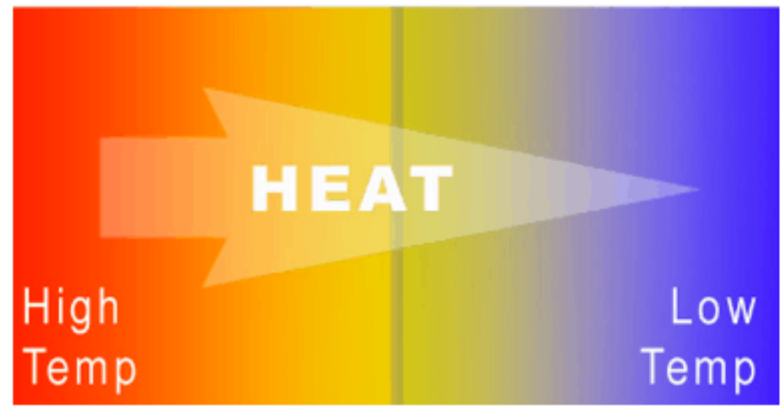
Surprises in Large Nc Thermodynamics



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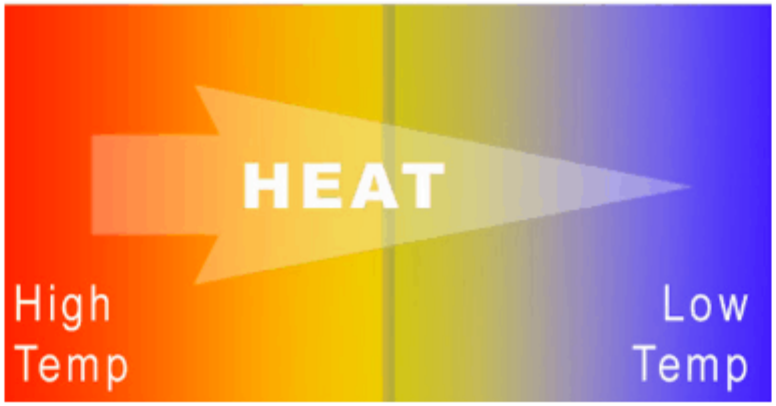

TDC, Scott Lawrence & Yukari Yamauchi (In Preparation)

An Overview

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rwoni i zielony szlak
Ski-Stop Grill Bar
BoardZone
Jaworzyna Krynicka
1114m npm
Gospoda na
Jaworzynie Krynickiej
Narciarskie Trasy
Biegowe "u Leśników"
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Jaworzyna Krynicka -
wyciąg krzesełkowy 4 os
Krynica Zdrój
PARK LINOWY
Paintball Krynica
Fabryka Endorfiny
Gościniec Czarny Potok
Hotel Czarny Potok

An Overview

- Introduction
  - Some cautions
  - Standard results
  - Assumptions
- Some surprises
  - A metastable supercooled phase with **negative absolute pressure**.
  - A **clean** demonstration of a strongly coupled regime of plasma.
  - Peculiar behavior at the endpoint of the hadronic phase; existence of locally unstable but long-lived regime.

A Motto for $1/N_c$ practitioners

We may well be wrong but at least we are systematic

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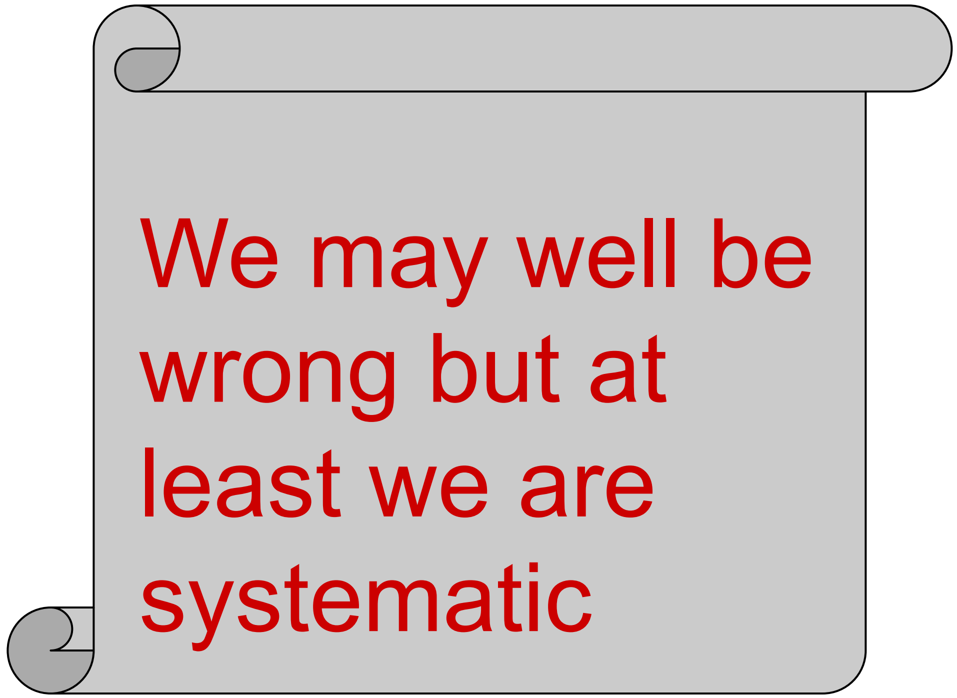
To the extent that $1/N_c$ corrections are modest, the large N_c world *may* be a useful cartoon version of the physical world.

A Motto for $1/N_c$ practitioners



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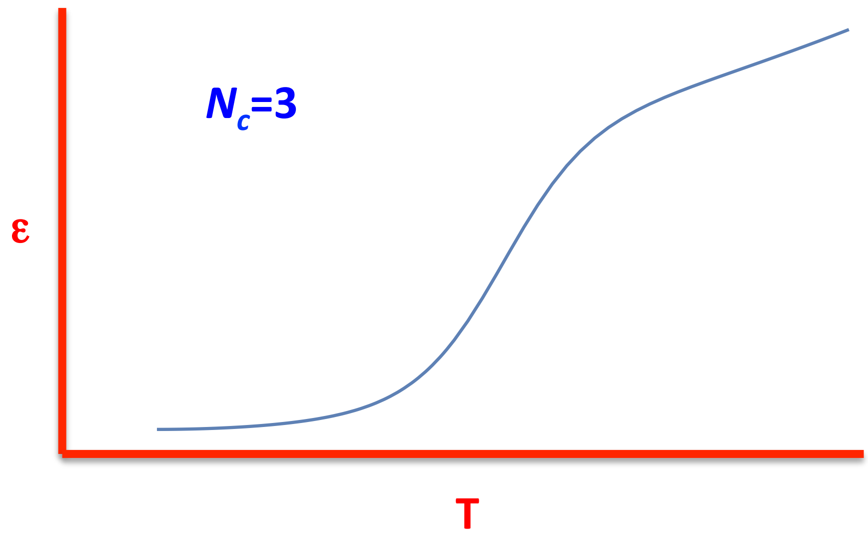
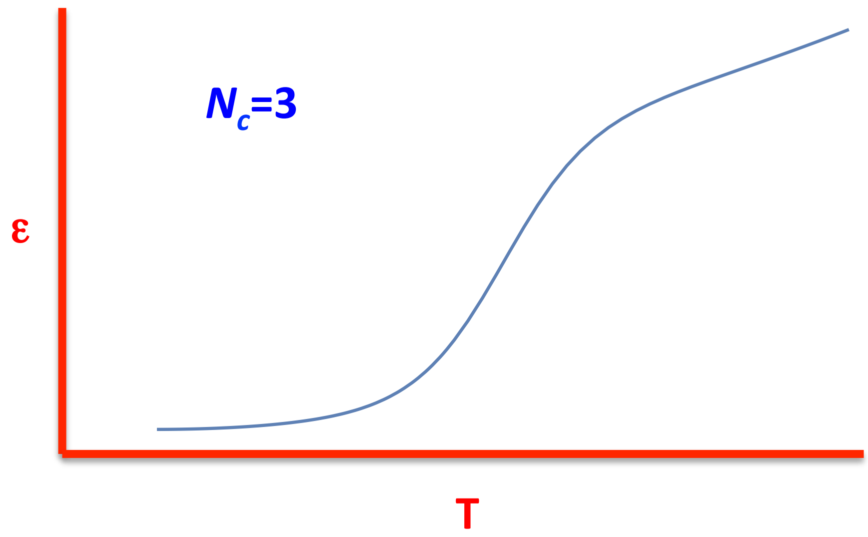
However thermodynamic properties around phase transitions or rapid cross-overs are likely to be cases where the cartoon is insufficient.


N_c=3
ε
T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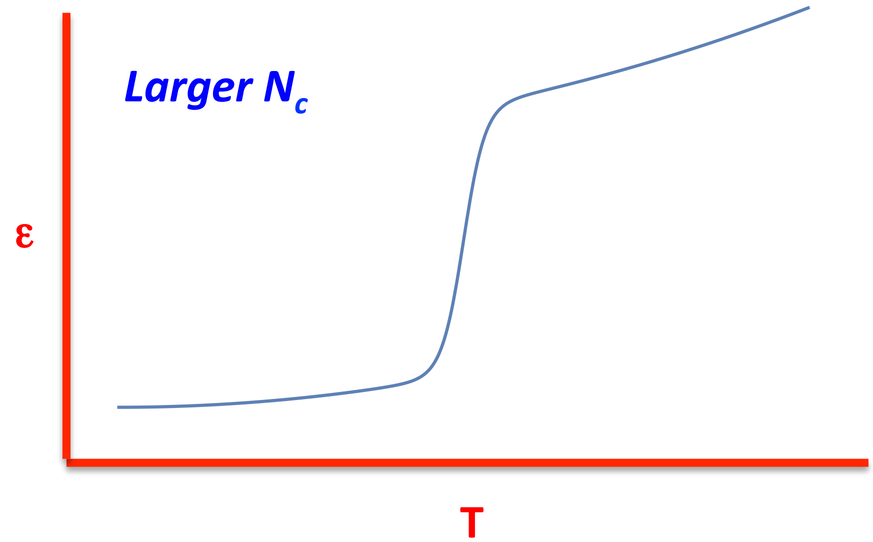
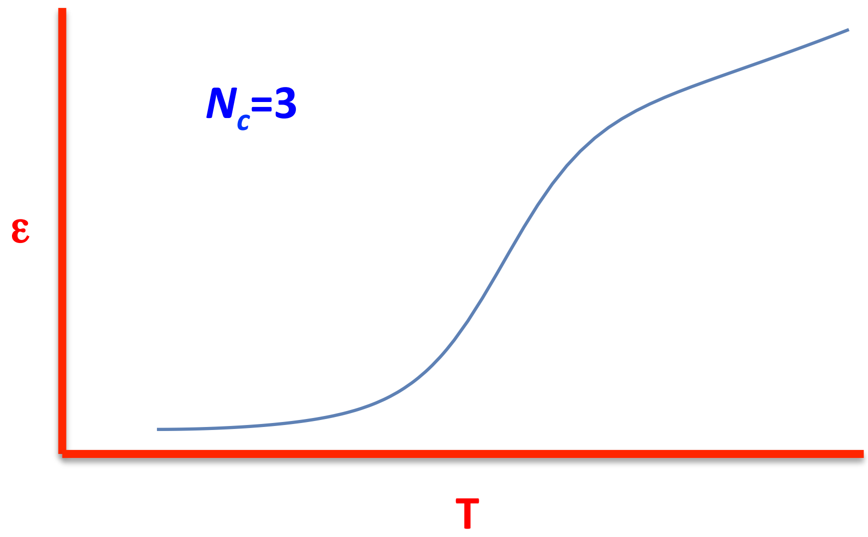

Larger N_c
ε
T



N_c=3
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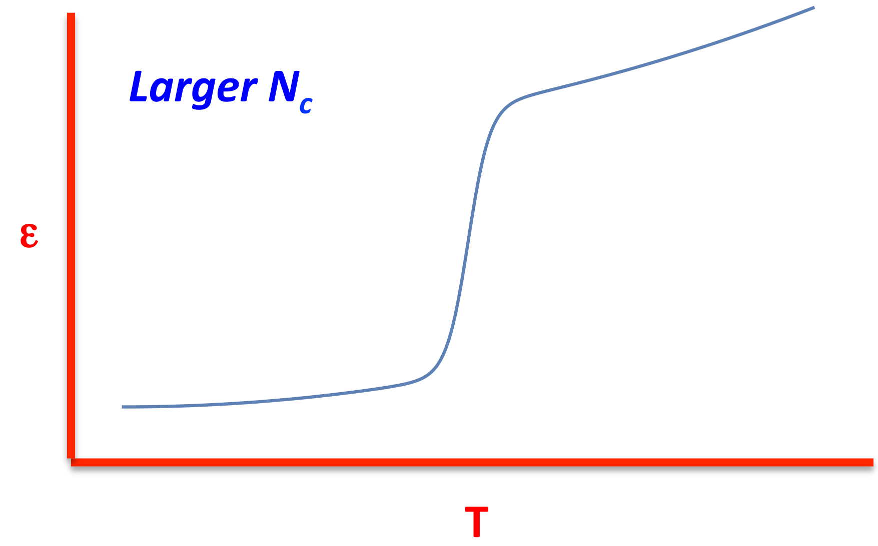
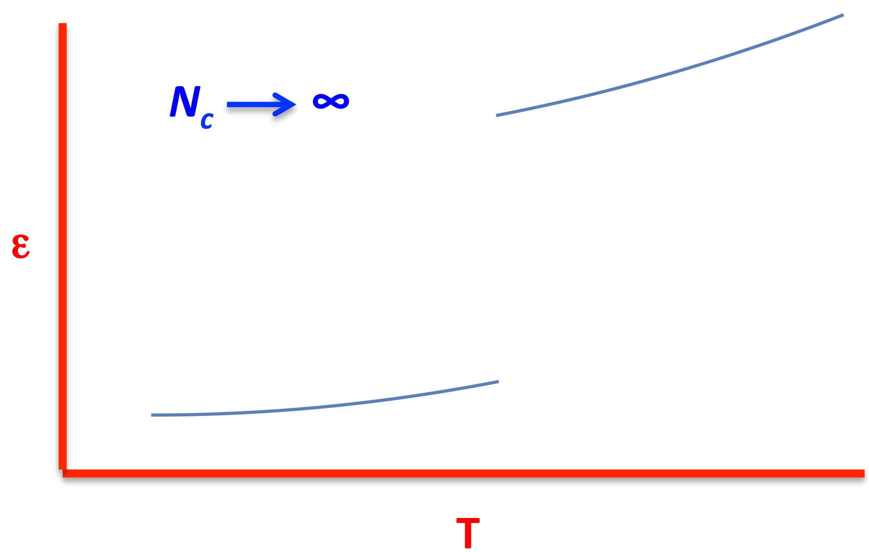
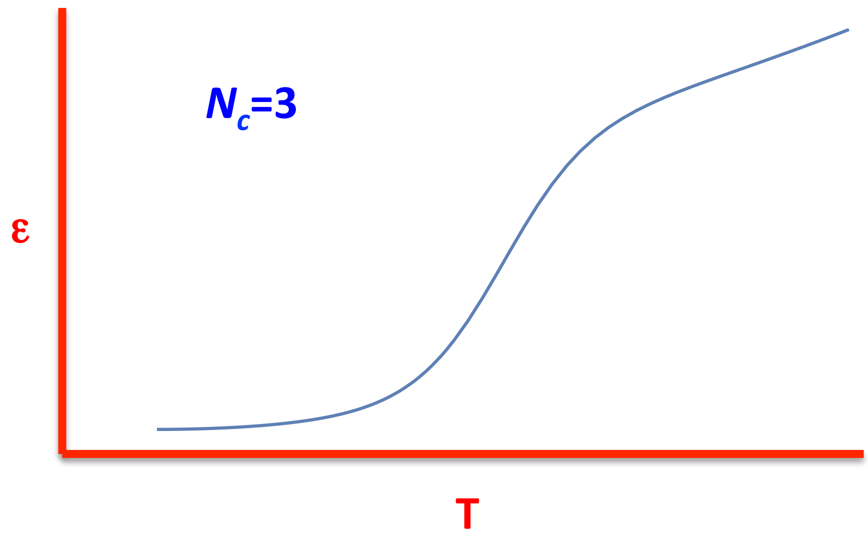
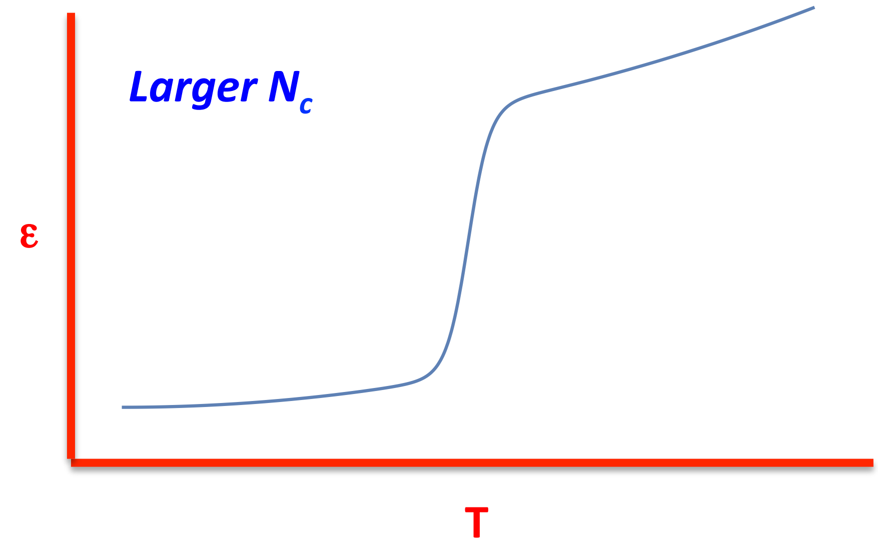


