Fluctuations of conserved quantities in heavy-ion collisions at high energies

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Excited QCD 2020

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What do we mean by "event-by-event"

In central Pb-Pb collisions at LHC energies, ~2000 particles within $|\eta|<0.5$.

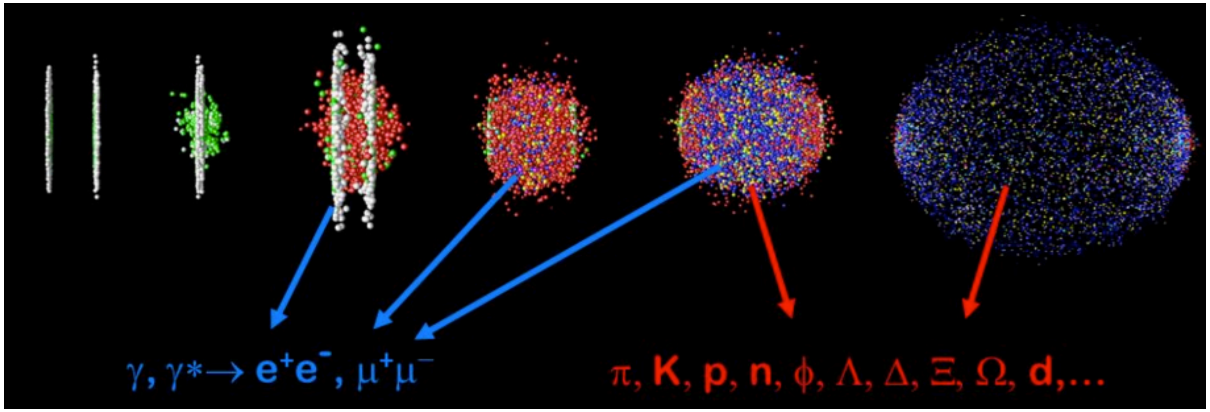
Many "event-averaged" observables can be studied:
particle yields, spectra, flow harmonics…

Event-by-event measurements:
when a given observable is measured on *an event-by-event basis,*
and the fluctuations are studied over the ensemble of the events.

- fluctuating net-charge, number of protons, mean p_T, forward-backward yields, etc.

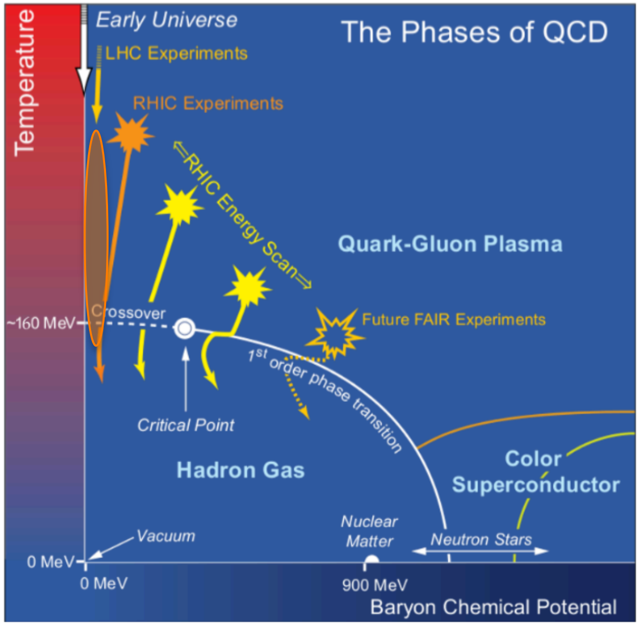
Why e-by-e fluctuations:

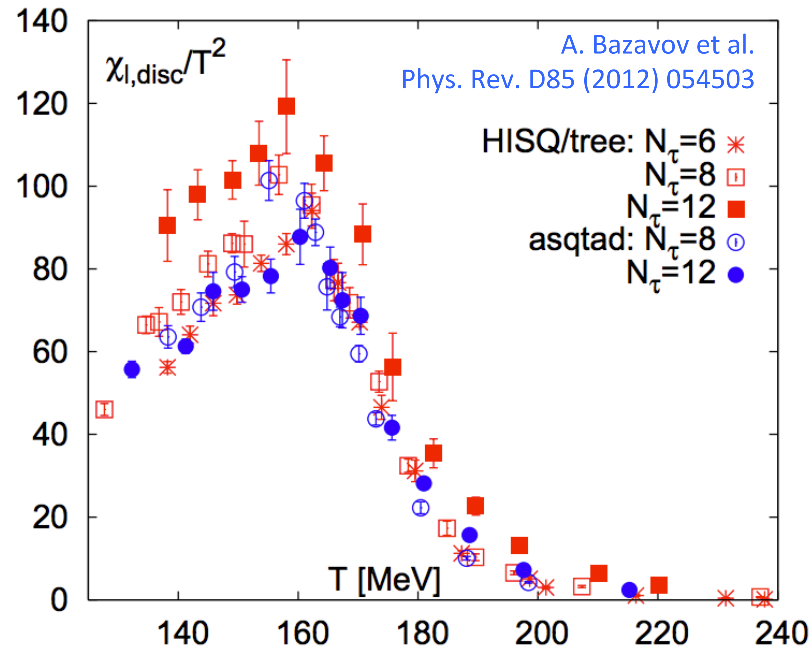
- they help to characterize the **properties of the "bulk" of the system**
- fluctuations also are closely related to **dynamics of the phase transitions**



Phase transitions at the LHC

Event-by-event fluctuations in heavy-ion collisions at the LHC make it possible to verify lattice QCD calculations at small values of baryon chemical potential (*$\mu_B \simeq 0$ at LHC energies*).





$T_c^{LQCD} = 156.5 \pm 1.5$ MeV, $T_{fo}^{ALICE} = 156.5 \pm 3$ MeV

A. Andronic et al., Nature 561, 321–330 (2018)

Thermodynamic susceptibilities χ:

- describe the response of a thermal system to changes in external conditions, fundamental properties of the medium
- can be calculated within lattice QCD
- within the Grand Canonical Ensemble, are related to e-by-e fluctuations of conserved charges: electric charge, strangeness, baryon number

$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$$

1 Connection between theory and experiment

Theory: susceptibilities

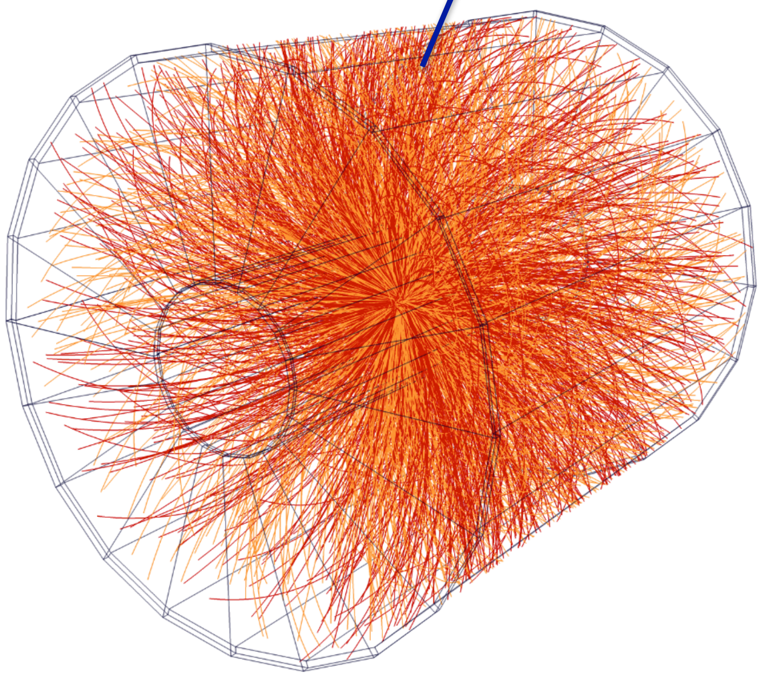
$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$$

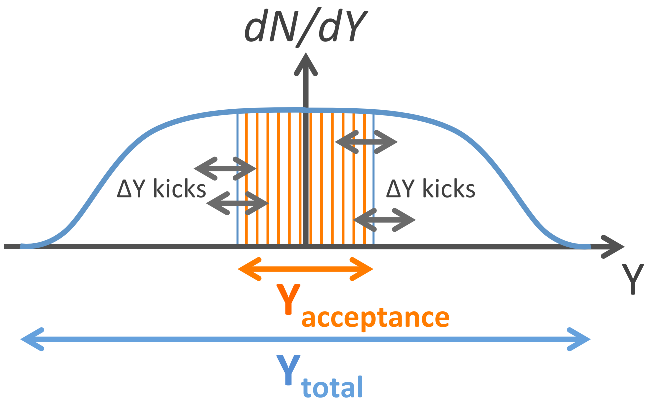
(fixed volume, particles in GCE)

Experiment:
moments of net-particle multiplicity distributions

$$\Delta N_B = N_B - N_{\bar{B}}$$

in each collision





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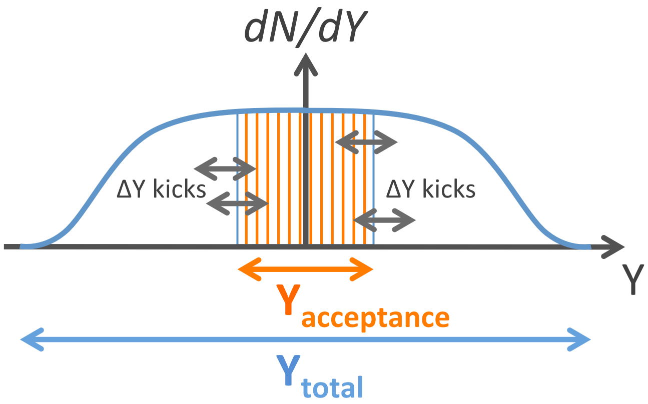


κ_n – cumulants

$$\chi_n^B = \frac{\kappa_n(\Delta N_B)}{VT^3}$$

MEASURE CUMULANTS!

