

# 3-flavor extension of the excluded volume model for the hard-core repulsion

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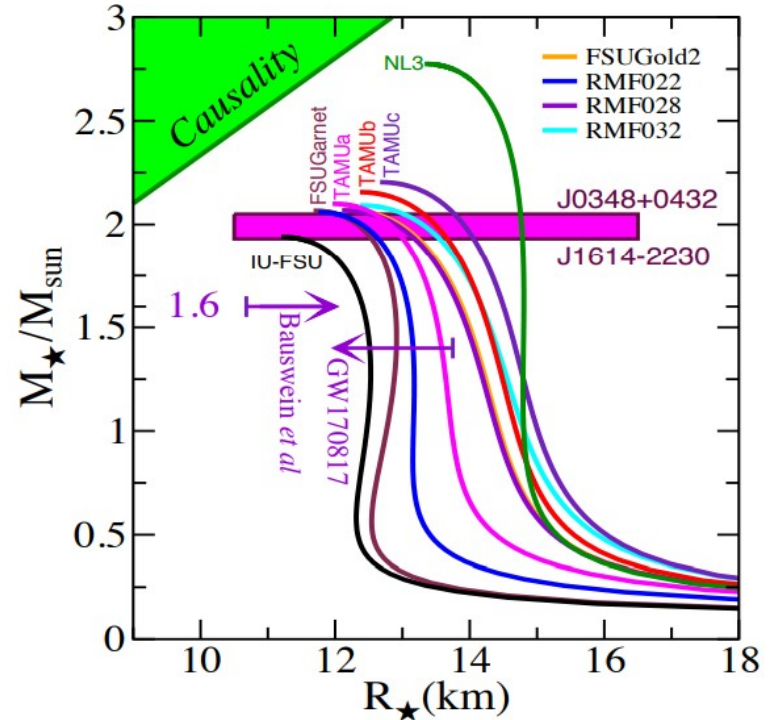
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# Outline

- Motivation;
- Quarkyonic matter;
- Excluded Volume Model;
- Some Numerical Results;
- Final Remarks

# Motivation

- Observation and analysis of GW170817: Important clues to understand cold and dense matter.
- EoS should be hard enough to support  $2M_{\odot}$  and soft enough to satisfy  $R_{1.4} \leq 13.5$  km.
- This is also reflected in sound velocity, that should increase rapidly and can be greater than its conformal value  $c_s^2 \geq 1/3$ .



F. J. Fattoyev et. al, PRL 120, 172702 (2018)

# Motivation

$R_{1.4} \leq 13.5$  km inferred by tidal deformability observed in neutron star inspiral with 90% credence, and subsequent analyses suggests even more compact neutron stars: **Need for not too hard EoS for intermediate density.**

- Inclusion of some other particle?
- Repulsive nuclear interaction is enough?
- Phase transition to quark matter?

# Motivation

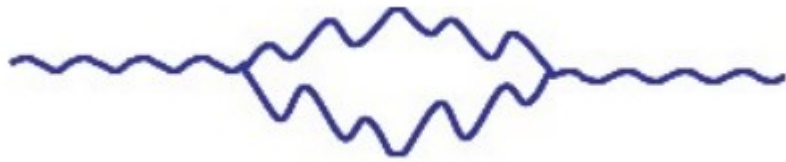
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- Inclusion of some other particle?
- Repulsive nuclear interaction is enough?
- Phase transition to quark matter?

Another candidate: **Quarkyonic matter**

# Quarkyonic Matter

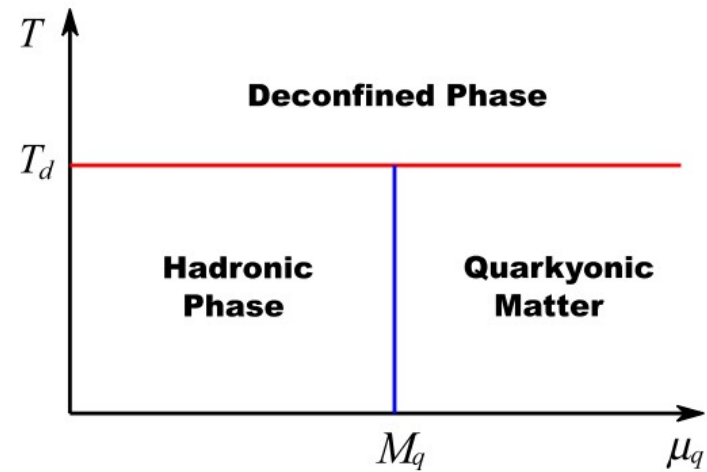
Phase of dense matter, argued from large  $N_c$  approximation and model computations.



Gluon loop

→  $g^2 N_c T^2 \sim T^2$ ;

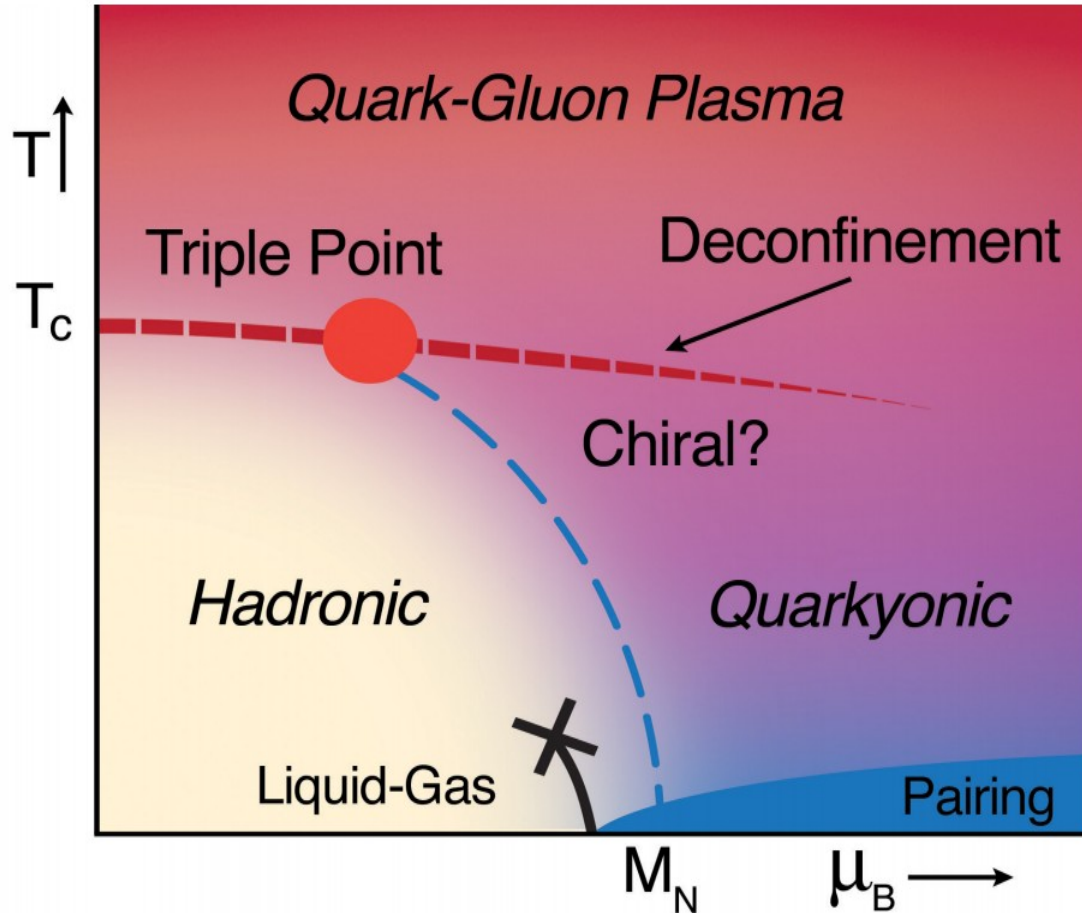
- Dynamics not affected by quarks;
- Debye screening at large distances.



