.*,
PRL 123, 102501 (2019)
PLB 754, 254 (2016)





- ^{76}Ge: unique example of determination of softness in γ from experimental data

- ^{72}Ge: much higher number of transitions observed in a new measurement $\rightarrow$ slight change of the deduced invariants due to extra states entering the sum

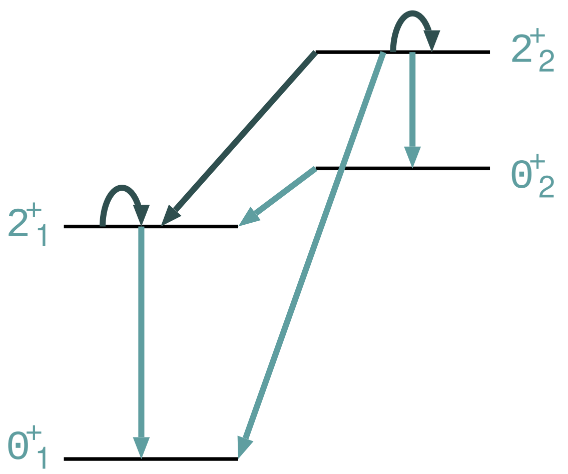
- observed shapes of $0^+_{1,2}$ states in ^{72}Ge are very similar in terms of β and γ – can it still be called shape coexistence?

Two-state mixing model

• we assume that physical states are linear combinations of pure spherical and deformed configurations:

$| I_1^+ \rangle = +\cos\theta_I \times | I_d^+ \rangle + \sin\theta_I \times | I_s^+ \rangle$

$| I_2^+ \rangle = -\sin\theta_I \times | I_d^+ \rangle + \cos\theta_I \times | I_s^+ \rangle$



with transitions between the pure spherical and deformed states forbidden:

$\langle 2_d^+ \| E2 \| 0_s^+ \rangle = \langle 2_d^+ \| E2 \| 2_s^+ \rangle = \langle 2_s^+ \| E2 \| 0_d^+ \rangle = 0$

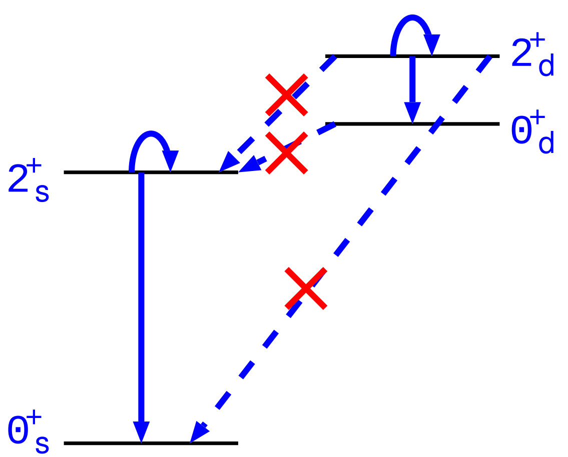
• the measured matrix elements can be expressed in terms of the “pure” matrix elements and the mixing angles:

$\langle 2_1^+ \| E2 \| 0_1^+ \rangle =$
$\sin\theta_0 \sin\theta_2 \ \langle 2_s^+ \| E2 \| 0_s^+ \rangle + \cos\theta_0 \cos\theta_2 \ \langle 2_d^+ \| E2 \| 0_d^+ \rangle$

$\langle 2_1^+ \| E2 \| 0_2^+ \rangle =$
$\cos\theta_0 \sin\theta_2 \ \langle 2_s^+ \| E2 \| 0_s^+ \rangle - \sin\theta_0 \cos\theta_2 \ \langle 2_d^+ \| E2 \| 0_d^+ \rangle$

