Hybrid star construction with the extended linear sigma model: preliminary results

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Overview

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Most of their times compact stars living at $T \approx 0$ MeV.

Investigation of compact stars can help to understand strongly interacting matter in medium and vice versa.

$\rightarrow$ Calculation of observable quantities: mass, radius, tidal deformability
Structure of compact stars

- Quark-hybrid star
- Hyperon star
- Strange star
- Neutron star with pion condensate
- Nucleon star

Absolutely stable strange quark matter

\[ \mu, u, d, s \]

\[ n, p, e, \mu \]

\[ M \sim 1.4 M_\odot \]

\[ R \sim 10 \text{ km} \]

Fig. from *F. Weber, J. Phys. G* 27, 465 (2001)
Various $M - R$ curves for different compact star EoS’s

- QCD directly unsolvable at finite density
- One can use effective models in the zero temperature finite density region
- Neutron star observations restrict such models [1,2]

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# Neutron star models (two flavours)

<table>
<thead>
<tr>
<th>EoS Name</th>
<th>Reference</th>
<th>( \rho &lt; \rho_{\text{ND}} )</th>
<th>( \rho_{\text{ND}} &lt; \rho &lt; \rho_0 )</th>
<th>( \rho_0 &lt; \rho )</th>
<th>Method</th>
<th>Comments</th>
</tr>
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<tbody>
<tr>
<td>BPS</td>
<td>Baym et al. (1971)</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>Empirical binding energies</td>
<td>Commonly used for low-density regime</td>
</tr>
<tr>
<td>NV</td>
<td>Negele &amp; Vauth. (1974)</td>
<td>X</td>
<td>Hartree-Fock</td>
<td>X</td>
<td>Variational</td>
<td>Used for intermediate densities</td>
</tr>
<tr>
<td>WFF (WFF1)</td>
<td>Wiringa et al. (1988)</td>
<td>X</td>
<td>X</td>
<td>np+eµ</td>
<td>Variational</td>
<td>A14+UVII</td>
</tr>
<tr>
<td>APR (AP4)</td>
<td>Akmal &amp; Pand. (1997)</td>
<td>X</td>
<td>X</td>
<td>np+eµ</td>
<td>Variational</td>
<td>A18+UIX*+δν</td>
</tr>
<tr>
<td>MPA (MPA1)</td>
<td>Müther et al. (1987)</td>
<td>X</td>
<td>X</td>
<td>Rel. Brueckner HF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ENG</td>
<td>Engvik et al. (1996)</td>
<td>X</td>
<td>X</td>
<td>Rel. Brueckner HF</td>
<td></td>
<td></td>
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<tr>
<td>PAL</td>
<td>Prakash et al. (1988)</td>
<td>X</td>
<td>X</td>
<td>np+e(μ?)</td>
<td>Schematic potential</td>
<td>Parameterizing E and S (symmetry energy)</td>
</tr>
<tr>
<td>GM</td>
<td>Glend. &amp; Moszk. (1991)</td>
<td>X</td>
<td>X</td>
<td>npH+eµ</td>
<td>Field theoretical</td>
<td>Same as the model in Glendenning’s book</td>
</tr>
<tr>
<td>H4</td>
<td>Lackey et al. (2006)</td>
<td>X</td>
<td>X</td>
<td>npH+eµ</td>
<td>Field theoretical</td>
<td>Stiffest EoS compatible with nuclear constraints</td>
</tr>
<tr>
<td>MS (MS1)</td>
<td>Müller &amp; Serot (1996)</td>
<td>X</td>
<td>X</td>
<td>np(+eµ?)</td>
<td>Field theoretical</td>
<td>Nonlinear mesonic potentials</td>
</tr>
<tr>
<td>SQM</td>
<td>Farhi &amp; Jaffe (1984)</td>
<td>uds+e</td>
<td></td>
<td></td>
<td>Bag model</td>
<td></td>
</tr>
</tbody>
</table>

Combinations of these models are used for the entire neutron star. **BPS, NV and APR** are commonly used together in astroph. appl.
Hybrid star models

