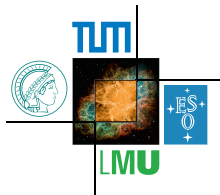


# Clustering of $\bar{B} \rightarrow D\tau^- \bar{\nu}_\tau$ kinematic distributions with ClusterKinG

Presented by Jason Aebischer

Excellence Cluster  
Technische Universität München



# Outline

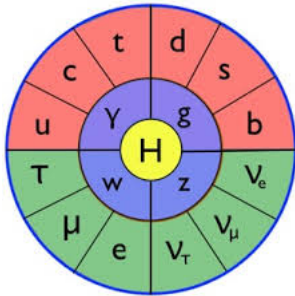
- 1 Motivation
- 2 Clustering
- 3 ClusterKinG
- 4  $\bar{B} \rightarrow D_{\mathcal{T}}^{-1} \bar{V}_{\mathcal{T}}$
- 5 Summary

Based on: [JA, Thomas Kuhr, Kilian Lieret \[1907.xxxx\]](#)

# Outline

- 1 Motivation
- 2 Clustering
- 3 ClusterKinG
- 4  $\bar{B} \rightarrow D_{\tau}^{-} \bar{\nu}_{\tau}$
- 5 Summary

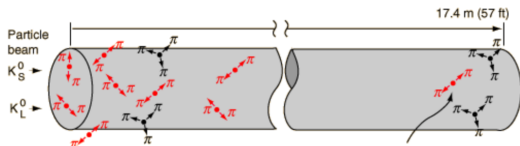
# SM and New Physics



~~CP~~



# CPV in $\varepsilon'/\varepsilon$



## Measurement

$$(\varepsilon'/\varepsilon)_{\text{exp}} = (16.6 \pm 2.3) \times 10^{-4}$$

NA48: hep-ex/0208009, KTeV: hep-ex/0208007

## SM prediction

$$\text{Lattice: } (1.4 \pm 6.9) \times 10^{-4}$$

$$\text{DQCD: } \leq (6 \pm 2.4) \times 10^{-4}$$

$$\chi_{\text{PT}}: (15 \pm 7) \times 10^{-4}$$

RBC-UKQCD: 1502.00263, 1505.07863

Buras, Gérard 1507.06326

Gisbert, Pich 1712.06147

# $b \rightarrow s \mu^+ \mu^-$ anomaly

## Measurements

Angular observable  $P'_5$  in  $B \rightarrow K^* \mu^+ \mu^-$ .

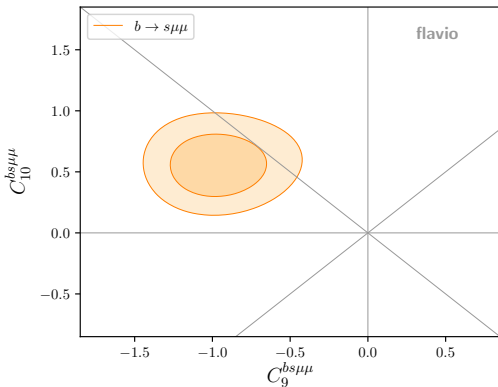
LHCb, 1512.04442

Branching ratios of  $B \rightarrow K \mu^+ \mu^-$ ,  $B \rightarrow K^* \mu^+ \mu^-$ , and  $B_s \rightarrow \phi \mu^+ \mu^-$ .

LHCb, 1403.8044, 1506.08777, 1606.04731

$$O_9^{bs\mu\mu} = (\bar{s}\gamma_\nu P_L b)(\bar{\mu}\gamma^\nu \mu)$$

$$O_{10}^{bs\mu\mu} = (\bar{s}\gamma_\nu P_L b)(\bar{\mu}\gamma^\nu \gamma_5 \mu)$$



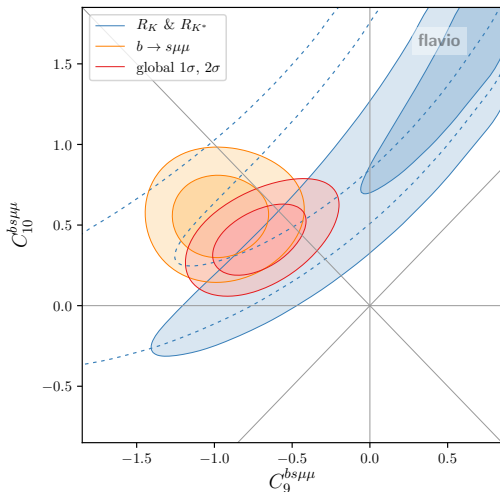
# LFU violation in neutral currents

## Measurements

$$R_K^{[1,6]}, R_{K^*}^{[0.045,1.1]}, R_{K^*}^{[1.1,6]}$$

LHCb: 1406.6482, 1705.05802, 1903.09252, Belle: 1904.0244

$$R_{K^{(*)}} = \frac{BR(B \rightarrow K^{(*)} \mu^+ \mu^-)}{BR(B \rightarrow K^{(*)} e^+ e^-)}$$