$$\kappa_1 = \langle \Delta N_B \rangle = VT^3 \chi_1^B$$

$$\kappa_2 = \sigma^2 = \left\langle \left(\Delta N_B - \langle \Delta N_B \rangle\right)^2 \right\rangle = VT^3 \chi_2^B$$

variance

$$\kappa_3/\sigma^3 = S = \left\langle \left(\Delta N_B - \langle \Delta N_B \rangle\right)^3 \right\rangle \Big/ \sigma^3 = \frac{VT^3 \chi_3^B}{\left(VT^3 \chi_2^B\right)^{3/2}}$$

skewness

$$\kappa_4/\sigma^4 = k = \left\langle \left(\Delta N_B - \langle \Delta N_B \rangle\right)^4 \right\rangle \Big/ \sigma^4 - 3 = \frac{VT^3 \chi_4^B}{\left(VT^3 \chi_2^B\right)^2}$$

kurtosis

Ratios from experiment:

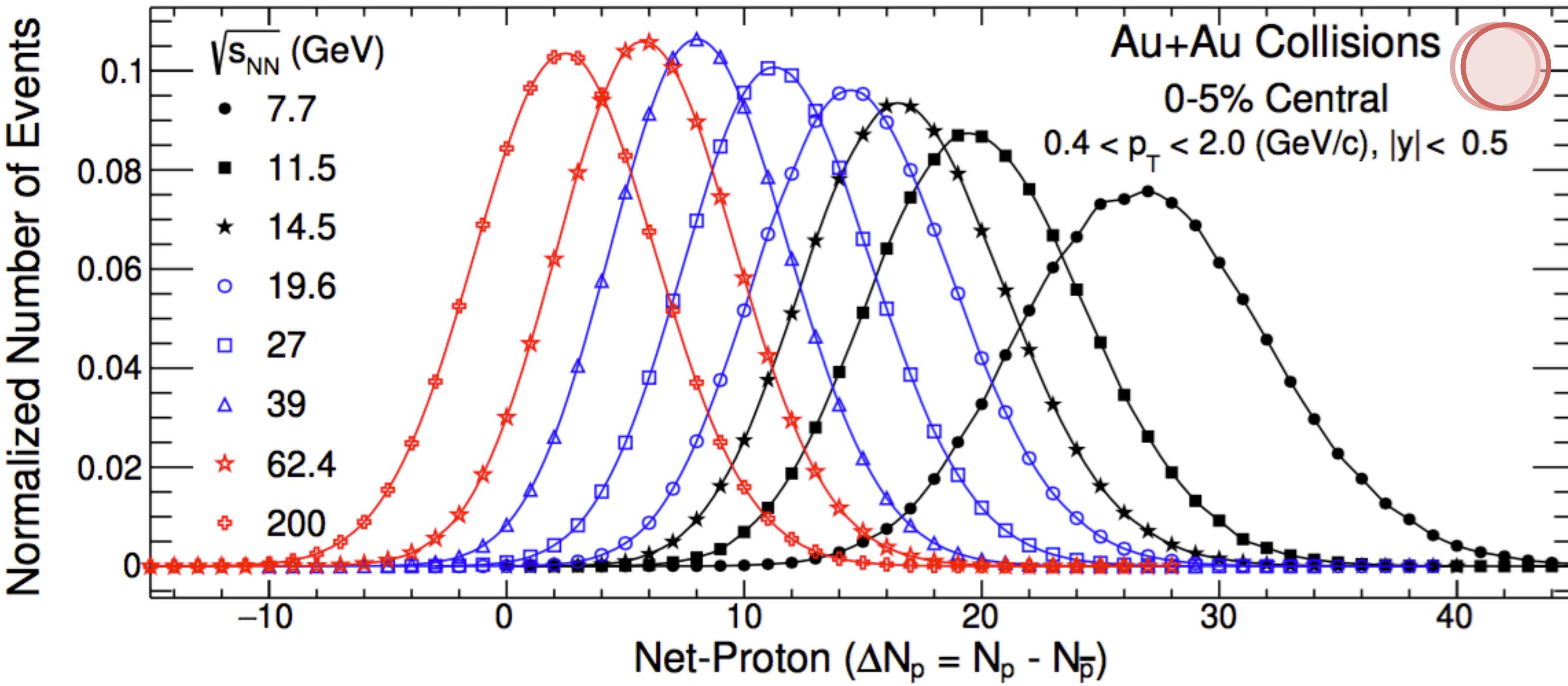
$$\frac{\kappa_3(\Delta N_B)}{\kappa_2(\Delta N_B)} \Leftrightarrow \frac{\chi_3^B}{\chi_2^B}$$

$$\frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} \Leftrightarrow \frac{\chi_4^B}{\chi_2^B}$$

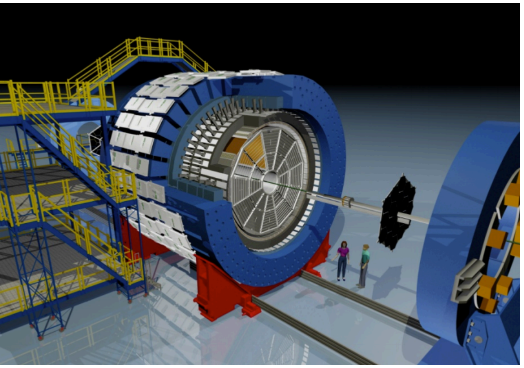
(cancel VT^3)

Event-by-event net-proton distributions from STAR

STAR, arXiv:2001.02852



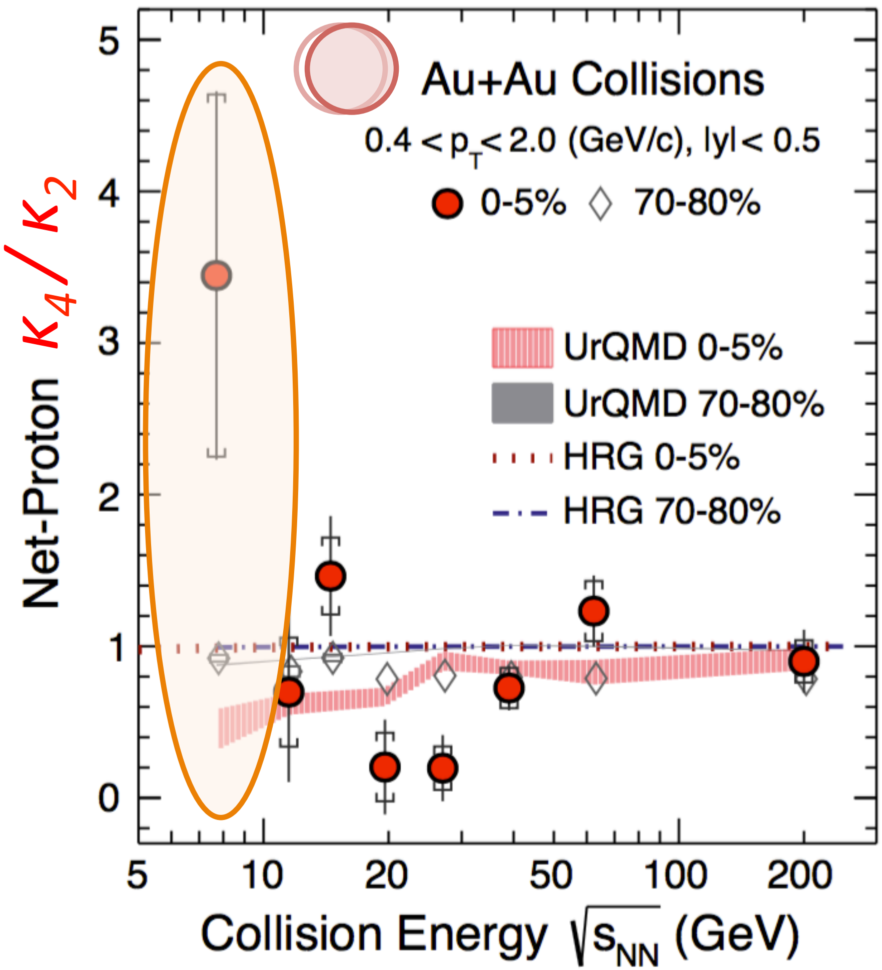
STAR detector:

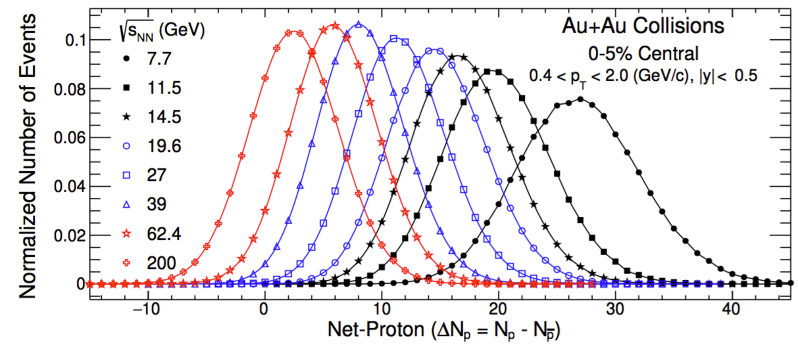


Shape of the event-by-event net-proton distribution is characterized by **cumulants κ_n** of various orders.

Net-proton cumulant ratios from STAR

STAR, arXiv:2001.02852





- **Head-on (central) Au+Au collisions: non-monotonic variation in 4-to-2 cumulant vs $\sqrt{s_{NN}}$!**
- Non-central Au+Au collisions show a *monotonic* variation as a function of √sNN.

Models without a critical point do not describe data at the lowest energy.

