Excited QCD 2020
Krynica Zdrój

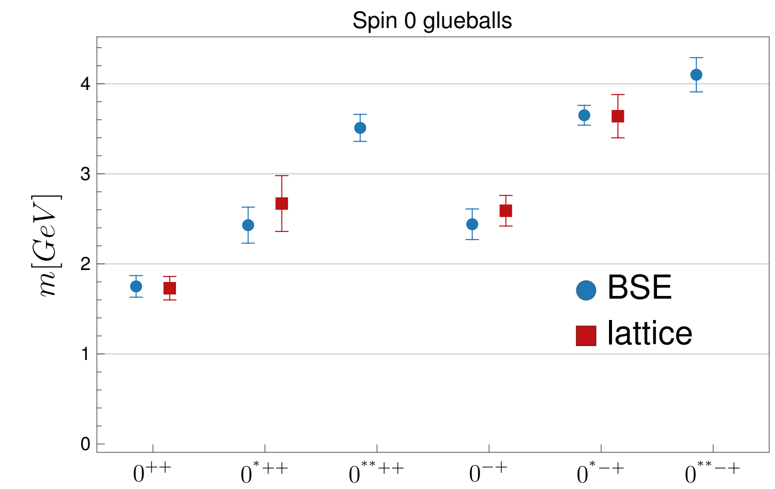
Exotic hadrons from functional methods

Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]
Wallbott, Eichmann and CF, in preparation
CF, Huber, Sanchis-Alepuz, in preparation

Overview

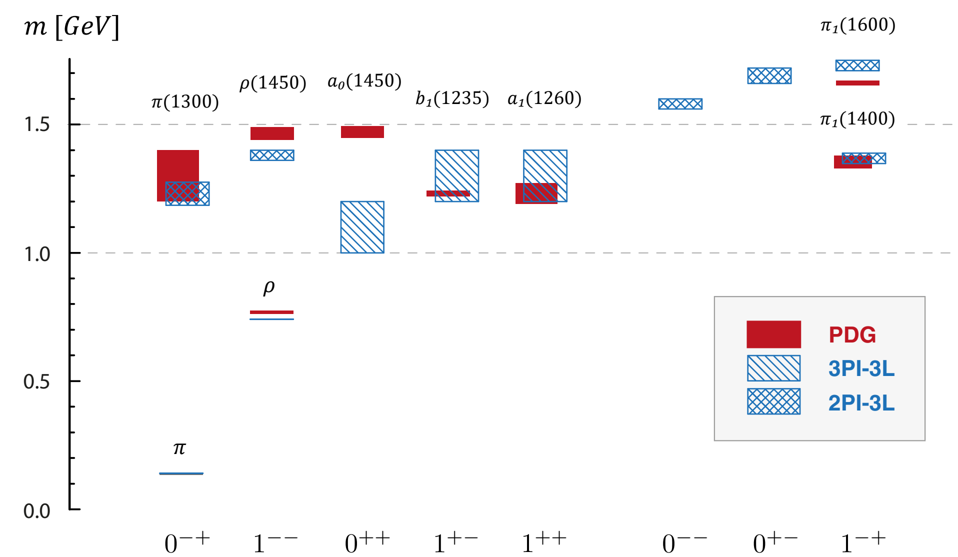
1. Glueballs in Yang-Mills theory

CF, Huber, Sanchis-Alepuz, in preparation



2. Quark masses and light meson spectroscopy

Williams, CF, Heupel, PRD93 (2016) 034026



3. Heavy-light tetraquarks: X(3872) and more…

Wallbott, Eichmann and CF, PRD100 (2019) no.1, 014033, arXiv:1905.02615
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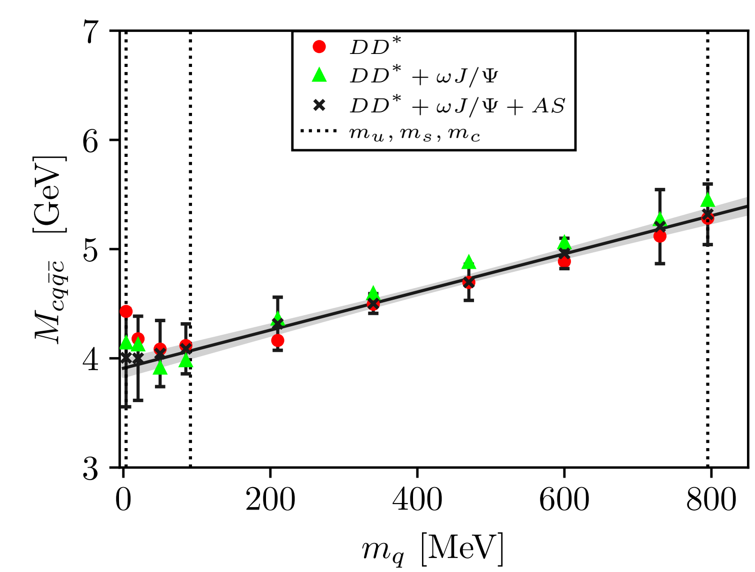
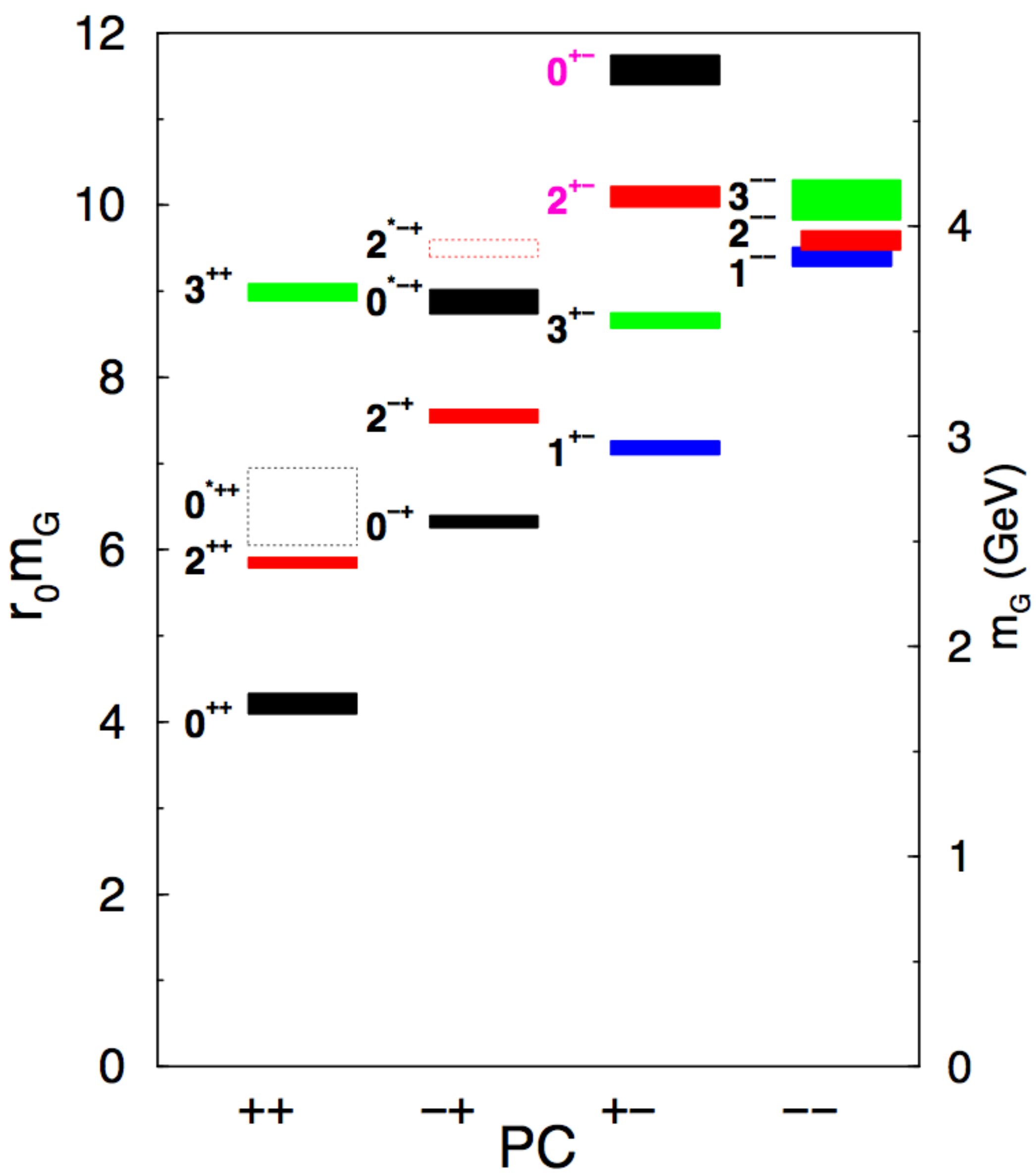


Glueballs



Morningstar and Peardon, PRD 60 (1999) 034509
Y.~Chen et al., PRD 73 (2006) 014516

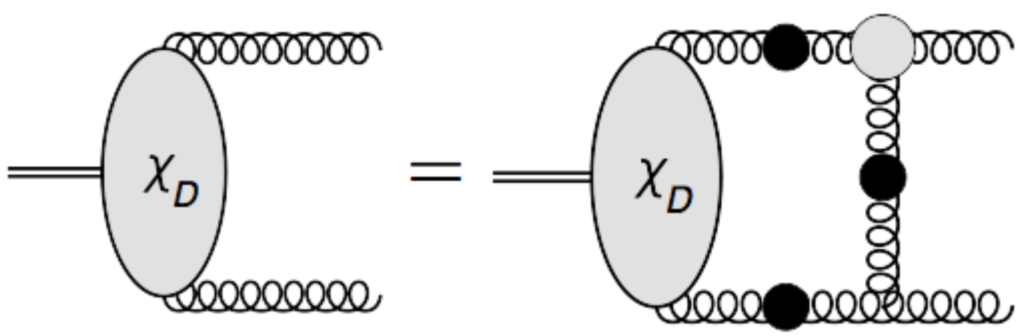
Lattice:

- States in the light and heavy quark energy regions
- Most calculations quenched
- Unquenched calculations very involved

Gregory et al., JHEP 1210 (2012) 170

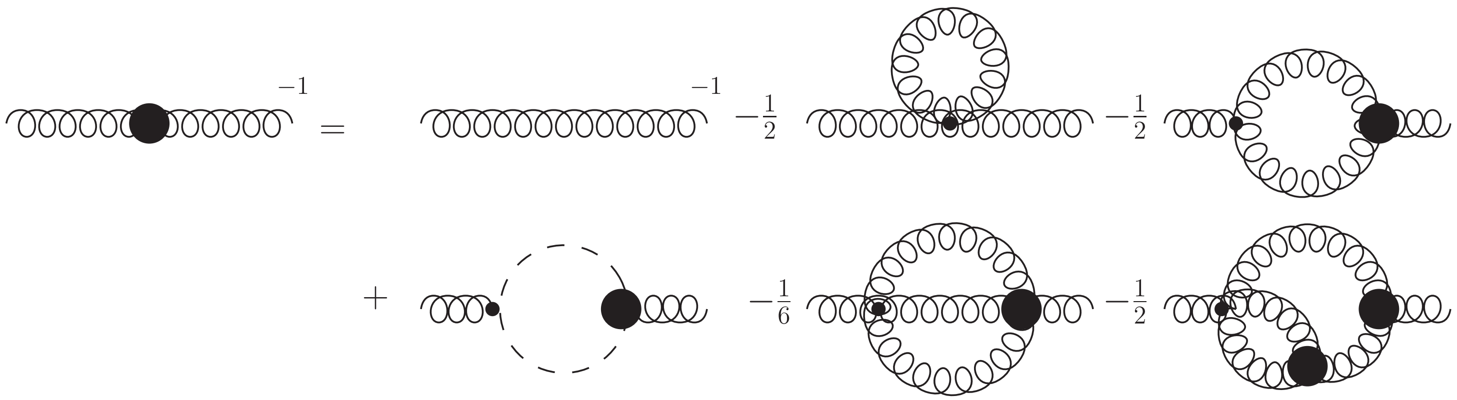
DSE:

- structural information



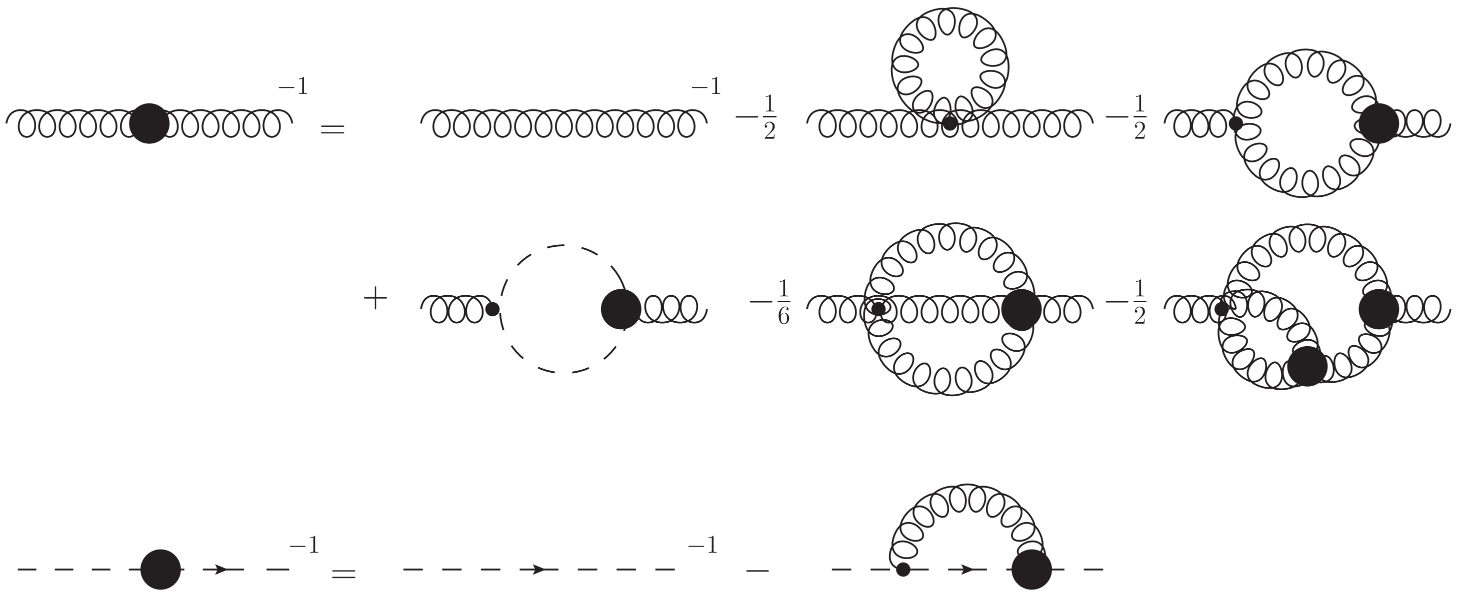
Meyers, Swanson, PRD 87 (2013) 3, 036009
Sanchis-Alepuz, CF, Kellermann and von Smekal, PRD 92 (2015) 3, 034001

Landau gauge - 3PI truncation



Huber, in preparation

Landau gauge - 3PI truncation



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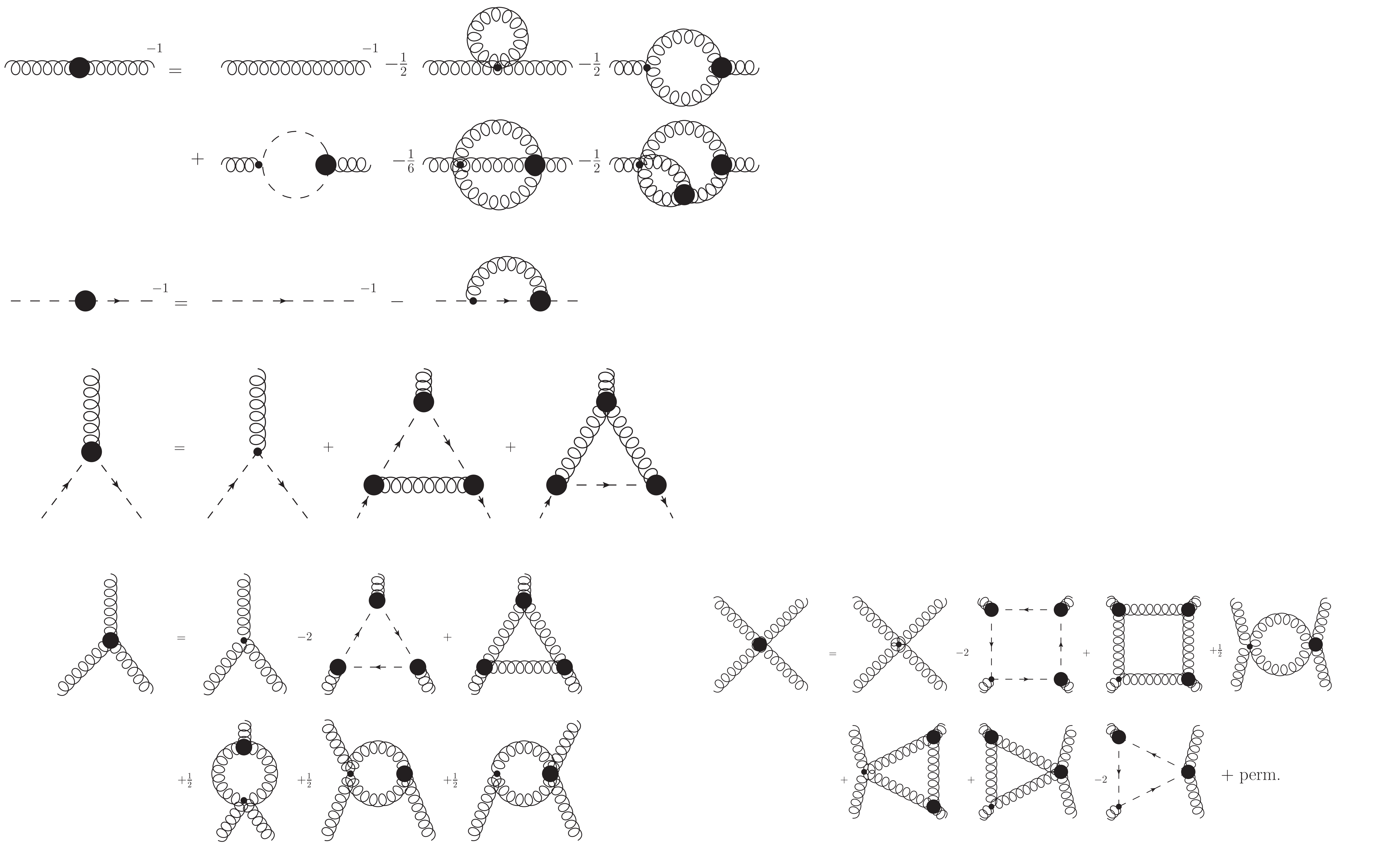
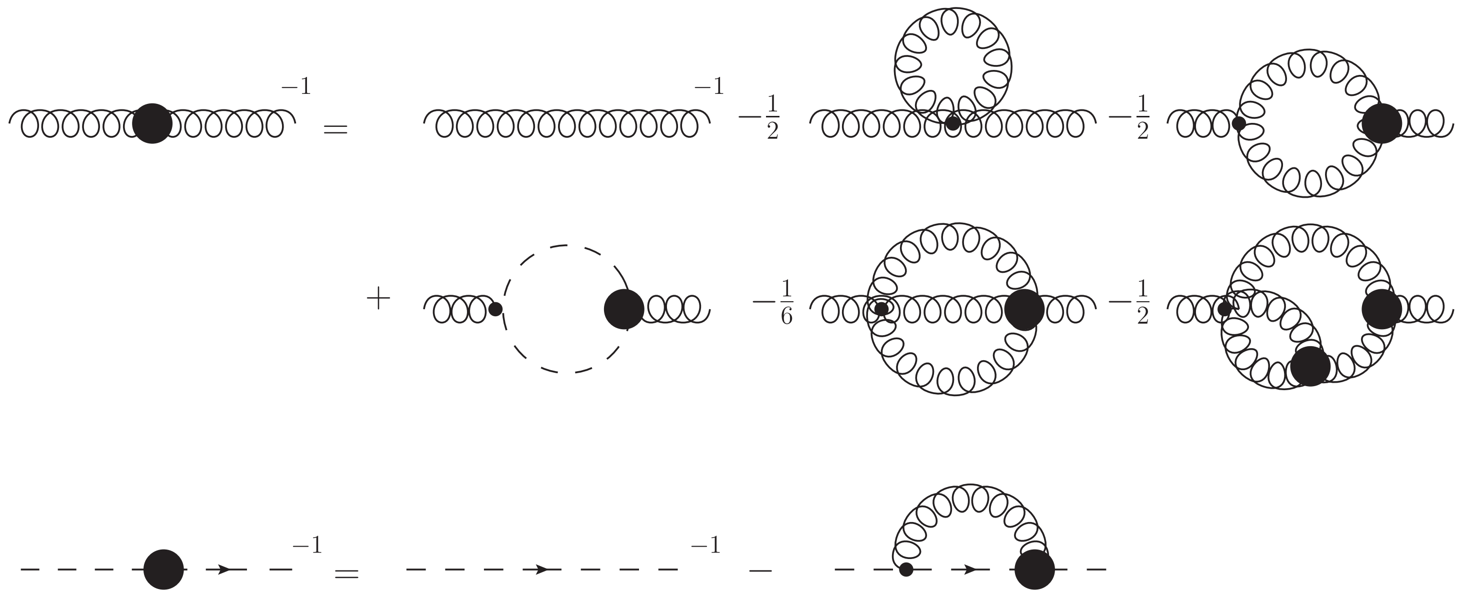


Huber, in preparation

Landau gauge gluon propagator

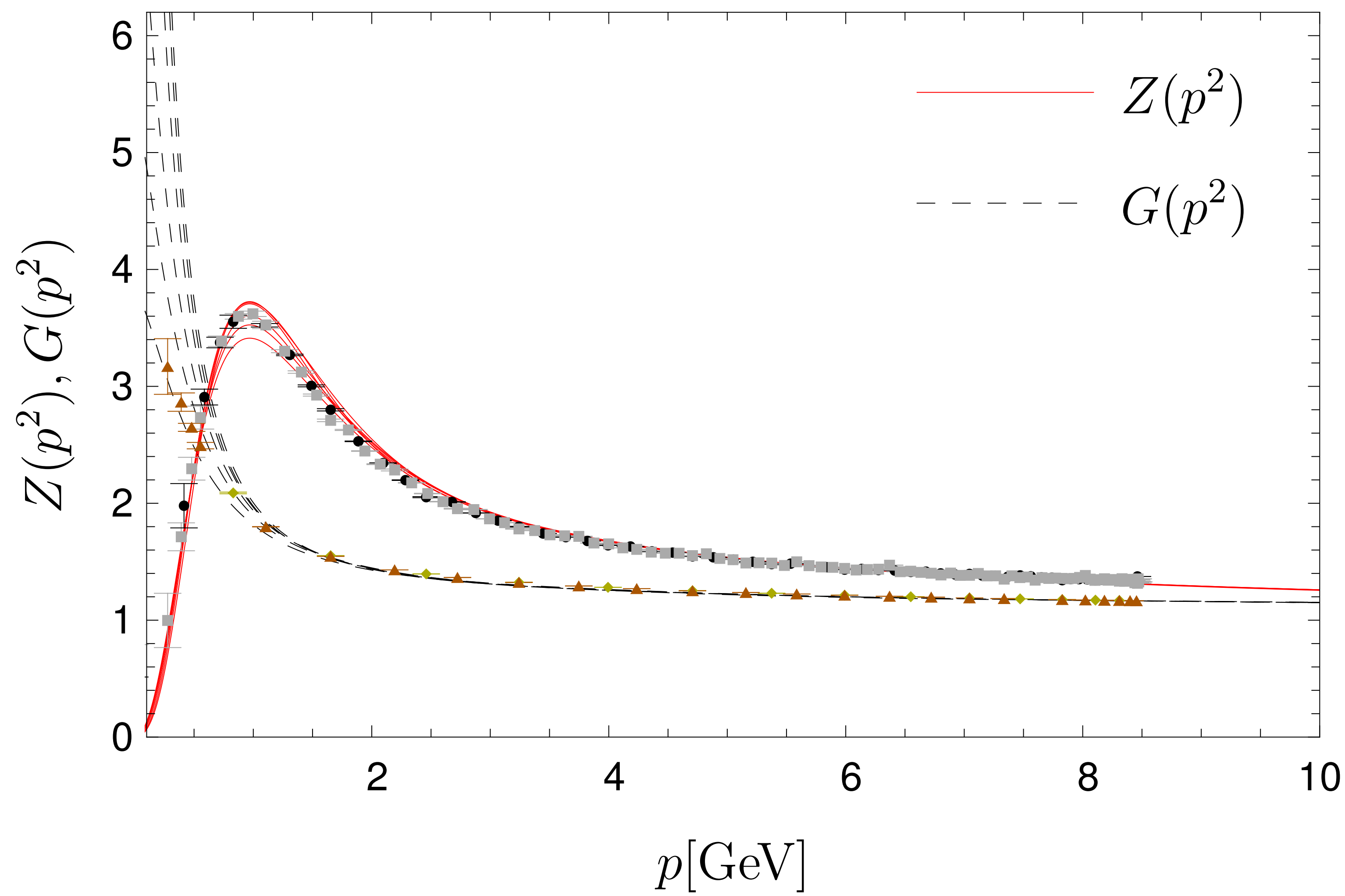


$$D_{\mu\nu}(p) = \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) \frac{Z(p^2)}{p^2}$$

- spacelike momenta: good agreement with lattice
- fully dressed gluon appears massive

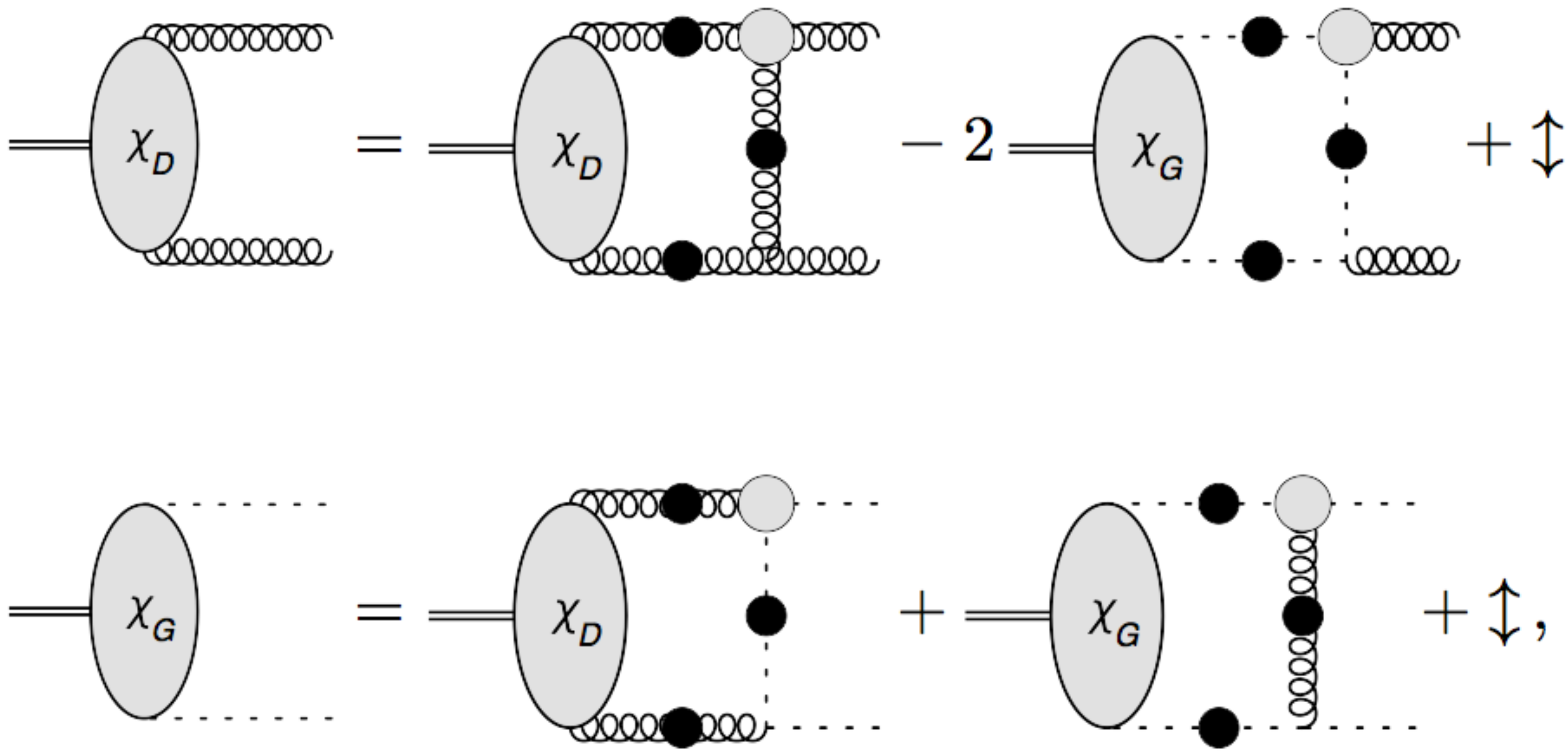
Cornwall PRD 26 (1982);
Cucchieri, Mendes PoS Lat2007 297
Aguilar, Binosi, Papavassiliou, PRD 78, 025010 (2008);
Boucaud et al. JHEP 0806 (2008) 099;
CF, Maas, Pawlowski, Annals Phys. 324 (2009) 2408

- time-like momenta: work in progress



DSE: Huber, in preparation
Lattice: Sternbeck et al.

