# Baryon fluctuations in Extended linear sigma model

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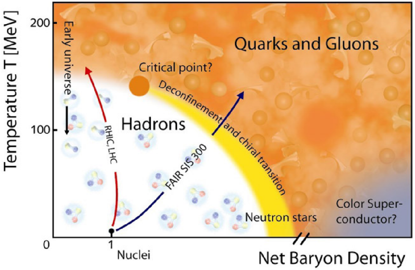
# Outline

- Motivations.
- Extended linear sigma model and its thermodynamics.
- Baryonfluctuations in the EL$\sigma$M.
- (Axial) vector curvature masses.
- Outlook.

# Critical end point

The phase structure of the strongly interacting matter, especially the region of chiral phase transition, is a heavily studied field of particle physics.




- There might be a first order phase transition.
- At $\mu_q = 0$ there is a crossover according to lattice calculations.
- There is a Critical end point (CEP) between the two regions.
- Effective models shows the existence of a CEP.
- The upcomming FAIR experiment will hopefully give insight into the energy region, where the CEP might be. PS **94**, no. 3, 033001 (2019)

# Uncertainty of CEP

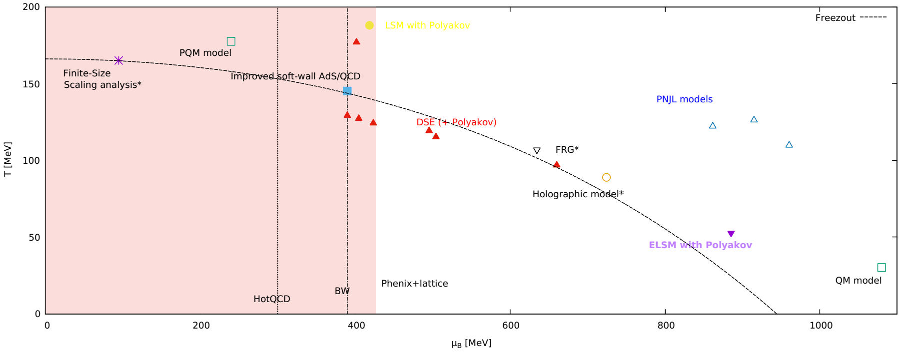



Figure: Location of CEP according to different models and the freeze-out line for comparison. The shadowed region and the vertical lines corresponds to lower limits of the CEP.

| | | | |
|---|---|---|---|
| Budapest-Wupperal | Phys. Lett. B **751**, 559 (2015) | Phenix | Phys. Rev. C **93**, no. 1, 011901 (2016) |
| HotQCD | Phys. Lett. B **795**, 15 (2019) | Freezout | J. Phys. G **32**, S165 (2006) |

# Extended linear sigma model

Already existing model, published in: Phys. Rev. D **93**, no. 11, 114014 (2016) .

- Linear sigma model with full Scalar, Pseudoscalar, Vector and Axialvector nonets.
- Isospin symmetric case: 16 mesonic degrees of freedom.
- The Lagrangian build up from the fields

$$L^\mu = \sum_a (V_a^\mu + A_a^\mu)\, T_a, \quad R^\mu = \sum_a (V_a^\mu - A_a^\mu)\, T_a, \quad M = \sum_a (S_a + iP_a)\, T_a,$$

with terms up to fourth order, taking care of the symmetry properties.

- Constituent quarks (2+1 flavors) are included in a Yukawa type term,

$$\mathcal{L}_{Yukawa} = \bar{\psi}\left[i\gamma_\mu D^\mu - \mathcal{M}\right]\psi,$$

where $\mathcal{M} = g_F\,(\mathbb{1} M_S + i\gamma_5 M_P)$ is the quark mass matrix and $M_S = \sum_a S_a T^a$, $M_P = \sum_a P_a T^a$.

# Extended linear sigma model

- The (15) parameters of the Lagrangian are fitted with $\chi^2$ test. The used (30) physical parameters are the (pseudo)scalar meson masses, decay widths and the $T_C(\mu = 0)$ critical temperature, getting from lattice simulations.
- All parameters and the fitting procedure can be found in PRD **93**, no. 11, 114014 (2016) .
- In the 0-8 sector N/S bases is used

$$\xi_N = \frac{1}{\sqrt{3}}\left(\sqrt{2}\xi_0 + \xi_8\right) \qquad \xi_S = \frac{1}{\sqrt{3}}\left(\xi_0 - \sqrt{2}\xi_8\right).$$

- Due to SSB fields can be written as $\varphi(x) := \phi + \varphi'(x)$, where $\varphi'(x)$ has no vacuum expectation value (vev).
  Only the scalar-isoscalar sector has nonzero vev denoted as: $\phi_N, \phi_S$.
- Including Polyakov loop variables $\langle\Phi\rangle, \langle\bar{\Phi}\rangle$ as further order parameters, which mimic the deconfinment phase transition.