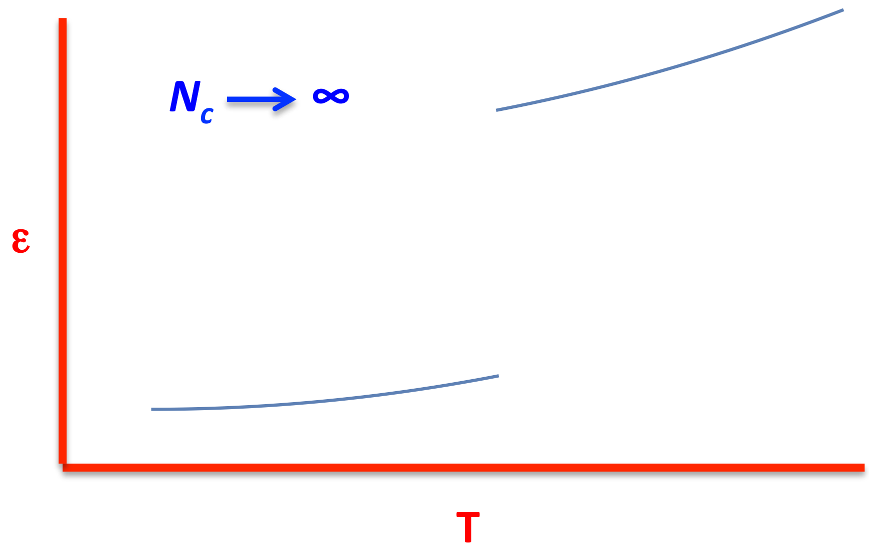
Larger N_c
ε
T



$N_c \longrightarrow \infty$
ε
T

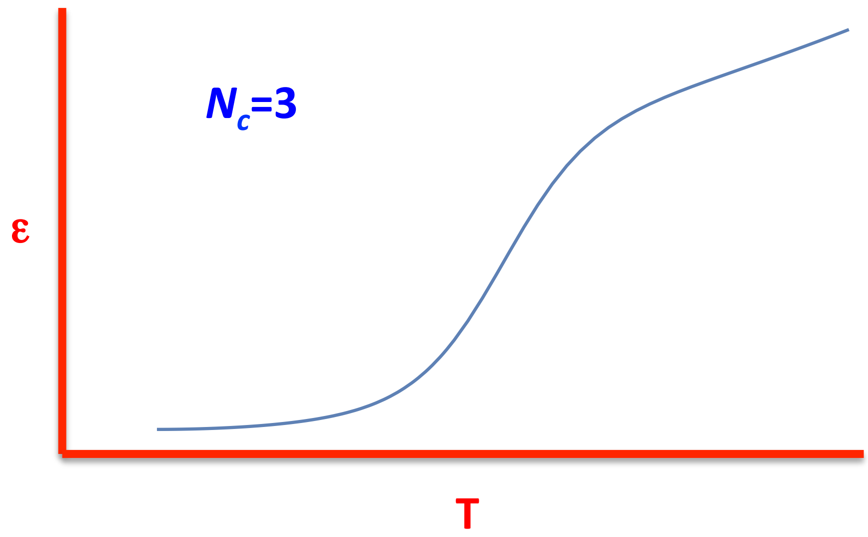


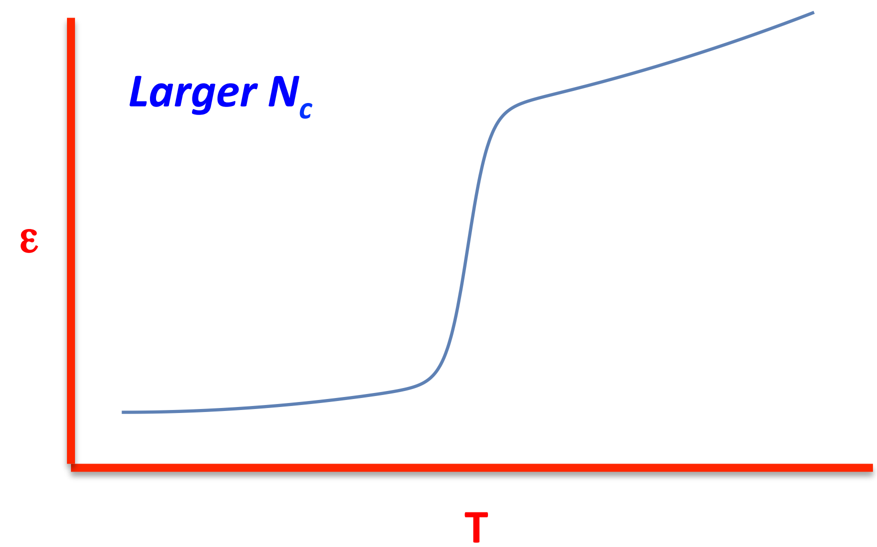


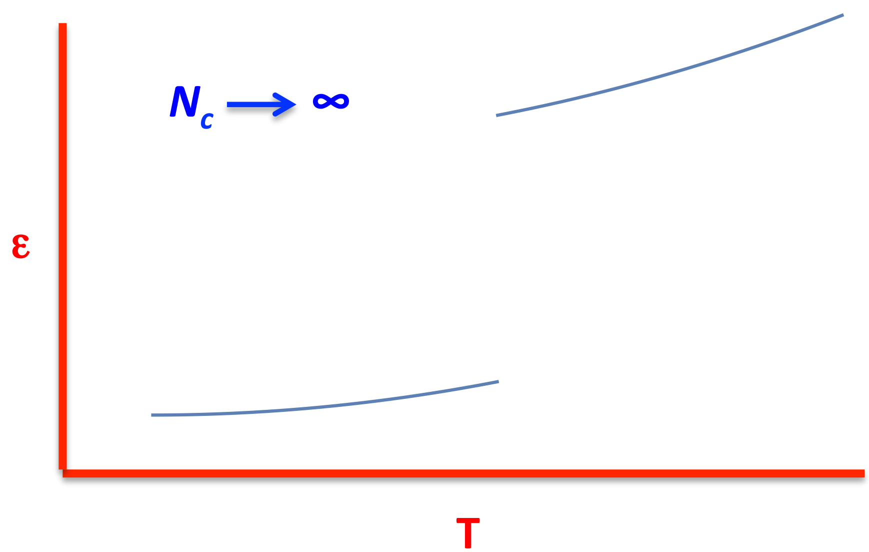


A crossover for N_c=3 can become increasing sharp as N_c increases and as it goes to ∞, the qualitative behavior can change from being a crossover to a first order transition—a qualitatively different behavior.

This is precisely what we believe happens for QCD.







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Despite the qualitative differences there may be useful insights by considering the large N_c limit.

Some standard large N_c results (Witten, 't Hooft 1970s)

– Mesons and glueballs exist as unmixed narrow states with masses of order unity in a $1/N_c$ expansion:

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  - Widths scales: $\Gamma_{meson} \sim N_c^{-1}$, $\Gamma_{glueball} \sim N_c^{-2}$
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- Baryons have masses that scale as $m_{\text{baryon}} \sim N_c^1$

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- Previous results imply that in a hadronic phase the system becomes a weakly coupled hadronic gas composed of mesons and glueballs with the energy density scaling as $\sim N_c^0$

- RG analysis indicates that the QCD becomes weakly coupled at a momentum transfer that scales as $\sim N_c^0$.
  - The system enters a quark-gluon plasma regime at temperature that scales as $\sim N_c^0$.
  - The energy density and entropy density in the quark-gluon plasma regime scale as $\sim N_c^2$: $s(\varepsilon)=N_c^2 f(\varepsilon/N_c^2)$
- The discrepancy between the N_c^0 behavior in the hadronic regime and the N_c^2 behavior in the plasma regime implies that there must be a **phase transition** (first or second order)—at least as $N_c \rightarrow \infty$.

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  - Yang Mills is known to have a first order transition at $N_c=3$.
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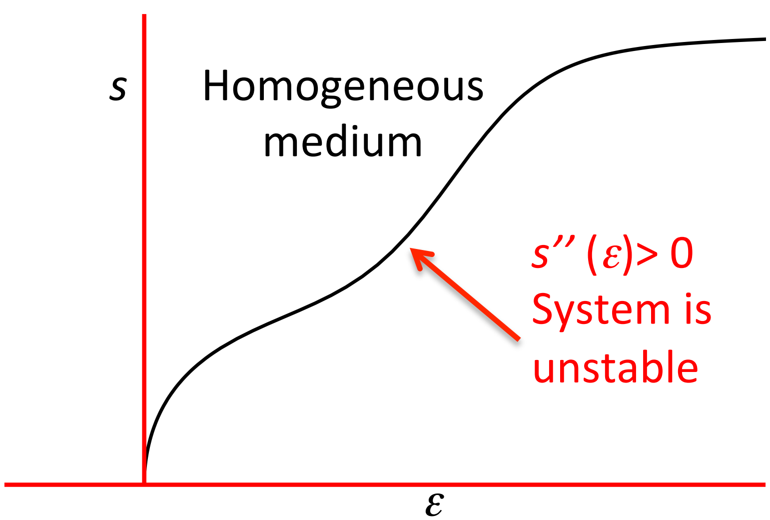
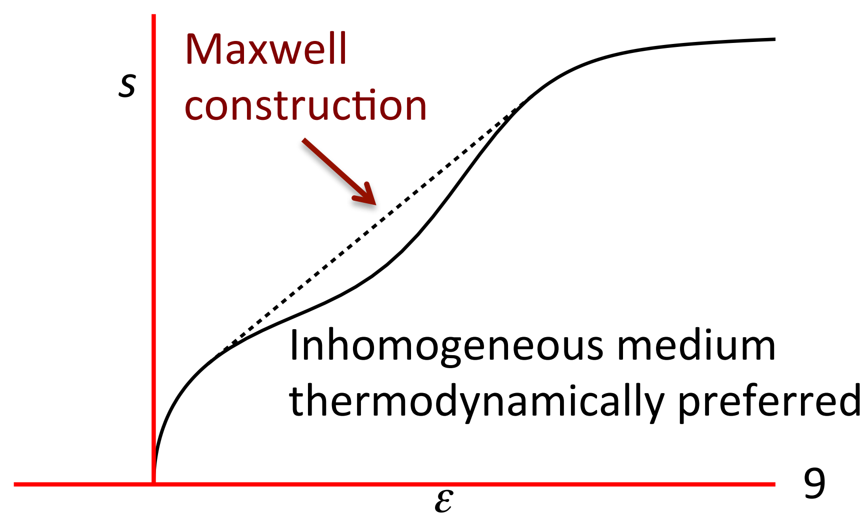
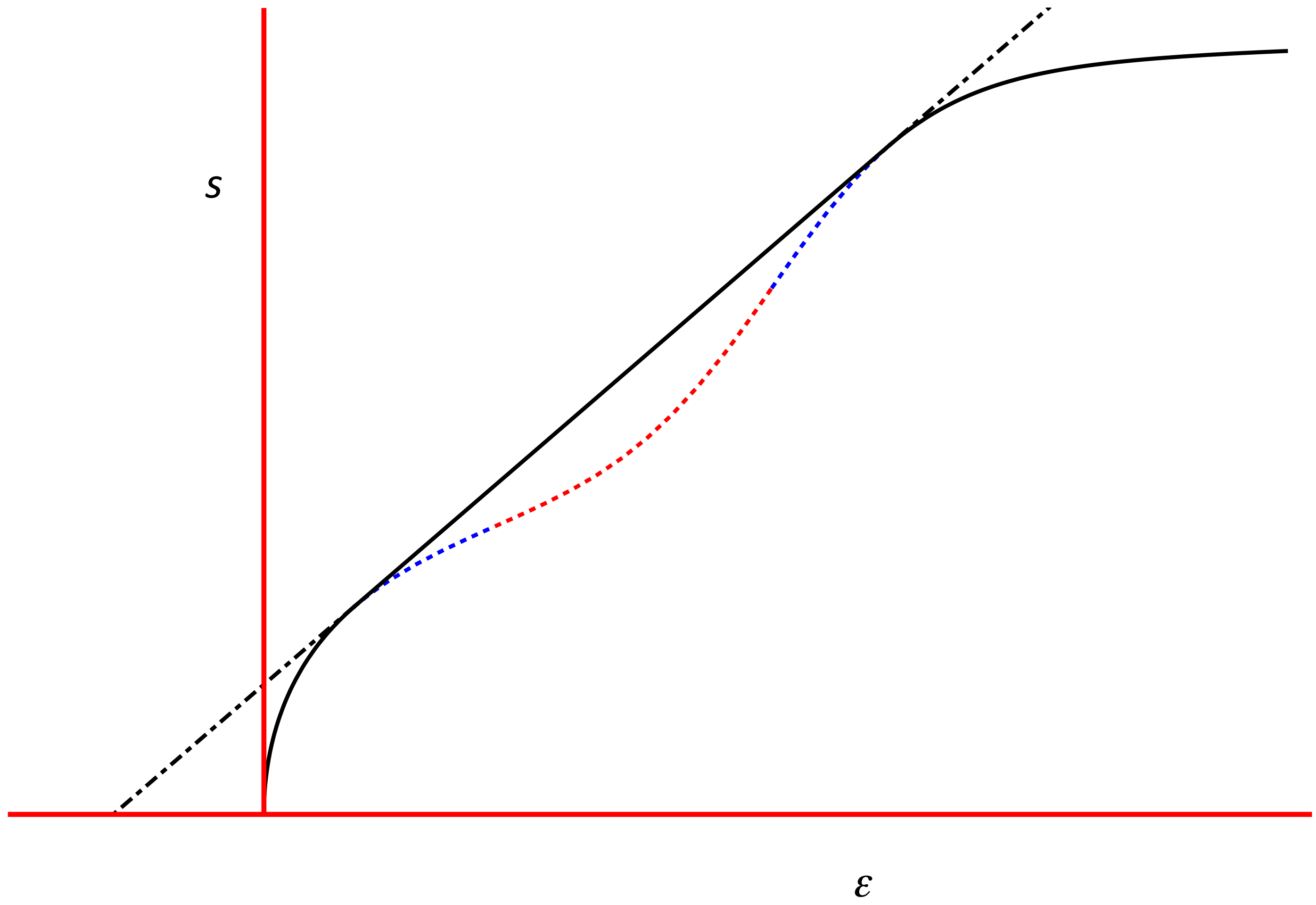
Throughout this talk, it will be assumed that a first order transition exists between a hadron and plasma phase

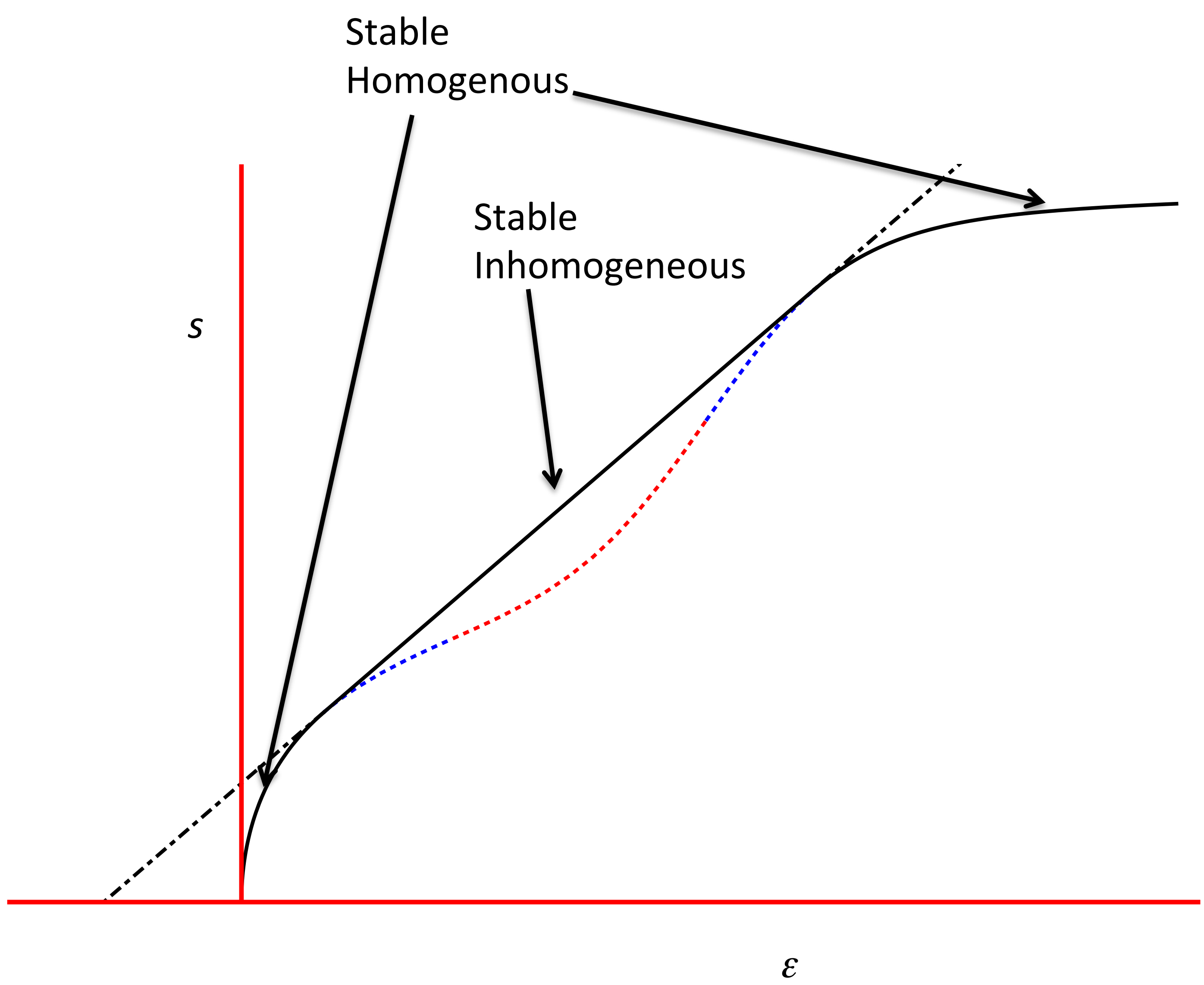
- This talk will use the microcanonical ensemble as this is the most fundamental.
  - Key quantity $S(E)$ where $S(E)$ is the log of the number of accessible states at E.
  - $S'(E)=1/T$
  - In thermodynamic limit of large volumes relevant quantities are entropy density, s, and energy density ε : $s(\varepsilon) = \mathrm{Lim}_{V\to\infty} S(\varepsilon V)/V$
  - Thermodynamic stability implies $s''(\varepsilon)\leq 0$.