Glueballs from DSE/BSEs



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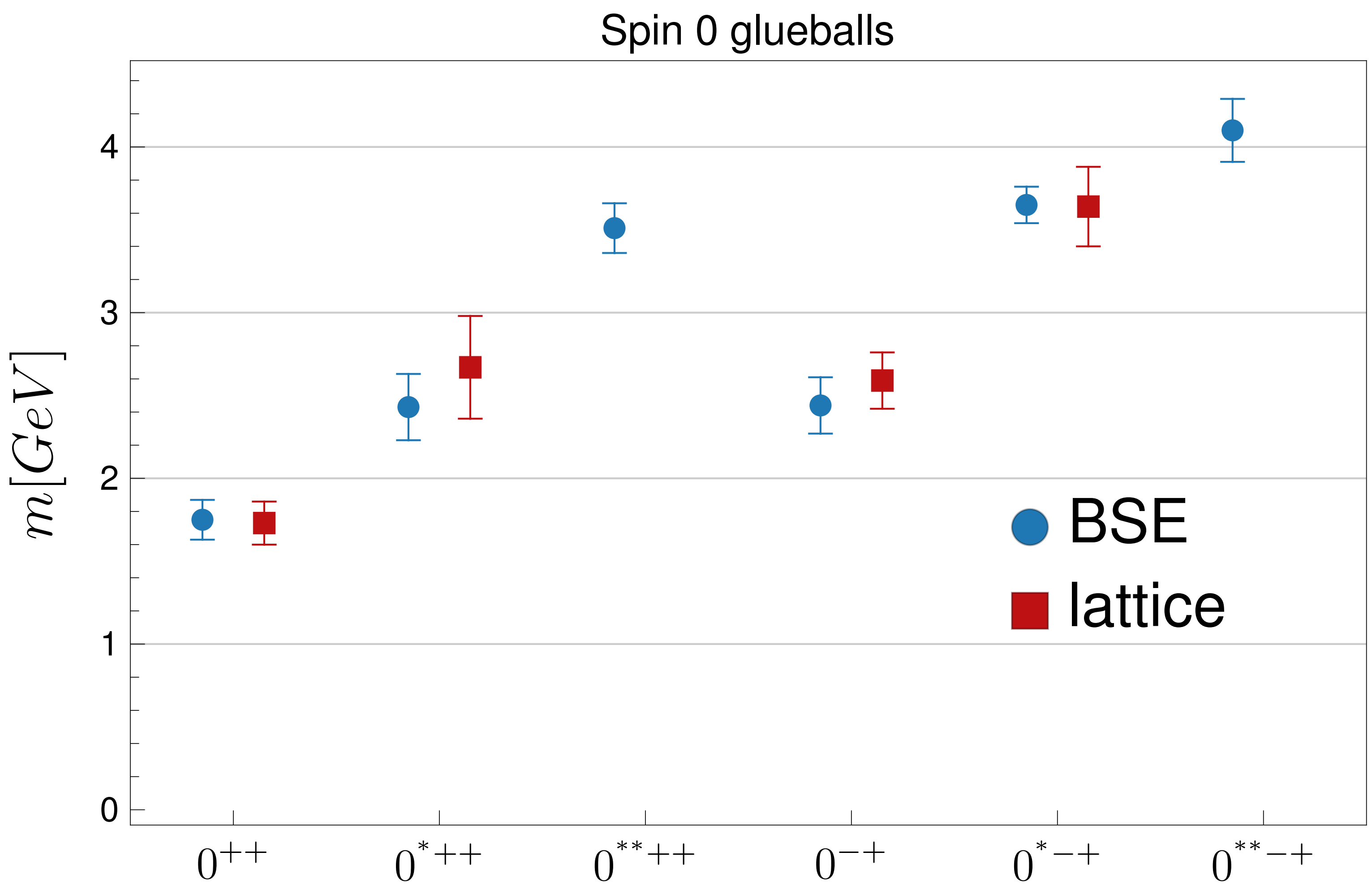
- Mixing of two-gluon amplitudes with ghost-antighost
- Probes analytical structure of gluons and ghosts

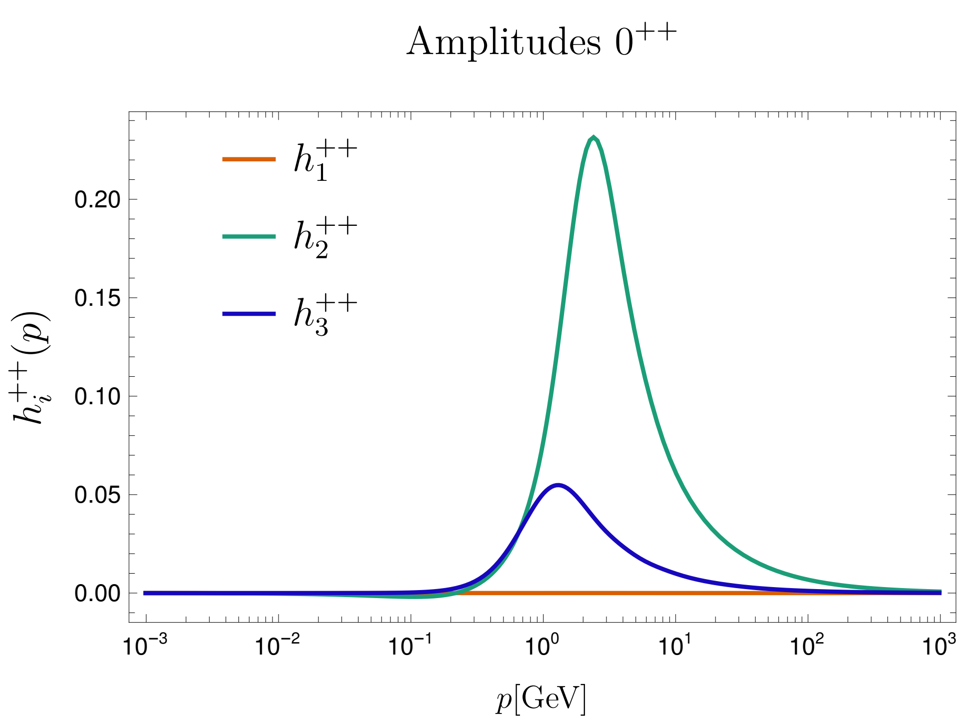
Glueballs: results

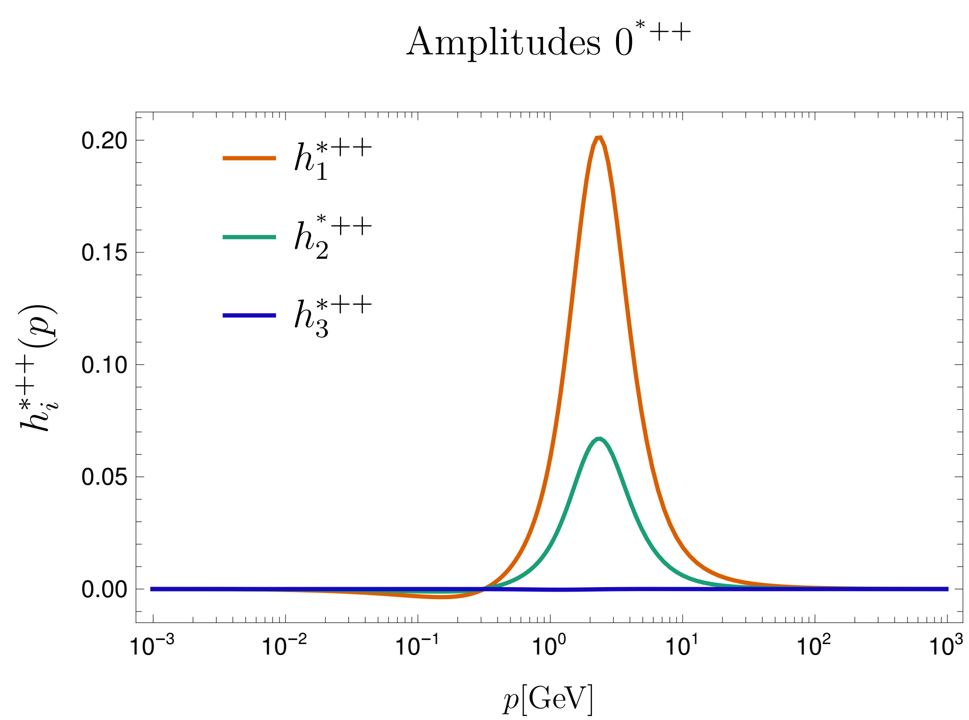


- excellent agreement lattice vs. DSE/BSE

Lattice: Morningstar and Peardon, PRD 60 (1999) 034509
Y. Chen et al., PRD 73 (2006) 014516

BSE: CF, Huber, Sanchis-Alepuz, in preparation





- different internal structure

Overview

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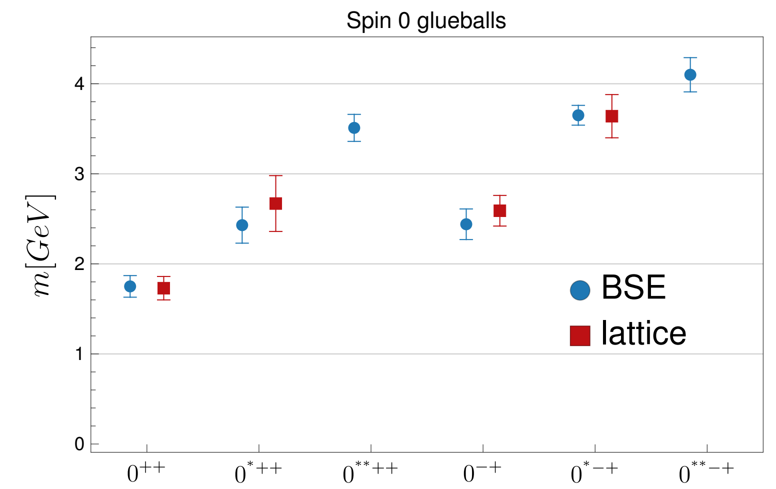

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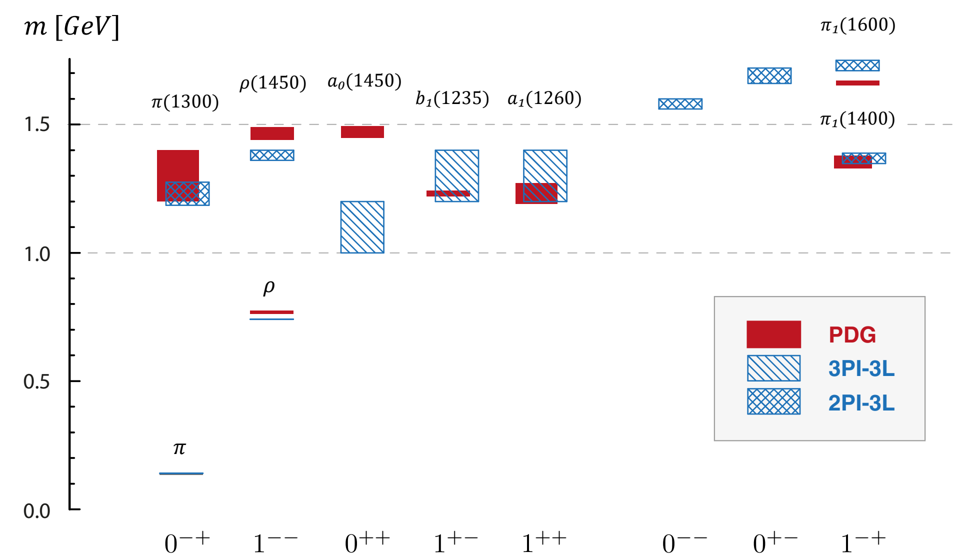

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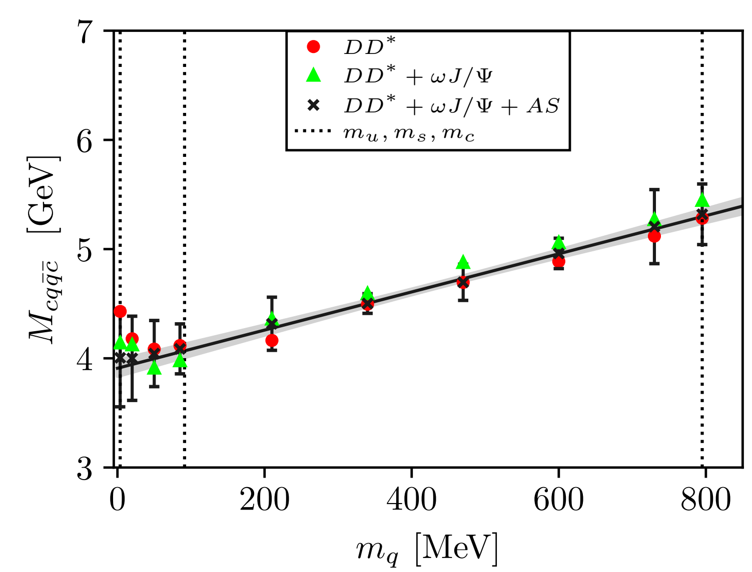
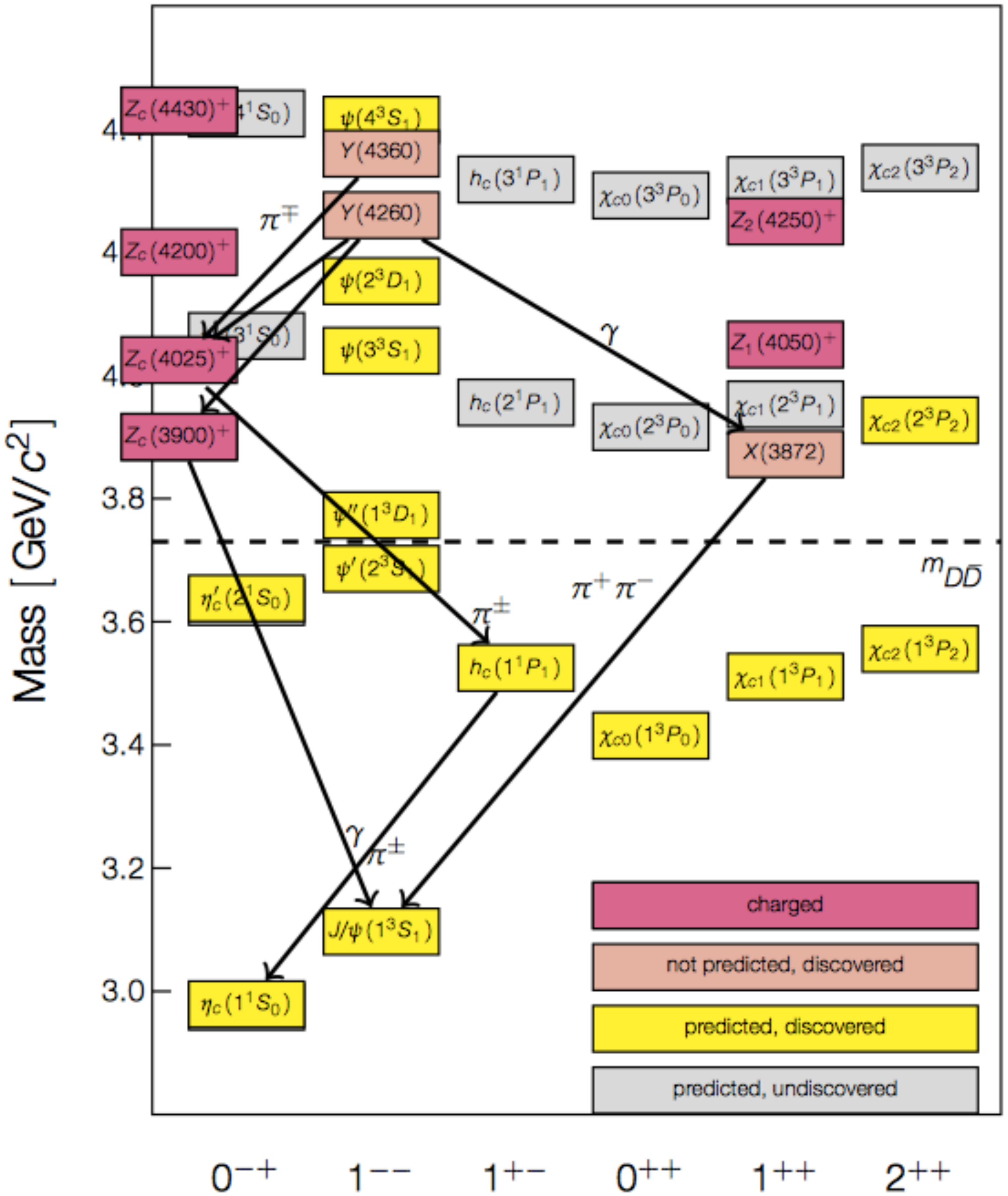
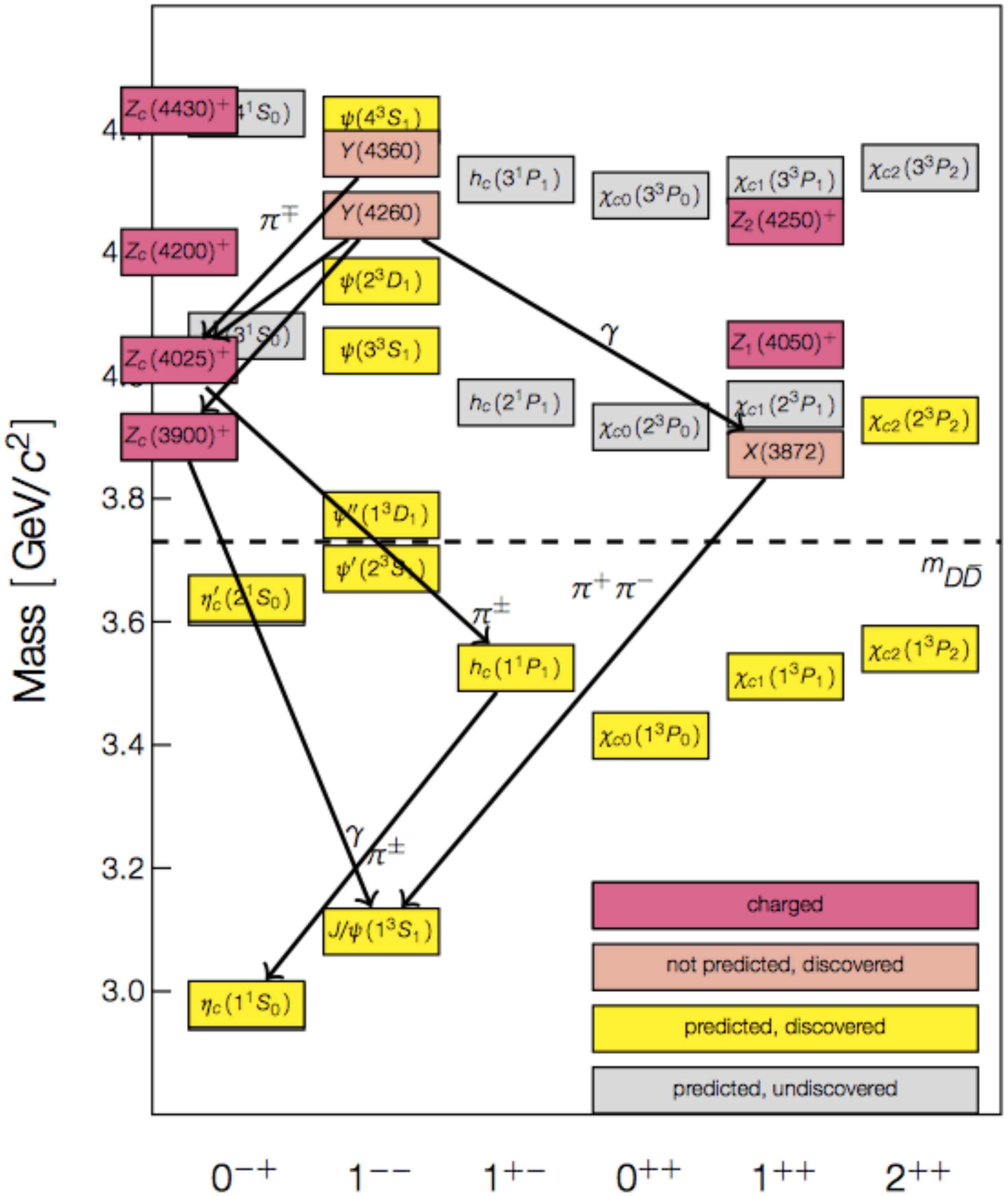


Tetraquark candidates with cq̄q̄c̄-content



Many new unexpected states found: Belle, BABAR, BES, LHCb …



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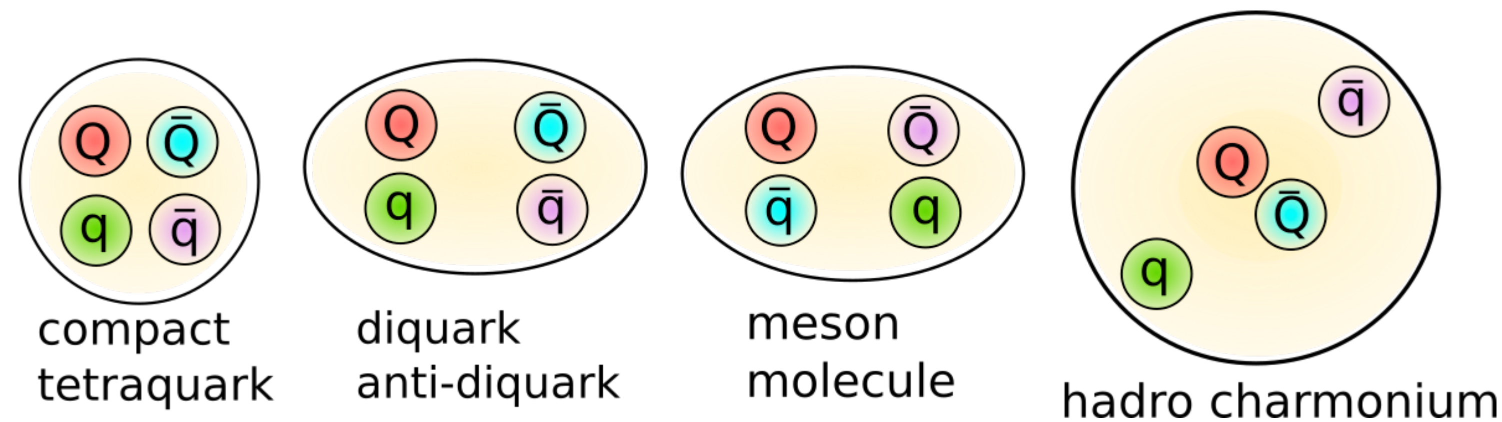
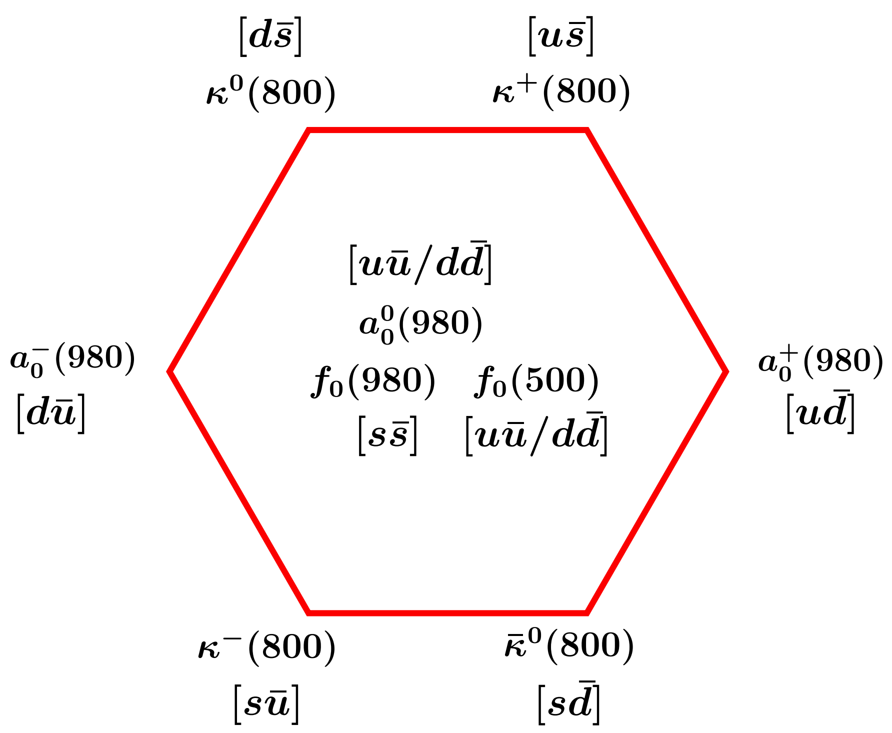
Internal structure ??