# LFU violation in charged currents

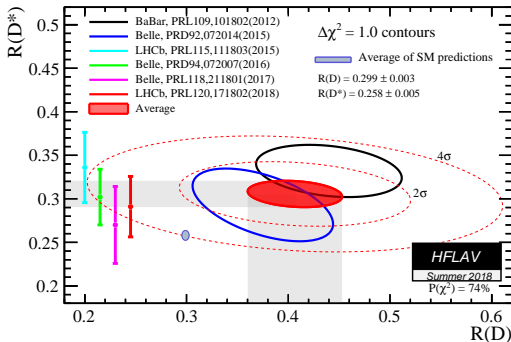
## Measurement

$R_D$  and  $R_{D^*}$

BaBar, 1205.5442, 1303.0571, LHCb, 1506.08614, 1708.08856  
Belle, 1507.03233, 1607.07923, 1612.00529

$$R_{D^{(*)}} = \frac{BR(B \rightarrow D^{(*)} \tau \nu)}{BR(B \rightarrow D^{(*)} \ell \nu)}$$

$$\ell \in \{e, \mu\}$$



HFLAV, 1612.07233



# NP in kinematic distributions

## Shapes influenced by NP parameters

→ Wilson coefficients

## Theory

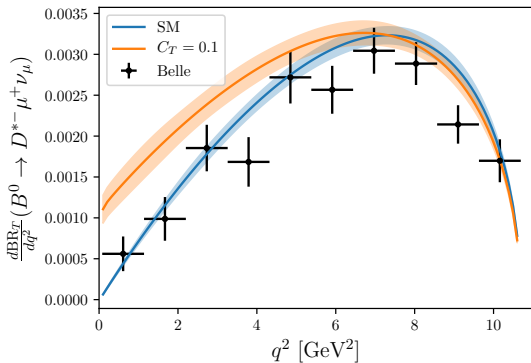
Predict kinematic distribution

## Experiment

Exclusions limits (under NP assumptions)