Quark loop

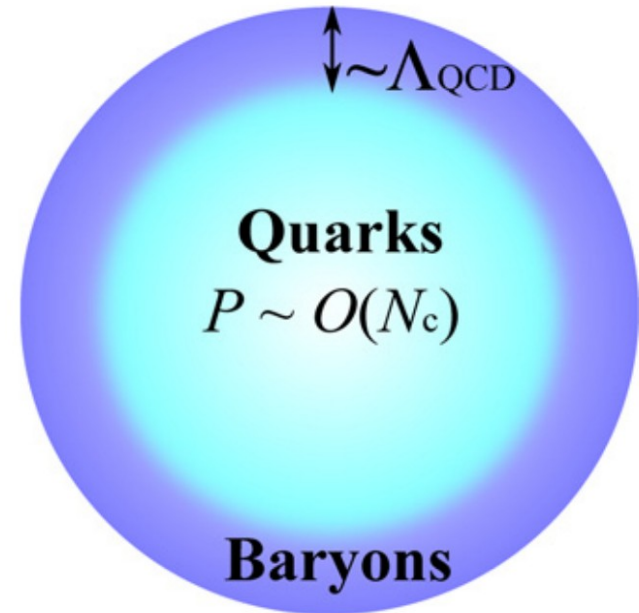
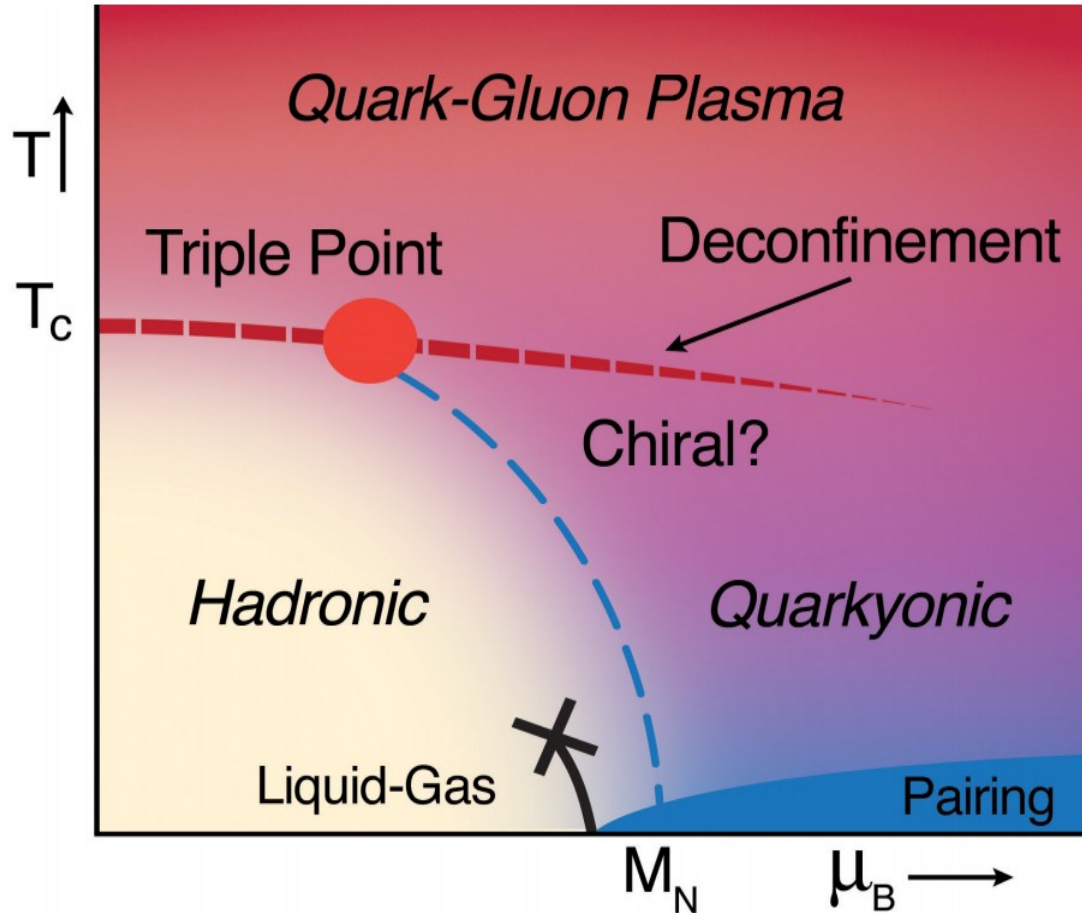
- $\sim \mu_Q^2 g^2 \Rightarrow$  Suppressed by  $1/N_c$  at large  $N_c$ .
- High density limit:  $\mu_Q \gg \Lambda_{\text{QCD}}$ , so quarks are important when  $\mu_Q \sim N_c^{1/2} \Lambda_{\text{QCD}}$ .
- Debye screen mass  $m_D \cong g \mu_Q$

# Quarkyonic Matter



- Different phase in confined world, appear when  $\mu_q > M_q$  and  $n_B$  becomes nonzero.
- Pressure changes suddenly from  $O(N_c^0) \rightarrow O(N_c)$ .
- Weakly interacting quark system or baryonic system?

# Quarkyonic Matter





# Quarkyonic Matter

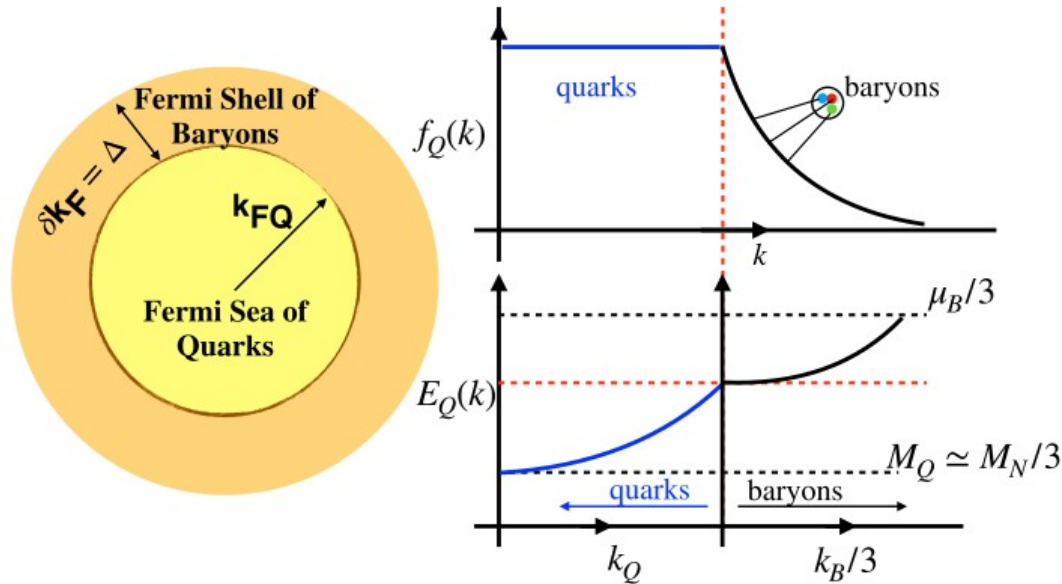
Nuclear  $\longrightarrow$  Quarkyonic  
(at few times  $\rho_0$ )

{ For  $k_F^B < \Lambda_{\text{QCD}}$ : Quarks confined in nucleons.  
For  $\Lambda_{\text{QCD}} \lesssim k_F^B \leq N_c \Lambda_{\text{QCD}}$ : Quarks starts to take low phase space, and a shell-like structure is formed.  
For  $k_F^B \simeq N_c^{3/2} \Lambda_{\text{QCD}}$ : Confinement disappears.

- Total baryon density has smooth behavior and chemical potential for confined states enhance suddenly, then pressure suddenly increases. **This is not an usual phase transition!**

# Quarkyonic Matter

- Alternative that could take into account the hardness of EoS at low densities and its softness when the density increases.



Quarks and nucleons are quasiparticles:  
 At low  $T$  quark states near Fermi surface are confined in the baryon-like states.