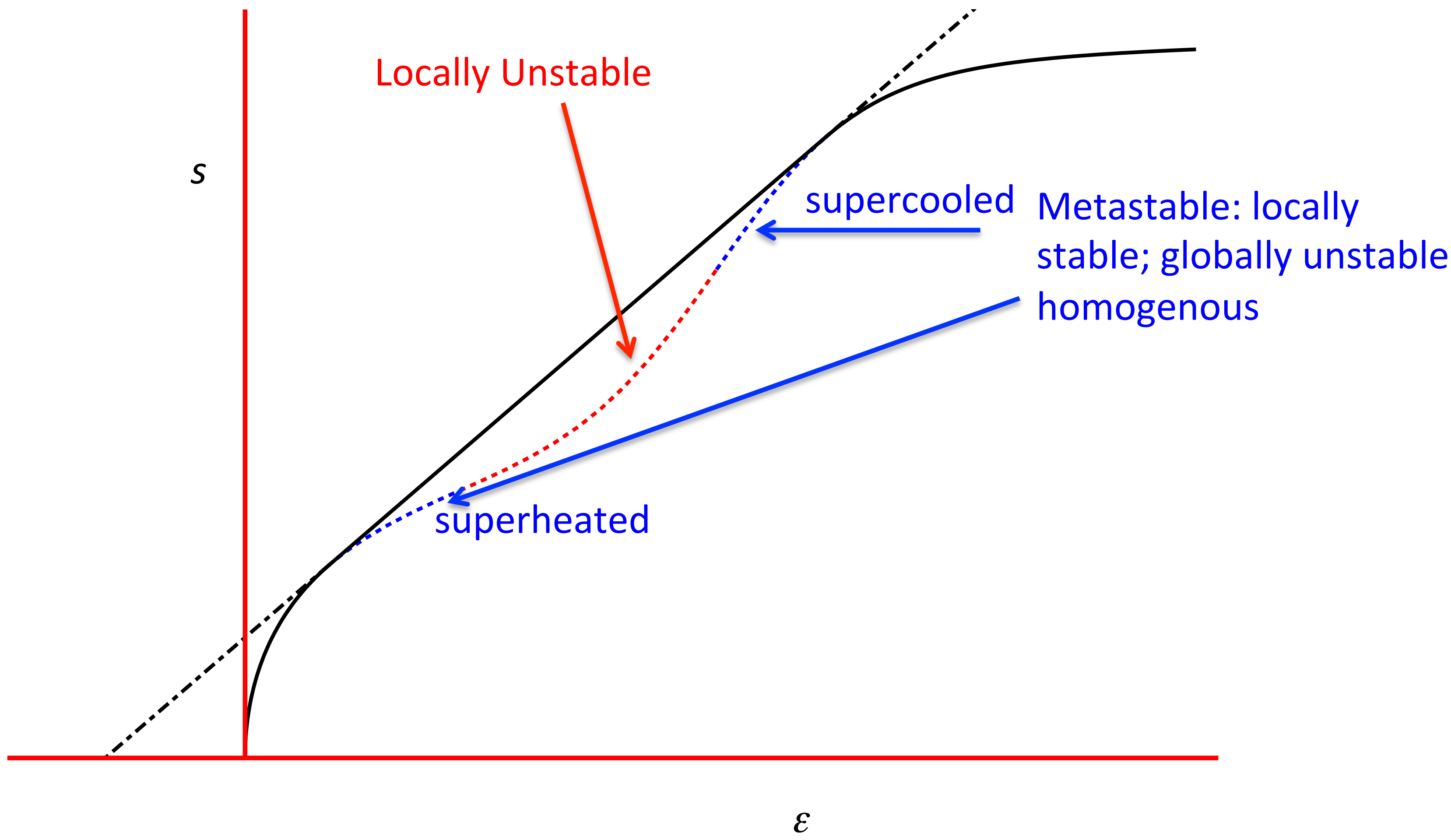


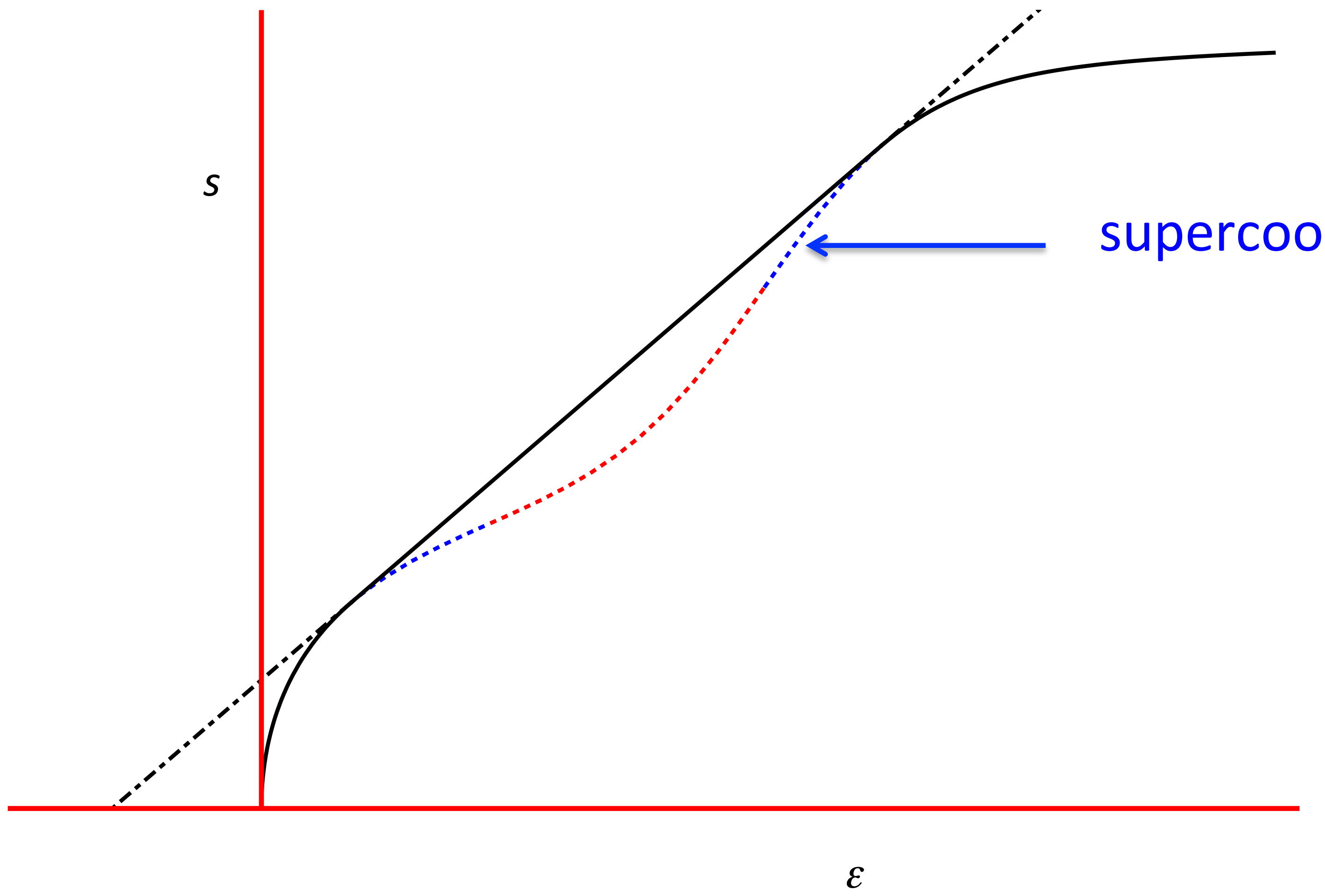
Generic first order transition



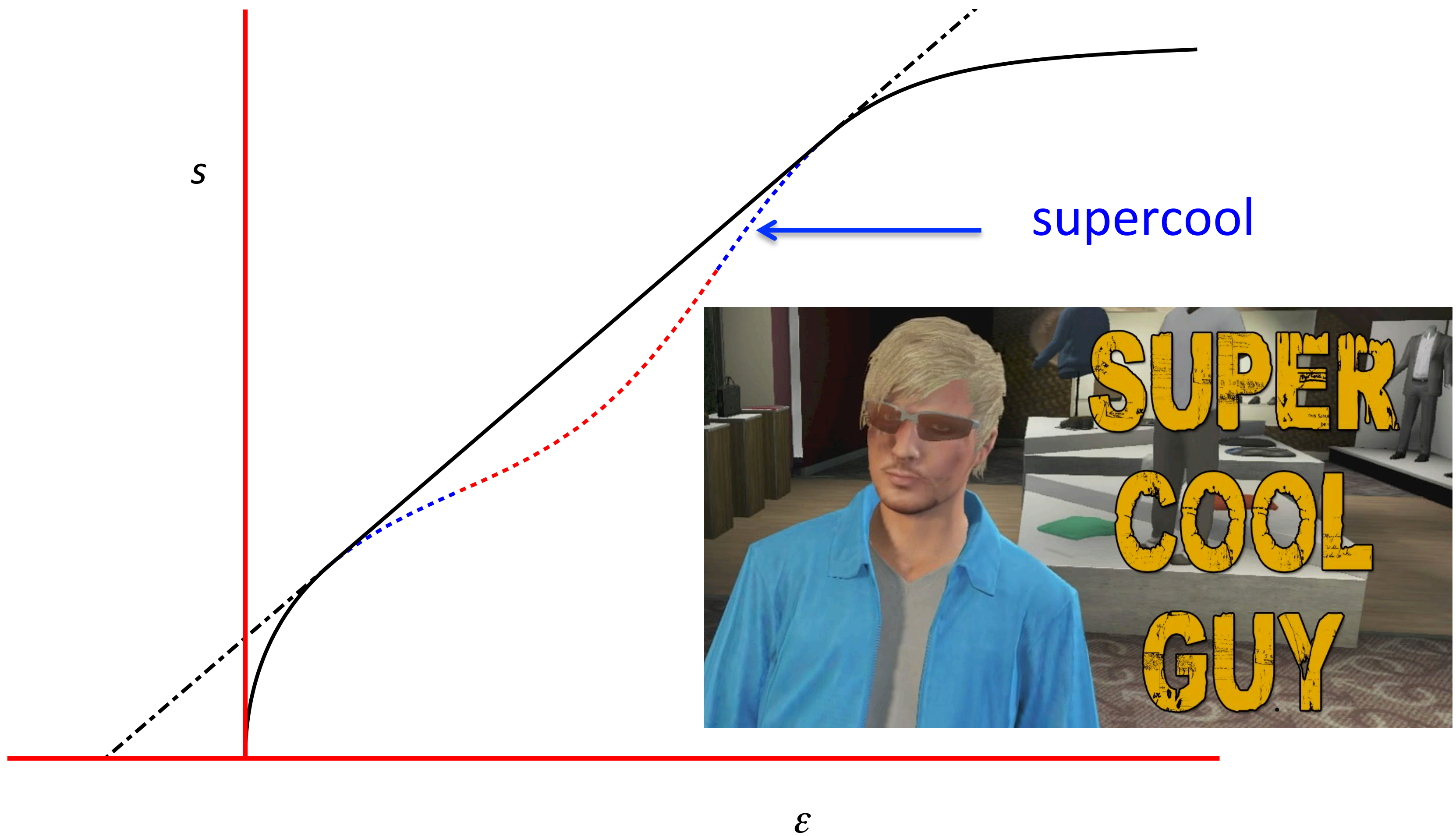


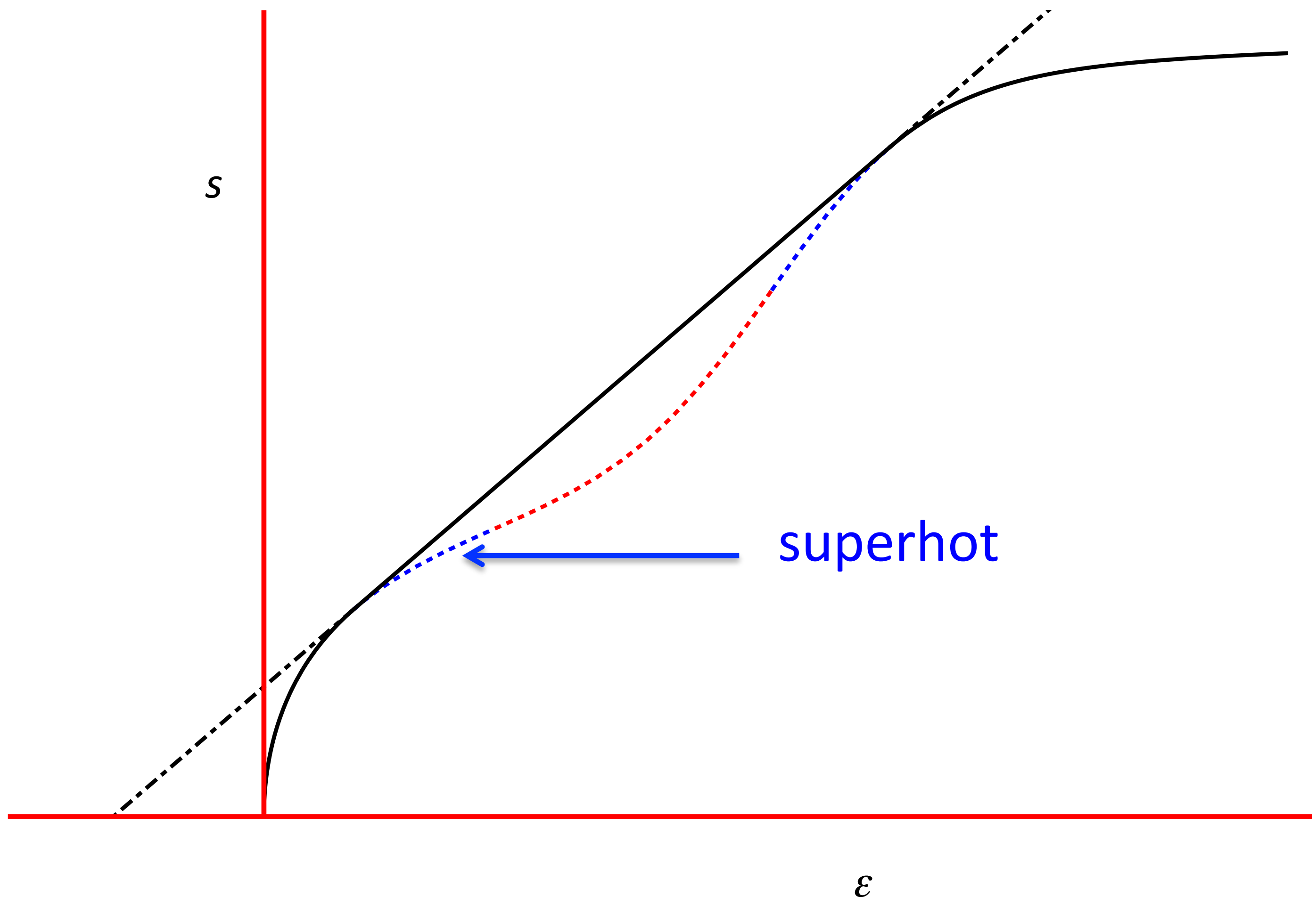
Stable
Homogenous
Stable
Inhomogeneous
s
ε


Locally Unstable
s
supercooled
Metastable: locally stable; globally unstable homogenous
superheated
ε

