

Electric Dipole Moment (EDM) of Charged Particles

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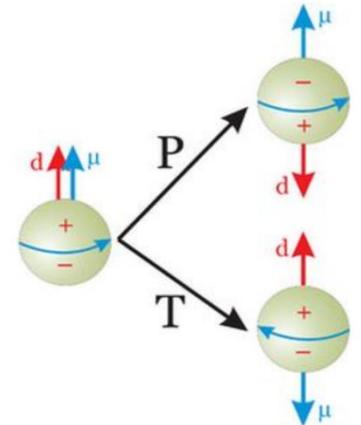
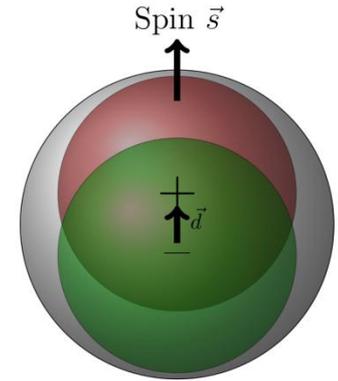
CERN, Geneva, Switzerland

on behalf of the  collaboration

Geneva, PBC Working Group meeting
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Motivation

- The Electric Dipole Moment (EDM) of a fundamental particle or a subatomic system is a **measure of the asymmetric charge distribution within the particle volume**.
- EDM exists only via violations of time reversal and parity symmetry and is **aligned with the particle spin**.
- If the particle is **charged**, then **a non zero EDM implies that the centre of charge of the particle is displaced from its centre of mass**.
- For hadrons, Standard Model expects EDM below 10^{-31} e.cm which is too weak to explain the matter-antimatter asymmetry.
- Axion-like particles create oscillating EDMs.



⇒ EDM is one of the few low energy measurements sensitive to fundamental particle physics at a scale of few TeV and above.

EDM: Aimed sensitivity

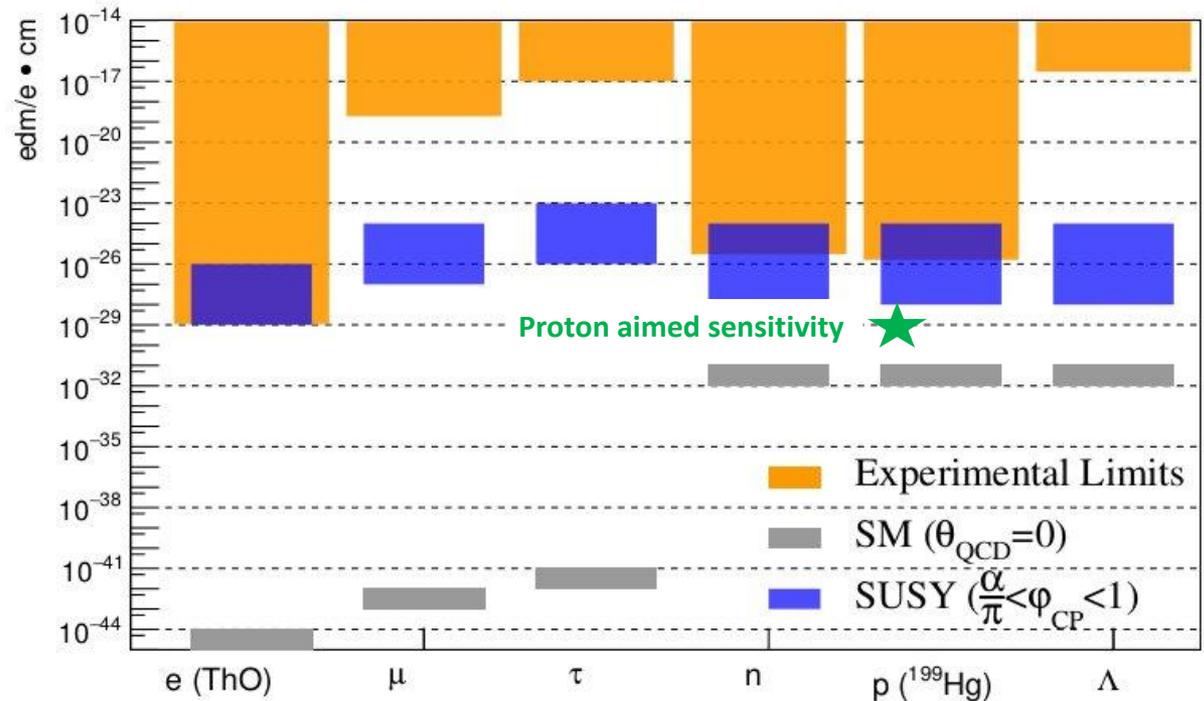
- Best upper limit for proton EDM from indirect measurement:

$$d_p < 7.9 \times 10^{-25} \text{ e} \cdot \text{cm} \quad (95 \% \text{ CL})$$

[W. C. Griffith et al., Phys. Rev. Lett. 102 \(2009\) 101601.](#)

- Aimed improvement by 5 orders of magnitude, equivalent to measuring a separation between the proton centre of charge and its centre of mass down to $\sim 10^{-31} \text{ m}$.

