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Lightest strange resonance precision determination from a dispersive analysis of data

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A.Rodas

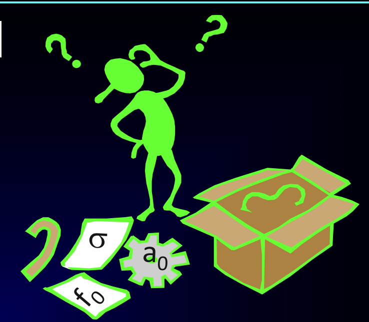
arXiv:2001.08153
and Physics Reports in preparation

Excited QCD 2020, Krynica Zroj- Poland 2-8/01/2020.

Supported by:

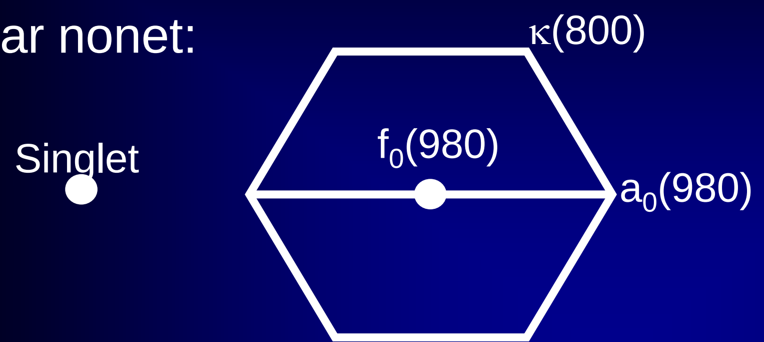
MOTIVATION: The light scalar controversy.

- Scalar SU(3) multiplets identification controversial



- Too many or too few resonances for decades
  But there is an emerging picture

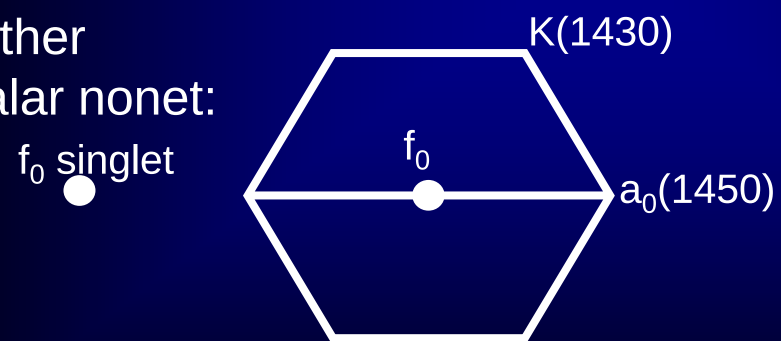
A Light scalar nonet:



Non-strange heavier!!
Inverted hierarchy problem
For quark-antiquark

f_0(500) and $f_0(980)$ are really octet/singlet mixtures

+ Another heavier scalar nonet:



+ glueball

f_0

Enough f_0 states have been observed: $f_0(1370)$, $f_0(1500)$, $f_0(1700)$.
The whole picture is complicated by mixture between them (lots of works here)

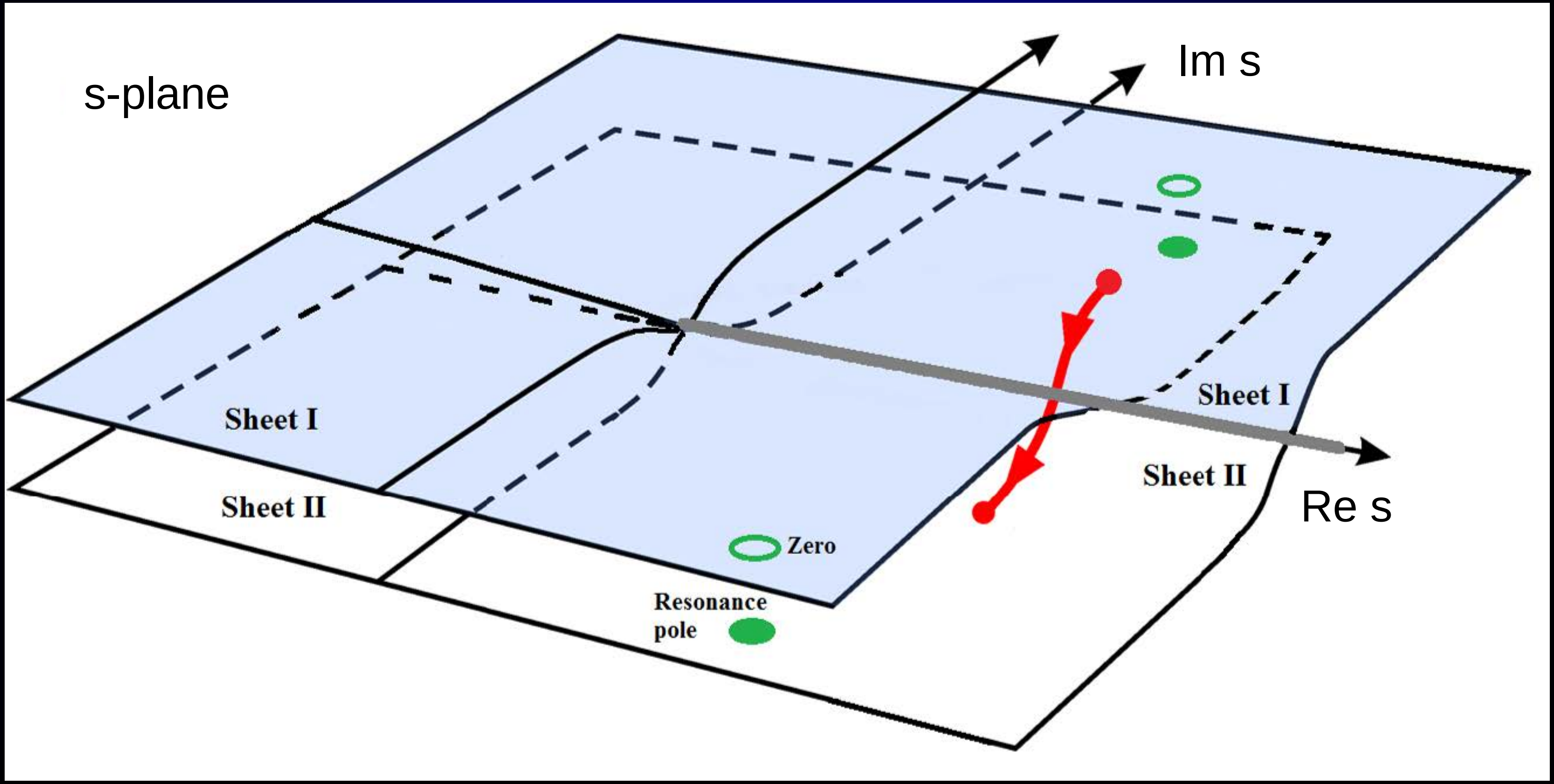
Today only the κ(800) or K0*(800) still "Needs Confirmation" @ PDG

The Breit-Wigner shape is just an approximation for narrow and isolated resonances

The universal features of resonances are their pole positions and residues *

$\sqrt{s_{pole}} \approx M - i\,\Gamma/2$

*in the Riemann sheet obtained from an analytic continuation through the physical cut



Overview of the K_0*(800) or “kappa” meson until 2018 @PDG

- Omittted from the 2017PDG summary table since, “needs confirmation”

But, all descriptions of data respecting unitarity and chiral symmetry find a pole at M=650-770 MeV and Γ~550 MeV or larger.

Best determination comes from a SOLUTION (NO I=1/2, J=0 data below 800 MeV) of a Roy-Steiner dispersive formalism, consistent with UChPT

Decotes Genon et al 2006

2017PDG dominated by such a SOLUTION

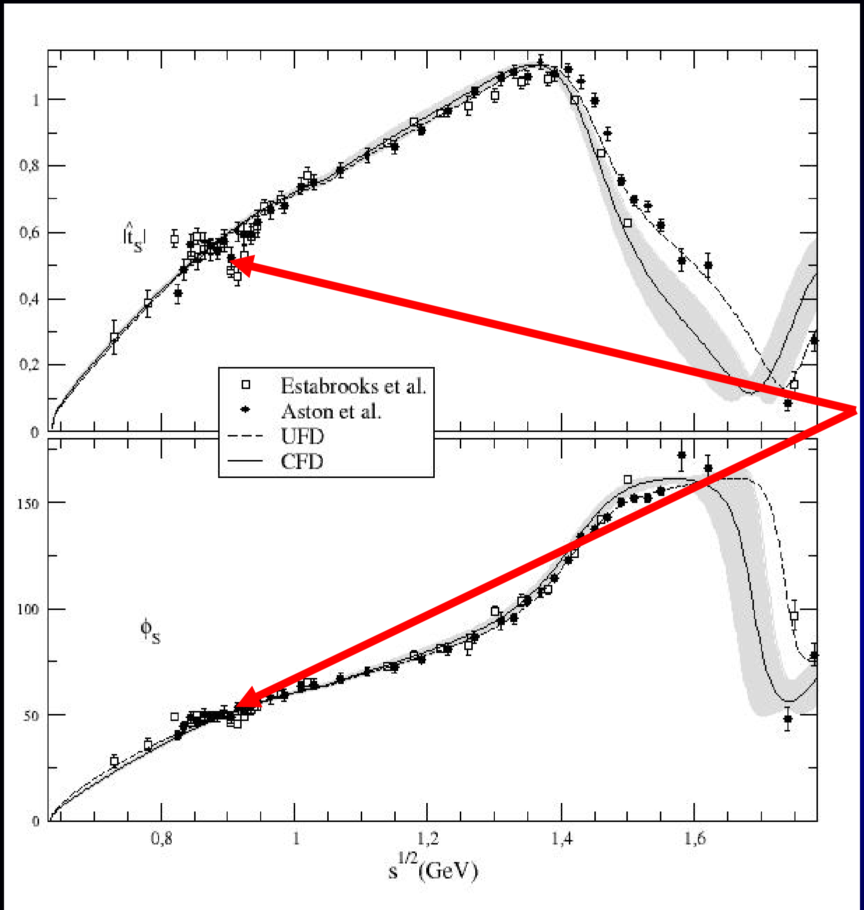
$$M-i\,\Gamma/2=(682\pm 29)-i(273\pm i12)\ \text{MeV} \quad @PDG2017$$

PDG willing to reconsider situation.. **if** additional independent dispersive DATA analysis.

We were encouraged

by PDG members to do it.

Data on πK scattering: S-channel



Most reliable sets:
Estabrooks et al. 78 (SLAC)
Aston et al.88 (SLAC-LASS)

I=1/2 and 3/2 combination

No clear "peak" or phase movement of $\kappa/K_0^*(800)$ resonance

Definitely NO BREIT-WIGNER shape

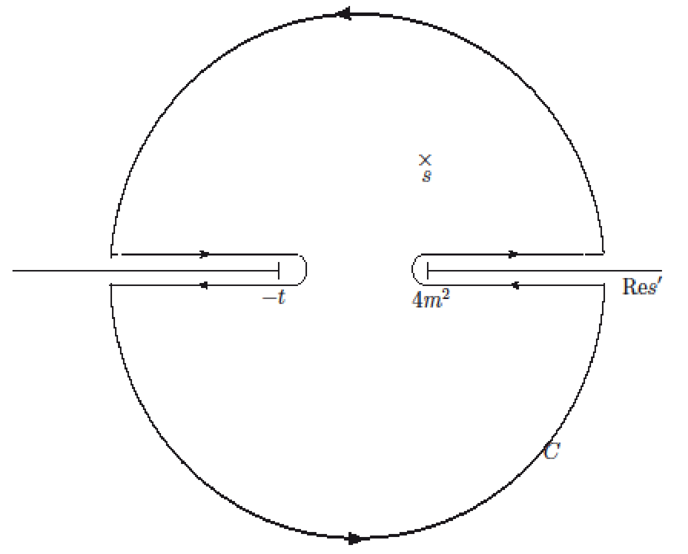
Mathematically correct to use POLES

Strong support for K_0*(700) from decays of heavier mesons, but rigorous model-independent extractions absent. Often inadequate Breit-Wigner formalism

POLE extraction rigorous when using Dispersion Relations
or complex-analyticity properties

Why use dispersion relations?

