Hybrid star construction with the extended linear sigma model: preliminary results

Péter Kovács
Senior Research Fellow
Wigner RCP

Collaborator:
János Takátsy

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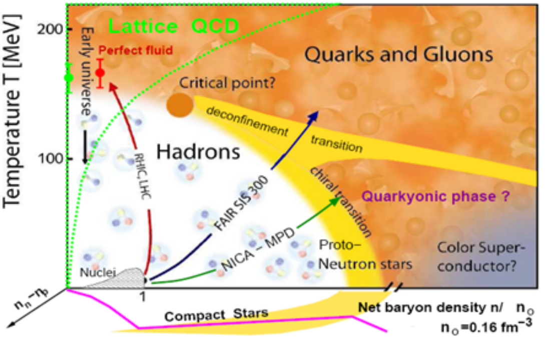
Overview

Envisaged phase diagram of QCD



Most of their times compact stars living at $T \approx 0$ MeV.

Investigation of compact stars can help to understand strongly interacting matter in medium and vice versa.

$\longrightarrow$ Calculation of observable quantities: mass, radius, tidal deformability

Structure of compact stars

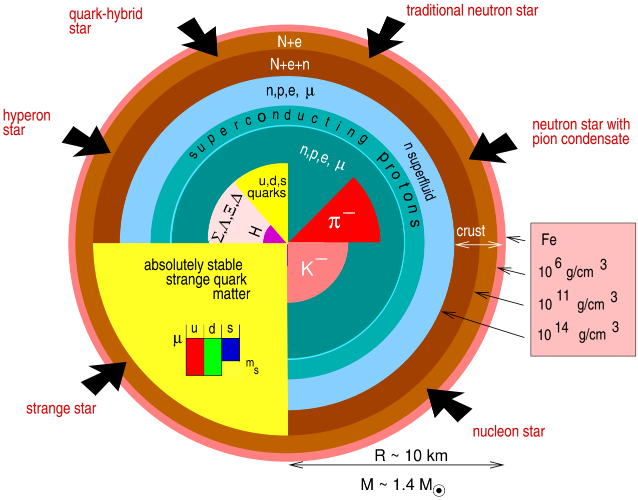
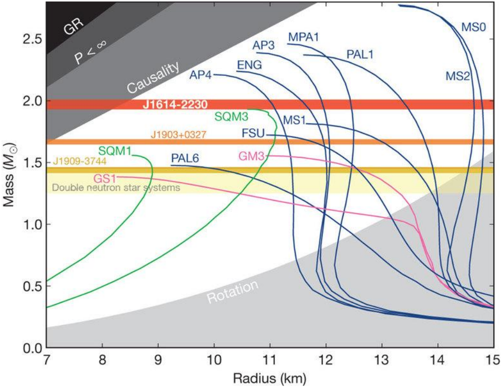


Fig. from *F. Weber, J. Phys. G* **27**, *465 (2001)*

Various $M - R$ curves for different compact star EoS's

- QCD directly unsolvable at finite density
- One can use effective models in the zero temperature finite density region
- Neutron star observations restrict such models [1,2]



$M - R$ relations from various models [1]

[1] *P. Demorest et al. (2010), Nature* **467**, *1081*
[2] *J. Antoniadis et al. (2013), Science* **340**, *6131*

Neutron star models (two flavours)

EoS Name	Reference	$\rho < \rho_{ND}$	$\rho_{ND} < \rho < \rho_0$	$\rho_0 < \rho$	Method	Comments
BPS	Baym et al. (1971)	B.En.+El.+ Lattice	X	X	Empirical binding energies	Commonly used for low-density regime
HP (HP94)	Haensel & Pichon (1994)	B.En.+El.+ Lattice	X	X	Empirical binding energies	Includes electron screening
NV	Negele & Vauth. (1974)	X	Hartree-Fock	X	Variational	Used for intermediate densities
FPS	Lorenz (1991)	CLDM	CLDM	np+eμ	Density functional	Unified EoS with FP Skyrme potential
Sly	Douchin & Haensel (2001)	CLDM	CLDM	np+eμ	Density functional	Unified EoS with Skyrme Lyon potential
WFF (WFF1)	Wiringa et al. (1988)	X	X	np+eμ	Variational	A14+UVII
APR (AP4)	Akmal & Pand. (1997)	X	X	np+eμ	Variational	A18+UIX*+δv
MPA (MPA1)	Müther et al. (1987)	X	X		Rel. Brueckner HF	
ENG	Engvik et al. (1996)	X	X		Rel. Brueckner HF	
PAL	Prakash et al. (1988)	X	X	np+e(μ?)	Schematic potential	Parameterizing E and S (symmetry energy)
GM	Glend. & Moszk. (1991)	X	X	npH+eμ	Field theoretical	Same as the model in Glendenning's book
H4	Lackey et al. (2006)	X	X	npH+eμ	Field theoretical	Stiffest EoS compatible with nuclear constraints
MS (MS1)	Müller & Serot (1996)	X	X	np(+eμ?)	Field theoretical	Nonlinear mesonic potentials
SQM	Farhi & Jaffe (1984)	uds+e			Bag model	

Combinations of these models are used for the entire neutron star.