**Hybrid stars**: Compact stars with quark matter in the core. Different approaches in the literature:

- BPS or BPS + NV at very low $\rho_B$
- Some nuclear model at low $\rho_B$ (2 or 3 flavour): Walecka model, Parity doublet model, Relativistic Mean-Field (RMF) models
- Quark matter at high $\rho_B$ (2 or 3 flavour): Nambu-Jona-Lasinio (NJL) model, Linear sigma model (LSM)

**How to combine models at low density with models at high density?**

$\rightarrow$ Various approaches exist: Quark-Meson-Nucleon model (QMN) with statistical confinement; Hadron-quark crossover with P-interpolation; Energy minimization method; Coexisting phases method; Gibbs construction; Maxwell construction
QMN with statistical confinement


Features of the model:

- Two flavour parity doublet model with mirror assignment ($N(938)$, $N(1500)$, $\pi(138)$, $f_0(500)$ (or $\sigma$), $\omega(782)$, $\rho(770)$)
- Linear sigma model ($u, d$ constituent quarks, $\pi(138)$, $f_0(500)$), quarks are not coupled to vectors
- Tree-level mesons, one-loop fermions (mean-field approximation)

Grand canonical potential:

$$\Omega = \sum_{x \in (p^\pm, n^\pm, u, d)} \Omega_x + V_\sigma + V_\omega + V_b + V_\rho$$

$$\Omega_x = \gamma_x \int \frac{d^3p}{(2\pi)^3} T \left[ \ln \left( 1 - n_x \right) + \ln \left( 1 - \bar{n}_x \right) \right].$$
Statistical confinement with auxiliary field

Concatenation at the level of the grand potential. *Nucleons* have to be suppressed at high $\rho_B$, while *quarks* at low $\rho_B$

$\implies$ Modified Fermi-Dirac distributions:

\[
\begin{align*}
n_\pm &= \theta (\alpha^2 b^2 - p^2) \, f_\pm \\
\bar{n}_\pm &= \theta (\alpha^2 b^2 - p^2) \, \bar{f}_\pm \\
n_q &= \theta (p^2 - b^2) \, f_q \\
\bar{n}_q &= \theta (p^2 - b^2) \, \bar{f}_q
\end{align*}
\]

$b$ is a $T$ and $\mu_B$ dependent bag field with $\langle b \rangle = b_0$

$b$ might be associated with chromoelectric part of the gluon sector
Chiral phase transition is shown, deconfinement starts at very large $\rho_B$; Other EoSs are shown for comparison

Fig. from *M. Marczenko et al., Phys. Rev. D, 98, 103021 (2018)*
Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein’s equation for spherically symmetric case and homogeneous matter → TOV eqs.:

\[
\frac{dp}{dr} = - \left[ p(r) + \varepsilon(r) \right] \frac{[M(r) + 4\pi r^3 p(r)]}{r[r - 2M(r)]}
\]  

with

\[
\frac{dM}{dr} = 4\pi r^2 \varepsilon(r)
\]

These are integrated numerically for a specific \( p(\varepsilon) \)

- For a fixed \( \varepsilon_c \) central energy density Eq. (1) is integrated until \( p = 0 \)
- Varying \( \varepsilon_c \) a series of compact stars is obtained (with given \( M \) and \( R \))
- Once the maximal mass is reached, the stable series of compact stars ends
$M - R$ curves for the QMN model

Fig. from M. Marczenko et al., Phys. Rev. D, 98, 103021 (2018)

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In the crossover region hadrons starts to overlap → both low and high $\rho_B$ models loose their validity. Gibbs condition (extrapolation from the dashed lines) can be misleading.
Features of the model:

- **H-EOS**: Three flavour hadronic EoSs with $Y$-mixing:
  - TNI2, TNI3, TNI2u, TNI3u, AV18+TBF, SCL3ΛΣ
- **Q-EOS**: NJL-model with $u,d,s$ quarks and vector interaction
- **mean-field approximation**