2 Connection between theory and experiment

Theory: susceptibilities

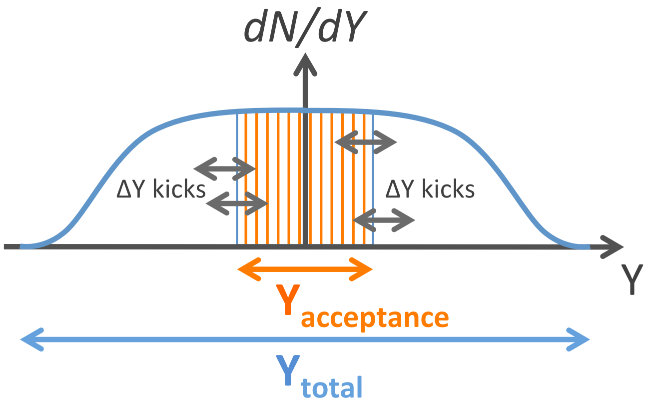
$$\chi_n^B = \frac{\partial^n \left(P/T^4\right)}{\partial\left(\mu_B/T\right)^n}$$

(fixed volume, particles in GCE)

⇔

Experiment:
moments of net-particle multiplicity distributions

$$\Delta N_B = N_B - N_{\bar{B}}$$



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(cancel VT^3)

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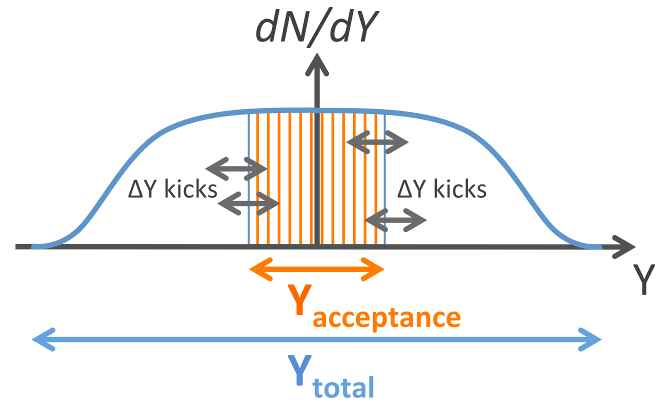
$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$$

(fixed volume, particles in GCE)

Experiment:
moments of net-particle multiplicity distributions

$$\Delta N_B = N_B - N_{\bar{B}}$$

(volume fluctuations, global conservation laws)



κ_n – cumulants

$$\chi_n^B \neq \frac{\kappa_n(\Delta N_B)}{VT^3}$$

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Ratios from experiment:

$$\frac{\kappa_3(\Delta N_B)}{\kappa_2(\Delta N_B)} \not\Leftrightarrow \frac{\chi_3^B}{\chi_2^B}$$

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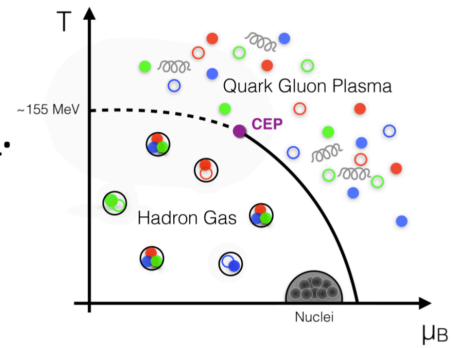
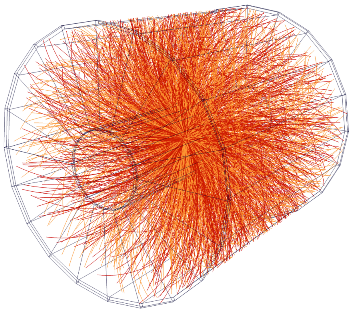
~~(cancel VT^3)~~

A.Rustamov et al., Nucl.Phys.A 960 (2017) 114

2 Cumulant measurements: experimental challenges



VS



Theory:
lattice calculations are done within Grand Canonical Ensemble.

Reality:
- resonance decays
- jets
- global charge conservation

} **estimations from models**

Experiment:
- depends on centrality selection methods
- finite efficiency of particle registration
- need correction for particle mis-identification

} **correction methods are developed**

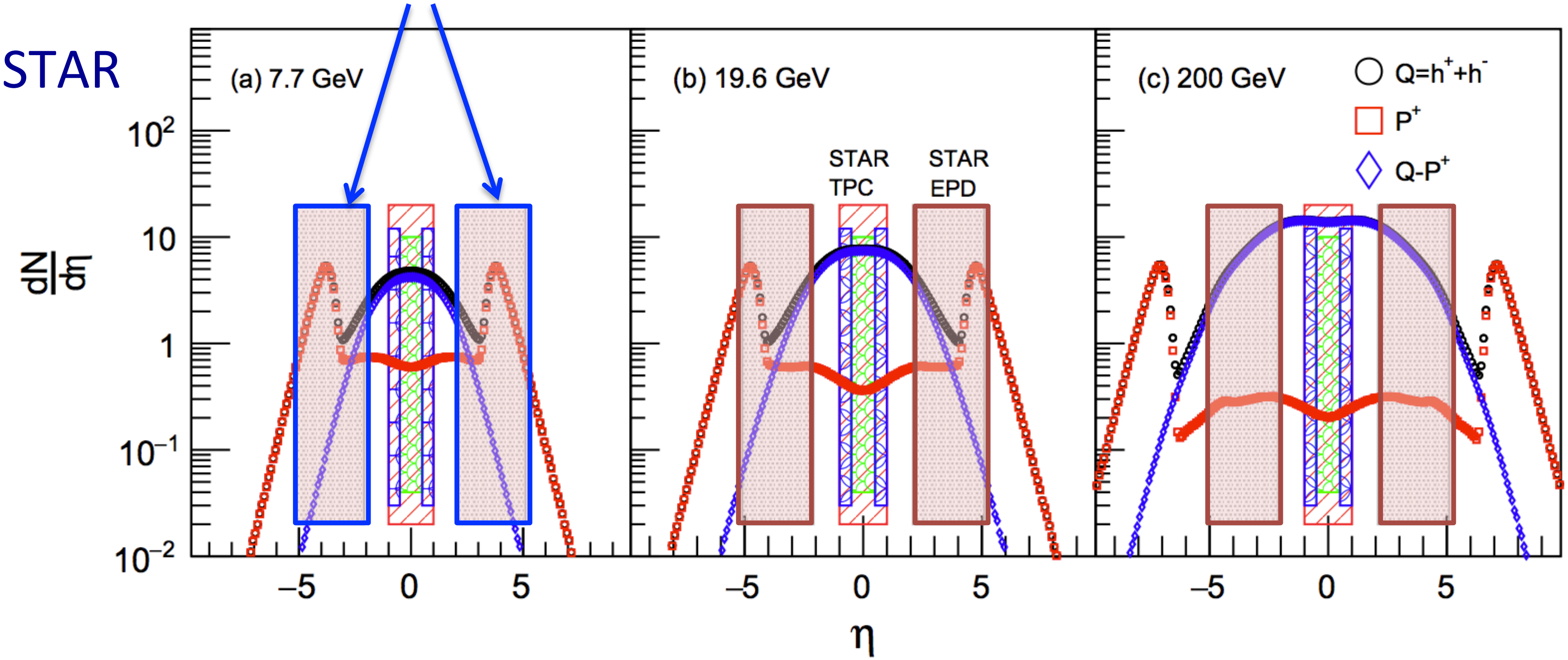
… Moreover, usually measure net-proton number as a *proxy* for net-baryon number (we typically don't see neutrons, Λ-hyperons are more difficult to measure, etc.).

2 Example of the problem: centrality determination

A. Chatterjee et al., arXiv:1910.08004

Here we count particles → determine **centrality**

STAR



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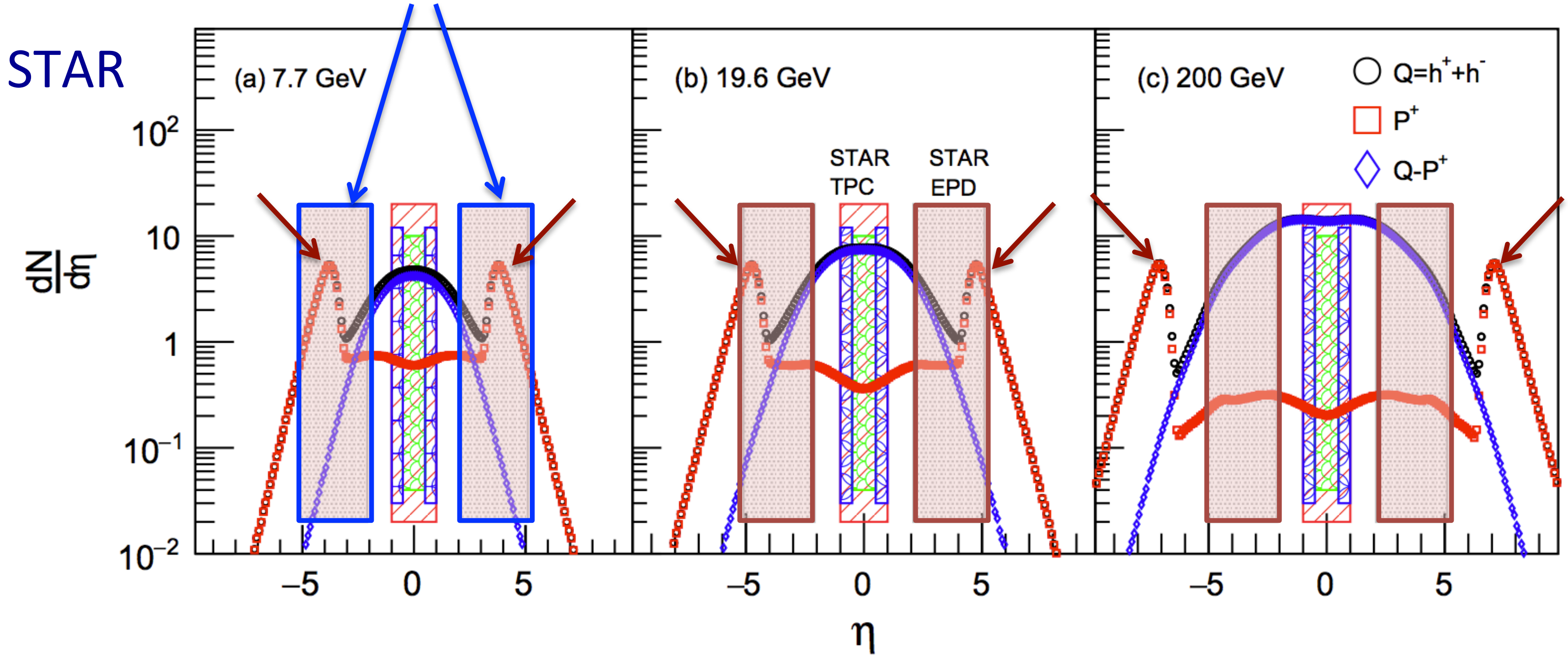

At low energies (~< 100 GeV), protons from incoming beams start hitting detectors used for centrality determination.

