Dualities of the $(N_c = 2, 3, \infty)$ QCD phase diagram: chiral imbalance, baryon density

Roman N. Zhokhov

IZMIRAN, IHEP

Excited QCD 2020, Krynica Zdrój, Poland, 2020

Russian Science Foundation

БАЗИС
Фонд развития теоретической физики и математики

K.G. Klimenko, IHEP
T.G. Khunjua, University of Georgia, MSU

in the broad sense our group stems from
Department of Theoretical Physics, Moscow State University
Prof. V. Ch. Zhukovsky

strong connections with
prof. D. Ebert, Humboldt University of Berlin

details can be found in

Phys.Rev. D100 (2019) no.3, 034009 arXiv: 1904.07151 [hep-ph]
JHEP 1906 (2019) 006 arXiv:1901.02855 [hep-ph]
Eur.Phys.J. C79 (2019) no.2, 151, arXiv:1812.00772 [hep-ph],
Phys.Rev. D98 (2018) no.5, 054030 arXiv:1804.01014 [hep-ph],
Phys.Rev. D97 (2018) no.5, 054036 arXiv:1710.09706 [hep-ph]
Phys.Rev. D95 (2017) no.10, 105010 arXiv:1704.01477 [hep-ph]

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БАЗИС
Фонд развития теоретической физики и математики

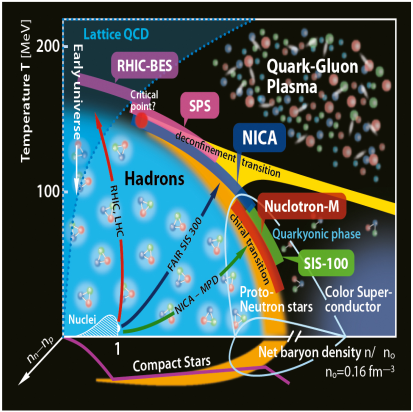
QCD Dhase Diagram and Methods

QCD at T and μ
(QCD at extreme conditions)

- neutron stars
- heavy ion collisions
- Early Universe

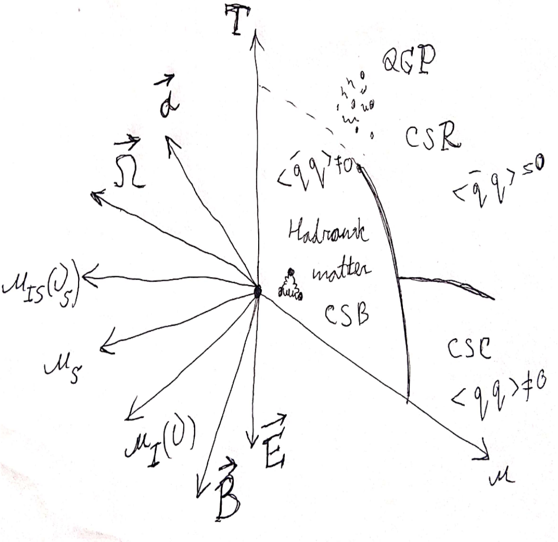
Methods of dealing with QCD

- First principle calcultion – lattice QCD
- Effective models
- DSE, FRG
- .....



More than just QCD at (μ, T)

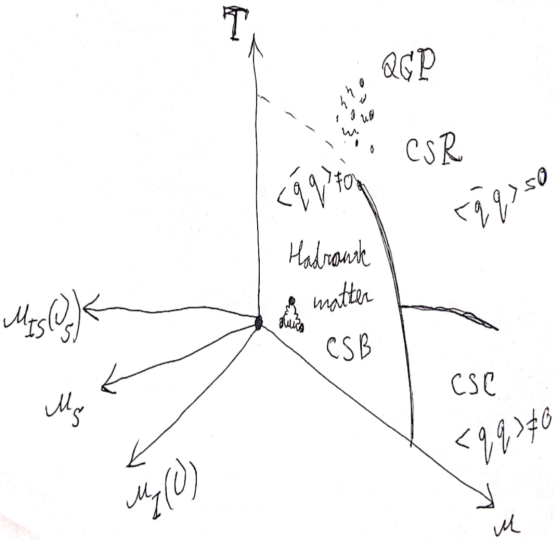
- more chemical potentials μ_i
- magnetic fields *(see talk by A. Kotov)*
- rotation of the system $\vec{\Omega}$
- acceleration $\vec{a}$
- finite size effects (finite volume and boundary conditions)