## Example: Theory prediction

$$O_T = (\bar{c}\sigma_{\rho\nu}P_L b)(\bar{\mu}\sigma^{\rho\nu}P_L\nu_\mu)$$



# Example: Measurement

**BaBar**

1205.5442

$R(D)$  and  $R(D^*)$

**SM values**

From  $m_{\text{miss}}^2 - |\vec{p}_\ell|$  distribution

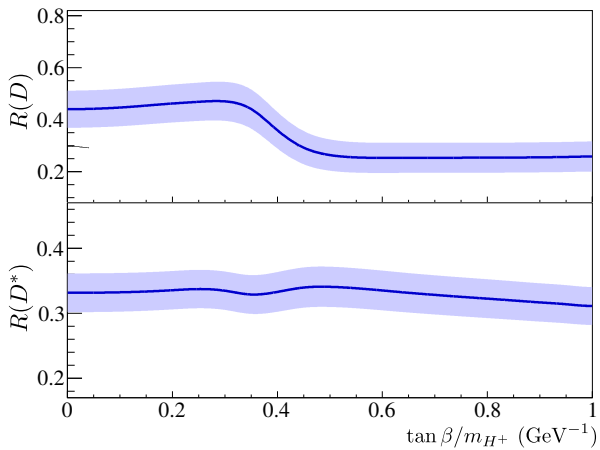
**Two-Higgs-Doublet model**

Charged Higgs mass:  $m_{H^\pm}$

Mixing angle:  $\beta$

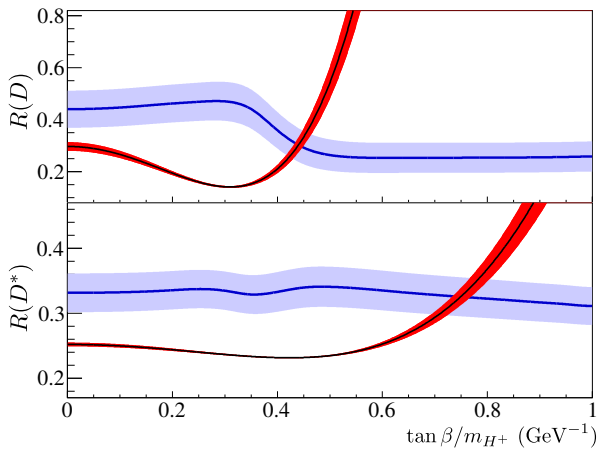
# Example: Measurement

BaBar: 1205.5442



# Example: Measurement

BaBar: 1205.5442



# Caveats

## **Many NP parameters**

SM effective theory: 2499

Weak effective theory: 5963

## **Model-dependent extraction**

Relies on NP assumptions

## **Multidimensional problem**

Hard to visualise

# Solution: Clustering

## Parameter space

Devided into regions of similar kinematic final states

## Clustering

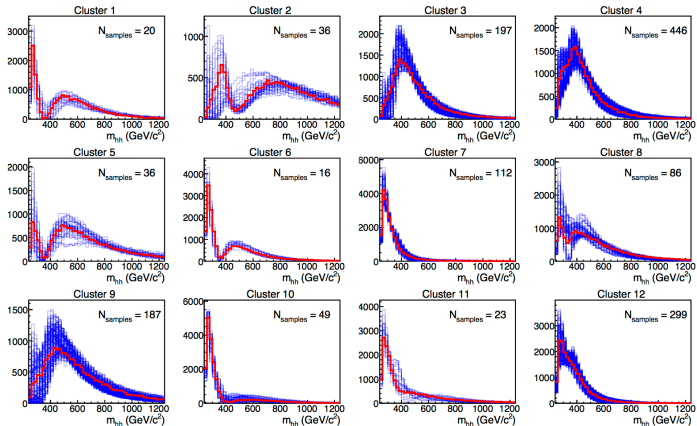
Group all similar points into a cluster

## Benchmark point

Most representative point in cluster → used for analysis

# Example: Higgs pair production

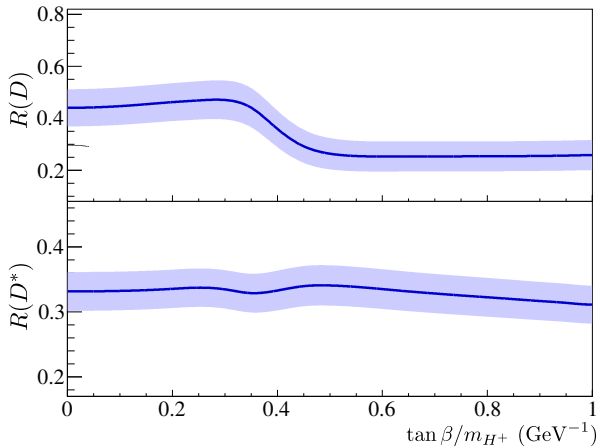
1507.02245, 1608.06578, 1710.08261



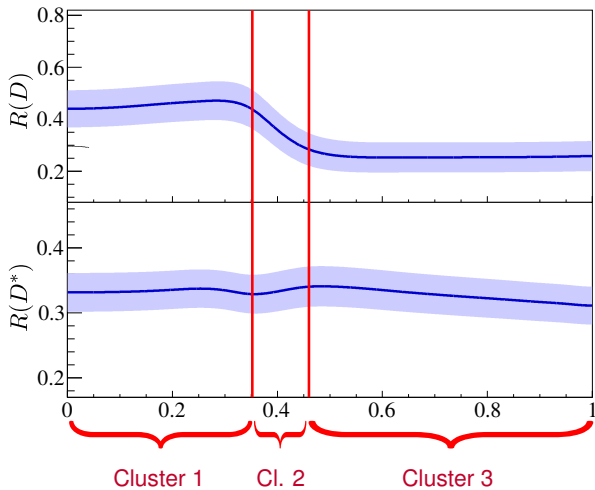
Carvalho, Dall'Osso, Dorigo, Goertz, Gottardo, Tosi: 1507.02245



## Example: $R(D^{(*)})$ from BaBar



## Example: $R(D^{(*)})$ from BaBar



# Outline

- 1 Motivation
- 2 Clustering
- 3 ClusterKinG
- 4  $\bar{B} \rightarrow D_T^- \bar{\nu}_T$
- 5 Summary

# Clustering: Prerequisites

## Similarity between distributions

Metric:  $\chi^2$  test, Kolmogorov test, etc.

## Benchmark points

Criterion: closest to all other points

## Clustering algorithm

Example: Hierarchical, K-Means

# Clustering algorithm: Hierarchical

## Step 1

Each point in own cluster

## Step 2

Merge nearest points

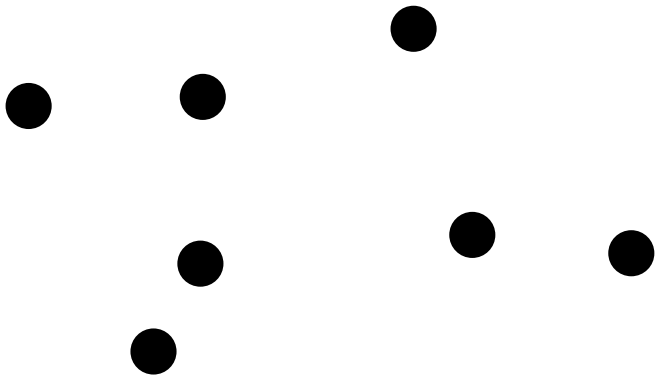
## Step 3

Compute benchmark point of each cluster

## Step 4 and following

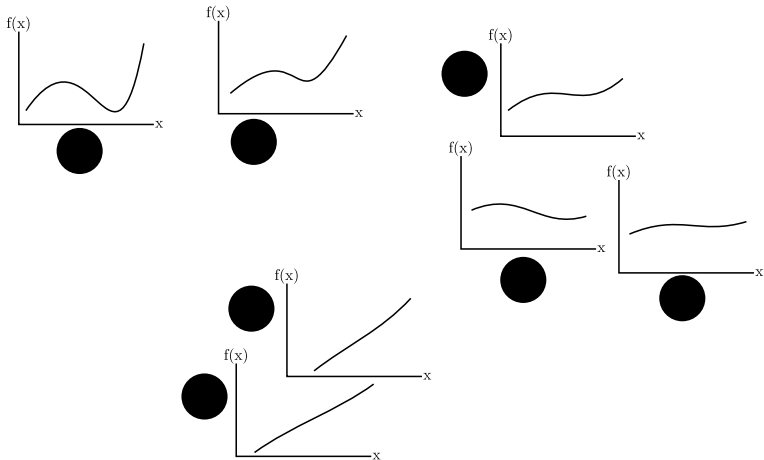
Repeat 2 and 3 until desired  $N_{\text{clusters}}$  reached

