Center regions as a solution to the Gribov problem of the center vortex model*

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*All data presented is taken from SU(2) gluonic lattice-QCD in Wilson action but the algorithms presented can be easily modified for SU(3)

The centre vortex model - some of the successes:

- behaviour of Wilson and Polyakov loops → phase transition

  ➜ *DelDebbio, 1998l; Nishimura, 2017*

- Casimir scaling of heavy-quark potentials (due to thick vortices)

  ➜ *M.F, 1997*

- spontaneous breaking of scale invariance

  ➜ *Langfeld, 1997*

- color structure of vortices → instantons, lumps of topological charge

  ➜ *Schweigler; Nejad, 2016; Höllwieser, 2011*

- quark condensate → chiral symmetry breaking

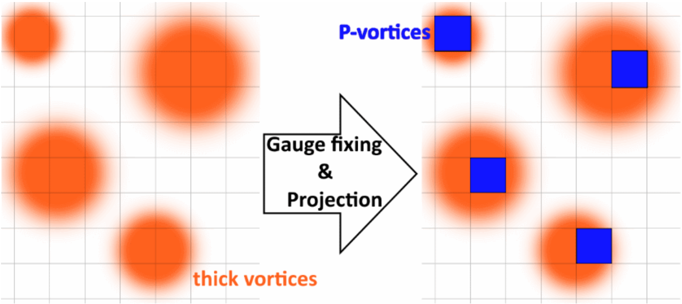
  ➜ *Höllwieser; M.F, 2017*

Z_N vortex condensation theory:

➜ *'t Hooft, Vinciarelli, Yoneya, 1978; Cornwall, Nielsen, Olesen, Mack, Petkova, 1979*

Detection of thick centre vortices

Finding the best fit to a configuration of thick vortices by P-vortices:



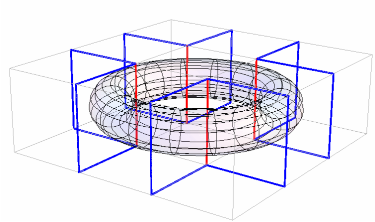
Direct maximal center gauge and centre projection

Gauge fixing: find $\Omega(x)$, s.t. $\acute{U}_\mu(x) = \Omega(x) U_\mu(x) \Omega^\dagger(x+e_\mu)$ maximizes

$$R_{SA}^2 = \sum_x \sum_\mu |\ \mathrm{Tr}[\acute{U}_\mu(x)]\ |^2$$

Centre projection:

$$\acute{U}_\mu(x) \to Z_\mu(x) = \mathrm{sign}\ \mathrm{Tr}[\acute{U}_\mu(x)]$$



Non-trivial Links build up the *Dirac volume*, whose surface is the vortex (transparent). They are detected by non-trivial plaquettes. (3-dimensional slice through a 4-dimensional lattice)

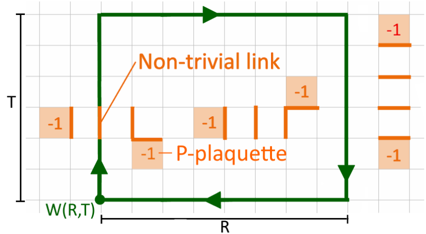
➜ *Del Debbio, M.F., Greensite, Ǒlejník (1996–1998)*

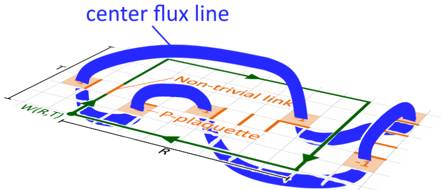
BUT: numerical methods can only detect local maxima → Gribov problem

➜ *Bornaykov et al. (2000)*

P-vortices and the string tension

Each P-plaquette contributes a factor -1 to enclosing Wilson loops:





With vortex density ϱ, that is, the proportion of P-plaquettes, the string tension can be calculated:

$$\langle W(R,T)\rangle = [(-1)\varrho + (+1)(1-\varrho)]^{\overbrace{R \times T}^{A}} = e^{-\ln(1-2\varrho)A} = e^{-\sigma A}$$