**$P$-interpolation ($\rho = \rho_B$):**

\[
P(\rho) = P_H(\rho)f_-(\rho) + P_Q(\rho)f_+(\rho), \tag{2}
\]

\[
f_{\pm}(\rho) = \frac{1}{2} \left( 1 \pm \tanh \left( \frac{\rho - \bar{\rho}}{\Gamma} \right) \right) \tag{3}
\]

\[
\varepsilon(\rho) = \varepsilon_H(\rho)f_-(\rho) + \varepsilon_Q(\rho)f_+(\rho) + \Delta \varepsilon \tag{4}
\]

\[
\Delta \varepsilon = \rho \int_{\bar{\rho}}^{\rho} \left( \varepsilon_H(\rho') - \varepsilon_Q(\rho') \right) \frac{g(\rho')}{\rho'} d\rho' \tag{5}
\]
$P(\rho)$ for hadronic matter for diff. models

Fig. from *K. Masuda et al., PTEP 2013 073D01*
$P(\rho)$ for pure quark matter for diff. $g_V$.

Fig. from K. Masuda et al., PTEP 2013 073D01
Interpolated pressure

Fig. from *K. Masuda et al., PTEP 2013 073D01*
Hyperons make the EoS softer $\rightarrow 2M_\odot$ limit not reached. On the left fig. $g_V/G_S = 1$

Fig. from K. Masuda et al., PTEP 2013 073D01
Energy minimization (EM) method

based on: \textit{X. H. Wu et al., PRC 99, 065802 (2019)}

Features of the model:

\begin{itemize}
  \item Hadronic matter: Relativistic mean-field (RMF) model (2 flavours, nucleons interacting through $\sigma$, $\omega$ and $\rho$ mesons, mesons treated at tree-level, additional $\omega - \rho$ interaction)
  \item Quark matter: NJL-model with $u$, $d$, $s$ quarks and vector interaction
  \item mean-field approximation
\end{itemize}

Total energy of the mixed phase:

$$
\epsilon_{\text{MP}} = u\epsilon_{\text{QP}} + (1-u)\epsilon_{\text{HP}} + \epsilon_{\text{surf}} + \epsilon_{\text{Coul}} \quad (6)
$$

$$
u = V_{\text{QP}}/(V_{\text{QP}} + V_{\text{HP}}) \quad (7)
$$

minimization w.r.t. the densities ($n_p$, $n_n$, $n_u$, $n_d$, $n_s$, $n_e$, $n_{\mu}$) and $u$ gives equilibrium conditions (under global charge neutrality and baryon number conservation)
Special cases of the EM method

- Coexisting phases method: $\varepsilon_{\text{surf}}$ and $\varepsilon_{\text{Coul}}$ are treated perturbatively (minimization or Gibbs condition without surf. and Coul. terms) $\Rightarrow P_{\text{HP}} = P_{\text{QP}}$, and
  $$\varepsilon_{\text{MP}} = u\varepsilon_{\text{QP}} + (1-u)\varepsilon_{\text{HP}} + \varepsilon_{\text{surf}} + \varepsilon_{\text{Coul}}$$

- Gibbs construction: $\varepsilon_{\text{surf}}$ and $\varepsilon_{\text{Coul}}$ are neglected, $\sigma \approx 0$, global charge neutrality, hadronic and quark phases can be charged separately, $\Rightarrow P_{\text{HP}} = P_{\text{QP}}$, and
  $$\varepsilon_{\text{MP}} = u\varepsilon_{\text{QP}} + (1-u)\varepsilon_{\text{HP}}$$

- Maxwell construction: $\varepsilon_{\text{surf}}$ and $\varepsilon_{\text{Coul}}$ are neglected, $\sigma >> 0$, local charge neutrality, both hadronic and quark phases charge neutral $\Rightarrow P_{\text{HP}} = P_{\text{QP}}$, and
  $$\varepsilon_{\text{MP}} = u\varepsilon_{\text{QP}} + (1-u)\varepsilon_{\text{HP}}$$
Pressure with the EM method

Fig. from X. H. Wu et al., PRC 99, 065802 (2019)
$M - R$ curves with the EM method

Fig. from X. H. Wu et al., PRC 99, 065802 (2019)

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**$M - R$ curves with the EM method**

![Graph showing $M - R$ curves for different values of $L$.](image)

Fig. from *X. H. Wu et al., PRC 99, 065802 (2019)*
Introduction
Hybrid stars
Hybrid star in eLSM
Conclusion