Related to details of underlying QCD forces between quarks and gluons

Tetraquark candidates with qqq̄q̄-content

Light scalar mesons:



wrong level ordering

R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

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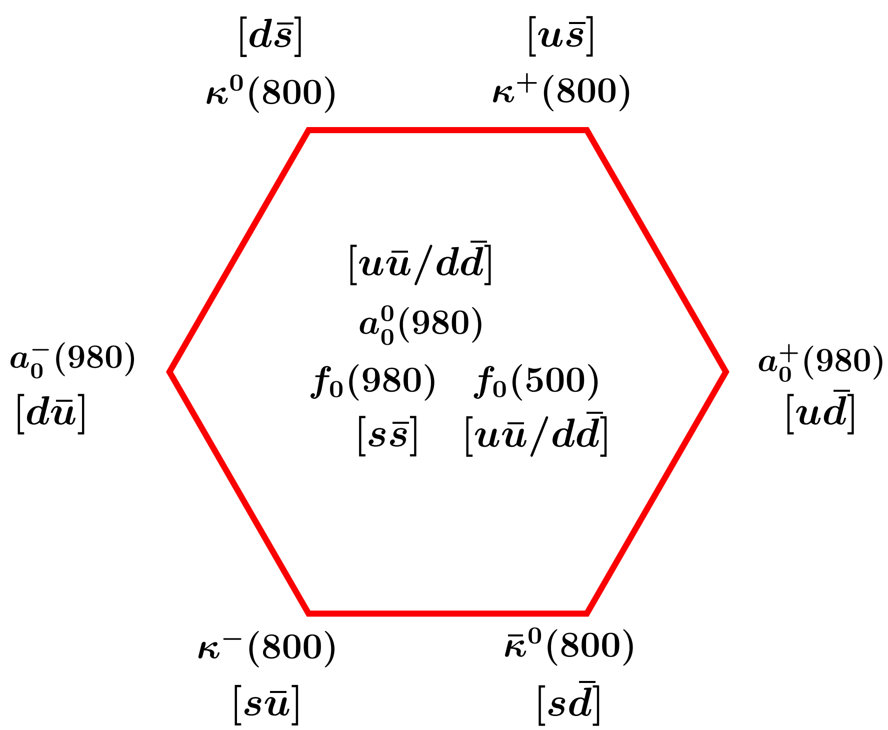

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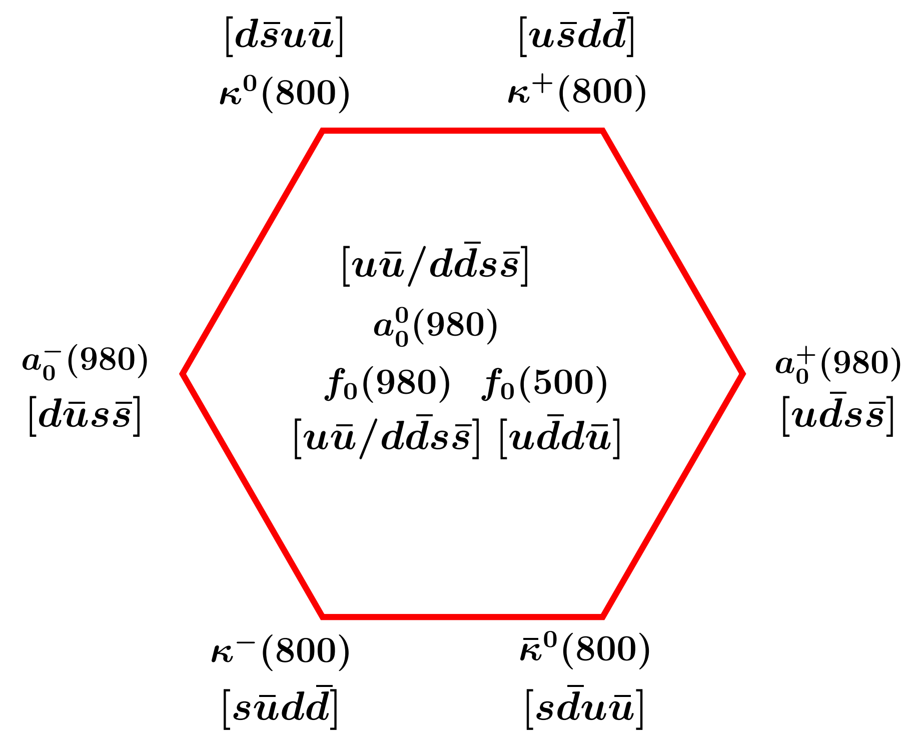
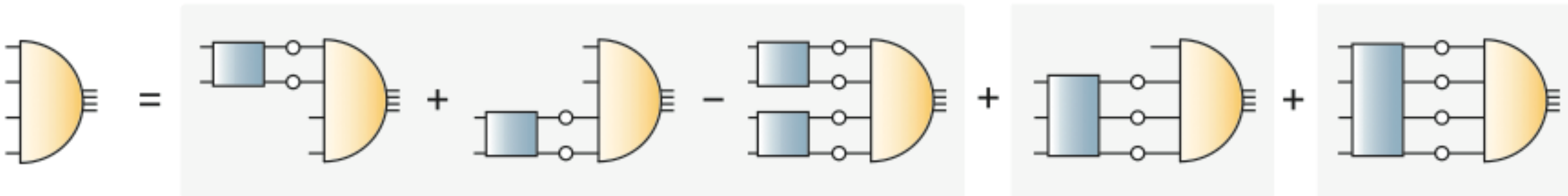


correct level ordering

R. L. Jaffe, Phys. Rev. D 15, 267 (1977)

Tetraquarks from the four-body equation

Exact equation:



Two-body interactions

Three- and four-body interactions

Kvinikhidze & Khvedelidze, Theor. Math. Phys. 90 (1992)

Heupel, Eichmann, CF, PLB 718 (2012) 545-549

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

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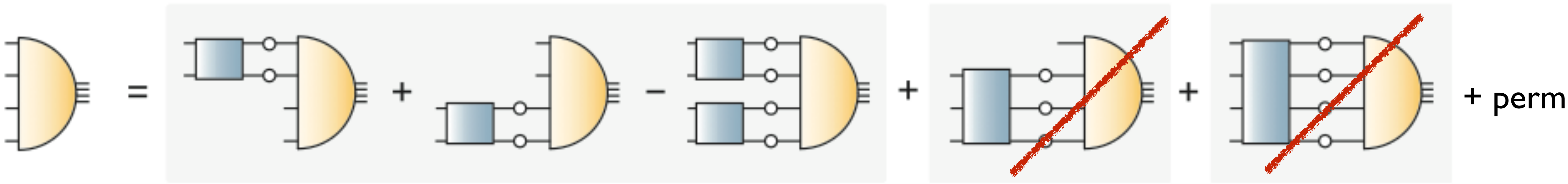

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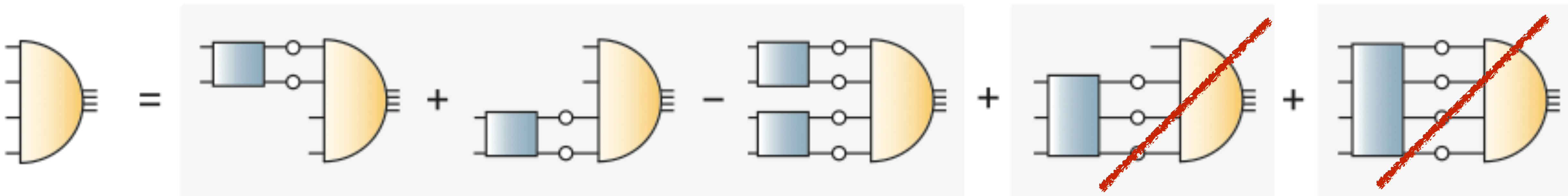

+ perm.

Two-body interactions

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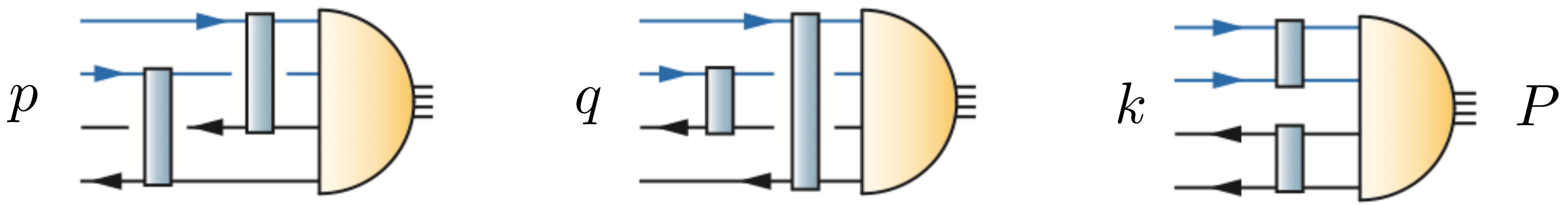
- Two-body interactions: allow for internal clustering
- use rainbow-ladder approximation…

Structure of the amplitude

Scalar tetraquark:



$$\Gamma(P,p,q,k) = \sum_i f_i(s_1,\cdots,s_9) \times \tau_i(P,p,q,k) \times color \times flavor$$

9 Lorentz scalars (built from P,p,q,k)

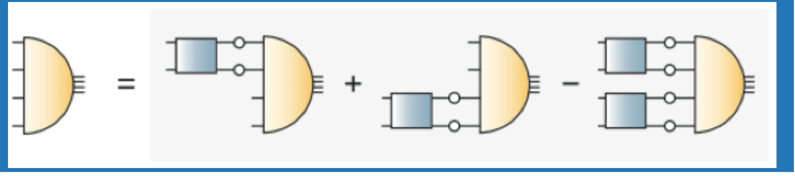
256 tensor structures (scalar tetra)

$3 \otimes \bar{3}, 6 \otimes \bar{6}$ or $1 \otimes 1, 8 \otimes 8$

- reduce # tensor structures guided by physics:

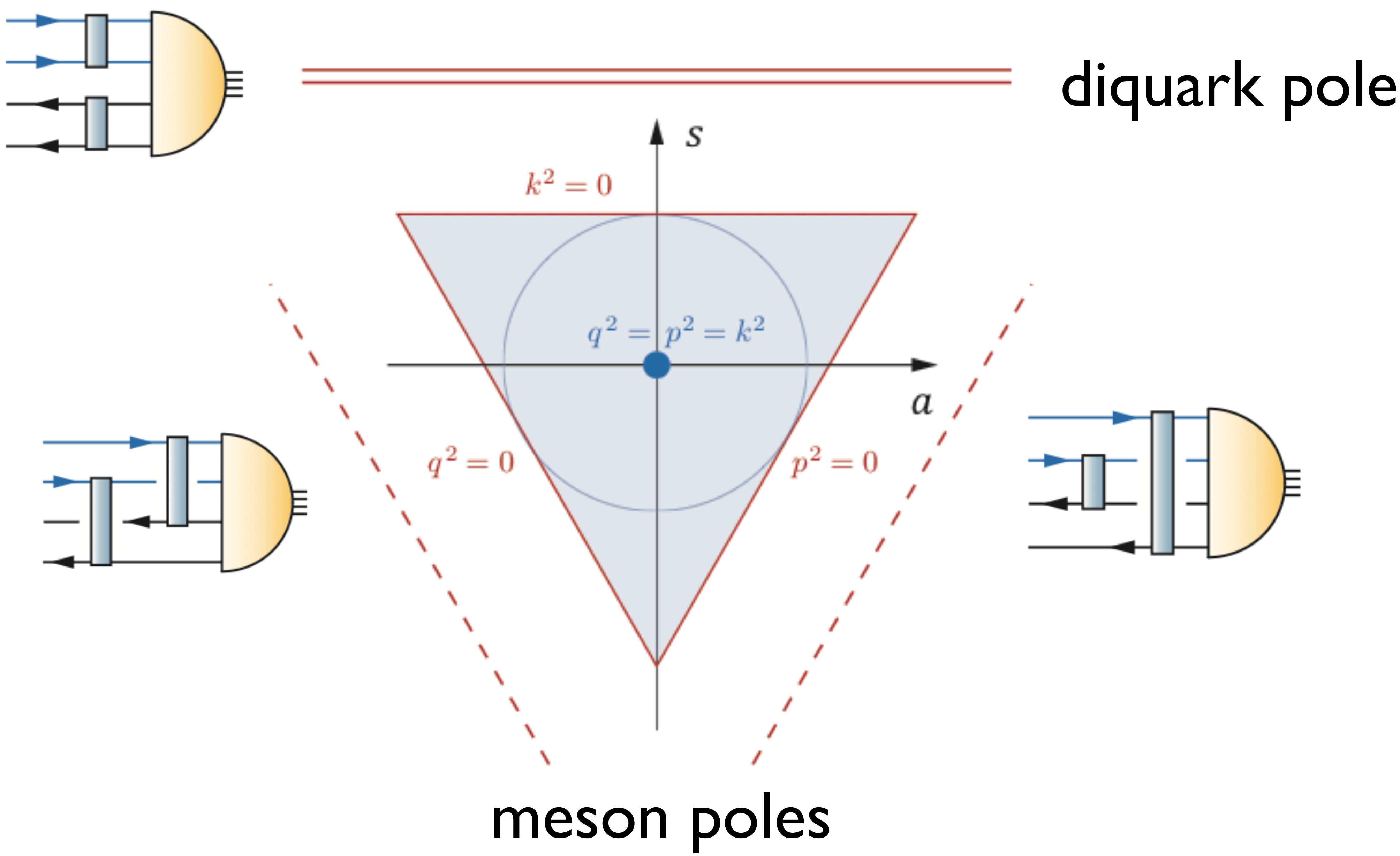
  → ~20 tensor structures

Four-body equation: permutations



- Singlet: $S_0 = (p^2 + q^2 + k^2)/4$
- Doublet: $s \sim p^2 + q^2 - 2k^2$
  $a \sim q^2 - p^2$

p,q,k: relative momenta



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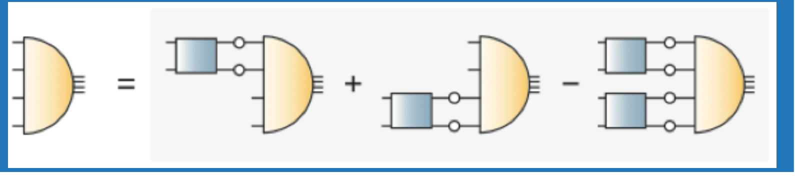

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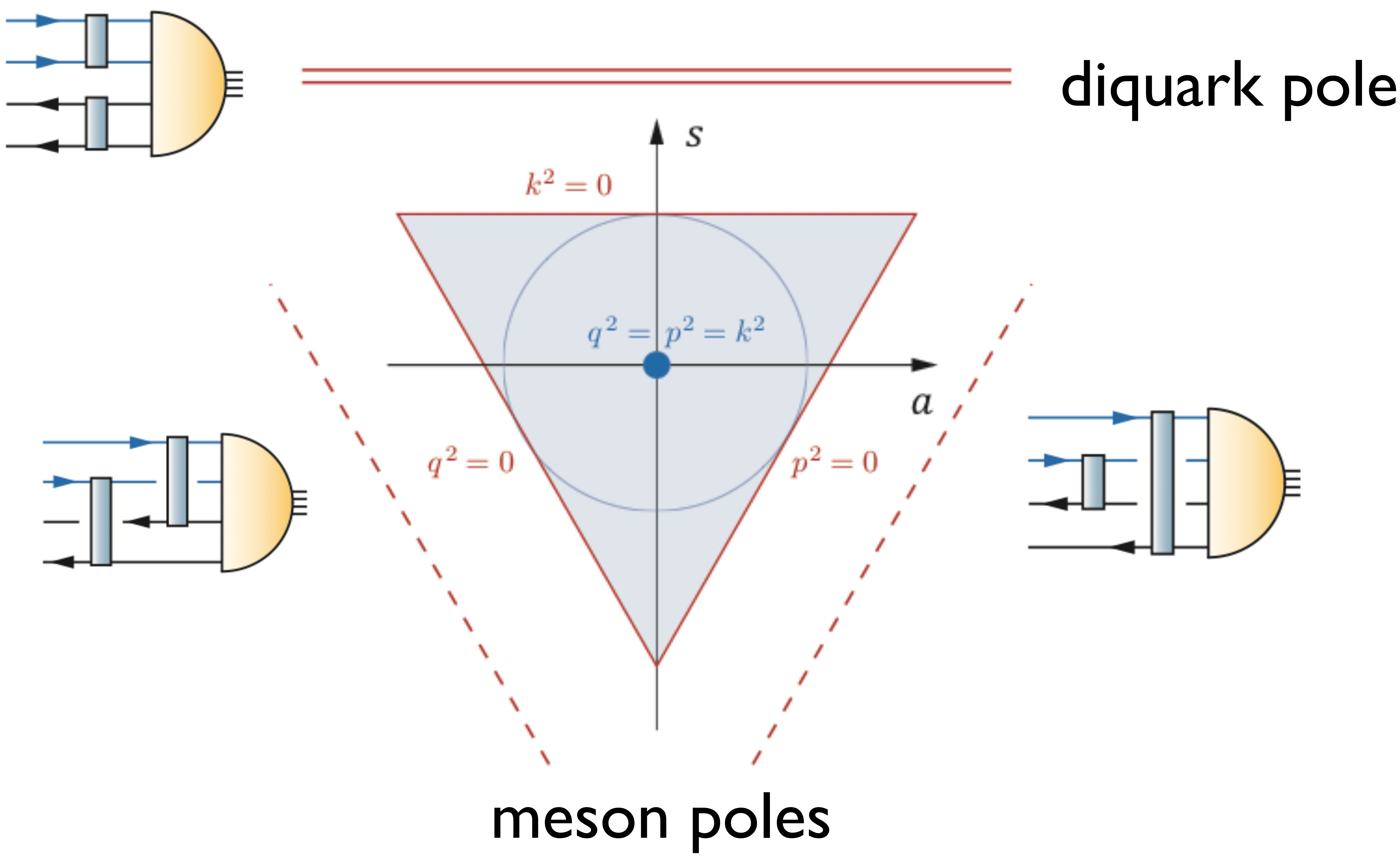

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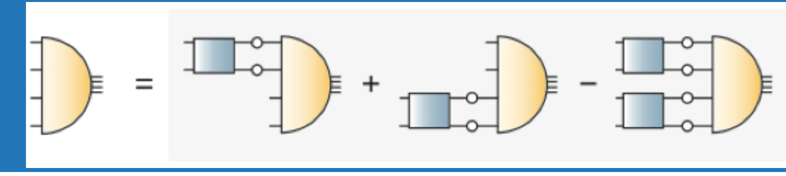

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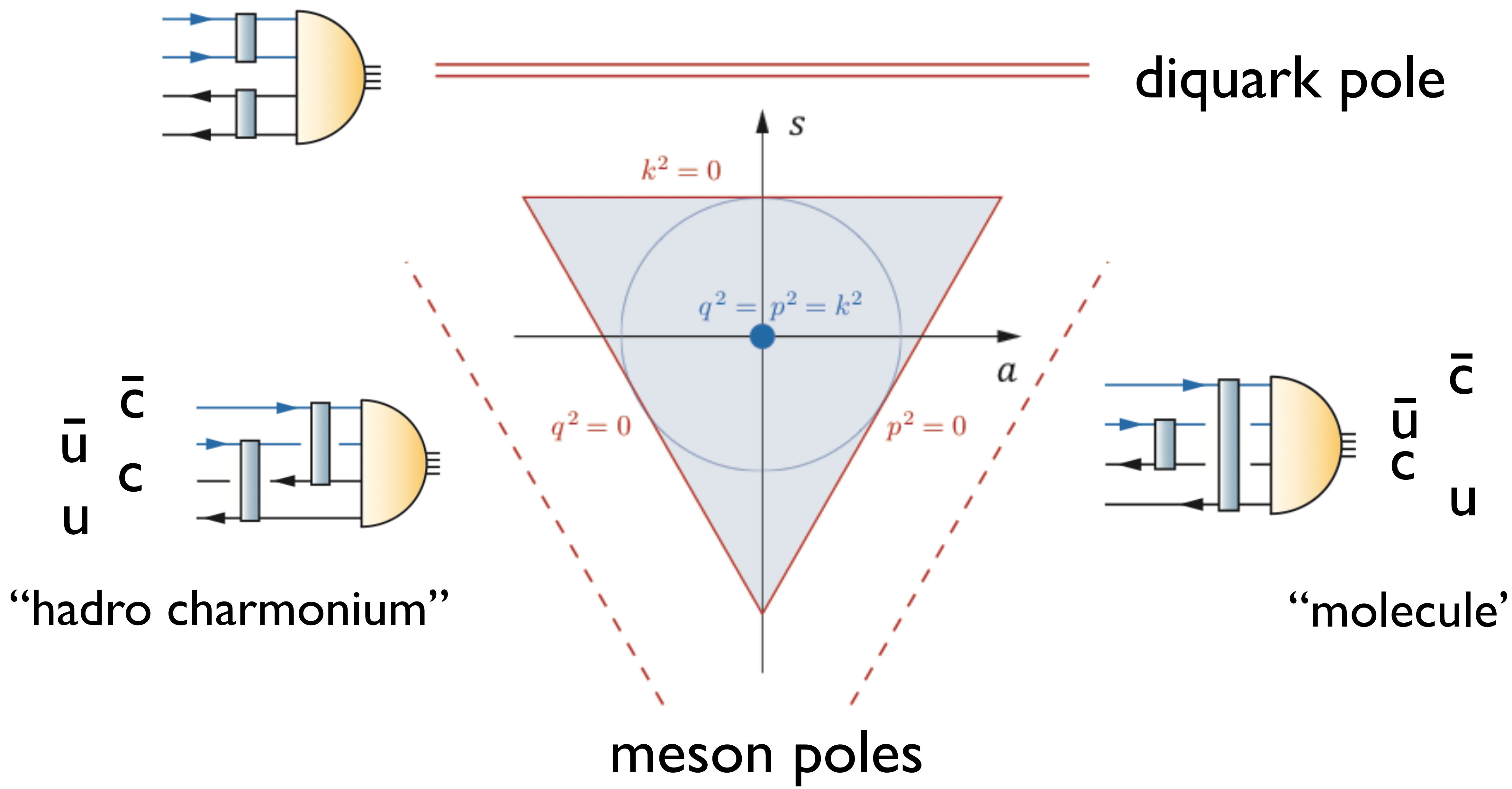
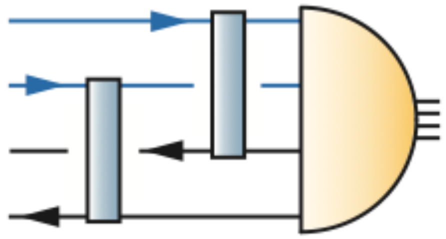
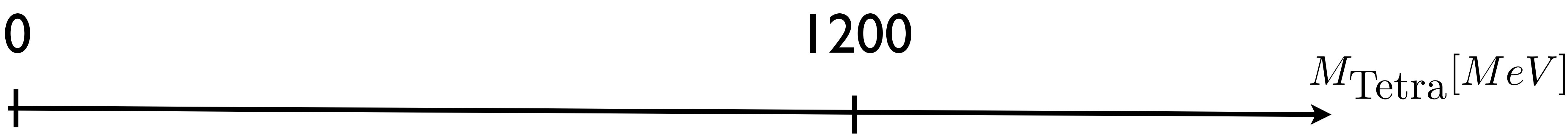


Bound state vs resonance: scalar tetraquarks



$\Gamma(S_0,$ ~~$s, a, \ldots$~~$)$

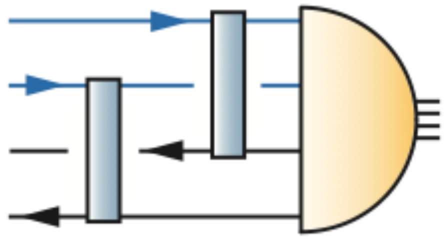
without π-clustering



Bound state of
four massive quarks

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance: scalar tetraquarks



$$\Gamma(S_0, s, a, \ldots)$$

without π-clustering

0 400 1200 $M_{\mathrm{Tetra}}[MeV]$

with π-clustering

Two-pion resonance

Bound state of
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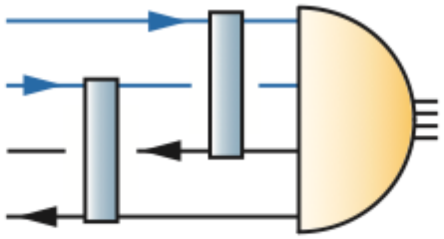
Eichmann, CF, Heupel, PLB 753 (2016) 282-287

Bound state vs resonance: scalar tetraquarks



$$\Gamma(S_0, s, a, \ldots)$$

without π-clustering

0 — 400 — 1200 — $M_{\mathrm{Tetra}}[MeV]$

with π-clustering

Two-pion resonance

Bound state of four massive quarks

⟶ identify with $f_0(500)$ ('σ-meson')