BPS, NV and APR are commonly used together in astroph. appl.

Hybrid star models

Hybrid stars: Compact stars with quark matter in the core.
Different approaches in the literature:

- BPS or BPS + NV at very low ρ_B
- Some nuclear model at low ρ_B (2 or 3 flavour): Walecka model, Parity doublet model, Relativistic Mean-Field (RMF) models
- Quark matter at high ρ_B (2 or 3 flavour): Nambu-Jona-Lasinio (NJL) model, Linear sigma model (LSM)

How to combine models at low density with models at high density?

$\longrightarrow$ Various approaches exist: Quark-Meson-Nucleon model (QMN) with statistical confinement; Hadron-quark crossover with P-interpolation; Energy minimization method; Coexisting phases method; Gibbs construction; Maxwell construction

QMN with statistical confinement

based on: *S. Benić et al., Phys. Rev. D*, **91**, *125034 (2015)*;
M. Marczenko et al., Phys. Rev. D, **98**, *103021 (2018)*

Features of the model:

- Two flavour parity doublet model with mirror assignment ($N(938)$, $N(1500)$, $\pi(138)$, $f_0(500)$ (or σ), $\omega(782)$, $\rho(770)$)
- Linear sigma model (u, d constituent quarks, $\pi(138)$, $f_0(500)$), quarks are not coupled to vectors
- Tree-level mesons, one-loop fermions (mean-field approximation)

Grand canonical potential:

$$\Omega = \sum_{x\in(p_\pm,n_\pm,u,d)} \Omega_x + V_\sigma + V_\omega + V_b + V_\rho$$

$$\Omega_x = \gamma_x \int \frac{\mathbf{d}^3 p}{(2\pi)^3} T \left[\ln\left(1 - n_x\right) + \ln\left(1 - \bar{n}_x\right)\right].$$

Statistical confinement with auxiliary field

Concatenation at the level of the grand potential. Nucleons have to be suppressed at high ρ_B, while quarks at low ρ_B

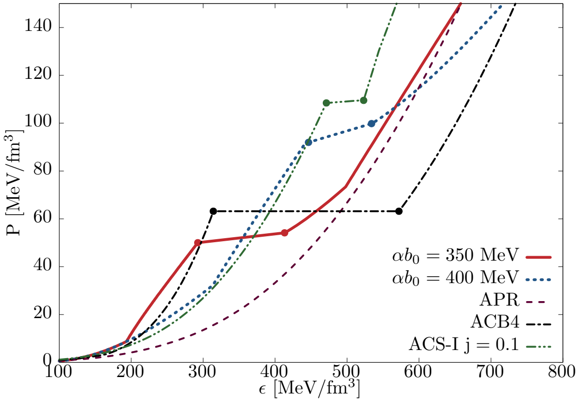
$\Longrightarrow$ Modified Fermi-Dirac distributions:

$$\begin{aligned} n_\pm &= \theta\left(\alpha^2 b^2 - \boldsymbol{p}^2\right) f_\pm \\ \bar{n}_\pm &= \theta\left(\alpha^2 b^2 - \boldsymbol{p}^2\right) \bar{f}_\pm \\ n_q &= \theta\left(\boldsymbol{p}^2 - b^2\right) f_q \\ \bar{n}_q &= \theta\left(\boldsymbol{p}^2 - b^2\right) \bar{f}_q \end{aligned}$$

b is a T and μ_B dependent bag field with $\langle b \rangle = b_0$

b might be associated with chromoelectric part of the gluon sector

EoSs for the QMN model



Chiral phase transition is shown, deconfinement starts at very large ρ_B; Other EoSs are shown for comparison

Fig. from *M. Marczenko et al., Phys. Rev. D,* **98**, *103021 (2018)*

Tolman-Oppenheimer-Volkoff (TOV) equation

Solving the Einstein's equation for spherically symmetric case and homogeneous matter $\longrightarrow$ TOV eqs.:

$$\frac{\mathrm{d}p}{\mathrm{d}r} = -\frac{[p(r)+\varepsilon(r)]\,[M(r)+4\pi r^3 p(r)]}{r[r-2M(r)]} \quad (1)$$

with

$$\frac{\mathrm{d}M}{\mathrm{d}r} = 4\pi r^2 \varepsilon(r)$$

These are integrated numerically for a specific $p(\varepsilon)$

- For a fixed ε_c central energy density Eq. (1) is integrated until $p=0$
- Varying ε_c a series of compact stars is obtained (with given M and R)
- Once the maximal mass is reached, the stable series of compact stars ends

