Center regions as a solution to the Gribov problem of the center vortex model

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*All data presented is taken from SU(2) gluonic lattice-QCD in Wilson action but the algorithms presented can be easily modified for SU(3)
The centre vortex model - some of the successes:

- behaviour of Wilson and Polyakov loops $\rightarrow$ phase transition
  $\rightarrow$ DelDebbio, 1998; Nishimura, 2017

- Casimir scaling of heavy-quark potentials (due to thick vortices)
  $\rightarrow$ M.F, 1997

- spontaneous breaking of scale invariance
  $\rightarrow$ Langfeld, 1997

- color structure of vortices $\rightarrow$ instantons, lumps of topological charge
  $\rightarrow$ Schweigler; Nejad, 2016; Höllwieser, 2011

- quark condensate $\rightarrow$ chiral symmetry breaking
  $\rightarrow$ Höllwieser; M.F, 2017

$\mathbb{Z}_N$ vortex condensation theory:

$\rightarrow$ ’t Hooft, Vinciarelli, Yoneya, 1978; Cornwall, Nielsen, Olesen, Mack, Petkova, 1979
Detection of thick centre vortices

Finding the best fit to a configuration of thick vortices by P-vortices:
Direct maximal center gauge and centre projection

Gauge fixing: find $\Omega(x)$, s.t. $\dot{U}_\mu(x) = \Omega(x) U_\mu(x) \Omega^\dagger(x + e_\mu)$ maximizes

$$R^2_{SA} = \sum_x \sum_\mu | \text{Tr}[\dot{U}_\mu(x)]|^2$$

Centre projection:

$$\dot{U}_\mu(x) \rightarrow Z_\mu(x) = \text{sign} \ \text{Tr}[\dot{U}_\mu(x)]$$

Non-trivial Links build up the Dirac volume, whose surface is the vortex (transparent). They are detected by non-trivial plaquettes. (3-dimensional slice through a 4-dimensional lattice)

$\Rightarrow$ Del Debbio, M.F., Greensite, Řelejník (1996–1998)

BUT: numerical methods can only detect local maxima $\rightarrow$ Gribov problem

$\Rightarrow$ Bornaykov et al. (2000)
P-vortices and the string tension

Each P-plaquette contributes a factor -1 to enclosing Wilson loops:

$$\langle W(R, T) \rangle = \left[ (-1)\varrho + (+1)(1 - \varrho) \right]^{A \times T} = e^{-\ln(1 - 2\varrho)A} = e^{-\sigma A}$$

$$\rightarrow$$ string tension $$\sigma = -\ln(1 - 2\varrho)$$

BUT: Small fluctuations, that is, correlated P-plaquettes, can lead to an mistakenly high vortex density ⇒ smoothing procedures are necessary! Or use Creutz ratios.
String tension via Creutz ratios

\[ \chi(R, T) = -\ln \frac{\langle W(R+1, T+1) \rangle \langle W(R, T) \rangle}{\langle W(R, T+1) \rangle \langle W(R+1, T) \rangle} \]

(With \( \langle W(R, T) \rangle \approx e^{-\sigma R T - 2 \mu (R+T)+C} \) and \( R, T \to \infty \Rightarrow \sigma = \chi \))

⇒ also usable without smoothing procedures

→ Calculation based on full SU(2) links, \( U_\mu(x) \):
  prediction of the full theory

→ Calculation based on center degrees of freedom, \( Z_\mu(x) \):
  prediction of the center vortex model
Problems concerning the predicted string tension

Improved value of gauge functional leads to a loss of string tension:

- 300 Wilson configurations at $\beta = 2.3$, sizes $12^4$ (left), $12^4$ (middle) and $14^4$ (right)

String tension calculated via: $\bullet \chi(R)_{Z2}$  

300 Wilson configurations at $\beta = 2.3$, sizes $12^4$ (left), $12^4$ (middle) and $14^4$ (right)

Resolution based on the vortex finding property

The vortex finding property needs to be preserved during the gauge fixing procedure.
Our improvements: Usage of center regions

Non-trivial Regions are to thick vortices, what P-Plaquettes are to P-vortices.

\[ = (-1)^{22} \times \]

Preserve the sign of the trace of Wilson loops enclosing them during gauge fixing
Identifying non-trivial center regions

Start with a plaquette and enlarge so that the bigger regions evaluation is nearer the non-trivial center (1-3).

1)  
2)  
3)  

When no more enlargement results in an improved evaluation, store the region and start with a new plaquette (4-5)

4)  
5)  
6)  

When a new region grows into an older one, the better one survives (6-7).

7a)  
7b)
A first set of regions is identified by finding a tangent through the point \((0, 1.1 \times \text{tracefactor}_{\text{lowest}})\). The regions below this tangent point are further filtered.
Selecting non-trivial center regions - part II

Within the selected set of regions an inflection point is identified numerically. It is assumed that within the set only one real inflection point occurs and within the numerical error the lowest possible inflection point is taken.

The regions below this inflection point comprise the set of non-trivial center regions used at the gauge fixing procedure.
Results of our improvements: Creutz ratios

Small decrease of the gauge functional, increase of the errorbars - BUT: We reproduce the literature value!

300 Wilson configurations, lattices size: $12^4$ (left), $14^4$ (middle) and $12^4$ (right).
Results of our improvements: simulated annealing

We stay on the literature value!

String tension in dependency of simulated annealing steps

\[ \langle \chi(R) \rangle_{R \in \{1, 2\}} \]

100 Wilson configurations per data point, lattices size: \(12^4\).
Results of our improvements: gauge functional

The decreases of the gauge functional weakens with stronger simulated annealing!

Gauge functional over simulated annealing steps

$R_{SA}$

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Results of our improvements: Creutz ratios (smoothed)

String tension at $\beta = 2.1$

String tension at $\beta = 2.3$

String tension at $\beta = 2.4$

300 Wilson configurations, lattices size $18^4$, with vortex smoothing.
Recovering thick vortices from P-vortices

Use the P-plaquettes of the $Z_{\mu}(x)$ configuration to recover thick vortices in the original, SU(2) $U_{\mu}(x)$ configurations:

Taking the algorithm for detecting center regions we start the enlargement procedure at the position of P-plaquettes in the full SU(2) configuration but allow overlapping regions (for now).

1) identifies those plaquette, that are positioned within the thick flux building up the thick center vortex.

⇒ Enables further studies of size and inner structure of thick vortices.
The road to go: properties of thick vortices

plaqette-count

tracefactor

β

β

PRELIMINARY

PRELIMINARY
An interesting property: The S2-homogeneity

Taking two plaquettes, \( W_1 \) and \( W_2 \), related to the same lattice point, they can be factorized using Pauli matrices \( \sigma_k, n_j \in S^2, |n_j| = 1 \) as

\[
W_j = \cos(\alpha_j) \sigma_0 + i \sum_{k=1}^{3} \sin(\alpha_j) (n_j)_k \sigma_k.
\]

S2-homogeneity is defined as

\[
h_{S2} := \frac{1}{2} |\vec{n}_1 + \vec{n}_2| \in [0, 1].
\]

⇒ gauge independent; measures the homogeneity of the flux with respect to the color-space.
Distinguish 3 scenarios in dependency whether the plaquettes compared are within the thick vortex or not:
The road to go: S2-homogeneity along the vortex

$h_{S2}$

- off line
- leaving line
- on line

off line: none of the plaquettes is a P-plaquette
leaving line: only one of the plaquettes is a P-plaquette
on line: both plaquettes are P-plaquettes

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The road to further investigations of thick vortices is clear ...

... and we will continue our way!

Thank you!