## Example: Hierarchical Clustering



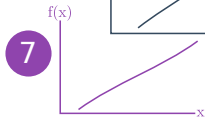
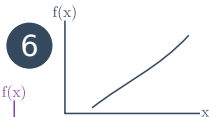
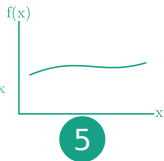
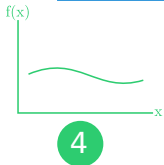
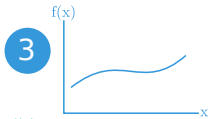
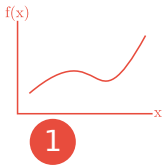
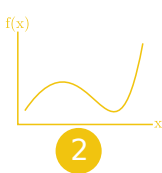
points in the parameter space

# Example: Hierarchical Clustering



compute corresponding distributions

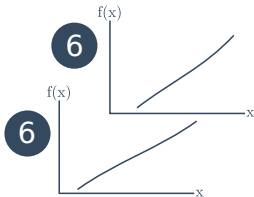
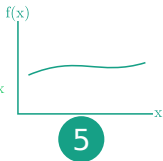
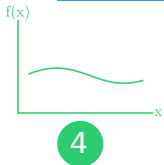
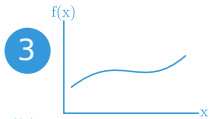
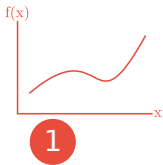
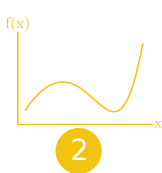
## Example: Hierarchical Clustering



Step 1: Every point in own cluster

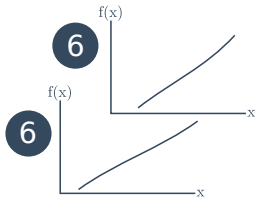
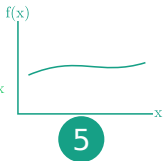
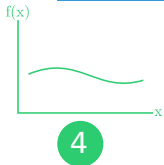
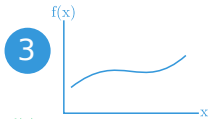
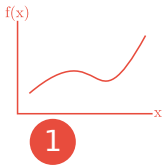
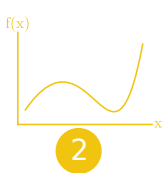


# Example: Hierarchical Clustering



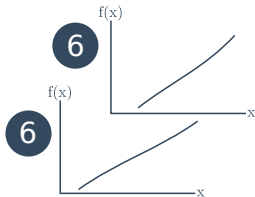
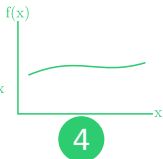
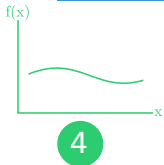
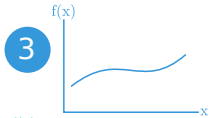
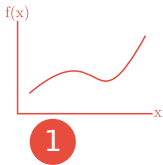
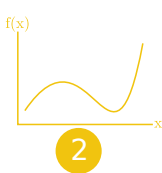
Step 2: Distributions from 6 and 7 merged

## Example: Hierarchical Clustering



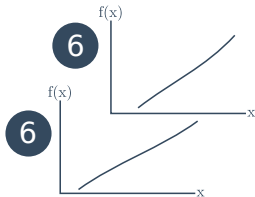
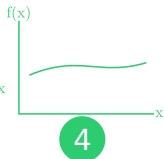
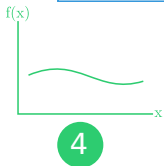
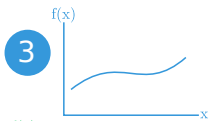
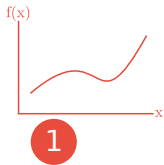
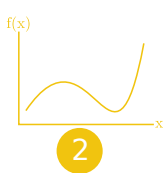
Step 3: Compute Benchmark points

## Example: Hierarchical Clustering



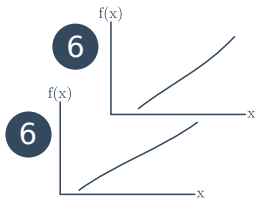
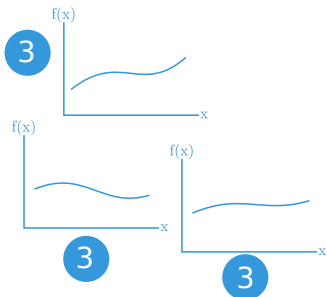
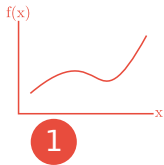
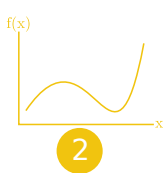
Step 4: Distributions from 4 and 5 merged