Problem with determining centrality classes (i.e. most central).
Analyzers are forced to use data from other acceptance regions.

2

Another challenge: efficiency correction

Nonaka et al. PRC 95, 064912 (2017)

if ***p*** – efficiency of particle registration:

Even for this simple case,
correction for cumulants of net-charge $\Delta N=N^{+}-N^{-}$ looks like this:

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Even for this simple case,
correction for cumulants of net-charge *$\Delta N=N^{+}-N^{-}$* looks like this:

n_{net} and n_{tot} – measured (uncorrected) values

$$\kappa_1 = \frac{1}{p}\langle n_{\rm net}\rangle_{\rm c}, \quad \text{(A1)}$$

$$\kappa_2 = \frac{1}{p^2}\langle n_{\rm net}^2\rangle_{\rm c} + \left(-\frac{1}{p^2}+\frac{1}{p}\right)\langle n_{\rm tot}\rangle, \quad \text{(A2)}$$

$$\kappa_3 = \frac{1}{p^3}\langle n_{\rm net}^3\rangle_{\rm c} + \left(-\frac{3}{p^3}+\frac{3}{p^2}\right)\langle n_{\rm net}n_{\rm tot}\rangle_{\rm c} + \left(\frac{2}{p^3}-\frac{3}{p^2}+\frac{1}{p}\right)\langle n_{\rm net}\rangle_{\rm c}, \quad \text{(A3)}$$

$$\kappa_4 = \frac{1}{p^4}\langle n_{\rm net}^4\rangle_{\rm c} + \left(-\frac{6}{p^4}+\frac{6}{p^3}\right)\langle n_{\rm net}^2 n_{\rm tot}\rangle_{\rm c} + \left(\frac{8}{p^4}-\frac{12}{p^3}+\frac{4}{p^2}\right)\langle n_{\rm net}^2\rangle_{\rm c} + \left(\frac{3}{p^4}-\frac{6}{p^3}+\frac{3}{p^2}\right)\langle n_{\rm tot}^2\rangle_{\rm c}$$
$$+\left(-\frac{6}{p^4}+\frac{12}{p^3}-\frac{7}{p^2}+\frac{1}{p}\right)\langle n_{\rm tot}\rangle_{\rm c}, \quad \text{(A4)}$$

$$\kappa_5 = \frac{1}{p^5}\langle n_{\rm net}^5\rangle_{\rm c} + \left(-\frac{10}{p^5}+\frac{10}{p^4}\right)\langle n_{\rm net}^3 n_{\rm tot}\rangle_{\rm c} + \left(\frac{20}{p^5}-\frac{30}{p^4}+\frac{10}{p^3}\right)\langle n_{\rm net}^3\rangle_{\rm c} + \left(\frac{15}{p^5}-\frac{30}{p^4}+\frac{15}{p^3}\right)\langle n_{\rm net}n_{\rm tot}^2\rangle_{\rm c}$$
$$+\left(-\frac{50}{p^5}+\frac{110}{p^4}-\frac{75}{p^3}+\frac{15}{p^2}\right)\langle n_{\rm net}n_{\rm tot}\rangle_{\rm c} + \left(\frac{24}{p^5}-\frac{60}{p^4}+\frac{50}{p^3}-\frac{15}{p^2}+\frac{1}{p}\right)\langle n_{\rm net}\rangle_{\rm c}, \quad \text{(A5)}$$

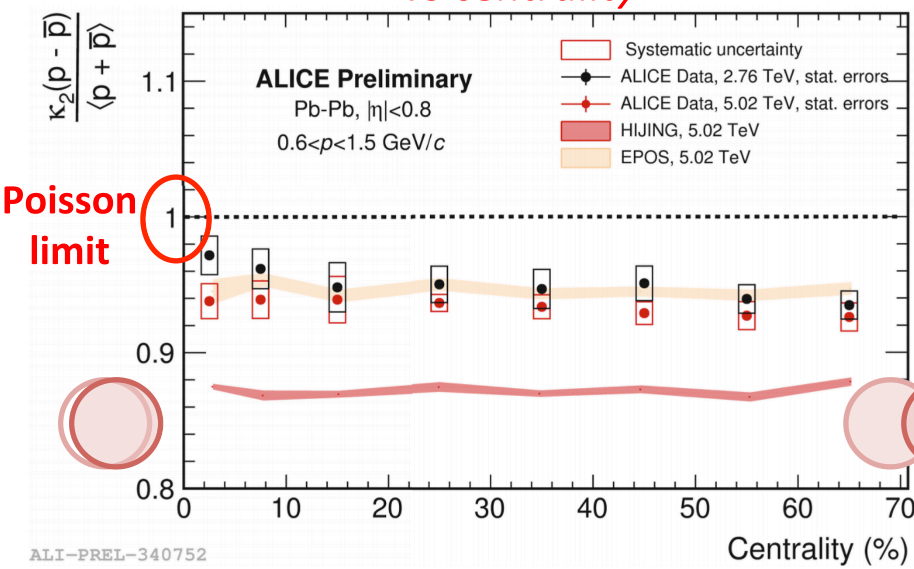
$$\kappa_6 = \frac{1}{p^6}\langle n_{\rm net}^6\rangle_{\rm c} + \left(-\frac{15}{p^6}+\frac{15}{p^5}\right)\langle n_{\rm net}^4 n_{\rm tot}\rangle_{\rm c} + \left(\frac{40}{p^6}-\frac{60}{p^5}+\frac{20}{p^4}\right)\langle n_{\rm net}^4\rangle_{\rm c}$$
$$+\left(\frac{45}{p^6}-\frac{90}{p^5}+\frac{45}{p^4}\right)\langle n_{\rm net}^2 n_{\rm tot}^2\rangle_{\rm c} + \left(-\frac{15}{p^6}+\frac{45}{p^5}-\frac{45}{p^4}+\frac{15}{p^3}\right)\langle n_{\rm tot}^3\rangle_{\rm c} + \left(-\frac{210}{p^6}+\frac{480}{p^5}-\frac{345}{p^4}+\frac{75}{p^3}\right)\langle n_{\rm net}^2 n_{\rm tot}\rangle_{\rm c}$$
$$+\left(\frac{184}{p^6}-\frac{480}{p^5}+\frac{430}{p^4}-\frac{150}{p^3}+\frac{16}{p^2}\right)\langle n_{\rm net}^2\rangle_{\rm c} + \left(\frac{90}{p^6}-\frac{270}{p^5}+\frac{285}{p^4}-\frac{120}{p^3}+\frac{15}{p^2}\right)\langle n_{\rm tot}^2\rangle_{\rm c}$$
$$+\left(-\frac{120}{p^6}+\frac{360}{p^5}-\frac{390}{p^4}+\frac{180}{p^3}-\frac{31}{p^2}+\frac{1}{p}\right)\langle n_{\rm tot}\rangle_{\rm c}. \quad \text{(A6)}$$

… scaring…

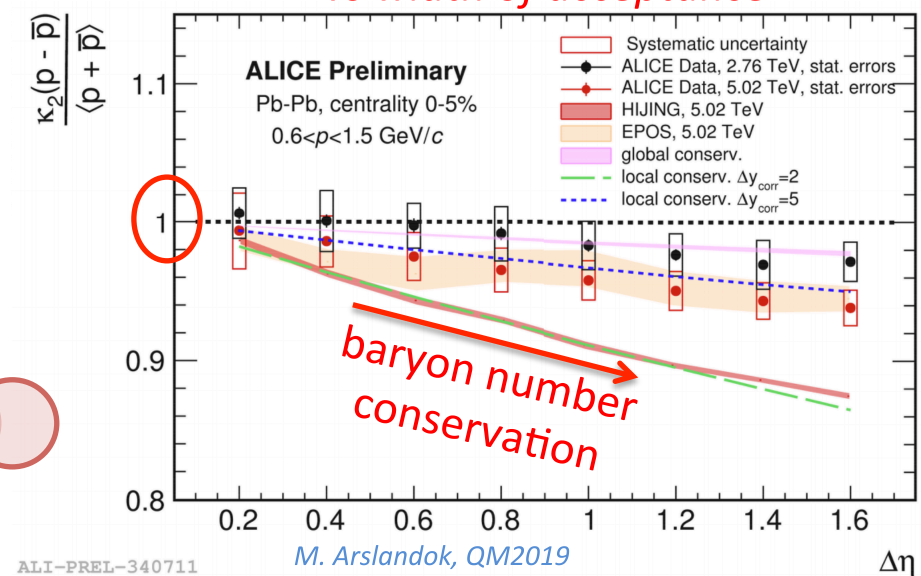
3 **Net-proton** fluctuations in ALICE (Pb-Pb): *the 2nd moment*

Why measure at LHC: we probe fluctuations at the cross-over region of the Phase Diagram.