$\rightarrow$ string tension $\boldsymbol{\sigma = -\ln(1-2\varrho)}$

BUT: Small fluctuations, that is, correlated P-plaquettes, can lead to an mistakenly high vortex density $\Rightarrow$ smoothing procedures are necessary! **Or use Creutz ratios.**

String tension via Creutz ratios

$$\chi(R, T) = -\ln \frac{\langle W(R+1, T+1)\rangle \, \langle W(R, T)\rangle}{\langle W(R, T+1)\rangle \, \langle W(R+1, T)\rangle}$$

(With $\langle W(R, T)\rangle \approx e^{-\sigma \, R \, T - 2\,\mu\,(R+T)+C}$ and $R, T \to \infty \; \Rightarrow \sigma \; = \; \chi$)

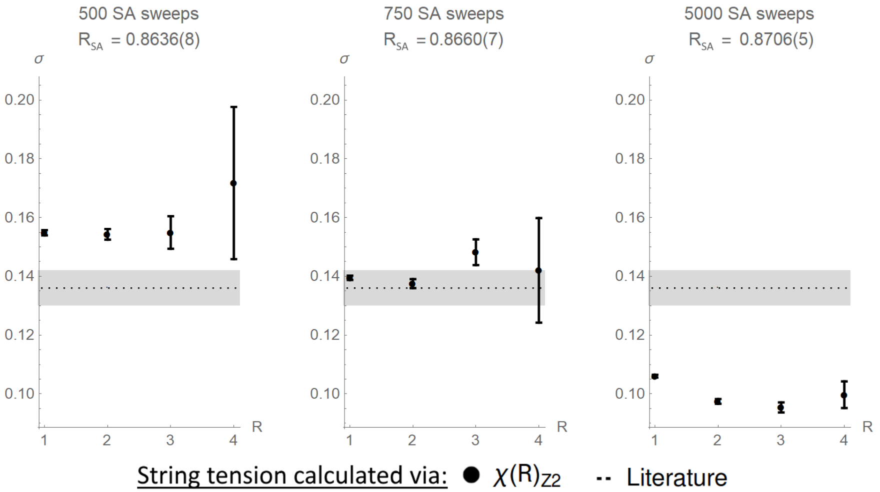
$\Rightarrow$ also usable without smoothing procedures

$\rightarrow$ Calculation based on full SU(2) links, $U_\mu(x)$:
prediction of the full theory

$\rightarrow$ Calculation based on center degrees of freedom, $Z_\mu(x)$:
prediction of the center vortex model

Problems concerning the predicted string tension

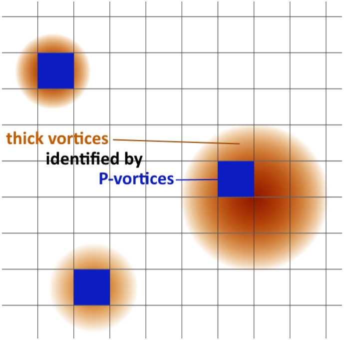
Improved value of gauge functional leads to a loss of string tension:



300 Wilson configurations at $\beta = 2.3$, sizes 12^4 (left), 12^4 (middle) and 14^4 (right)

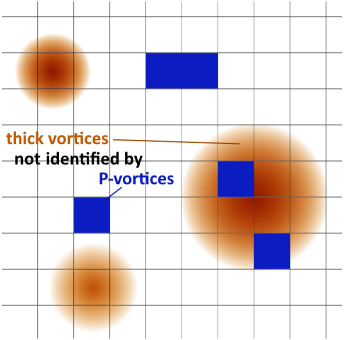
➜ *see: Bornyakov et al 2002, Faber et al 2001 (literature value based on Bali 1995)*

Resolution based on the vortex finding property



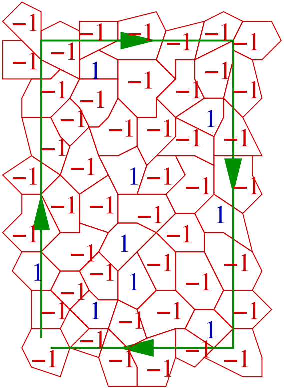
$\Longrightarrow$

loosing the vortex finding property

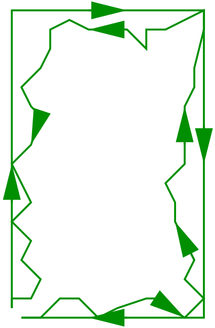


The vortex finding property needs to be preserved during the gauge fixing procedure

Our improvements: Usage of center regions



$$= (-1)^{22} \times$$



Centre regions — Area law — Perimeter law

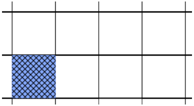
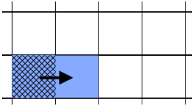
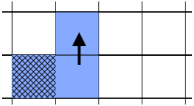
Non-trivial Regions are to thick vortices, what P-Plaquettes are to P-vortices.