## Example: Hierarchical Clustering



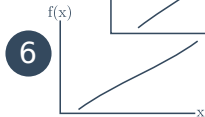
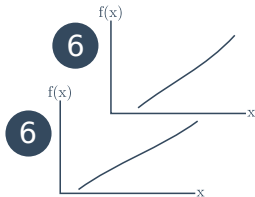
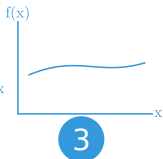
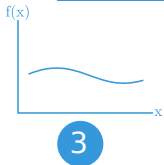
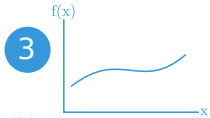
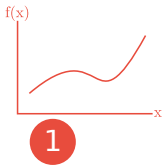
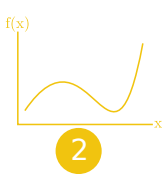
Step 5: Compute Benchmark points

## Example: Hierarchical Clustering



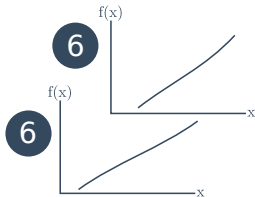
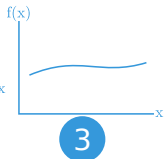
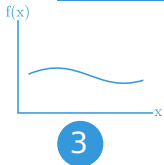
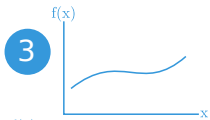
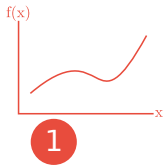
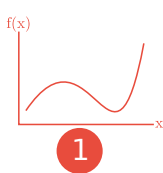
Step 6: Distributions from 4 and 3 merged

## Example: Hierarchical Clustering



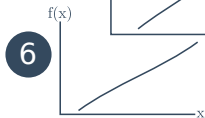
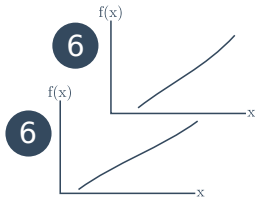
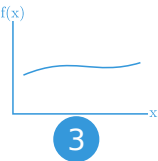
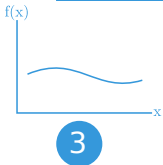
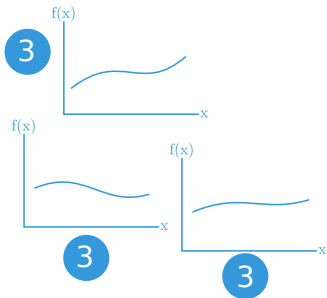
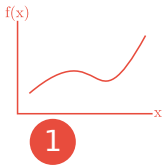
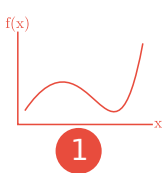
Step 7: Compute Benchmark points

# Example: Hierarchical Clustering



Step 8: Distributions from 1 and 2 merged

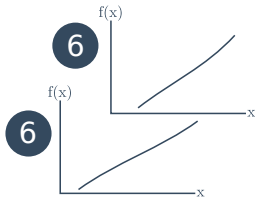
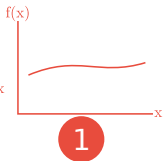
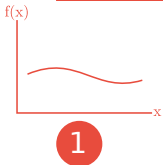
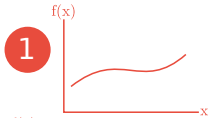
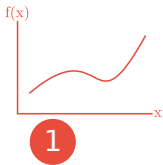
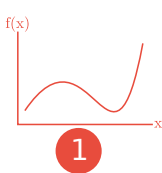
# Example: Hierarchical Clustering



Step 9: Compute Benchmark points

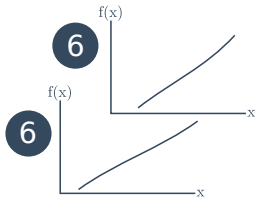
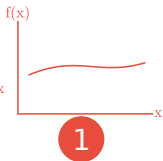
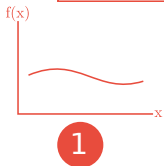
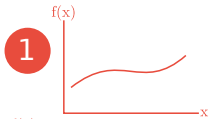
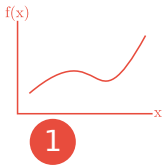
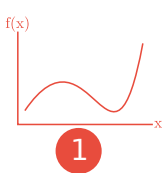