Courtesy Jörg Pretz



Experimental method : storage ring based EDM search

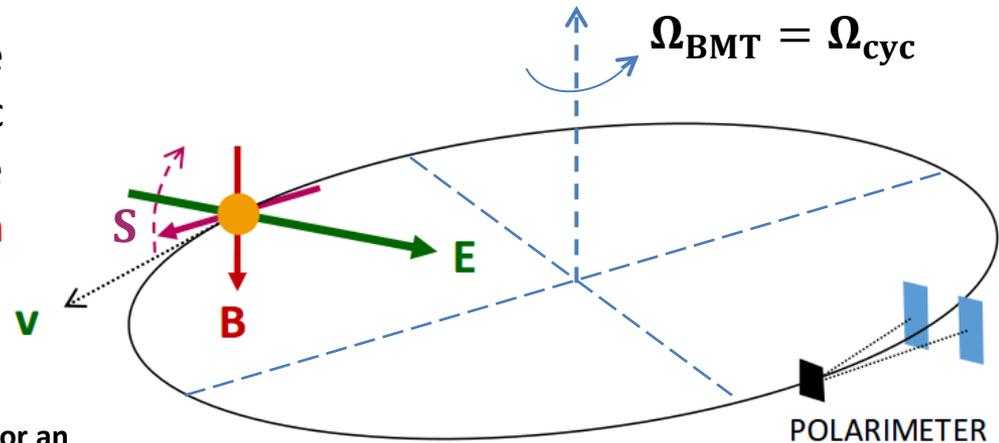
- An EDM generates a vertical spin component by coupling with radial E field:

$$\frac{d\vec{S}}{dt} = d\vec{E} \times \vec{S}$$

- The stored beam must be spin polarized in the longitudinal direction. The best sensitivity involves rotations as small as micro-radians \Rightarrow **begin with spins aligned along the particle momentum.**

- For zero EDM, the beam energy fixes the ratio between the (bending) magnetic and electric fields to keep the spin in the longitudinal direction \Rightarrow **frozen spin lattice.**

- For instance, for protons the frozen spin can be satisfied for an all electric ring, for a specific energy, the so-called magic energy, corresponding to $E_{kin} = 232.8$ MeV.



Simplified schematic of the ring.

Sensitivity scaling

- For an extraction that keeps the same number of detected particles as a function of time, the statistical error of the measurement is given by:

$$\sigma_{stat} = \frac{2\hbar}{PAE \sqrt{N f T_{tot} \tau_{sct}}}$$



$$\sigma_{stat}(1 \text{ year}) = 2.4 \times 10^{-29} \text{ e.cm}$$

- The main challenge is to reduce the systematic errors to the same level.**

Beam intensity	$N = 4.10^{10}$ particles per fill
Radial E-field	$E = 8$ MV/m
Spin coherence time	$\tau_{sct} = 1000$ s
Total running time	$T_{tot} = 10^7$ s (per year)
Polarimeter analyzing power	$A = 0.6$
Particle detection efficiency	$f = 0.005$
Beam polarization	$P = 0.8$

- The achievable statistical error is 3 orders of magnitude better than the neutron EDM.

Systematic errors

- Some of the leading sources of systematic errors in the storage ring concept:

Effect	Comment
Static radial magnetic fields (see next slide)	Mimics EDM (no cancellation between CW and CCW beams): An average radial magnetic field of $\sim 10 \text{ aT} = 10^{-17} \text{ T}$ yields the same aimed EDM signal corresponding to $d_p = 10^{-29} \text{ e.cm}$ ⇒ Measure the orbit separation between counter-rotating beams.
Gravity (balanced by the vertical E-fields).	Factor 30 larger than the one due to $d_p = 10^{-29} \text{ e.cm}$. Cancellation between CW and CCW beams.
Geometric phases	Second order effects. Needs two polarimeters at least to determine and eliminate. Some cancel with CW and CCW beams.
Polarimeter systematic error	Beam drifting on the target; use consecutive bunches of opposite helicity.

Counter-rotating beams

- The experiment can be changed to a time reverse of itself by:
 - inverting the direction of all velocities,
 - inverting the sign of all magnetic fields while keeping the electric field as is,
 - reversing all the spins.
- In this case where the time-reversed beam travels inside the same machine, the two results can be compared directly and the EDM can be extracted by taking the difference of the two signals.
- **If a residual radial magnetic field does not reverse, then it will yield a signal mimicking the EDM one. This is probably the most serious systematic imperfection.**
- Remedy: **multilayer shielding** (down to the nT level) and **measuring the vertical orbit separation between the two counter-rotating beams** \Rightarrow operate the machine with low vertical tune to maximize the orbit separation.
 - **With a vertical tune $Q_y = 0.1$, a residual radial magnetic field of 10 aT leads to an average orbit separation at the pm level \Rightarrow high precision beam control required.**

Lattice parameters

- For protons at the magic energy, $E_{\text{kin}} = 232.8 \text{ MeV}$, the full scale ring lattice is the all-electric “strong focusing” lattice proposed by V. Lebedev and used for our studies.