Eichmann, CF, Heupel, PLB 753 (2016) 282-287

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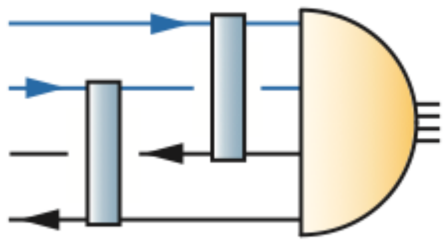


$$\Gamma(S_0, s, a, \ldots)$$

without π-clustering

0 — 400 — 1200 — $M_{\text{Tetra}}[MeV]$

with π-clustering

Two-pion resonance

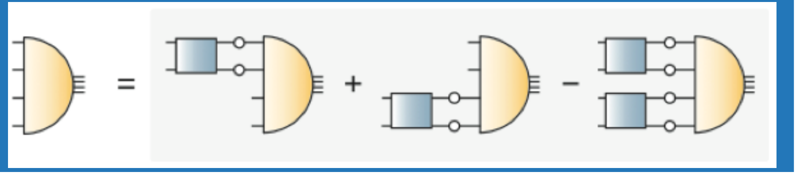
Bound state of four massive quarks

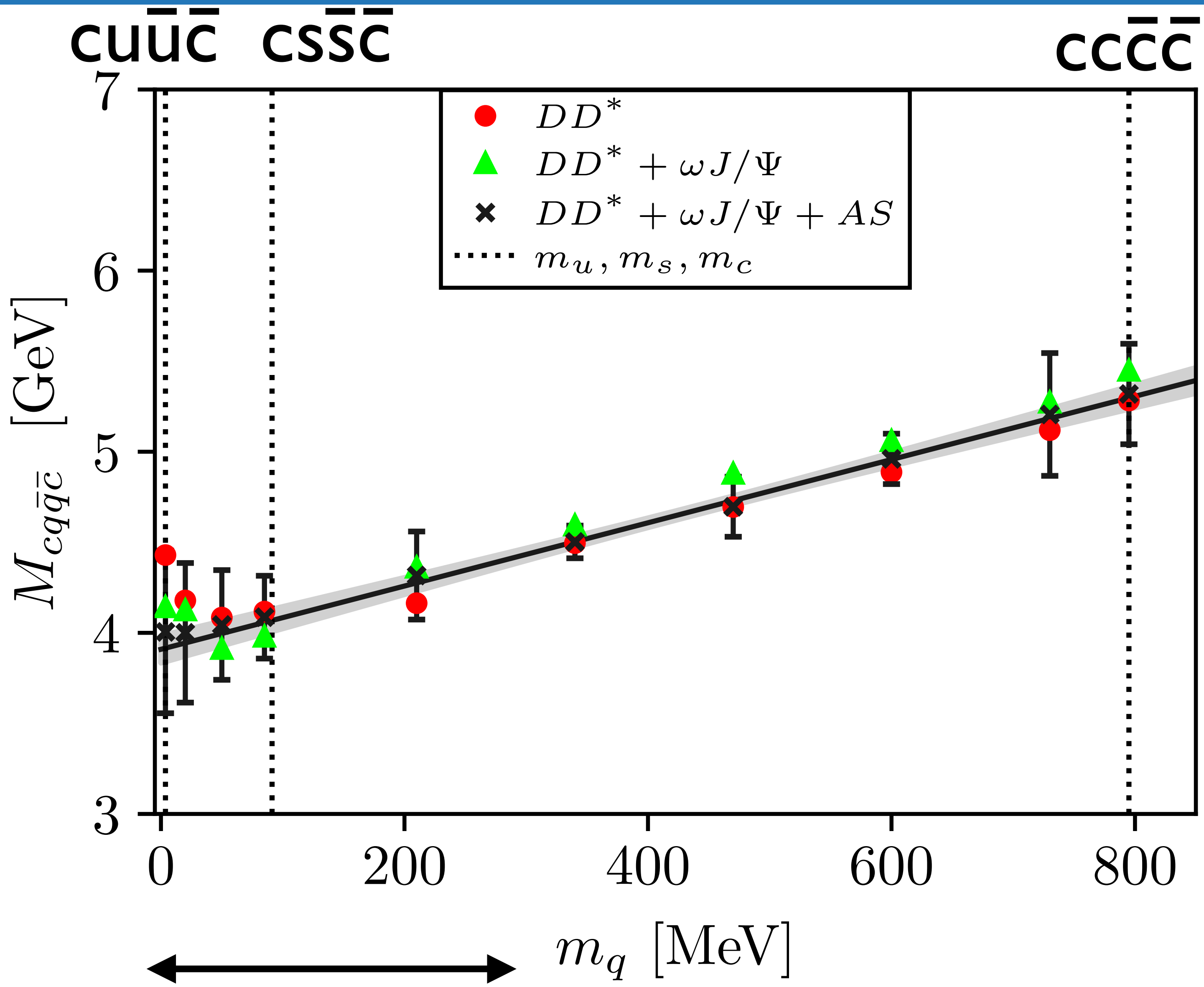
⟶ identify with $f_0(500)$ ('σ-meson')

with strange quarks: $m_\kappa \sim 750\,\text{MeV}$

$$m_{a_0, f_0} \sim 1080\,\text{MeV}$$

Eichmann, CF, Heupel, PLB 753 (2016) 282-287





m_c fixed

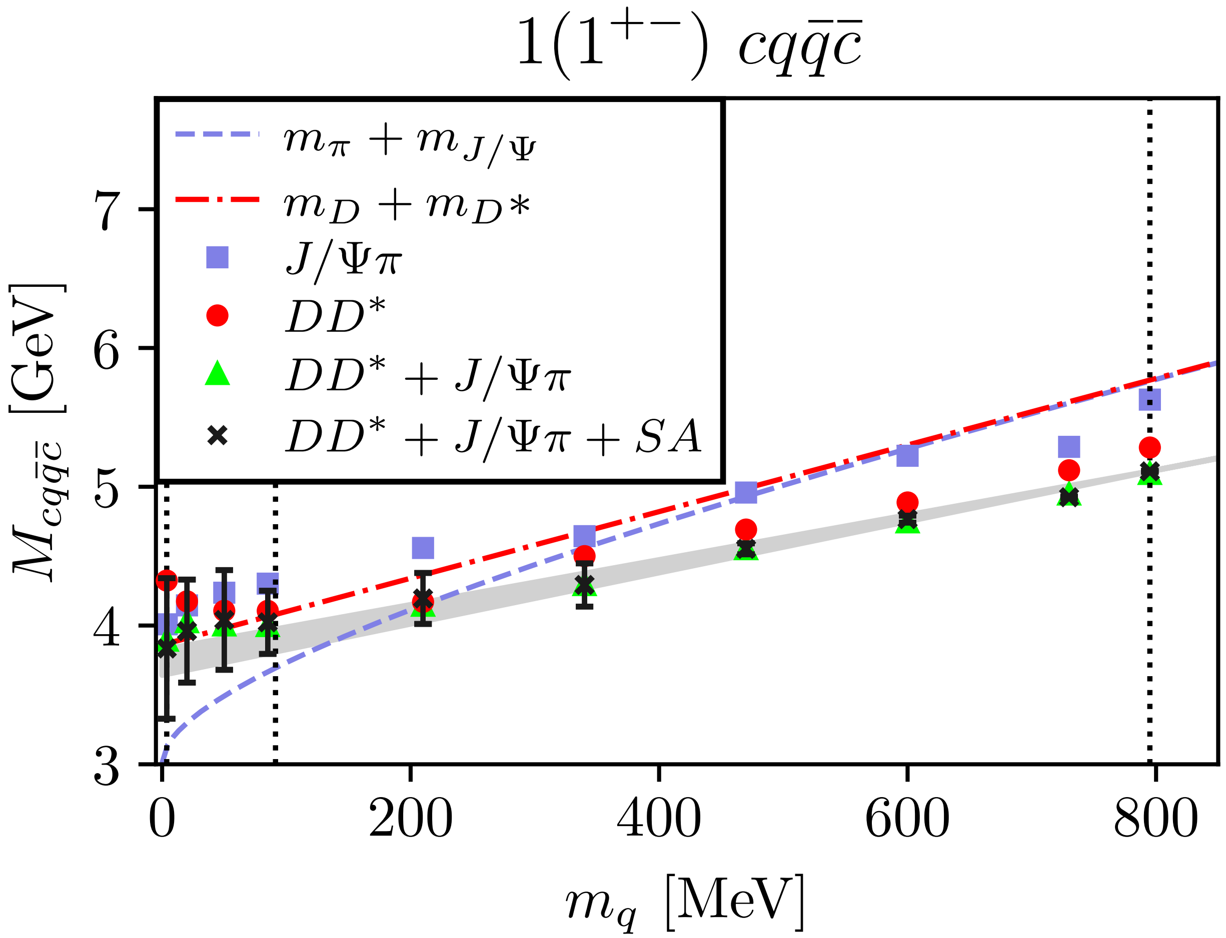
m_q varied

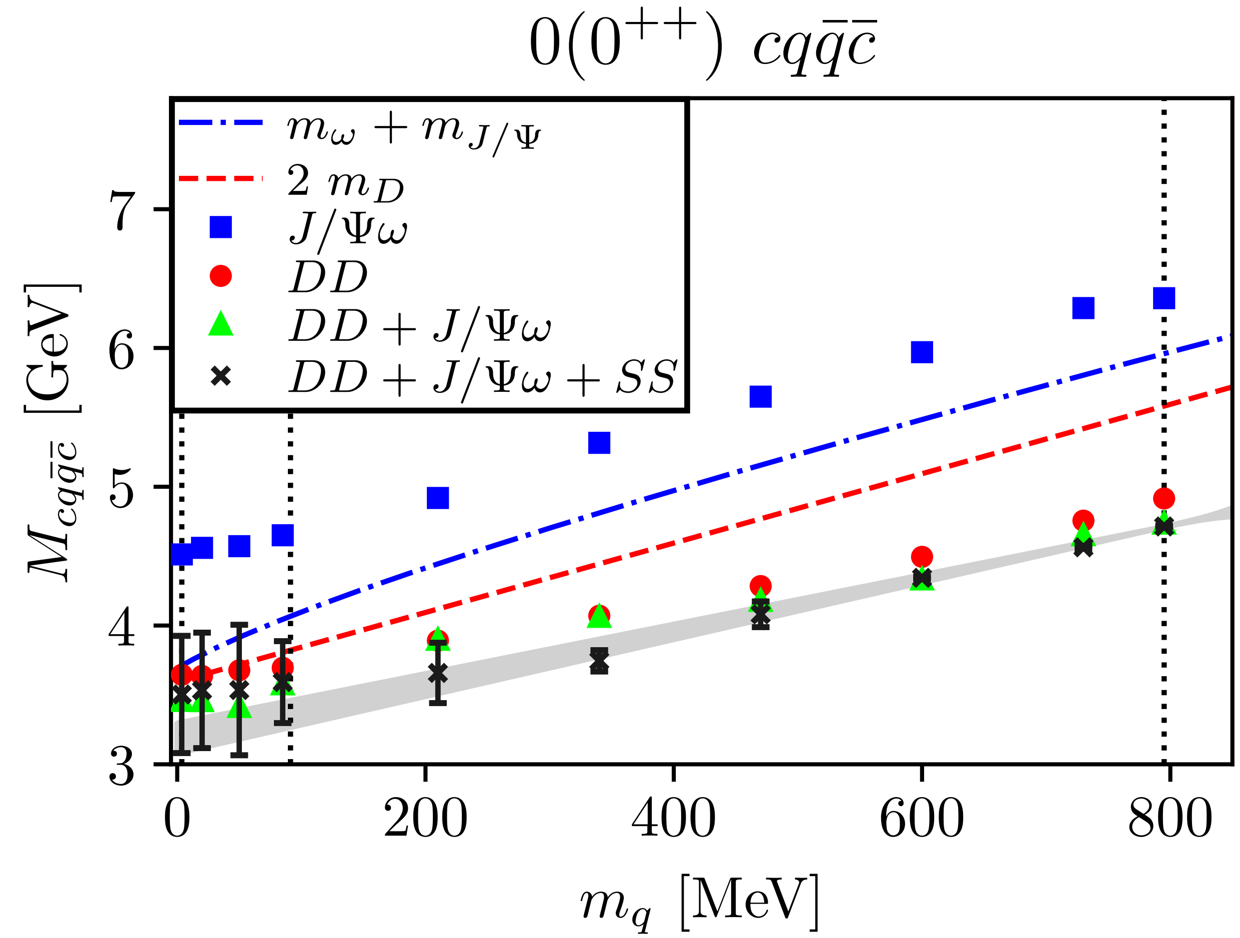
- DD* components dominate !

$$M_{1^{++}}^{cq\bar{q}\bar{c}} = 3916(74)\,\text{MeV} \longrightarrow X(3872)$$

Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

J^{PC} = 1⁺⁻ and 0⁺⁺



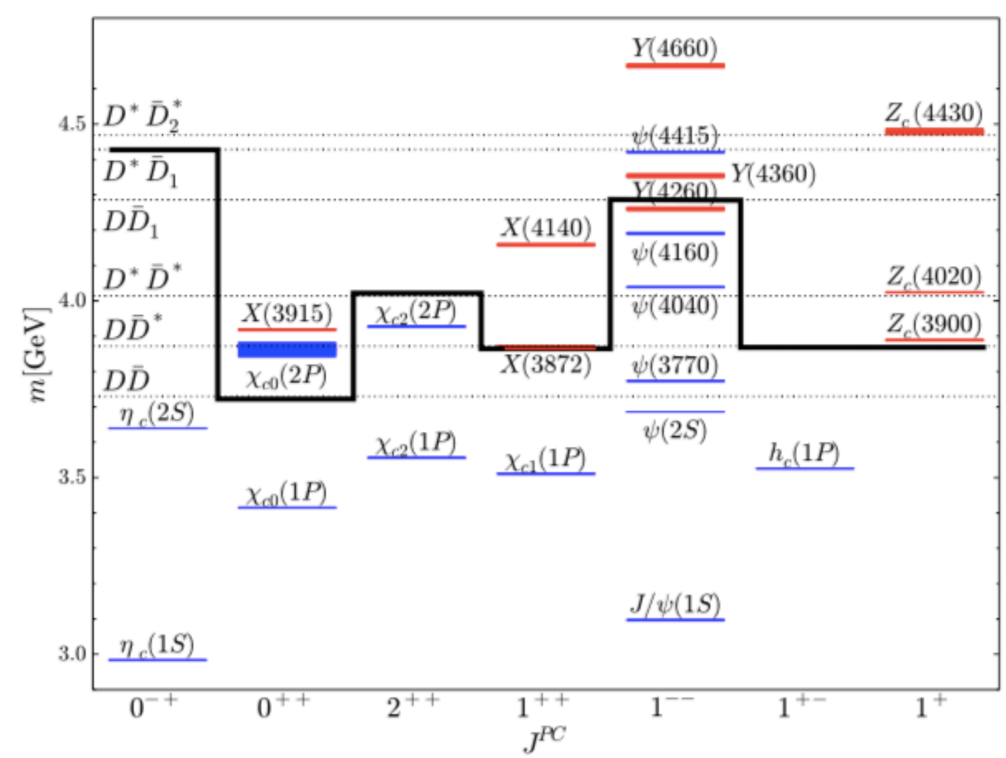


Wallbott, Eichmann and CF, in preparation

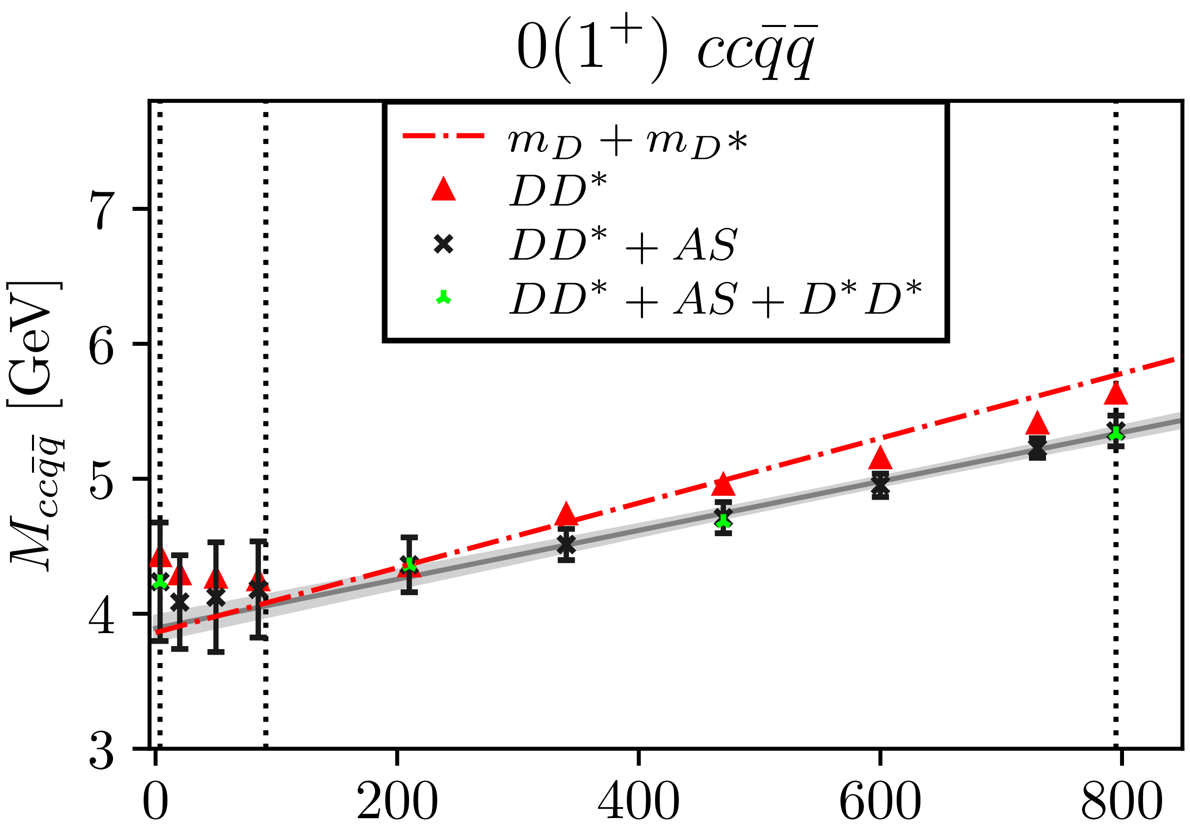
$$M_{1^{+-}}^{cq\bar{q}\bar{c}} = 3741(91) \ \rightarrow \ Z(3900)$$

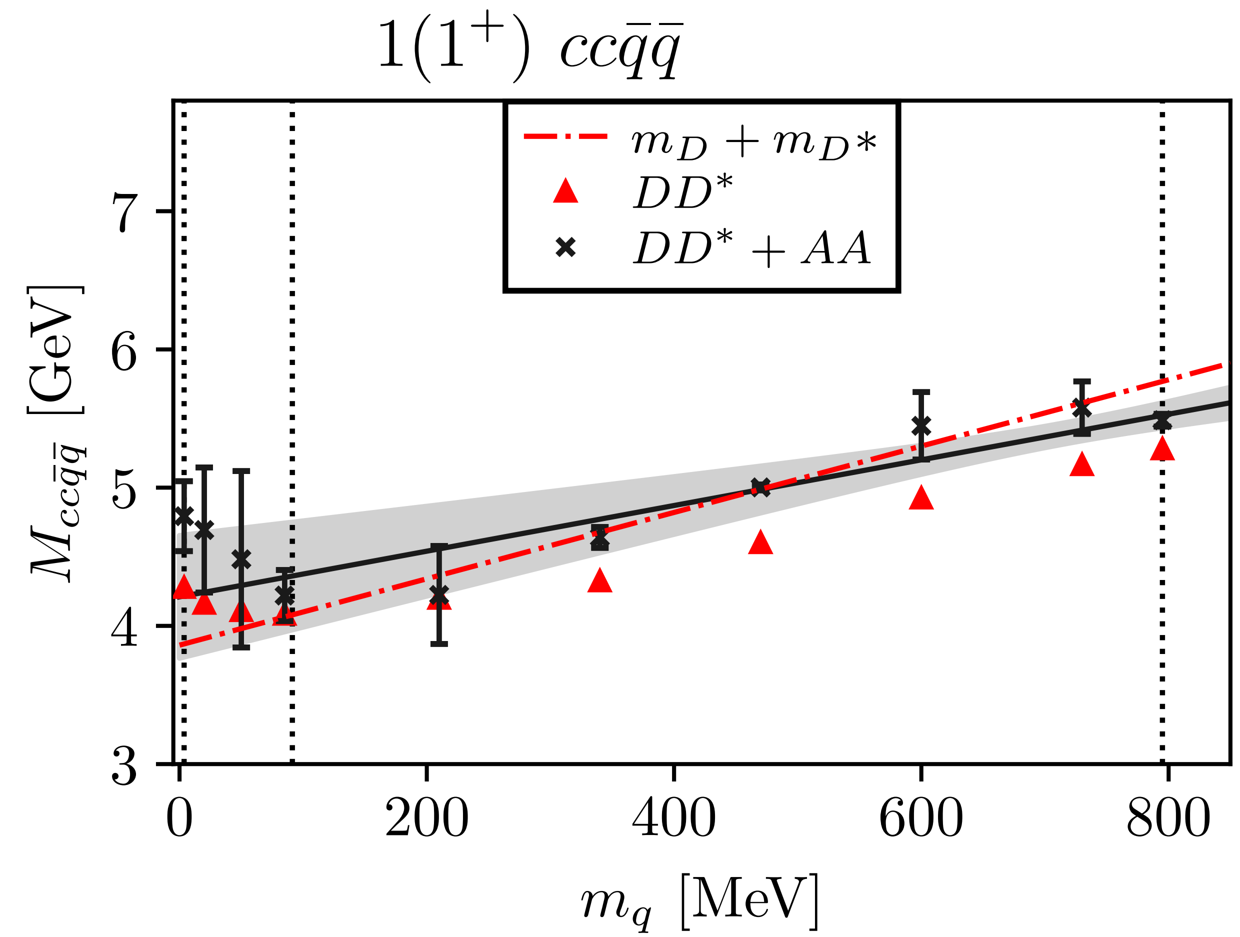
$$M_{0^{++}}^{cq\bar{q}\bar{c}} = 3195(107) \ \rightarrow ?$$

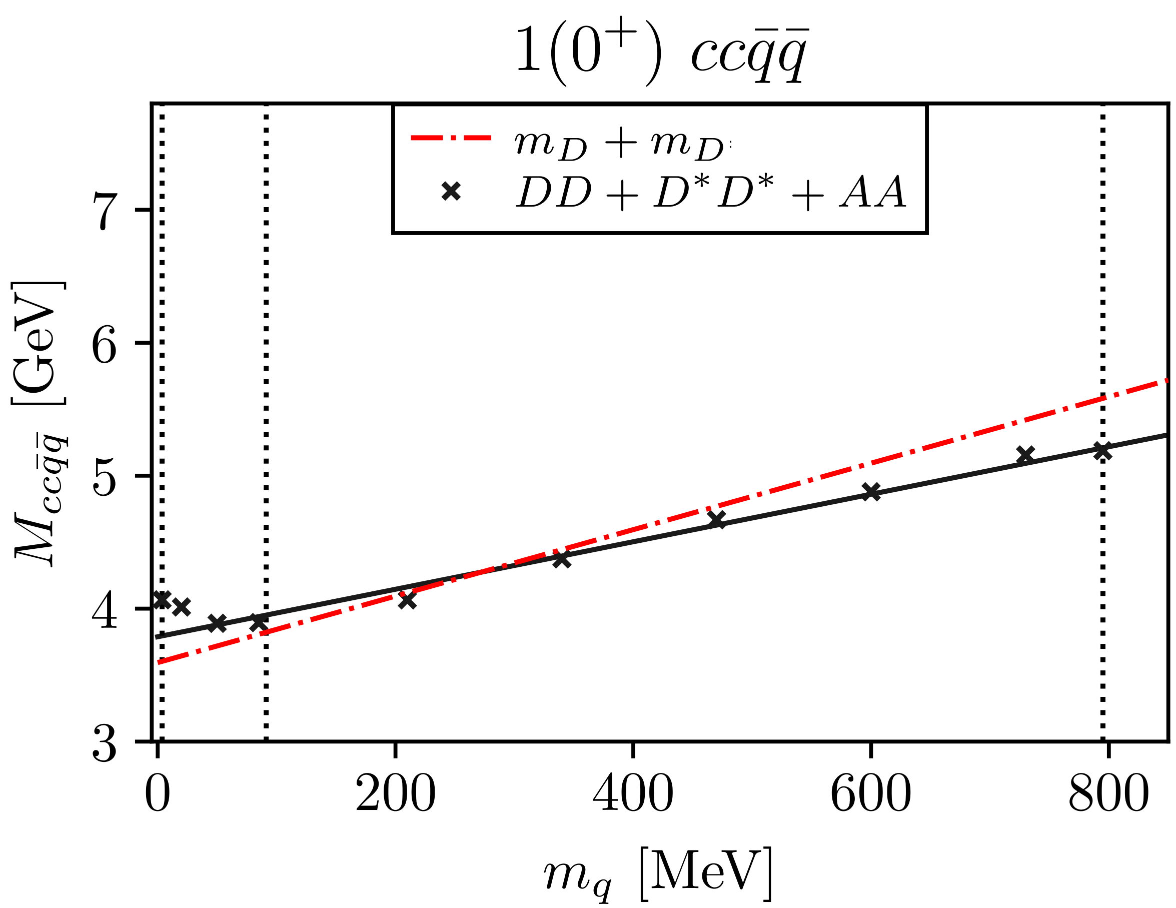
mass pattern matches molecule picture of Cleven et al. PRD 92 (2015) 014005:



Open charm four-quark states







- DD(*) and diquarks important!

Summary

Internal dynamics very important !!