# Grand potential

### Assumptions

- Symmetric quark matter: $\mu_u = \mu_d = \mu_s = \mu_q = \mu_B/3$
- Fermionic determinant calculated with neglecting mesonic fluctuations, ie. (pseudo)scalar mesons treated at tree-level only.

The Grand potential reads

$$\Omega(T, \mu_q) = U(\langle M \rangle) + U(\langle \Phi \rangle, \langle \bar{\Phi} \rangle) + \Omega^{(0)}_{\bar{q}q}(T, \mu_q)$$

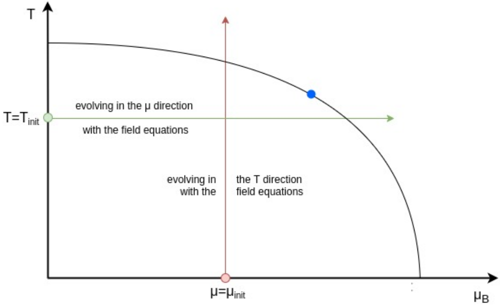
where the terms are the tree-level mesonic potential, the Polyakov loop potential, and the contribution of fermionic fluctuations respectively.

### Field equations

$$\frac{\partial \Omega}{\partial \bar{\Phi}} = \frac{\partial \Omega}{\partial \Phi} = \frac{\partial \Omega}{\partial \phi_N} = \frac{\partial \Omega}{\partial \phi_S} = 0$$

# Solutions in $T$ and $\mu_B$

- We need initial values for solving the field equations. It is possible at $T = \mu_B = 0$.
- Move into finite $T$ (or $\mu_B$), using the last solutions as initial values.




- One can calculate the pressure as $p(T, \mu_B) = \Omega(T = 0, \mu_B) - \Omega(T, \mu_B)$ for fixed $\mu_B$ as a function of temperature.

# Baryon fluctuations

- One need quantities accessible both from measurements and theoretical calculations and are sensitive to the critical behavior.
- Baryon number fluctuations (ratios, such as kurtosis).

One can characterize the baryon fluctuations with the susceptibilities.

**Baryon number susceptibilities**

$$\chi_n^B = \frac{\partial^n p/T^4}{\partial\left(\mu_B/T\right)^n} = T^{n-4}\frac{\partial^n p}{\partial \mu_B^n}$$

Ratios of susceptibilities do not include (static) finite volume effect. Finite volume fluctuations still should be included from the theoretical side.

# Baryon fluctuations

The (excess) kurtosis can be written as the ratio of the fourth and the square of the second order cumulants (variance),

$$\kappa = \frac{m_4}{m_2^2} = \frac{m_4}{m_2 \sigma_2} = \frac{\chi_4}{\chi_2 \sigma_2}$$

$\kappa$ can be rewritten with the number susceptibilities (in the grand canonical ensemble). Thus often the ratio of susceptibilities defined as kurtosis.

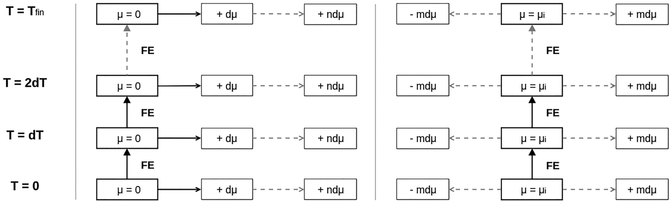
### Kurtosis

$$\sigma_2 \kappa = \frac{\chi_4}{\chi_2}$$

Further ratios can be defined. $\chi_3/\chi_1$ (skewness), $\chi_4/\chi_2$ (kurtosis), $\chi_6/\chi_2$, $\chi_8/\chi_2$ has higher importance in lattice calculations by the continuum extrapolation. Thus, these are good quantities to compare effective models with lattice results.

# Calculation of $\mu_B$ derivatives

In $T$ running: finite difference method or polynomial.