# Example: Hierarchical Clustering



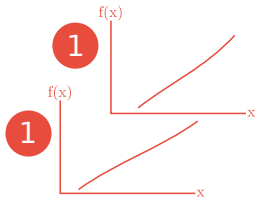
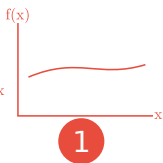
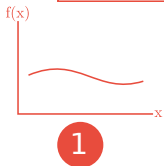
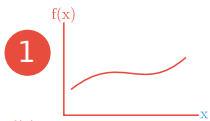
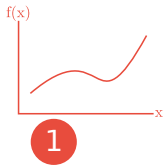
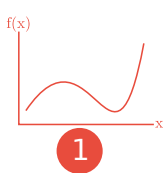
Step 10: Distributions from 1 and 3 merged

# Example: Hierarchical Clustering



Step 11: Compute Benchmark points

# Example: Hierarchical Clustering



Step 12: 1 cluster remaining

# Problems

## Dimensionality

Large number of points, many parameters

## Repetition

Redo clustering for new analysis

## Solution

Software: **ClusterKinG**

# Outline

- 1 Motivation
- 2 Clustering
- 3 ClusterKinG**
- 4  $\bar{B} \rightarrow D_T^- \bar{\nu}_T$
- 5 Summary




## Python module


Automatic clustering of parameter space

## Open source

Actively developed on  **GitHub**: <https://github.com/clusterking>

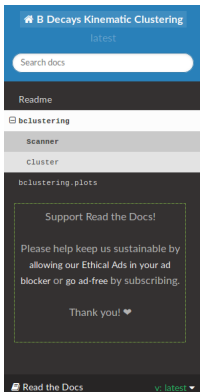
## Tutorials

Interactive using  jupyter notebooks

In browser using binder 

# Documentation

 Read the Docs:



The screenshot shows the Read the Docs interface for the `bclustering` project. At the top, it displays the project name "B Decays Kinematic Clustering" and the version "latest". Below this is a search bar labeled "Search docs". A navigation menu includes "Readme", "bclustering", "Scanner", and "Cluster". The "Scanner" page is currently selected. At the bottom, there is a "Support Read the Docs!" message with a call to action to help keep the project sustainable by allowing ethical ads or going ad-free by subscribing. The footer includes "Read the Docs" and a version selector set to "v: latest".

Docs » `bclustering`

[Edit on GitHub](#)

`bclustering`

`Scanner`

class `bclustering.scan.Scanner` [\[source\]](#)

Scans the NP parameter space in a grid and also q2, producing the normalized q2 distribution.

Usage example:

```
import flavio
from bclustering.scan import Scanner

# Initialize Scanner object
s = Scanner()

# Sample 4 points for each of the 5 Wilson coefficients
s.set_points_eqidist(
    [
        "CVL_bctaunuttau": (-1, 1, 4),
        "CSI_bctaunuttau": (-1, 1, 4),
        "CT_bctaunuttau": (-1, 1, 4)
    ],
    scale=5,
    eft='WEI',
```

► manual: coming soon

# Features

## Scanner

Scan automatically over parameter space

## Clustering

Algorithms: Hierarchical, K-Means

## Customizable

Various metrics, clustering criteria



# Visualisation

## Clusters

2D or 3D cuts through parameter space

## Benchmark points

Boxplots, histograms, bundle plots

## Errors

Systematic, statistical

# Outline

- 1 Motivation
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$$\bar{B} \rightarrow D\tau^- (\rightarrow \ell^- \bar{\nu}_\ell \nu_\tau) \bar{\nu}_\tau$$

### Charged current

Only 5 Wilson coefficients

### Distributions known

$q^2, \cos(\theta), E_\ell$

Alonso/Camalich/Kobach: 1602.07671

### Parameter ranges

can be constrained

## Parameter space

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cb} [C_{VL} O_{VL} + C_{VR} O_{VR} + C_{SL} O_{SL} + C_{SR} O_{SR} + C_T O_T] + \text{h.c.},$$

$V_{cb}$  = CKM element  $G_F$  = Fermi coupling constant

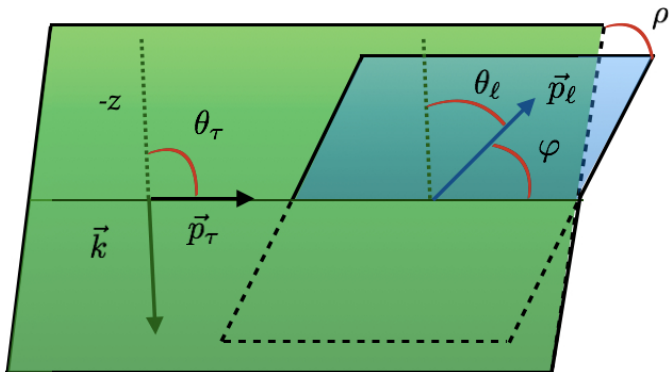
$$O_{VL} = (\bar{s} \gamma_\mu P_L b) (\bar{\tau} \gamma^\mu P_L \nu_\tau),$$