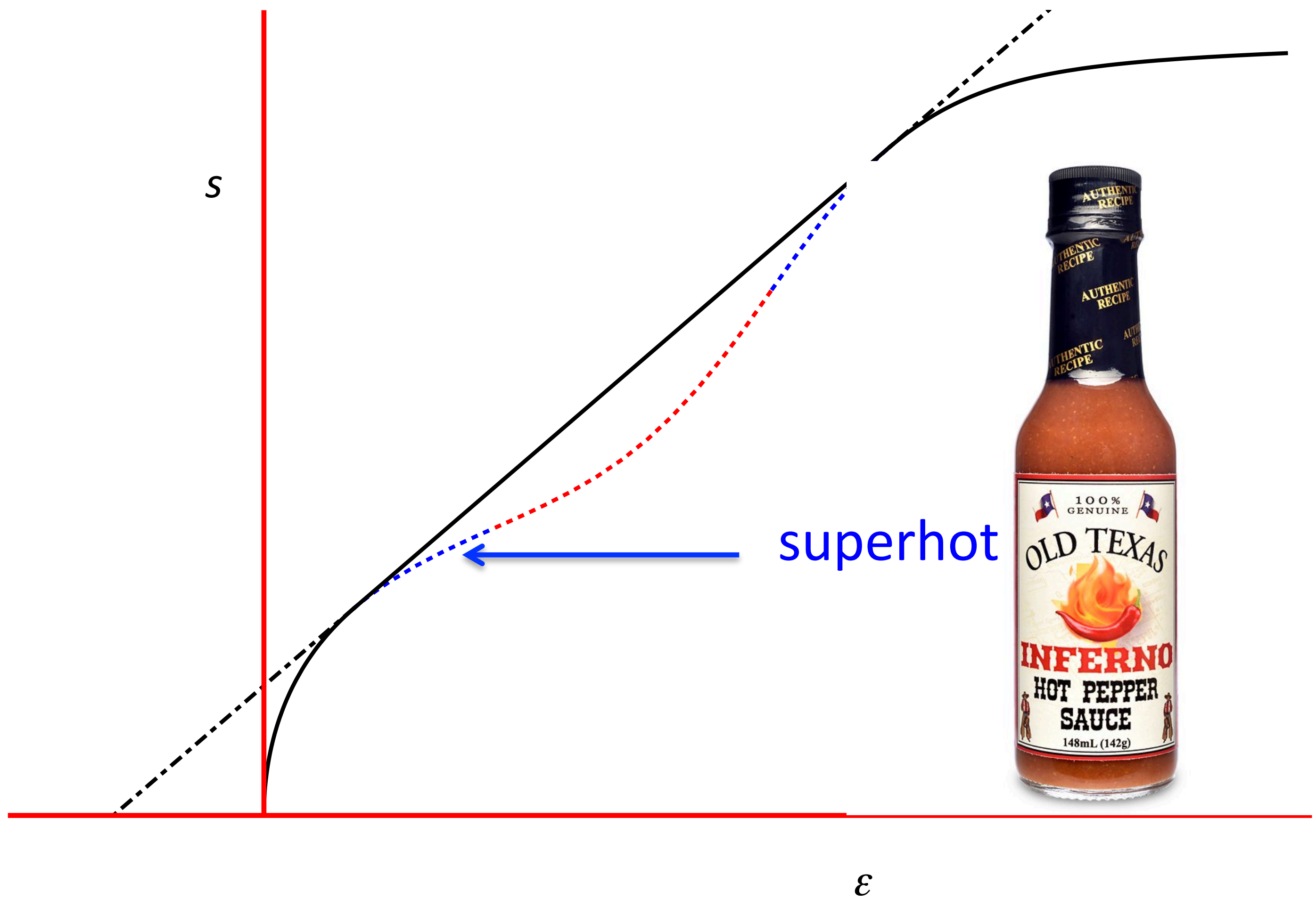


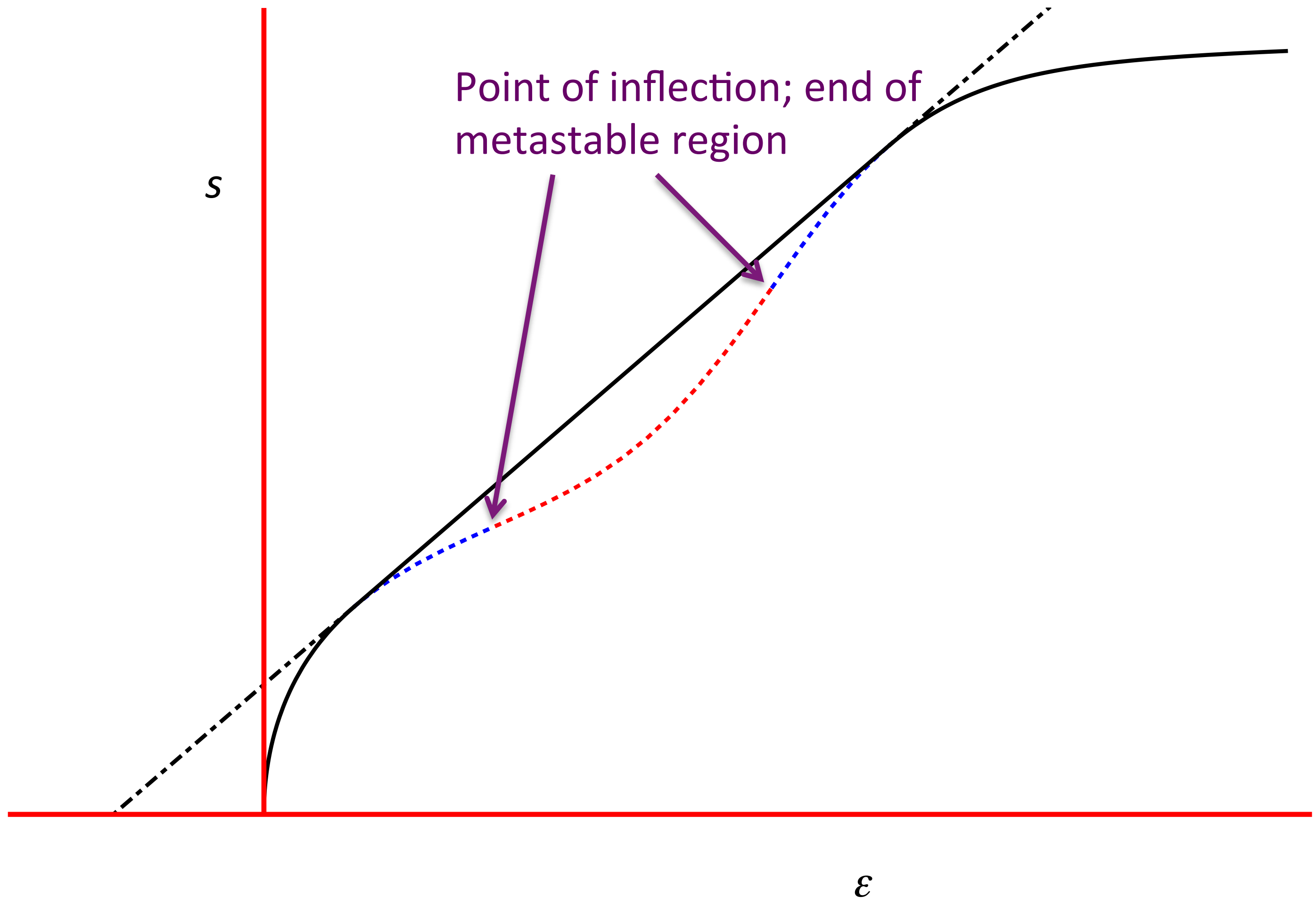
s
supercool
ε

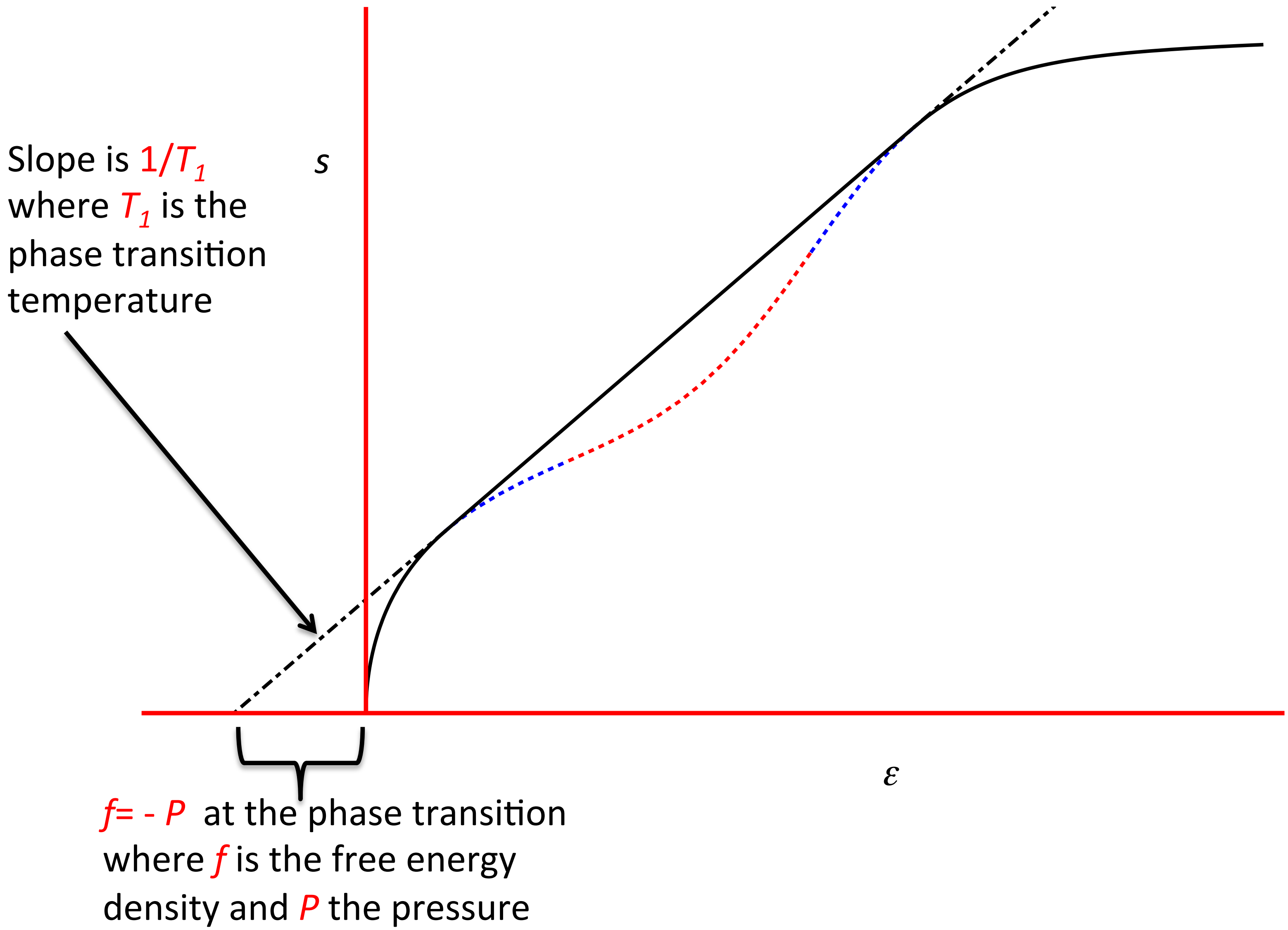

s
supercool
SUPER
COOL
GUY
ε



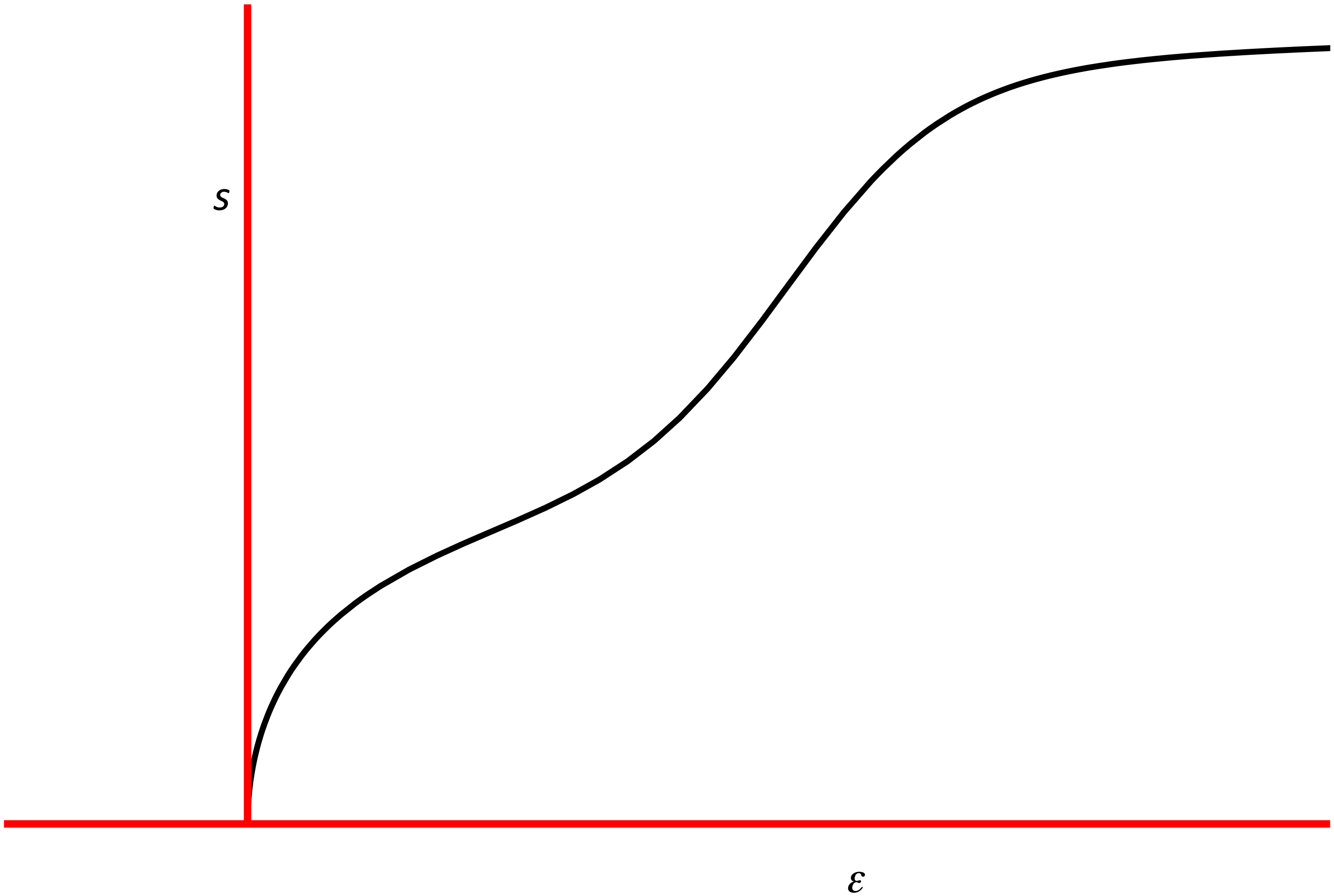
s
superhot
ε


s
superhot
ε
100%
GENUINE
OLD TEXAS
INFERNO
HOT PEPPER
SAUCE
148mL (142g)


Point of inflection; end of metastable region
s
ε


Slope is $1/T_1$
where T_1 is the
phase transition
temperature
s
ε
$f = -P$ at the phase transition
where f is the free energy
density and P the pressure

What happens at large N_c ?



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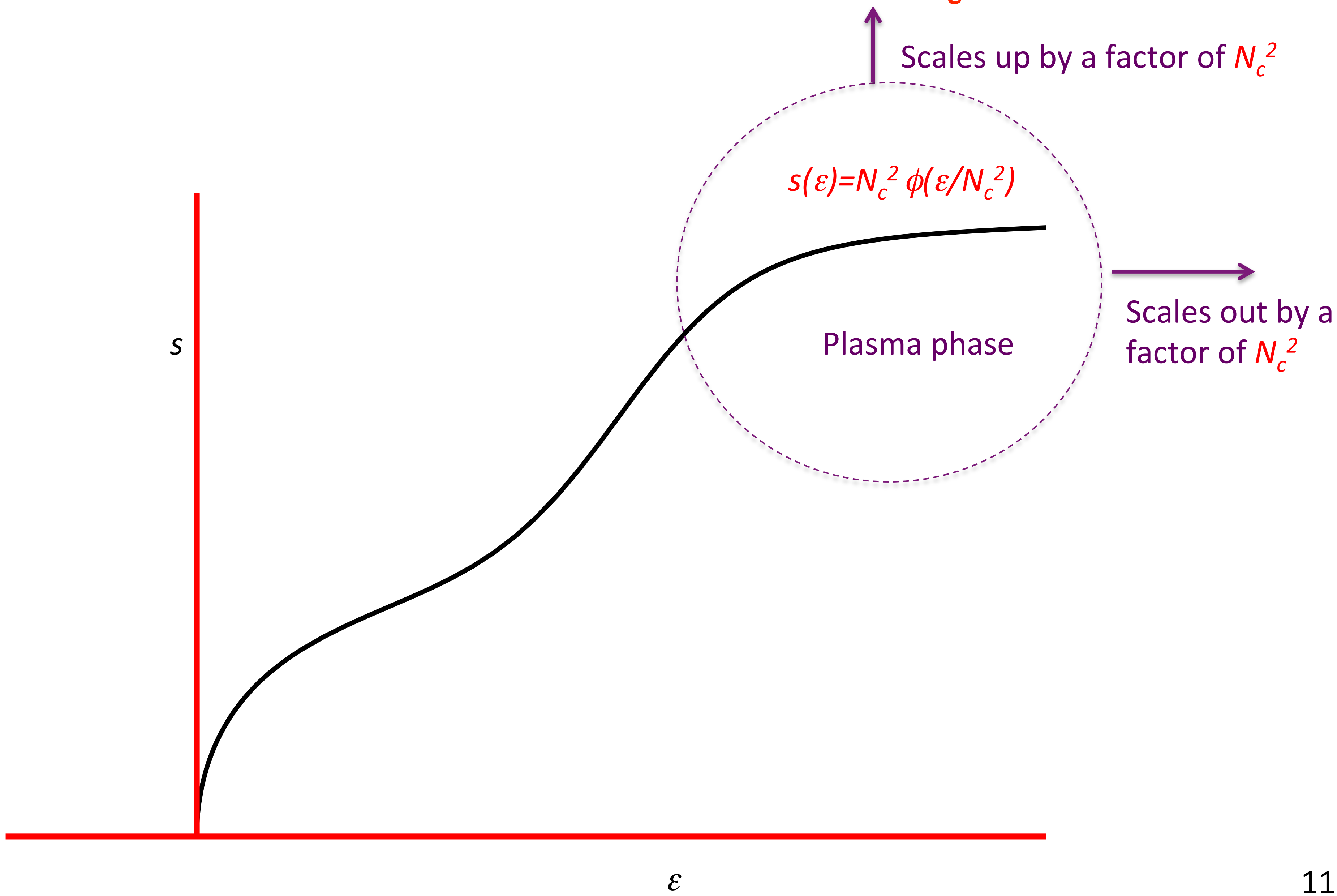

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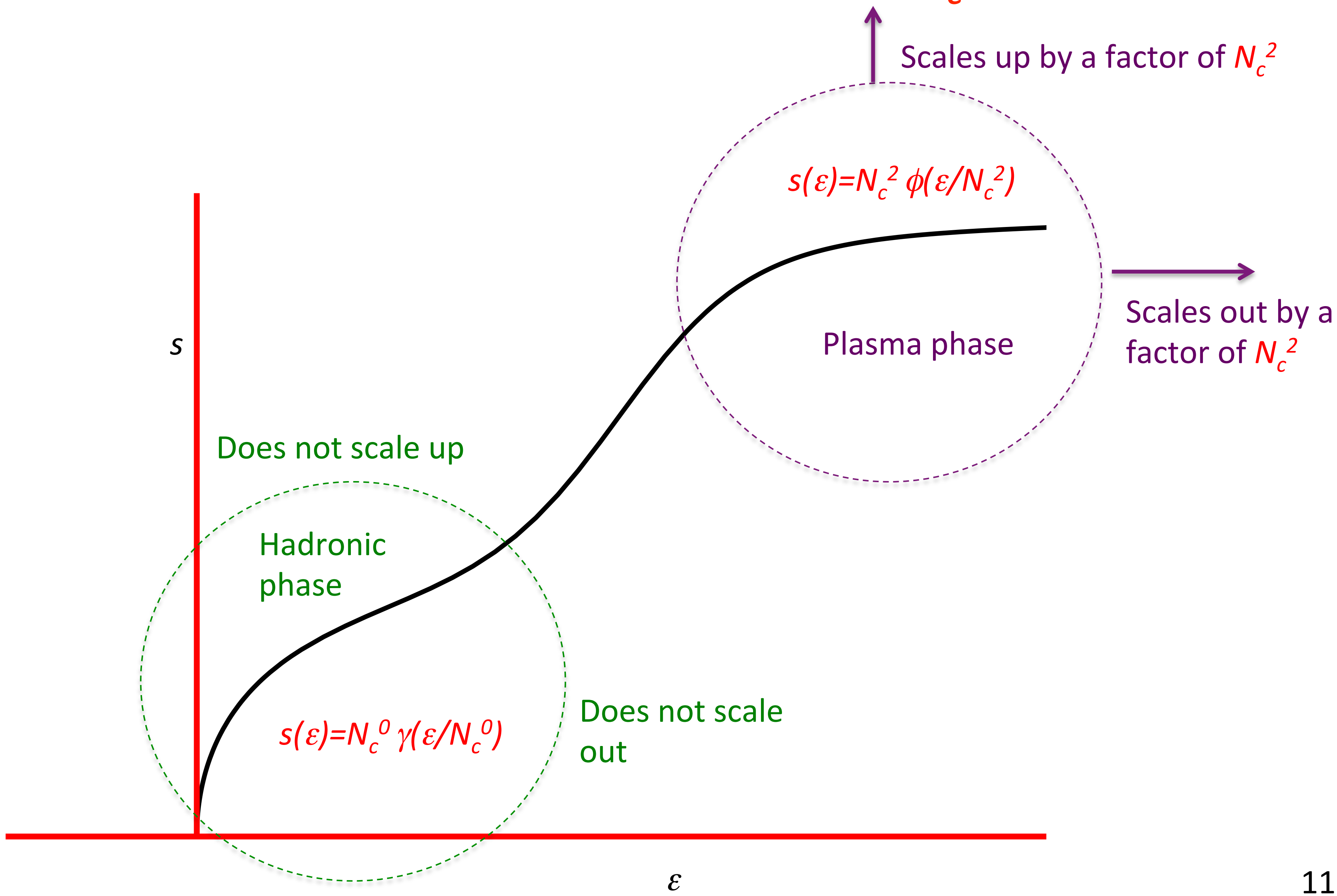