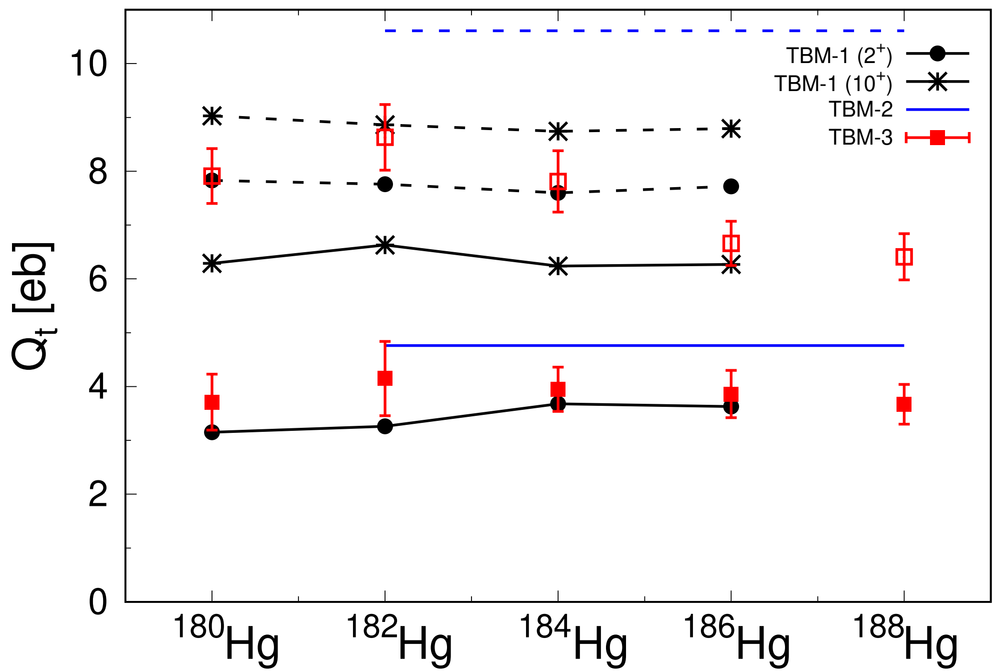
$\langle 2_2^+ \| E2 \| 0_1^+ \rangle =$
$\sin\theta_0 \cos\theta_2 \ \langle 2_s^+ \| E2 \| 0_s^+ \rangle - \cos\theta_0 \sin\theta_2 \ \langle 2_d^+ \| E2 \| 0_d^+ \rangle$

$\langle 2_2^+ \| E2 \| 0_2^+ \rangle =$
$\cos\theta_0 \cos\theta_2 \ \langle 2_s^+ \| E2 \| 0_s^+ \rangle + \sin\theta_0 \sin\theta_2 \ \langle 2_d^+ \| E2 \| 0_d^+ \rangle$



Dependence on additional assumptions

• two-state mixing parameters for ^{180,182,184,186,188}Hg derived under three different assumptions:

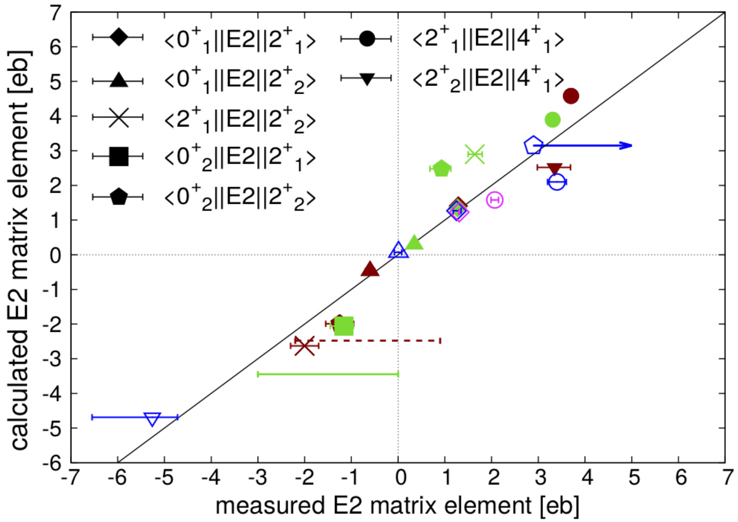


M. Siciliano *et al.*, PRC 102, 014318 (2020)

A) Q_t values the same for all four Hg isotopes and constant within bands

B) Q_t evolve within bands according to moments of inertia

C) Q_t calculated independently for each mass and spin



K. Wrzosek-Lipska *et al.*, EPJA 55, 130 (2019) (variant A)

• large difference in resulting Q_t values; Q_t for the less deformed configuration in variant B approaches values for the more deformed one in variant C

E0 strengths, shape coexistence and mixing

- E0 transitions are sensitive to the changes in the nuclear charge-squared radii

- their strengths depends on the mixing of configurations that have different mean-square charge radii:

$$\rho^2(E0) = \frac{Z^2}{R^4}\cos^2\theta_0\ \sin^2\theta_0\ \left(\langle r^2\rangle_A - \langle r^2\rangle_B\right)^2$$

$$= \left(\frac{3Z}{4\pi}\right)^2 \cos^2(\theta_0)\sin^2(\theta_0)\cdot\left[\left(\beta_1^2-\beta_2^2\right)+\frac{5\sqrt{5}}{21\sqrt{\pi}}\left(\beta_1^3\cos\gamma_1-\beta_2^3\cos\gamma_2\right)\right]^2$$