More external conditions to QCD

More than just QCD at (μ, T)

- **more chemical potentials** μ_i
- magnetic fields
  (see talk by A. Kotov)
- rotation of the system
- acceleration
- finite size effects (finite volume and boundary conditions)



Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

Baryon chemical potential μ_B

Allow to consider systems with non-zero baryon densities.

$$\frac{\mu_B}{3}\bar{q}\gamma^0 q = \mu\bar{q}\gamma^0 q,$$

Isotopic chemical potential μ_I

Allow to consider systems with isospin imbalance ($n_n \neq n_p$).

$$\frac{\mu_I}{2}\bar{q}\gamma^0\tau_3 q = \nu\left(\bar{q}\gamma^0\tau_3 q\right)$$

$$n_I = n_u - n_d \quad \longleftrightarrow \quad \mu_I = \mu_u - \mu_d$$

chiral (axial) chemical potential

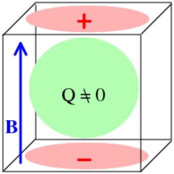
Allow to consider systems with chiral imbalance (difference between densities of left-handed and right-handed quarks).

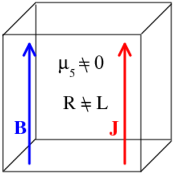
$$n_5 = n_R - n_L \quad \longleftrightarrow \quad \mu_5 = \mu_R - \mu_L$$

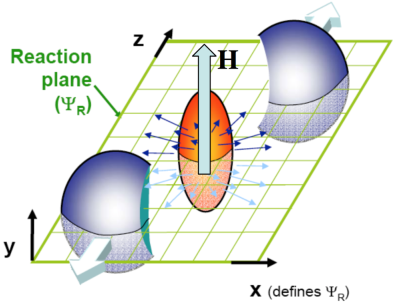
The corresponding term in the Lagrangian is

$$\mu_5 \bar{q} \gamma^0 \gamma^5 q$$

Chiral magnetic effect

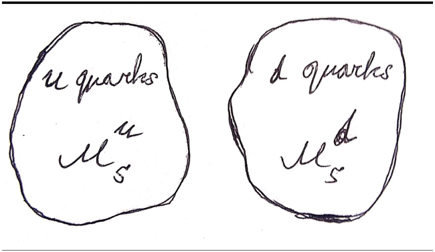






$$\vec{J} \sim \mu_5 \vec{B},$$

A. Vilenkin, PhysRevD.22.3080,
K. Fukushima, D. E. Kharzeev and H. J. Warringa, Phys. Rev. D **78** (2008) 074033



$$\mu_5^u \neq \mu_5^d \qquad \text{and} \qquad \mu_{I5} = \mu_5^u - \mu_5^d$$

Term in the Lagrangian — $\frac{\mu_{I5}}{2}\bar{q}\tau_3\gamma^0\gamma^5 q = \nu_5(\bar{q}\tau_3\gamma^0\gamma^5 q)$

$$n_{I5} = n_{u5} - n_{d5}, \qquad n_{I5} \longleftrightarrow \nu_5$$

Chiral imbalance n_5 and hence μ_5 can be generated in parallel magnetic and electric fileds

$$\vec{E} \,||\, \vec{B}$$

M. Ruggieri, M. Chernodub, H. Warringa et al

Chiral isospin imbalance generated in $\vec{E}\,||\,\vec{B}$

Chiral isospin imbalance n_{I5} and hence μ_{I5} can be generated in parallel magnetic and electric fileds $\vec{E}\,||\,\vec{B}$

μ_{I5} and μ_5 are generated by $\vec{E}\,||\,\vec{B}$

Generation of Chiral imbalance in dense quark matter

Chiral imbalance could appear in dense matter

- Chiral separation effect
  (Thanks for the idea to Igor Shovkovy)
- Chiral vortical effect

Different chemical potentials and matter content

$$\mu = \frac{\mu_B}{3}, \qquad \nu = \frac{\mu_I}{2}, \qquad \mu_5, \qquad \nu_5 = \frac{\mu_{I5}}{2}$$

Model and its Lagrangian

We consider a NJL model, which describes dense quark matter with two massless quark flavors (u and d quarks).

$$\mathcal{L} = \bar{q}\Big[\gamma^\nu \mathrm{i}\partial_\nu + \frac{\mu_B}{3}\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5\Big]q +$$

$$\frac{G}{N_c}\Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5\vec{\tau}q)^2\Big]$$

q is the flavor doublet, $q = (q_u, q_d)^T$, where q_u and q_d are four-component Dirac spinors as well as color N_c-plets; τ_k ($k = 1, 2, 3$) are Pauli matrices.