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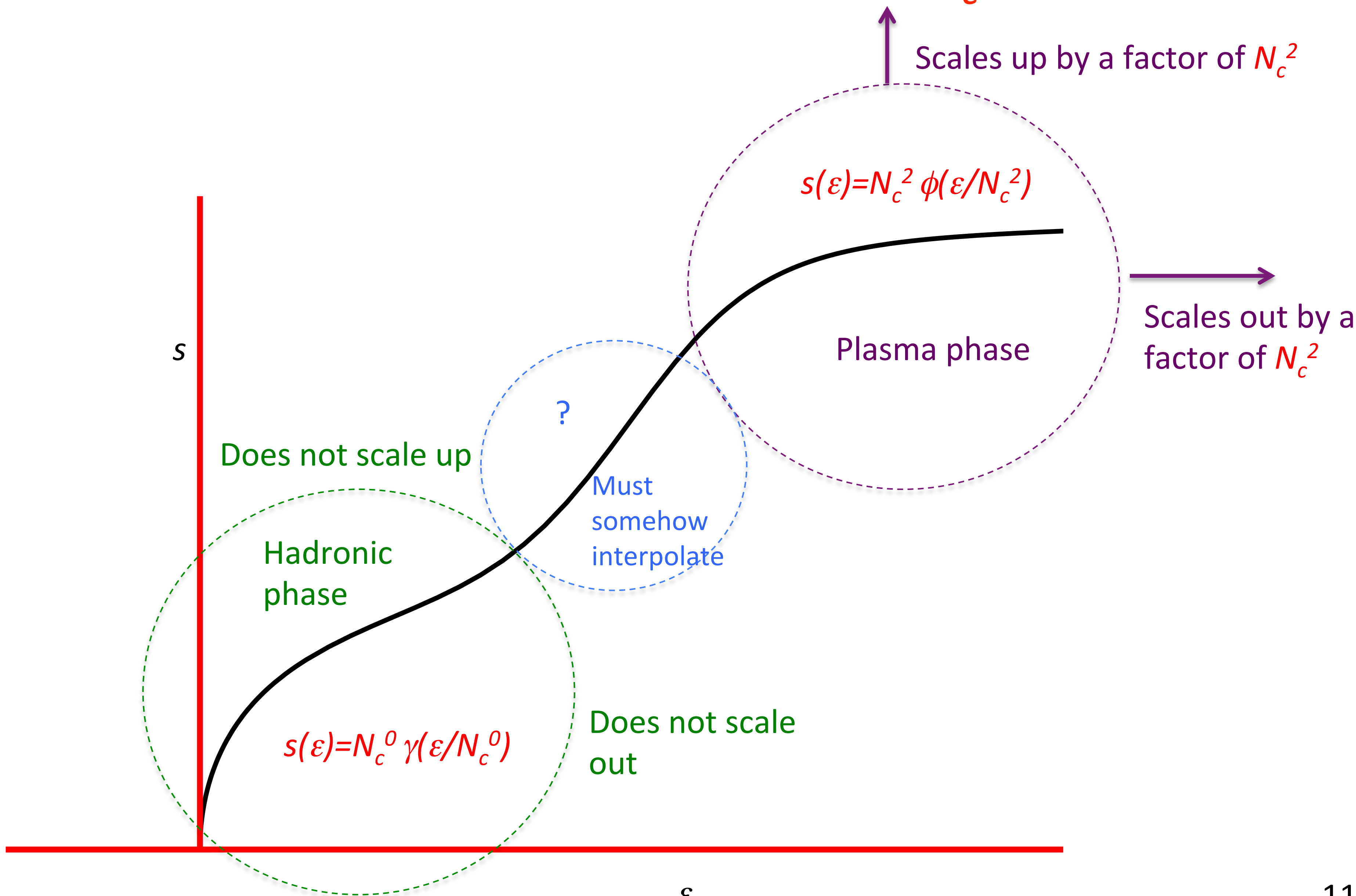

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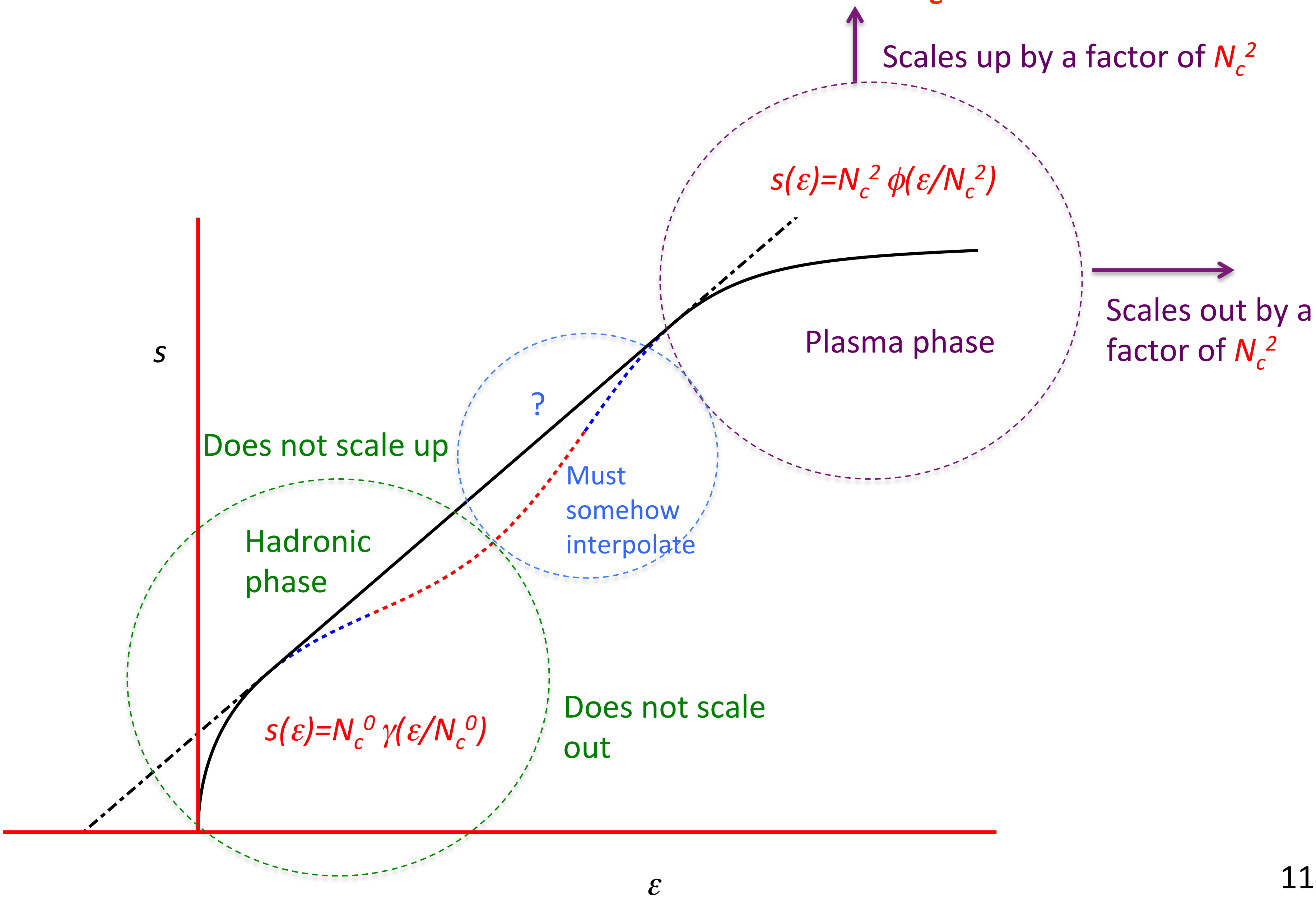

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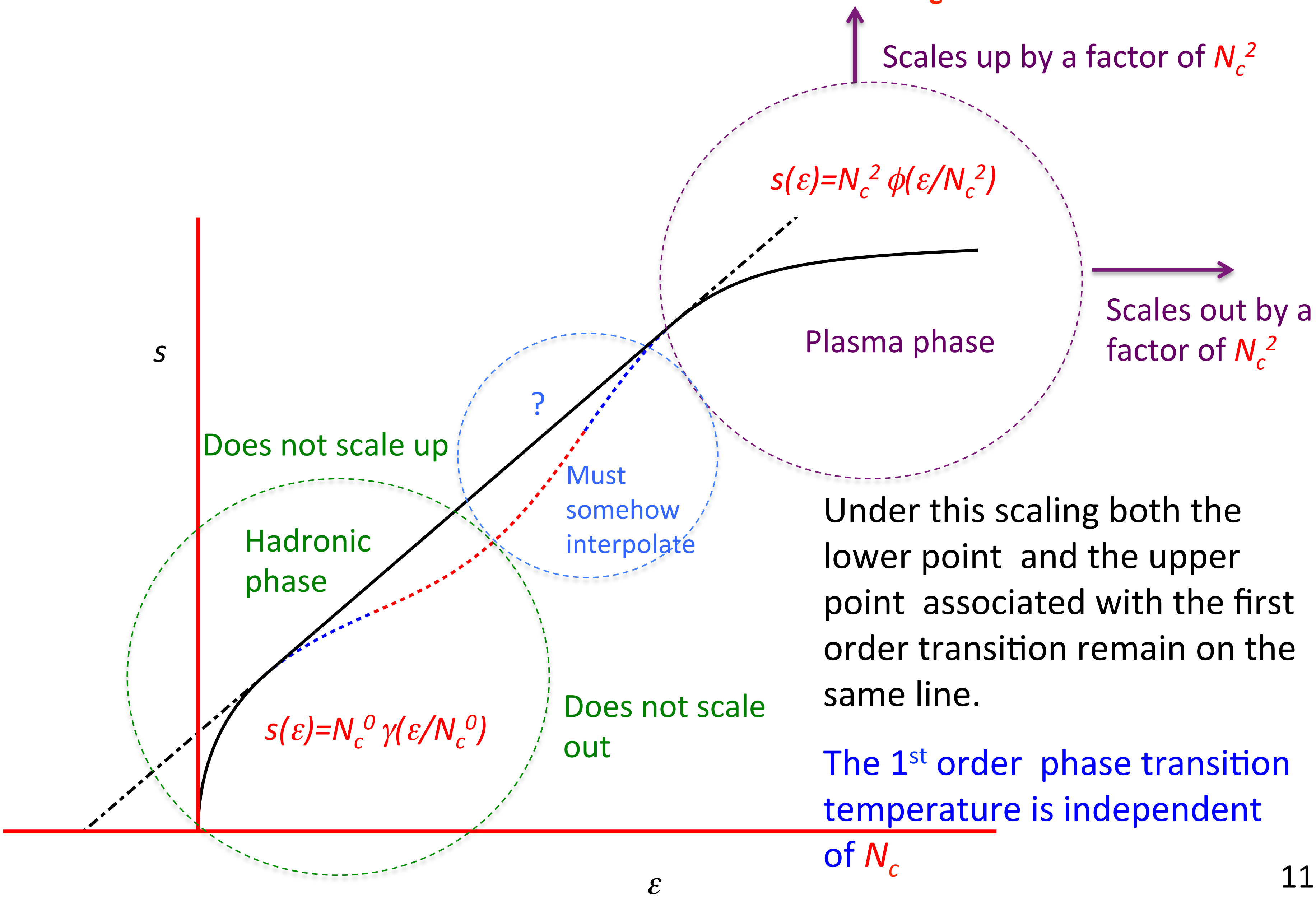
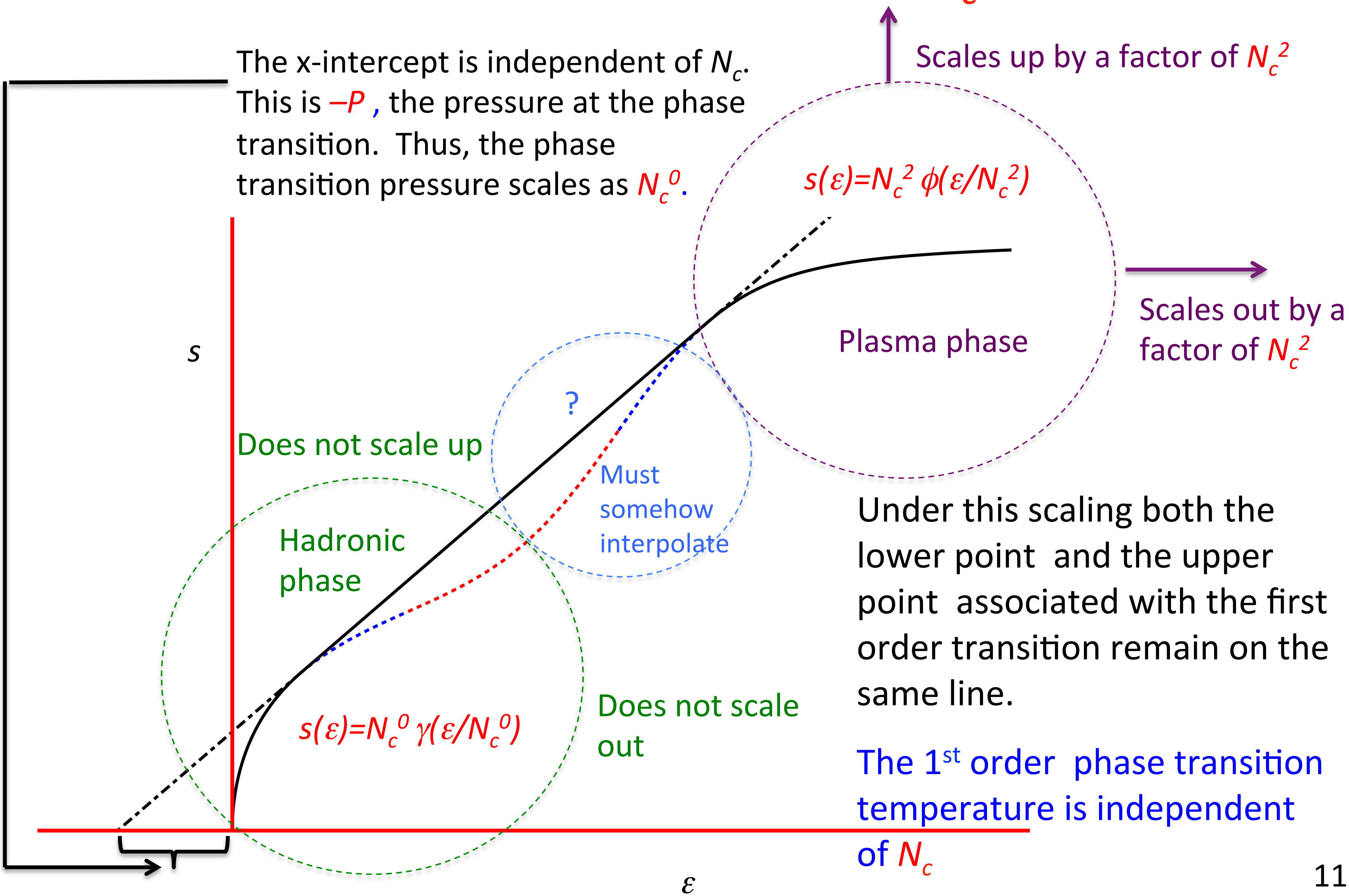


Under this scaling both the lower point and the upper point associated with the first order transition remain on the same line.

The 1st order phase transition temperature is independent of N_c

What happens at large N_c ?

The x-intercept is independent of N_c. This is $-P$, the pressure at the phase transition. Thus, the phase transition pressure scales as N_c^0.



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Implications for the plasma phase

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- However, near the phase transition but still in the plasma phase, Ts and ε cancel almost exactly, up to relative order N_c^{-2}.
- This cancelation is rather remarkable and leads to some quite surprising results.

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Others use different criteria for what constitutes a perfect fluid!

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    - Based on this people have described the medium formed in these collisions as a (nearly) perfect fluid.
  - While there is strong evidence that this medium is strongly coupled, the evidence that it is a "plasma" is more problematic.
    - There is no phase transition in QCD between the plasma and hadronic phases. The medium is called a plasma largely because it is far too dense to be a weakly couple hadronic gas.
    - But it is equally not a weakly couple quark-gluon plasma. So why "strongly coupled plasma" & not "strongly coupled hadronic gas"

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- Yes! The high temperature phase of Large N_c QCD just above the phase transition
  - Unlike QCD at $N_c=3$, there is a phase transition which cleanly delineates the hadronic from plasma phases. The high temperature phase is clearly a plasma.
  - While there is no practical way to test η/s for this system to demonstrates that the constituents were strongly coupled, if the plasma is composed of massless constituents (eg. gluons) there is another useful measure

$$\Omega = \frac{\varepsilon}{3P}$$

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At large N_c QCD unambiguously is both strongly coupled and in a plasma phase!

This is modulo the very reasonable assumption that a first order transition persists.