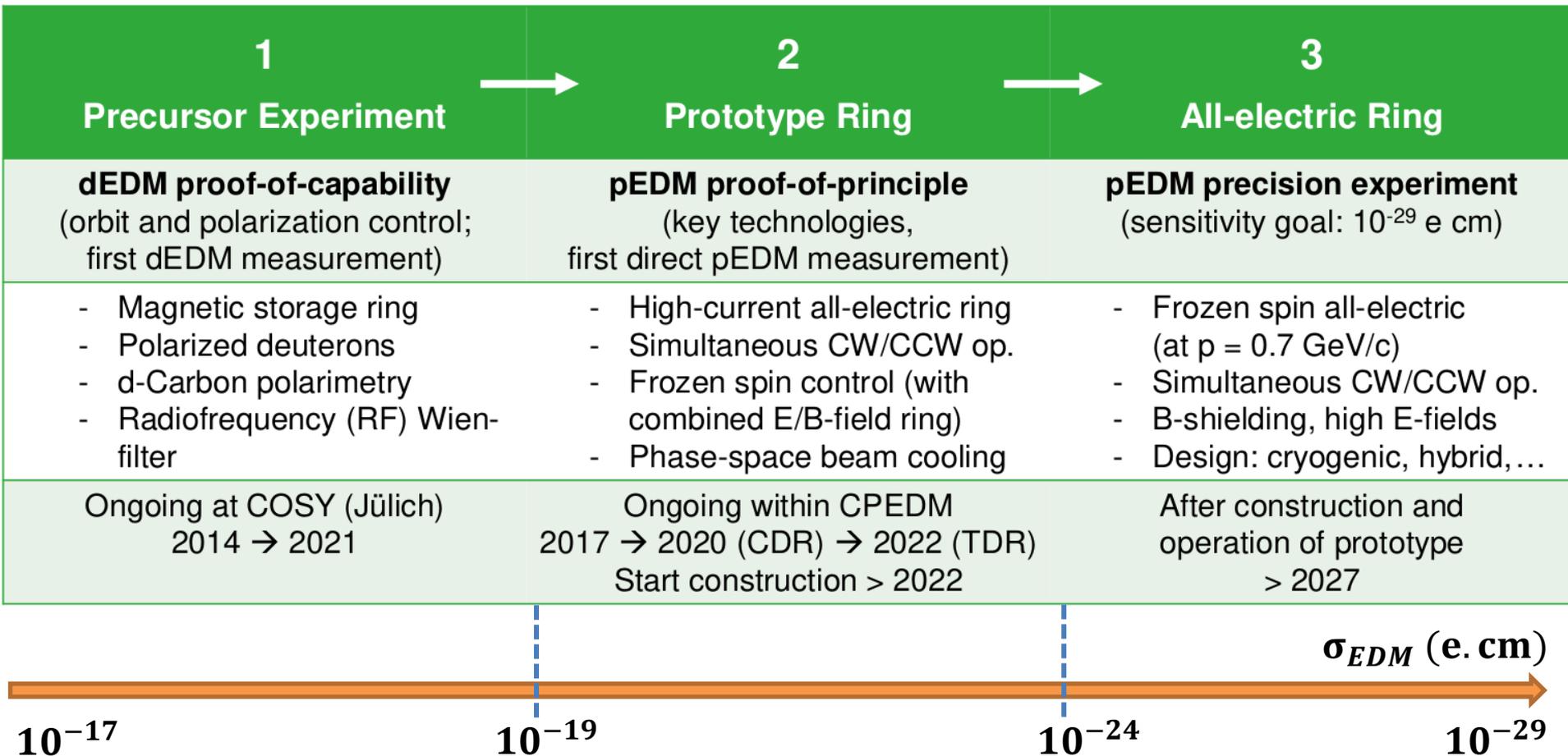
- The designed ring lattice requires electric gradient of 8 MV/m hence its circumference.**

- Operation below transition reduces IBS growth rates at the thermal equilibrium.

parameter	symbol	unit	full scale
bending radius	r_0	m	52.3
electrode spacing	g	cm	3
electrode height	d	cm	20
deflector shape			cylindrical
electrode index	m		0
radial electric field	E_0	Mv/m	8.0
number long straights			4
long strght sec. leng.	lss	m	20.8
polarimeter sections			2
injection sections			2
total circumference	C		500.0
harmonic number	h		100
RF frequency			35.878
number of bunches			100
particles per bunch			2.5e8
mom. spread(not/cooled)			$\pm 5e-4/1e-4$
max. horz. beta func.	$\beta_{x,\text{max}}$	m	47
max. vert. beta func.	$\beta_{y,\text{max}}$	m	216
dispersion	D	m	29.5
horizontal tune	Q_x		2.42
vertical tune	Q_y		0.44
horz. emit.(not/cooled)	ϵ_x	mm-mr	3.2/3
vert. emit.(not/cooled)	ϵ_y	mm-mr	17/3
slip-factor	η		-0.192