# Excluded Volume Model

- Need of repulsive interaction to support hard EoS at high density regime.
- Considering a hard core repulsion: Scale can be measured by the effective size of the baryon.
- Protons + Neutrons + Hyperons in an excluded volume  $v_0 = 1/n_0$  :

$$\varepsilon_N = \left(1 - \frac{n_{\tilde{B}}}{n_0}\right) \sum_{i=p,n,\Lambda} \int_0^{K_F^i} \frac{dk k^2}{\pi^2} \sqrt{k^2 + m_i^2} + \varepsilon_e$$

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Excluded volume model:  $K_F^i = \left(3\pi^2 \frac{n_i}{1 - n_{\tilde{B}}/n_0}\right)^{1/3}$

# 3 Flavor Excluded Volume Model

- Provides a hard-soft EoS in the case of single flavor<sup>‡</sup> when considering shell structure.
- Generalization for 3 flavor and inclusion of  $\beta$ -equilibrium and electromagnetic charge neutrality: **Possibility of making EoS softer.**

$$n_N^{ex} = \sum_{i=p,n,\Lambda} \frac{n_N}{1 - n_{\tilde{B}}/n_0} = 2 \int_0^{K_F^i} \frac{dk k^2}{2\pi^2}$$

$$n_N = n_p + n_n + n_\Lambda; \quad n_{\tilde{B}} = n_p + n_n + (1 + \alpha)n_\Lambda; \quad \mu_i = \frac{\partial \varepsilon}{\partial n_i}$$

<sup>‡</sup>K. S. J., L. M. and S. S., arXiv:1908.04799 [nucl-th]

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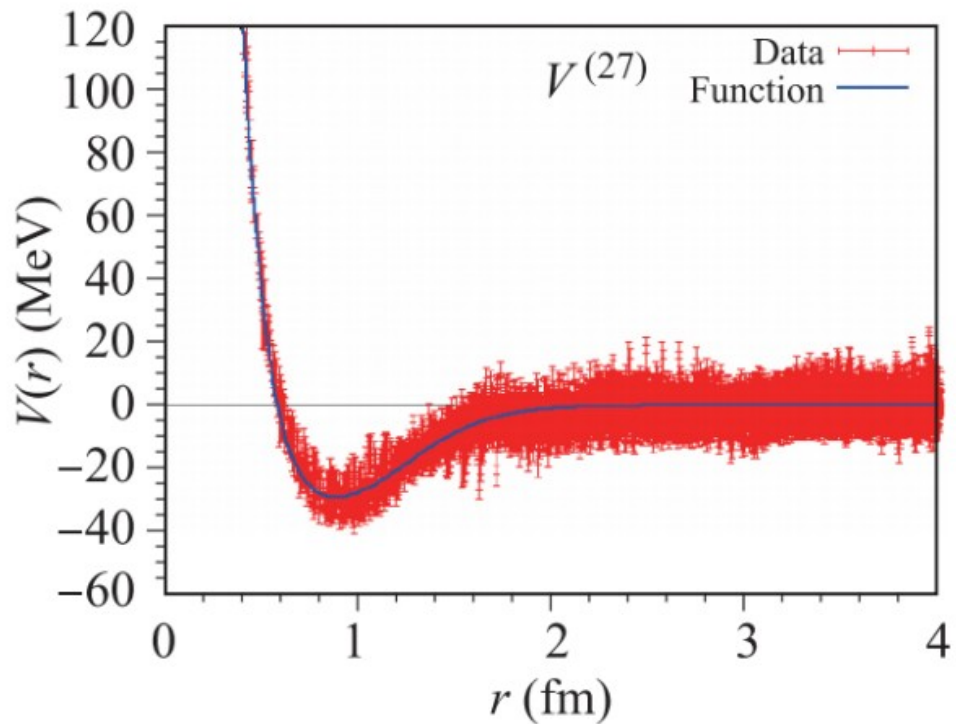
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Strength of  $\Lambda$   
repulsion

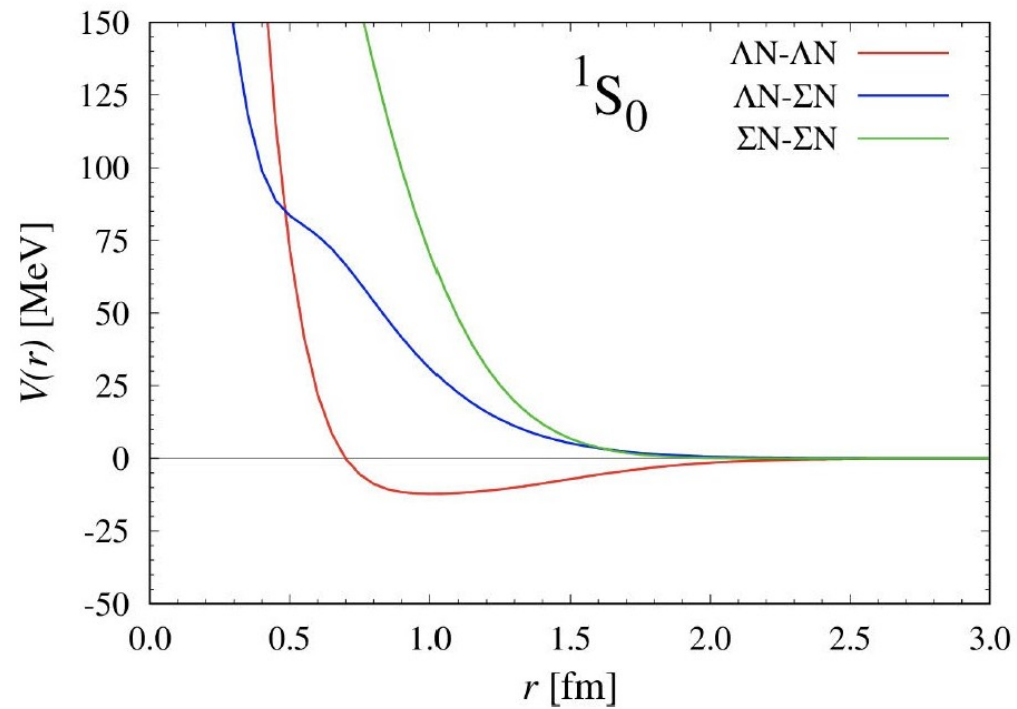


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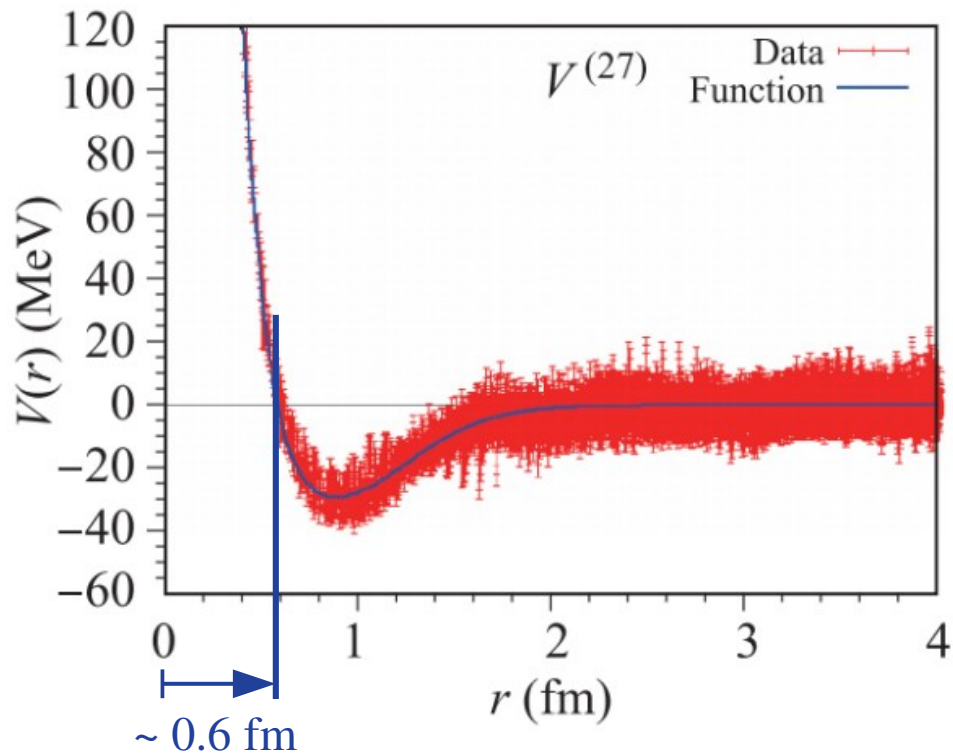
T. Hatsuda, Front. Phys. 13(6), 132105 (2018)