Glueballs:

- First quantitatively reliable results using very involved truncation

CF, Huber, Sanchis-Alepuz, in preparation

Four-quark states:

- Closed flavor states dominated by meson-meson clusters (diquarks are almost irrelevant !)
- Dynamical description of σ: π-π resonance Eichmann, CF, Heupel, PLB 753 (2016) 282-287
- Dynamical description of X(3872) and Z(3900): DD* dominated
- First results in open charm channels

Wallbott, Eichmann and CF, PRD100 (2019) 014033, [1905.02615]

INTERNATIONAL SCHOOL OF NUCLEAR PHYSICS
42nd Course
QCD under extreme conditions

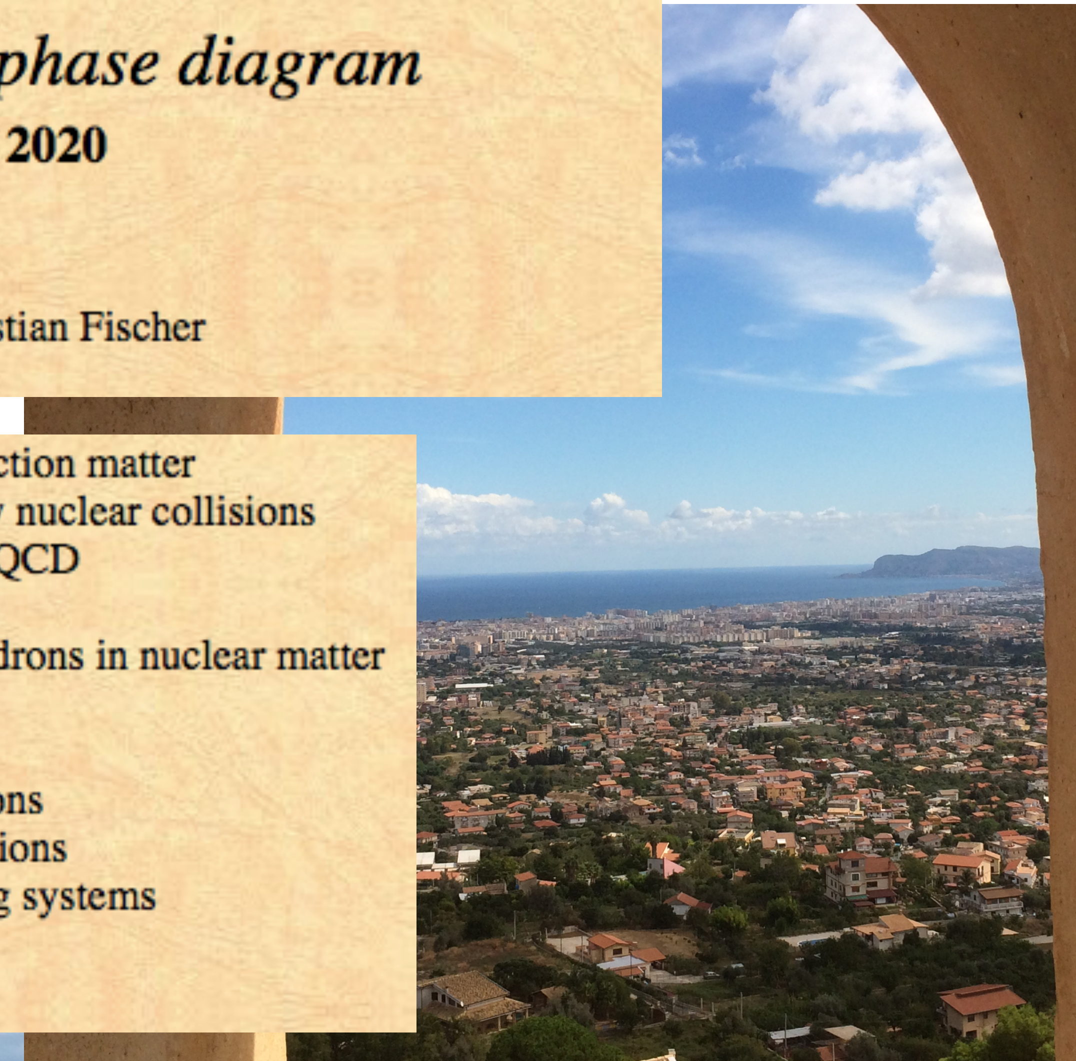
- from heavy-ion collisions to the phase diagram

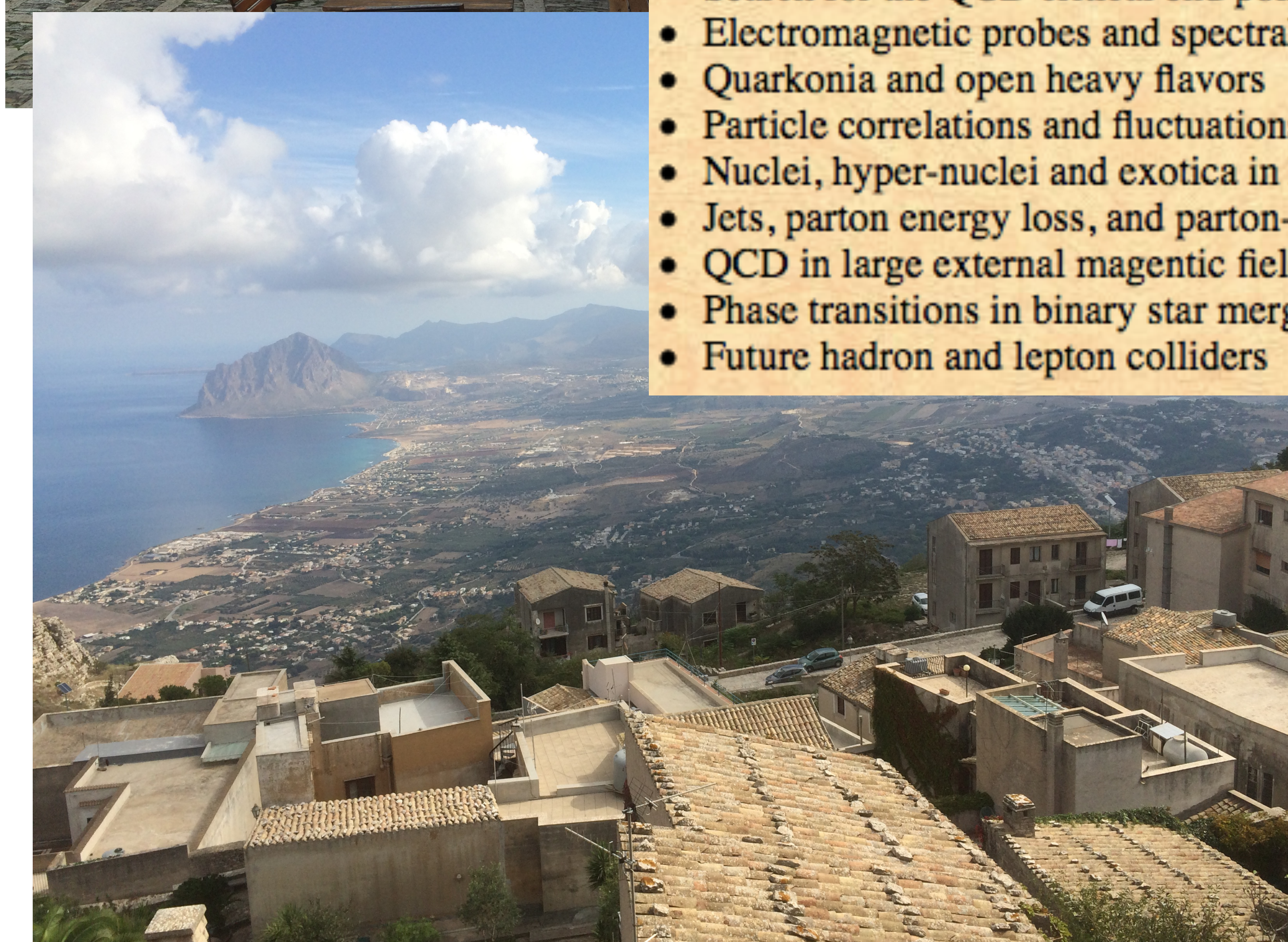
Erice-Sicily: September 16-24, 2020

Directors of the school

Michael Buballa, Amand Faessler, and Christian Fischer

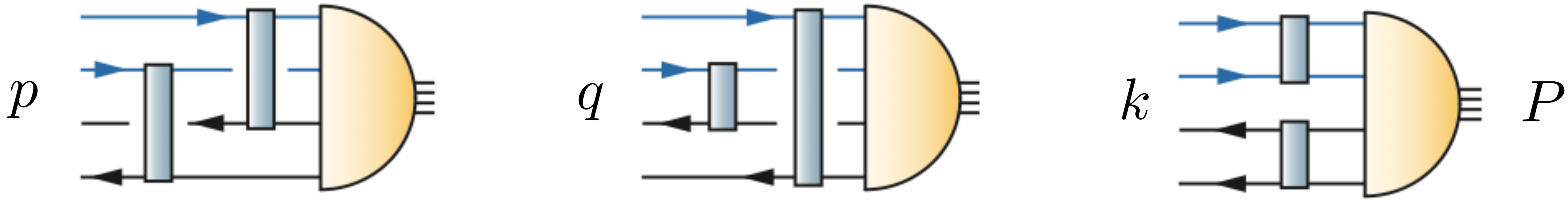
- Phase diagram and equation of state of strong interaction matter
- Phenomenology and size dependence of high-energy nuclear collisions
- Lattice and continuum approaches to hot and dense QCD
- Search for the QCD critical end point
- Electromagnetic probes and spectral functions of hadrons in nuclear matter
- Quarkonia and open heavy flavors
- Particle correlations and fluctuations
- Nuclei, hyper-nuclei and exotica in heavy ion collisons
- Jets, parton energy loss, and parton-medium interactions
- QCD in large external magentic fields and in rotating systems
- Phase transitions in binary star mergers
- Future hadron and lepton colliders





Backup Slides

Scalar tetraquark:



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9 Lorentz scalars (built from P,p,q,k)

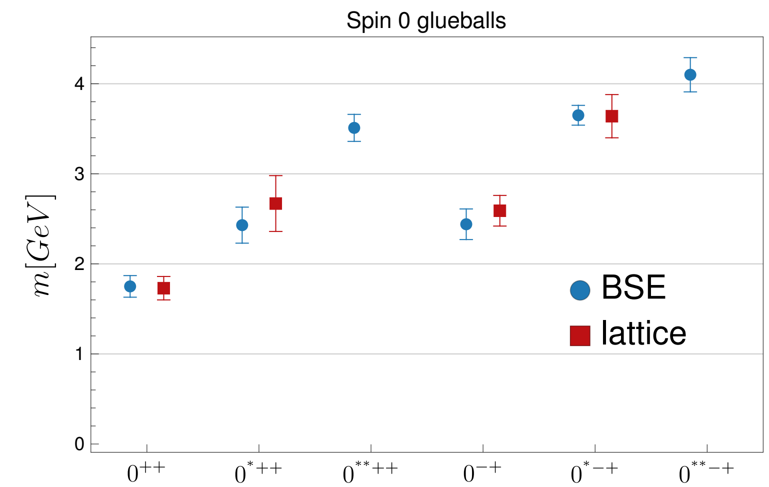
256 tensor structures (scalar tetra)

$3\otimes\bar{3}, 6\otimes\bar{6}$ or $1\otimes 1, 8\otimes 8$

- reasonable approximation: keep s-waves only;
  → 16 tensor structures
- $\Gamma(P,p,q,k) \to \Gamma(S_0,s,a,...)$

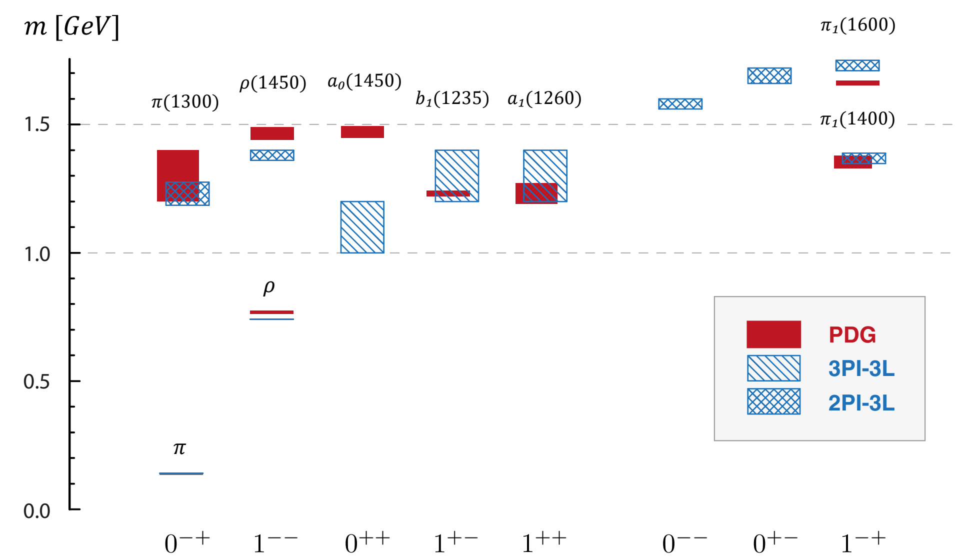
1. Glueballs in Yang-Mills theory

CF, Huber, Sanchis-Alepuz, in preparation



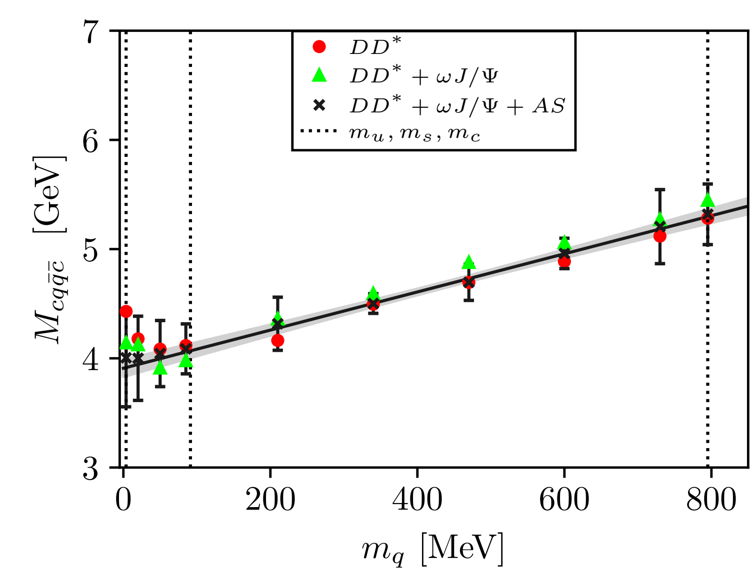
2. Quark masses and light meson spectroscopy

Williams, CF, Heupel, PRD93 (2016) 034026

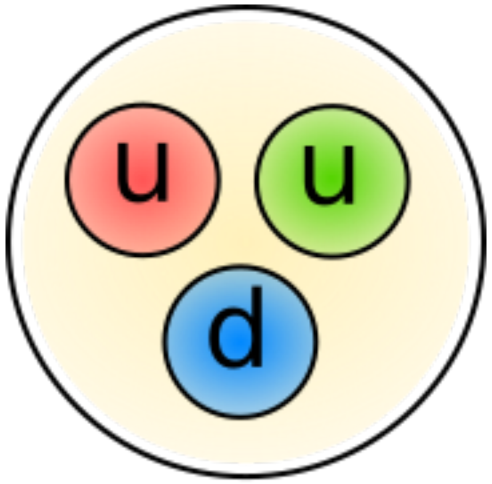


3. Heavy-light tetraquarks: X(3872) and more…

Wallbott, Eichmann and CF, PRD100 (2019) no.1, 014033, arXiv:1905.02615
Wallbott, Eichmann and CF, in preparation



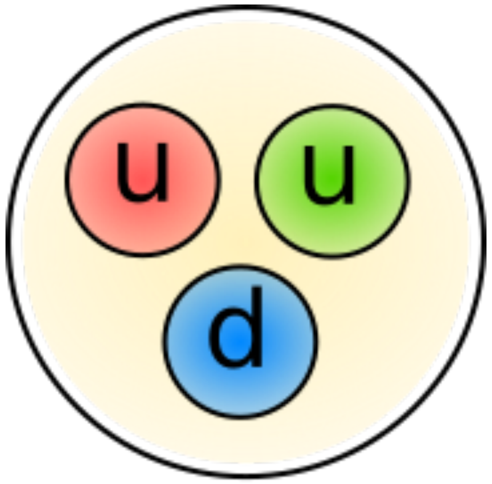
Properties of QCD: Dynamical mass generation



$m_{proton} = 938$ MeV

Dynamical quark masses via weak force

quarks	u	d	s	c	b	t
M_{weak} [MeV]	3	5	80	1200	4500	176000



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Dynamical quark masses via weak force and strong force:

quarks	u	d	s	c	b	t
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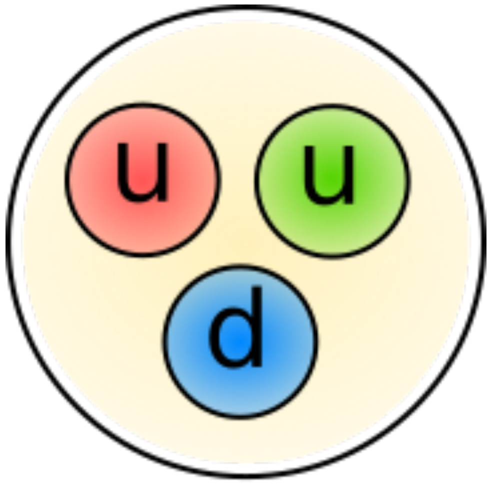

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Yoichiro Nambu,
Nobel prize 2008

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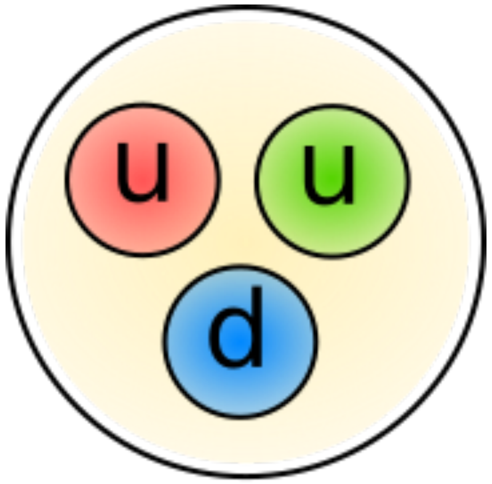

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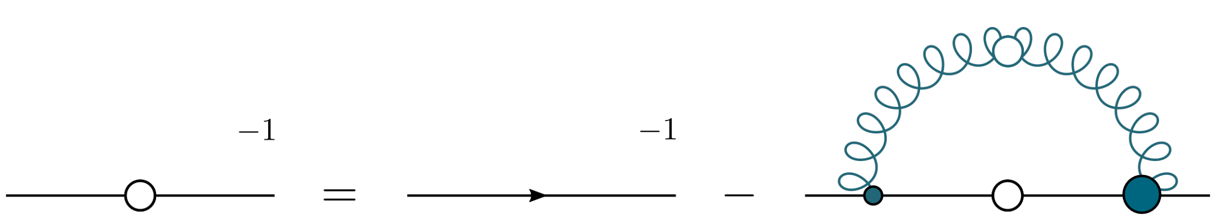
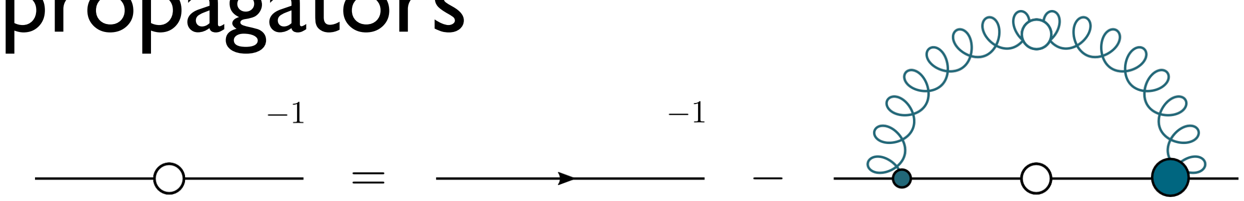


Dyson-Schwinger equations - "3PI vs RL"

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A] \exp\left\{ - \int d^4x \left(\bar{\Psi} \left(i\not{D} - m \right) \Psi - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 \right) \right\}$$

propagators



CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, EPJ C77 (2017) no.11, 733

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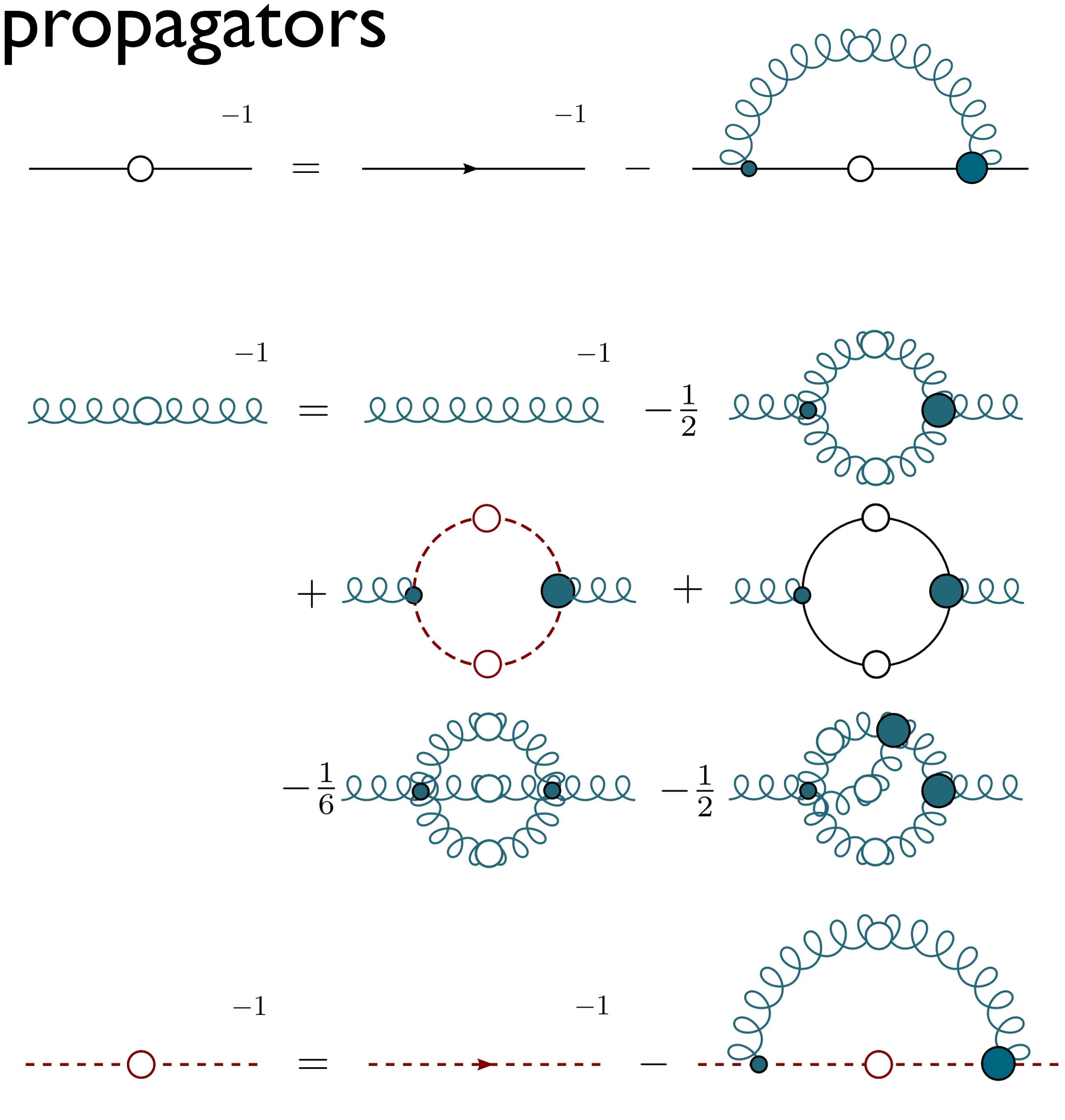

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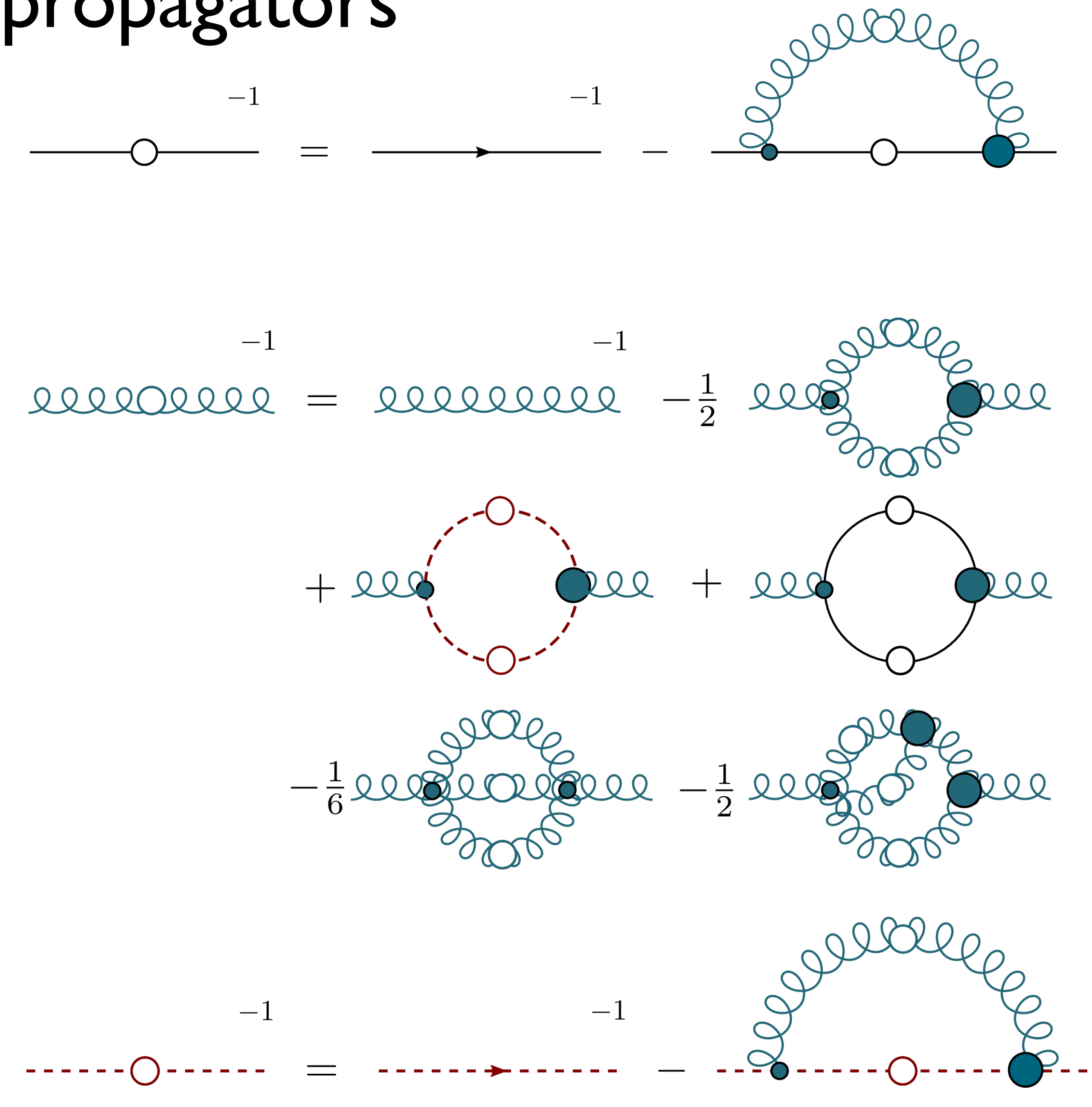

CF,Alkofer, PRD67 (2003) 094020
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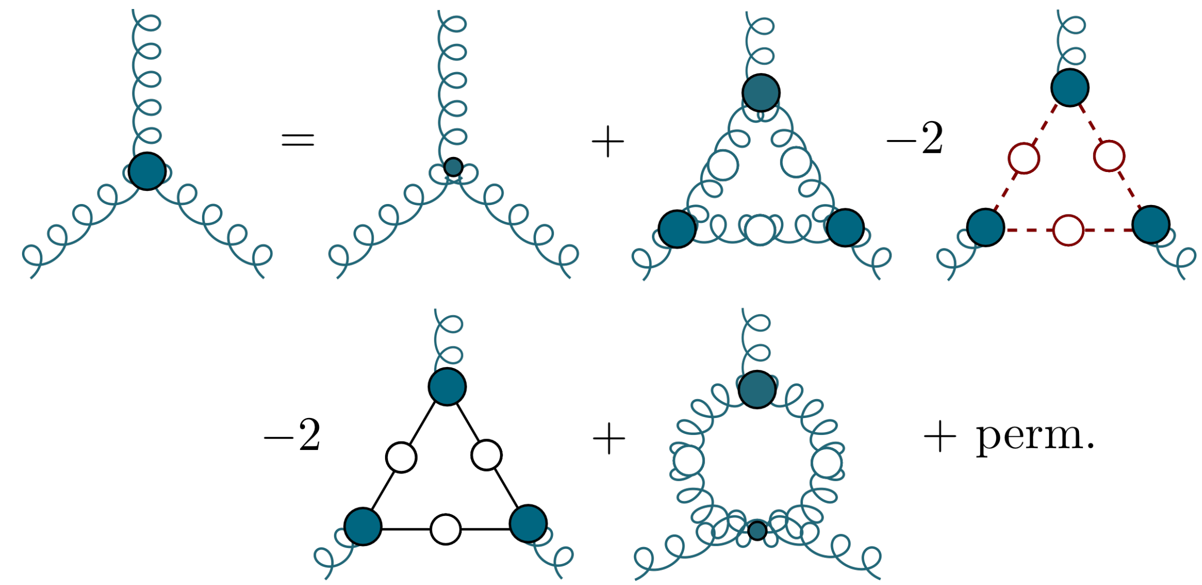
vertices