J.L. Wood *et al.*, NPA 651, 323 (1999)

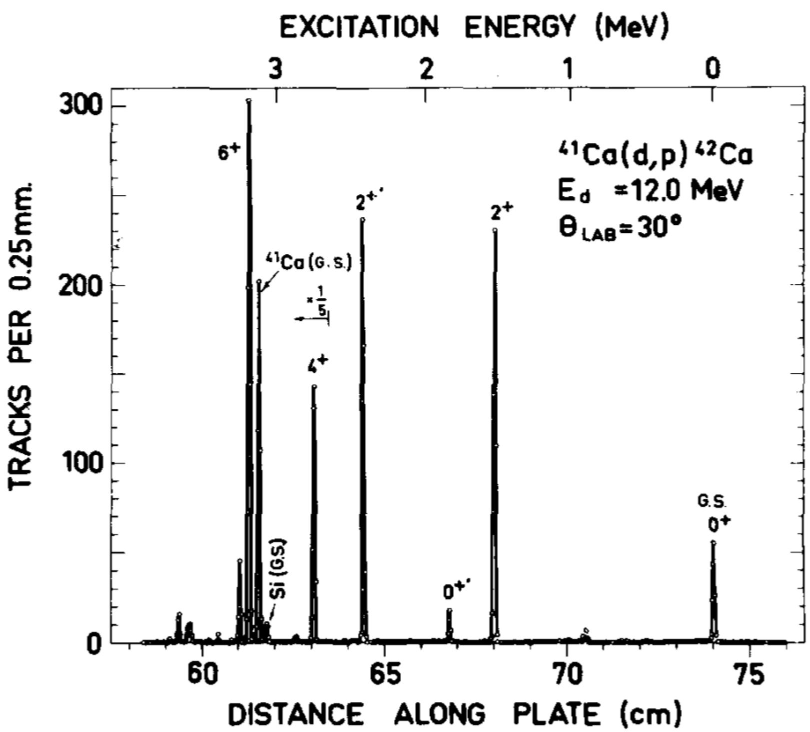
Example of ^{42}Ca: K. Hadyńska-Klęk *et al.*, PRC 97 (2018) 024326 (Coulomb excitation), J.L. Wood *et al.*, NPA 651, 323 (1999) (E0)

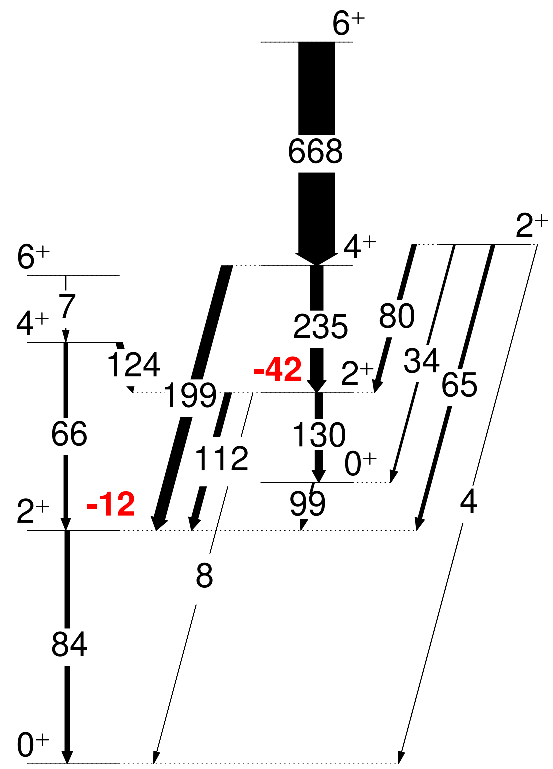
	from E2 matrix elements [KHK]	from $\rho^2(E0)$ [JLW] + sum rules results [KHK]
$\cos^2(\theta_0)$	0.88(4)	0.84(4)
$\cos^2(\theta_2)$	0.39(8)	-

- good agreement of the $\cos^2(\theta_0)$ values obtained with the two methods

- $\cos^2(\theta_2)<0.5$: two-state mixing model cannot be applied to 2^+ states in ^{42}Ca