Equivalent Lagrangian

To find the thermodynamic potential we use a semi-bosonized version of the Lagrangian

$$\widetilde{L} = \bar{q}\Big[i\partial\!\!\!/ + \mu\gamma^0 + \nu\tau_3\gamma^0 + \nu_5\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5 - \sigma - i\gamma^5\pi_a\tau_a\Big]q - \frac{N_c}{4G}\Big[\sigma^2 + \pi_a^2\Big].$$

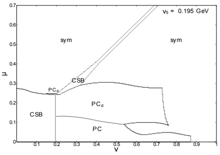
$$\sigma(x) = -2\frac{G}{N_c}(\bar{q}q); \quad \pi_a(x) = -2\frac{G}{N_c}(\bar{q}i\gamma^5\tau_a q).$$

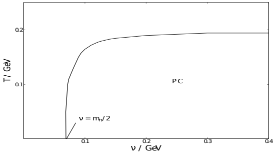
Condansates ansatz $\langle\sigma(x)\rangle$ and $\langle\pi_a(x)\rangle$ do not depend on spacetime coordinates

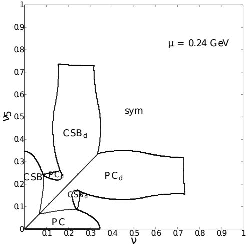
$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \Delta, \quad \langle\pi_2(x)\rangle = 0, \quad \langle\pi_3(x)\rangle = 0.$$

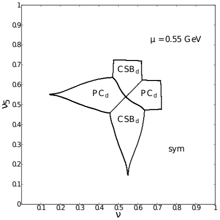
where M and Δ are already constant quantities.

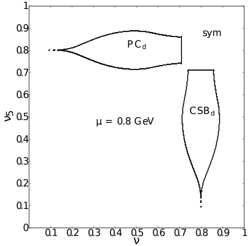
Phase diagram, lots of plots











Chiral imbalance leads to the generation of PC in dense quark matter (PC_d)

In the early 1970s Migdal (Sawyer, Scalapino, Kogut, Manassah) suggested the possibility of **pion condensation in a nuclear matter**

A.B. Migdal, Zh. Eksp. Teor. Fiz. 61, 2210 (1971) [Sov. Phys. JETP 36, 1052 (1973)]; A. B. Migdal, E. E. Saperstein, M. A. Troitsky and D. N. Voskresensky, Phys. Rept. 192, 179 (1990). R.F. Sawyer, Phys. Rev. Lett. 29, 382 (1972); J. Kogut, J.T. Manassah, Physics Letters A, 41, 2, 1972, Pages 129-131

(In medium pion mass properties and the RMF models.) **pion condensation** with zero momentum (s-wave condensation) is **highly unlikely to be realized** in nature in **matter of neutron star.**

A. Ohnishi D. Jido T. Sekihara, and K. Tsubakihara, Phys. Rev. C80, 038202 (2009) . .

Pion condensation in NJL model, chiral limit

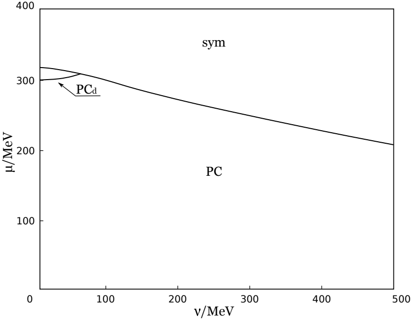


Figure: (ν, μ) phase diagram in NJL model in the chiral limit.

PC phenomenon maybe could be realized in **dense baryonic matter (non-zero baryon density**

K. G. Klimenko, D. Ebert
J.Phys. G32 (2006) 599-608;
Eur.Phys.J.C46:771-776,(2006)

Pion condensation in NJL model, chiral limit

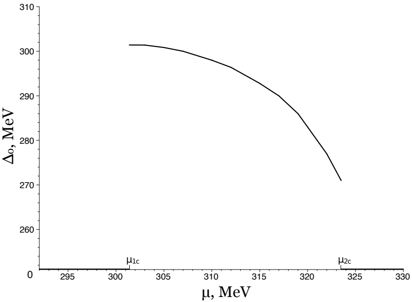
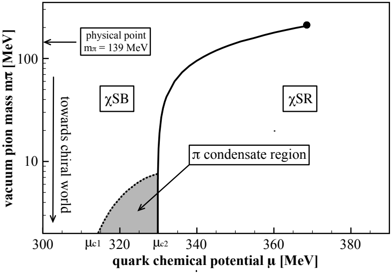


Figure: Pion condensate in dense quark matter in NJL model.