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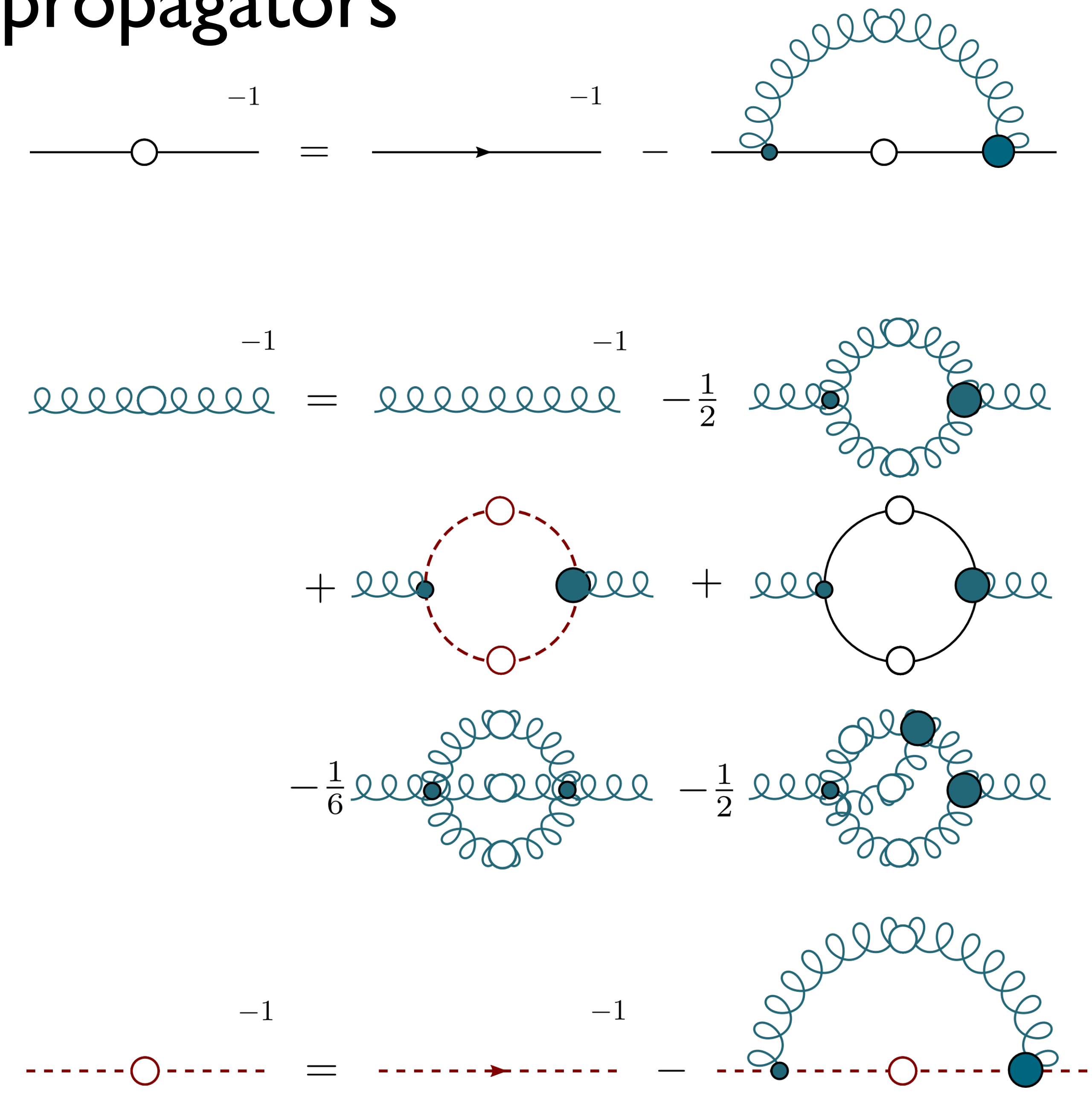

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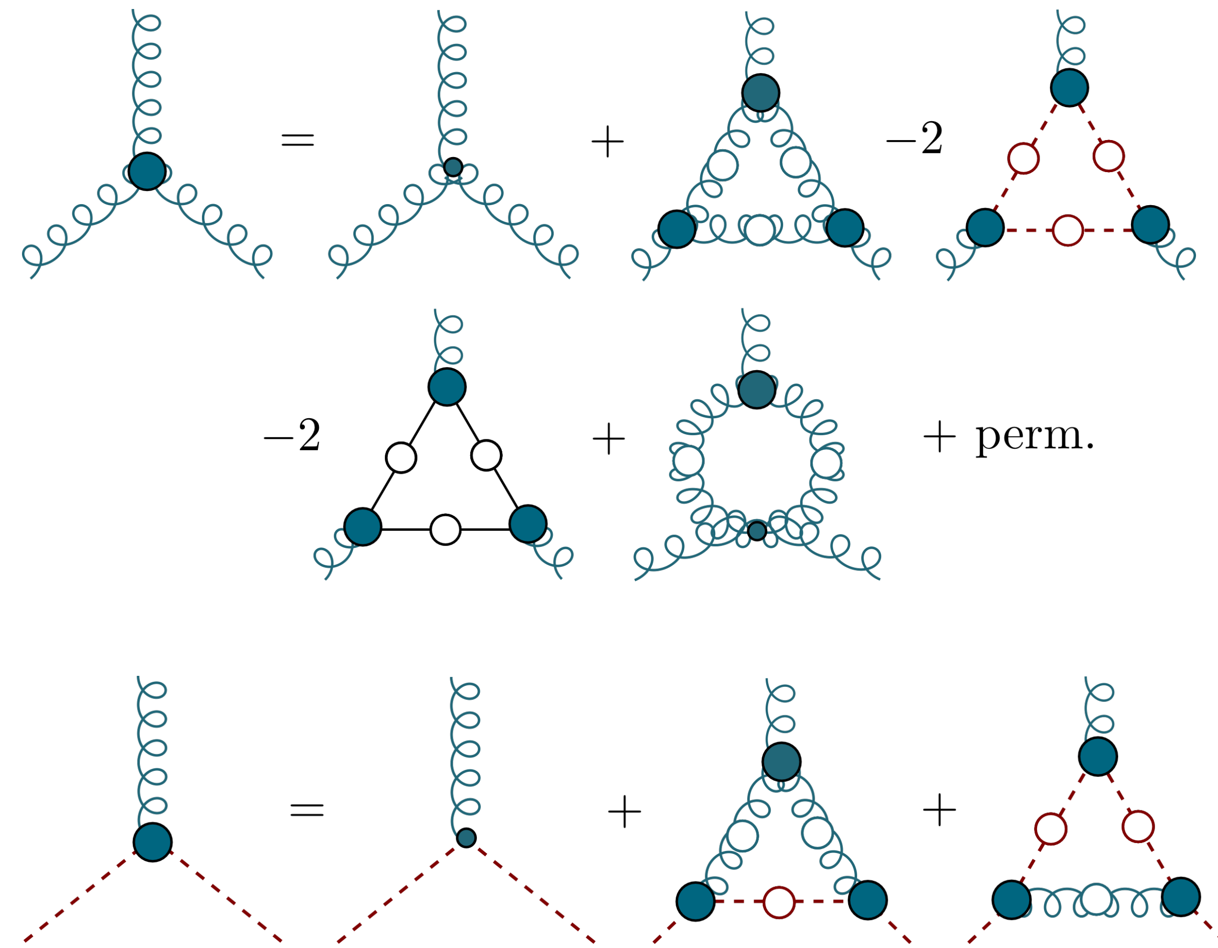

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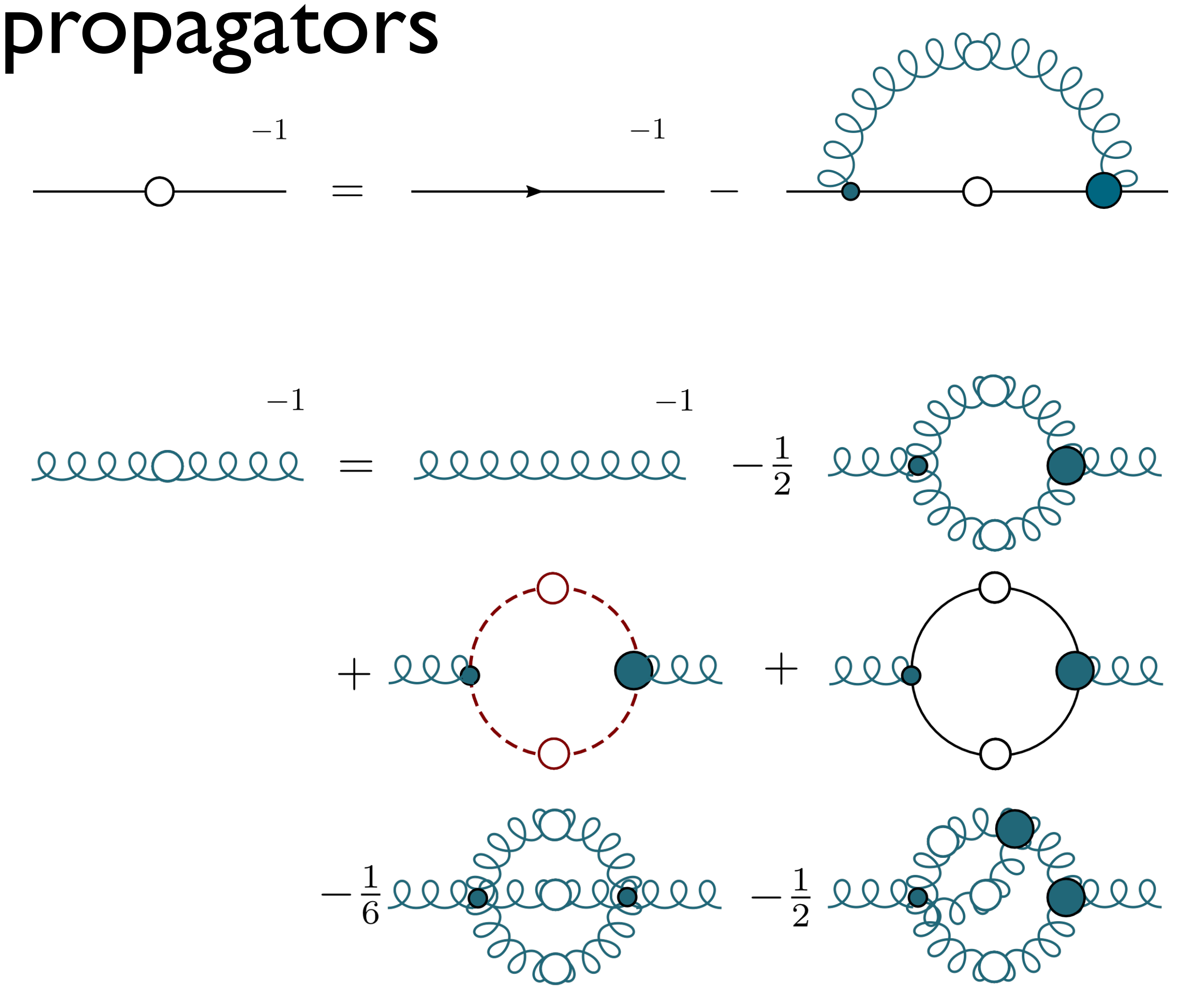

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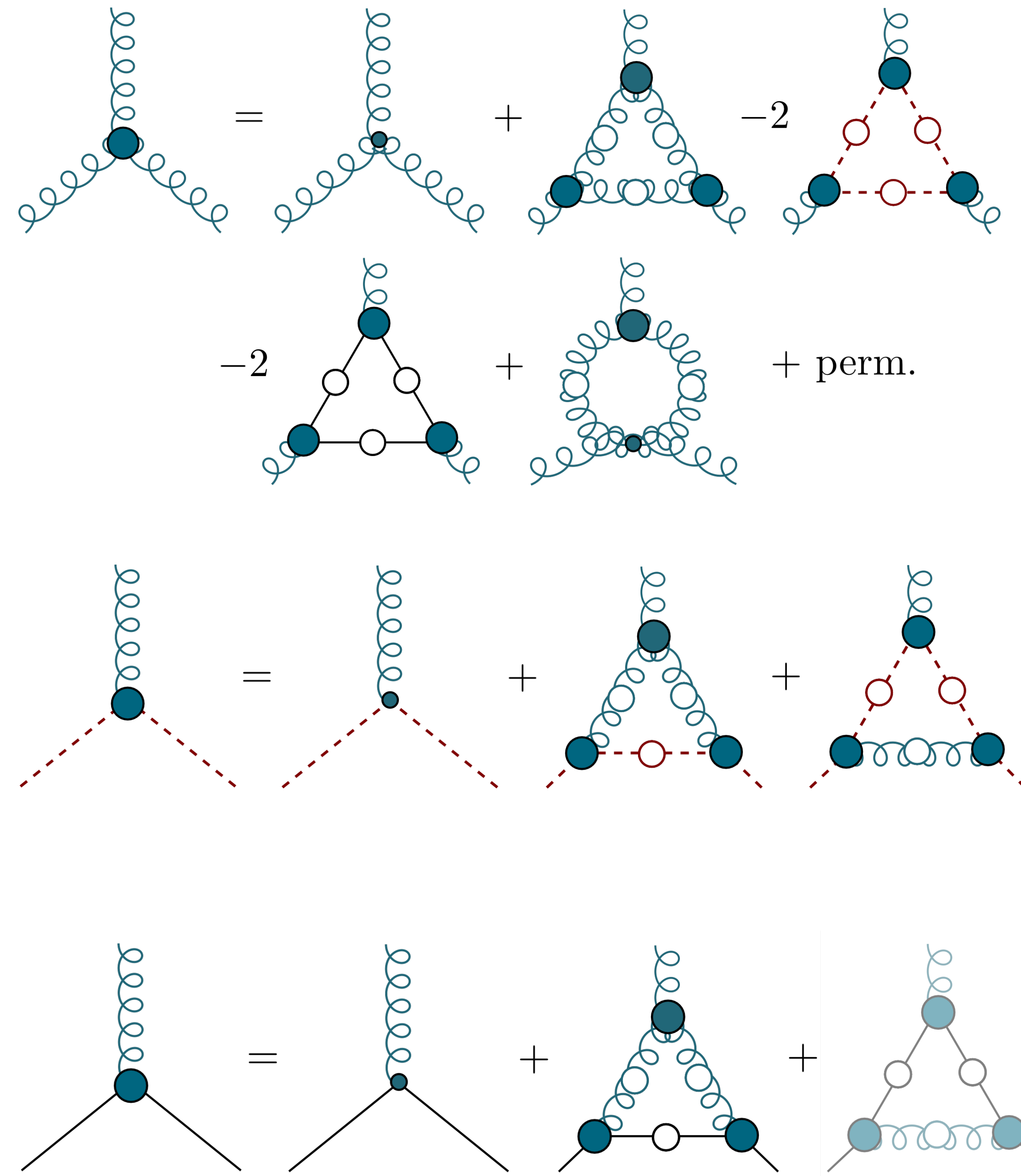

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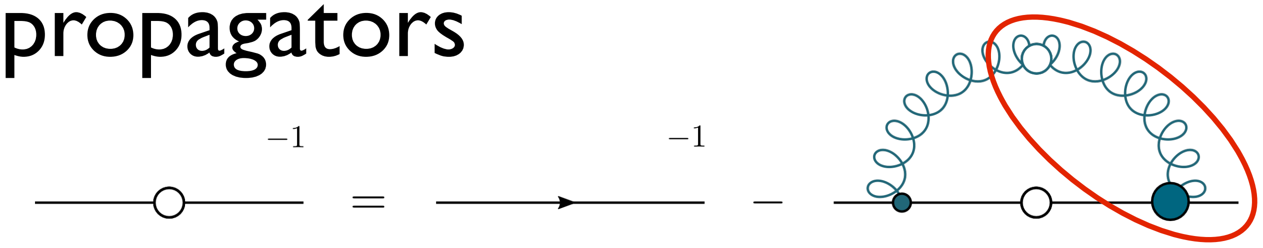

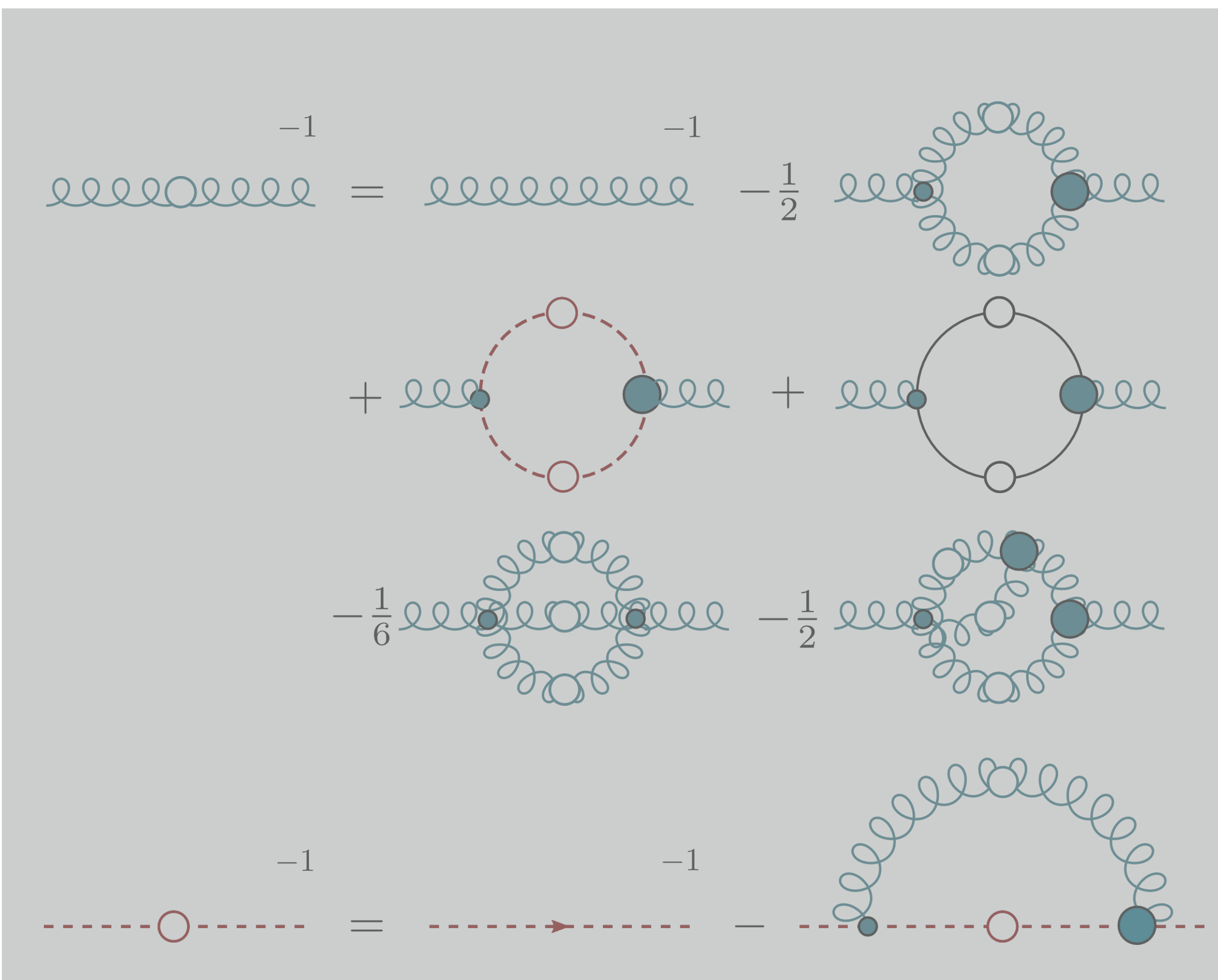

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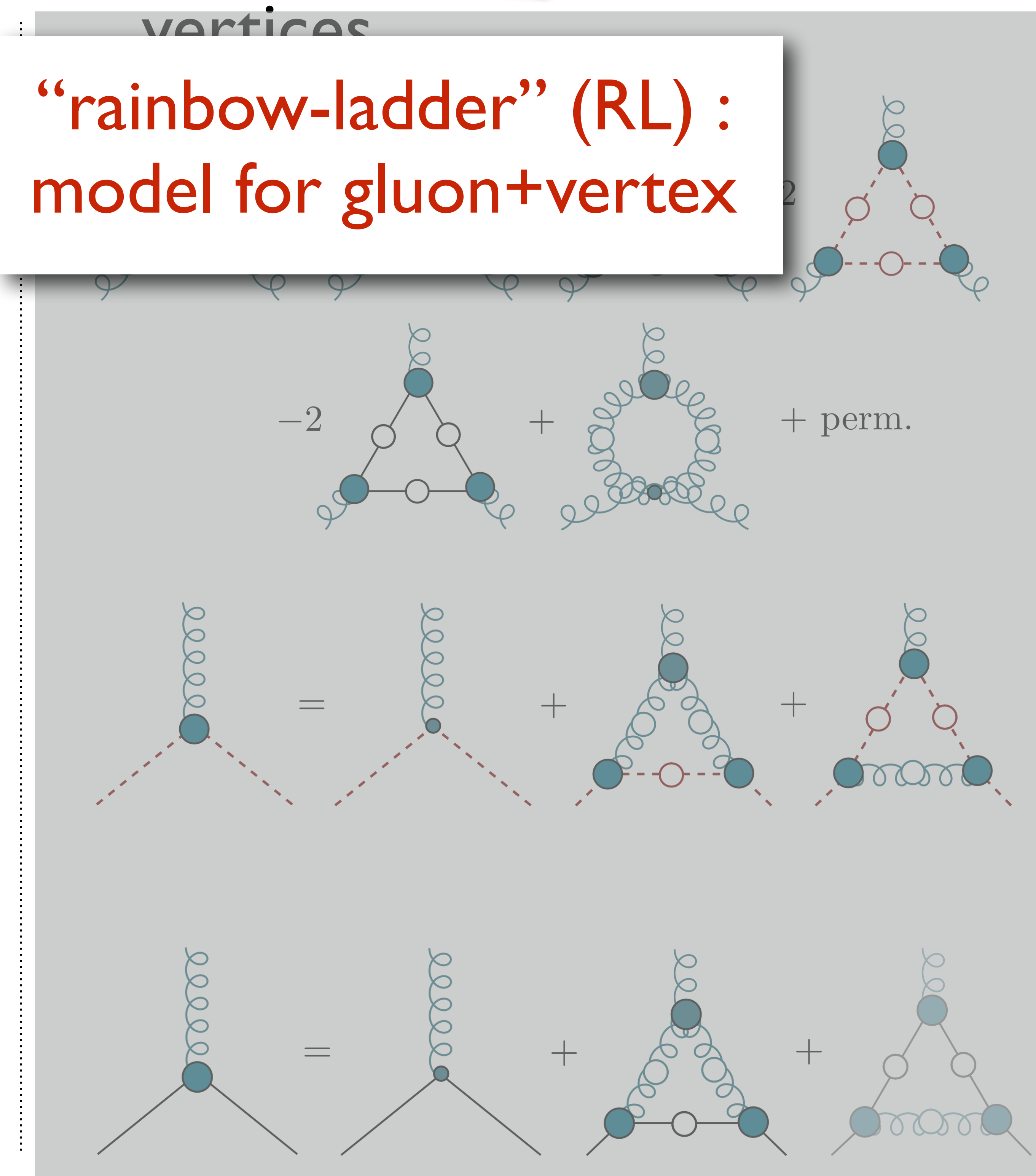
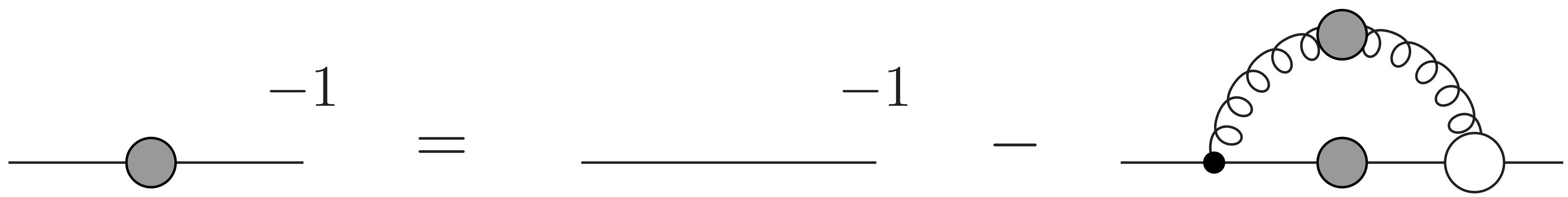


“rainbow-ladder” (RL) :
model for gluon+vertex

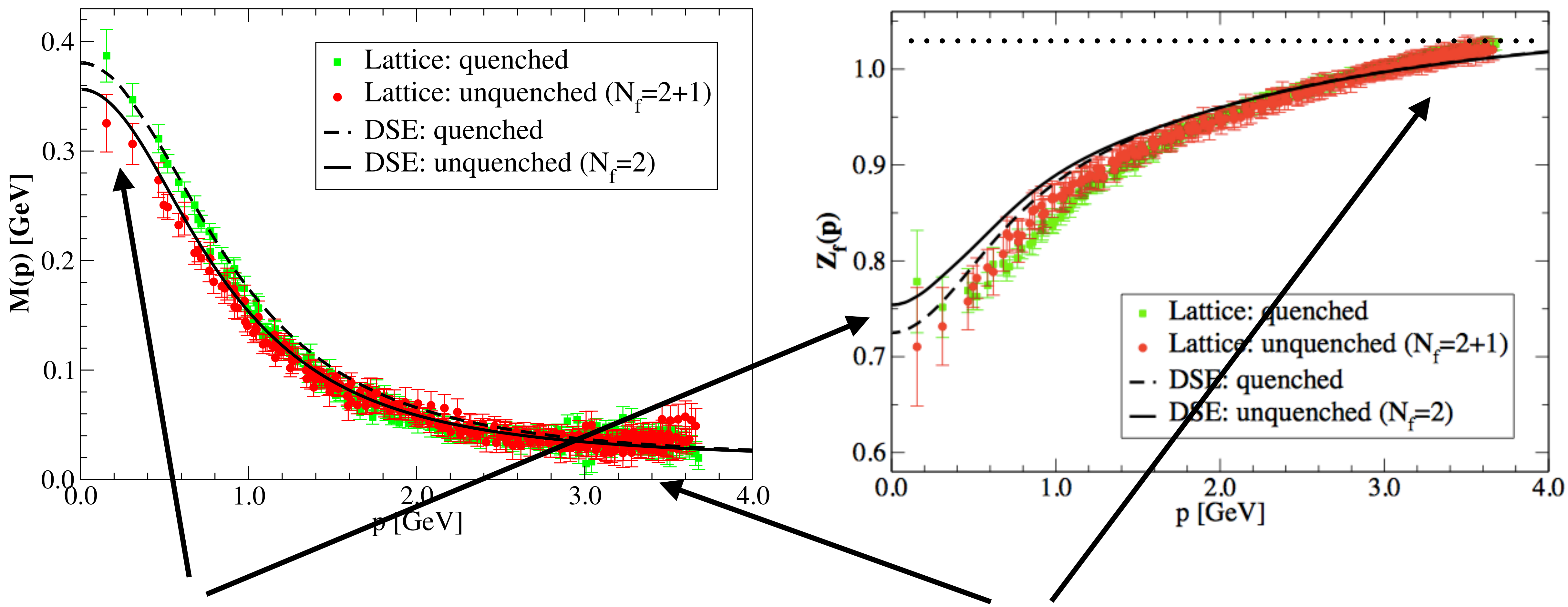
CF, Alkofer, PRD67 (2003) 094020
Williams, CF, Heupel, PRD93 (2016) 034026
Huber, EPJ C77 (2017) no.11, 733

Quarks: mass from interaction



$$S(p) = Z_f(p^2)\,\frac{-i\not{p} + M(p^2)}{p^2 + M^2(p^2)}$$

DSE: CF, Nickel, Williams, EPJ C 60 (2009) 47
Williams, CF, Heupel, PRD 93 (2016) 034026
Lattice: P. O. Bowman, et al PRD 71 (2005) 054507