In $\mu_B$ running: directly from interpolation (spline or polynomial).

# Results for cumulants

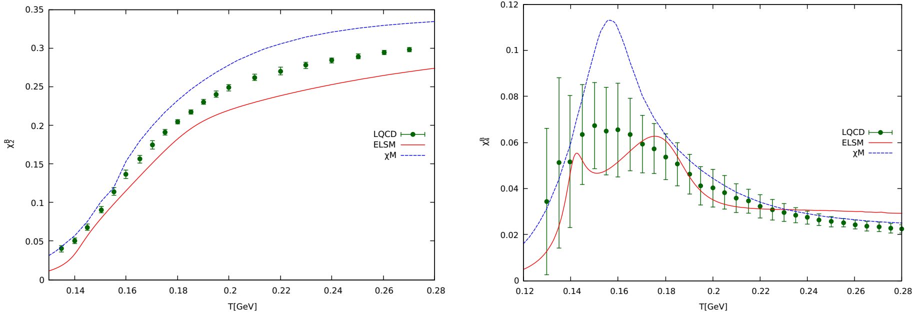



Figure: $\chi_2^B$ as the function of temperature in the extended linear sigma model, in the chiral matrix model, and in lattice calculations (left). The temperature dependence of $\chi_4^B$ compared again with the chiral matrix model and lattice data (right).

Chiral matrix model: Phys. Rev. D **94**, no. 3, 034015 (2016)

# Results for cumulants

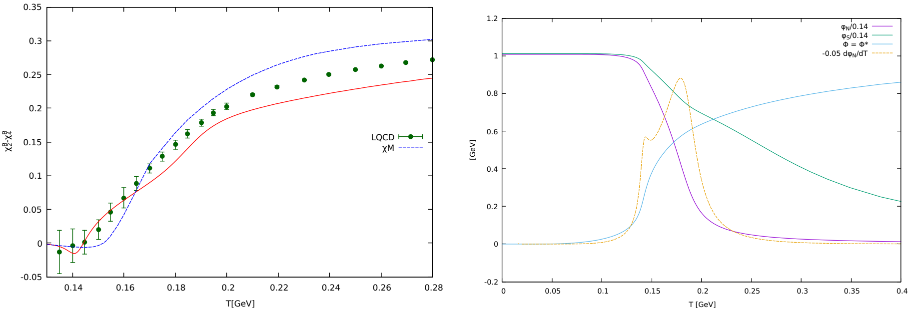



Figure: $\chi_2^B - \chi_4^B$ (left) in the extended linear sigma model, in the chiral matrix model, and in lattice calculations. The order parameters of ELSM (right).

# Results: kurtosis

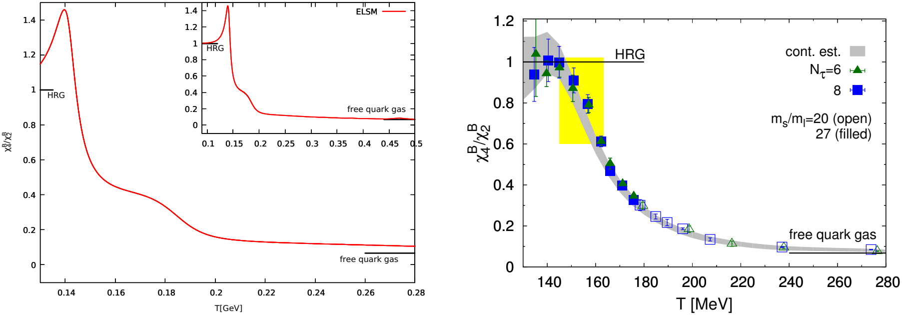



Figure: Kurtosis in the extended linear sigma model (left) compared with lattice data (right) as a function of temperature. The insert shows, how the kurtosis reaches the HRG and free quark gas limits in EL$\sigma$M.

Lattice: Phys. Rev. D **95**, no. 5, 054504 (2017)

# In finite baryon chemical potential

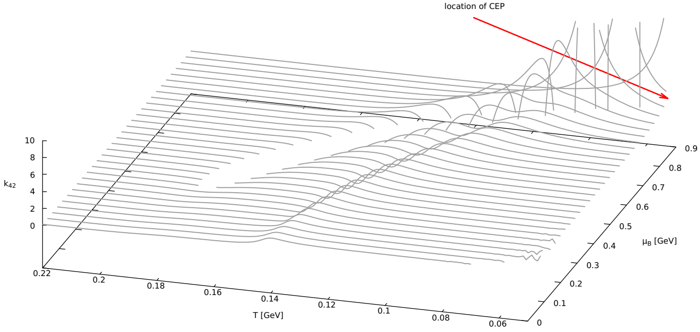



Figure: 3D plot of the kurtosis.