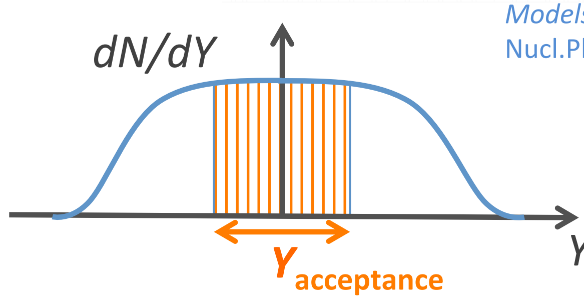
vs centrality

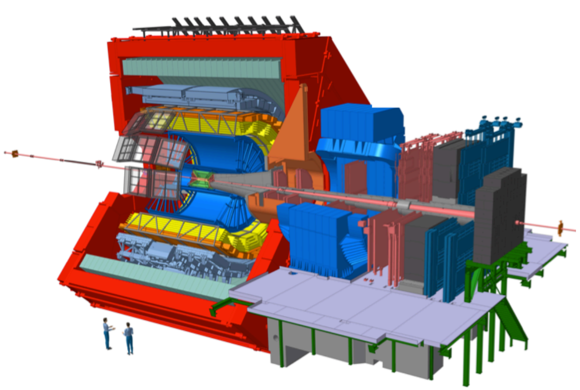


vs width of acceptance



M. Arslandok, QM2019
Models: P. Braun-Munzinger, A.Rustamov, J.Stachel, Nucl.Phys.A 960 (2017) 114; arXiv:1907.03032





Deviation from 1 (= from Poisson) is treated to be due to **baryon number conservation**
(*i.e.* no evidence for dynamical fluctuations so far).

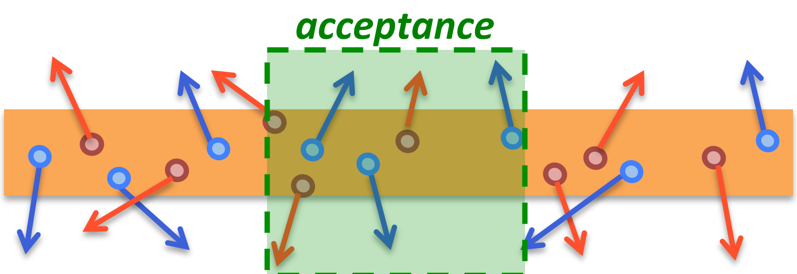
Global *vs* **local** charge conservation and cumulants

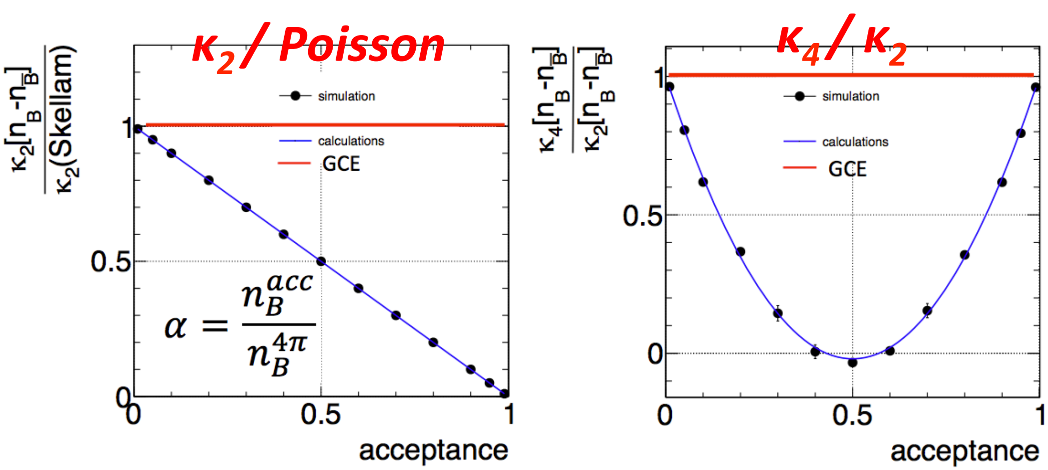
… So, to compare net-charge fluctuations with theory (=calculations in GCE), charge conservation in each event should be taken into account.

Options:

Global baryon number conservation:

pairs are produced independently with y_1^+ and y_2^- within acceptance.





P. Braun-Munzinger et al., NPA 982 (2019) 307–310

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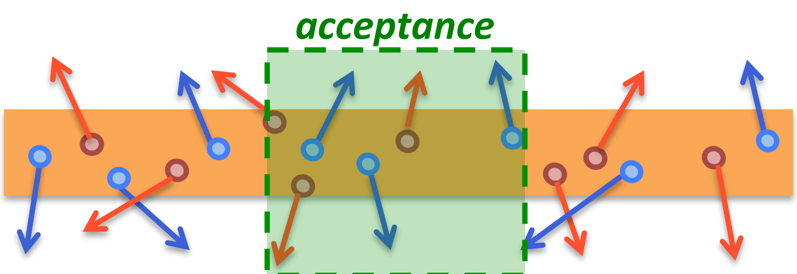
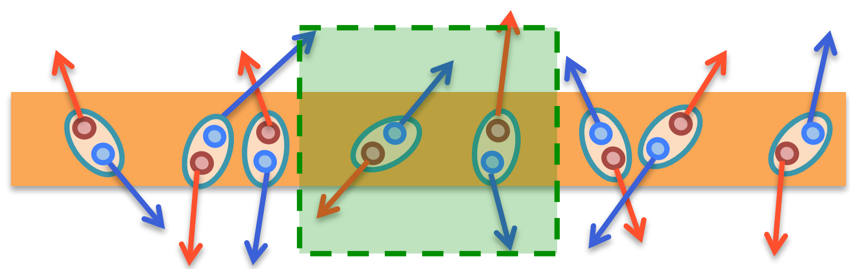


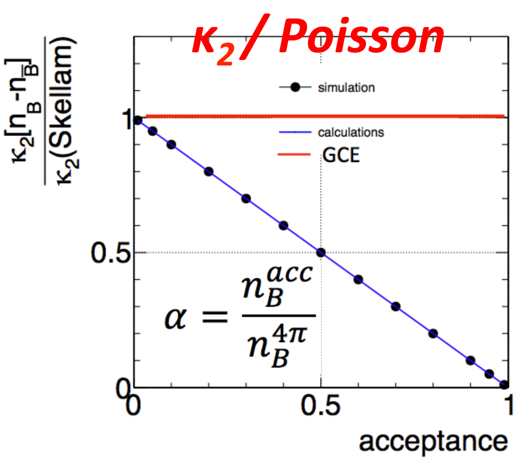
Local baryon number conservation:

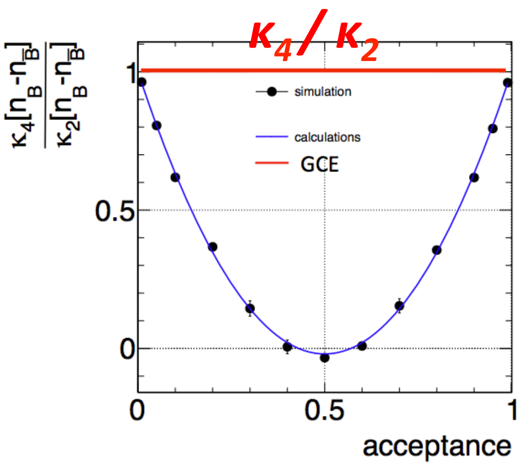
there is a **finite correlation length.**

(resonance decays, string fragmentation, …)



Example: simple correlation $|y_1^+ - y_2^-| < \Delta y_{corr}/2$:





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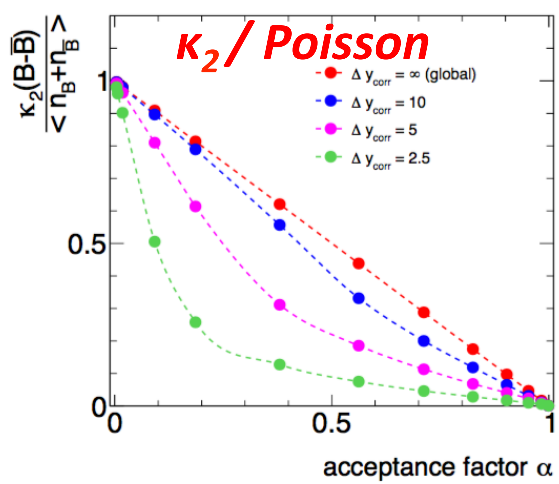
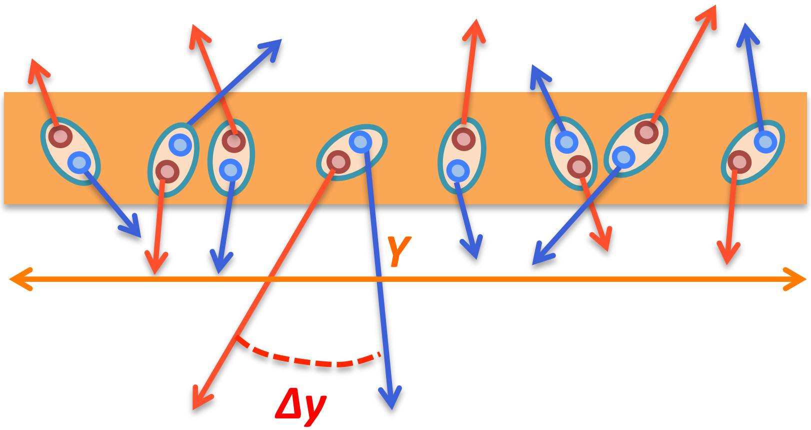


P.Braun-Munzinger et al., arXiv:1907.03032

Charge conservation expressed via **balance function**

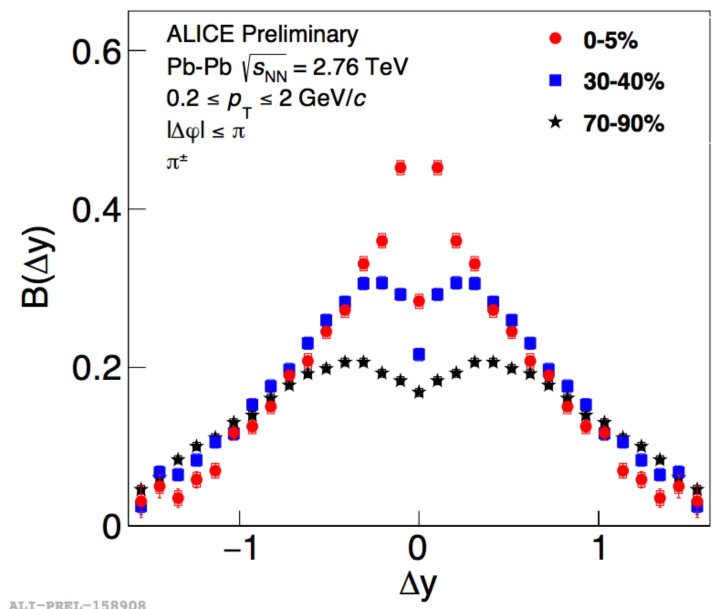


Balance Function is a measure of correlations between the opposite charges.