$\Rightarrow$

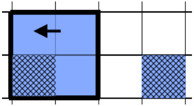
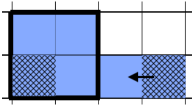
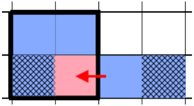
Preserve the sign of the trace of Wilson loops enclosing them during gauge fixing

Identifying non-trivial center regions

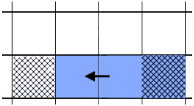
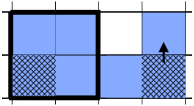
Start with a plaquette and enlarge so that the bigger regions evaluation is nearer the non-trivial center (1-3).

1)  2)  3) 

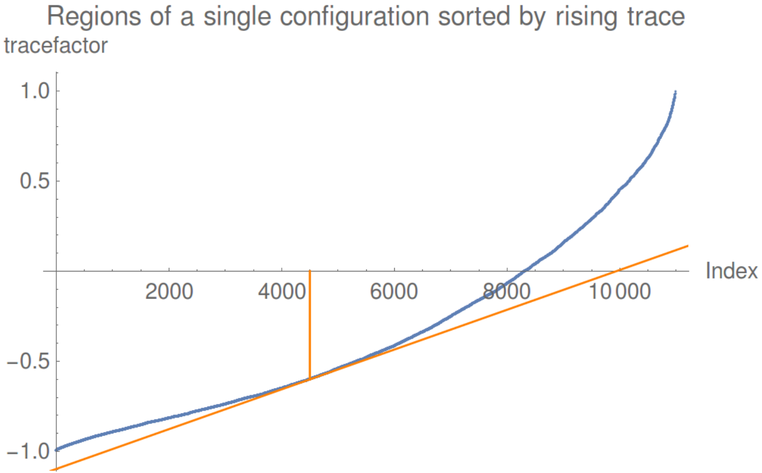
When no more enlargement results in an improved evaluation, store the region and start with a new plaquette (4-5)

4)  5)  6) 

When a new region grows into an older one, the better one survives (6-7).

7a)  7b) 

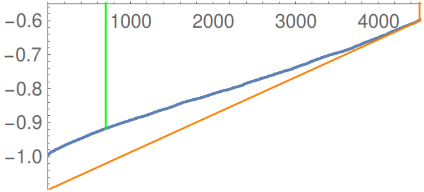
Selecting non-trivial center regions - part I

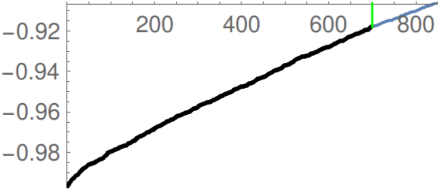


A first set of regions is identified by finding a tangent through the point $(0, 1.1 * \text{tracefactor}_{\text{lowest}})$. The regions below this tangent point are further filtered.

Selecting non-trivial center regions - part II

Within the selected set of regions an inflection point is identified numerically. It is assumed that within the set only one real inflection point occurs and within the numerical error the lowest possible inflection point is taken.