(Axial)vector meson extended linear $\sigma$ model (eLSM)
The Equation of State (EOS)
$M - R$ relations of the pure eLSM

Lagrangian of the eLSM


\[ \mathcal{L} = \text{Tr}[(D_\mu \Phi)\dagger (D_\mu \Phi)] - m_0^2 \text{Tr}(\Phi\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi\dagger \Phi)^2 \\
+ c_1 (\det \Phi + \det \Phi\dagger) + \text{Tr}[H(\Phi + \Phi\dagger)] - \frac{1}{4} \text{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2) \\
+ \text{Tr} \left[ \left( \frac{m_1^2}{2} \mathbb{1} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + i \frac{g_2}{2} \left( \text{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \text{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\} \right) \\
+ \frac{h_1}{2} \text{Tr}(\Phi\dagger \Phi)\text{Tr}(L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[(L_\mu \Phi)^2 + (\Phi R_\mu)^2] + 2h_3 \text{Tr}(L_\mu \Phi R_\mu \Phi\dagger) \\
+ \bar{\Psi} i\gamma_\mu D^\mu \Psi - g_F \bar{\Psi} (\Phi_S + i\gamma_5 \Phi_{PS}) \Psi, \]

\begin{align*}
D^\mu \Phi &= \partial^\mu \Phi - ig_1 (L^\mu \Phi - \Phi R^\mu) - ieA_\mu^\mu [T_3, \Phi], \\
L^{\mu\nu} &= \partial^\mu L^{\nu} - ieA_\mu^\mu [T_3, L^{\nu}] - \{ \partial^\nu L^\mu - ieA_\nu^\nu [T_3, L^\mu] \}, \\
R^{\mu\nu} &= \partial^\mu R^{\nu} - ieA_\mu^\mu [T_3, R^{\nu}] - \{ \partial^\nu R^\mu - ieA_\nu^\nu [T_3, R^\mu] \}, \\
D^\mu \Psi &= \partial^\mu \Psi - iG^\mu \Psi, \quad \text{with} \quad G^\mu = g_s G_\mu^a T_a.
\end{align*}

+ Polyakov loop potential (for $T > 0$)
Determination of the parameters

14 unknown parameters \((m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F)\) \(\rightarrow\) determined by the \text{min. of} \(\chi^2:\)

\[
\chi^2(x_1, \ldots, x_N) = \sum_{i=1}^{M} \left[ \frac{Q_i(x_1, \ldots, x_N) - Q_i^{\text{exp}}}{\delta Q_i} \right]^2,
\]

\((x_1, \ldots, x_N) = (m_0, \lambda_1, \lambda_2, \ldots), \ Q_i(x_1, \ldots, x_N) \rightarrow\) from the model, \(Q_i^{\text{exp}} \rightarrow\) PDG value, \(\delta Q_i = \max\{5\%, \text{PDG value}\}\)
multiparametric minimalization \(\rightarrow\) MINUIT

- \(\text{PCAC} \rightarrow\) 2 physical quantities: \(f_\pi, f_K\)
- \(\text{Curvature masses} \rightarrow\) 16 physical quantities:

\[
m_u/d, m_s, m_\pi, m_\eta, m_\eta', m_K, m_\rho, m_\Phi, m_{K^*}, m_{a_1}, m_{f_1^H}, m_{K_1},
m_{a_0}, m_{K_S}, m_{f_0^L}, m_{f_0^H}\]

- \(\text{Decay widths} \rightarrow\) 12 physical quantities:

\[
\Gamma_{\rho \rightarrow \pi\pi}, \Gamma_{\Phi \rightarrow KK}, \Gamma_{K^* \rightarrow K\pi}, \Gamma_{a_1 \rightarrow \pi\gamma}, \Gamma_{a_1 \rightarrow \rho\pi}, \Gamma_{f_1 \rightarrow KK^*}, \Gamma_{a_0}, \Gamma_{K_S \rightarrow K\pi},
\Gamma_{f_0^L \rightarrow \pi\pi}, \Gamma_{f_0^L \rightarrow KK}, \Gamma_{f_0^H \rightarrow \pi\pi}, \Gamma_{f_0^H \rightarrow KK}\]