# In finite $\mu_B$

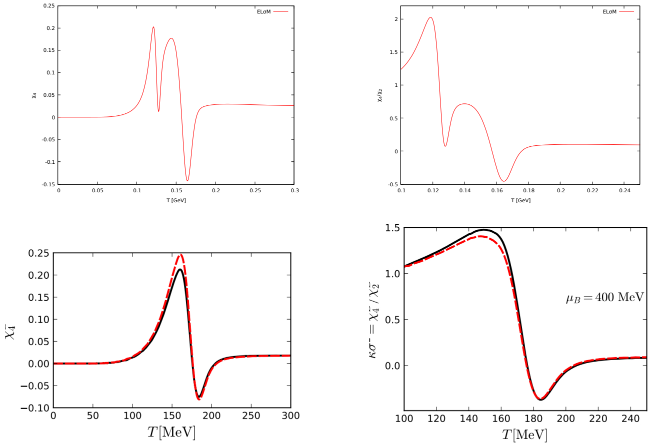



Figure: $\chi_4$ (left) and the kurtosis (right) in finite $\mu_B$, in the EL$\sigma$M (top) and in FRG calulation (bottom). Phys. Rev. D **100**, no. 9, 094029 (2019)

# In finite $\mu_B$

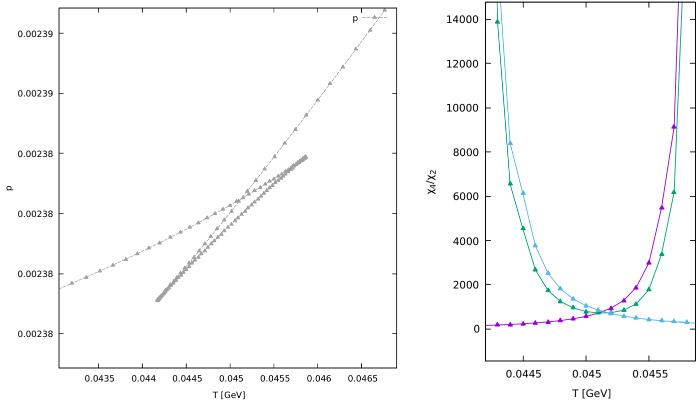



Figure: The pressure and the kurtosis at $T_I = 0.9$ GeV.

# Curvature masses

**Curvature meson masses**

$$m^2_{i,ab} = \frac{\partial^2 \Omega(T,\mu)}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\bigg|_{\{\varphi_j=0\}_j} = \mathfrak{m}^2_{i,ab} + \Delta m^2_{i,ab} + \delta m^2_{i,ab}$$

The three terms are the tree-level mass and the corrections coming from the fermionic vacuum and thermal fluctuations, respectively. $a, b \in \{0, \ldots, 8\}$

- $i = S, P$ firstly calculated by Schaefer and Wagner in PRD **79**, 014018 (2009)
- $i = V, A$: a work in progress.

# Recall the grand potential

$U(\langle M \rangle)$ depends on the (up to fourth order of) scalar-isoscalar background and the parameters of the Lagrangian. It also contains the counterterms for the renormalization of the fermionic contribution.

$U(\langle \Phi \rangle, \langle \bar{\Phi} \rangle)$ depends only on the Polyakov loop parameters.

$\Omega_{\bar{q}q}^{(0)}(T, \mu_q)$ coming from the fermion determinant. It can be written as

$$\Omega_{\bar{q}q}^{(0)}(T, \mu_q) = \Omega_{\bar{q}q}^{(0)v} + \Omega_{\bar{q}q}^{(0)T}(T, \mu_q)$$

where $v$ denotes the zero temperature vacuum part, while $T$ refers to the thermal part.