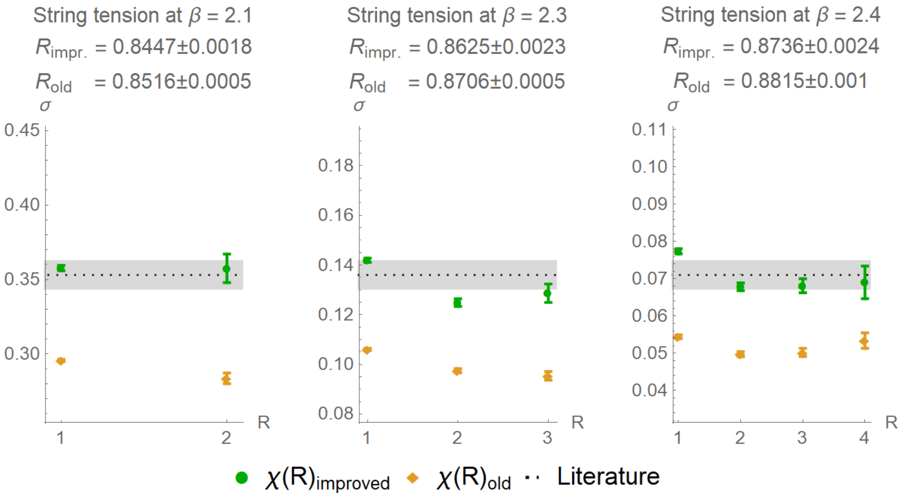




The regions below this inflection point comprise the set of non-trivial center regions used at the gauge fixing procedure.

Results of our improvements: Creutz ratios

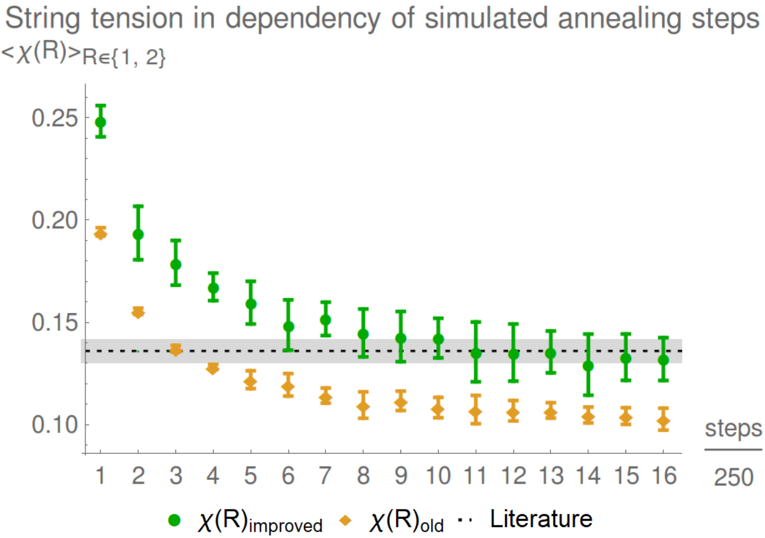
Small decrease of the gauge functional, increase of the errorbars - BUT:
We reproduce the literature value!



300 Wilson configurations, lattices size: 12^4 (left), 14^4 (middle) and 12^4 (right).

Results of our improvements: simulated annealing

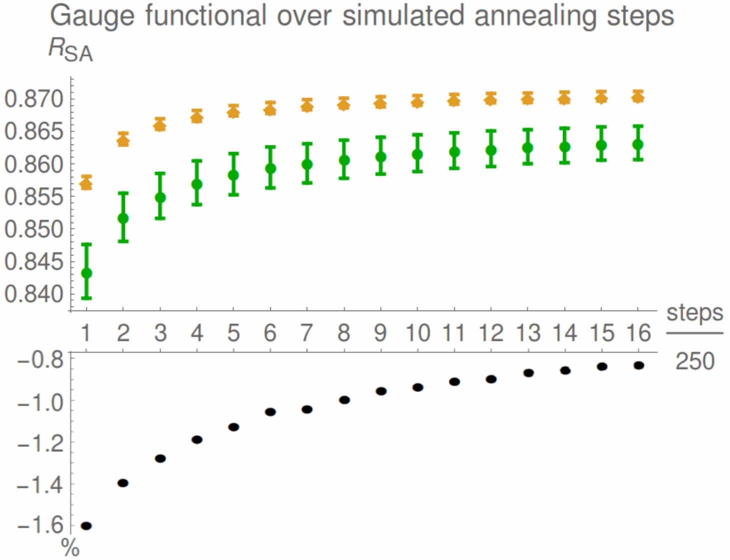
We stay on the literature value!



