Hybrids in a chiral approach

Excited QCD 2020

2-8/2/2020 – Krynica Zroj - Poland

Francesco Giacosa

Based on: **arXiv:2001.061**

in collaboration with W. Eshraim, C. Fischer, and D. Parganlija

Outline

- Symmetries of QCD
- An hadronic model of QCD: the eLSM. Recall of the mesonic sector
- Glueballs (briefly)
- **Hybrids**
- Conlcuions and outlook

Symmetries of QCD

	Giuseppe Lodovico Lagrangia
Born	25 January 1736
	Turin
Died	10 April 1813 (aged 77)
	Paris

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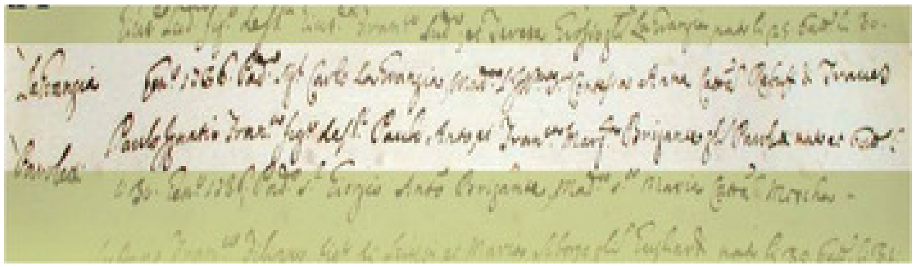


Qui accanto si può osservare
l'**Estratto dall'atto di nascita e di battesimo** tratto
dai registri parrocchiali della
Parrocchia di Sant'Eusebio,
dove risulta il nome di *Lagrangia.*

II

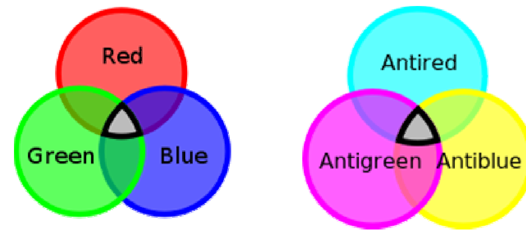
LETTERA
DI
LUIGI DE LA GRANGE TOURNIER
TORINESE
ALL' ILLUSTRISSIMO SIGNOR CONTE
GIULIO CARLO DA FAGNANO
Marchese de' Toschi, e di S. Onorio, Nobile Romano, e Senogagliese, Matematico celebratissimo.
Contenente una nuova serie per i differenziali, ed integrali di qualsivoglia grado corrispondente alla Newtoniana per le potestà, e le radici.

IN TORINO, MDCCLIV.
NELLA STAMPERIA REALE. Con lic. de' Sup.

Nel 1754 pubblicò la
Lettera a Giulio Carlo da Fagnano
il suo primo lavoro,
l'unico scritto in italiano,
che gli procurò il primo impiego di
sostituito del maestro di matematica
presso le Scuole di Artiglieria.

The QCD Lagrangian

Quark: u,d,s and c,b,t R,G,B

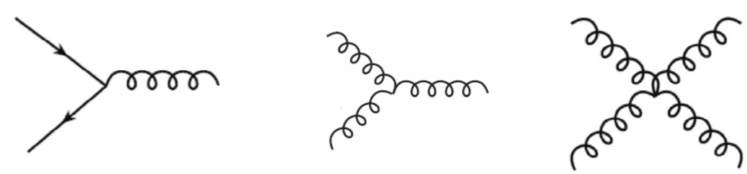


$$q_i = \begin{pmatrix} q_i^R \\ q_i^G \\ q_i^B \end{pmatrix} ; \; i = u,d,s \,,...$$

$$\boxed{\mathcal{L}_{QCD} = \sum_{i=1}^{N_f} \overline{q}_i (i\gamma^\mu D_\mu - m_i) q_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}}$$