CPEDM Strategy

- As emphasized in the previous slides, the proton EDM measurement is a challenging experiment. Thus, it was decided that the project will proceed in stages as outlined below:



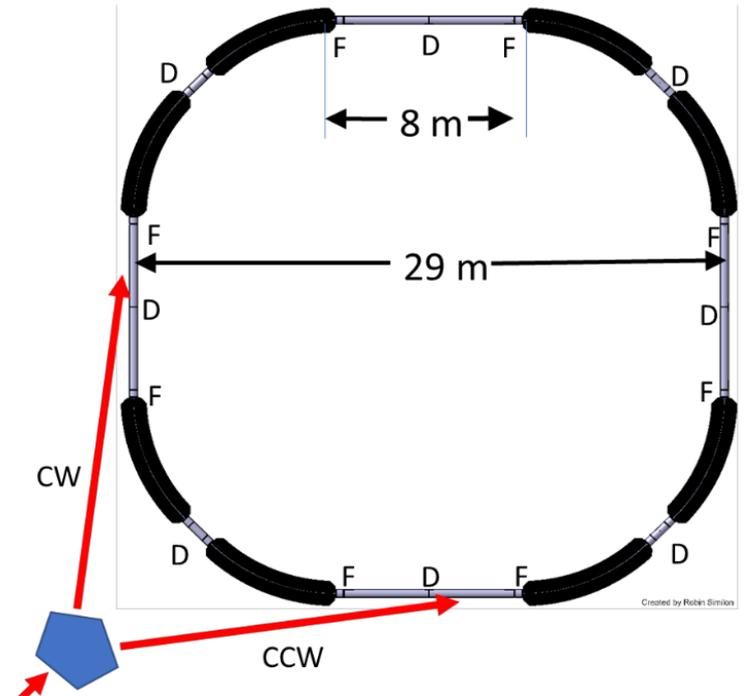
Proposal for a prototype

- Following the completion of the COSY precursor experiment (~ 2021), the next stage is to fund and build a prototype ring to address the critical questions of the EDM ring design:

	Stage 1	Stage 2	
	E only	E, B	unit
Kinetic energy	30	45	MeV
$\beta = v/c$	0.247	0.299	
γ (kinetic)	1.032	1.048	
Momentum	239	294	MeV/c
Magnetic rigidity $B\rho$		0.981	T·cm
Electric field only	6.67		MV/m
Electric field E (frozen spin)		7.00	MV/m
Magnetic field B (frozen spin)		0.0327	T

- Some of the goals of the PT ring:
 - Demonstrate the ability to store enough protons to perform the EDM measurement.
 - Demonstrate the ability to store and control simultaneously two counter-rotating beams.
 - **First direct measurement of the proton EDM.**

Cost estimate: 17 M€ for the ring only.



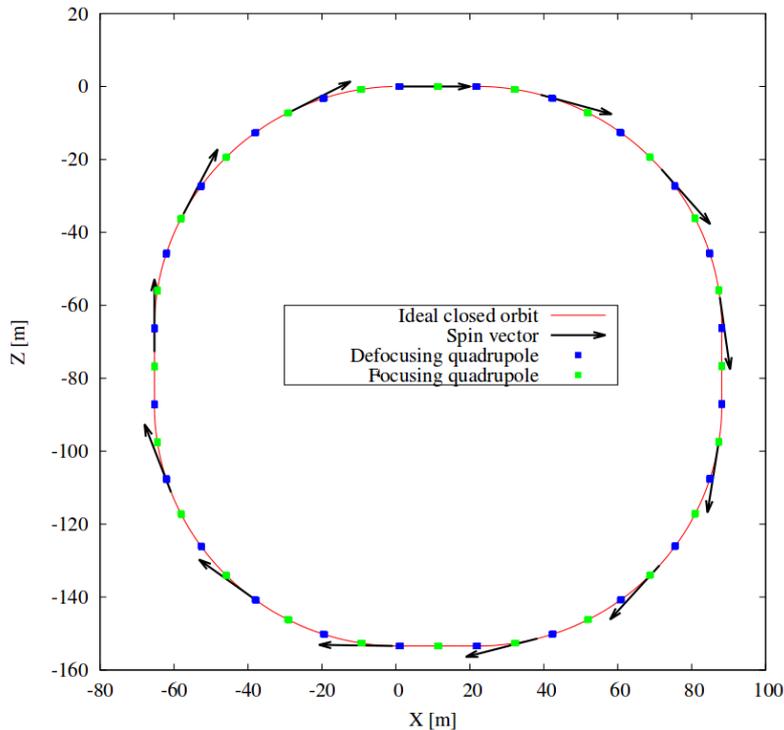
Lattice layout for the prototype ring consisting of 8 dual, superimposed electric and magnetic fields. The total circumference is about 100 m.