$M - R$ curves for the QMN model

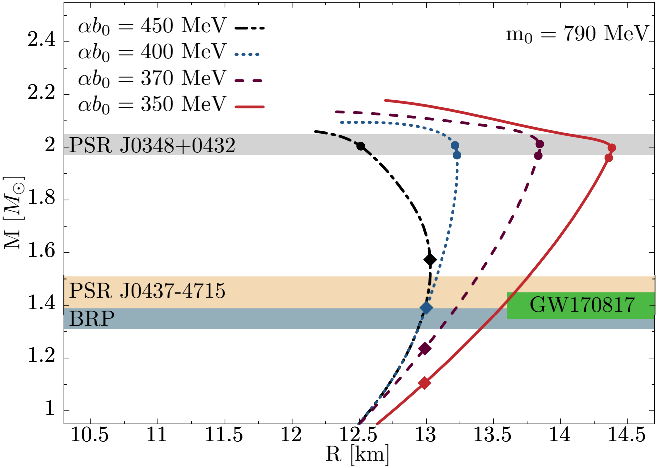
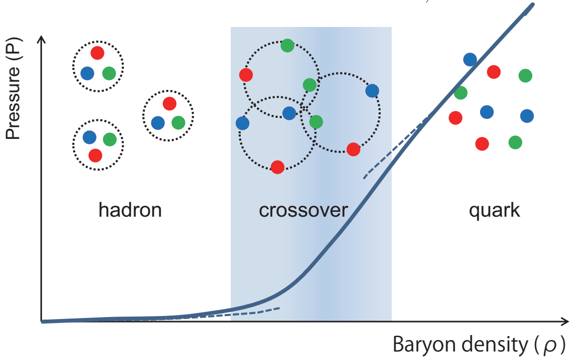


Fig. from *M. Marczenko et al., Phys. Rev. D,* **98**, *103021 (2018)*

Schematic picture of pressure (H-Q crossover)

based on: *K. Masuda et al., PTEP* **2013** *073D01*



In the crossover region hadrons starts to overlap
$\longrightarrow$ both low and high ρ_B models loose their validity.
Gibbs condition (extrapolation from the dashed lines) can be misleading.

Hadron-quark crossover with P-interpolation

Features of the model:

- H-EOS: Three flavour hadronic EoSs with Y-mixing: TNI2, TNI3, TNI2u, TNI3u, AV18+TBF, SCL3$\Lambda\Sigma$
- Q-EOS: NJL-model with u, d, s quarks and vector interaction
- mean-field approximation

P-interpolation ($\rho = \rho_B$):

$$P(\rho) = P_H(\rho) f_-(\rho) + P_Q(\rho) f_+(\rho), \tag{2}$$

$$f_\pm(\rho) = \frac{1}{2}\left(1 \pm \tanh\left(\frac{\rho - \bar{\rho}}{\Gamma}\right)\right) \tag{3}$$

$$\varepsilon(\rho) = \varepsilon_H(\rho) f_-(\rho) + \varepsilon_Q(\rho) f_+(\rho) + \Delta\varepsilon \tag{4}$$

$$\Delta\varepsilon = \rho \int_{\bar{\rho}}^{\rho} (\varepsilon_H(\rho') - \varepsilon_Q(\rho')) \frac{g(\rho')}{\rho'} d\rho' \tag{5}$$