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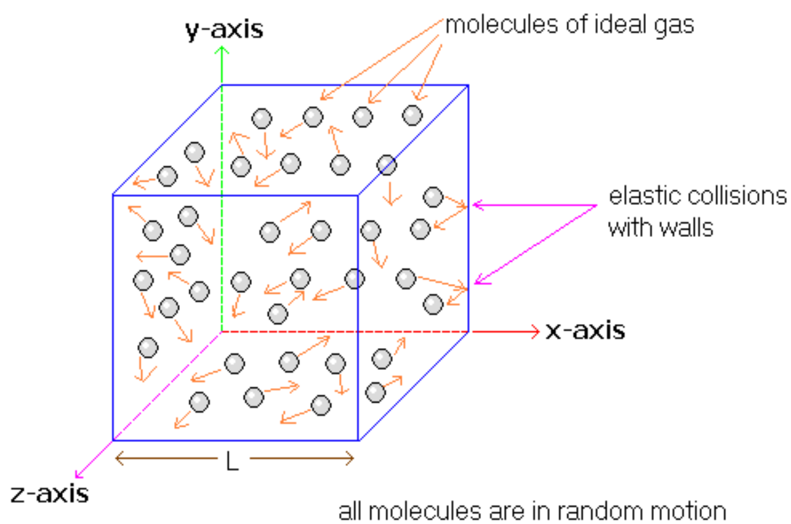

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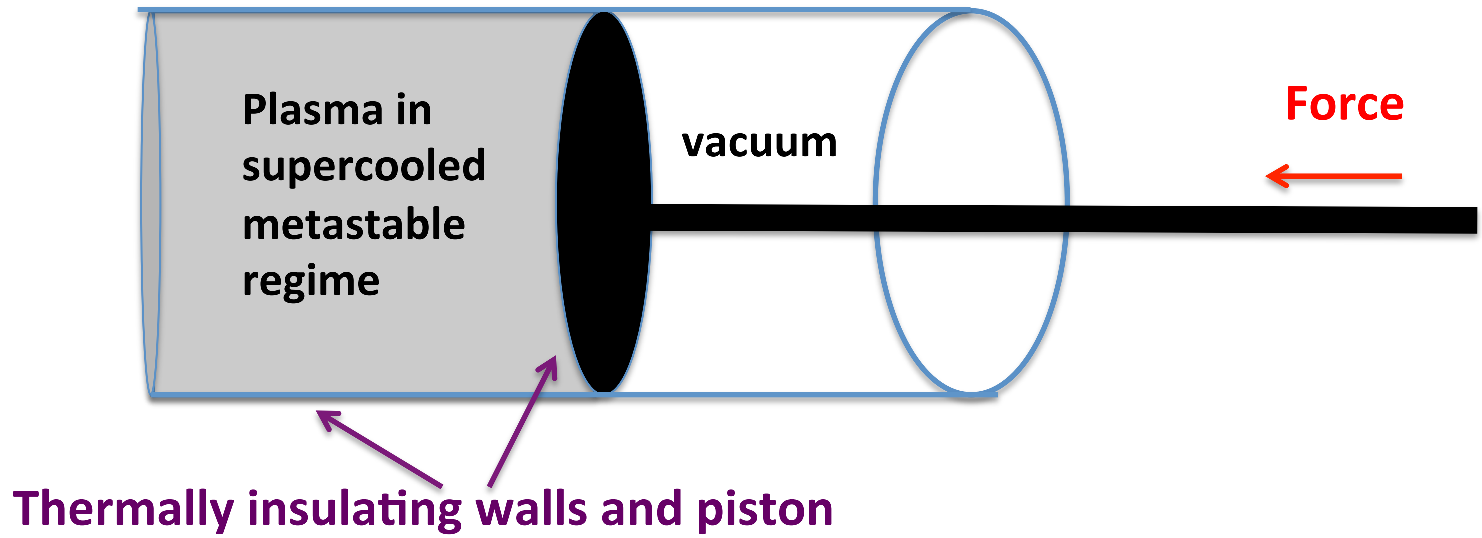
  - Negative pressure absolute pressure violates our naïve kinetic theory intuition based on particles bouncing around in a gas.
  - **No go theorem:** systems with no chemical potentials or fixed densities of conserved quantities in a stable phase cannot have negative absolute pressure.
    - This follows from the condition $s''(\varepsilon) \leq 0$, and the facts that $s'(\varepsilon) = T^{-1}$, and $P=-f=Ts-\varepsilon$.
    - **But this does not apply to the supercooled metastable phase.**

Negative absolute pressure is remarkable



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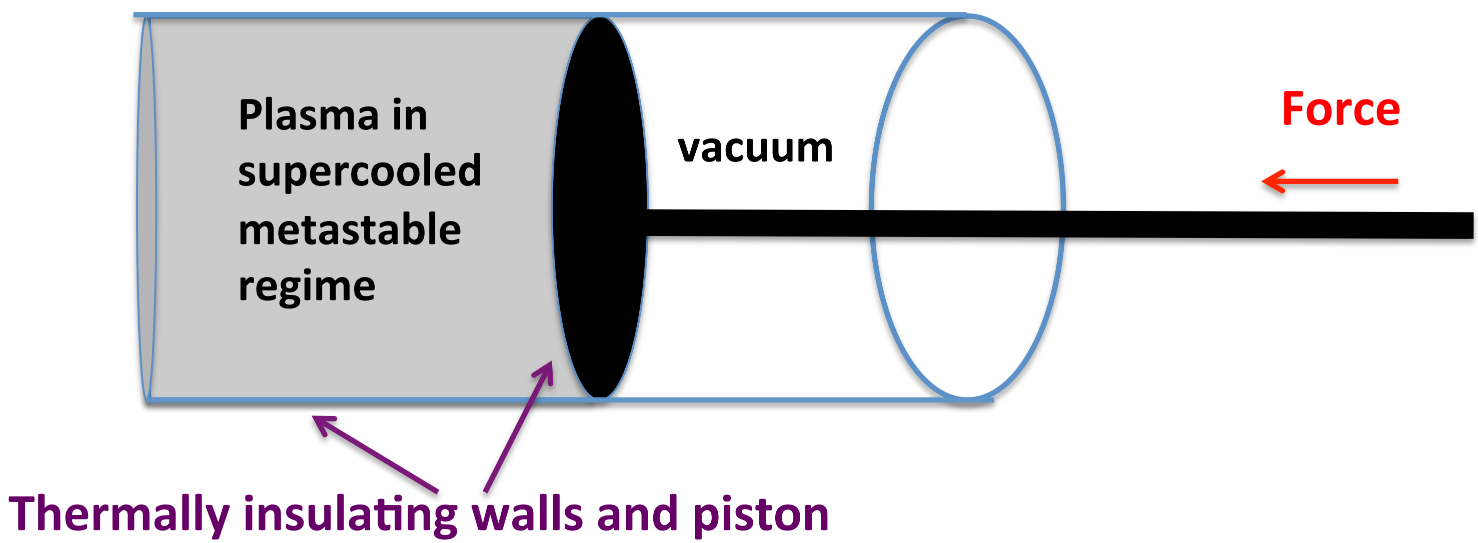

The medium is not just weird—it sucks!

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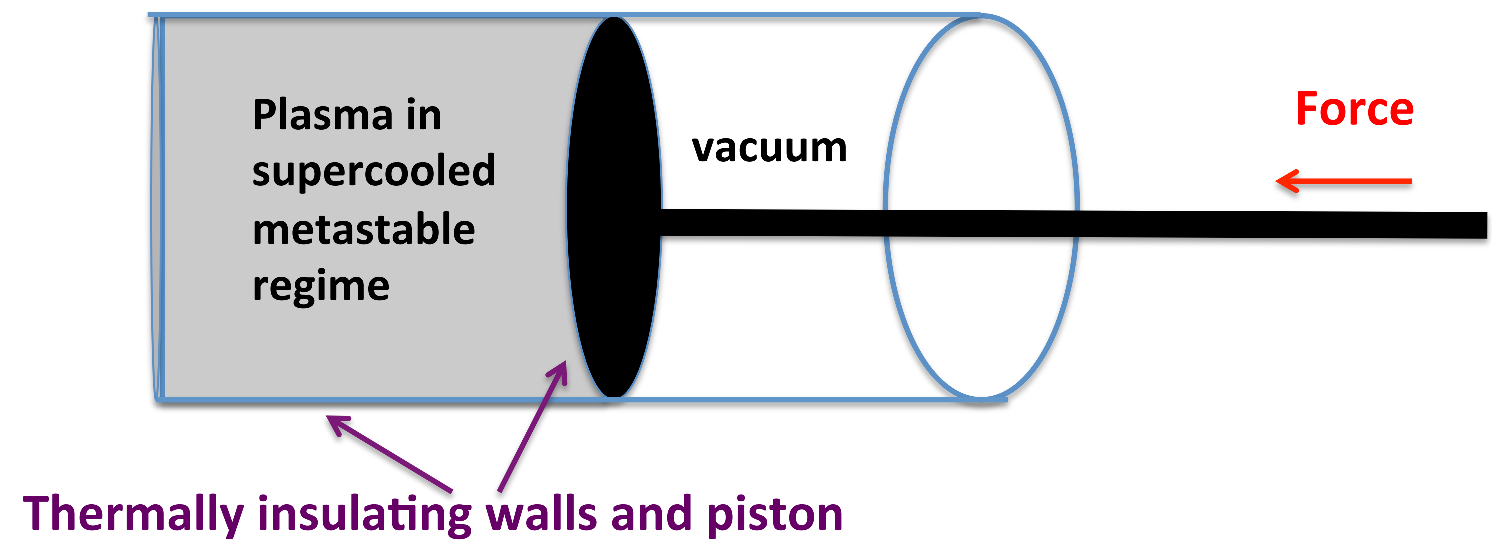

The medium is not just weird—it sucks!

The break down of intuition based on kinetic theory, indicates that whatever this medium is, the pressure is not describable in terms of particles or quasiparticles that strike the wall transferring momentum and imparting an outward pressure. **This requires a strongly coupled theory where the quasiparticle motion is not dominant.**

Why does the supercooled plasma have *P<0*?

$$s(\varepsilon) = N_c^2\,\phi\left(\varepsilon N_c^{-2}\right)\left(1 + O(N_c^{-2})\right)$$

$$T = \frac{1}{s'(\varepsilon)}$$

$$P = -f = T\,s - \varepsilon$$

Scaling in plasma phase

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So if **any** supercooled plasma phase exists at large N_c there is a region with $T<T_1$ so that $\zeta(T)<\zeta(T_1)=0$: Thus ***P(T)<0***

It has negative absolute pressure!

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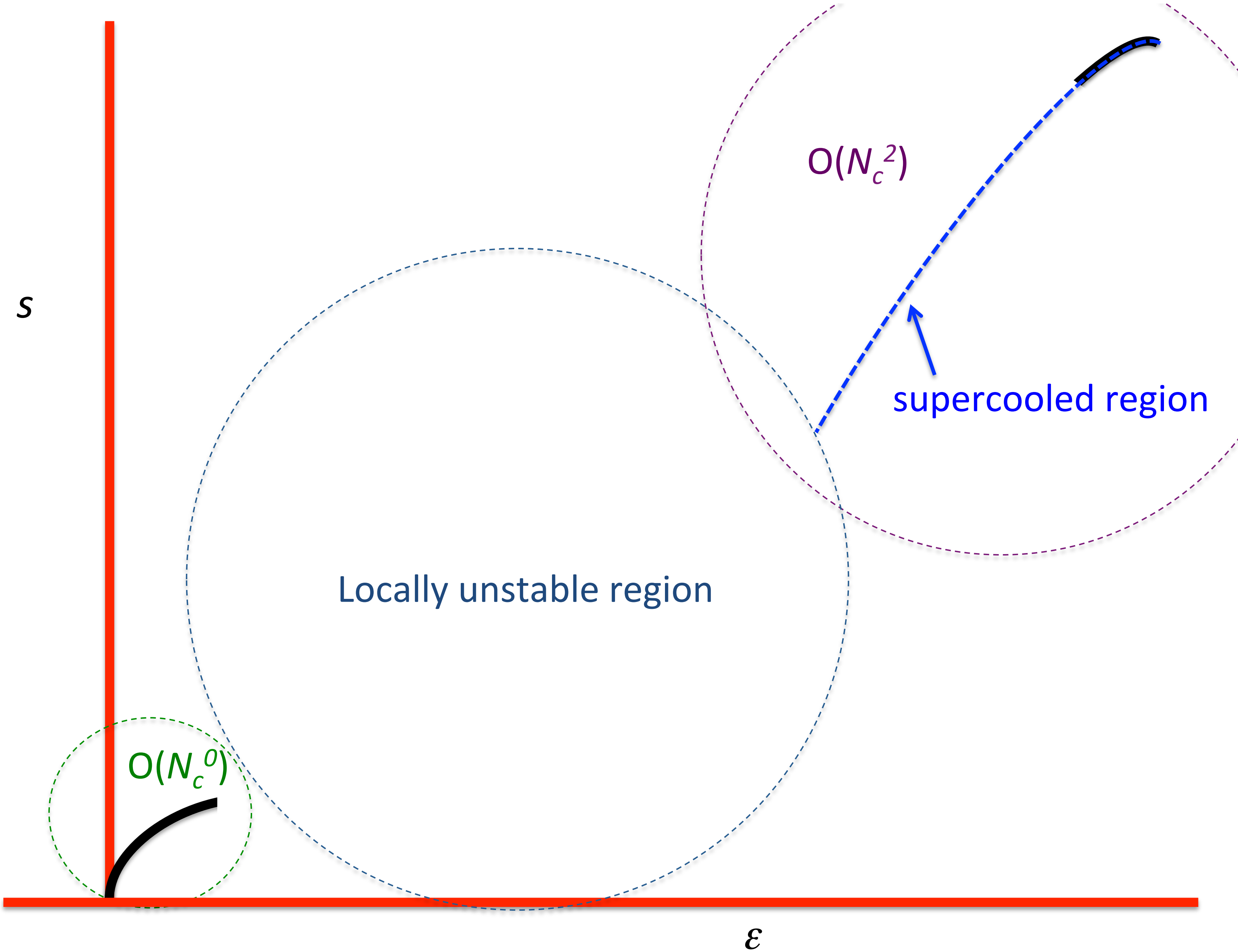
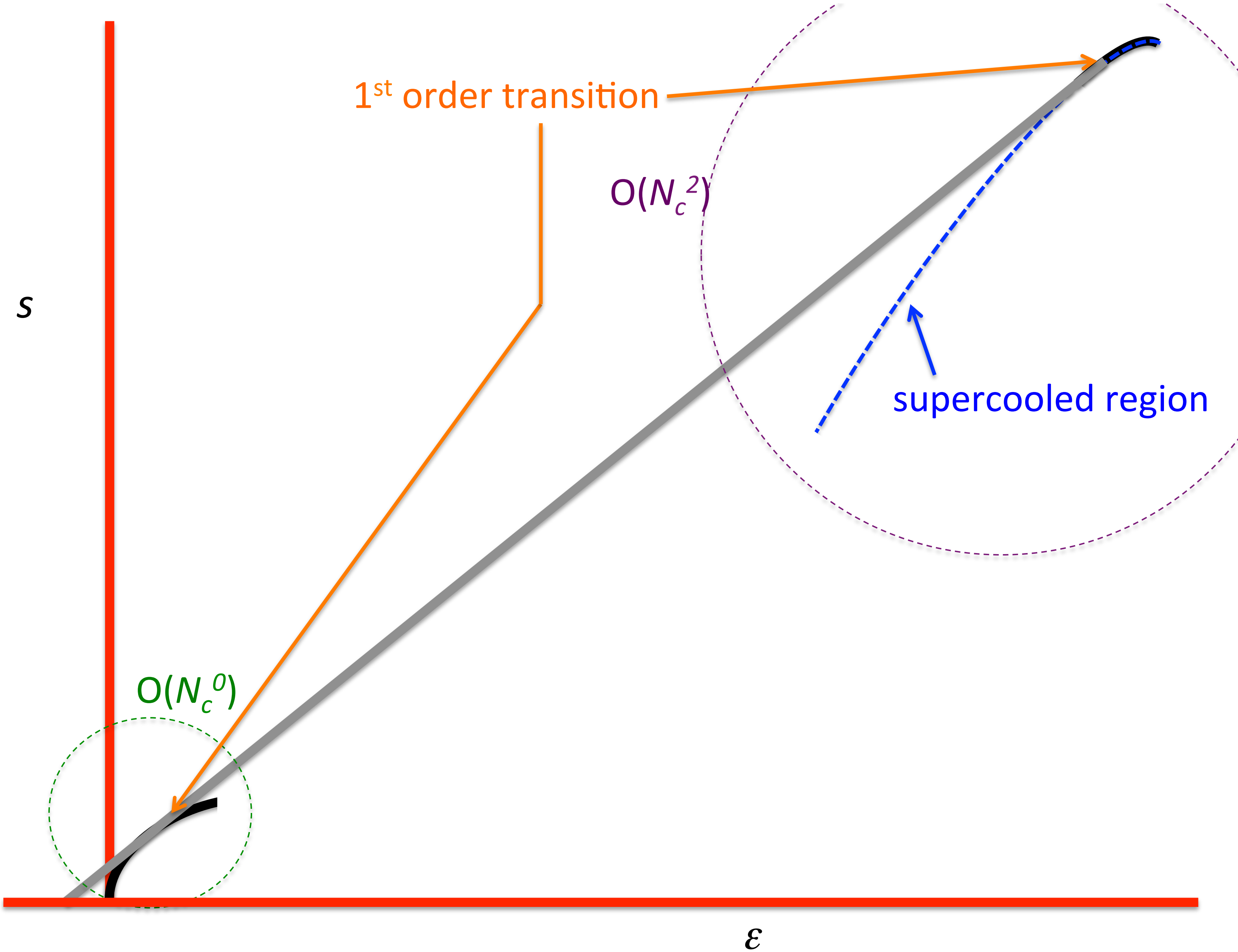
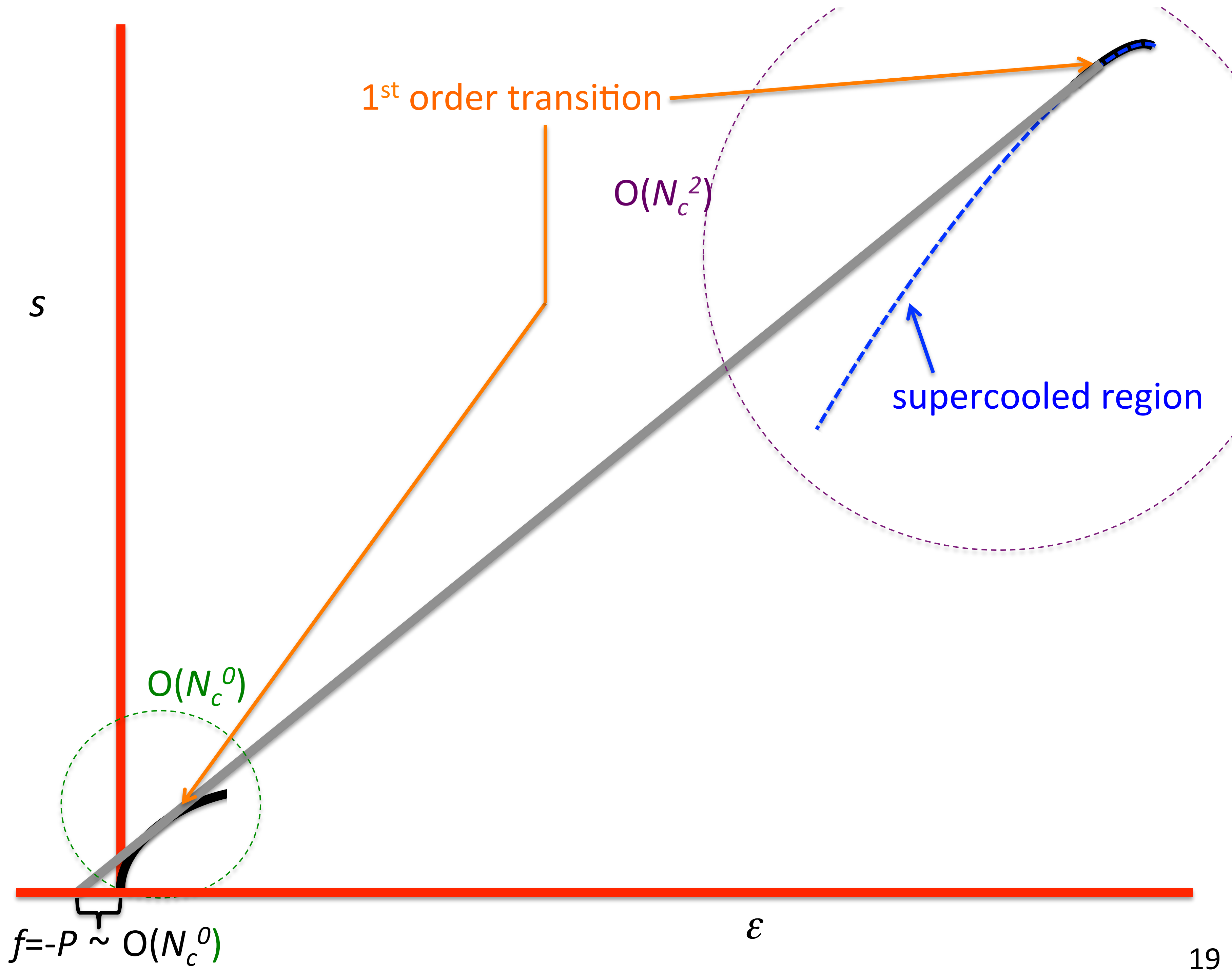