CAUSALITY:
Amplitudes T(s,t) are ANALYTIC in complex s plane but for cuts for thresholds.
Crossing implies left cut from u-channel threshold



Cauchy Theorem determines T(s,t) at ANY s, from an INTEGRAL on the contour

If T->0 fast enough at high s, curved part vanishes

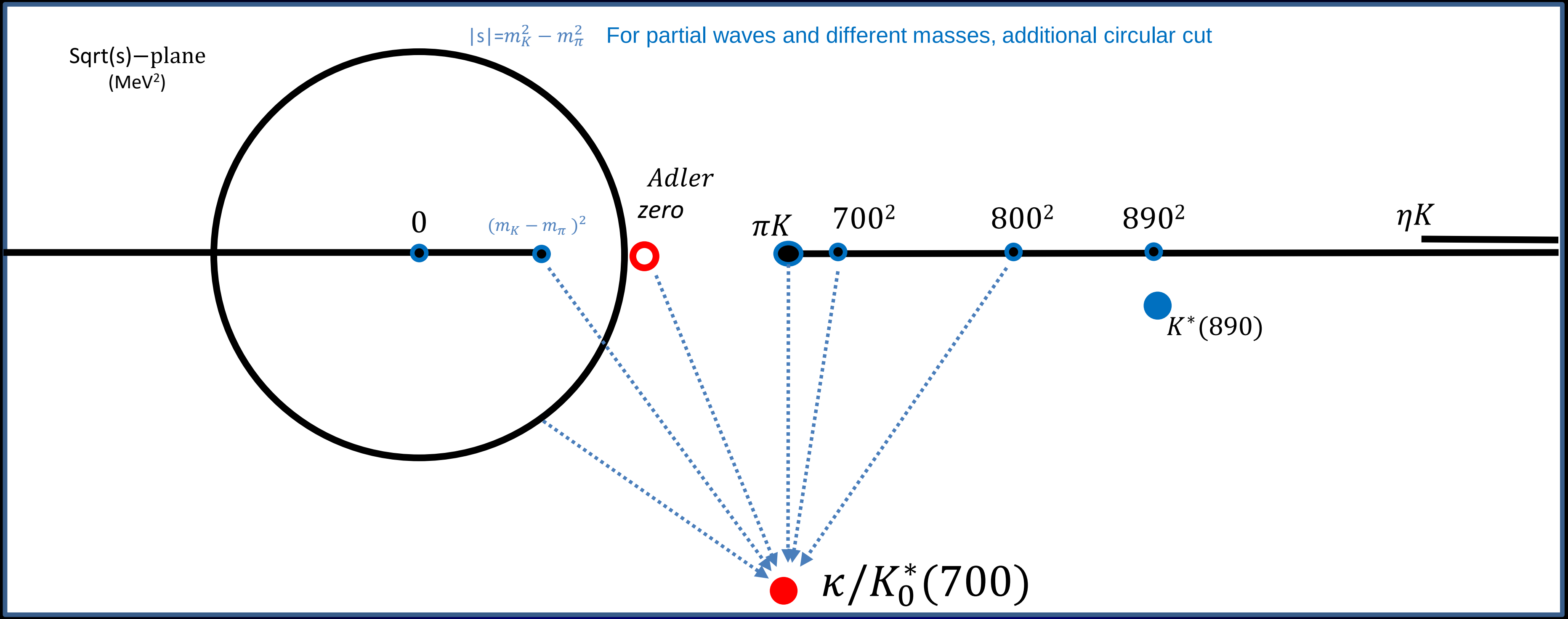
$$T(s,t,u) = \underbrace{\frac{1}{\pi}\int_{4m^2}^{\infty} ds' \frac{\mathrm{Im}\, T(s',t,u')}{s'-s}}_{\text{Right cut}} + \underbrace{\frac{1}{\pi}\int_{-\infty}^{-t} ds' \frac{\mathrm{Im}\, T(s',t,u')}{s'-s}}_{\text{Left cut}}$$

Otherwise, determined up to a polynomial (subtractions)
Left cut usually a problem

Good for:
1) Calculating T(s,t) where there is not data
2) Constraining data analysis
3) ONLY MODEL INDEPENDENT extrapolation to complex s-plane without extra assumptions

Why so much worries about low energy and CORRECT ANALYTIC STRUCTURE?

Analyticity is expressed in the *s*-variable, not in Sqrt(s)



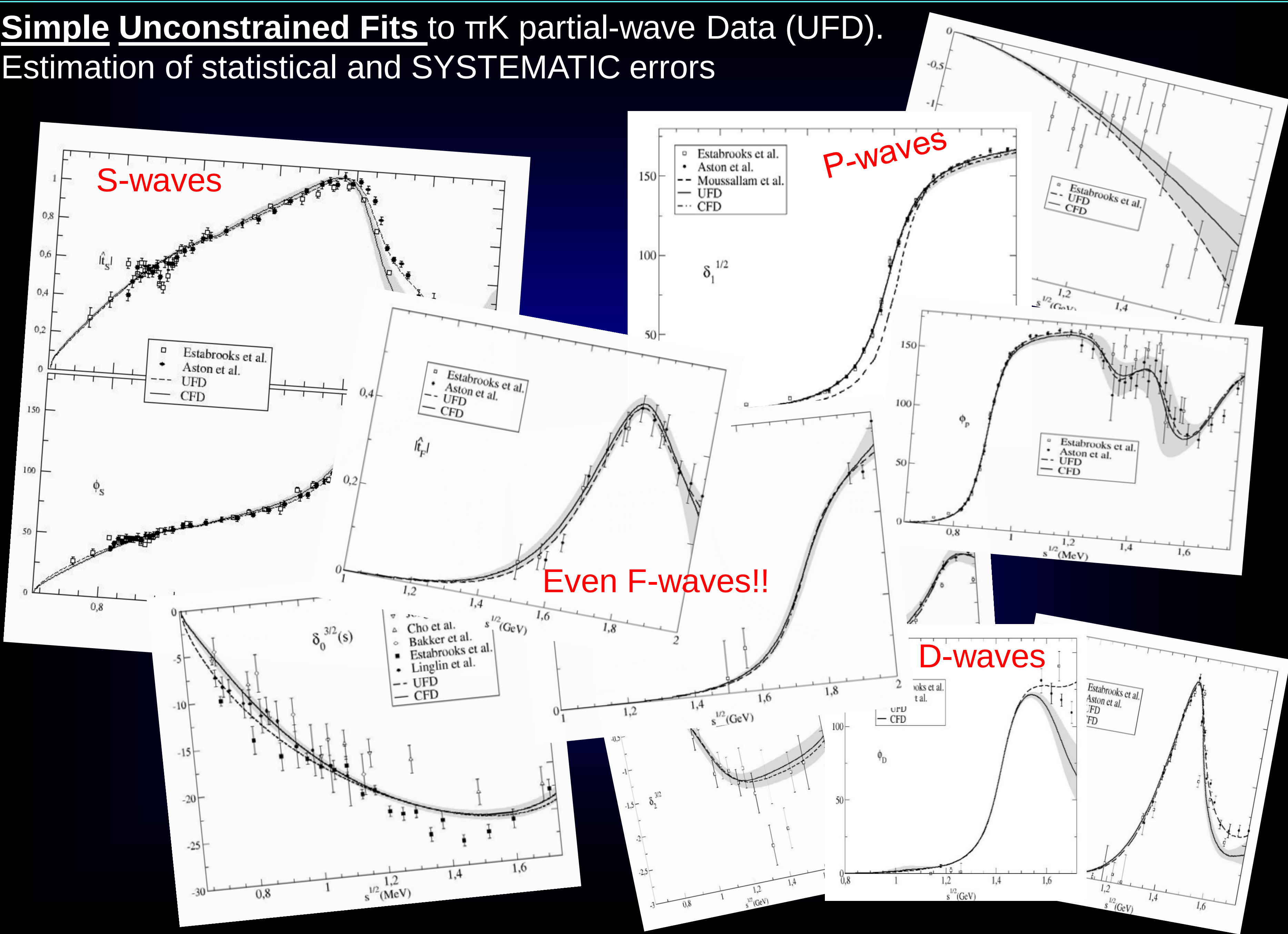
Important for the $\kappa/K_0^*(700)$

- Threshold behavior (chiral symmetry)
- Subthreshold behavior (chiral symmetry →Adler zeros)
- Other cuts (Left & circular)
- Avoid spurious singularities

Less important for other resonances…

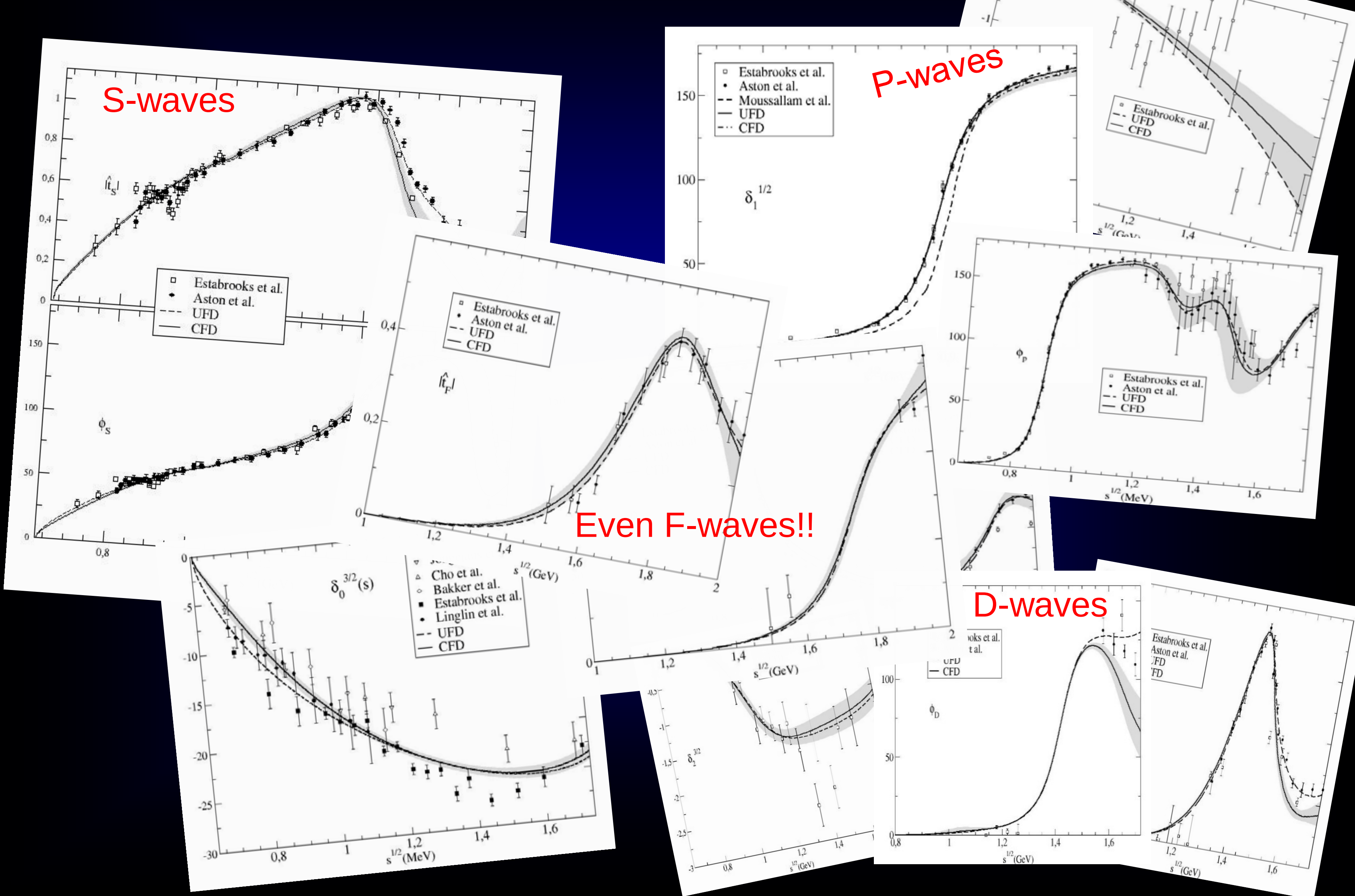
Simple Unconstrained Fits to πK partial-wave Data (UFD).

Estimation of statistical and SYSTEMATIC errors



Our Dispersive/Analytic Approach for πK and strange resonances

Simple Unconstrained Fits to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors



Simple **Unconstrained Fits** to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors

Forward Dispersion Relations:
Left cut easy to rewrite
Relate amplitudes, not partial waves
Not direct access to pole

Forward dispersion relations for K π scattering.

Since interested in the resonance region, we use minimal number of subtractions

Defining the s↔u symmetric and anti-symmetric amplitudes at t=0

$$T^{+}(s) = \frac{T^{1/2}(s) + 2T^{3/2}(s)}{3} = \frac{T^{I_t=0}(s)}{\sqrt{6}},$$

$$T^{-}(s) = \frac{T^{1/2}(s) - T^{3/2}(s)}{3} = \frac{T^{I_t=1}(s)}{2}.$$

We need one subtraction for the symmetric amplitude

$$\mathrm{Re}T^{+}(s) = T^{+}(s_{\mathrm{th}}) + \frac{(s - s_{\mathrm{th}})}{\pi} P \int_{s_{\mathrm{th}}}^{\infty} ds' \left[\frac{\mathrm{Im}T^{+}(s')}{(s'-s)(s'-s_{\mathrm{th}})} - \frac{\mathrm{Im}T^{+}(s')}{(s'+s-2\Sigma_{\pi K})(s'+s_{\mathrm{th}}-2\Sigma_{\pi K})} \right],$$

And none for the antisymmetric

$$\mathrm{Re}T^{-}(s) = \frac{(2s - 2\Sigma_{\pi K})}{\pi} P \int_{s_{\mathrm{th}}}^{\infty} ds' \frac{\mathrm{Im}T^{-}(s')}{(s'-s)(s'+s-2\Sigma_{\pi K})}.$$

where $\Sigma_{\pi K} = m_\pi^2 + m_K^2$

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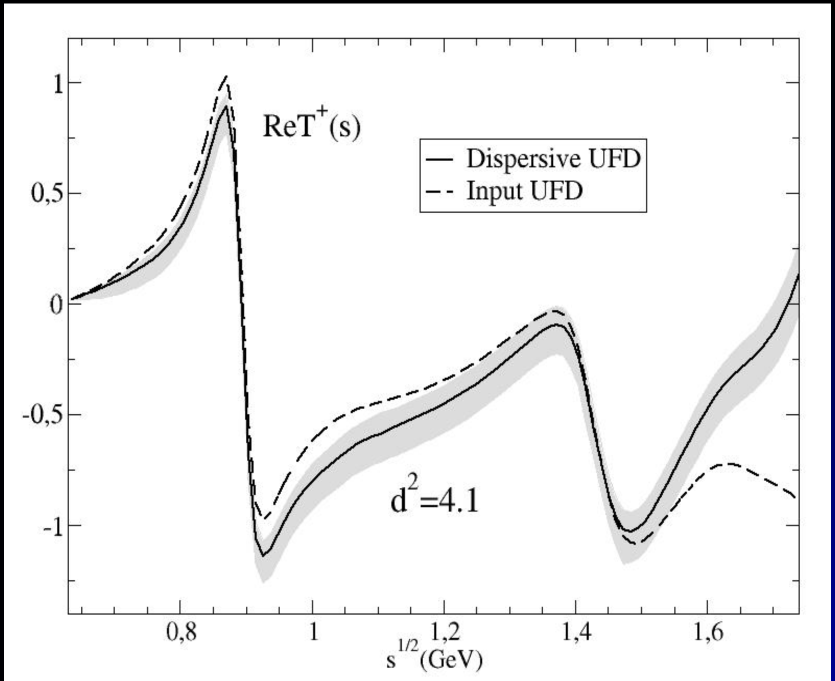