$P(\rho)$ for hadronic matter for diff. models

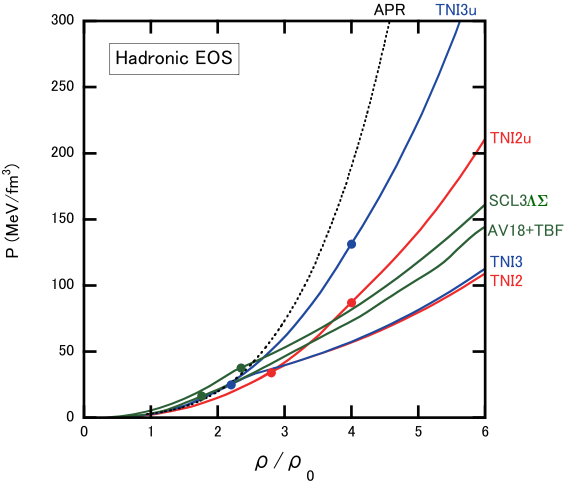


Fig. from *K. Masuda et al., PTEP* **2013** *073D01*

$P(\rho)$ for pure quark matter for diff. g_Vs

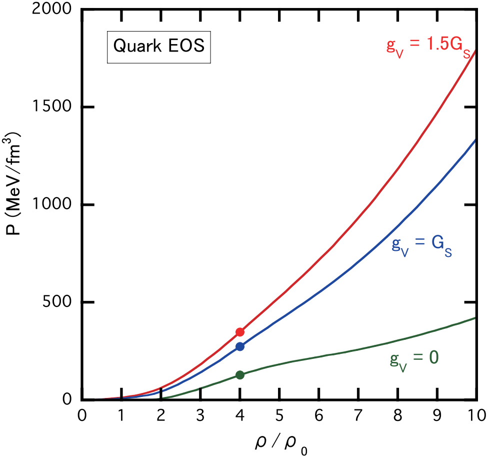


Fig. from *K. Masuda et al., PTEP* **2013** *073D01*

Interpolated pressure

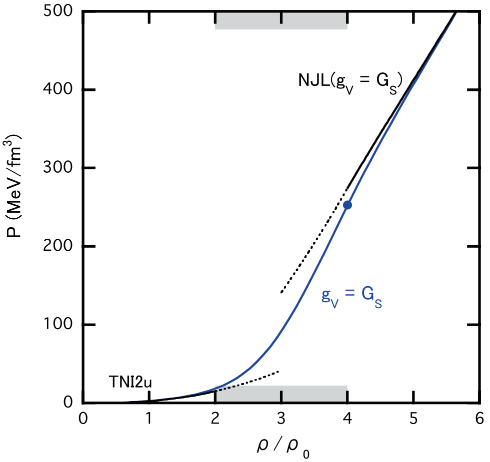
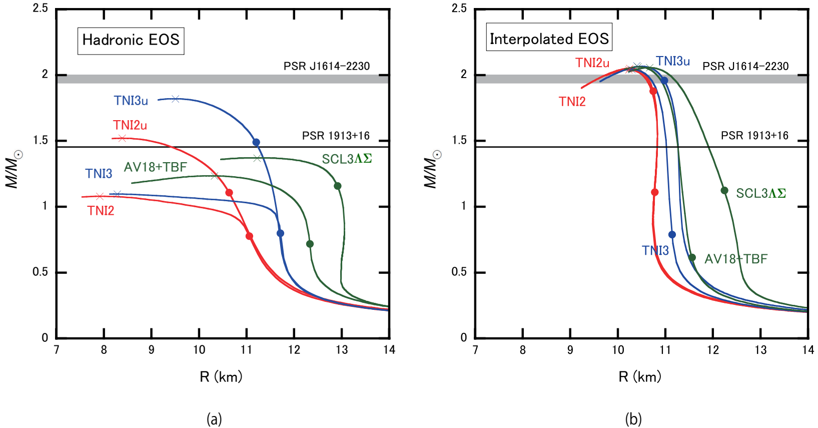


Fig. from *K. Masuda et al., PTEP* **2013** *073D01*

$M - R$ curves for hadron-quark crossover model



Hyperons make the EoS softer $\longrightarrow 2M_\odot$ limit not reached.

On the left fig. $g_V/G_S = 1$

Fig. from *K. Masuda et al., PTEP* **2013** *073D01*

Energy minimization (EM) method

based on: *X. H. Wu et al., PRC* **99**, *065802 (2019)*

Features of the model:

- Hadronic matter: Relativistic mean-field (RMF) model (2 flavours, nucleons interacting through σ, ω and ρ mesons, mesons treated at tree-level, additional $\omega - \rho$ interaction)
- Quark matter: NJL-model with u, d, s quarks and vector interaction
- mean-field approximation

Total energy of the mixed phase:

$$\varepsilon_{\mathrm{MP}} = u\varepsilon_{\mathrm{QP}} + (1-u)\varepsilon_{\mathrm{HP}} + \varepsilon_{\mathrm{surf}} + \varepsilon_{\mathrm{Coul}} \quad (6)$$

$$u = V_{\mathrm{QP}}/(V_{\mathrm{QP}} + V_{\mathrm{HP}}) \quad (7)$$

minimization w.r.t. the densities (n_p, n_n, n_u, n_d, n_s, n_e, n_μ) and u gives equilibrium conditions (under global charge neutrality and baryon number conservation)

Special cases of the EM method

- Coexisting phases method: $\varepsilon_{\rm surf}$ and $\varepsilon_{\rm Coul}$ are treated perturbatively (minimization or Gibbs condition without surf. and Coul. terms) $\Rightarrow P_{HP} = P_{QP}$, and $\varepsilon_{\rm MP} = u\varepsilon_{\rm QP} + (1-u)\varepsilon_{\rm HP} + \varepsilon_{\rm surf} + \varepsilon_{\rm Coul}$
- Gibbs construction: $\varepsilon_{\rm surf}$ and $\varepsilon_{\rm Coul}$ are neglected, $\sigma \approx 0$, global charge neutrality, hadronic and quark phases can be charged separately, $\Rightarrow P_{HP} = P_{QP}$, and $\varepsilon_{\rm MP} = u\varepsilon_{\rm QP} + (1-u)\varepsilon_{\rm HP}$
- Maxwell construction: $\varepsilon_{\rm surf}$ and $\varepsilon_{\rm Coul}$ are neglected, $\sigma >> 0$, local charge neutrality, both hadronic and quark phases charge neutral $\Rightarrow P_{HP} = P_{QP}$, and $\varepsilon_{\rm MP} = u\varepsilon_{\rm QP} + (1-u)\varepsilon_{\rm HP}$