‘constituent quark’:
large mass; very composite

‘current quark’:
- small mass; non-composite

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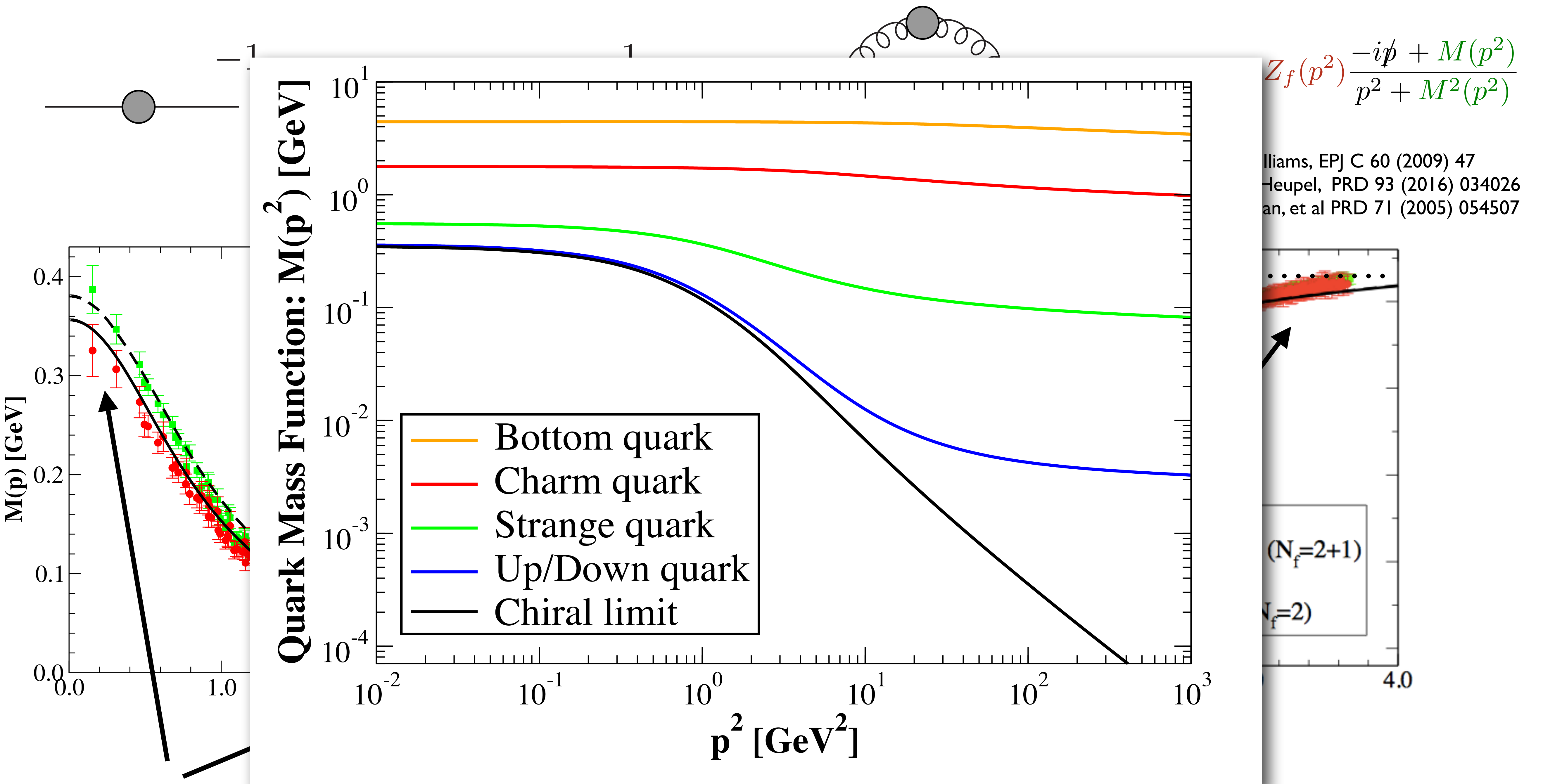

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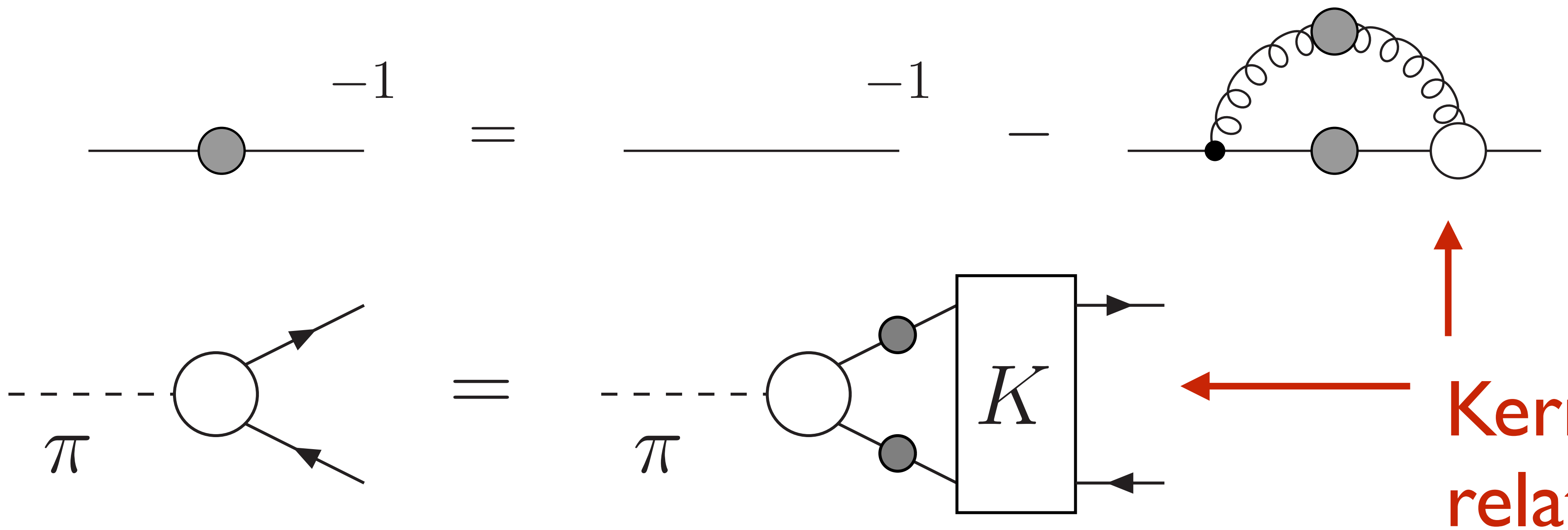
Theoretical Tools: DSEs and BSEs



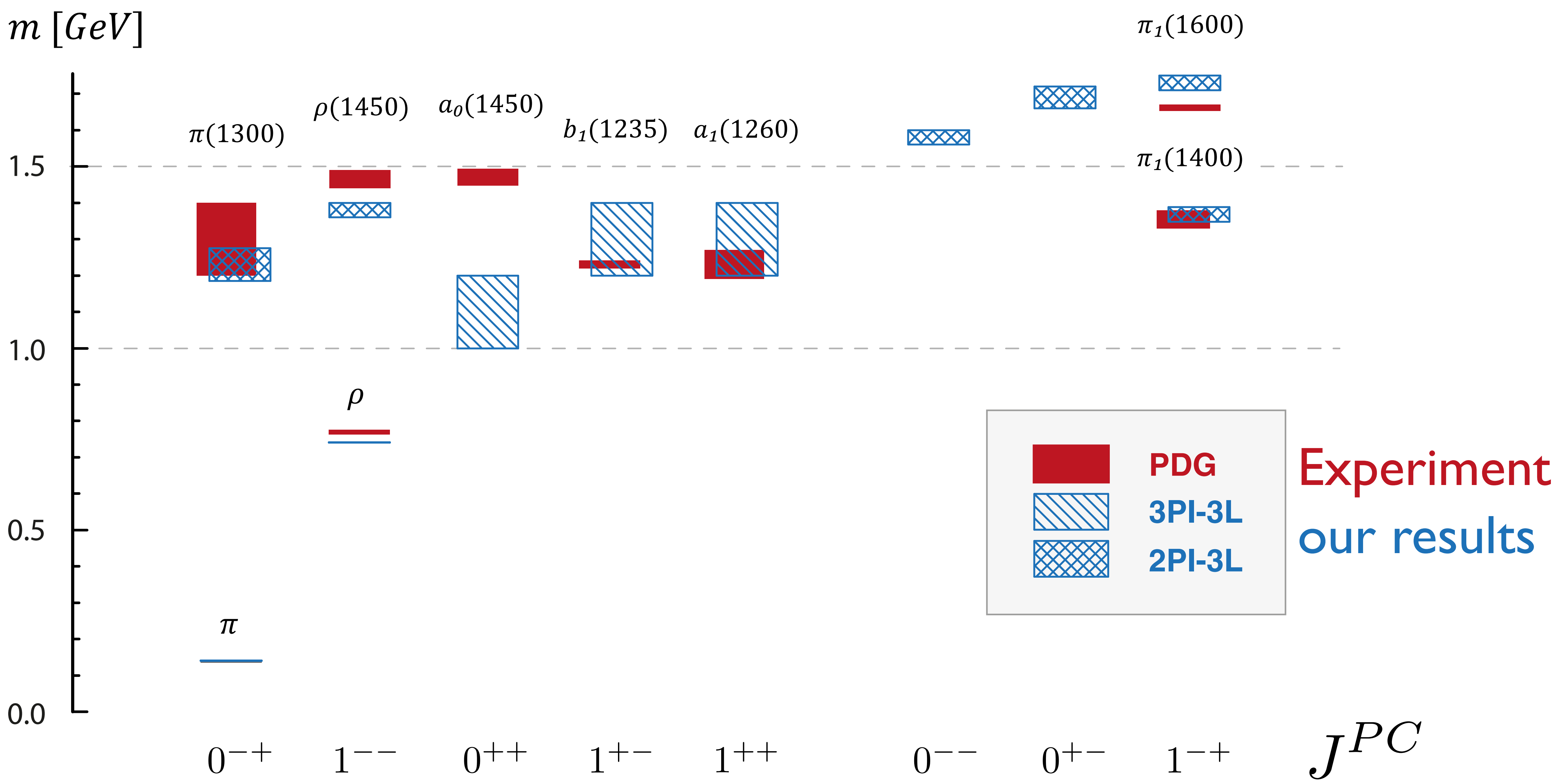
Kernel K uniquely related to quark-DSE via axWTI

→Pion is bound state **and** Goldstone boson

Maris, Roberts, Tandy, PLB 420 (1998) 267

- Determine gauge invariant spectrum from underlying, gauge dependent quark/gluon dynamics

Light meson spectrum - full 3PI-calculation



Williams, CF, Heupel, PRD93 (2016) 034026

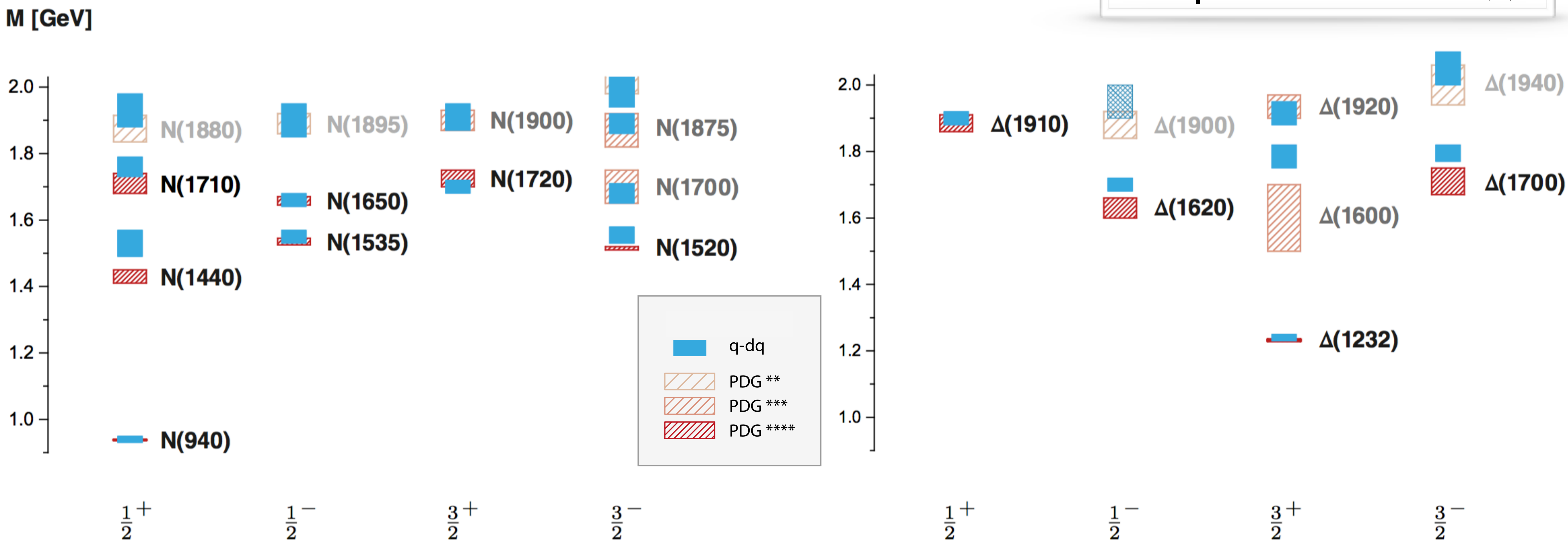
- good agreement with experiment in most channels
- special channels:

  pseudoscalar 0^{-+} : (pseudo-) Goldstone bosons

  scalar 0^{++} : complicated channel…

Light baryon spectrum: DSE-RL

- 3 parameters + $m_{u,d,s}$



Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016) [1607.05748]
Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2

- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)

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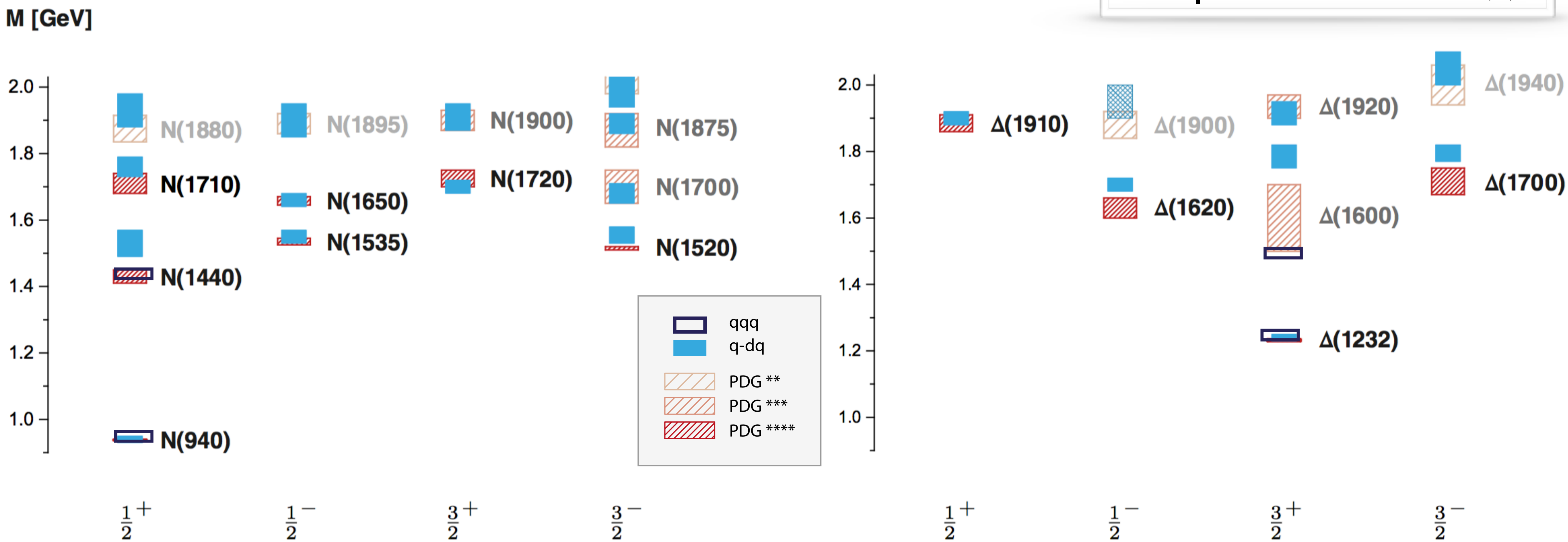

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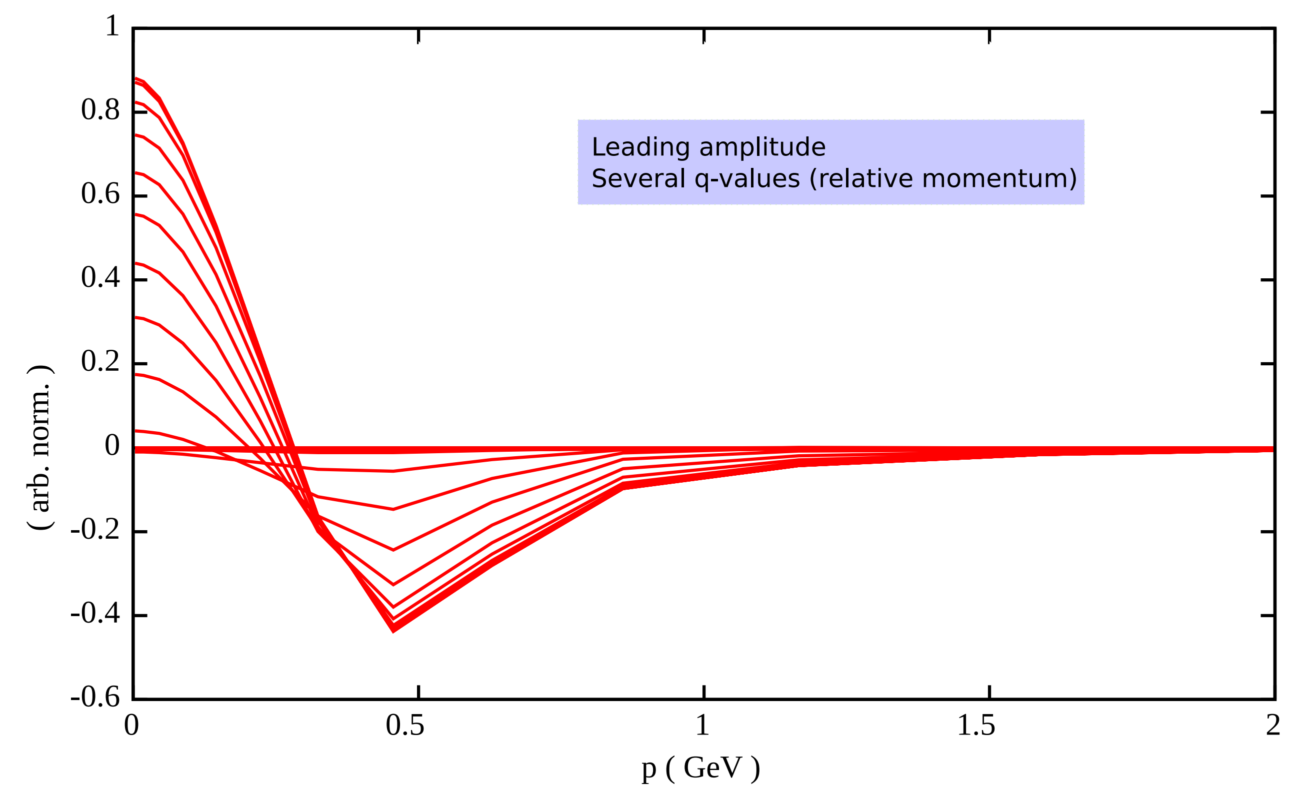
- spectrum in one to one agreement with experiment
- correct level ordering (without coupled channel effects…)
- three-body agrees with diquark-quark where applicable

Properties of the Roper

angular mom. decomposition

%	N	$N^*(1440)$	Δ	$\Delta^*(1600)$
s wave	66	15	56	10
p wave	33	61	40	33
d wave	1	24	3	41
f wave	–	–	< 0.5	16

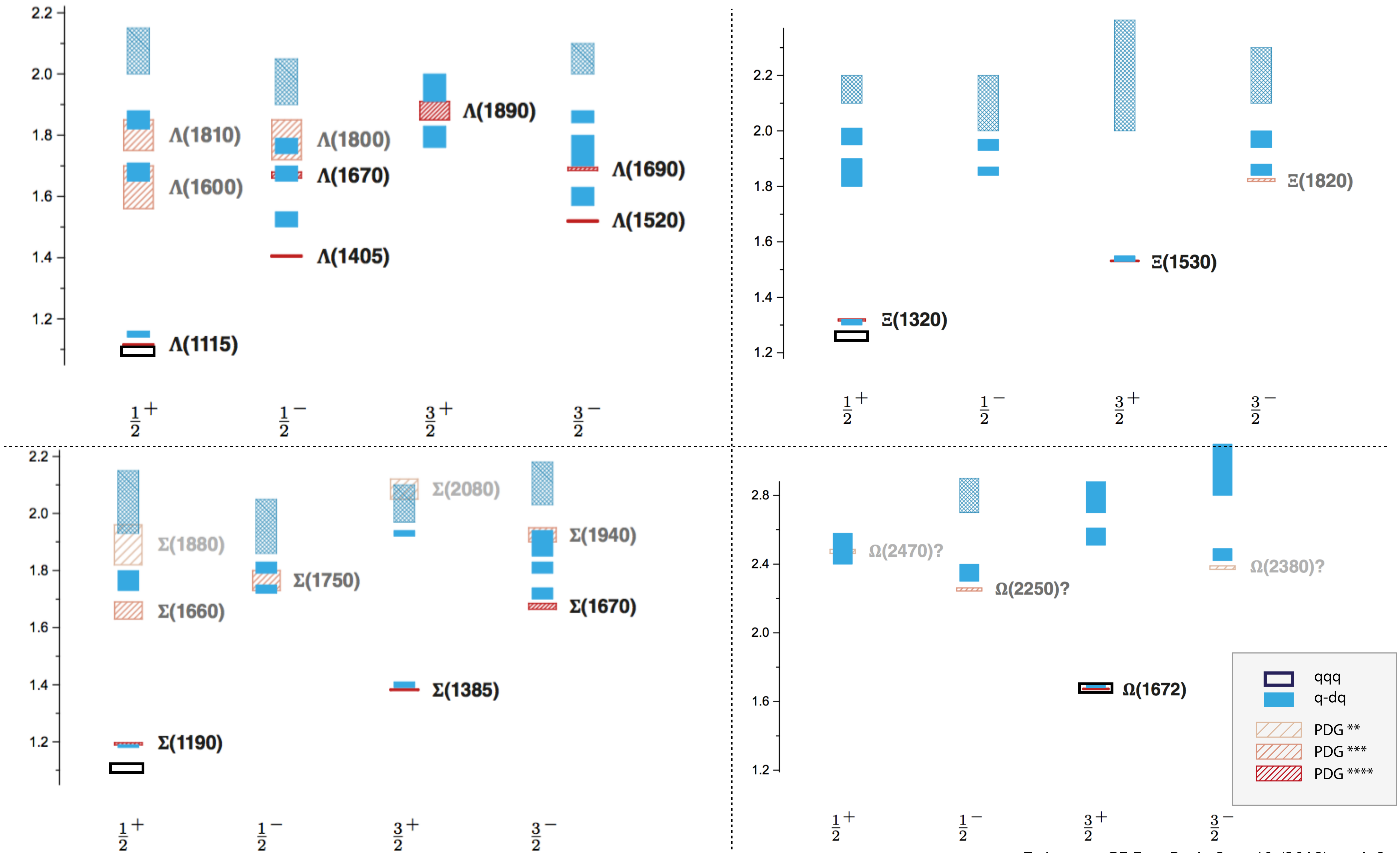


Eichmann, CF, Sanchis-Alepuz, PRD 94 (2016)

- zero crossing of wave function: 2s-state
- every state is mixture of several partial waves !
- different internal structure of radial excitations

tension with simpler calculations ('contact interaction', 'quark-diquark model'):
Wilson, Cloet, Chang and Roberts, PRC 85 (2012) 025205,
Segovia, El-Bennich, Rojas, Cloet, Roberts, Xu and Zong, PRL 115 (2015) 17
Lu, Chen, Roberts et al., PRC 96 (2017) 015208

Strange baryon spectrum: DSE-RL (preliminary !)