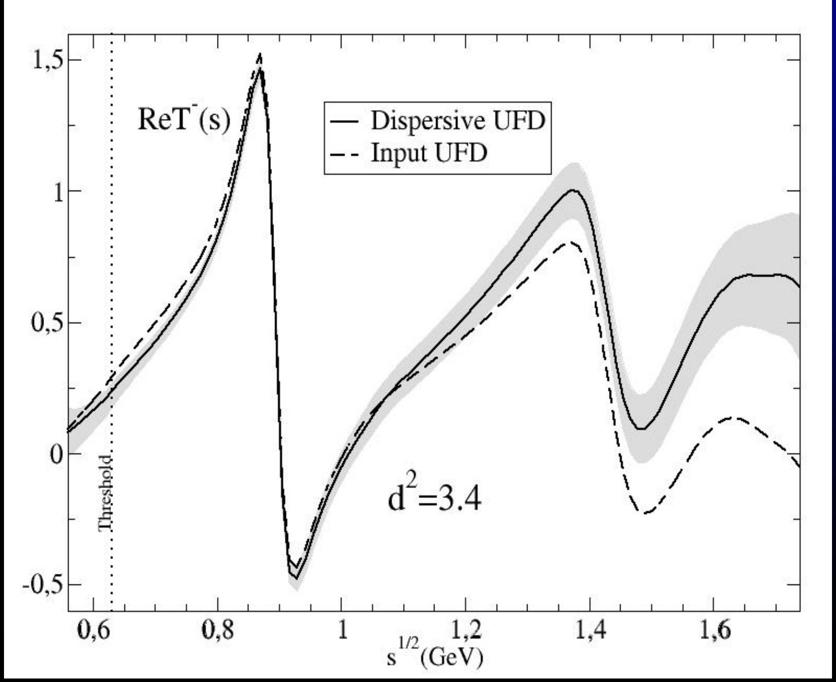

Forward Dispersion Relation
analysis of
πK scattering DATA
up to 1.6 GeV

(**not a solution** of dispersión relations,
but a constrained fit)

A.Rodas & JRP, PRD93,074025 (2016)

First observation:
Forward Dispersion relations
Not well satisfied by data
Particularly at high energies

So we use
Forward Dispersion Relations
as CONSTRAINTS on fits

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- As πK checks: Small inconsistencies.
- As constraints:
  πK consistent fits up to 1.6 GeV JRP, A.Rodas,Phys.Rev. D93 (2016)

Every 22 MeV calculate the difference between both sides of the DR /uncertainty

Define an averaged χ^2 over these points, that we call d^2

> d^2 close to 1 means that the relation is well satisfied
>
> d^2>> 1 means the data set is inconsistent with the relation.

This can be used to check DR

To obtain CONSTRAINED FITS TO DATA (CFD) we minimize:

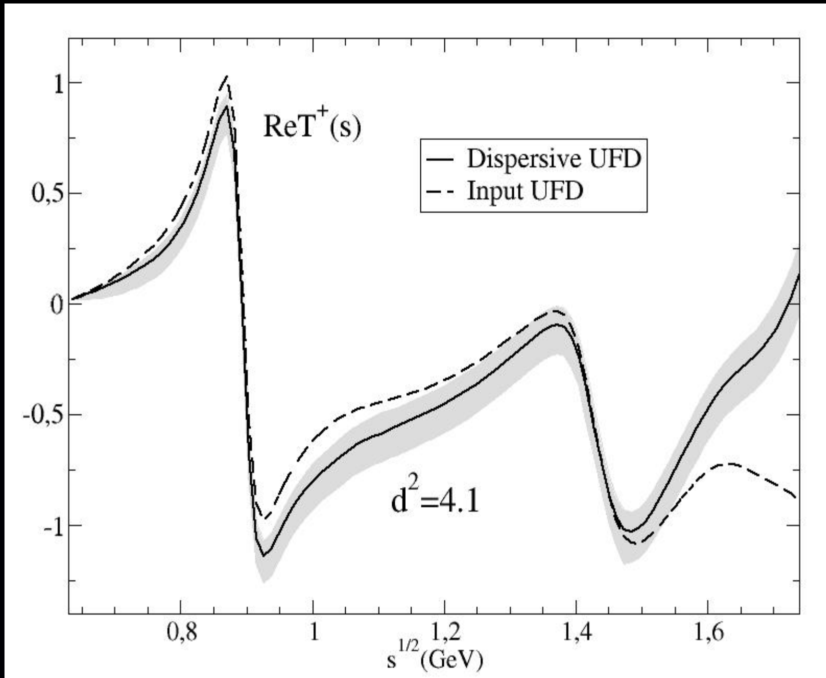
$$W^2\left(d^2_{T+} + d^2_{T-}\right) + \sum_{I=\frac{1}{2},\frac{3}{2}}\left(\frac{\Delta_I}{\delta\Delta_I}\right)^2 + \sum_k\left(\frac{p_k^{UFD} - p_k}{\delta p_k^{UFD}}\right)^2,$$

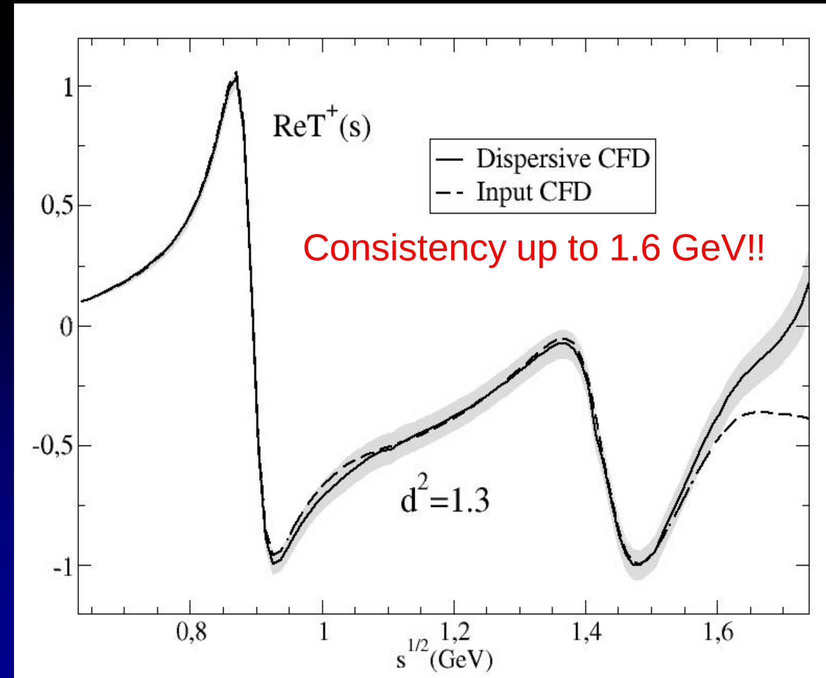
2 FDR's

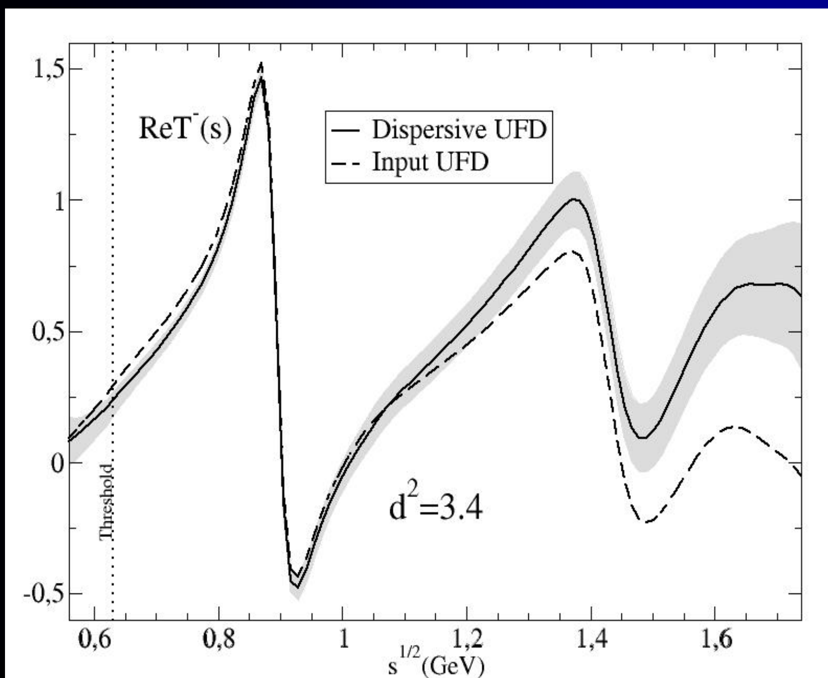
Sum Rules threshold

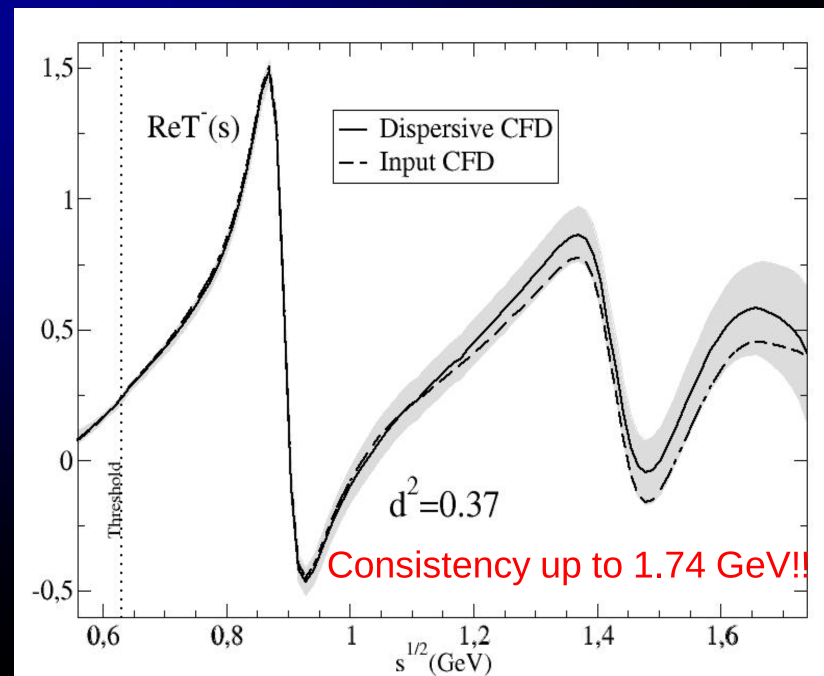
Parameters of the unconstrained data fits

W roughly counts the number
of effective degrees of freedom
(sometimes we add weight on certain energy regions)


ReT$^+$(s)
Dispersive UFD
Input UFD
d^2=4.1
$s^{1/2}$(GeV)


ReT$^+$(s)
Dispersive CFD
Input CFD
Consistency up to 1.6 GeV!!
d^2=1.3
$s^{1/2}$(GeV)


ReT$^-$(s)
Dispersive UFD
Input UFD
Threshold
d^2=3.4
$s^{1/2}$(GeV)

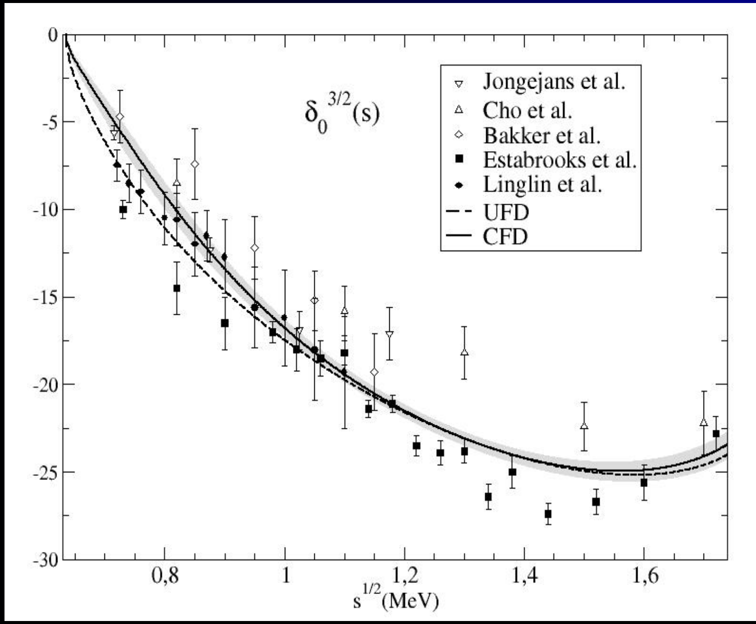

ReT$^-$(s)
Dispersive CFD
Input CFD
Threshold
d^2=0.37
Consistency up to 1.74 GeV!!
$s^{1/2}$(GeV)