$$B(\Delta\eta) = \frac{1}{2}\left(\frac{\rho_2^{(+,-)} - \rho_2^{(+,+)}}{\rho_1^{(+)}} + \frac{\rho_2^{(-,+)} - \rho_2^{(-,-)}}{\rho_1^{(-)}}\right)$$

Bass et al., Phys. Rev. Lett. 85 (2000) 2689

Example: ALICE, BF for pions:



J.Pan, NPA 982 315-318 (2019)

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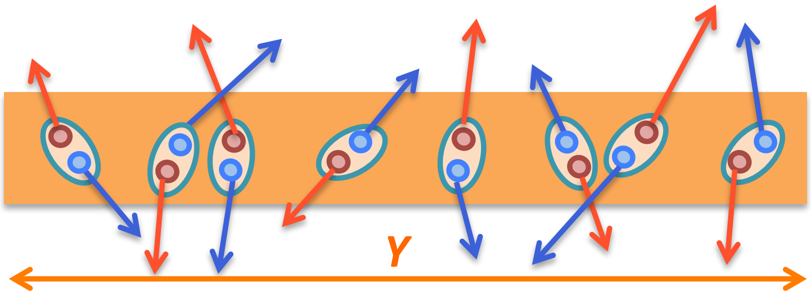


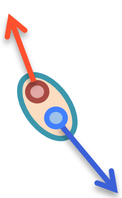
Balance Function is a measure of correlations between the opposite charges.

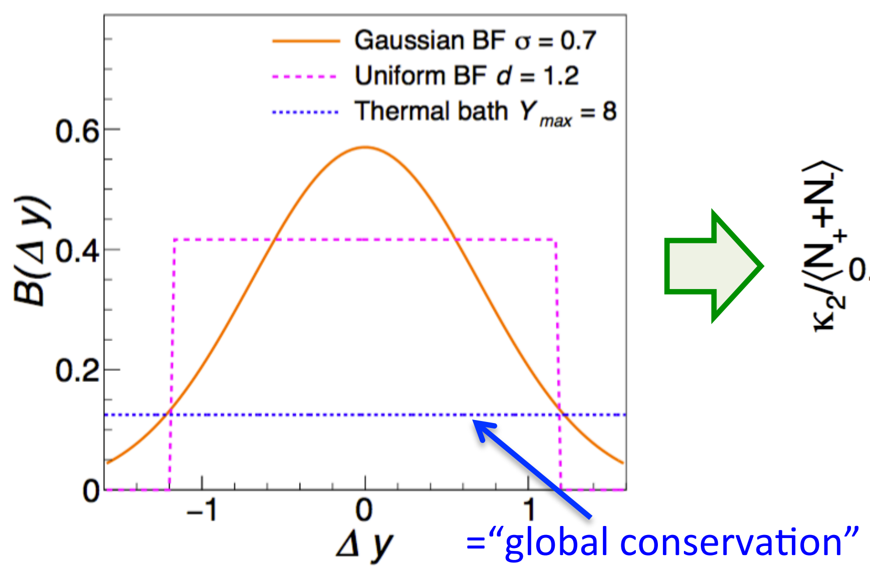
There is a simple straightforward connection between 2nd cumulant and BF:

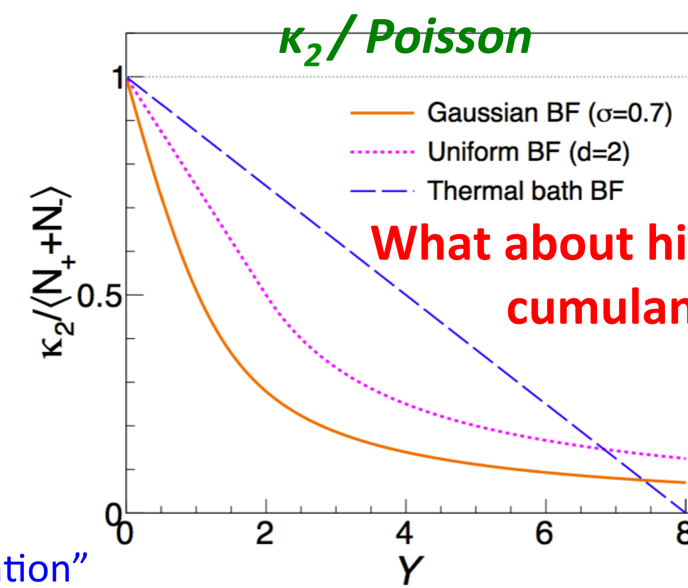
$$\frac{\kappa_2(\Delta N_p)}{\langle N\rangle + \langle \overline{N}\rangle} = 1 - \int_{-Y}^{Y} B(\Delta y)\left(1 - \frac{|\Delta y|}{Y}\right) d\Delta y$$

*first discussed (with some caveats) in C.Pruneau, PRC 100, 034905 (2019).

This work: from balance functions → to cumulant ratios

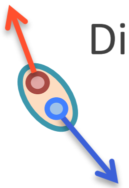




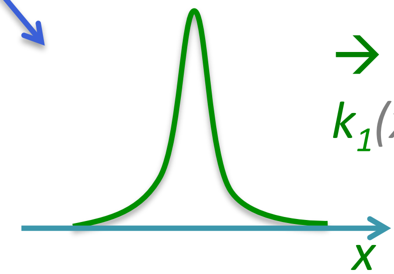


What about higher-order cumulants?..

5 Cumulants of a system of independent sources

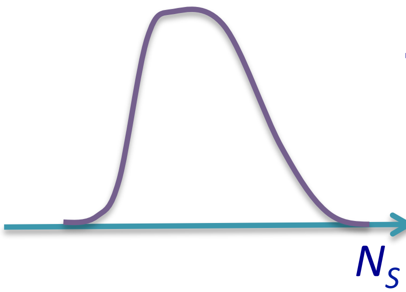


Distribution of a quantity *x* for a single source:

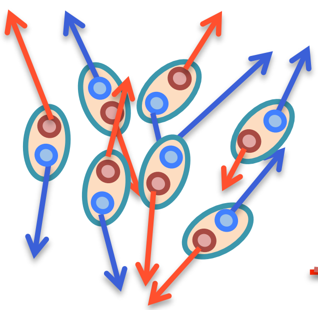


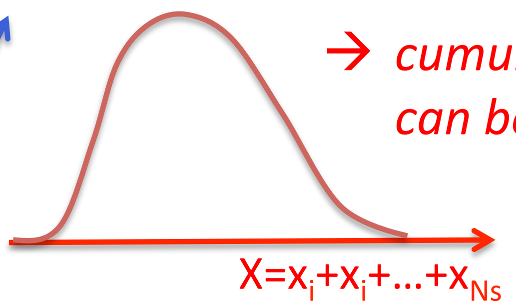
→ *cumulants:*
$k_1(x), k_2, k_3, k_4, \ldots$

Distribution of number of sources N_S:



→ *cumulants:*
$K_1(N_S), K_2, K_3, K_4, \ldots$





→ *cumulants* κ_n *of* ***total event-wise distribution*** *of X can be calculated via derivatives of MGF:*

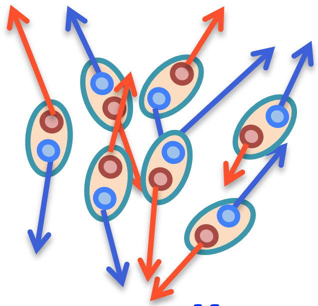
$$M_X(t) = [M_x(t)]^{N_S}$$

Rustamov et al., Nucl.Phys.A 960 (2017) 114

$$\kappa_1 = k_1 K_1,$$
$$\kappa_2 = {k_1}^2 K_2 + k_2 K_1,$$
$$\kappa_3 = {k_1}^3 K_3 + 3k_2 k_1 K_2 + k_3 K_1,$$
$$\kappa_4 = {k_1}^4 K_4 + 6k_2 {k_1}^2 K_3 + k_4 K_1 + \left(3{k_2}^2 + 4k_1 k_3\right) K_2$$

*interesting prospects – strongly intensive cumulants
Broniowski, Olszewski, PRC 95, 064910 (2017)

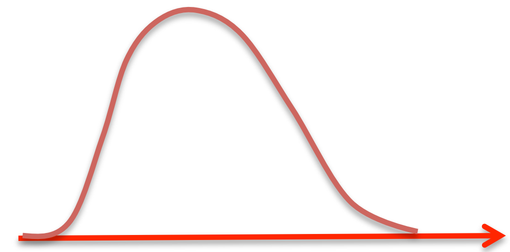
5 Cumulants of a system of independent sources



$X = x_i + x_j + \ldots + x_{N_s}$

source cumulants: $k_n(x)$,
cumulants of source distr.: $K_n(N_S)$

Event-wise cumulants of X-distribution:



$\kappa_1 \ldots \kappa_4$

Rustamov et al., Nucl.Phys.A 960 (2017) 114

$$\kappa_1 = k_1 K_1,$$
$$\kappa_2 = {k_1}^2 K_2 + k_2 K_1,$$
$$\kappa_3 = {k_1}^3 K_3 + 3 k_2 k_1 K_2 + k_3 K_1,$$
$$\kappa_4 = {k_1}^4 K_4 + 6 k_2 {k_1}^2 K_3 + k_4 K_1 + \left(3{k_2}^2 + 4 k_1 k_3\right) K_2$$