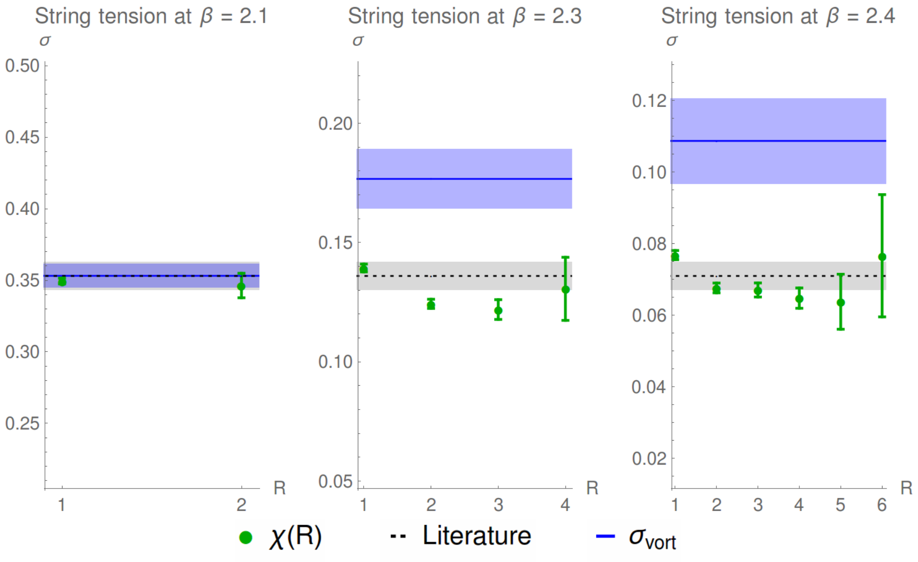
100 Wilson configurations per data point, lattices size: 12^4.

Results of our improvements: gauge functional

The decreases of the gauge functional weakens with stronger simluated annealing!



Results of our improvements: Creutz ratios (smoothed)

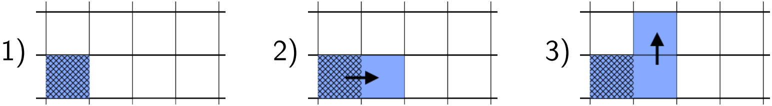


300 Wilson configurations, lattices size 18^4, with vortex smoothing.

Recovering thick vortices from P-vortices

⇒ Use the P-plaquettes of the $Z_\mu(x)$ configuration to recover thick vortices in the original, SU(2) $U_\mu(x)$ configurations:

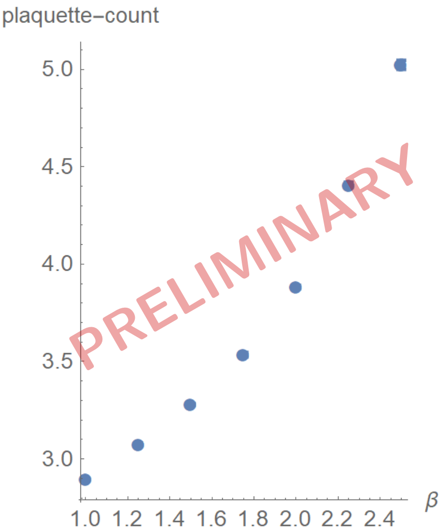
Taking the algorithm for detecting center regions we start the enlargement procedure at the position of P-plaquettes in the full SU(2) configuration but allow overlapping regions (for now).

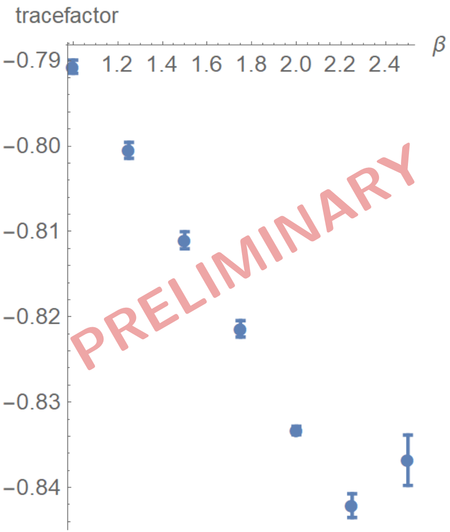


⇒ identifies those plaquette, that are positioned within the thick flux building up the thick center vortex.