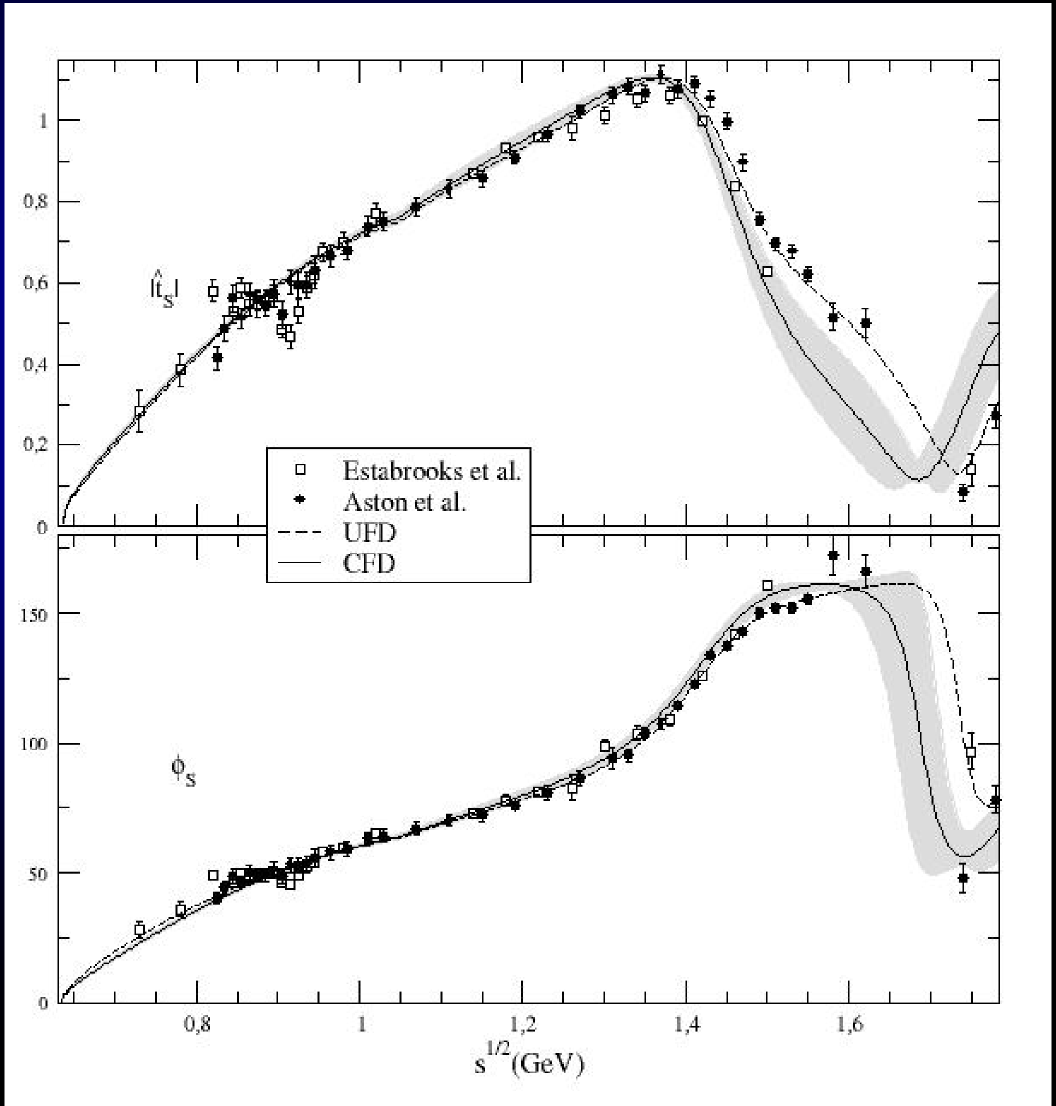
From Unconstrained (UFD) to Constrained Fits to data (CFD)

S-waves. The most interesting for the K_0^* resonances

Largest changes from UFD to CFD

at higher energies





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  eQCD16
- Analytic methods to extract poles: reduced model dependence on strange resonances
  eQCD19
  JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

Kappa pole from CFD

1) Extracted from our conformal CFD parameterization A.Rodas & JRP, PRD93,074025 (2016)

Fantastic analyticity properties, but not model independent

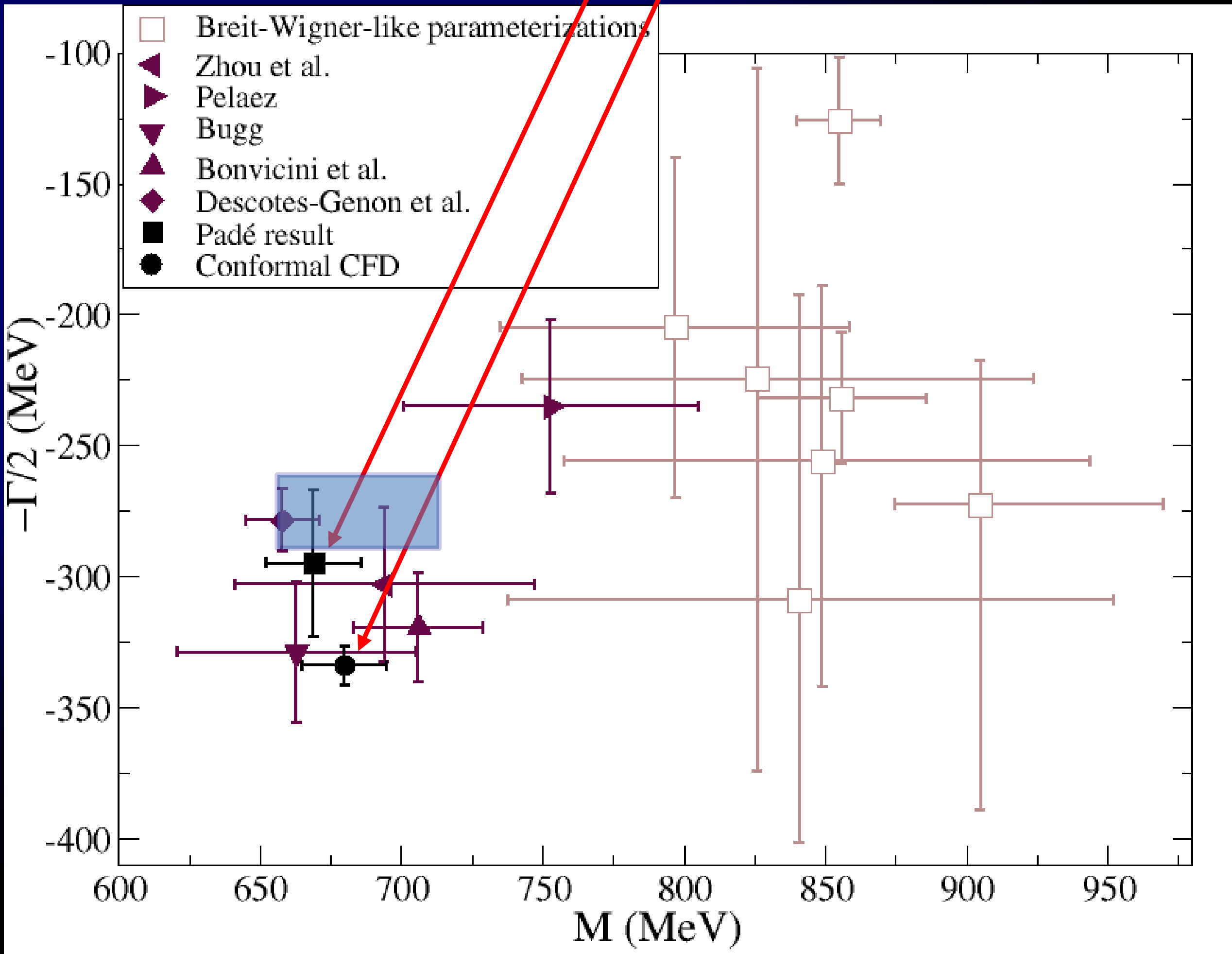
(680±15)-i(334±7.5) MeV

2) Using Padé Sequences…

JRP, A. Rodas & J. Ruiz de Elvira. Eur. Phys. J. C (2017) 77:91

(670±18)-i(295± 28) MeV

Compare to PDG2017:
(682±29)-i(273±12) MeV



The resonance is NO LONGER the κ nor the $K_0^*(800)$

But Still “Needs Confirmation” !

Best analysis so far:
Roy-Steiner
dispersion relations

Plenty of room for improvement on parameters

Our
Pade sequences

Kappa pole analytic determinations from constrained fits

1) Extracted from our conformal CFD parameterization A.Rodas & JRP, PRD93,074025 (2016)

Fantastic analyticity properties, but not model independent

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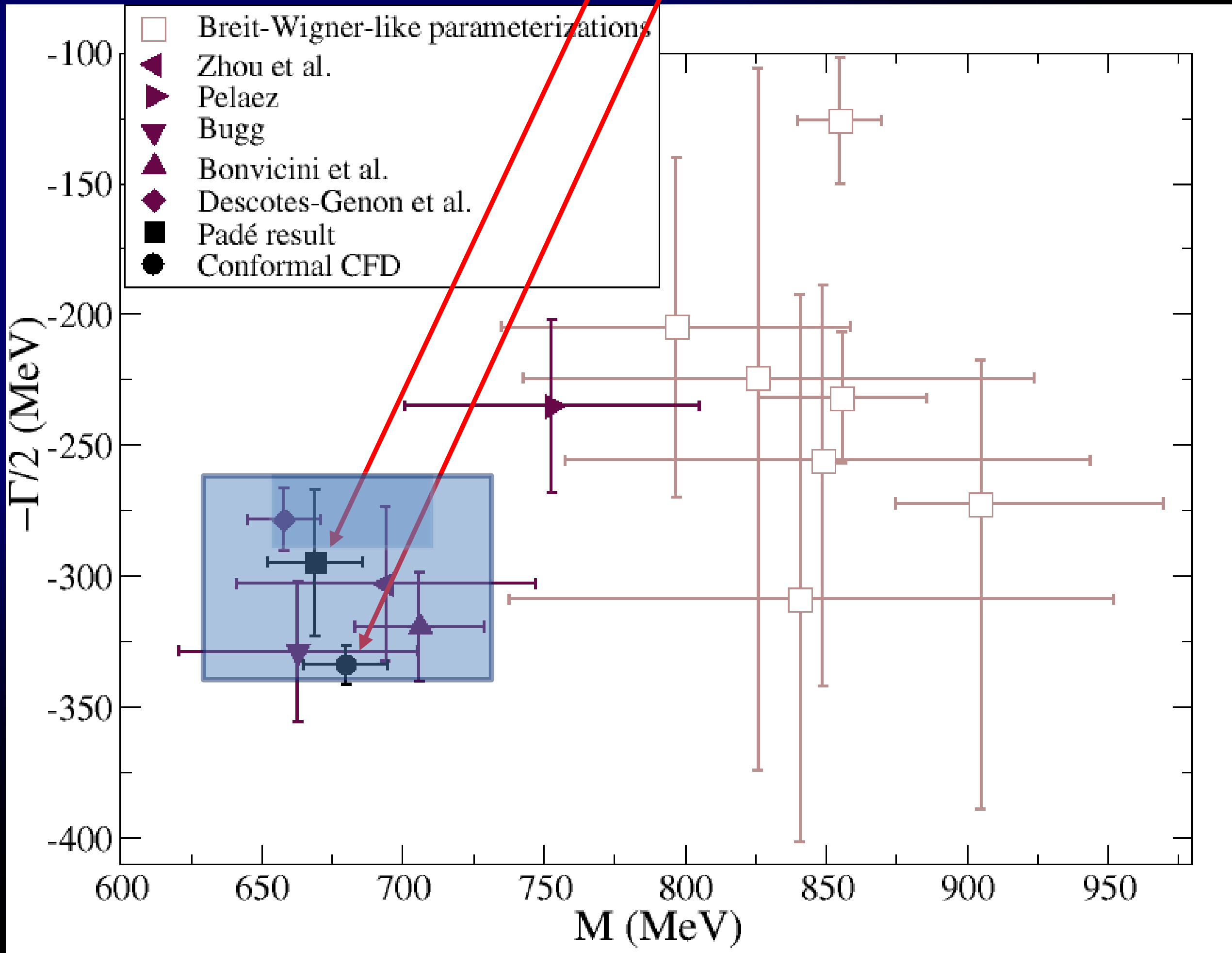

Compare to PDG2017:
(682±29)-i(273±12) MeV

New PDG2018:
(630-730)-i(260-340) MeV
And name changed
K_0^*(700)
Still "Needs Confirmation"

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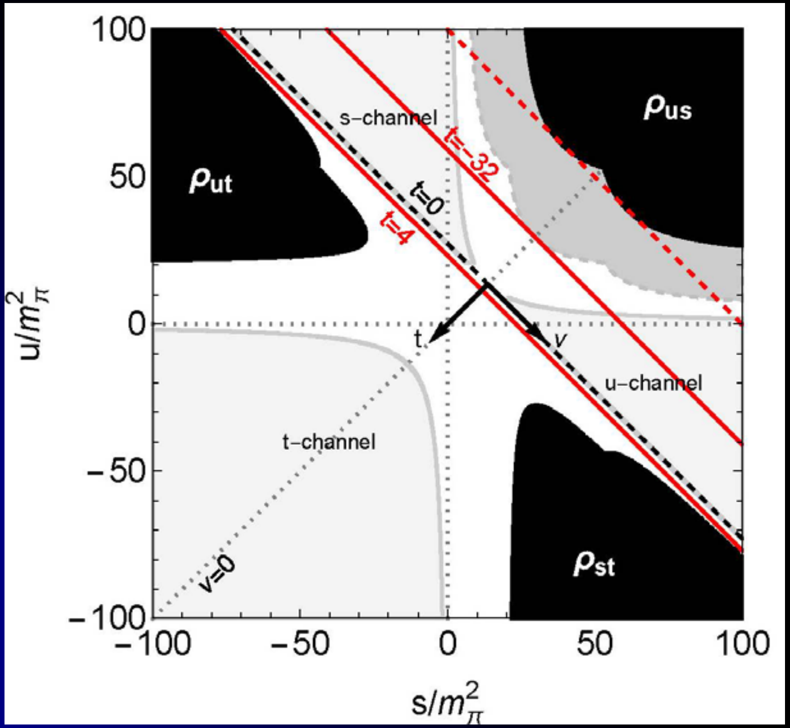
JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

Partial-wave πK Dispersion Relations
Need ππ→KK to rewrite left cut. Range optimized.

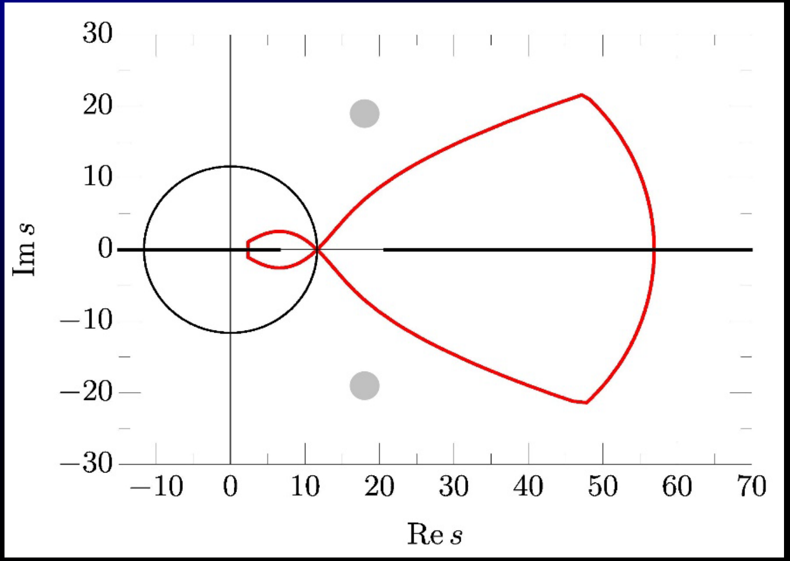
Partial Wave πK→πK and ππ→KK Dispersion Relations (Roy-Steiner eqs.)