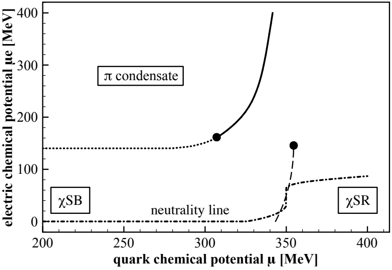
PC phenomenon is realized in **dense baryonic matter**

even in **charge neutral and β-equilibrated case**

K. G. Klimenko, D. Ebert
J.Phys. G32 (2006) 599-608;
Eur.Phys.J.C46:771-776,(2006)



(μ, m_0) phase portrait.



(μ, μ_e) phase portrait.

No PC condensation in the neutral case at the physical point

(H. Abuki, R. Anglani, M. Ruggieri etc.
Phys. Rev. D **79** (2009) 034032.

There are a number of **external parameters** such as **chiral imbalance** that can generate **PC in dense quark matter**.

See small review

Symmetry 2019, 11(6), 778
arXiv:1912.08635 [hep-ph]

Special Issue "Nambu-Jona-Lasinio model and its applications" of symmetry

(Thanks to Tomohiro Inagaki)

Pion condensation in NJL model, physical point

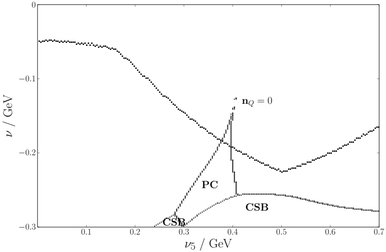
Charge neutrality and β-equilibrium can destroy the generation of PC

So it is interesting to see if chiral imbalance can generate PC in dense quark matter even in this case

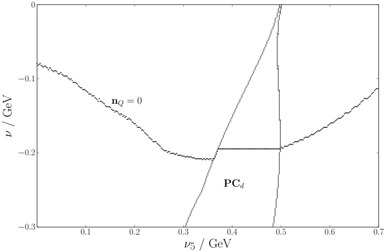
- Charge neutrality and β-equilibrium in neutron stars
- There are constraints in HIC ($n_Q = 0.4 n_B$)

  Hot QCD Collaboration arXiv:1812.08235 [hep-lat]
- Or β-equilibrium in neutron star mergers

  Mark Alford Phys. Rev. C 98, 065806 (2018); arXiv:1803.00662 [nucl-th]



$(\nu_5,\, \nu)$ phase portrait at $\mu =$ 450 MeV and $\mu_5 = 0$.



$(\nu_5,\, \nu)$ phase portrait at $\mu =$ 500 MeV and $\mu_5 = 150$ MeV.

Chiral imbalance generates the charged pion condensation in dense electric neutral matter.

There have been discussed several **mechanism of generation of chiral imbalance in neutron stars**.

It is interesting in light of new and expected data on masses and radii to extend the studies and

- find the **EOS** in the presence of **chiral imbalance**

- and explore **the M-R relation** for **neutron star with chiral imbalance**
(Consideration of phase structure of dense electric neutral baryonic matter with β-equilibrium is a first step in that direction.)

Dualities

Dualities

It is not related to holography or gauge/gravity duality

it is the dualities of the phase structures of different systems

The TDP

$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...)$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle\bar{q}q\rangle, ...) \qquad \Omega(T, \mu, \mu_i, ..., M, \Delta, ...)$$

The TDP

$$\Omega(T, \mu, \mu_i, ..., \langle \bar{q}q \rangle, ...) \qquad \Omega(T, \mu, \mu_i, ..., M, \Delta, ...)$$