Pressure with the EM method

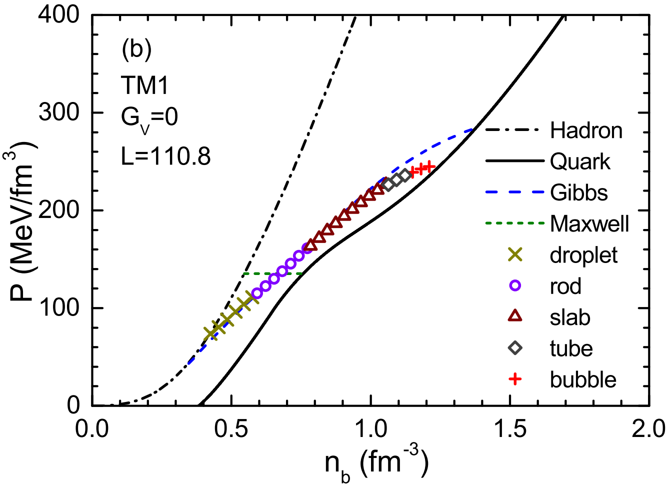


Fig. from *X. H. Wu et al., PRC* **99**, *065802 (2019)*

$M - R$ curves with the EM method

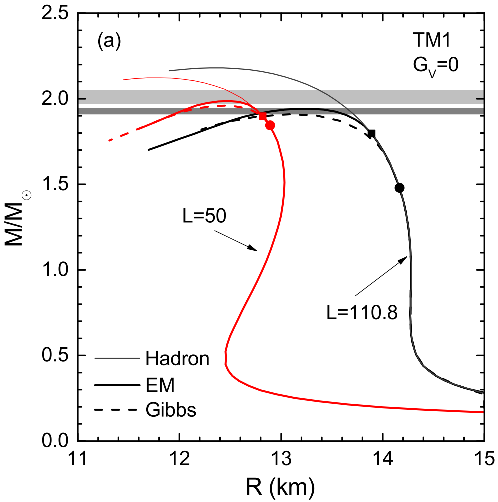


Fig. from *X. H. Wu et al., PRC* **99**, *065802 (2019)*

$M - R$ curves with the EM method

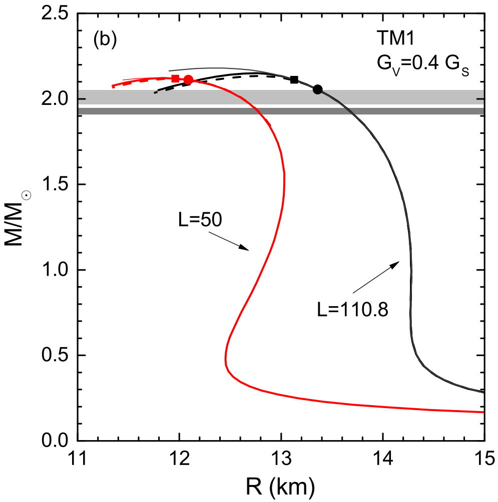


Fig. from *X. H. Wu et al., PRC* **99**, *065802 (2019)*

Lagrangian of the eLSM

P. Kovács et al. Phys. Rev. D **93***, no. 11, 114014 (2016), J. Takátsy et al., Universe* **5***, 174 (2019)*

$$\begin{aligned}\mathcal{L} &= \mathrm{Tr}[(D_\mu\Phi)^\dagger(D_\mu\Phi)] - m_0^2\mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2\\ &+ c_1(\det\Phi + \det\Phi^\dagger) + \mathrm{Tr}[H(\Phi+\Phi^\dagger)] - \frac{1}{4}\mathrm{Tr}(L_{\mu\nu}^2 + R_{\mu\nu}^2)\\ &+ \mathrm{Tr}\left[\left(\frac{m_1^2}{2}\mathbb{1} + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + i\frac{g_2}{2}(\mathrm{Tr}\{L_{\mu\nu}[L^\mu, L^\nu]\} + \mathrm{Tr}\{R_{\mu\nu}[R^\mu, R^\nu]\})\\ &+ \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}(L_\mu^2 + R_\mu^2) + h_2\mathrm{Tr}[(L_\mu\Phi)^2 + (\Phi R_\mu)^2] + 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger)\\ &+ \bar{\Psi}i\gamma_\mu D^\mu\Psi - g_F\bar{\Psi}(\Phi_S + i\gamma_5\Phi_{PS})\Psi,\end{aligned}$$

$$\begin{aligned}D^\mu\Phi &= \partial^\mu\Phi - ig_1(L^\mu\Phi - \Phi R^\mu) - ieA_e^\mu[T_3,\Phi],\\ L^{\mu\nu} &= \partial^\mu L^\nu - ieA_e^\mu[T_3,L^\nu] - \{\partial^\nu L^\mu - ieA_e^\nu[T_3,L^\mu]\},\\ R^{\mu\nu} &= \partial^\mu R^\nu - ieA_e^\mu[T_3,R^\nu] - \{\partial^\nu R^\mu - ieA_e^\nu[T_3,R^\mu]\},\\ D^\mu\Psi &= \partial^\mu\Psi - iG^\mu\Psi, \quad \text{with} \quad G^\mu = g_s G_a^\mu T_a.\end{aligned}$$

+ Polyakov loop potential (for $T > 0$)

Determination of the parameters

14 unknown parameters $(m_0, \lambda_1, \lambda_2, c_1, m_1, g_1, g_2, h_1, h_2, h_3, \delta_S, \Phi_N, \Phi_S, g_F)$ $\longrightarrow$ determined by the min. of χ^2:

$$\chi^2(x_1,\ldots,x_N) = \sum_{i=1}^{M}\left[\frac{Q_i(x_1,\ldots,x_N) - Q_i^{\exp}}{\delta Q_i}\right]^2,$$