# Curvature masses

Derivatives of vacuum and thermal part of $\Omega^{(0)}_{\bar{q}q}(T,\mu_q)$:

$$\Delta m^2_{i,ab,\nu} = \frac{-3}{8\pi^2}\sum_{f=u,d,s}\left[\left(\frac{3}{2}+\log\frac{m_f^2}{M_0^2}\right)m^{2(i)}_{f,a,\mu}m^{2(i)}_{f,b,\mu} + m_f^2\left(\frac{1}{2}\log\frac{m_f^2}{M_0^2}\right)m^{2(i)}_{f,ab,\mu}\right]$$

$$\delta m^2_{i,ab,\nu} = 6\sum_{f=u,d,s}\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_f(p)}\left[\left(f_f^+(p)+f_f^-(p)\right)\left(m^{2(i)}_{f,ab,\mu}-\frac{m^{2(i)}_{f,a,\mu}m^{2(i)}_{f,b,\mu}}{2E_f^2(p)}\right) + \left(B_f^+(p)+B_f^-(p)\right)\frac{m^{2(i)}_{f,a,\mu}m^{2(i)}_{f,b,\mu}}{2TE_f(p)}\right]$$

where

$$f_f^{\pm}(p) = \frac{\Phi^{\pm}e^{-\beta E_f^{\pm}(p)}+2\Phi^{\mp}e^{-2\beta E_f^{\pm}(p)}+e^{-3\beta E_f^{\pm}(p)}}{1+3\left(\Phi^{\pm}+\Phi^{\mp}e^{-\beta E_f^{\pm}(p)}\right)e^{-\beta E_f^{\pm}(p)}+e^{-3\beta E_f^{\pm}(p)}}$$

and

$$B_f^{\pm}(p) = 3\left(f_f^{\pm}(p)\right)^2 - \frac{\Phi^{\pm}e^{-\beta E_f^{\pm}(p)}+4\Phi^{\mp}e^{-2\beta E_f^{\pm}(p)}+3e^{-3\beta E_f^{\pm}(p)}}{1+3\left(\Phi^{\pm}+\Phi^{\mp}e^{-\beta E_f^{\pm}(p)}\right)e^{-\beta E_f^{\pm}(p)}+e^{-3\beta E_f^{\pm}(p)}}$$

# Curvature masses

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$$m^{2(i)}_{f,a} \equiv \left.\frac{\partial m_f^2}{\partial\varphi_{i,a}}\right|_{\{\varphi_j=0\}_j} \qquad m^{2(i)}_{f,ab} \equiv \left.\frac{\partial^2 m_f^2}{\partial\varphi_{i,a}\partial\varphi_{i,b}}\right|_{\{\varphi_j=0\}_j}$$

# Derivatives in the curvature masses

### The fermion mass matrix

$$\mathcal{M} = \gamma_0 \left( \mathbb{1} M_S + \gamma_5 M_P + \gamma_\mu V^\mu + \gamma_\mu \gamma_5 A^\mu \right)$$

# Derivatives in the curvature masses

## The fermion mass matrix

$$\mathcal{M} = \gamma_0 \left( \mathbb{1} M_S + \gamma_5 M_P + \gamma_\mu V^\mu + \gamma_\mu \gamma_5 A^\mu \right)$$

| (i) | ab | $m^{2(i)\mu}_{l,ab}$ | $m^{2(i)\mu}_{s,ab}$ |
|---|---|---|---|
| V | 11 | $g_V^2$ | 0 |
| | NN | $g_V^2$ | 0 |
| | SS | 0 | $g_V^2$ |

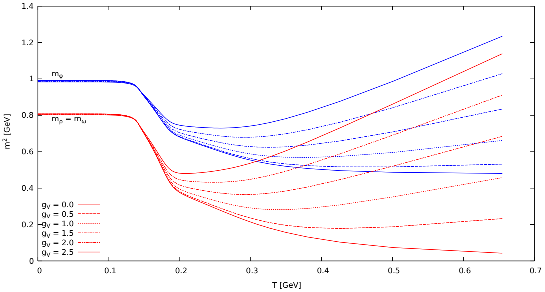


Figure: Vector masses with different couplings.

# Summary and outlook

- EL$\sigma$M with Polyakow loop. Gives CEP at $(885, 52.7)$ $MeV$.
- Baryon fluctuations and Kurtosis results are close to lattice data in $\mu_B = 0$ and can be calculated in finite $\mu_B$.

Ways of improvement

- Improve the calculation of derivatives with respect to $\mu_B$.
- To compere to experimental results further improvements needed. (Like finite volume effects)
- Extend the approximation for the field equation and the curvature masses selfconsistently. (Including meson fluctuations)