The TDP (phase daigram) is invariant under

Interchange of - condensates - matter content

$$\Omega(M, \Delta, \mu_I, \mu_{I5})$$

$$M \longleftrightarrow \Delta, \qquad \nu \longleftrightarrow \nu_5$$

$$\Omega(M, \Delta, \mu_I, \mu_{I5}) = \Omega(\Delta, M, \mu_{I5}, \mu_I)$$

Duality in the phase portrait

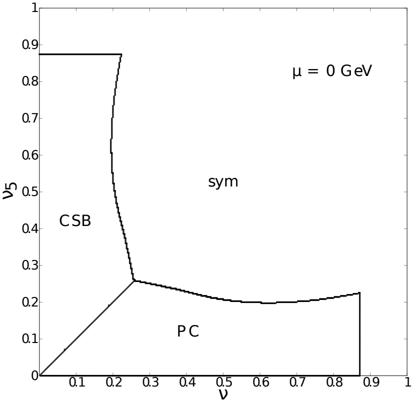


Figure: NJL model results

$$\mathcal{D}: \; M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5$$

Duality between chiral symmetry breaking and pion condensation

$$\text{PC} \longleftrightarrow \text{CSB} \quad \nu \longleftrightarrow \nu_5$$

Duality, Chiral limit

$$\mathcal{L}_{\text{QCD}} = \sum_{f=u,d} \bar{q}_f (i\not{D} - m_f) q_f - \frac{1}{4} \mathcal{G}_{\mu\nu,a} \mathcal{G}^{\mu\nu}_a.$$

$$\mathcal{L}_{\text{NJL}} = \sum_{f=u,d} \bar{q}_f \Big[\mathrm{i}\gamma^\nu \partial_\nu - m_f \Big] q_f + \frac{G}{N_c} \Big[(\bar{q}q)^2 + (\bar{q}\mathrm{i}\gamma^5 \vec{\tau} q)^2 \Big]$$

m_f is current quark masses

In the chiral limit $m_f = 0$ the Duality $\mathcal{D}$ is exact

m_f : $\frac{m_u + m_d}{2} \approx 3.5\text{MeV}$
In our case typical values of $\mu, \nu, ..., T, .. \sim 10 - 100s$ MeV, for example, 200-400 MeV
One can work in the chiral limit $m_f \to 0$
$m_f = 0 \quad \to \quad m_\pi = 0$
physical m_f a few MeV $\quad \to \quad$ physical $m_\pi \sim 140$ MeV

Duality between CSB and PC is **approximate** in **physical point**

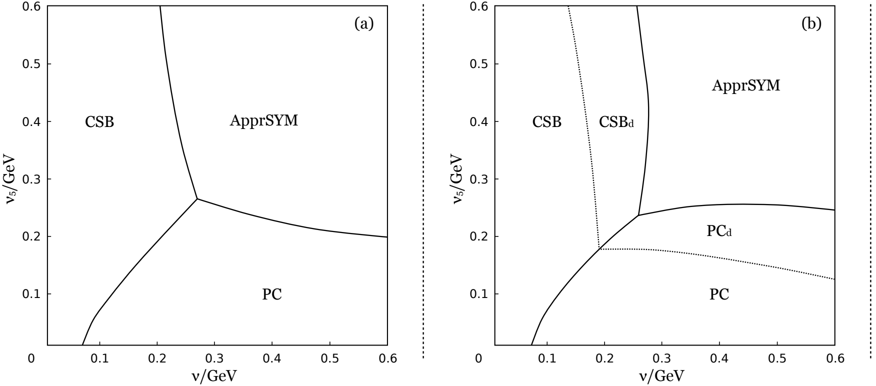


Figure: (ν, ν_5) phase diagram

Other Dualities

They are not that strong but still...

They could still be usefull

The TDP

$$\Omega(T, \mu, \nu, \nu_5, \mu_5;\ M, \Delta)$$

Let us assume that there is no PC

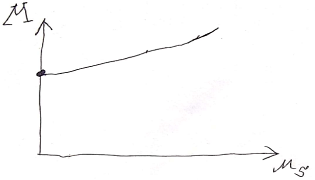
$$\Delta = 0$$

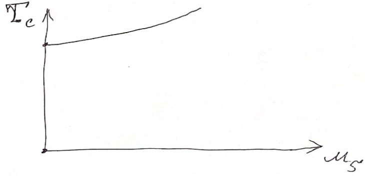
The TDP (phase diagram) is invariant under

$$\mu_5 \longleftrightarrow \nu_5$$

Example of use of duality: catalysis of CSB

QCD at non-zero μ_5





catalysis of CSB by chiral imbalance:

discussed by a number of authors, A. Kotov, V. Braguta, et al

- increase of $\langle \bar{q}q \rangle$ as μ_5 increases
- increase of critical temperature T_c of chiral phase transition (crossover) as μ_5 increases