$(x_1,\ldots,x_N) = (m_0, \lambda_1, \lambda_2, \ldots)$, $Q_i(x_1,\ldots,x_N) \longrightarrow$ from the model, $Q_i^{\exp} \longrightarrow$ PDG value, $\delta Q_i = \max\{5\%, \text{PDG value}\}$
multiparametric minimalization $\longrightarrow$ MINUIT

- PCAC $\to$ 2 physical quantities: f_π, f_K
- Curvature masses $\to$ 16 physical quantities:
  $m_{u/d}, m_s, m_\pi, m_\eta, m_{\eta'}, m_K, m_\rho, m_\Phi, m_{K^\star}, m_{a_1}, m_{f_1^H}, m_{K_1}, m_{a_0}, m_{K_s}, m_{f_0^L}, m_{f_0^H}$
- Decay widths $\to$ 12 physical quantities:
  $\Gamma_{\rho\to\pi\pi}, \Gamma_{\Phi\to KK}, \Gamma_{K^\star\to K\pi}, \Gamma_{a_1\to\pi\gamma}, \Gamma_{a_1\to\rho\pi}, \Gamma_{f_1\to KK^\star}, \Gamma_{a_0}, \Gamma_{K_S\to K\pi}, \Gamma_{f_0^L\to\pi\pi}, \Gamma_{f_0^L\to KK}, \Gamma_{f_0^H\to\pi\pi}, \Gamma_{f_0^H\to KK}$
- Pseudocritical temperature T_c at $\mu_B = 0$

Features of our approach

- D.O.F's: – scalar, pseudoscalar, vector, and axial-vector nonets
  – u, d, s constituent quarks ($m_u = m_d$)
  – Polyakov loop variables $\Phi, \bar{\Phi}$ with $\mathcal{U}_{\log}^{\mathrm{YM}}$ or $\mathcal{U}_{\log}^{\mathrm{glue}}$
- no mesonic fluctuations, only fermionic ones

$$\mathcal{Z}=e^{-\beta V\Omega(T,\mu_q)}=\int\limits_{\mathrm{PBC}}\prod_a \mathcal{D}\xi_a \int\limits_{\mathrm{APBC}}\prod_f \mathcal{D}q_f\mathcal{D}q_f^\dagger \exp\left[-\int_0^\beta d\tau \int_V d^3x\left(\mathcal{L}+\mu_q\sum_f q_f^\dagger q_f\right)\right]$$

approximated as $\Omega(T,\mu_q) = U_{\mathrm{meson}}^{\mathrm{tree}}(\langle M\rangle) + \Omega_{\bar{q}q}^{(0)}(T,\mu_q) + \mathcal{U}_{\log}(\Phi,\bar{\Phi})$, $\tilde{\mu}_q=\mu_q-iG_4$

$$e^{-\beta V\Omega_{\bar{q}q}^{(0)}} = \int\limits_{\mathrm{APBC}}\prod_{f,g}\mathcal{D}q_g\mathcal{D}q_f^\dagger \exp\left\{\int_0^\beta d\tau\int_x q_f^\dagger\left[\left(i\gamma_0\vec{\gamma}\cdot\vec{\nabla}-\frac{\partial}{\partial\tau}+\tilde{\mu}_q\right)\delta_{fg}-\gamma_0\mathcal{M}_{fg}|_{\xi_a=0}\right]q_g\right\}$$

- tree-level (axial)vector masses
- fermionic vacuum and thermal fluctuations included in the (pseudo)scalar curvature masses used to parameterize the model
- 4 coupled T/μ_B-dependent field equations for the condensates $\phi_N, \phi_S, \Phi, \bar{\Phi}$
- thermal contribution of π, K, f_0^L included in the pressure, however their curvature mass contains no mesonic fluctuations

Inclusion of vector meson Yukawa term

$$\mathcal{L}_{\text{Yukawa-vec}} = -g_v\sqrt{6}\bar{\Psi}\gamma_\mu V_0^\mu \Psi$$

$$V_0^\mu = \frac{1}{\sqrt{6}}\text{diag}\left(v_0 + \frac{v_8}{\sqrt{2}}, v_0 + \frac{v_8}{\sqrt{2}}, v_0 - \sqrt{2}v_8\right)$$

mean-field treatment

$$< v_0^\mu >= v_0\delta^{0\mu}, \; < v_8^\mu >= 0^\mu$$

Modification of the grand canonical potential

$$\Omega(T=0,\mu_q) \to \Omega(T=0,\tilde{\mu}_q) - \frac{1}{2}m_v^2 v_0^2, \text{ with } \tilde{\mu}_q = \mu_q - g_v v_0$$