$$O_{VR} = (\bar{s} \gamma_\mu P_R b) (\bar{\tau} \gamma^\mu P_L \nu_\tau),$$

$$O_{SL} = (\bar{s} P_L b) (\bar{\tau} P_L \nu_\tau),$$

$$O_{SR} = (\bar{s} P_R b) (\bar{\tau} P_L \nu_\tau),$$

$$O_T = (\bar{s} \sigma_{\mu\nu} P_L b) (\bar{\tau} \sigma^{\mu\nu} P_L \nu_\tau).$$



$$\frac{d^3\Gamma_5}{dq^2 dE_\ell d(\cos\theta_\ell)} = B[\tau_\ell] \frac{G_F^2 |V_{cb}|^2 |\vec{k}|}{32\pi^3 m_B^2} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \frac{E_\ell^2}{m_\tau^3} \left[ l_0(q^2, E_\ell) + l_1(q^2, E_\ell) \cos\theta_\ell + l_2(q^2, E_\ell) \cos^2\theta_\ell \right]$$

# Parameter regions

## CP conserving limit

$$C_i \in \mathbb{R}$$

## Linear combinations

only  $C_{VL} + C_{VR}$  and  $C_{SL} + C_{SR}$

## Perturbativity

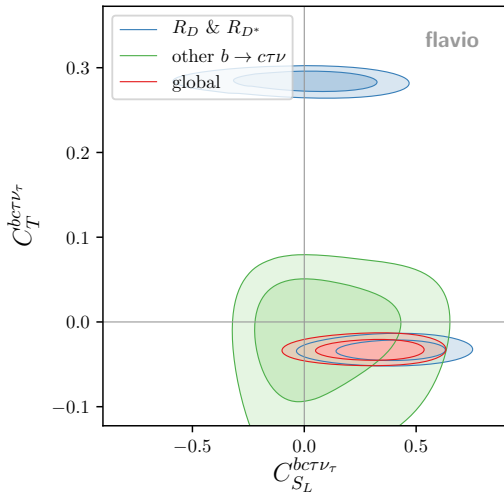
$$C_i \sim \mathcal{O}(1)$$

# Parameter regions

$C_T, C_{SL}$

Belle: 1901.06380

Constrained from  $F_L$ :  $C_T \in [-0.1, 0.1]$ ,  $C_{SL} \in [-0.5, 0.5]$



JA/Kumar/Stangl/Straub: 1810.07698

## Setting up scan

```
1 import clusterking as ck
2
3 d = ck.Data()
4 s = ck.WilsonScanner(scale=5, eft='WET', basis='flavio')
```



## Setting up scan

```
1 import clusterking as ck
2
3 d = ck.Data()
4 s = ck.WilsonScanner(scale=5, eft='WET', basis='flavio')
5
6 s.set_dfunction(
7     dGq2,
8     binning=linspace(q2min, q2max, 10),
9 )
```

## Setting up scan

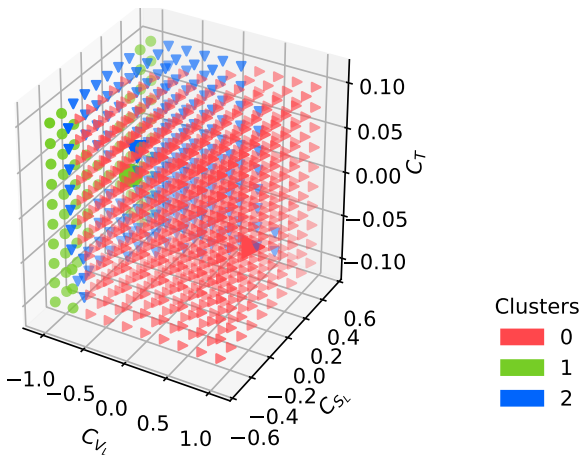
```
1 import clusterking as ck
2
3 d = ck.Data()
4 s = ck.WilsonScanner(scale=5, eft='WET', basis='flavio')
5
6 s.set_dfunction(
7     dGq2,
8     binning=linspace(q2min, q2max, 10),
9 )
10
11 s.set_points_equidist({
12     "CVL_bctaunuttau": (-1, 1, 10),
13     "CSL_bctaunuttau": (-0.5, 0.5, 10),
14     "CT_bctaunuttau": (-0.1, 0.1, 10)
15 })
16
17 s.run(d)
```

# Clustering

```
1 c = ck.cluster.HierarchyCluster(d)
2 c.build_hierarchy()      # Build up clustering hierarchy
3 c.write()                # Write results to d
4 d.plot_clusters_scatter(["CVL_bctauntau", "CSL_bctauntau",
                          "CT_bctauntau"])
```

# Clustering

```
1 c = ck.cluster.HierarchyCluster(d)
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                          CT_bctauntau"])
```