- \(\text{Pseudocritical temperature} \ T_c \text{ at } \mu_B = 0\)

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Features of our approach

- **D.O.F’s:**
  - scalar, pseudoscalar, vector, and axial-vector nonets
  - $u$, $d$, $s$ constituent quarks ($m_u = m_d$)
  - Polyakov loop variables $\Phi$, $\bar{\Phi}$ with $U_{YM}^{\log}$ or $U_{log}$

- **no mesonic fluctuations**, only fermionic ones

\[ Z = e^{-\beta \Omega(T, \mu_q)} = \int_{PBC} \prod_a D\xi_a \int_{APBC} \prod_f Dq_f Dq_f^\dagger \exp \left[ - \int_0^\beta d\tau \int_V d^3 x \left( L + \mu_q \sum_f q_f^\dagger q_f \right) \right] \]

approximated as $\Omega(T, \mu_q) = U_{meson}^{\text{tree}}(\langle M \rangle) + \Omega_{qq}^{(0)}(T, \mu_q) + U_{log}(\Phi, \bar{\Phi})$; $\bar{\mu}_q = \mu_q - iG_4$

\[ e^{-\beta \Omega_{qq}^{(0)}} = \int_{APBC} \prod_{f,g} Dq_g Dq_f^\dagger \exp \left\{ \int_0^\beta d\tau \int_x q_f^\dagger \left[ (i\gamma_0 \vec{\nabla} - \frac{\partial}{\partial \tau} + \bar{\mu}_q) \right] \delta_{fg} - \gamma_0 \mathcal{M}_{fg}|_{\xi_a=0} \right\} q_g \]

- **tree-level (axial)vector masses**

- **fermionic vacuum** and **thermal** fluctuations included in the (pseudo)scalar curvature masses used to parameterize the model

- **4 coupled** $T/\mu_B$-dependent field equations for the condensates $\phi_N, \phi_S, \Phi, \bar{\Phi}$

- **thermal contribution of** $\pi, K, f_0^L$ included in the pressure, however their curvature mass contains no mesonic fluctuations
Inclusion of vector meson Yukawa term

\[ \mathcal{L}_{\text{Yukawa-vec}} = -g_v \sqrt{6} \bar{\psi} \gamma_\mu V_0^\mu \psi \]

\[ V_0^\mu = \frac{1}{\sqrt{6}} \text{diag}(v_0 + \frac{v_8}{\sqrt{2}}, v_0 + \frac{v_8}{\sqrt{2}}, v_0 - \sqrt{2}v_8) \]

mean-field treatment

\[ < v_0^\mu > = v_0 \delta^{0\mu}, \quad < v_8^\mu > = 0^\mu \]

Modification of the grand canonical potential

\[ \Omega(T = 0, \mu_q) \rightarrow \Omega(T = 0, \tilde{\mu}_q) - \frac{1}{2} m_v^2 v_0^2, \quad \text{with} \quad \tilde{\mu}_q = \mu_q - g_v v_0 \]

While the field equations

\[ \left. \frac{\partial \Omega}{\partial \phi_N} \right|_{\phi_N = \bar{\phi}_N} = \left. \frac{\partial \Omega}{\partial \phi_S} \right|_{\phi_S = \bar{\phi}_S} = 0 \quad \text{and} \quad \left. \frac{\partial \Omega}{\partial v_0} \right|_{v_0 = \bar{v}_0} = 0, \]
The EOS of the pure eLSM

- Pure eLSM compared to Walecka and free quark models
- At low energies the EoS of the eLSM is close to the EoS of the Walecka-model
- At higher energies it tends to the EoS of the free quark model
- note: Walecka Int means
  \[ \mathcal{L}_{W, \text{Int}} = -\frac{b}{3} m_n (g_\sigma \sigma)^3 - \frac{c}{4} (g_\sigma \sigma)^4 \]

- In the Walecka models electrons and \( \rho \) mesons are also included (dashed-dotted lines)
$M - R$ relations of the pure eLSM (quark star)

- eLSM, $g_v = 0$
- eLSM, $g_v = 2$
- Free quark
- Walecka
- Walecka (e, $\rho$), BPS
- Walecka (Int)
- Walecka (Int, e, $\rho$), BPS