The TDP

$$\Omega(T, \mu, \nu, \nu_5, \mu_5;\ M, \Delta)$$

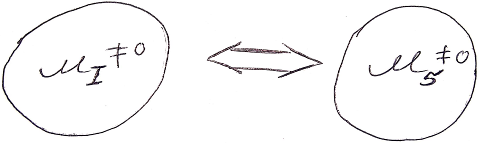
Let us assume that there is no CSB

$$M = 0$$

The TDP (phase diagram) is invariant under

$$\mu_5 \longleftrightarrow \nu$$

Two completely different systems



Dualities on the lattice

$(\mu_B, \mu_I, \mu_{I5}, \mu_5)$

$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

$\mu_B \neq 0$ impossible on lattice but if $\mu_B = 0$

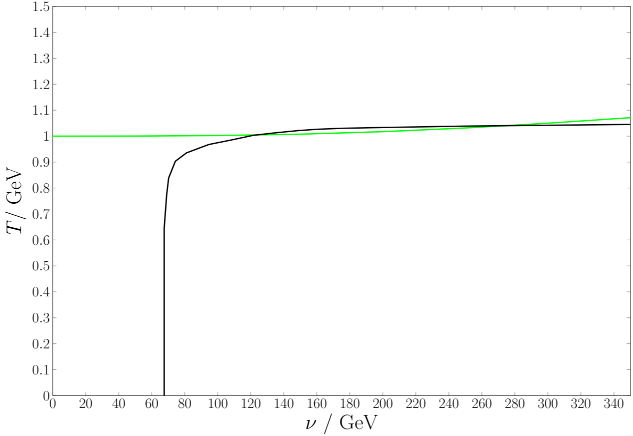
- **QCD at** μ_5 —- (μ_5, T)

  V. Braguta, A. Kotov et al, ITEP lattice group

- **QCD at** μ_I —- (μ_I, T)

  G. Endrodi, B. Brandt et al, Emmy Noether junior research group, Goethe-University Frankfurt, Institute for Theoretical Physics ()

Dualities on the lattice



T_c^M as a function of μ_5 (green line) and $T_c^\Delta(\nu)$ (black)

Uses of Dualities

How (if at all) it can be used

discussed in Particles 2020, 3(1), 62-79

A number of papers predicted **anticatalysis** (T_c decrease with μ_5) of dynamical chiral symmetry breaking

A number of papers predicted **catalysis** (T_c increase with μ_5) of dynamical chiral symmetry breaking

lattice results show the **catalysis**

(ITEP lattice group, V. Braguta, A. Kotov, et al)

But unphysically large pion mass

Duality $\Rightarrow$ catalysis of chiral symmetry beaking

- **Large N$_c$ orbifold equivalences** connect gauge theories with different gauge groups and **matter content** in the large N$_c$ limit.

M. Hanada and N. Yamamoto, JHEP 1202 (2012) 138, arXiv:1103.5480 [hep-ph], PoS LATTICE **2011** (2011), arXiv:1111.3391 [hep-lat]

- two gauge theories with gauge groups G_1 and G_2

- but with different μ_1 and μ_2

Duality

$$G_1 \longleftrightarrow G_2, \qquad \mu_1 \longleftrightarrow \mu_2$$

or

$$\text{Phase structure } (G_1 \text{ at } \mu_1) \quad \longleftrightarrow \quad \text{Phase structure } (G_2 \text{ at } \mu_2)$$

Circumvent the sign problem

Duality

QCD at $\mu_1 \longleftrightarrow$ QCD at μ_2

- QCD with μ_2 —- sign problem free,
- QCD with μ_1 —- sign problem (no lattice)

Investigations of (QCD with $\mu_2)_{\text{on lattice}} \Longrightarrow$ (QCD with μ_1)

Inhomogeneous phases (case)

Homogeneous case

$$\langle \sigma(x) \rangle \text{ and } \langle \pi_a(x) \rangle$$

$$\langle \sigma(x) \rangle = M, \quad \langle \pi_\pm(x) \rangle = \Delta, \quad \langle \pi_3(x) \rangle = 0.$$

In vacuum the quantities $\langle \sigma(x) \rangle$ and $\langle \pi_a(x) \rangle$ do not depend on space coordinate x.

in a dense medium the ground state expectation values of bosonic fields might depend on spatial coordinates