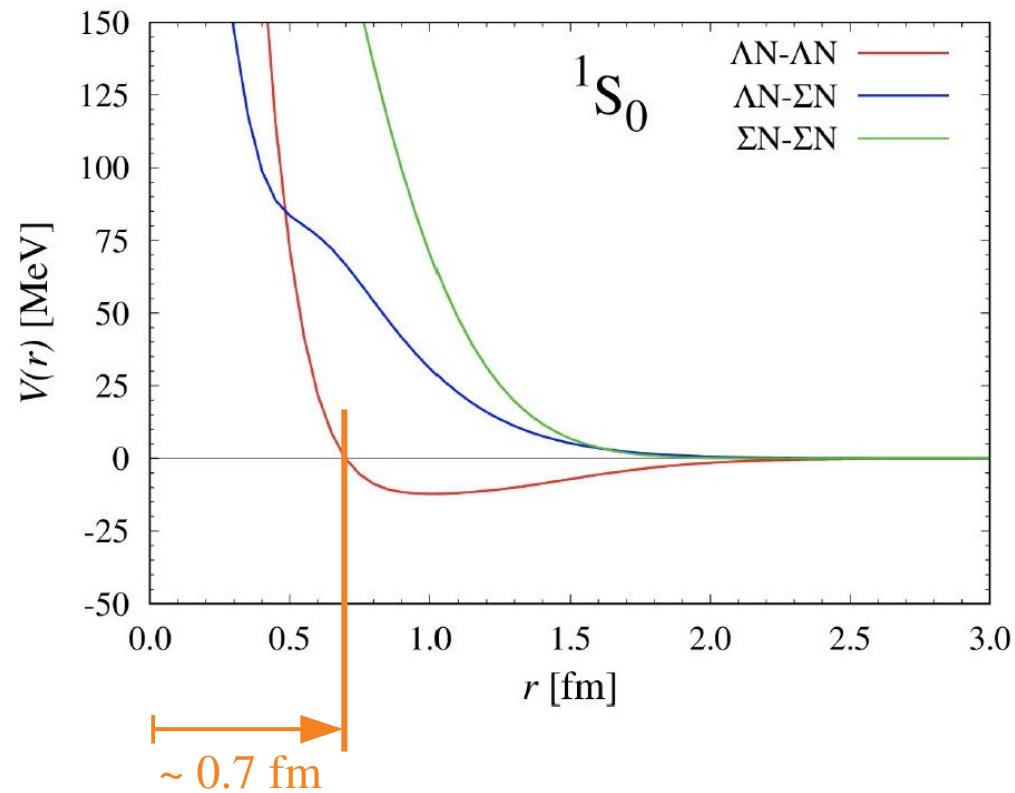
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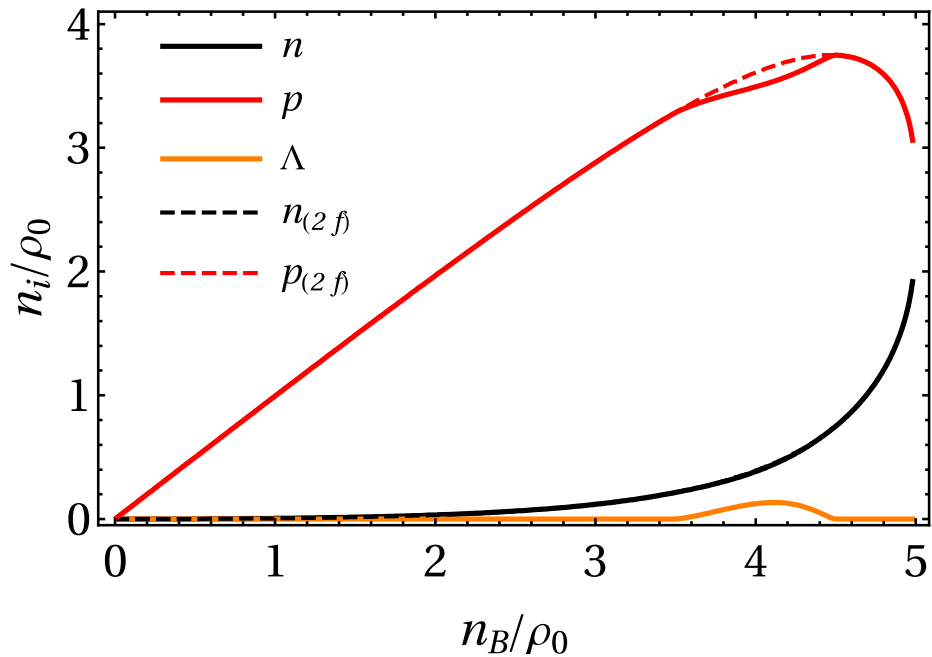