$\kappa_5 \ldots \kappa_8$

this work

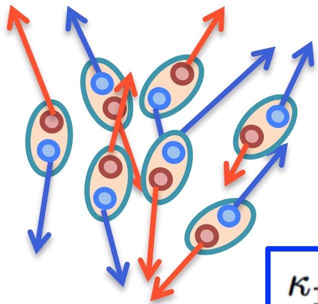
$$\kappa_5 = k_5 K_1 + 5(2k_2k_3 + k_1k_4)K_2 + {k_1}^5 K_5 + 10 k_2 {k_1}^3 K_4 + 5\left(3{k_2}^2 k_1 + 2k_1^2 k_3\right) K_3,$$
$$\kappa_6 = {k_1}^6 K_6 + 15 k_2 {k_1}^4 K_5 + 20 k_3 {k_1}^3 K_4 + 15 k_4 {k_1}^2 K_3 + 45 {k_2}^2 {k_1}^2 K_4 + 60 k_2 k_3 k_1 K_3 + k_6 K_1 + \left(10{k_3}^2 + 15 k_2 k_4 + 6 k_1 k_5\right) K_2 + 15 {k_2}^3 K_3,$$
$$\kappa_7 = {k_1}^7 K_7 + 21 k_2 {k_1}^5 K_6 + 35 k_3 {k_1}^4 K_5 + 35 k_4 {k_1}^3 K_4 + 105 {k_2}^2 {k_1}^3 K_5 + 21 k_5 {k_1}^2 K_3 + 210 k_2 k_3 {k_1}^2 K_4 + 70 {k_3}^2 k_1 K_3 + 105 k_2 k_4 k_1 K_3 + 105 {k_2}^3 k_1 K_4 + k_7 K_1 + 7(5k_3k_4 + 3k_2k_5 + k_1k_6)K_2 + 105 {k_2}^2 k_3 K_3,$$
$$\kappa_8 = {k_1}^8 K_8 + 28 k_2 {k_1}^6 K_7 + 56 k_3 {k_1}^5 K_6 + 70 k_4 {k_1}^4 K_5 + 210 {k_2}^2 {k_1}^4 K_6 + 56 k_5 {k_1}^3 K_4 + 560 k_2 k_3 {k_1}^3 K_5 + 28 k_6 {k_1}^2 K_3 + 280 {k_3}^2 {k_1}^2 K_4 + 420 k_2 k_4 {k_1}^2 K_4 + 420 {k_2}^3 {k_1}^2 K_5 + 280 k_3 k_4 k_1 K_3 + 168 k_2 k_5 k_1 K_3 + 840 {k_2}^2 k_3 k_1 K_4 + k_8 K_1 + \left(35{k_4}^2 + 56 k_3 k_5 + 28 k_2 k_6 + 8 k_1 k_7\right) K_2 + 280 k_2 {k_3}^2 K_3 + 210 {k_2}^2 k_4 K_3 + 105 {k_2}^4 K_4$$

… Fortunately, we are interested in the case when

$k_1 = \langle \Delta n \rangle = \langle n^+ - n^- \rangle = 0$

(at LHC, same number of baryons
and antibaryons at mid-rapidity)

Cumulants of a system of independent sources



$X = x_i + x_i + \ldots + x_{Ns}$

source cumulants: $k_n(x)$,
cumulants of source distr.: $K_n(N_S)$

Event-wise cumulants $\boldsymbol{\kappa_1} \ldots \boldsymbol{\kappa_8}$ *when* $\boldsymbol{\langle \Delta n \rangle = 0}$*:*

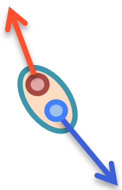
$$
\begin{aligned}
\kappa_1 &= 0, \\
\kappa_2 &= k_2 K_1, \\
\kappa_3 &= k_3 K_1, \\
\kappa_4 &= 3{k_2}^2 K_2 + k_4 K_1, \\
\kappa_5 &= k_5 K_1 + 10 k_2 k_3 K_2, \\
\kappa_6 &= 15{k_2}^3 K_3 + k_6 K_1 + \left(10{k_3}^2 + 15 k_2 k_4\right) K_2, \\
\kappa_7 &= 105 k_3 {k_2}^2 K_3 + k_7 K_1 + 7(5 k_3 k_4 + 3 k_2 k_5) K_2, \\
\kappa_8 &= 105{k_2}^4 K_4 + 210 k_4 {k_2}^2 K_3 + 280 {k_3}^2 k_2 K_3 + k_8 K_1 + \left(35{k_4}^2 + 56 k_3 k_5 + 28 k_2 k_6\right) K_2.
\end{aligned}
$$

Cumulant ratios, for instance, **4-to-2**:

$$
\frac{\kappa_4}{\kappa_2}(\Delta N) = \frac{k_4}{k_2}(\Delta n) + 3 k_2(\Delta n) \frac{K_2(N_S)}{\langle N_S \rangle}
$$

… but we need cumulants k_4, k_2 of a *single source*…

5 Cumulants of a system with independent + – sources



If the sources are particle-antiparticle pairs, then for a single source:

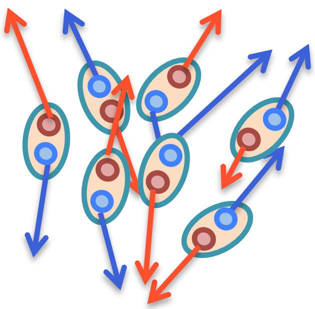
$$k_4(\Delta n) = k_2(\Delta n) - 3k_2^2(\Delta n)$$

Δn – for one source
ΔN – for whole system

… it can be shown using *factorial moments of **Δn:***

$$K_4 = N - 6K_1^4 + \cancel{F_{04}} + 6\cancel{F_{03}} + 7\cancel{F_{02}} - 2F_{11} - 6\cancel{F_{12}} - 4\cancel{F_{13}} + 7\cancel{F_{20}} - 6\cancel{F_{21}} + 6\cancel{F_{22}} + 6\cancel{F_{30}} - 4\cancel{F_{31}} + \cancel{F_{40}}$$
$$+ 12K_1^2(N + F_{02} - 2F_{11} + \cancel{F_{20}}) - 3(N + \cancel{F_{02}} - 2F_{11} + \cancel{F_{20}})^2 - 4K_1(K_1 - \cancel{F_{03}} - 3\cancel{F_{02}} + 3\cancel{F_{12}} + 3\cancel{F_{20}} - 3\cancel{F_{21}} + \cancel{F_{30}})$$

Bzdak, Koch, PRC86 044904 (2012)

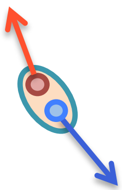


For the whole system:

$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 - 3k_2(\Delta n)\left(1 - \frac{K_2(N_S)}{\langle N_S \rangle}\right)$$

V.F.

Cumulants of a system with independent + – sources



If the sources are particle-antiparticle pairs, then for a single source:

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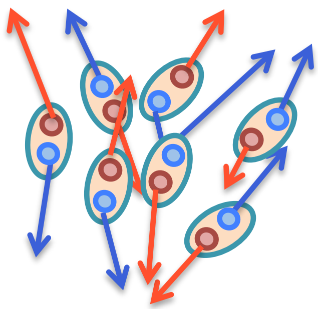
$$+ 12K_1^2(N + F_{02} - 2F_{11} + \cancel{F_{20}}) - 3(N + \cancel{F_{02}} - 2F_{11} + \cancel{F_{20}})^2 - 4K_1(K_1 - \cancel{F_{03}} - 3\cancel{F_{02}} + 3\cancel{F_{12}} + 3\cancel{F_{20}} - 3\cancel{F_{21}} + \cancel{F_{30}})$$

Bzdak, Koch, PRC86 044904 (2012)



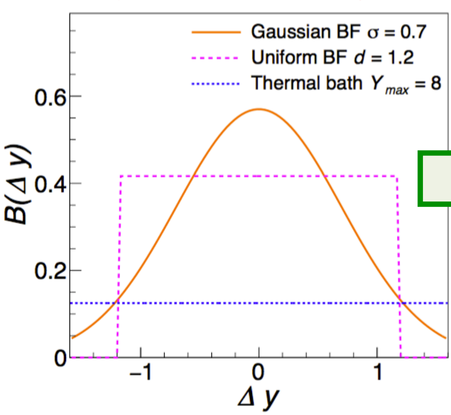
For the whole system:

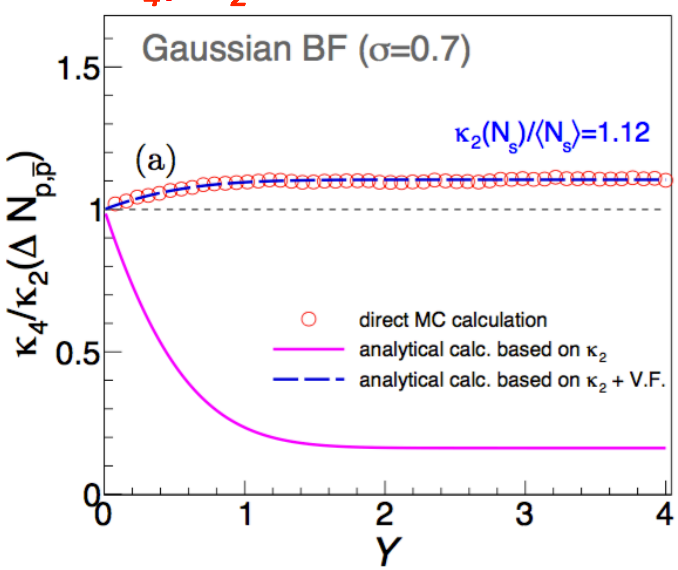
$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 - 3k_2(\Delta n)\left(1 - \frac{K_2(N_S)}{\langle N_S\rangle}\right)$$

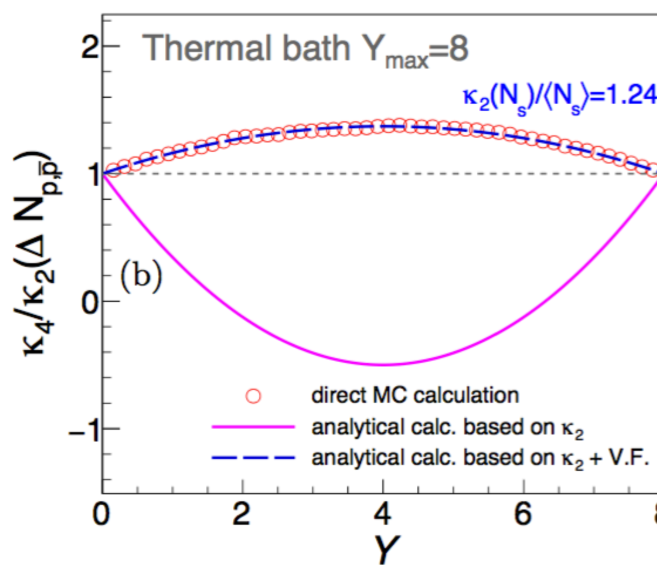
V.F.