Beam and Spin simulations

- Limited number of codes that can perform beam tracking simulations in **electrostatic elements**.
 - Even so, not so many codes perform spin tracking. Our numerical simulations so far are based on BMAD code developed by D. Sagan.
- Benchmarking is very demanding: the aimed sensitivity of the experiment is equivalent to detecting a spin buildup of 1.6 nrad/s or $4.4 * 10^{-15}$ rad/turn.
 - Objective: understand the systematic imperfections that can mimic an EDM signal to be measured.
- Several studies so far. In what follows, we focus our analysis on the all electric proton EDM ring.

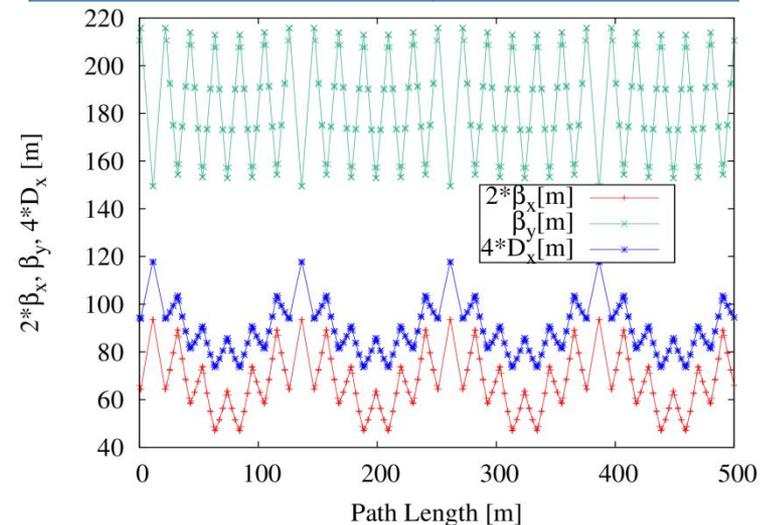
Beam dynamics simulations

- Implemented the Lebedev lattice in BMAD and benchmarked the tracking results with the analytical calculations.



Overview of the ideal ring. The beam is circulating clockwise such as the particle momentum vector is aligned with the spin vector.

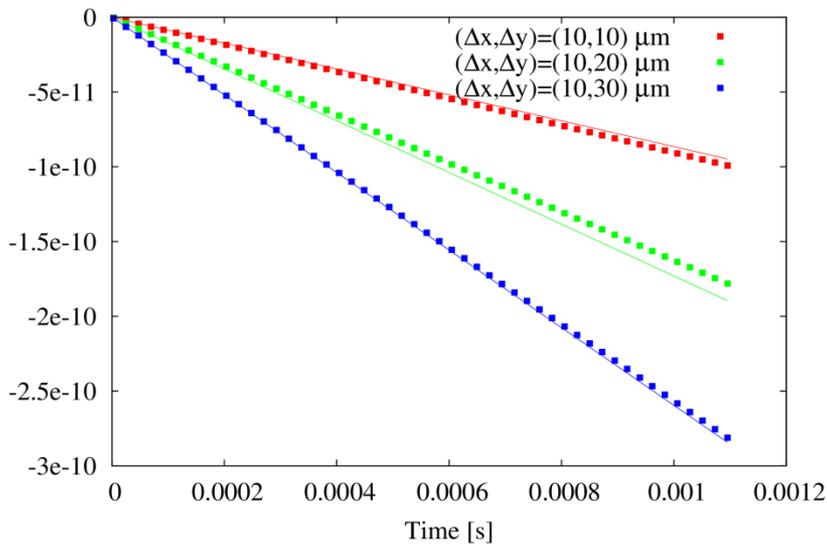
Beam energy	232.792 MeV
Bending radius	52.3 m
Radial E-field	$E = 8$ MV/m
Tunes Q_x/Q_y	2.42/0.44
Slip factor η	-0.192
Harmonic number	100
Particles/bunch	$2.5e8$



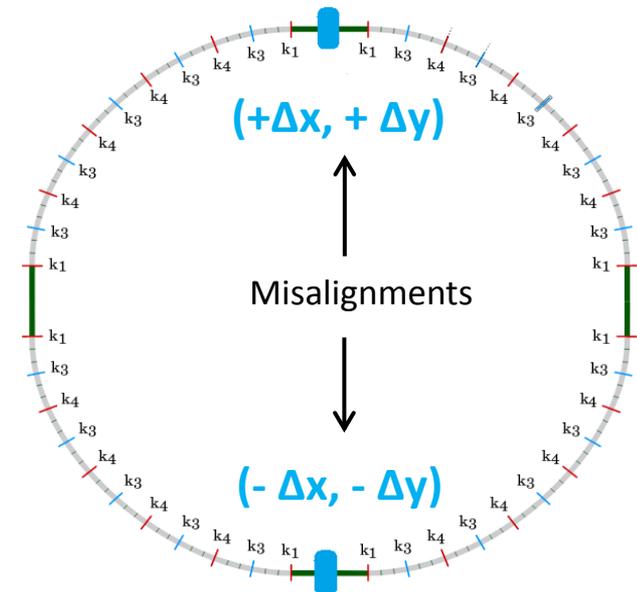
Spin tracking simulations

■ Benchmarking/validation tests: several cases were simulated to understand the different imperfections that can contribute to a fake EDM signal and to ensure reliable modeling tools. For instance,