$$n_{\tilde{B}} = n_p + n_n + (1 + \alpha)n_{\Lambda}$$



# 3 Flavor Excluded Volume Model: Nucleons only

$\alpha = 0.2$  case

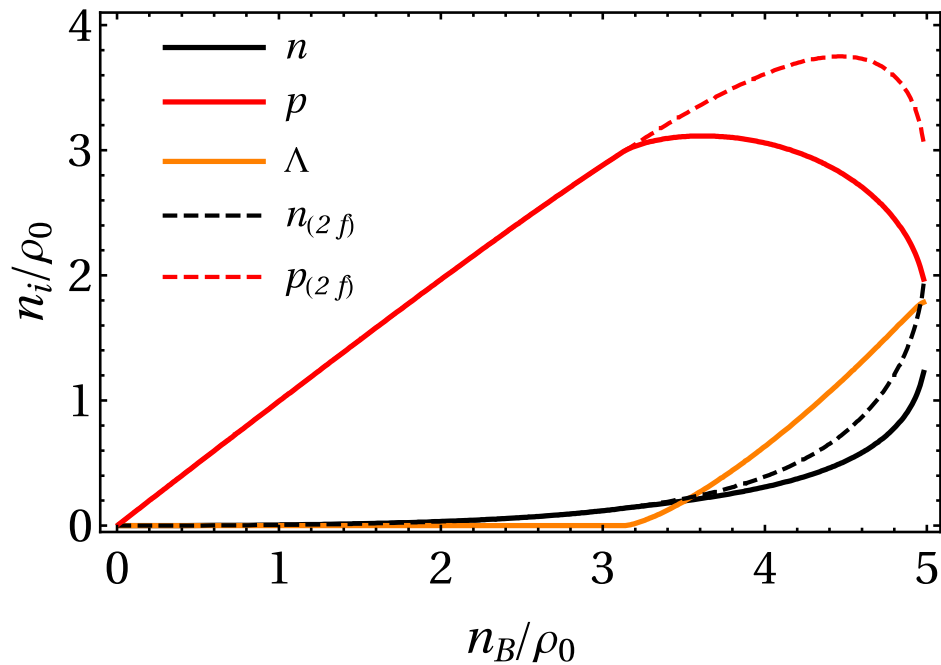


$$\mu_n = \mu_p + \mu_e$$

and

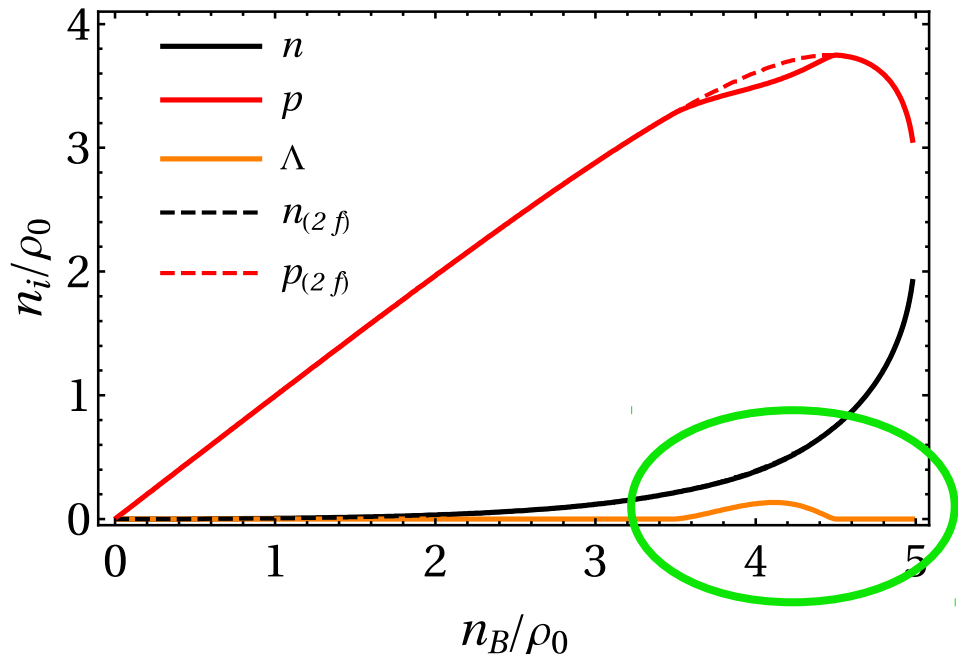
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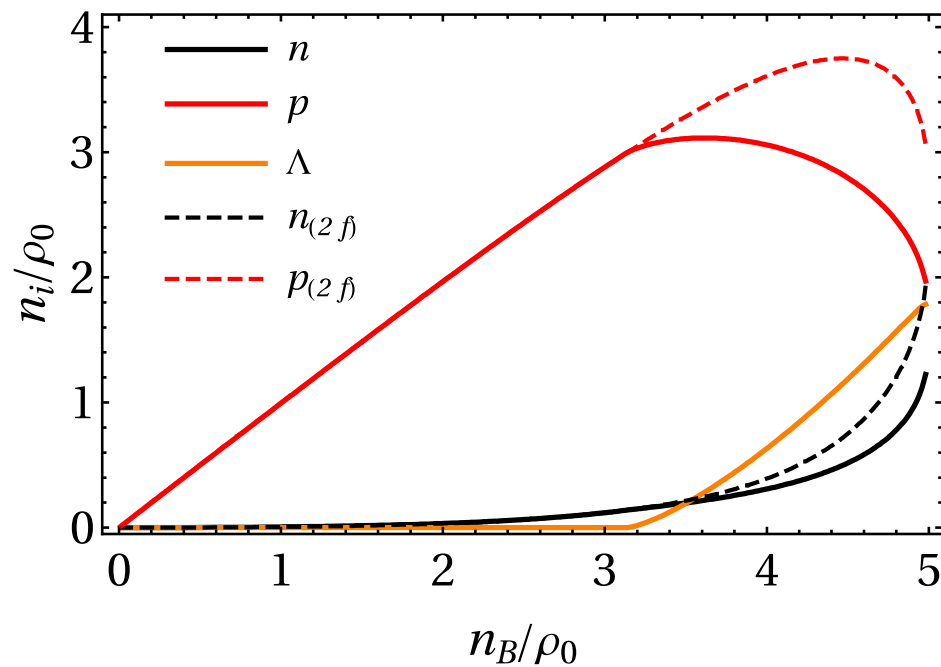


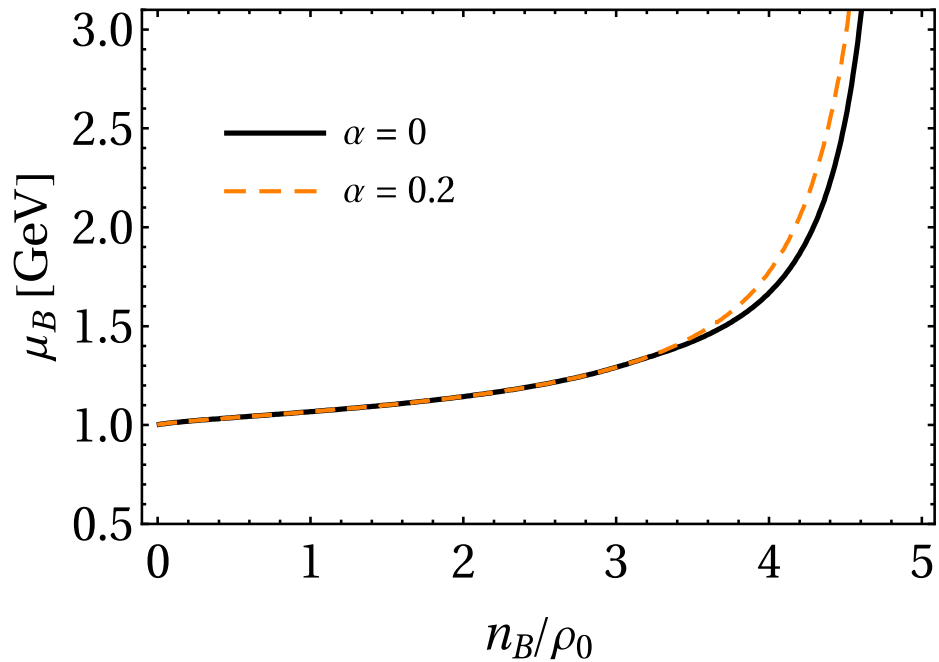
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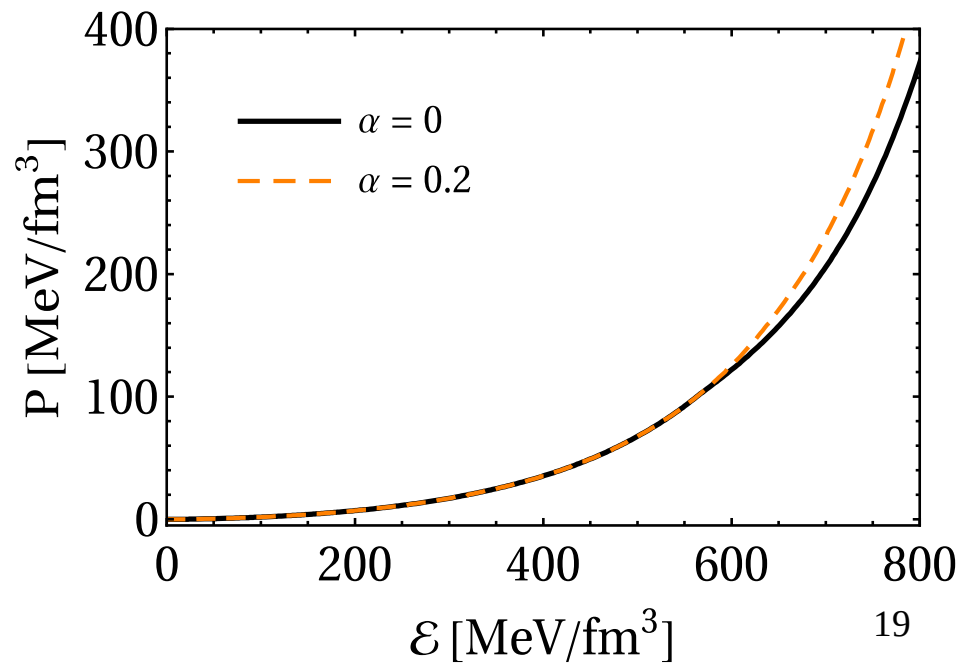
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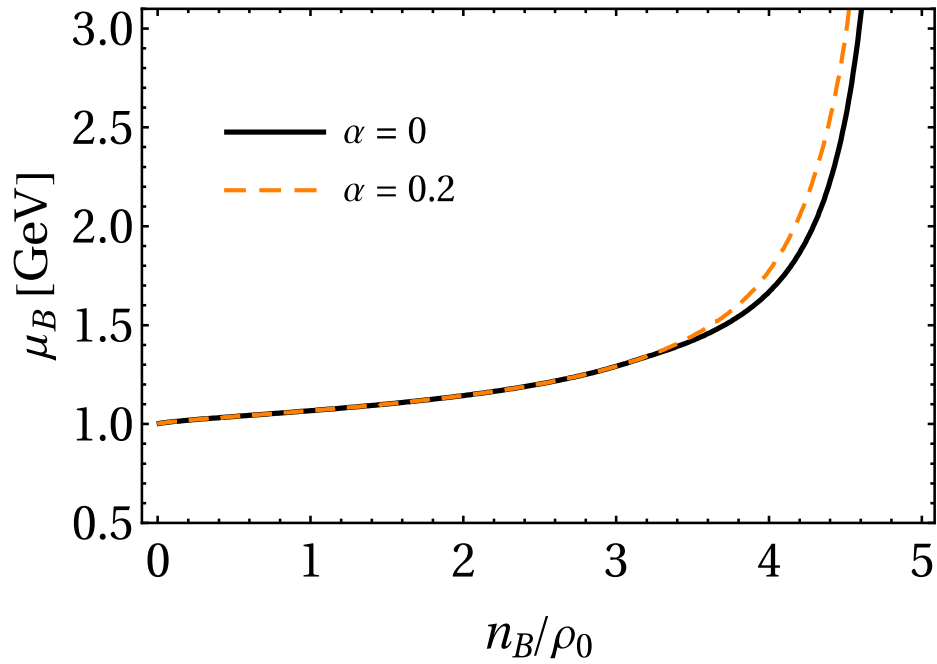