Population of the deformed structure in one-neutron transfer





C. Ellegaard *et al.*, Phys. Lett. 40B (1972) 641

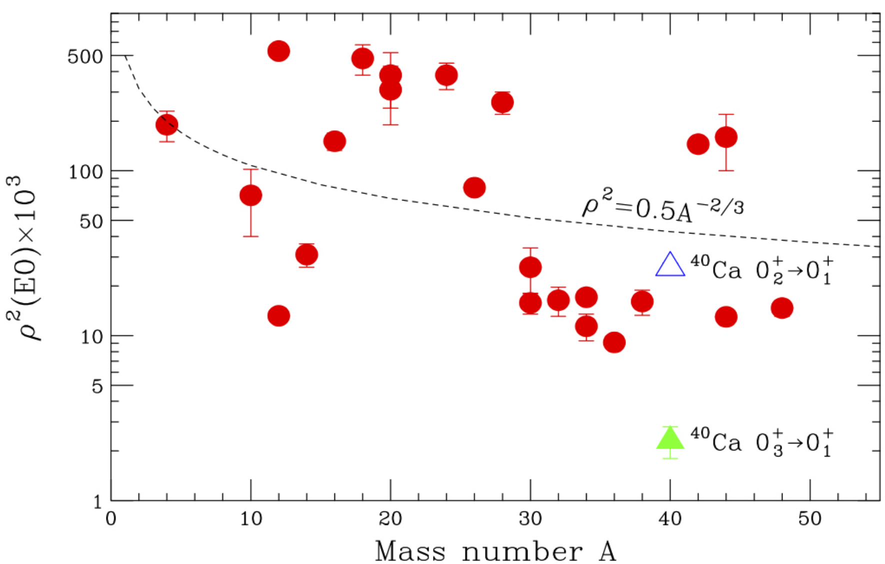
- equal population of 2_1^+ and 2_2^+ in ^{41}Ca(d,p)^{42}Ca – the same admixture of $(f_{7/2})^2$, while the quadrupole moments are very different!

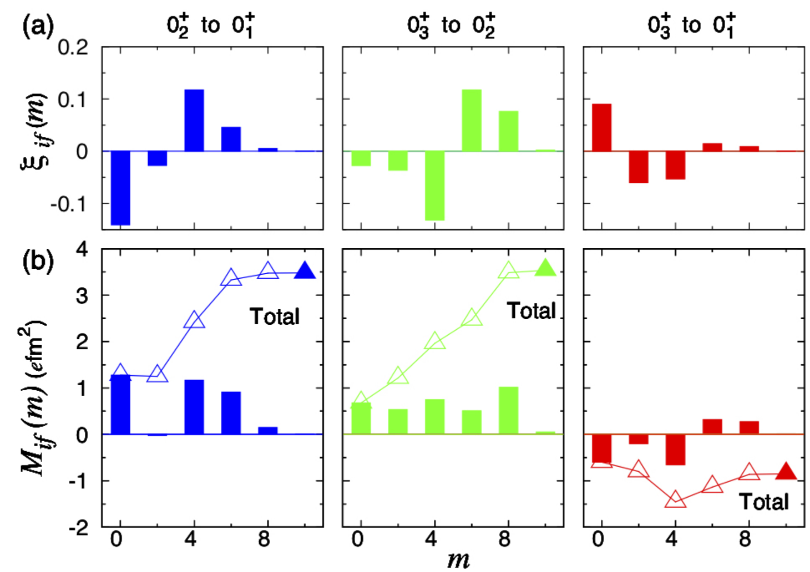
$\rightarrow$ the remaining admixtures to the 2_1^+ and 2_2^+ wave functions must be different $\rightarrow$ another configuration must enter the mixing

Three-state mixing

• three-state mixing provides good reproduction of B(E2) values and transfer cross sections for ^{30,32}Mg (A. Machiavelli, Phys. Scr. 92, 064001 (2017))

• future challenge: identification of the predominantly 0p-0h 0^+ state in ^{32}Mg that would confirm this scenario (two $(0,2)^+$ states observed recently in a knockout study, N. Kitamura *et al.*, PLB 221, 136682 (2021))





E. Ideguchi *et al.*, PRL 128, 252501 (2022)

• destructive interference in three-state mixing proposed as the reason for an anomalously low $\rho^2(\mathrm{E0};\, 0^+_3 \rightarrow 0^+_1)$ value

Take-away message

- multiple observables can point to shape coexistence in more or less direct way
- they can be measured using various experimental techniques, each of them having different limitations
- use of complementary probes improves our understanding and provides necessary consistency checks