1. Particle injected at the magic energy in a perfect machine. Frozen spin condition.
2. Particle injected off the magic energy in a perfect machine.
3. Particle injected at the magic energy in an imperfect machine: if the particle no longer lies in the median plane of the accelerator i.e. exhibits vertical oscillations, the vertical polarization component develops:



Vertical spin buildup from tracking simulations.



Lattice layout with imperfections.

- $\frac{dS_y}{dt} \propto \Delta x \cdot \Delta y$: we refer to this as second order effects. How to explain?

Modified Bogoliubov Krylov Mitropolski (BKM) method of averages

- The spin precession equation can be written in the matrix form $\frac{dP}{dt} = M(t)P(t)$
 - This is a first order linear differential equation with periodic coefficients for which in general a closed form solution is not known. $M(t)$ is a skew-symmetric matrix and the problem is formulated as follows:

$$\begin{pmatrix} dP_r/dt \\ dP_y/dt \\ dP_l/dt \end{pmatrix} = \begin{pmatrix} 0 & -\Omega_l(t) & \Omega_y(t) \\ \Omega_l(t) & 0 & -\Omega_r(t) \\ -\Omega_y(t) & \Omega_r(t) & 0 \end{pmatrix} \begin{pmatrix} P_r \\ P_y \\ P_l \end{pmatrix}$$

- It can be shown that the second order approximation of the vertical polarization buildup is given by:

$$P_{y,2}(t) = \xi_{y,2}(t) + \phi_{y,2}(t)$$

EDM + radial B fields

Geometric phases

where

$$\xi_{y,2}(t) = -\langle \Omega_r \rangle t + \langle \Omega_l \tilde{\Omega}_y \rangle t - \langle \Omega_y \rangle \langle \tilde{\Omega}_l \rangle t + \frac{\langle \Omega_y \rangle \langle \Omega_l \rangle}{2} t^2$$

and

$$\phi_{y,2}(t) = -\tilde{\Omega}_r(t) + \Omega_l(t) \widetilde{\tilde{\Omega}_y}(t) + \langle \Omega_y \rangle \left[t \tilde{\Omega}_l(t) - \tilde{\tilde{\Omega}}_l(t) \right]; \quad \tilde{\Omega}_i(t) = \int_0^t [\Omega_i(\tau) - \langle \Omega_i \rangle] d\tau; \quad i = r, y, l$$

- $\xi_{y,2}(t)$ is the frozen spin solution, i.e. the vertical polarization signal measured at the location of the polarimeter. This does explain the tracking simulations.

Conclusion

- Storage ring based EDM search is one of the few low energy measurements sensitive to fundamental particle physics at a scale of few TeV and above.
- Significant effort and progress is made worldwide.
- Studies of systematic imperfections helped understand the main sources of systematic errors.
- A stepwise approach for the project has been decided: precursor at COSY, prototype ring (100 m), all-electric dedicated ring (500 m).
- A CERN yellow report has been drafted by the CPEDM collaboration and will be released soon.