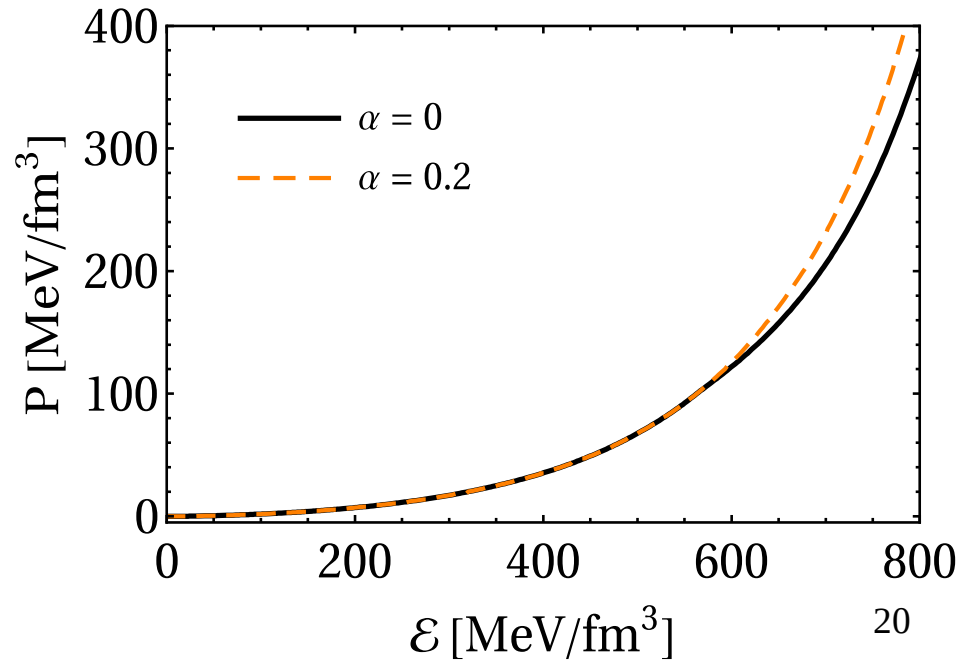
$$\mu_B = \sum_{f=p,n,\Lambda,e} \mu_f \frac{\partial n_f}{\partial n_B}$$



- Very soft EoS for low densities;
- Generate singular sound velocity



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→ Very soft EoS for low densities;

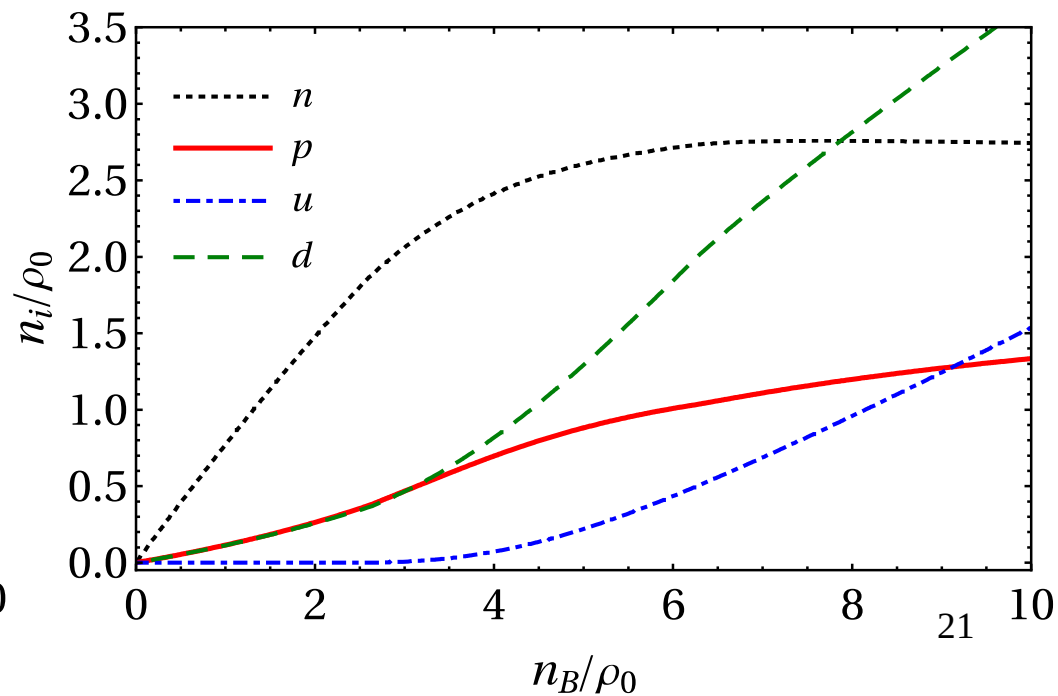
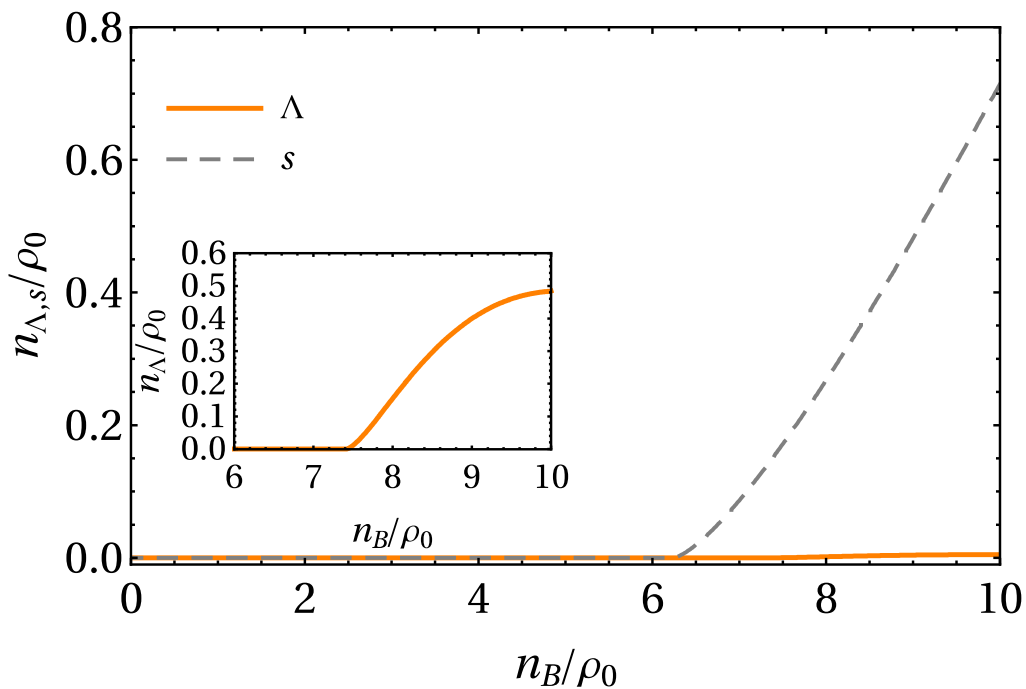
→ Generate singular sound velocity

$$n_N^{ex} = \sum_{i=p,n,\Lambda} \frac{n_N}{1 - n_{\tilde{B}}/n_0}$$

# Mean Field Mixture: $n_B = n_Q + n_N$ in each point

Protons + Neutrons + Hyperons in an excluded volume +  $u, d, s$  quarks.