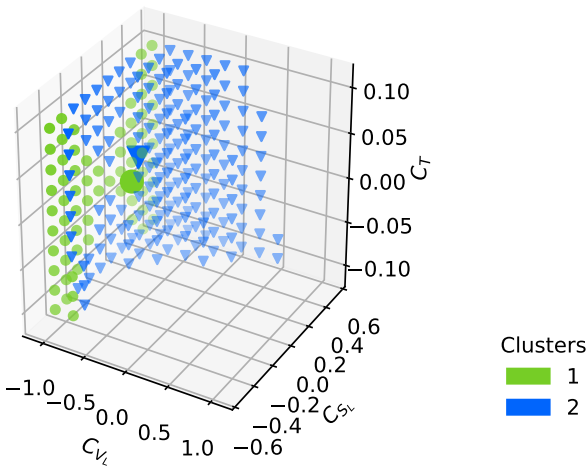


## 3D plot

```
1 d.plot_clusters_scatter(["CVL_bctauntau", "CSL_bctauntau",  
    CT_bctauntau"], clusters=[1,2])
```

## 3D plot

```
1 d.plot_clusters_scatter(["CVL_bctauntau", "CSL_bctauntau",  
    CT_bctauntau"], clusters=[1,2])
```

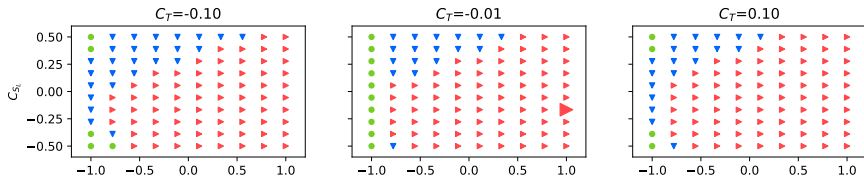


## 2D plots

```
1 d.plot_clusters_scatter(['CVL_bctauntau', 'CSL_bctauntau'])
```

## 2D plots

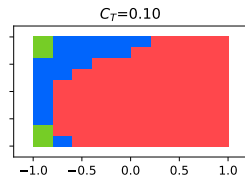
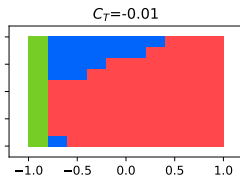
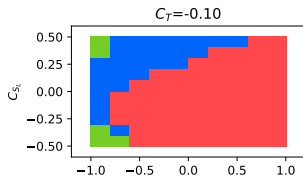
```
1 d.plot_clusters_scatter(['CVL_bctaunuttau', 'CSL_bctaunuttau'])
```





## 2D plots filled

```
1 d.plot_clusters_fill(['CVL_bctaunutau', 'CSL_bctaunutau'])
```

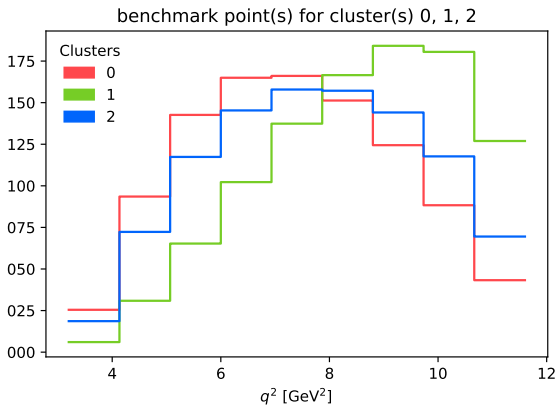


## Distribution plots

```
1 d.plot_dist()
```

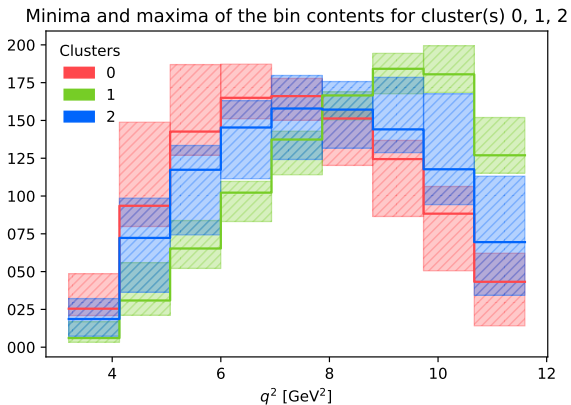
# Distribution plots

```
1 d.plot_dist()
```



# Distribution plots

```
1 d.plot_dist_minmax()
```



# Outline

- 1 Motivation
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# Summary

## NP in kinematical distributions

Few benchmark points



Clustering and visualisation

## Example

$\bar{B} \rightarrow D\tau^- \bar{\nu}_\tau$  distribution

# Project outline

## Identify observable(s)

Find models, which ones are doable

## Implementation

flavio, wcx, wilson, tree-models




## Clustering

Metric, Benchmark points,  $N_{Clusters}$

## Do analysis

For given Benchmark points

# Tools for the numerical analysis

- ▶ Computing hundreds of relevant flavour observables properly accounting for theory uncertainties
  - ▶  **flavio** <https://flav-io.github.io> Straub, 1810.08132
- ▶ Representing and exchanging thousands of Wilson coefficient values, different EFTs, possibly different bases
  - ▶  **Wilson coefficient exchange format (WCxf)** <https://wcxf.github.io/>  
JA et al., 1712.05298
- ▶ RG evolution above and below the EW scale, matching from SMEFT to the weak effective theory (WET)
  - ▶  **wilson** <https://wilson-eft.github.io>  
JA, Kumar, Straub, 1804.05033