To get a resonance pole we need PARTIAL-WAVE dispersion relations.

Their applicability is limited
-by the double spectral regions
-by the Lehmann ellipses
(way too technical. See our apendices)



Two possibilities in the literature:

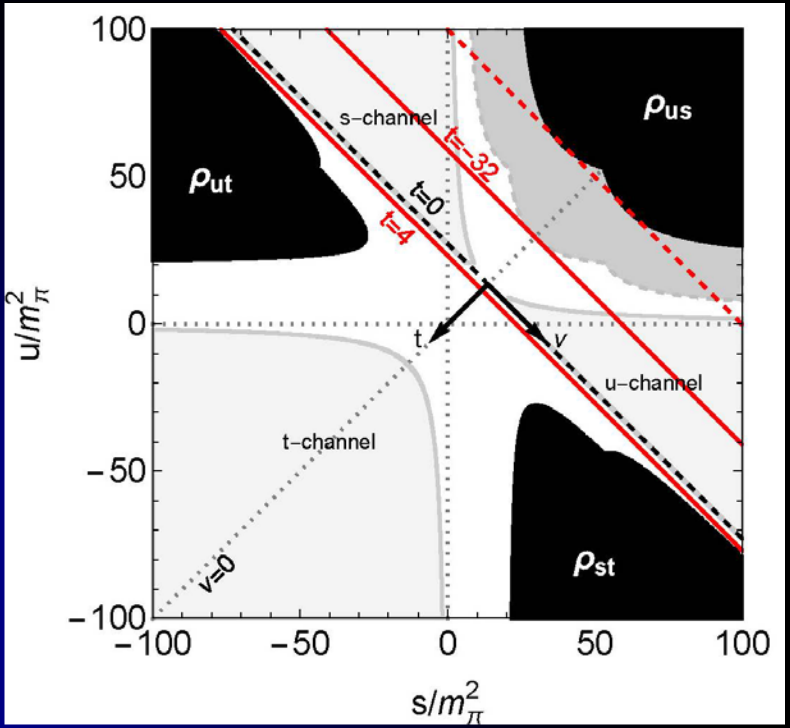


1) Integrate "t" for fixed-t dispersion relations.
Fine for the real axis (1.1 GeV)
but bad to reach the pole.
Were used to obtain solutions by the Paris Group
We will only used them as constraints on data

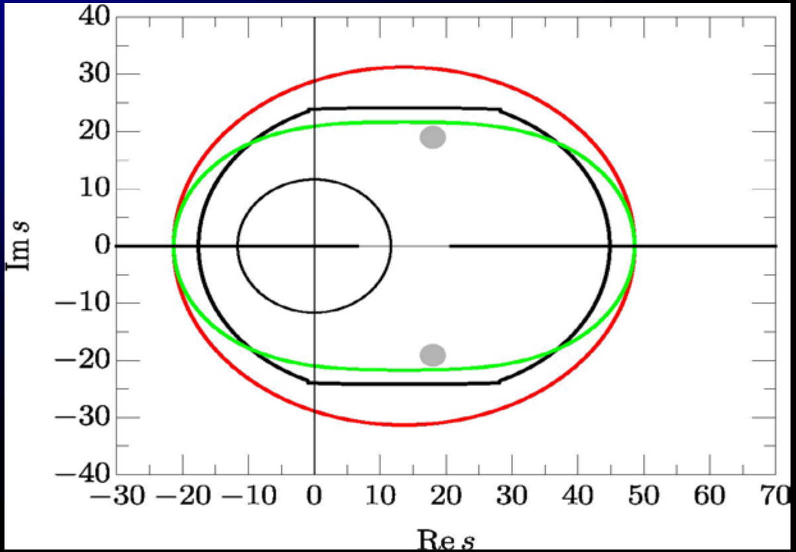
πK→πK and ππ→KK **Hyperbolic Dispersion Relations (HDR)**

2) Integrate along (s-a)(u-a)=b hyperbolae in the Mandelstam plane

We tuned a to maximize applicability for ππ→KK



Applicability range slightly smaller in real axis but covers the kappa pole if a chosen appropriately



We will use them as constraints and to get the pole.

a=-9m_π^2 chosen to include also error bars inside applicability region

πK→πK and ππ→KK Hyperbolic Dispersion Relations (HDR)

g^I_J = ππ → KK partial waves. We study (I,J)=(0,0),(1,1),(0,2)

f^I_J = Kπ → Kπ partial waves. Taken from previous dispersive study

JRP, A. Rodas PRD 2016

$$g_0^0(t) = \frac{\sqrt{3}}{2} m_+ a_0^+ + \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{\mathrm{Im}\, g_0^0(t')}{t'(t'-t)}dt' + \frac{t}{\pi}\sum_{\ell\geq 2}\int_{4m_\pi^2}^\infty \frac{dt'}{t'} G^0_{0,2\ell-2}(t,t')\,\mathrm{Im}\, g^0_{2\ell-2}(t') + \sum_\ell \int_{m_+^2}^\infty ds'\, G^+_{0,\ell}(t,s')\,\mathrm{Im}\, f^+_\ell(s'),$$

$$g_1^1(t) = \frac{1}{\pi}\int_{4m_\pi^2}^\infty \frac{\mathrm{Im}\, g_1^1(t')}{t'-t}dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^\infty dt'\, G^1_{1,2\ell-1}(t,t')\,\mathrm{Im}\, g^1_{2\ell-1}(t') + \sum_\ell \int_{m_+^2}^\infty ds'\, G^-_{1,\ell}(t,s')\,\mathrm{Im}\, f^-_\ell(s'),$$

$$g_2^0(t) = \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{\mathrm{Im}\, g_2^0(t')}{t'(t'-t)}dt' + \sum_{\ell\geq 2}\int_{4m_\pi^2}^\infty \frac{dt'}{t'} G'^0_{2,4\ell-2}(t,t')\,\mathrm{Im}\, g^0_{4\ell-2}(t') + \sum_\ell \int_{m_+^2}^\infty ds'\, G'^+_{2,\ell}(t,s')\,\mathrm{Im}\, f^+_\ell(s'). \quad (39)$$

$G^I_{J,J'}$(t,t') =integral kernels, depend on a parameter

Lowest # of subtractions. Odd pw decouple from even pw.

$$g_\ell^0(t) = \Delta_\ell^0(t) + \frac{t}{\pi}\int_{4m_\pi^2}^\infty \frac{dt'}{t'}\frac{\mathrm{Im}\, g_\ell^0(t)}{t'-t}, \quad \ell = 0, 2,$$

$$g_1^1(t) = \Delta_1^1(t) + \frac{1}{\pi}\int_{4m_\pi^2}^\infty dt' \frac{\mathrm{Im}\, g_1^1(t)}{t'-t}, \quad (40)$$

Δ(t) depend on higher waves or on Kπ→Kπ.

Integrals from 2π threshold !

Solve in descending J order

We have used models for higher waves, but give very small contributions

For unphysical region below KK threshold, we used Omnés function

$$\Omega_\ell^I(t) = \exp\left(\frac{t}{\pi}\int_{4m_\pi^2}^{t_m} \frac{\phi_\ell^I(t')dt'}{t'(t'-t)}\right),$$

$$\Omega_\ell^I(t) \equiv \Omega_{l,R}^I(t) e^{i\phi_\ell^I(t)\theta(t-4m_\pi^2)\theta(t_m-t)},$$

This is the form of our HDR: Roy-Steiner+Omnés formalism

$$g_0^0(t) = \Delta_0^0(t) + \frac{t\Omega_0^0(t)}{t_m - t}\left[\alpha + \frac{t}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{(t_m - t')\Delta_0^0(t')\sin\phi_0^0(t')}{\Omega_{0,R}^0(t')t'^2(t'-t)} + \frac{t}{\pi}\int_{t_m}^{\infty} dt' \frac{(t_m - t')|g_0^0(t')|\sin\phi_0^0(t')}{\Omega_{0,R}^0(t')t'^2(t'-t)}\right]$$

$$g_1^1(t) = \Delta_1^1(t) + \Omega_1^1(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta_1^1(t')\sin\phi_1^1(t')}{\Omega_{1,R}^1(t')(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g_1^1(t')|\sin\phi_1^1(t')}{\Omega_{1,R}^1(t')(t'-t)}\right],$$

$$g_2^0(t) = \Delta_2^0(t) + t\Omega_2^0(t)\left[\frac{1}{\pi}\int_{4m_\pi^2}^{t_m} dt' \frac{\Delta_2^0(t')\sin\phi_2^0(t')}{\Omega_{2,R}^0(t')t'(t'-t)} + \frac{1}{\pi}\int_{t_m}^{\infty} dt' \frac{|g_2^0(t')|\sin\phi_2^0(t')}{\Omega_{2,R}^0(t')t'(t'-t)}\right].$$

We can now check how well these HDR are satisfied

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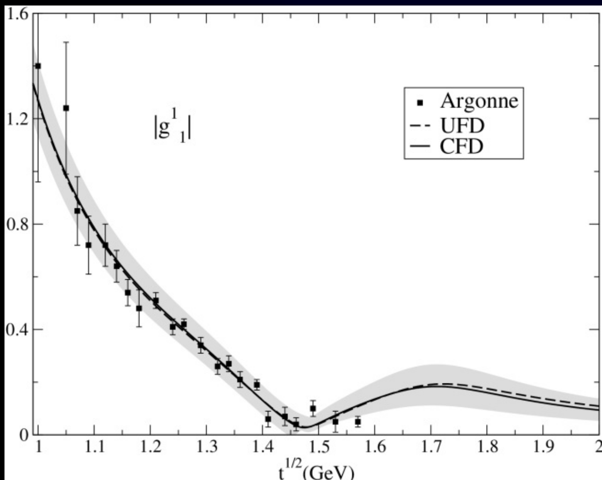
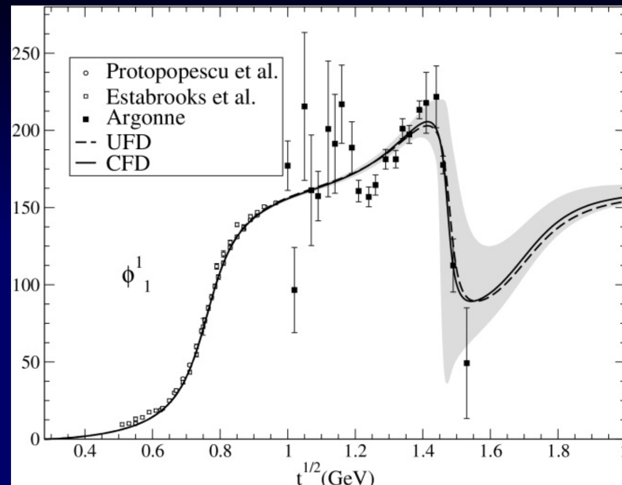
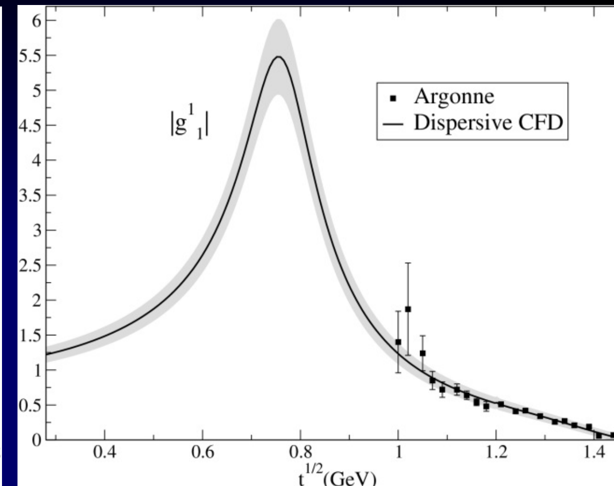
Partial-wave πK Dispersion Relations
Need ππ→KK to rewrite left cut. Range optimized.

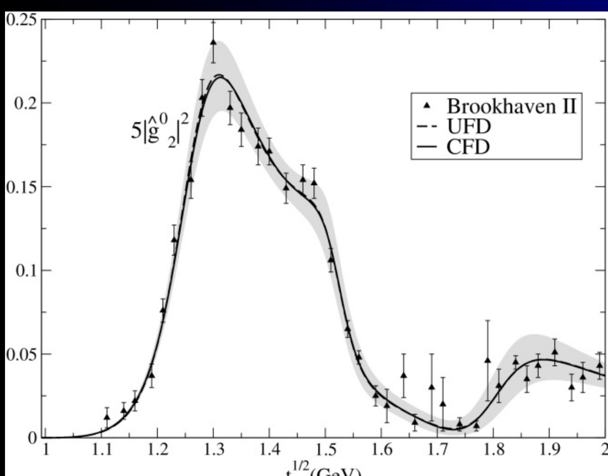
- As ππ→KK checks: Small inconsistencies.
- As constraints:
  ππ→KK consistent fits up to 1.5 GeV
  eQCD19
  JRP, A.Rodas, Eur.Phys.J. C78 (2018)