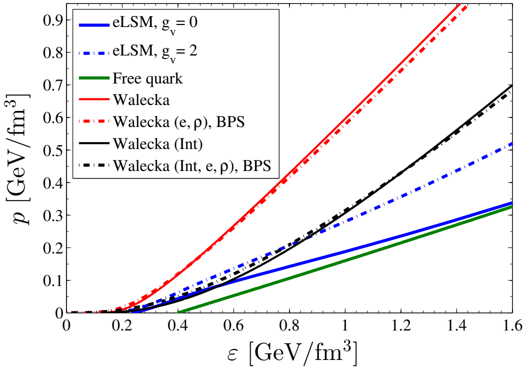
While the field equations

$$\left.\frac{\partial\Omega}{\partial\phi_{\mathrm{N}}}\right|_{\phi_{\mathrm{N}}=\bar{\phi}_{\mathrm{N}}} = \left.\frac{\partial\Omega}{\partial\phi_{\mathrm{S}}}\right|_{\phi_{\mathrm{S}}=\bar{\phi}_{\mathrm{S}}} = 0 \quad \text{and} \quad \left.\frac{\partial\Omega}{\partial v_0}\right|_{v_0=\bar{v}_0} = 0\,,$$

The EOS of the pure eLSM

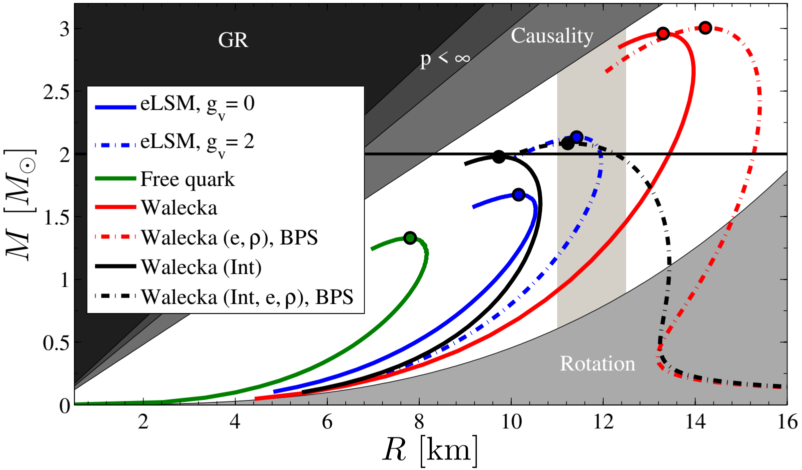
- Pure eLSM compared to Walecka and free quark models
- At low energies the EoS of the eLSM is close to the EoS of the Walecka - model
- At higher energies it tends to the EoS of the free quark model
- note: Walecka Int means

$$\mathcal{L}_{W,Int} = -\frac{b}{3} m_n (g_\sigma \sigma)^3 - \frac{c}{4} (g_\sigma \sigma)^4$$

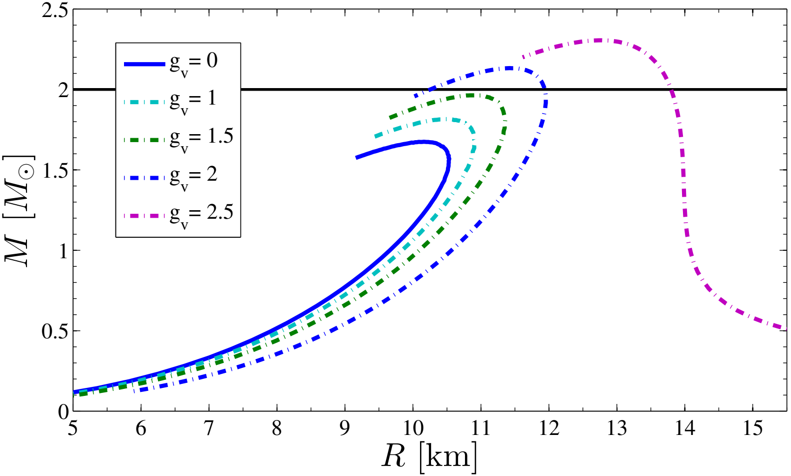


- In the Walecka models electrons and ρ mesons are also included (dashed-dotted lines)

$M - R$ relations of the pure eLSM (quark star)