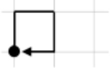
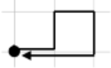
Enables further studies of size and inner structure of thick vortices.

The road to go: properties of thick vortices





An interesting property: The S2-homogeneity

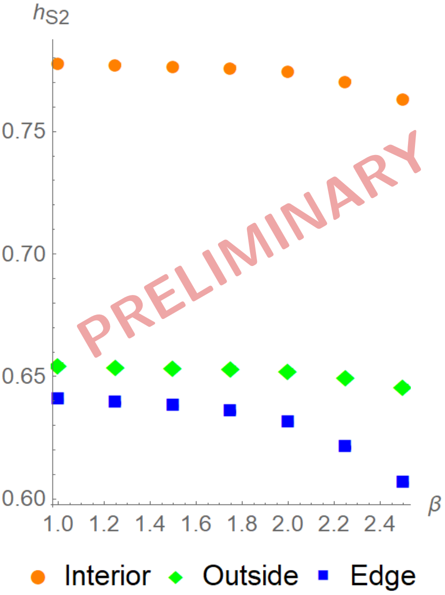
Taking two plaquettes,  W_1 and  W_2, related to the same lattice point, they can be factorized using Pauli matrizes σ_k, $n_j \in \mathbb{S}^2$, $\mid n_j \mid = 1$ as

$$W_j = \cos(\alpha_j)\, \sigma_0 + \mathrm{i} \textstyle\sum_{k=1}^{3} \sin(\alpha_j)\, (n_j)_k\, \sigma_k .$$

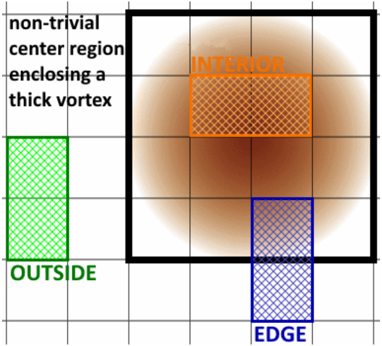
S2-homogeneity is defined as $\mathbf{h_{S2}} := \frac{\mathbf{1}}{\mathbf{2}} \, |\vec{\mathbf{n}}_1 + \vec{\mathbf{n}}_2| \in [\mathbf{0}, \mathbf{1}]$.

$\Rightarrow$ gauge independent; measures the homogeneity of the flux with respect to the color-space.

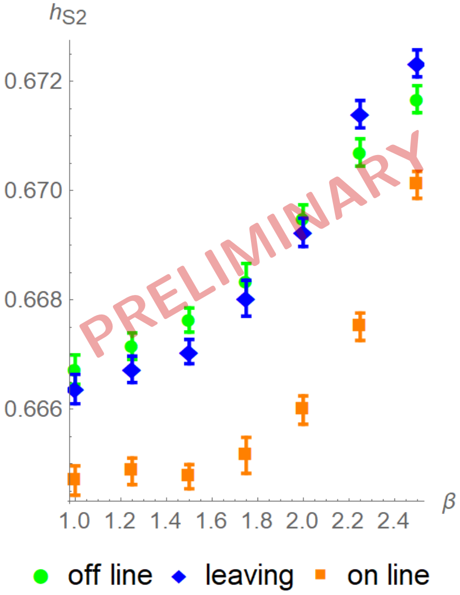
The road to go: S2-homogeneity of thick vortices

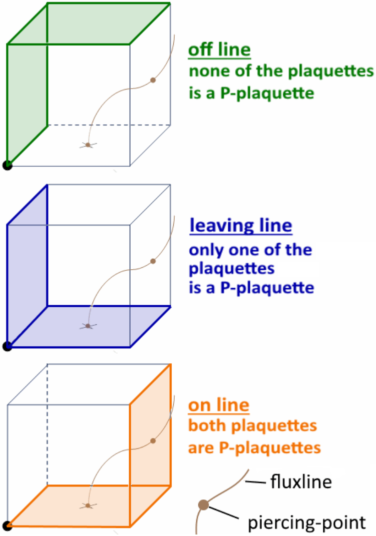


Distinguish 3 scenarios in dependency whether the plaquettes compared are within the thick vortex or not:



The road to go: S2-homogeneity along the vortex





The road to further investigations of thick vortices is clear ...

... and we will continue our way!

Thank you!