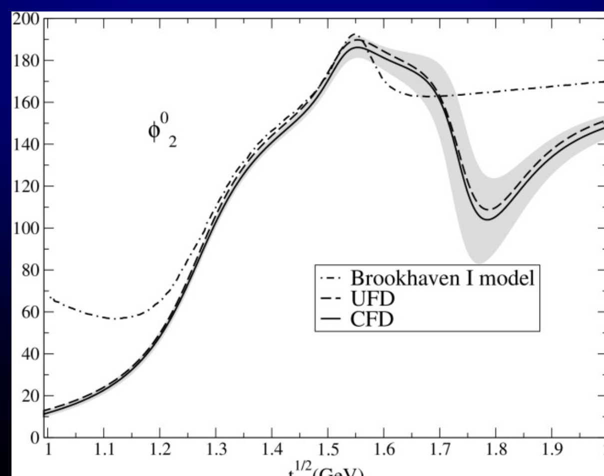
Once agin SIMPLE FITS TO ππ→KK DATA





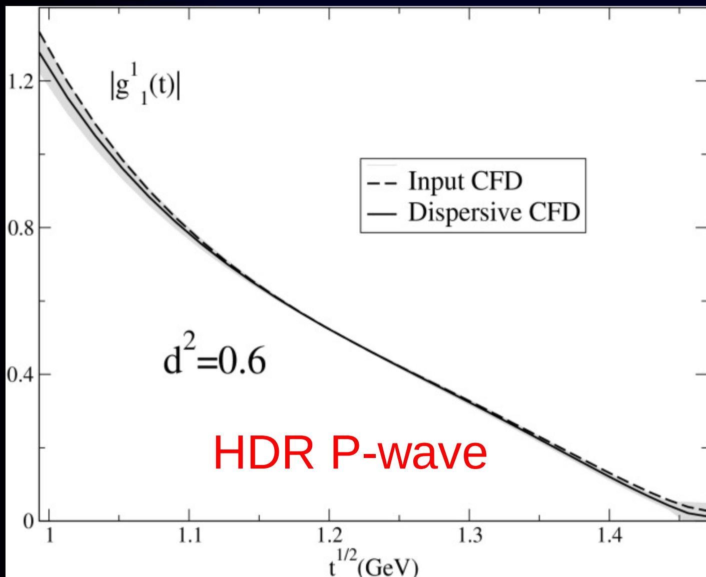


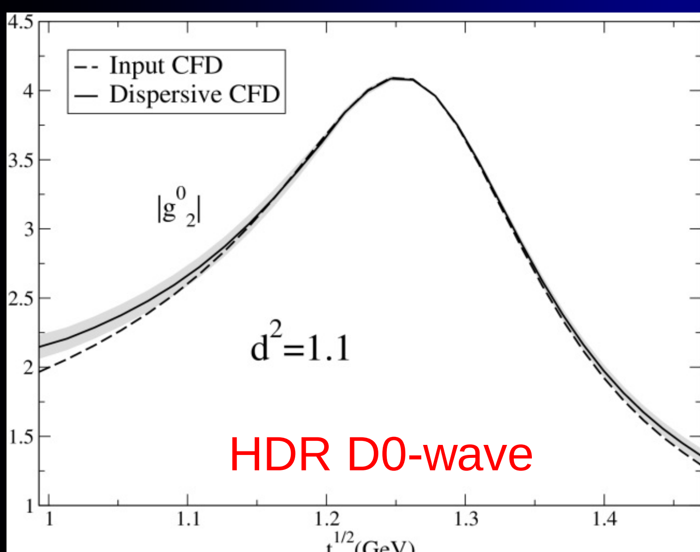




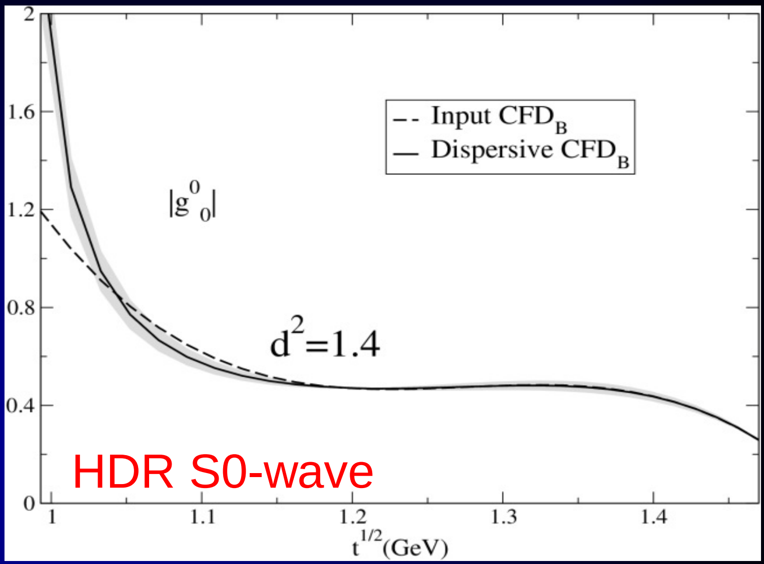
Inconsistent with HDR
If not constrained

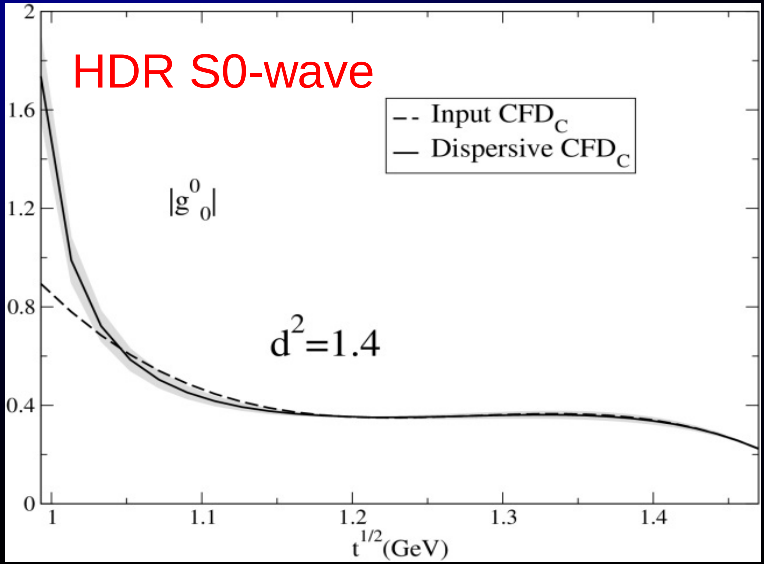
But consistent after HDR used as constraints



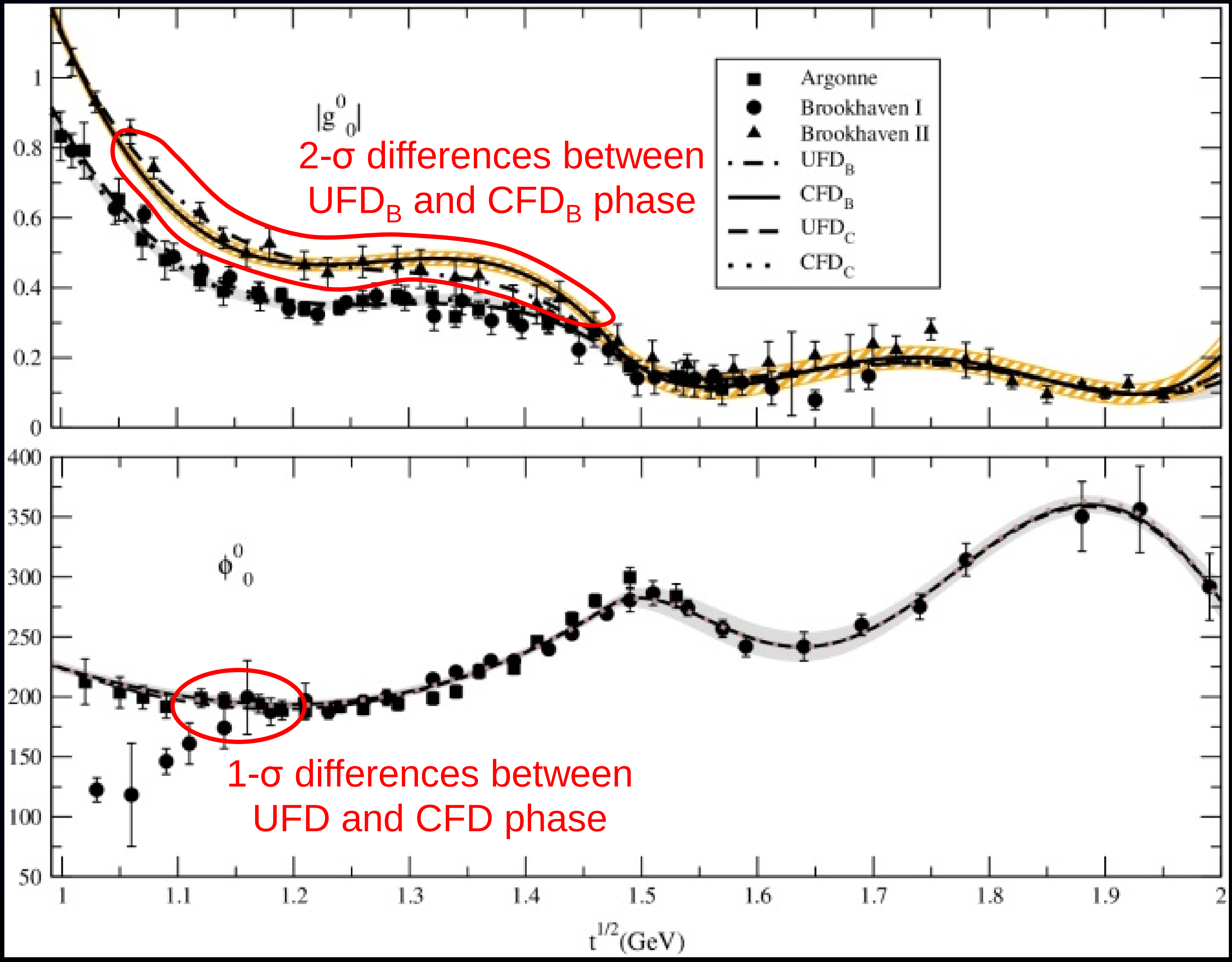


Two possible solutions for S0 wave





Some 2-σ level differences between UFD_B and CFD_B between 1.05 and 1.45 GeV
CFD_C consistent within 1-σ band of UFD_C



Our Dispersive/Analytic Approach for πK and strange resonances

Simple Unconstrained Fits to πK partial-wave Data (UFD).
Estimation of statistical and SYSTEMATIC errors

Forward Dispersion Relations:
Left cut easy to rewrite
Relate amplitudes, not partial waves
Not direct access to pole

- As πK checks: Small inconsistencies.
- As constraints:
  πK consistent fits up to 1.6 GeV JRP, A.Rodas,Phys.Rev. D93 (2016) — eQCD16
- Analytic methods to extract poles: reduced model dependence on strange resonances — eQCD19
  JRP, A. Rodas. J. Ruiz de Elvira, Eur.Phys.J. C77 (2017)

Partial-wave πK Dispersion Relations
Need ππ→KK to rewrite left cut. Range optimized.

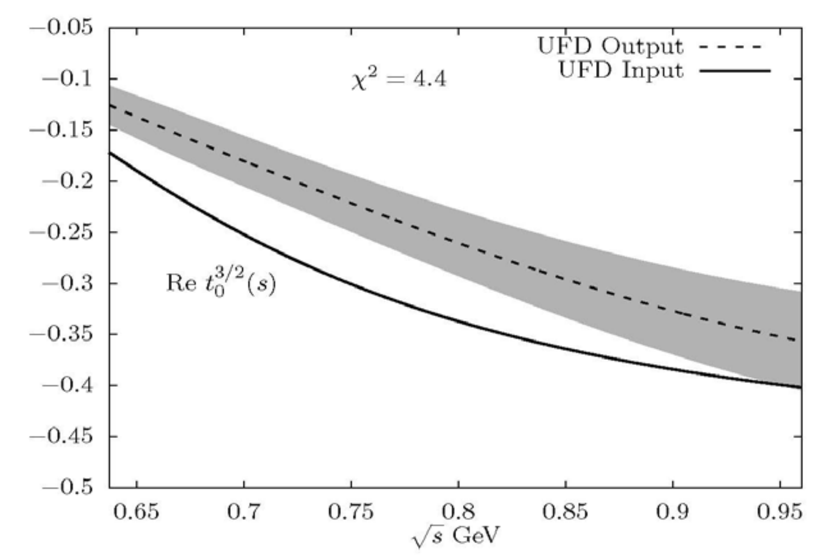
- As ππ→KK checks: Small inconsistencies.
- As constraints:
  ππ→KK consistent fits up to 1.5 GeV — eQCD19
  JRP, A.Rodas, Eur.Phys.J. C78 (2018)

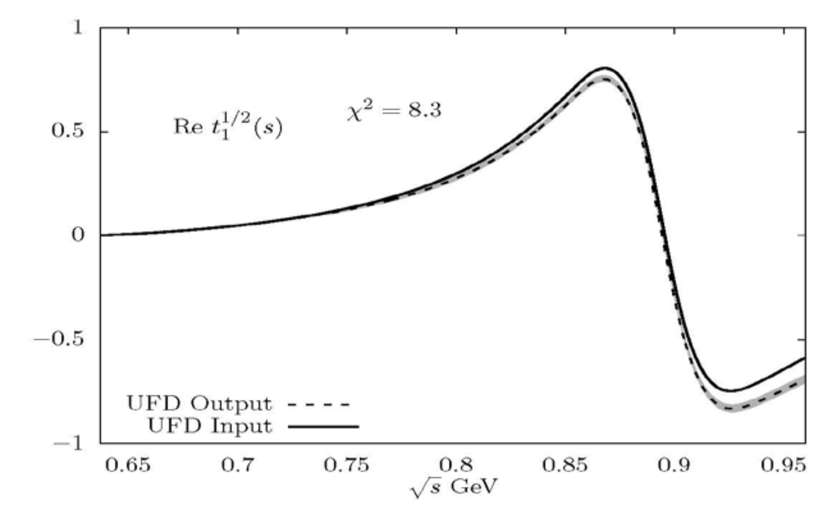
- From fixed-t DR:
  ππ→KK influence small.
  $\kappa/K_0^*(700)$ out of reach
- From Hyperbolic DR:
  ππ→KK influence important.
  JRP, A.Rodas, in progress. PRELIMINARY results shown here

- As πK Checks: Large inconsistencies.