Thank you

Feasibility Study for an EDM Storage Ring - Addendum

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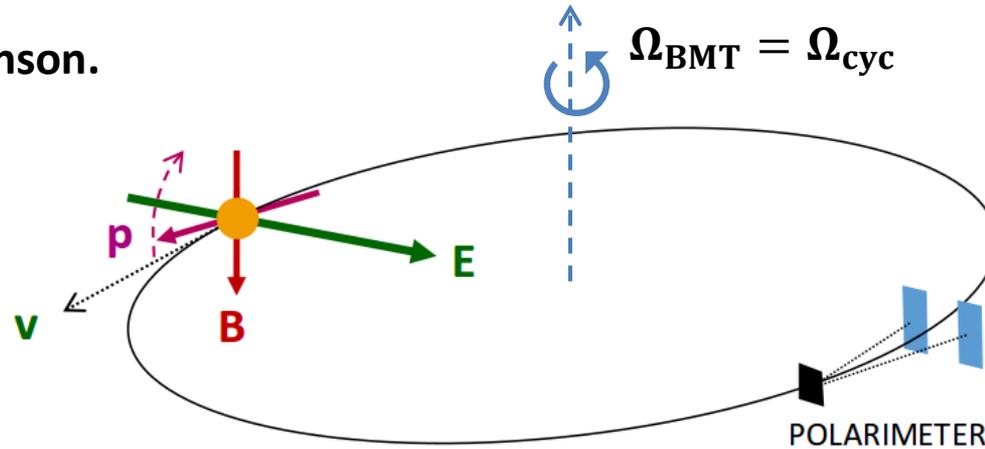
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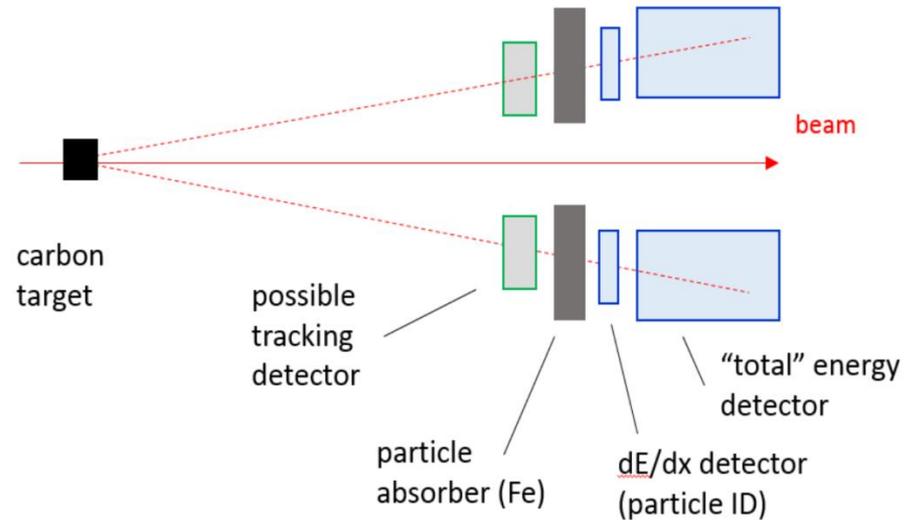
Backup slides

Polarimeter

Courtesy Ed. Stephenson.



Schematic layout showing the important components of an EDM polarimeter. The beam passes through a thick carbon target. Scattered particles first encounter a tracking detector that traces rays back to the target. Next an absorber removes unwanted events. Lastly, a detector pair identifies the energy of the particles of interest along with the particle type.



Thomas-BMT equation in inertial frame

- The Thomas Bargman-Michel-Telegdi (T-BMT) equation gives the precession rate of the angle between the spin and momentum vectors of a relativistic particle in the presence of electromagnetic fields:

$$\frac{d\mathbf{P}}{dt} = (\boldsymbol{\Omega}_{MDM} + \boldsymbol{\Omega}_{EDM}) \times \mathbf{P}$$

where the precession vector due to the particle's Magnetic Dipole Moment (MDM) is:

$$\boldsymbol{\Omega}_{MDM} = -\frac{e}{mc} \left[\left(a + \frac{1}{\gamma} \right) c\mathbf{B} - \frac{a\gamma c}{\gamma + 1} (\boldsymbol{\beta} \cdot \mathbf{B})\boldsymbol{\beta} - \left(a + \frac{1}{\gamma + 1} \right) \boldsymbol{\beta} \times \mathbf{E} \right]$$

and the precession vector due to the particle's Electric Dipole Moment (EDM) is:

$$\boldsymbol{\Omega}_{EDM} = -\frac{e}{mc} \frac{\eta}{2} \left[\mathbf{E} - \frac{\gamma}{\gamma + 1} (\boldsymbol{\beta} \cdot \mathbf{E})\boldsymbol{\beta} + c\boldsymbol{\beta} \times \mathbf{B} \right]$$

- The spin is defined in the inertial rest frame of the particle while the electromagnetic field vectors are expressed in the laboratory frame. The MDM is usually very well known.**

Thomas-BMT equation in non inertial frame

- The accelerator frame is, by definition, a non-inertial frame. In a storage ring, it can be shown that the T-BMT equation transforms into:

$$\frac{d\mathbf{P}}{dt} = (\boldsymbol{\Omega}_{MDM} - \boldsymbol{\Omega}_{cyc} + \boldsymbol{\Omega}_{EDM}) \times \mathbf{P}$$

where $\boldsymbol{\Omega}_{cyc}$ is the cyclotron angular frequency describing the rotation of the coordinate system and \mathbf{P} is the polarization vector projected in such a frame.