Eichmann, CF, Few Body Syst. 60 (2019) no.1, 2
CF, Eichmann PoS Hadron 2017 (2018) 007
Sanchis-Alepuz, CF, PRD 90 (2014) 096001

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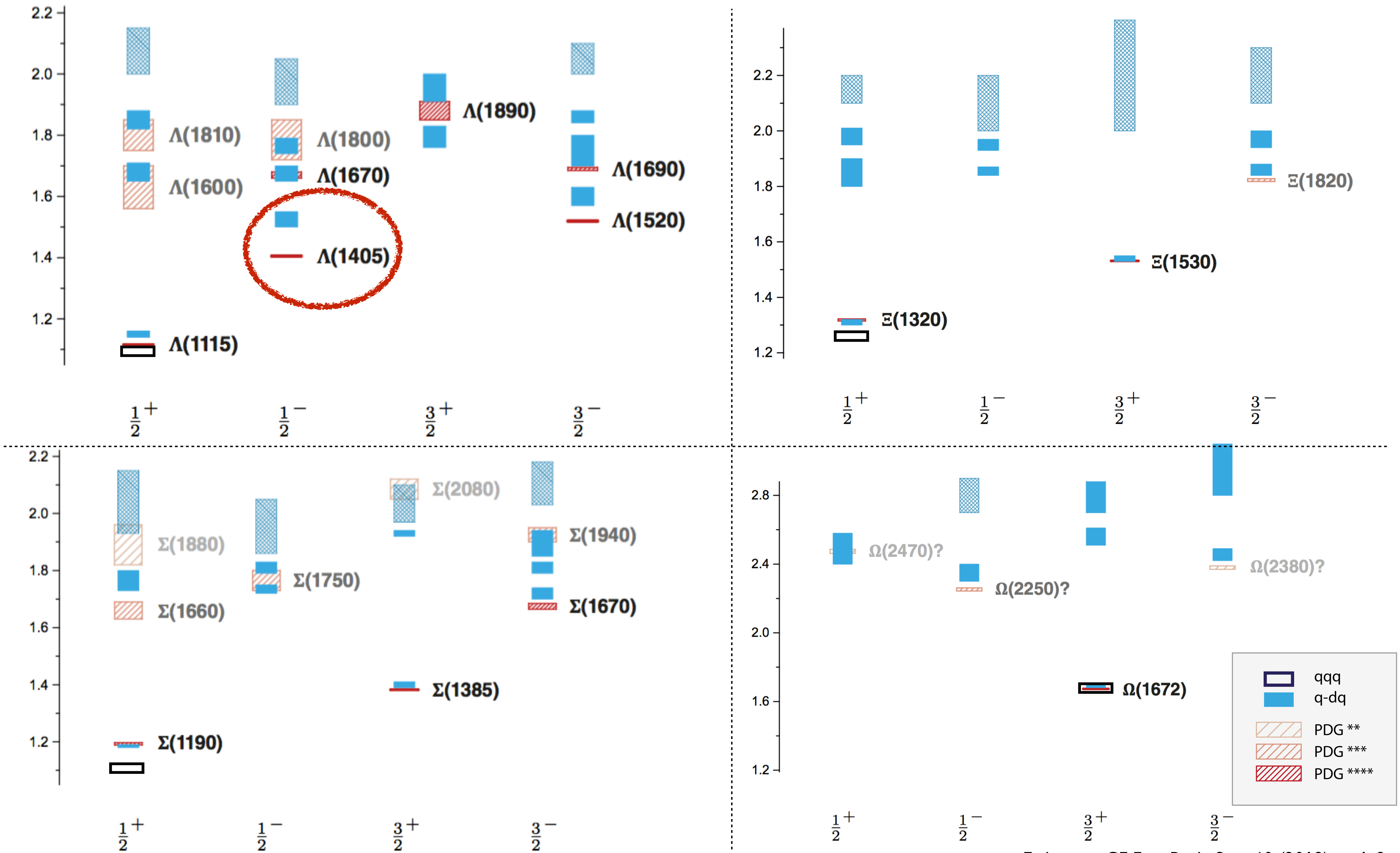

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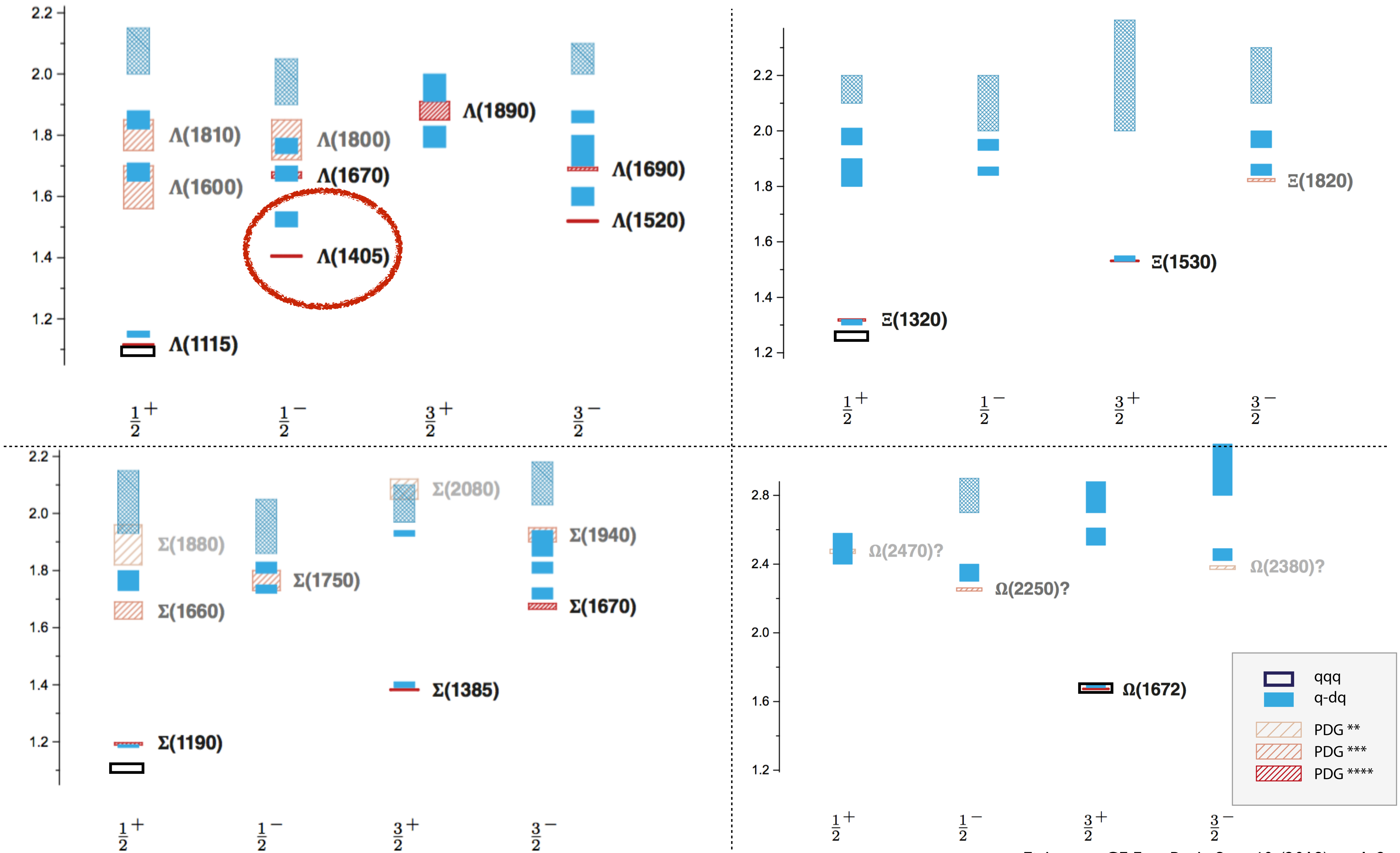

New states: Bonn-Gatchina (talk of M. Matveev)

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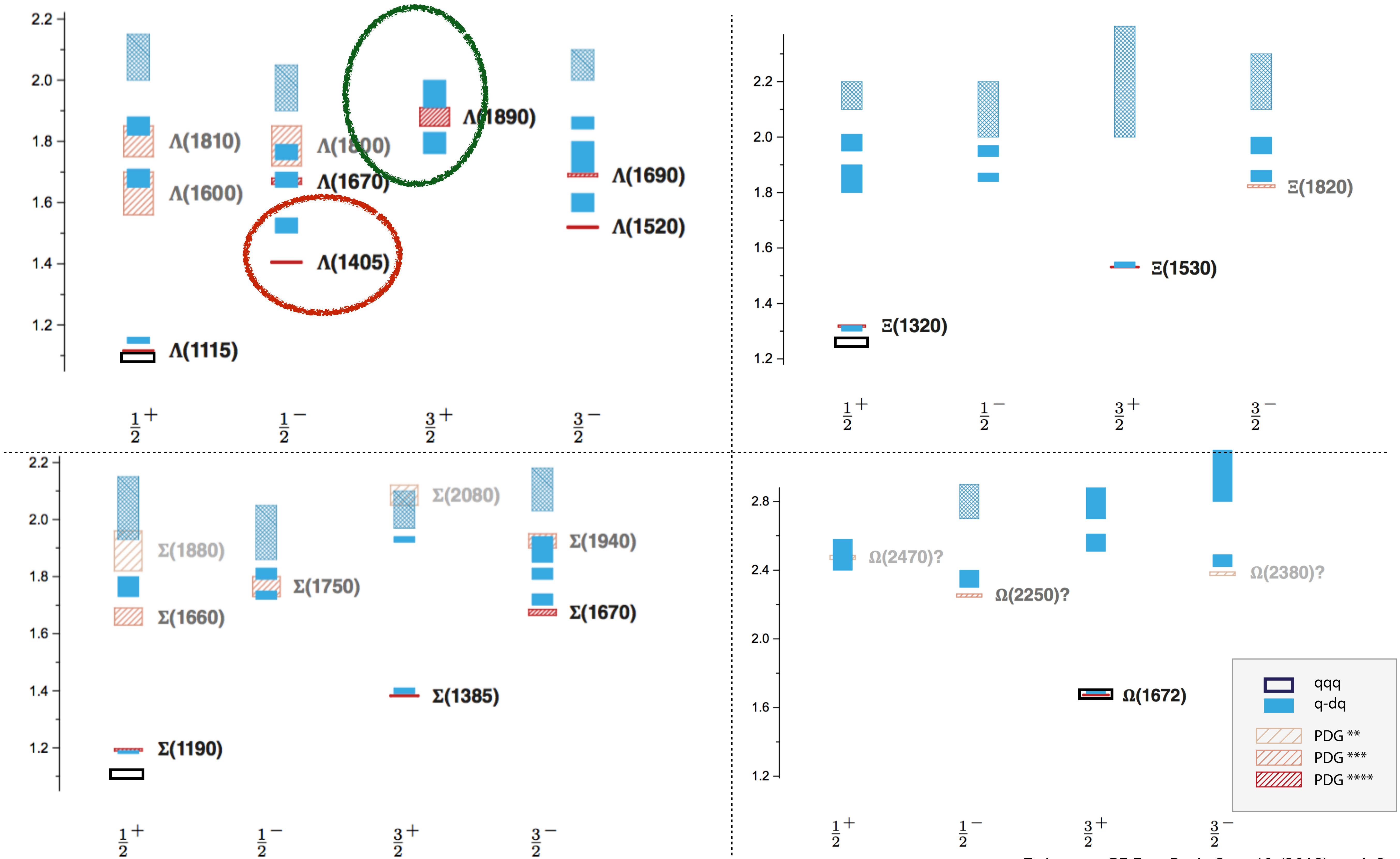

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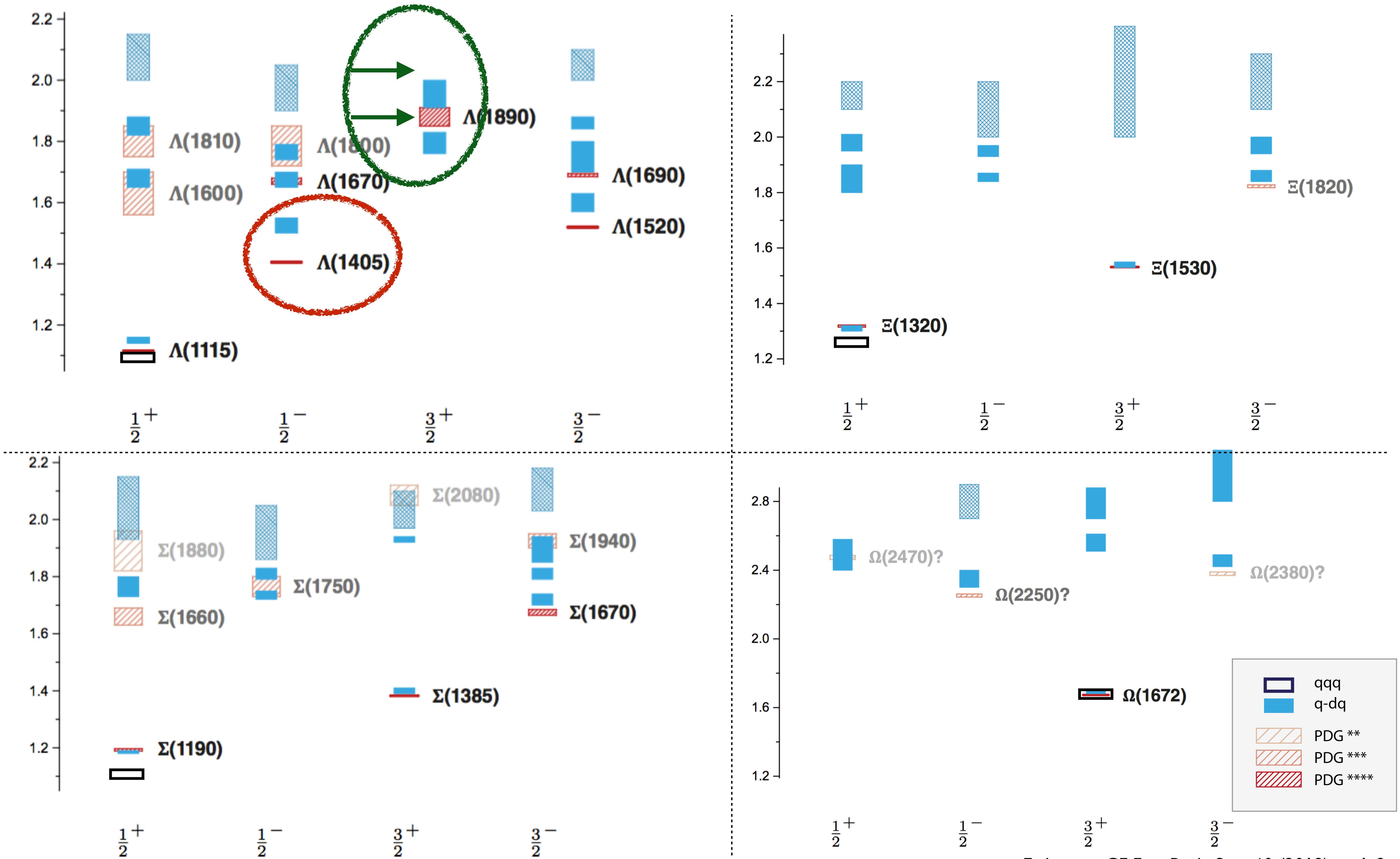

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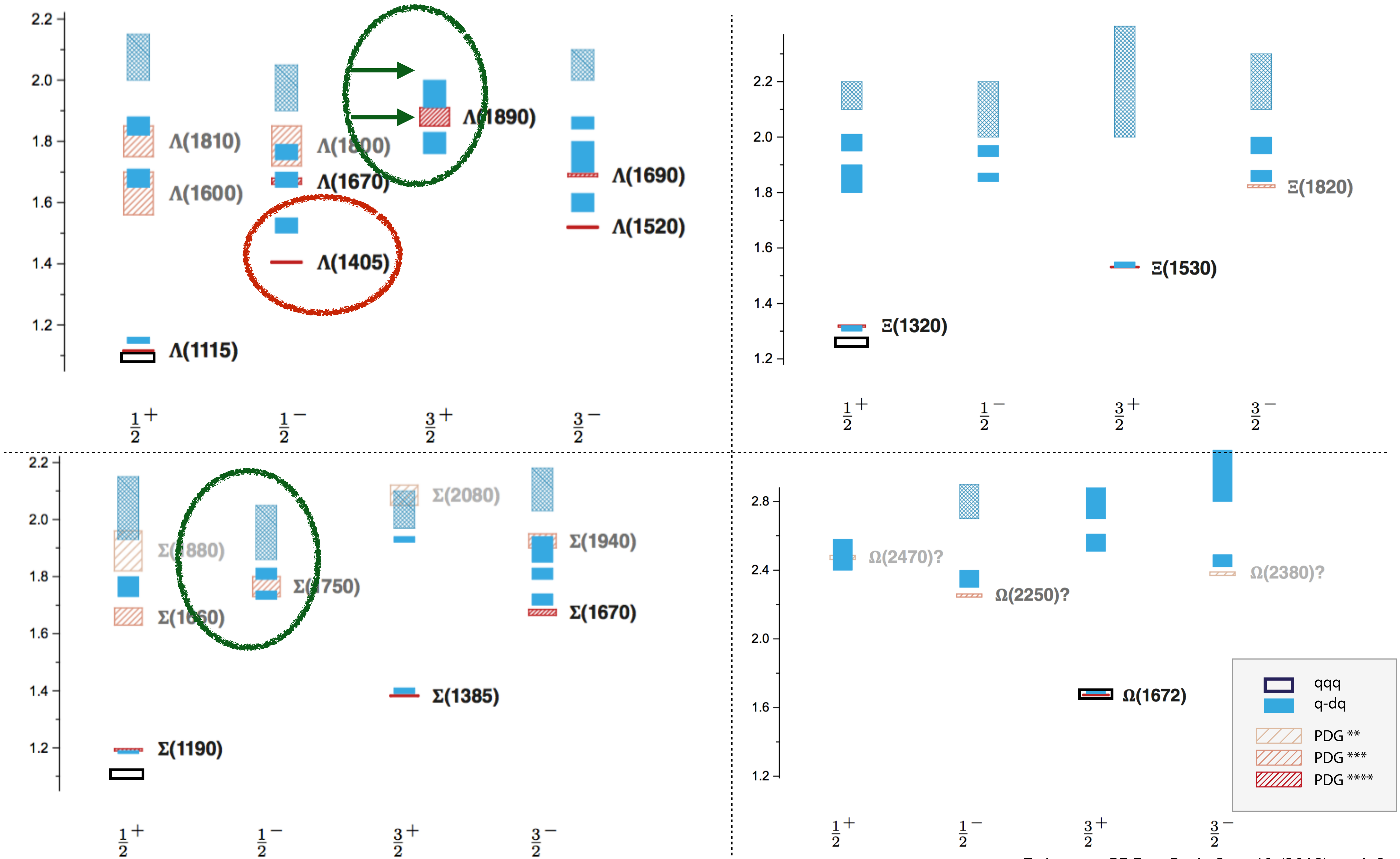

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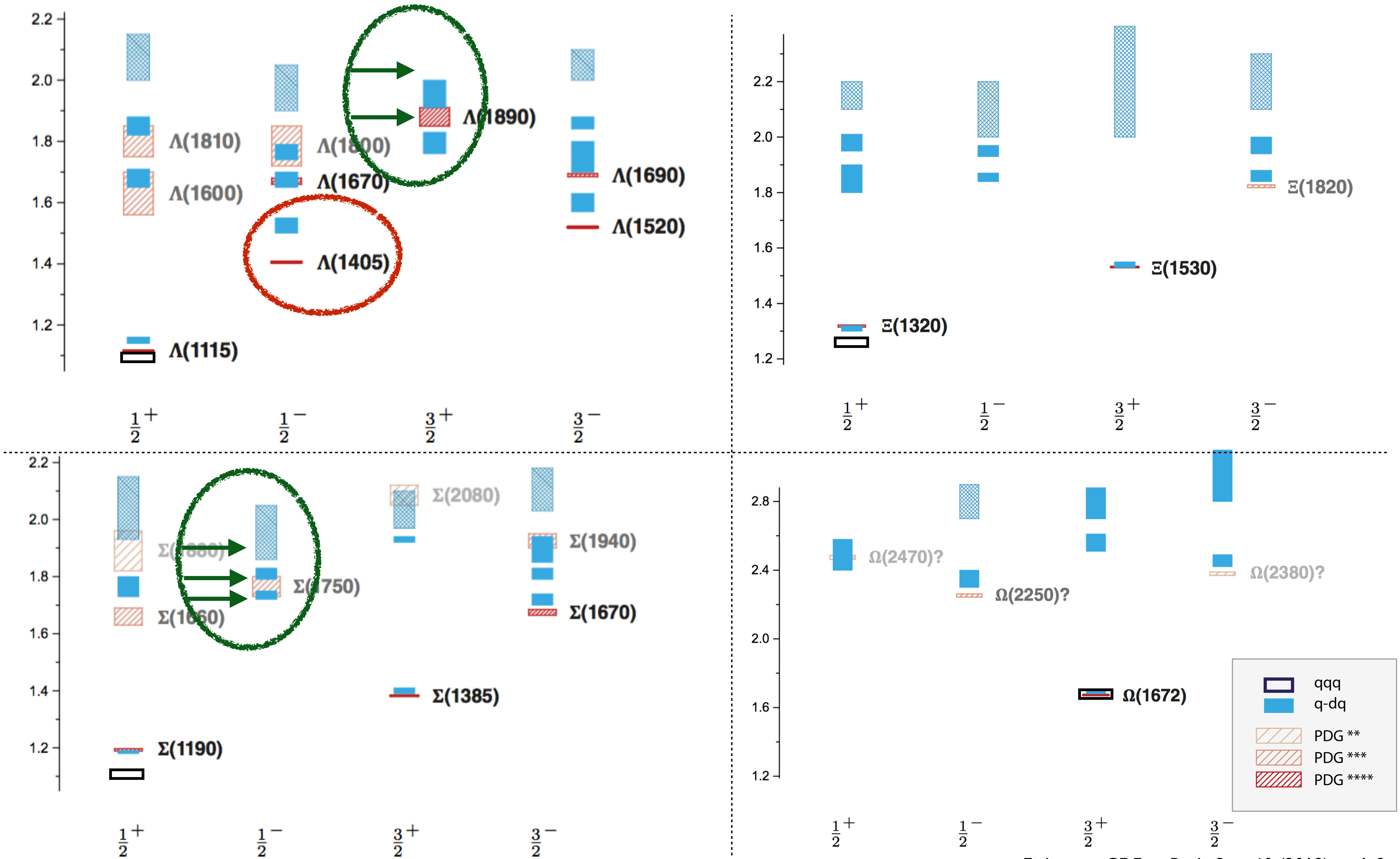

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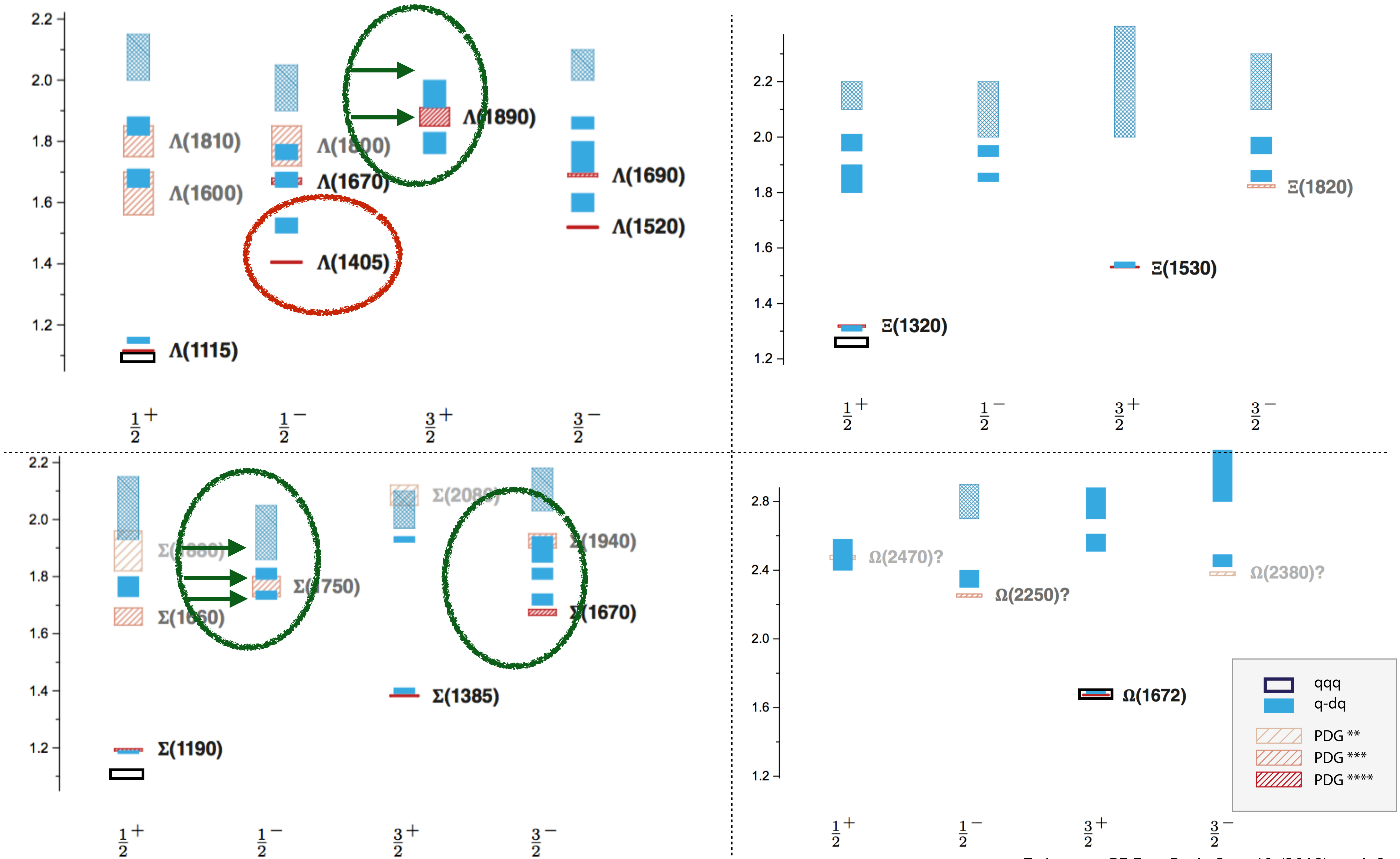

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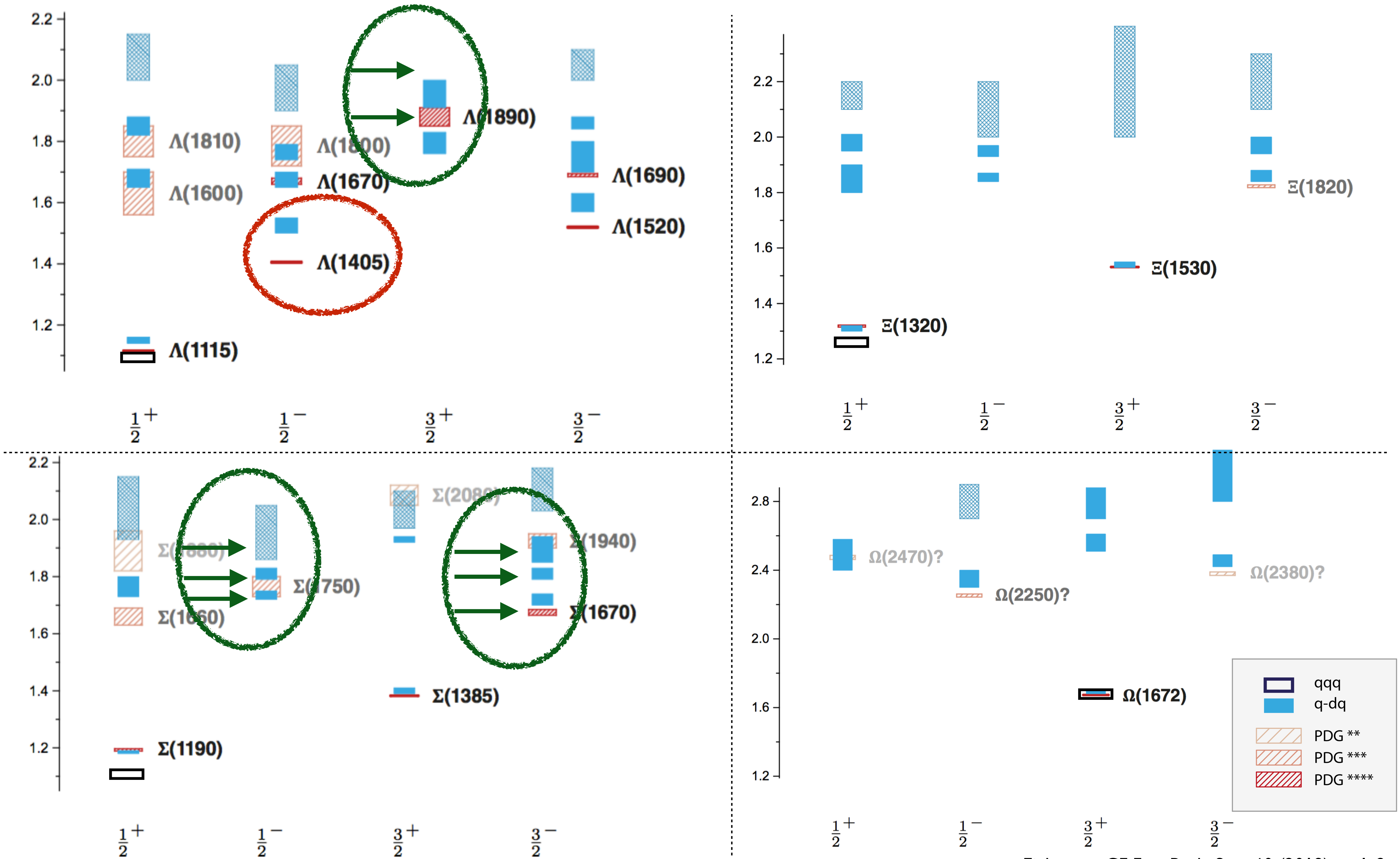

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