πK Hiperbolic Dispersion Relations I=3/2, J=0 and I=1/2, J=0

SIZABLE inconsistencies of unconstrained fits with the minimal number of subtractions (shown here). Fairly consistent with one more subtraction for F^-



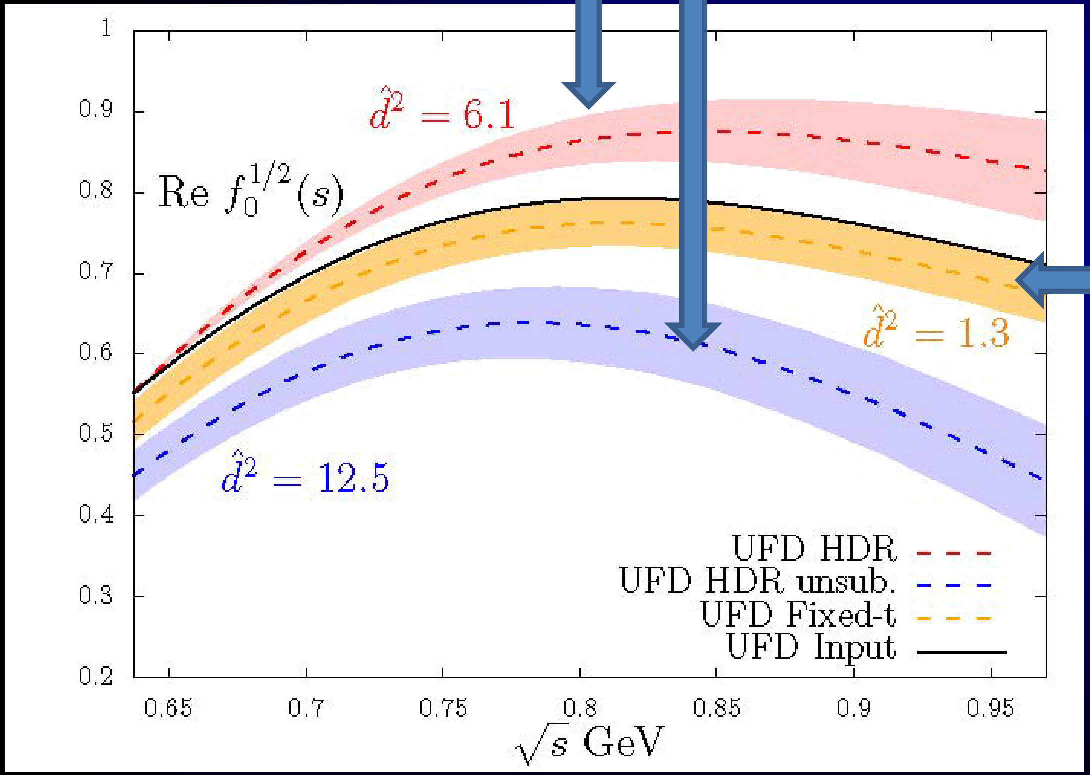


πK Hiperbolic Dispersion Relations I=1/2, J=0

FINAL eQCD2020

The most relevant wave for the kappa resonance.

LARGE inconsistency of HDR Roy-Steiner from unconstrained fits

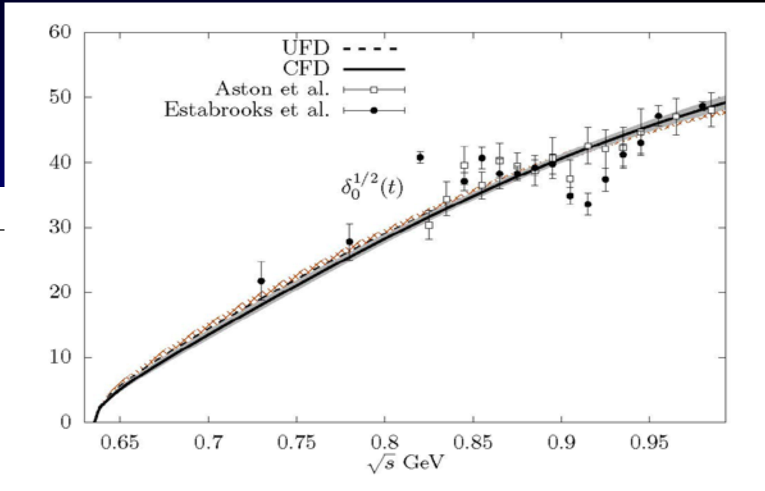


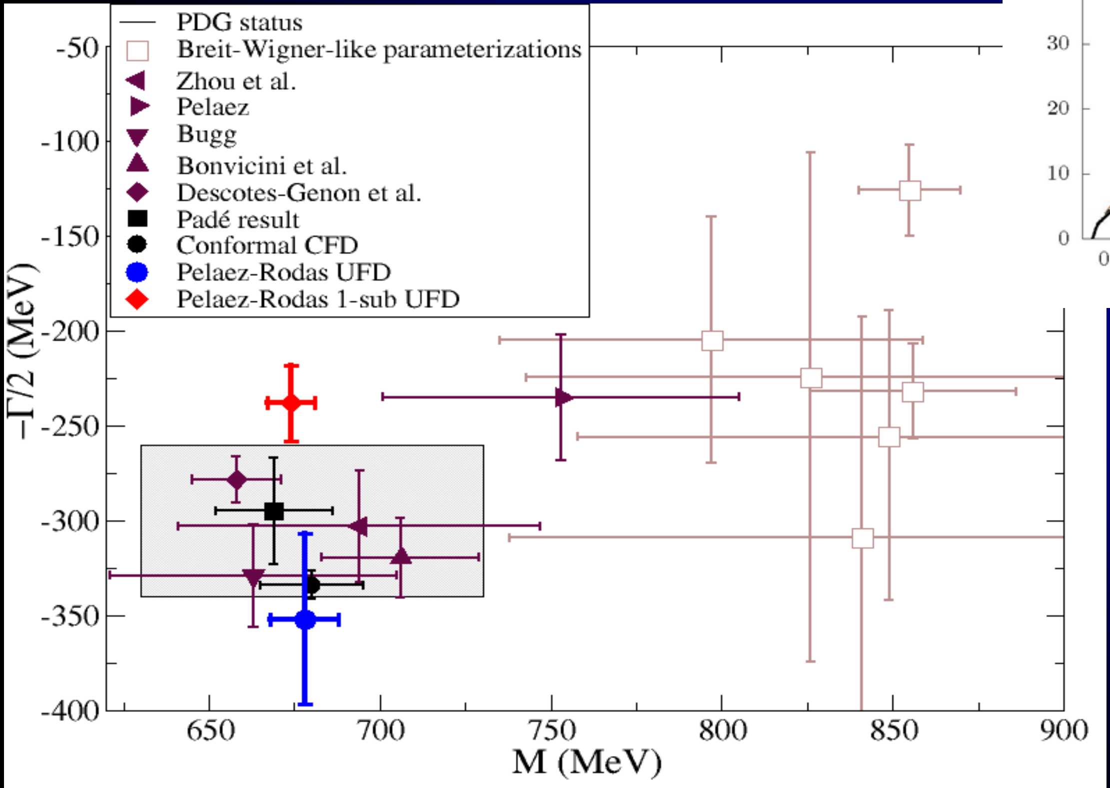
Fixed-t Roy-Steiner is OK but kappa pole outside their applicability region

We have chosen the hyperbolae family so that the kappa pole and its uncertainties lie within their applicability region

WARNING ABOUT THE PRECISION OF UNCONSTRAINED FITS

Before imposing Roy Eqs. incompatible results with different # of subtractions !!
This is part of the left cut.





Looking fine on data is not enough to get an stable and precise continuation to the complex plane

You can imagine what precision you get if you use simple models only of piK, without left cut or without dispersión relations…

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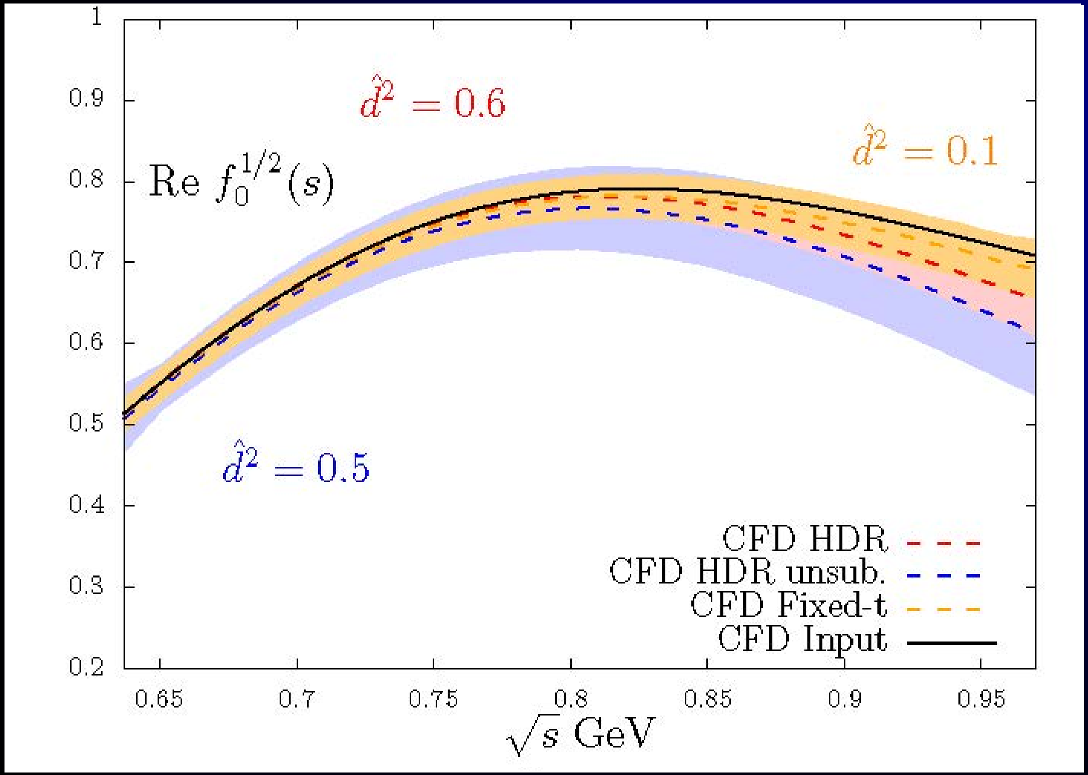
  JRP, A.Rodas, arXiv:2001.08153

- As πK Checks: Large inconsistencies.
- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV

πK Hiperbolic Dispersion Relations I=1/2, J=0

FINAL eQCD2020

Thus, a constrained data analysis stisfying Dispersion Relations is needed.
This is what we have finally completed, satisfying all of them.
(16 in our case, FDRs, fixed-t, HDR, different # subtractions)

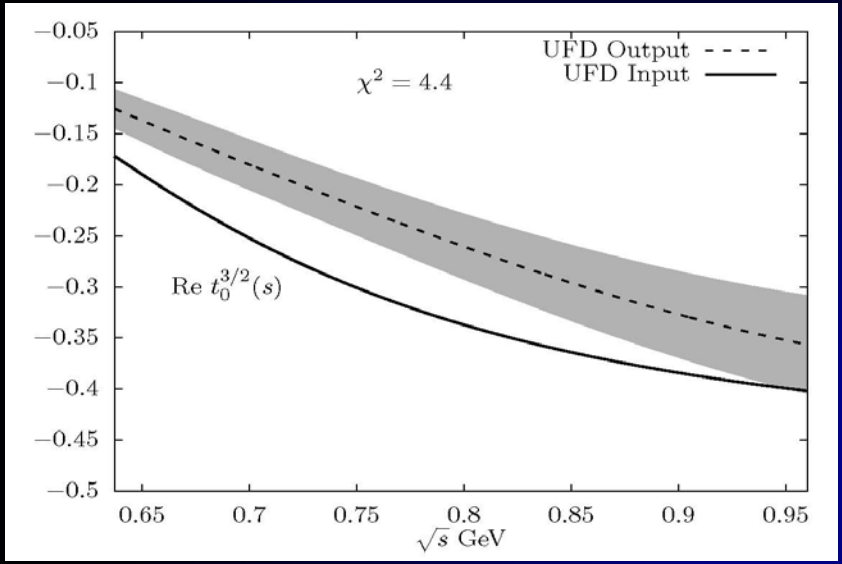


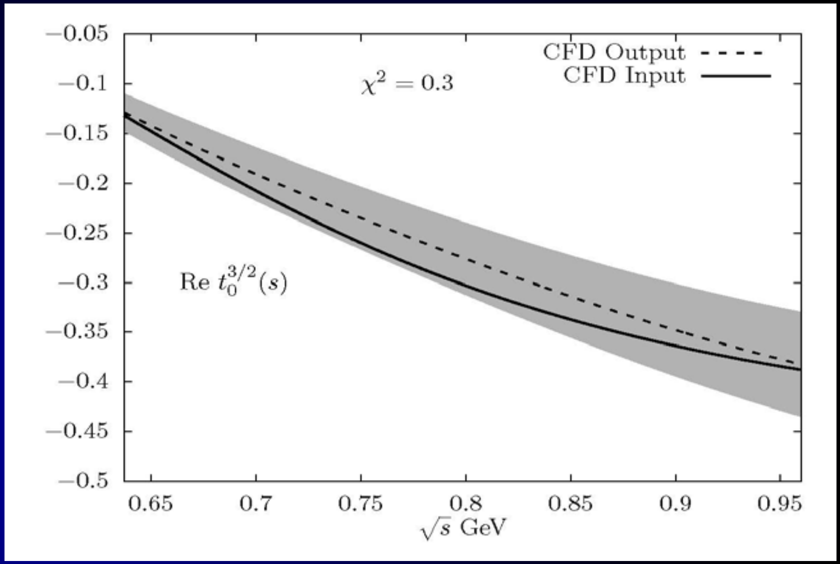
Our Constrained parameterization now yields consistent output for all Dispersion Relations

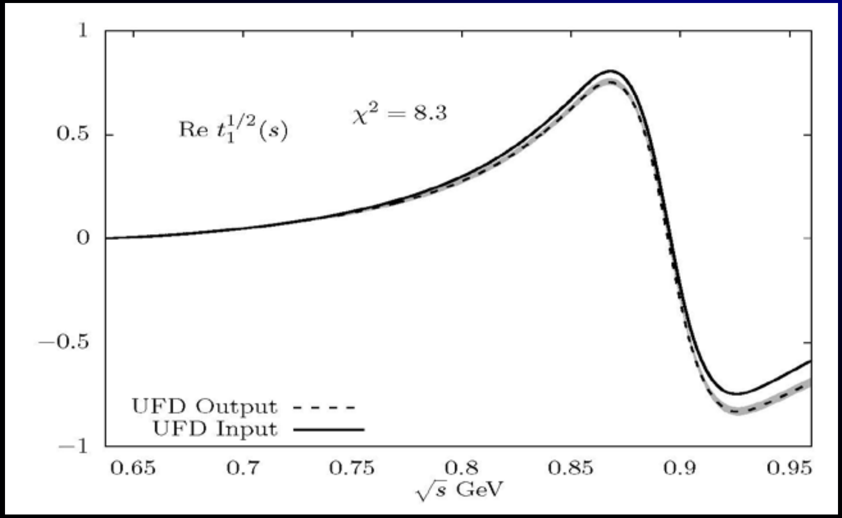
πK Hiperbolic Dispersion Relations I=3/2, J=0 and I=1/2, J=0