- The idea is to maximize the EDM signal by minimizing the MDM contribution to the spin buildup. Such a condition is set when:

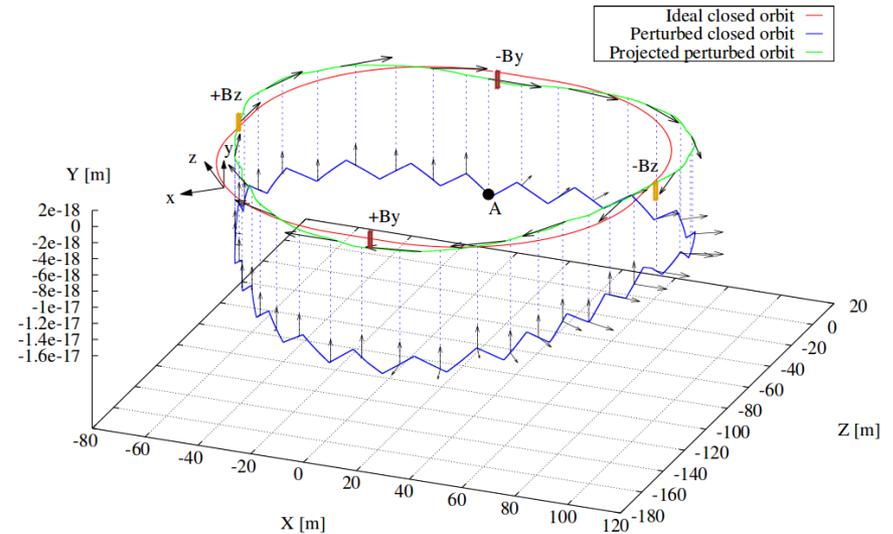
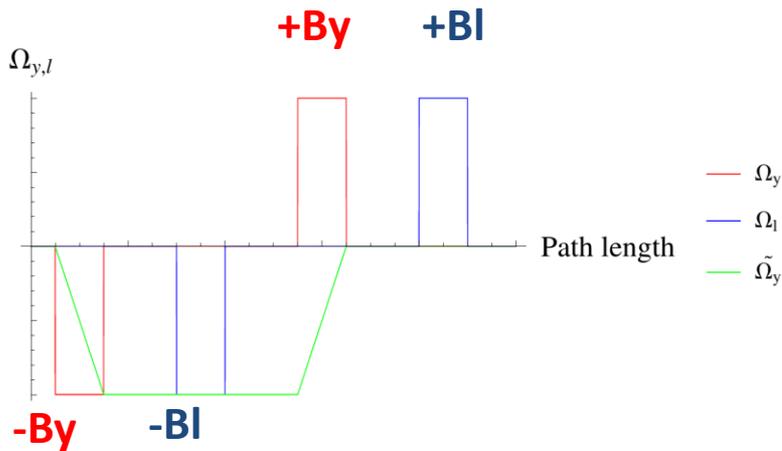
$$\boldsymbol{\Omega}_{MDM} - \boldsymbol{\Omega}_{cyc} = \mathbf{0}$$

- For protons, such a condition is fulfilled for an all-electric storage ring for a specific energy, the so called magic energy corresponding to $\mathbf{E}_{kin} = 232.8 \text{ MeV}$ and the lattice is referred to as the **frozen spin lattice**.

There is an on-going discussion among the members of the CPEDM collaboration regarding the choice of the coordinate system to properly describe the spin precession. $\boldsymbol{\Omega}_{BMT} - \boldsymbol{\Omega}_{cyc}$ shall be interpreted with care.

Example of systematic errors: Geometric phases

- Due to the non-commutativity of spin rotations around different axes, spin buildup mimicking the EDM signal can occur even if $\langle \Omega_x \rangle = \langle \Omega_y \rangle = \langle \Omega_z \rangle = 0$. Such a contribution is generally referred to as **geometric phase** or **Berry phase**:



Schematic of the consecutive rotations around different axes yielding a vertical spin buildup.

$$\langle \Omega_l \tilde{\Omega}_y \rangle \approx \frac{-1}{c\beta_l C} \left(\frac{e}{m} \right)^2 \left(a + \frac{1}{\gamma} \right) \frac{1+a}{\gamma} (B_y L)(B_l L)$$

- If $B_y L = B_l L = \pm 100$ nT.m, this yields $\frac{dP_y}{dt} = 2.4$ nrad/s for a beam at the magic energy.
- Shall be eliminated with counter-rotating beams.

Spin motion and synchrotron oscillations

- In order to increase the spin coherence time, it is necessary to employ an RF cavity without acceleration \Rightarrow **particle energy oscillates around the magic energy.**
- A direct relation between the horizontal spin and the longitudinal position of the particle was derived:

$$s_r = 2 \frac{\beta_l c}{\gamma [C/(2\pi)]} \alpha_s \frac{\tau}{-\eta} = -\frac{4\pi}{\gamma} \alpha_s \frac{z/C}{-\eta} ; \alpha_s = \frac{1}{2\pi} \int_0^C \frac{1}{\rho(s)} \left(1 - \frac{D(s)}{\rho(s)} \right)$$

- The horizontal spin precesses with a frequency equal to the synchrotron frequency of the particle (no betatron oscillations considered).

Assuming $V_{rf} = 6 \text{ kV}$ and $h=100$ yields

$$T_{\text{syn}} = \frac{T_{\text{rev}}}{Q_s} = \mathbf{0.421 \text{ ms}}$$

- Verified through tracking simulations.