CDW ansatz for CSB
the single-plane-wave LOFF ansatz for PC

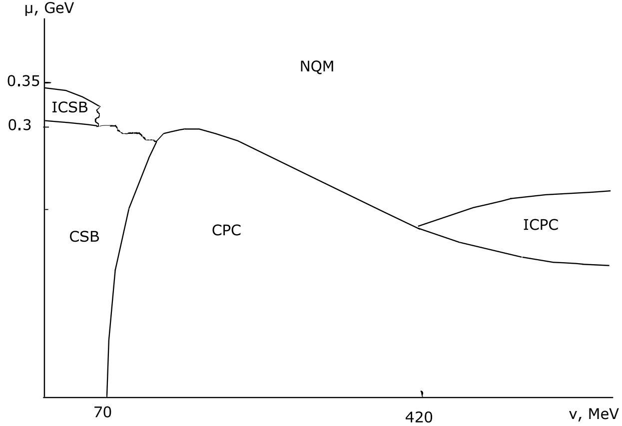
$$\langle \sigma(x) \rangle = M \cos(2kx^1), \quad \langle \pi_3(x) \rangle = M \sin(2kx^1),$$
$$\langle \pi_1(x) \rangle = \Delta \cos(2k'x^1), \quad \langle \pi_2(x) \rangle = \Delta \sin(2k'x^1)$$

equivalently

$$\langle \pi_\pm(x) \rangle = \Delta e^{\pm 2k'x^1}$$

Duality in inhomogeneous case is shown

$$\mathcal{D}_I : \qquad M \longleftrightarrow \Delta, \quad \nu \longleftrightarrow \nu_5, \quad k \longleftrightarrow k'$$



Combined schematic (ν, μ)-phase diagram.

M. Buballa, S. Carignano, J. Wambach, D. Nowakovski, Lianyi He et al.

schematic (ν_5, μ)-phase diagram

- exchange axis ν to the axis ν_5,
- rename the phases ICSB $\leftrightarrow$ ICPC, CSB $\leftrightarrow$ CPC, and NQM phase stays intact here

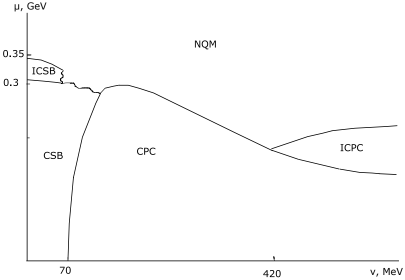


Figure: (ν, μ)-phase diagram

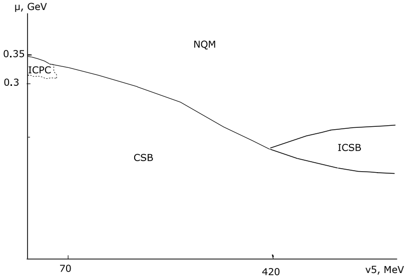


Figure: (ν_5, μ)-phase diagram

Two colour QCD case

QC_2D

Instead of chiral symmetry

$$SU_L(2) \times SU_R(2)$$

one has Pauli-Gursey flavor symmetry

$$SU(4)$$

Two colour NJL model

$$L = \bar{q}\Big[i\hat{\partial} - m_0\Big]q + H\Big[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 + \big(\bar{q}i\gamma^5\sigma_2\tau_2 q^c\big)\big(\overline{q^c}i\gamma^5\sigma_2\tau_2 q\big)\Big]$$

$$L = \bar{q}\left[i\hat{\partial} - m_0\right]q + H\left[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2 + \left(\bar{q}i\gamma^5\sigma_2\tau_2 q^c\right)\left(\overline{q^c}i\gamma^5\sigma_2\tau_2 q\right)\right]$$

If you use Habbard-Stratanovich technique and auxiliary fileds

$$\sigma(x) = -2H(\bar{q}q),\ \vec{\pi}(x) = -2H(\bar{q}i\gamma^5\vec{\tau}q)$$
$$\Delta(x) = -2H\left[\overline{q^c}i\gamma^5\sigma_2\tau_2 q\right] = -2H\left[q^T C i\gamma^5\sigma_2\tau_2 q\right]$$
$$\Delta^*(x) = -2H\left[\bar{q}i\gamma^5\sigma_2\tau_2 q^c\right] = -2H\left[\bar{q}i\gamma^5\sigma_2\tau_2 C\bar{q}^T\right]$$

Condensates are

$$\langle\sigma(x)\rangle = M, \quad \langle\pi_1(x)\rangle = \pi_1, \quad \langle\Delta(x)\rangle = \Delta, \quad \langle\Delta^*(x)\rangle = \Delta^*.$$