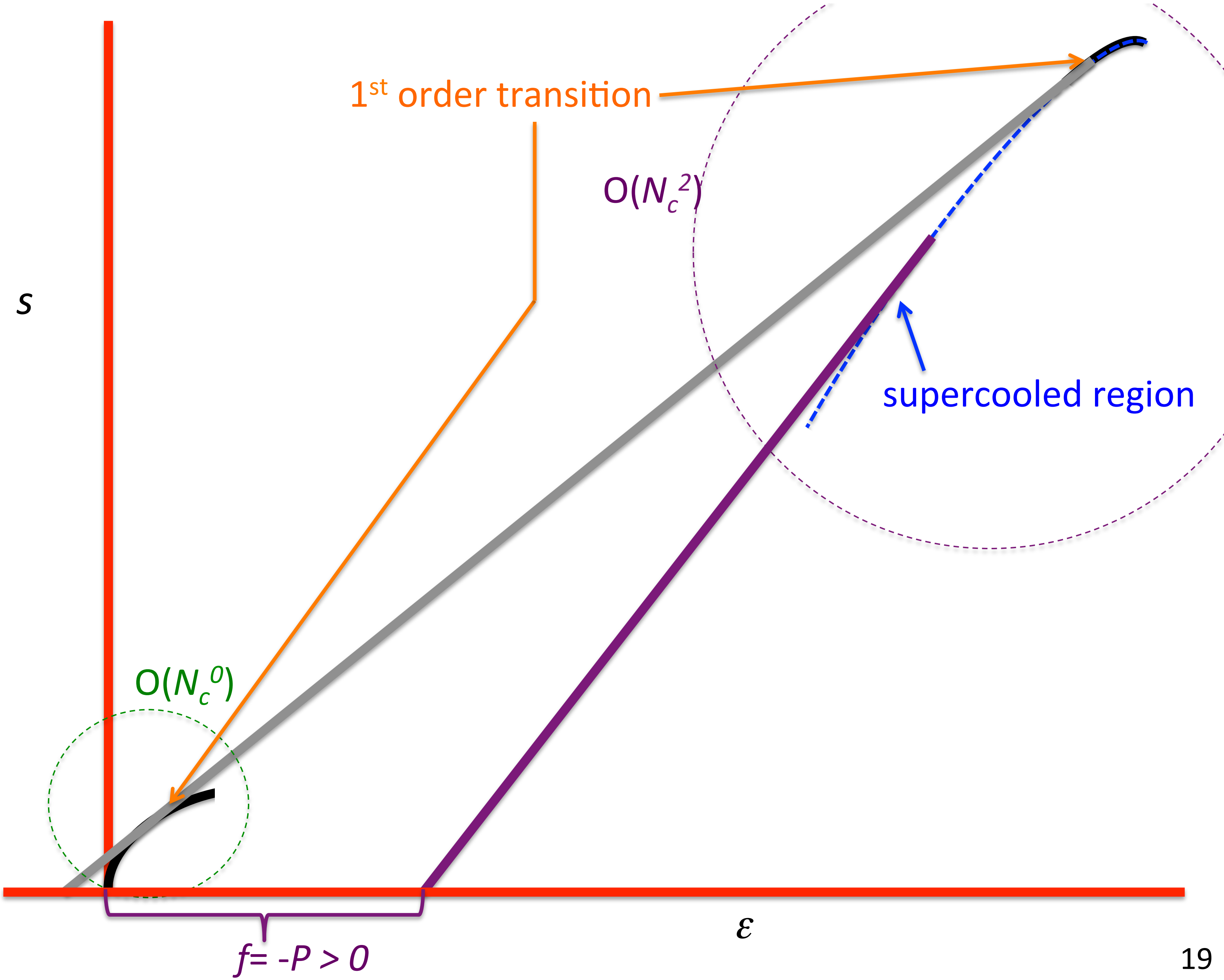
$O(N_c^2)$
supercooled region
s
Locally unstable region
$O(N_c^0)$
ε



1st order transition
O(N_c^2)
supercooled region
O(N_c^0)
s
ε



1st order transition
$O(N_c^2)$
s
supercooled region
$O(N_c^0)$
ε
f=-P ~ $O(N_c^0)$



1st order transition
O(Nc2)
s
supercooled region
O(Nc0)
ε
f= -P > 0

- There is a caveat.
  - There must be a supercooled regime with negtive absolute pressure at large N_c, **provided that a metastastable supercooled regime exists.**
  - Logically a first order transition could exist in which the phase transition point happens to coincide with a point of inflection; if this were to happen no metastable regime would exist.
  - There is no reason to expect this to happen based on large N_c analysis; and it would be even more interesting then negative absolute pressure.
- **So we can conclude something cool happens!**

Either the metastable supercooled phase does not exist or it has negative absolute pressure.

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- The key to understanding them is the fact that large N_c QCD must have a Hagedorn spectrum (TDC 2009)
  - a Hagedorn spectrum for the asymptotic density of hadrons $N(m) \sim m^{-d} \exp(m/T_H)$, where $N(m)$ is the number of mesons and glueballs with mass less than m, T_H, the Hagedorn temperature and is a mass parameter and $-d$ fixes power law prefactor.
  - In the large N_c limit, T_H corresponds to an upper bound on the temperature of hadronic matter

The Hagedorn Spectrum

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Strictly, it only makes sense when the number of colors is large

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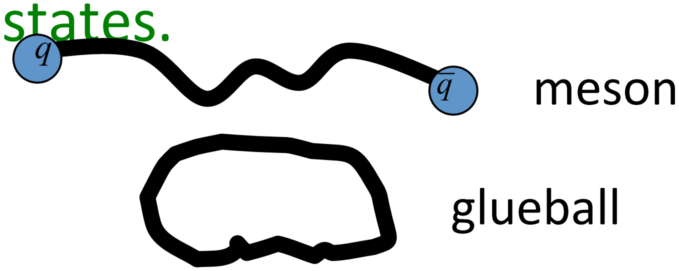
  - There is a good reason to believe that $d = 4$.
    - $d = 4$ is the result for a bosonic string.
    - Highly excited mesons and glueballs are expected to look like excitations of flux tubes which become increasingly stringy as the flux tubes get long—as they do for highly excited states.



Modern string theory grew out of the failed attempt in pre-QCD days to treat strong interactions as a string theory.

It was ultimately abandoned

- **Phenomenological issues (a pesky massless spin-2 meson etc.)**
- **Theoretical consistency (negative norm states, tachyons)**
- **Emergence of QCD as a viable field theory for strong interactions**

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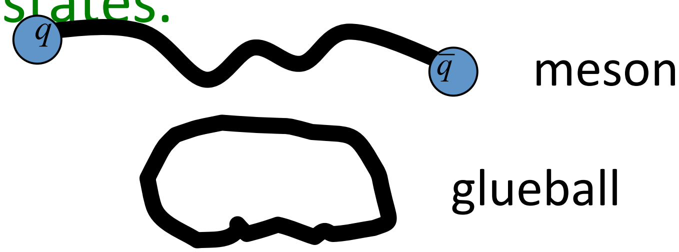

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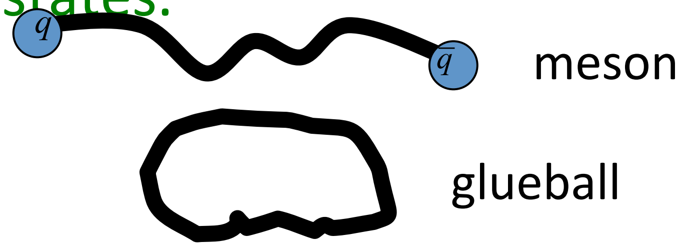

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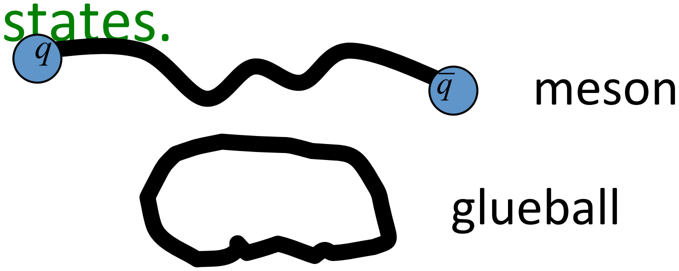

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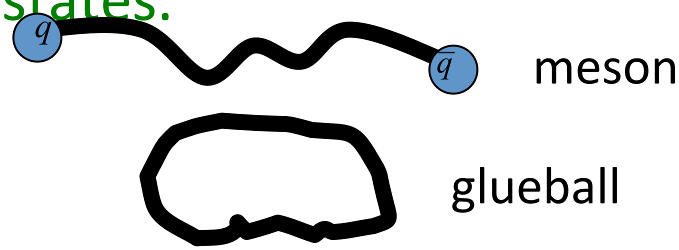
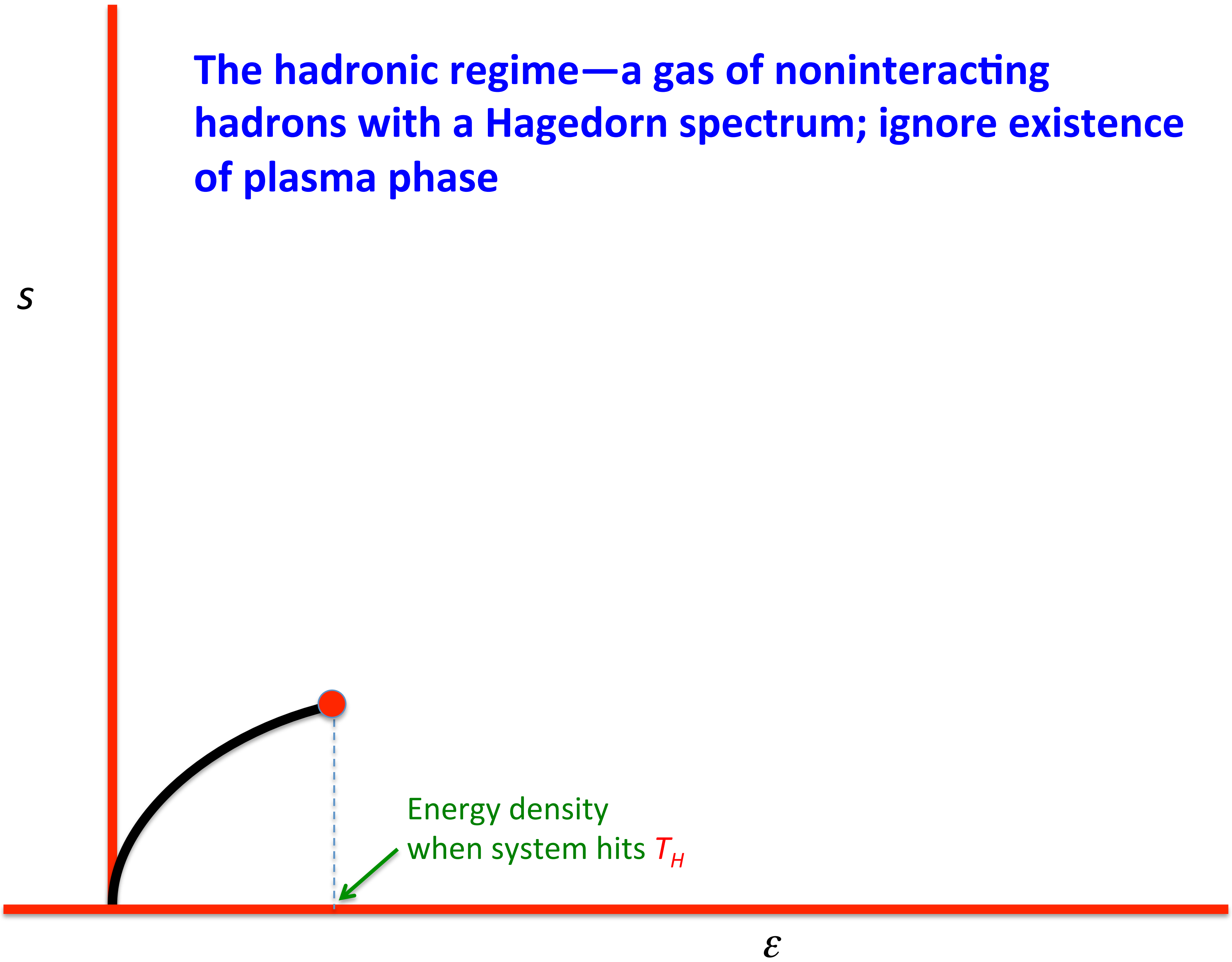


- *d = 4* is assumed in what follows.

The hadronic regime—a gas of noninteracting hadrons with a Hagedorn spectrum; ignore existence of plasma phase



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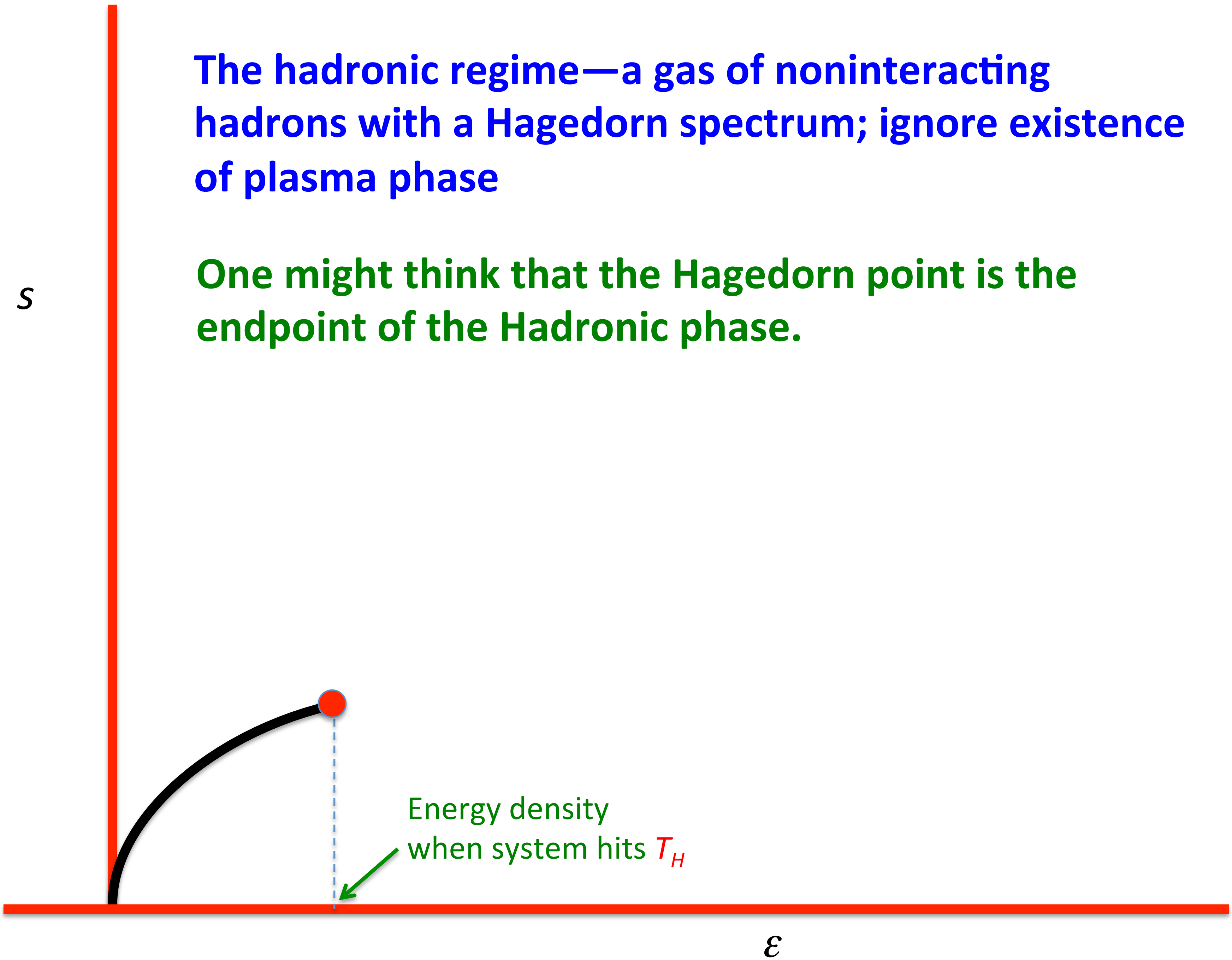
One might think that the Hagedorn point is the endpoint of the Hadronic phase.



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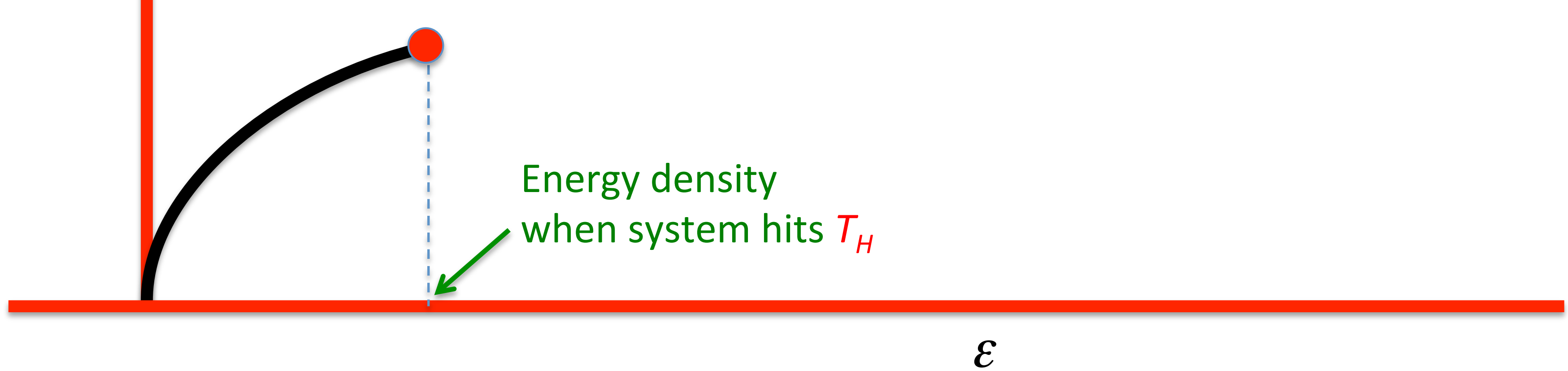
But this is problematic; I can create hadronic states with higher energy densities. What happens if I do?