$$\varepsilon_{\text{mix}} = \left(1 - \frac{n_{\tilde{B}}}{n_0}\right) \sum_{i=p,n,\Lambda} \int_0^{K_F^i} \frac{dk k^2}{\pi^2} \sqrt{k^2 + m_i^2} + \varepsilon_e + \sum_{j=u,d,s} \int_0^{k_F^{Qj}} \frac{dk k^2}{\pi^2} \sqrt{k^2 + m_j^2}$$



$$n_Q = 2 \sum_{j=u,d,s} \int_0^{k_F^{Qj}} \frac{dk k^2}{2\pi^2}$$

→ **Electromagnetic charge neutrality**

$$n_e = n_p + 2n_{\tilde{u}} - n_{\tilde{d}} - n_{\tilde{s}}$$

→ **Beta equilibrium conditions**

$$\mu_n = \mu_p + \mu_e$$

$$\mu_{\tilde{d}} = \mu_{\tilde{u}} + 3\mu_e$$

→ **Existence of  $\Lambda$  hyperon**

$$\mu_\Lambda = \mu_n$$

→ **Existence of s quark**

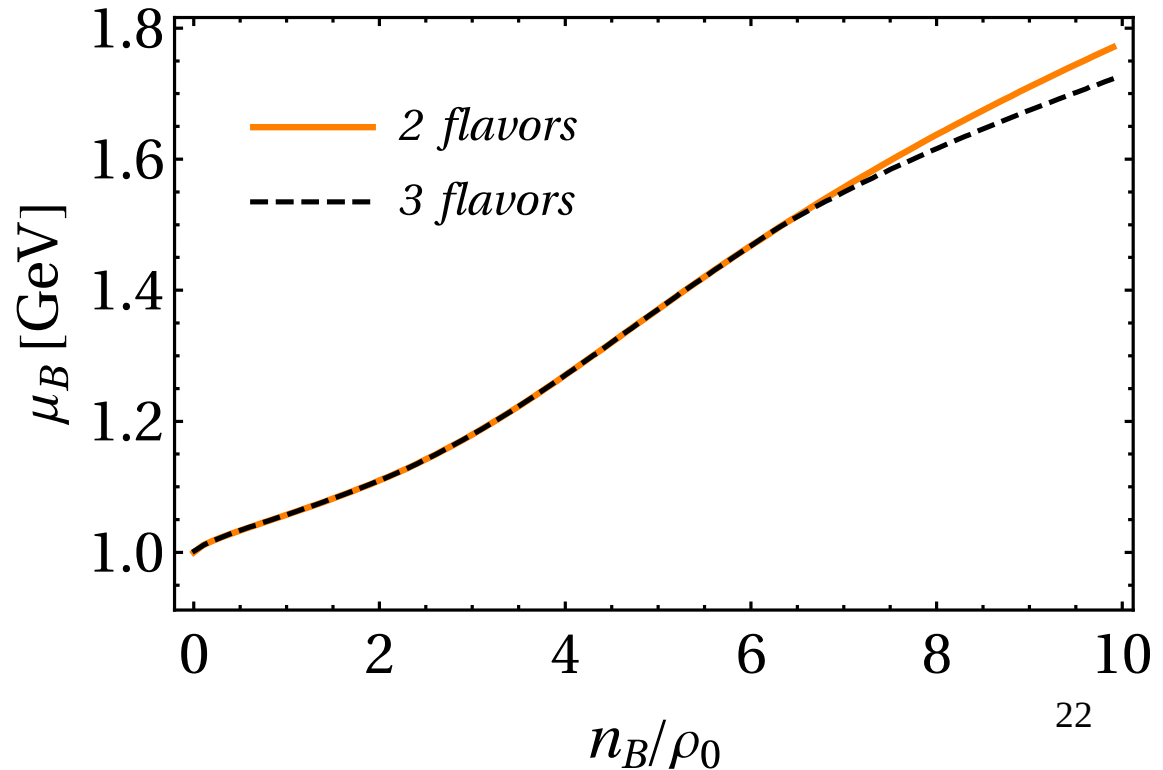
$$\mu_{\tilde{s}} = \mu_{\tilde{d}}$$

Condition for the minimum of energy density:

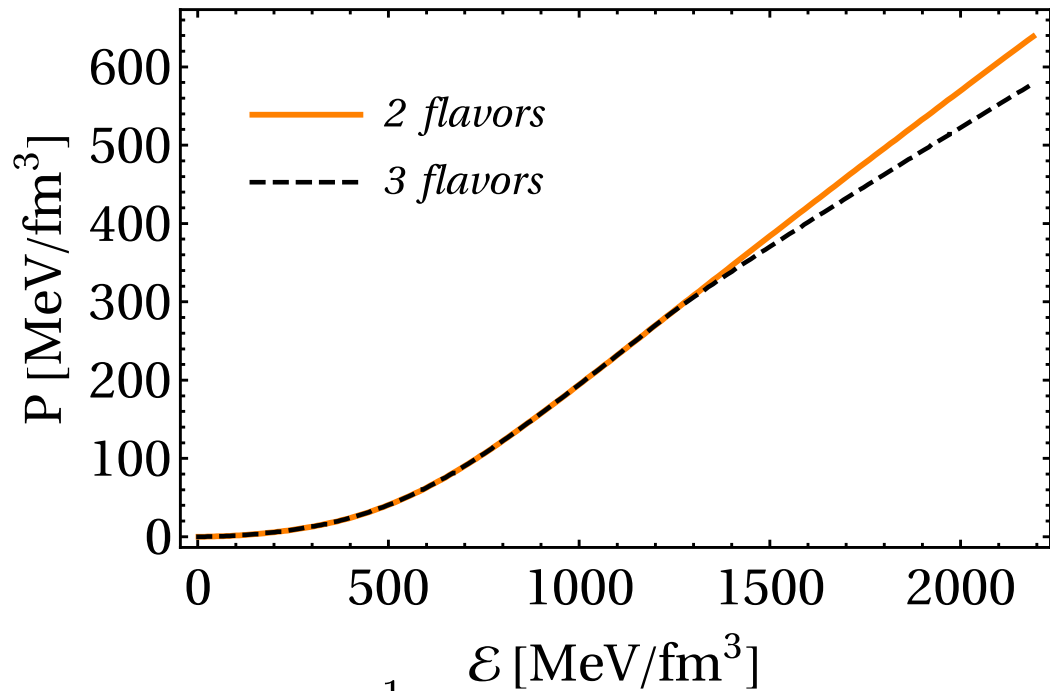
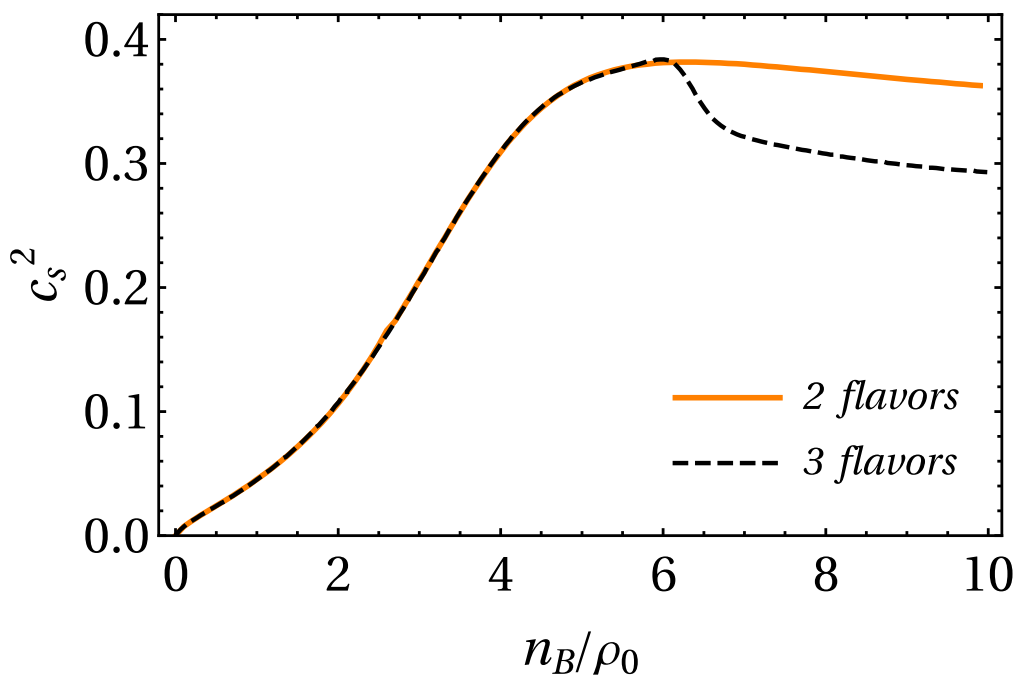
$$dn_B = dn_n + dn_Q = 0$$