κ₄ / κ₂:

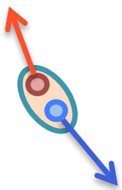
simple models of BF:







…Can go higher: **6-to-2 cumulant ratio**



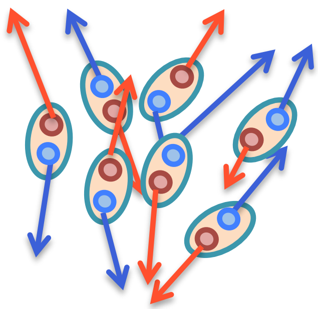
If the sources are particle-antiparticle pairs, then for a single source:

$$k_4(\Delta n) = k_2(\Delta n) - 3k_2^2(\Delta n)$$

Δn – for one source

ΔN – for whole system

$$k_6(\Delta n) = k_2(\Delta n)\left[1 - 15k_2(\Delta n) + 30k_2^2(\Delta n)\right]$$



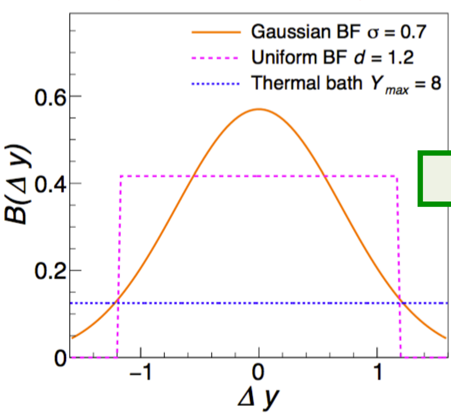
$$\kappa_6(\Delta N) = 15{k_2}^3 K_3 + \kappa_6 K_1 + (15\kappa_2\kappa_4)K_2$$

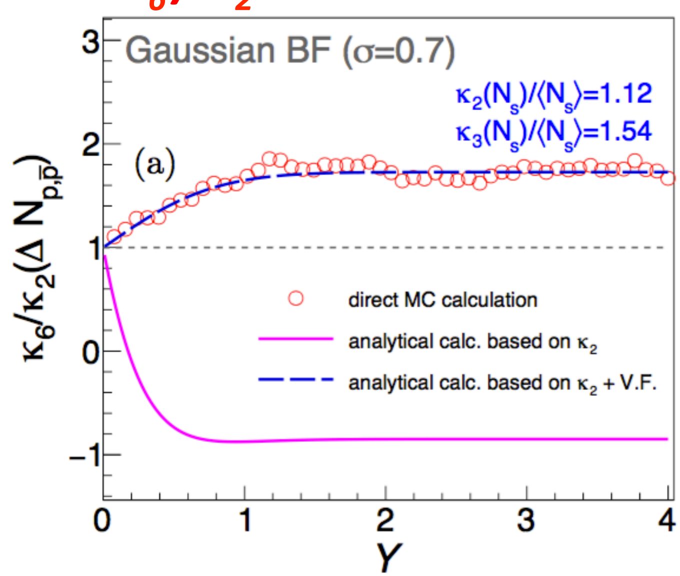
$$\frac{\kappa_6}{\kappa_2}(\Delta N) = 1 - 15k_2 + 30k_2^2 + 15{k_2}^2\frac{K_3(N_S)}{\langle N_S\rangle} + 15k_2(1-3k_2)\frac{K_2(N_S)}{\langle N_S\rangle}$$

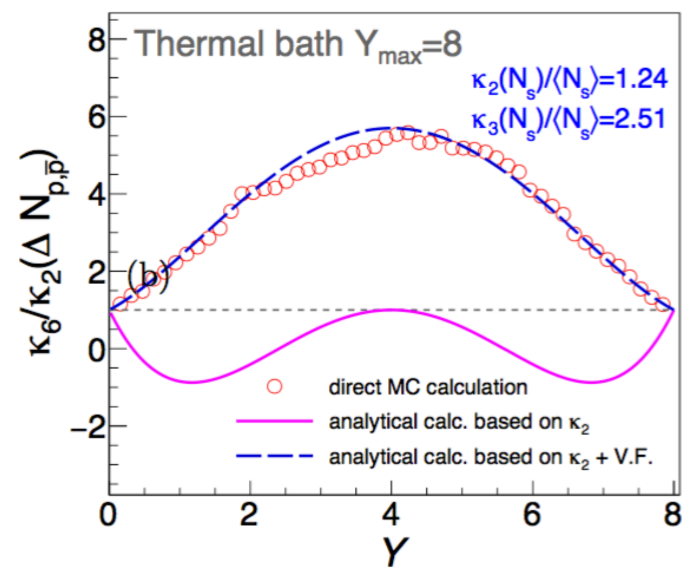
V.F. V.F.

κ_6/κ_2:

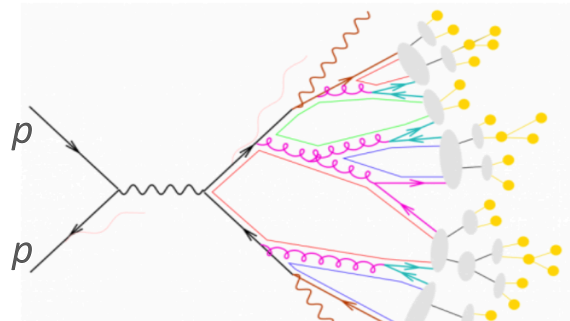
simple models of BF:







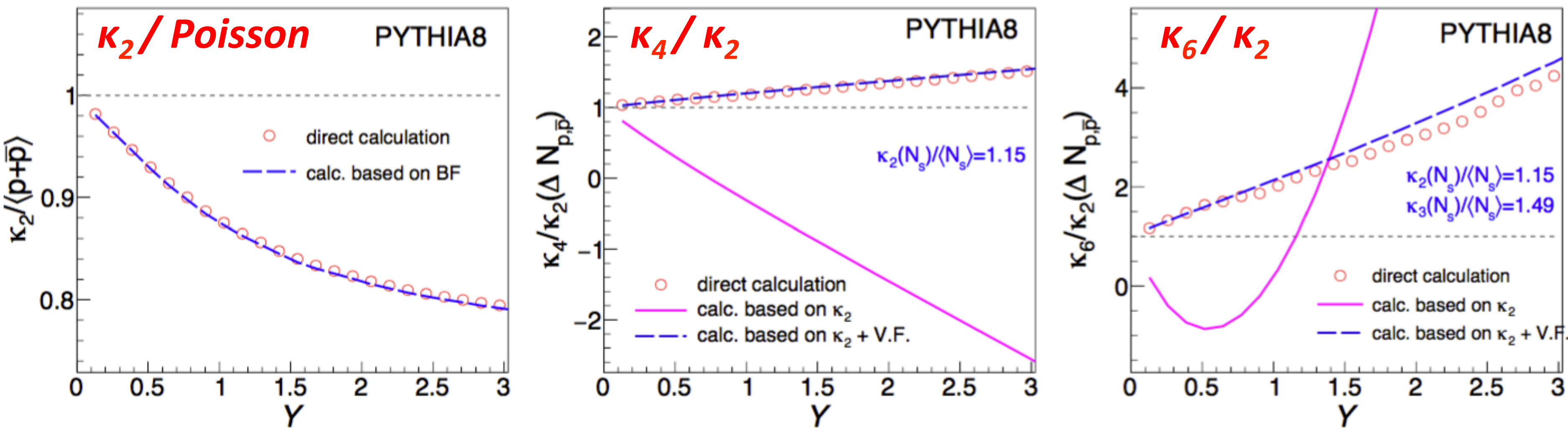
Does this picture hold in *realistic* model (PYTHIA)?



Sjöstrand et al. JHEP05 (2006) 026
Comput. Phys. Comm. 178 (2008) 852

Procedure:

- Calculate κ_2 (for example, via balance function)
- Take number of protons in some acceptance as a *proxy* for number of sources.
- Calculate 4-to-2 and 6-to-2 ratios by formulae shown above.



- Good correspondance between *direct* analysis of higher-order cumulants and *calculations based on* κ_2 .
- Locality of particle-antiparticle pairs creation allows, via balance function, to estimate the higher-order cumulant ratios. These ratios can be considered as baseline when comparing with experiment.

Summary

- Event-by-event measurements help to characterize the properties of the "bulk" of the system and also are closely related to dynamics of the phase transitions.

- **The cumulants of net-protons** are used as a proxy for net-baryons.
  - measured up to 4th order at RHIC – non-monotonic behaviour with energy observed.
  - very experimentally challenging analysis – need sophisticated corrections and modeling.
  - **baryon number conservation** and **volume fluctuations** influence the experimental measurements, while usually not taken into account in theoretical calculations.

- At LHC energies, there is a direct connection between the 2nd cumulants and the balance functions, which are determined by charge conservation. In this work, higher order cumulant ratios κ_4/κ_2, κ_6/κ_2 were calculated via balance function integral under assumption of local independent production of particle-antiparticle pairs (**local baryon conservation).**

- Obtained values may be considered as a baseline for comparison with experimental data.

paper: I.A., arXiv:2002.11398

Thank you for your attention!

This work is supported by the Russian Science Foundation, grant 17-72-20045.