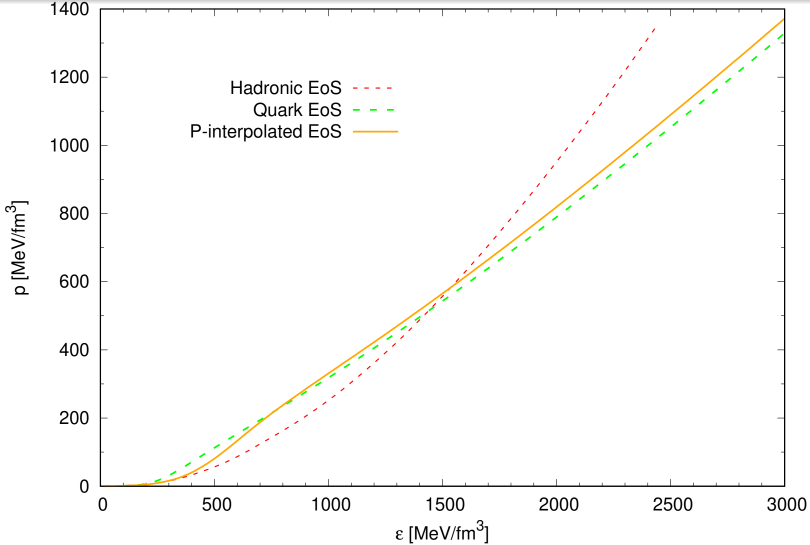


Changing the g_v vector coupling in pure ELSM

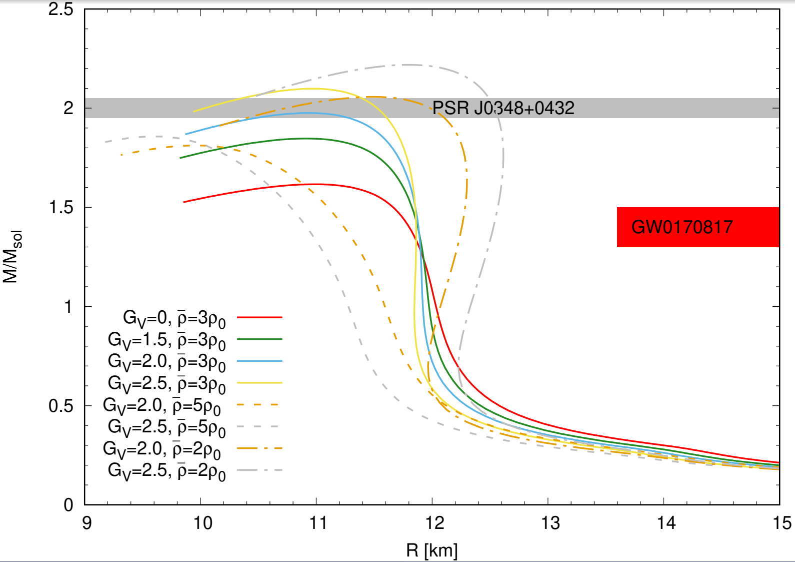


For $g_v \gtrsim 2.5$ $p(\varepsilon)$ becomes zero at $\varepsilon = 0 \Rightarrow$ small M, large R

EOS of the hybrid BPS+TNI3u+eLSM model



$M - R$ relations of the BPS+TNI3u+eLSM



Conclusion

Conclusion

- Current astrophys. restrictions (like $2M_\odot$ limit) can be fulfilled in many ways
- Precise radius measurements can select models
- The quark–vector meson Yukawa interactions are very important
- Reasonable hybrid stars can be constructed with eLSM, however even pure quark stars are not excluded currently

Plans

- Other hybrid star constructions with eLSM
- Better (more consistent) approximations in the eLSM part
- Inclusion of the total vector-quark Yukawa term, consistent treatment
- Beyond mean-field calculations
- Tidal deformability (in progress)

Thank you for your attention!

Péter Kovács kovacs.peter@wigner.mta.hu

Particle content

- Vector and Axial-vector meson nonets

$$V^{\mu}=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{\omega_N+\rho^0}{\sqrt{2}} & \rho^+ & K^{\star+}\\ \rho^- & \frac{\omega_N-\rho^0}{\sqrt{2}} & K^{\star0}\\ K^{\star-} & \bar{K}^{\star0} & \omega_S\end{pmatrix}^{\mu} \quad A^{\mu}=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{f_{1N}+a_1^0}{\sqrt{2}} & a_1^+ & K_1^+\\ a_1^- & \frac{f_{1N}-a_1^0}{\sqrt{2}} & K_1^0\\ K_1^- & \bar{K}_1^0 & f_{1S}\end{pmatrix}^{\mu}$$

$\rho\to\rho(770), K^\star\to K^\star(894)$
$\omega_N\to\omega(782), \omega_S\to\phi(1020)$

$a_1\to a_1(1230), K_1\to K_1(1270)$
$f_{1N}\to f_1(1280), f_{1S}\to f_1(1426)$

- Scalar $(\sim\bar{q}_i q_j)$ and pseudoscalar $(\sim\bar{q}_i\gamma_5 q_j)$ meson nonets

$$\Phi_S=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{\sigma_N+a_0^0}{\sqrt{2}} & a_0^+ & K_0^{\star+}\\ a_0^- & \frac{\sigma_N-a_0^0}{\sqrt{2}} & K_0^{\star0}\\ K_0^{\star-} & \bar{K}_0^{\star0} & \sigma_S\end{pmatrix} \quad \Phi_{PS}=\frac{1}{\sqrt{2}}\begin{pmatrix}\frac{\eta_N+\pi^0}{\sqrt{2}} & \pi^+ & K^+\\ \pi^- & \frac{\eta_N-\pi^0}{\sqrt{2}} & K^0\\ K^- & \bar{K}^0 & \eta_S\end{pmatrix}$$

unknown assignment
mixing in the $\sigma_N-\sigma_S$ sector

$\pi\to\pi(138), K\to K(495)$
mixing: $\eta_N,\eta_S\to\eta(548),\ \eta'(958)$

Spontaneous symmetry breaking: $\sigma_{N/S}$ acquire nonzero expectation values $\phi_{N/S}$

fields shifted by their expectation value: $\sigma_{N/S}\to\sigma_{N/S}+\phi_{N/S}$