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Changing the $g_v$ vector coupling in pure ELSM

For $g_v \gtrsim 2.5$ $p(\varepsilon)$ becomes zero at $\varepsilon = 0 \Rightarrow$ small $M$, large $R$
**EOS of the hybrid BPS+TNI3u+eLSM model**

![Graph showing the equation of state (EoS) for the hybrid BPS+TNI3u+eLSM model. The graph compares hadronic EoS, quark EoS, and P-interpolated EoS.]
**M – R relations of the BPS+TNI3u+eLSM**

![Graph showing M – R relations](image)

- **GV=0, \( \bar{\rho}=3\rho_0 \)**
- **GV=1.5, \( \bar{\rho}=3\rho_0 \)**
- **GV=2.0, \( \bar{\rho}=3\rho_0 \)**
- **GV=2.5, \( \bar{\rho}=3\rho_0 \)**
- **GV=2.0, \( \bar{\rho}=5\rho_0 \)**
- **GV=2.5, \( \bar{\rho}=5\rho_0 \)**
- **GV=2.0, \( \bar{\rho}=2\rho_0 \)**
- **GV=2.5, \( \bar{\rho}=2\rho_0 \)**

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Conclusion

- Current astrophys. restrictions (like $2M_\odot$ limit) can be fulfilled in many ways
- Precise radius measurements can select models
- The quark–vector meson Yukawa interactions are very important
- Reasonable hybrid stars can be constructed with eLSM, however even pure quark stars are not excluded currently

Plans

- Other hybrid star constructions with eLSM
- Better (more consistent) approximations in the eLSM part
- Inclusion of the total vector-quark Yukawa term, consistent treatment
- Beyond mean-field calculations
- Tidal deformability (in progress)
Thank you for your attention!
Particle content

- **Vector** and **Axial-vector** meson nonets

\[
V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{\omega_N + \rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\
\rho^- & \frac{\omega_N - \rho^0}{\sqrt{2}} & K^{*0} \\
K^{*-} & \omega_S & 0
\end{pmatrix}^\mu
\]

\[
A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{f_{1N} + a_0^0}{\sqrt{2}} & a_1^+ & K_1^+ \\
a_1^- & \frac{f_{1N} - a_1^0}{\sqrt{2}} & K_0^0 \\
K_1^- & \frac{1}{\sqrt{2}} & f_{1S}
\end{pmatrix}^\mu
\]

\[\rho \to \rho(770), K^* \to K^*(894)\]
\[\omega_N \to \omega(782), \omega_S \to \phi(1020)\]

\[a_1 \to a_1(1230), K_1 \to K_1(1270)\]
\[f_{1N} \to f_1(1280), f_{1S} \to f_1(1426)\]

- **Scalar** \((\sim \bar{q}_i q_j)\) and **pseudoscalar** \((\sim \bar{q}_i \gamma_5 q_j)\) meson nonets

\[
\Phi_S = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{\sigma_N + a_0^0}{\sqrt{2}} & a_0^+ & K_0^{*+} \\
a_0^- & \frac{\sigma_N - a_0^0}{\sqrt{2}} & K_0^{*0} \\
K_0^{*-} & \sigma_S & 0
\end{pmatrix}
\]

\[
\Phi_{PS} = \frac{1}{\sqrt{2}} \begin{pmatrix}
\frac{\eta_N + \pi^0}{\sqrt{2}} & \pi^+ & K^+ \\
\pi^- & \frac{\eta_N - \pi^0}{\sqrt{2}} & K^0 \\
K^- & \frac{1}{\sqrt{2}} & \eta_S
\end{pmatrix}
\]

unknown assignment
mixing in the \(\sigma_N - \sigma_S\) sector

\[\pi \to \pi(138), K \to K(495)\]

mixing: \(\eta_N, \eta_S \to \eta(548), \eta'(958)\)

Spontaneous symmetry breaking: \(\sigma_{N/S}\) acquire nonzero expectation values \(\phi_{N/S}\)
fields shifted by their expectation value: \(\sigma_{N/S} \to \sigma_{N/S} + \phi_{N/S}\)