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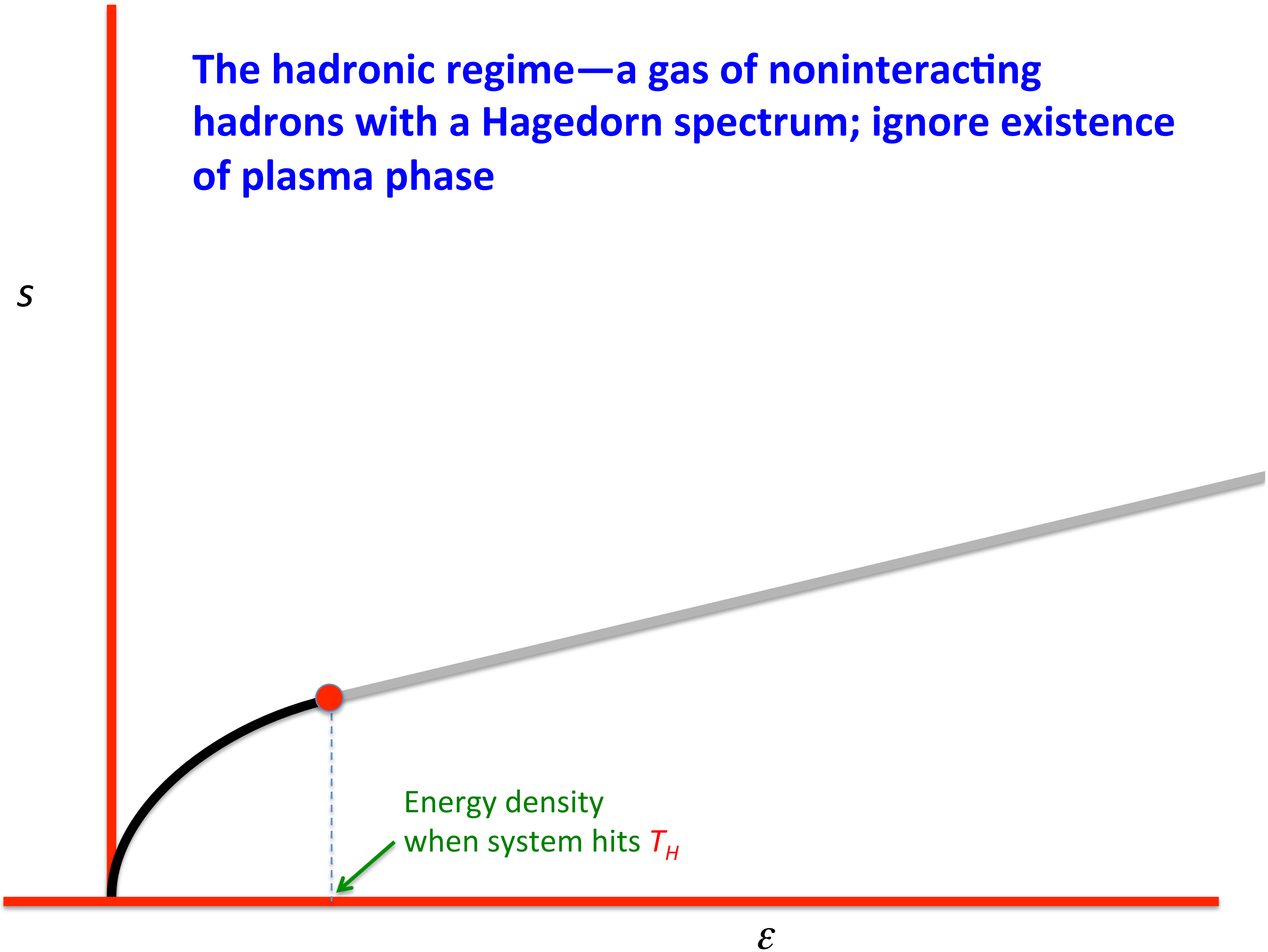

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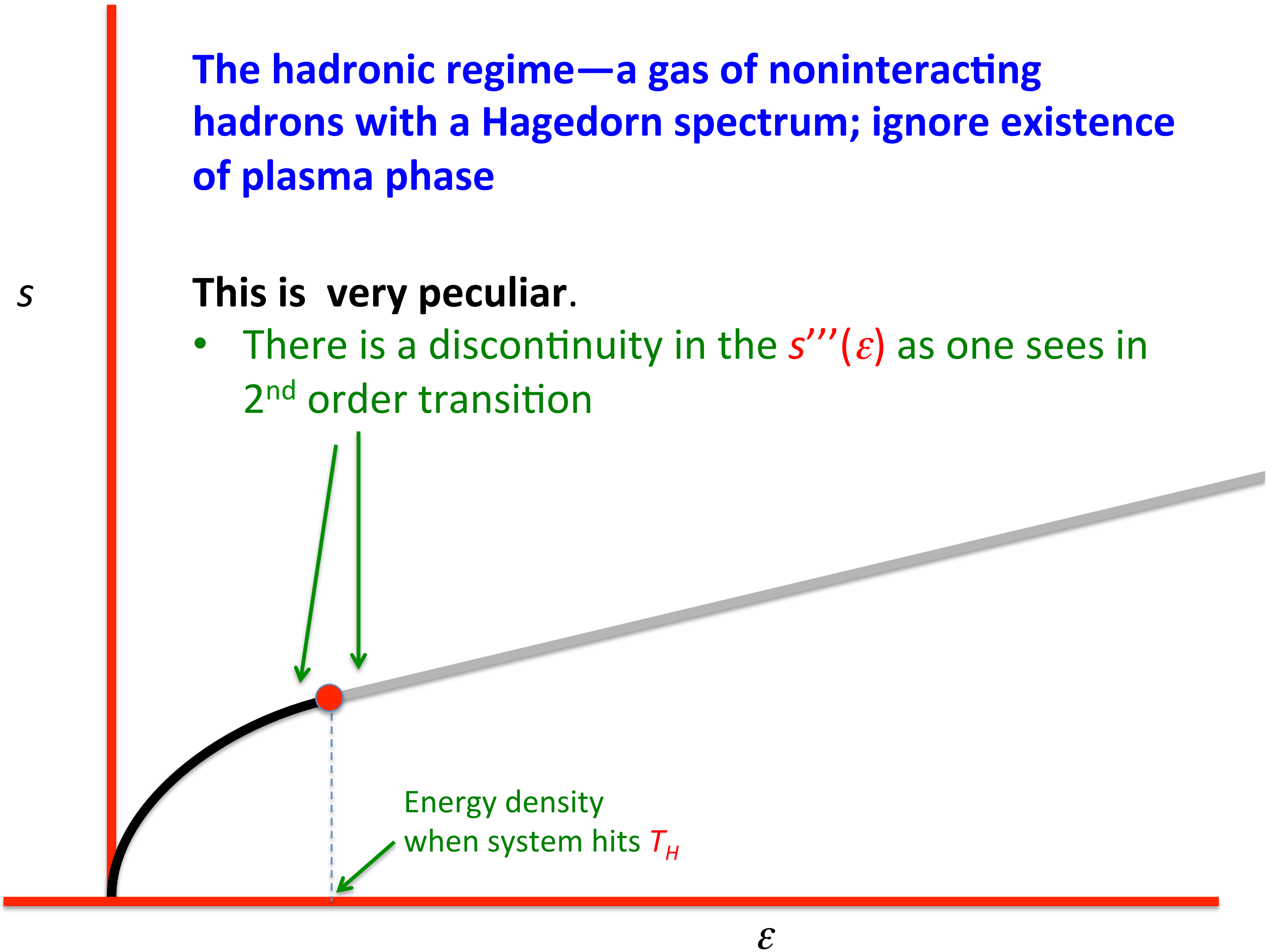
This is very peculiar.

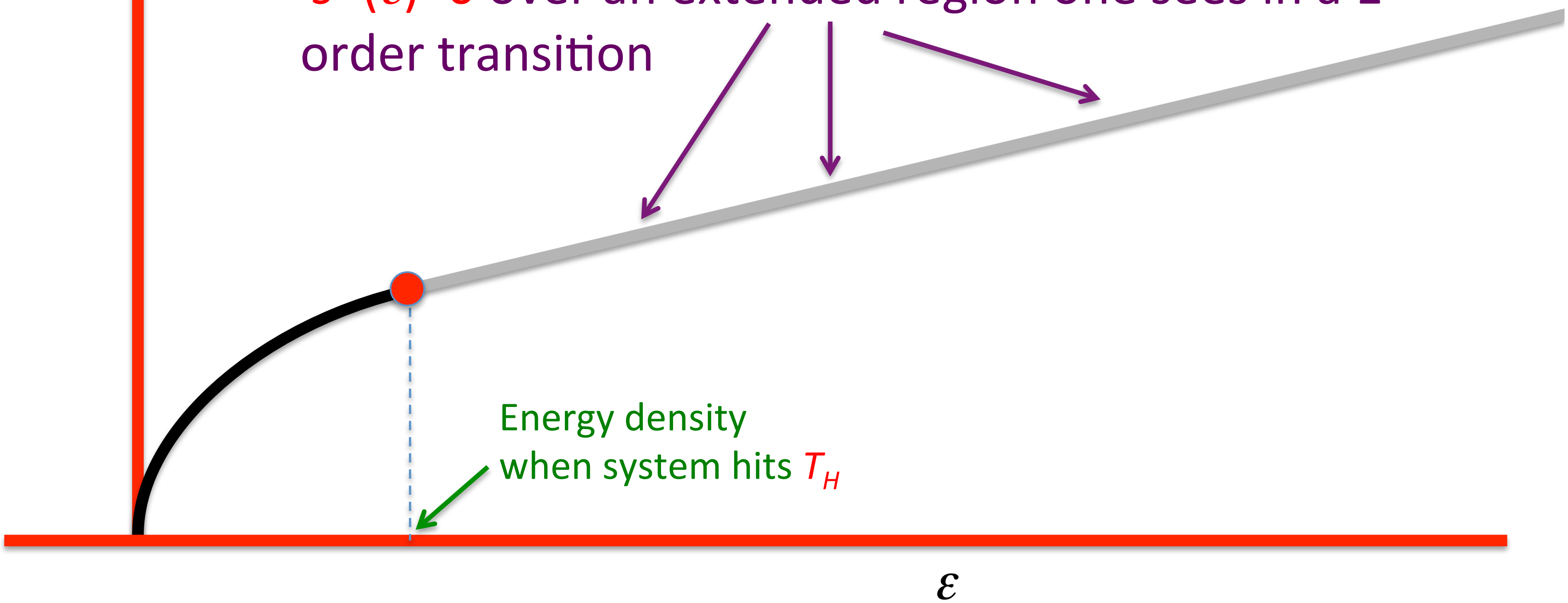
- There is a discontinuity in the $s'''(\varepsilon)$ as one sees in 2nd order transition



The hadronic regime—a gas of noninteracting hadrons with a Hagedorn spectrum; ignore existence of plasma phase

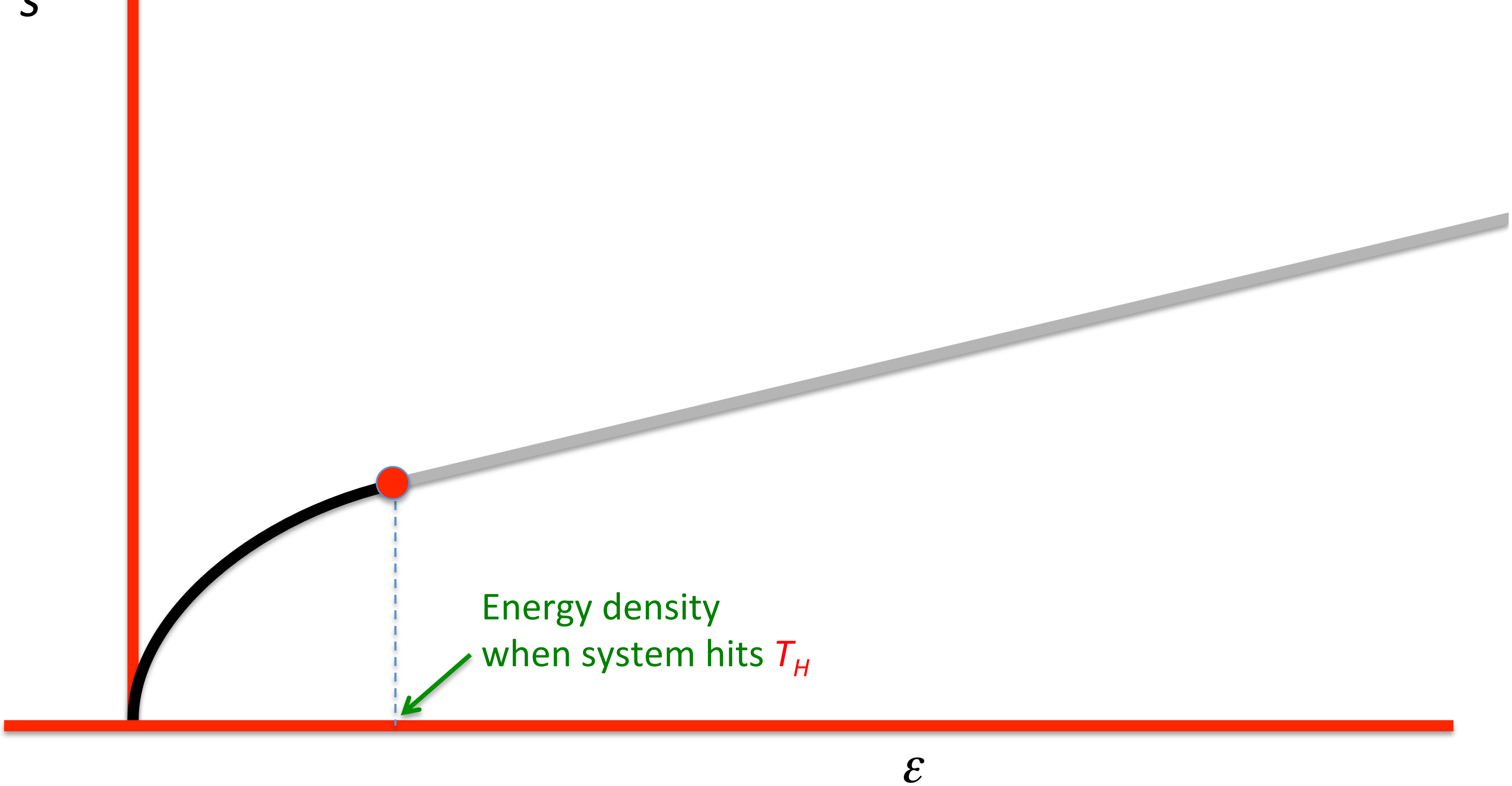
This is very peculiar.

- There is a discontinuity in the $s'''(\varepsilon)$ as one sees in 2nd order transition
- $s''(\varepsilon)=0$ over an extended region one sees in a 1st order transition



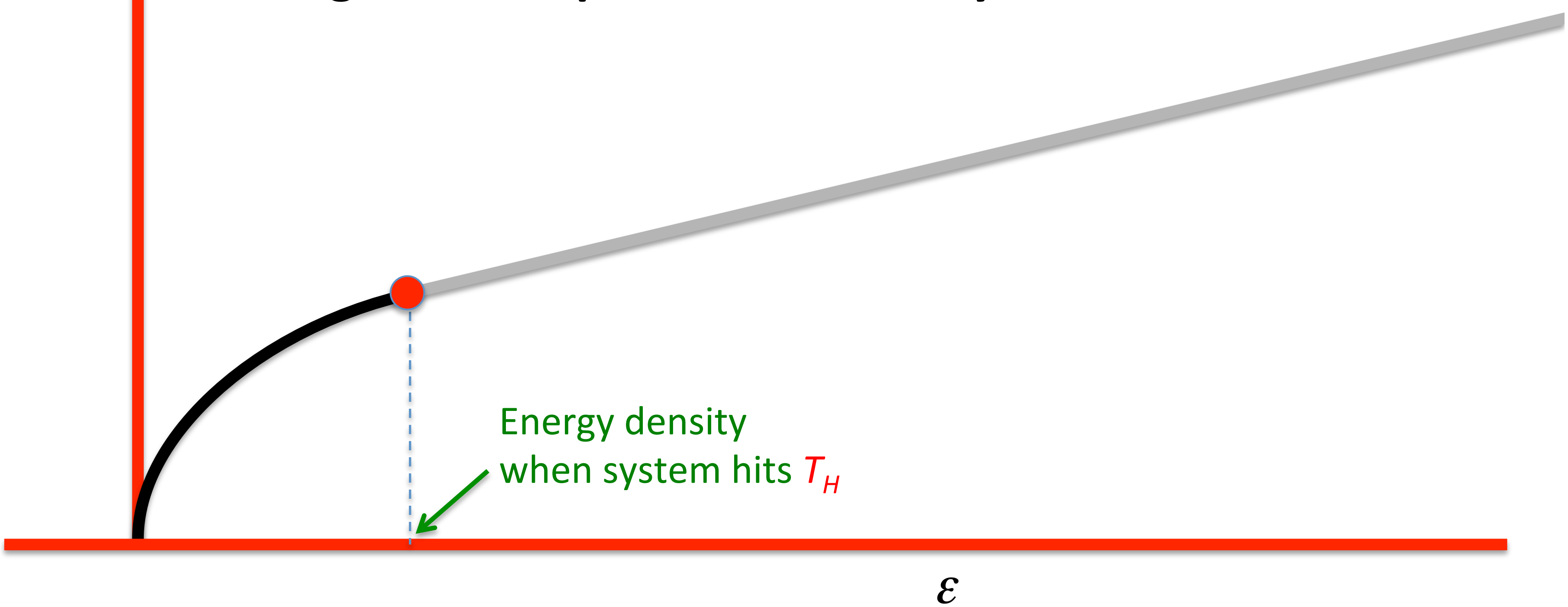
Presumably, N_c^{-2} corrections alter the behavior.

One expects that rather than $s''(\varepsilon)=0$ beyond the Hagedorn point one has $s''(\varepsilon)>0$ indicating local instability but with $s''(\varepsilon)\sim N_c^{-2}$



Implication:

- One can show that the characteristic time scale of the instability, τ_I, scales as $\tau_I \sim N_c^3$
- The equliberation τ_{Eq} , scales as $\tau_{Eq} \sim N_c^2$
- Thus as the large N_c limit is approached the system can be equilibrated and (parmetrically) long-lived despite the instability



Conclusions/Surprises

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The metastable supercooled plasma phase of large N_c QCD has negative absolute pressure.

Beyond the endpoint of the metastable hadronic phase of large N_c QCD is a regime that is locally unstable, but none-the-less long-lived.