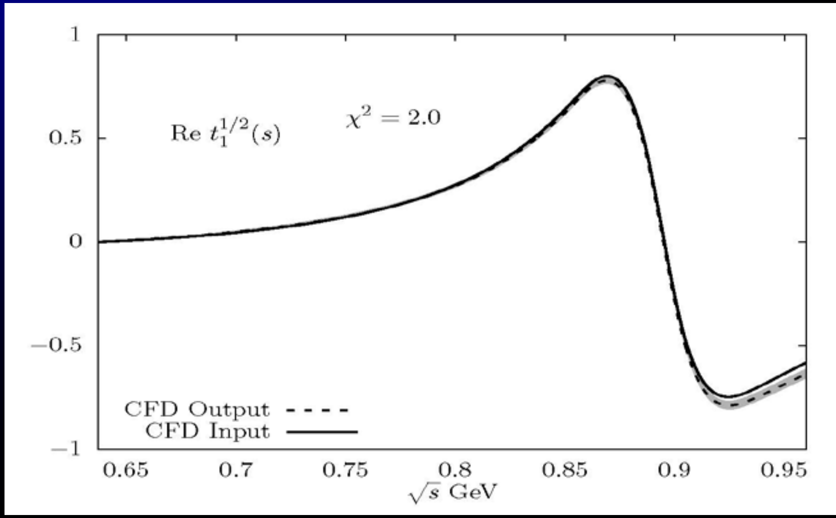
SIZABLE inconsistencies of unconstrained fits with the minimal number of subtractions (shown here). Fairly consistent with one more subtraction for F^-

Made consistent within uncertainties when we use the DR as constraints





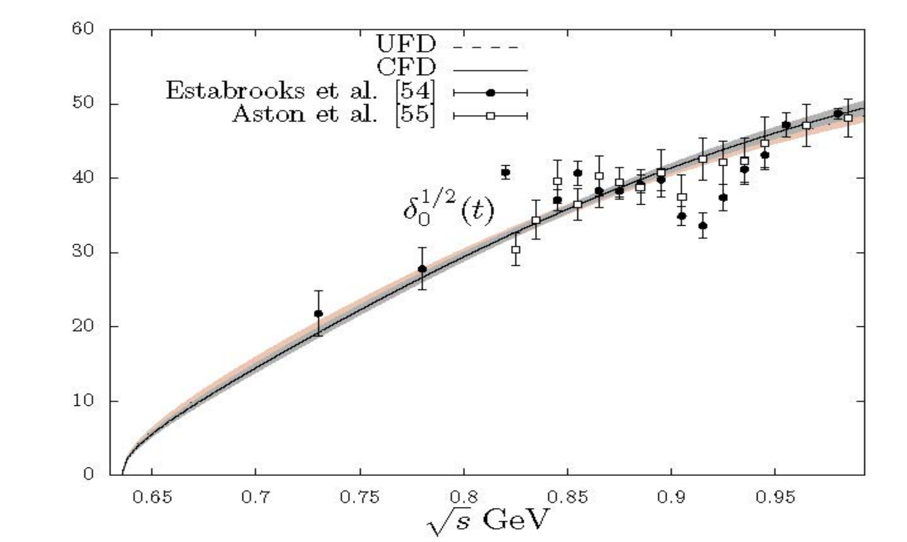


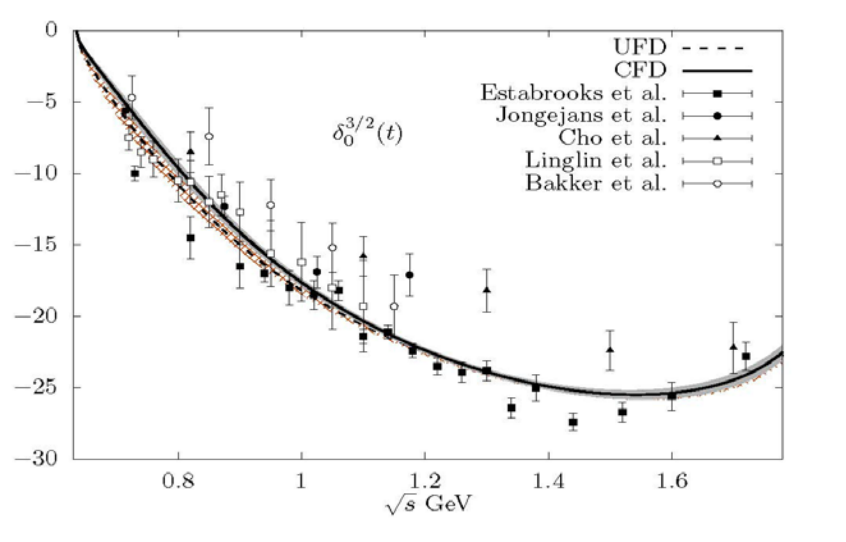


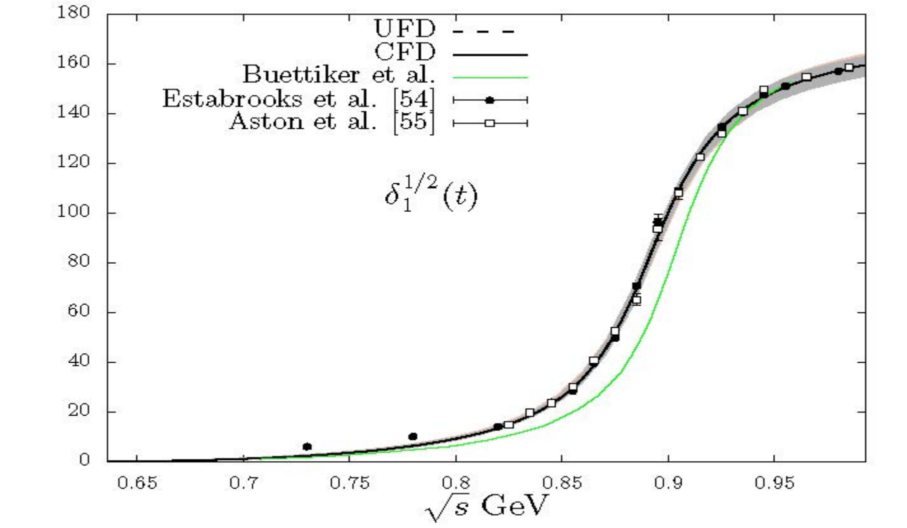
πK CFD vs. UFD

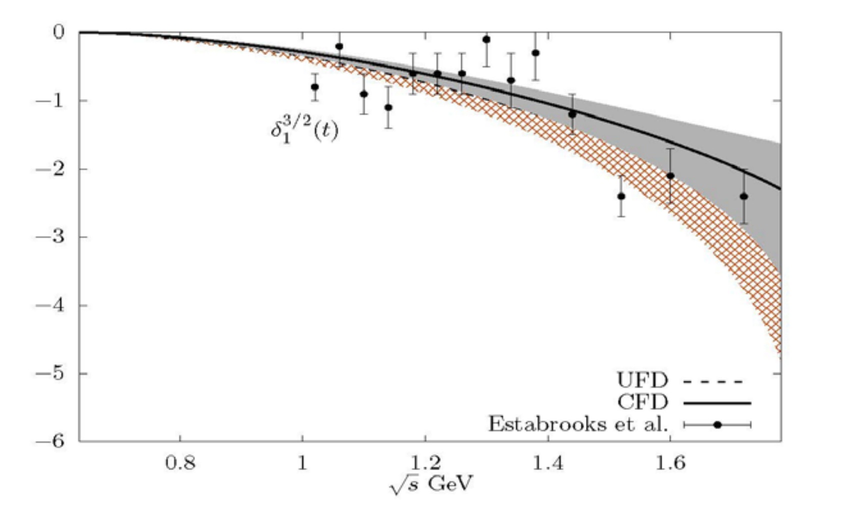
Final

Constrained parameterizations suffer minor changes but still describe πK data fairly well. Here we compare the unconstrained fits (UFD) versus the constrained ones (CFD)









The “unphysical” rho peak in ππ→KK grows by 10% from UFD to CFD

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- **ALL DR TOGETHER** as Constraints:
  πK consistent fits up to 1.1 GeV
- **Rigorous κ/K_0^*(700) pole** JRP, A.Rodas, arXiv:2001.08153

Dispersive pole analysis from constrained fit to data **JRP, A. Rodas** ArXiv:2001.08153

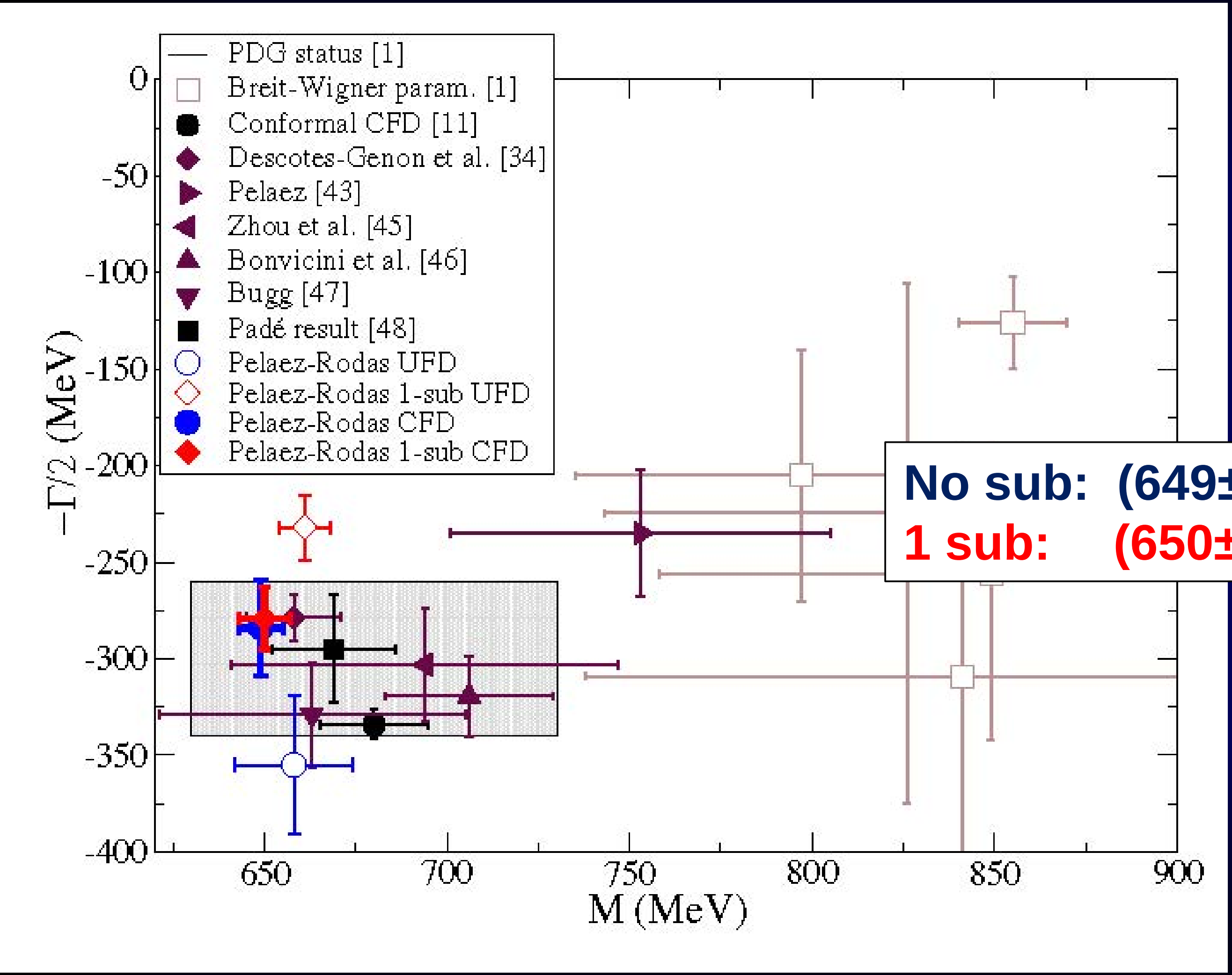
Now we have:

- Constrained **FIT TO DATA** (not solution but fit)
- Improved P-wave (consistent with data)
- Realistic $\pi\pi \rightarrow KK$ uncertainties (none before)
- Improved Pomeron

- Constrained $\pi\pi \rightarrow KK$ input with DR
- Unphysical region VERY RELEVANT
- FDR up to 1.6 GeV
- Fixed-t Roy-Steiner Eqs.
- Hyperbolic Roy Steiner Eqs.
  both in real axis (not before)
  and complex plane
- Both one and no-subtractions for F- HDR
  (only the subtracted one before)

Dispersive pole analysis from constrained fit to data **JRP, A. Rodas,** arXiv:2001.08153

When using the constrained fit to data both poles come out nicely compatible



FINAL RESULT eQCD2020

No sub: (649± 6)-i(284±25) MeV

1 sub: (650±7)-i(279±16) MeV

Compatible with Paris group

Decotes-Genon-Moussallam 2006

(658±13)-i(278.5±12) MeV

And our previous "Pade sequence" determination

(670±18)-i(295±28) MeV

Summary

- The πK and $\pi\pi \rightarrow KK$ data do not satisfy well basic dispersive constraints
- Using dispersion relations as constraints we provide **simple** and consistent data parameterizations.
- We have implemented partial-wave dispersion relations whose applicability range reaches the kappa pole.
- Our results confirm previous studies and provide a precise determination of its parameters **FROM DATA. A good control on the left cut is needed for this precision.**
- We believe this resonance should be considered “well-established”, completing the nonet of lightest scalars.