The TDP is invariant with respect to the so-called dual transformations $\mathcal{D}_1$, $\mathcal{D}_2$ and $\mathcal{D}_3$,

$$\mathcal{D}_1: \quad \mu \longleftrightarrow \nu, \quad \pi_1 \longleftrightarrow |\Delta|$$

$$\mathcal{D}_2: \quad \mu \longleftrightarrow \nu_5, \quad M \longleftrightarrow |\Delta|$$

$$\mathcal{D}_3: \quad \nu \longleftrightarrow \nu_5, \quad M \longleftrightarrow \pi_1$$

Duality between CSB and PC was found in

- In the framework of NJL model
- In the large N_c approximation (or mean field)
- In the chiral limit

QCD Lagrangian is

$$\mathcal{L} = i\bar{\psi}\gamma^\mu D_\mu\psi + \bar{\psi}\Big[\mu\gamma^0 + \frac{\mu_I}{2}\tau_3\gamma^0 + \frac{\mu_{I5}}{2}\tau_3\gamma^0\gamma^5 + \mu_5\gamma^0\gamma^5\Big]\psi$$

$$\mathcal{D}: \qquad \psi_R \to i\tau_1\psi_R$$

$$\mu_I \leftrightarrow \mu_{I5}$$

$$\bar{\psi}\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_1\psi$$

$$M \longleftrightarrow \Delta, \qquad \nu \longleftrightarrow \nu_5, \quad \mu_I \longleftrightarrow \mu_{I5}$$

$$i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi \leftrightarrow i\bar{\psi}^C\sigma_2\tau_2\gamma^5\psi, \qquad \bar{\psi}^C\sigma_2\tau_2\psi \leftrightarrow \bar{\psi}^C\sigma_2\tau_2\psi$$

$$\bar{\psi}\tau_2\psi \leftrightarrow \bar{\psi}\tau_3\psi, \qquad \bar{\psi}\tau_1\psi \leftrightarrow i\bar{\psi}\gamma^5\psi, \qquad i\bar{\psi}\gamma^5\tau_2\psi \leftrightarrow i\bar{\psi}\gamma^5\tau_3\psi$$

$$\mathcal{D} \;\in\; SU_R(2) \;\in\; SU_L(2)\times SU_R(2)$$

$$\mu_I \leftrightarrow \mu_{I5}$$

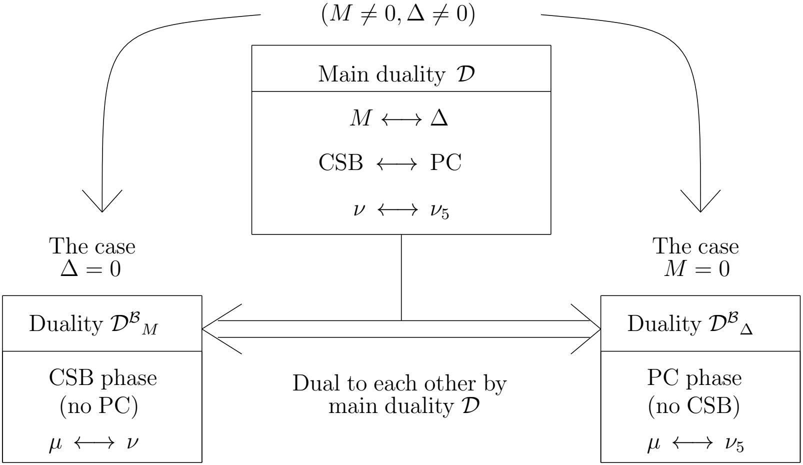
$M \neq 0$ breaks the chiral symmetry

Duality $\mathcal{D}$ is a remnant of chiral symmetry

Dualities concerning baryon density

They could be even more usefull

Dualities with baryon density



Let us assume that $\nu \neq 0$
as a rule there is PC and there is no CSB

$$M = 0, \qquad \mu \longleftrightarrow \nu_5$$

PC at non-zero μ $\longleftrightarrow$ PC at non-zero ν_5

Conclusions and Thanks for the Attention

- $(\mu_B, \mu_I, \nu_5, \mu_5)$ phase diagram was studied
  PC *in dense matter with chiral imbalance*
  in *in dense electic neutral matter in β-equilibrium*
- It was shown that there exist dualities
- There have been shown ideas how dualities can be used
- Duality is **not just entertaining mathematical property** but an **instrument with very high predictivity power**
- (μ_B, ν_5) phase diagram is **quite rich** and contains various **inhomogeneous phases**
- Richer structure of **Dualities in the two colour case**