from which we obtain

$$\mu_n = N_c \mu_{\tilde{d}} - \mu_e$$



# Hard-Soft EoS



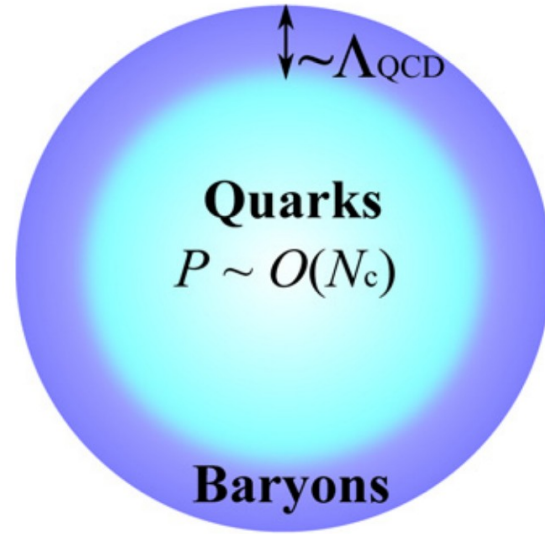
$$c_s^2 = \frac{\partial P}{\partial \mathcal{E}_{\text{mix}}} = \frac{n_B}{\mu_B} \left( \frac{\partial n_B}{\partial \mu_B} \right)^{-1}$$

# Final Remarks

- We study the baryon-quark mixture with three flavor particles in  $\beta$  equilibrium and considering electromagnetic charge neutrality.
- The mean field mixture generates a hard-soft EoS, and in the high density limit the 3-flavor EoS is softer.
- **Next step:** More realistic model, including the baryon-like shell structure that correctly describes quarkyonic matter (in progress).

$$\varepsilon_N = \left(1 - \frac{n_{\tilde{B}}}{n_0}\right) \sum_{i=p,n,\Lambda} \int_{K_F^i}^{K_F^i + \Delta_i} \frac{dk k^2}{\pi^2} \sqrt{k^2 + m_i^2} + \sum_{j=u,d,s} \int_0^{k_F^j} \frac{dk k^2}{\pi^2} \sqrt{k^2 + m_j^2}$$



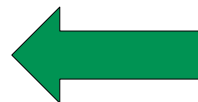
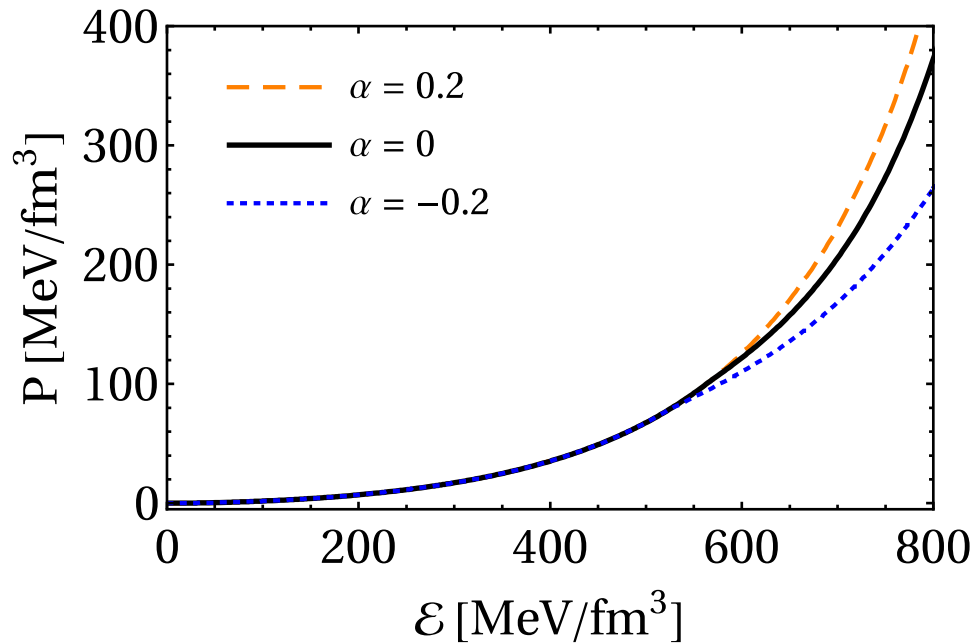
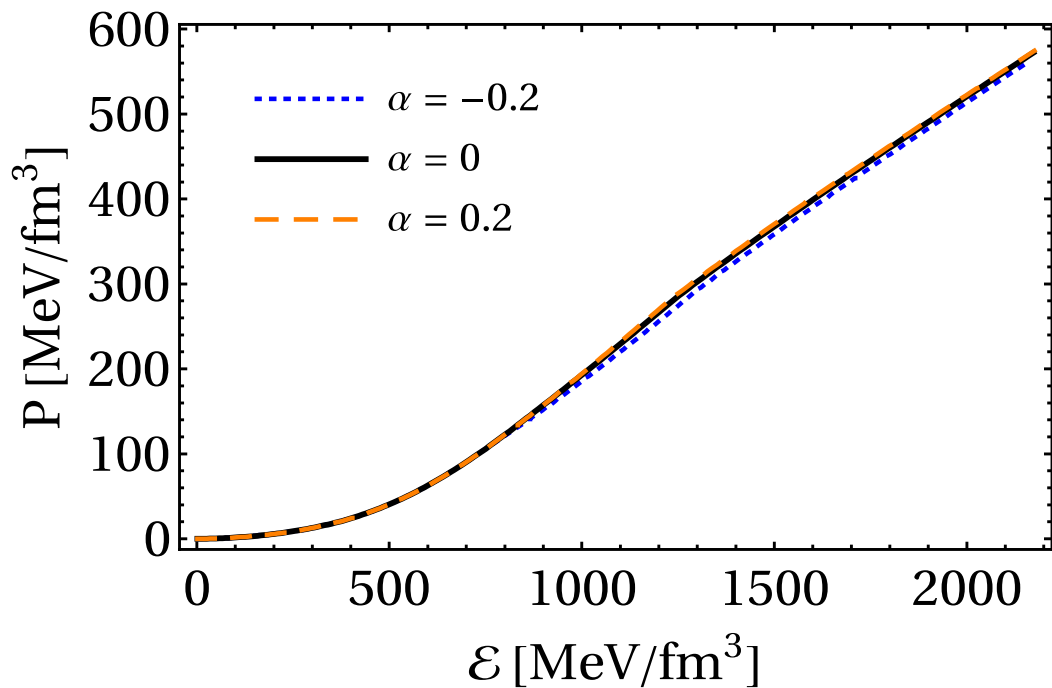


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**Thanks for your  
attention!**

Only Nucleons



Mean Field mixture