8 type of gluons (R$\overline{\text{G}}$,B$\overline{\text{G}}$,…)

$$A_\mu^a \; ; \; a = 1,...,8$$



Confinement: quarks never ‘seen’ directly. How they might look like ☺

Picture by Pawel Piotrowski

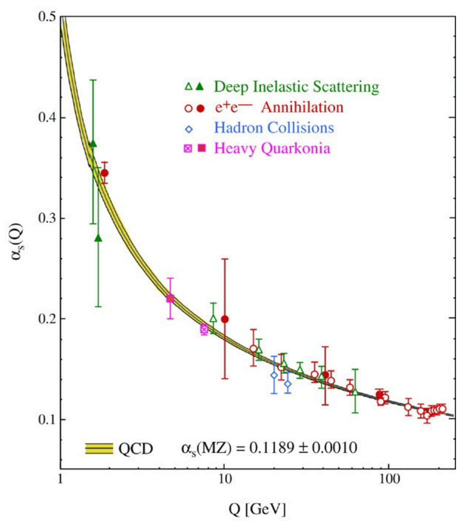
Trace anomaly: the emergence of a dimension

Chiral limit: $m_q = 0$

$x^\mu \to x'^\mu = \lambda^{-1} x^\mu$ is a classical symmetry broken by quantum fluctuations (trace anomaly)

Dimensional transmutation $\Lambda_{YM} \approx 250\ \mathrm{MeV}$

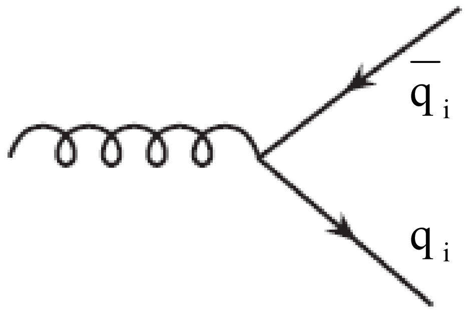
$$\alpha_S(\mu = Q) = \frac{g^2(Q)}{4\pi}$$



Effective gluon mass: $m_{gluon} = 0 \to m^*_{gluon} \approx 500 - 800\ \mathrm{MeV}$

Gluon condensate: $\left\langle G^a_{\mu\nu} G^{a,\mu\nu} \right\rangle \neq 0$

Flavor symmetry



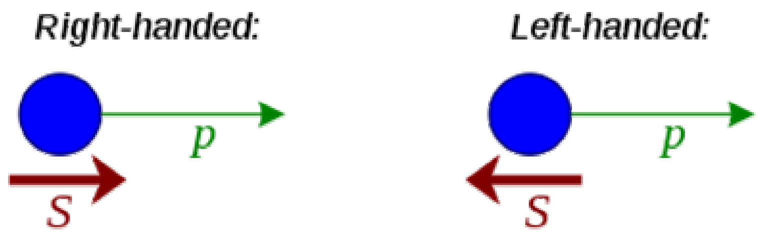
Gluon-quark-antiquark vertex

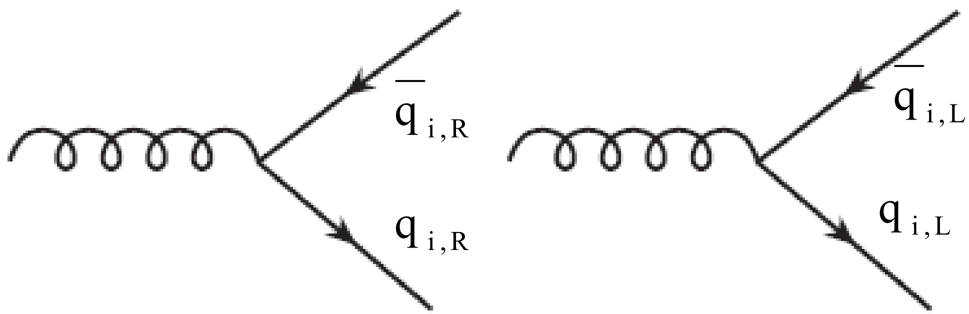
It is democratic! The gluon couples to each flavor with the same strength

$$q_i \rightarrow U_{ij} q_j$$

$$\mathrm{U} \in \mathrm{U}(3)_{\mathrm{V}} \rightarrow \mathrm{U}^{+}\mathrm{U} = 1$$

Chiral symmetry





$$q_i = q_{i,R} + q_{i,L}$$

$$q_{i,R} = \frac{1}{2}(1+\gamma^5)q_i$$

$$q_{i,L} = \frac{1}{2}(1-\gamma^5)q_i$$

$$\mathrm{q_i} = \mathrm{q_{i,R}} + \mathrm{q_{i,L}} \rightarrow \mathrm{U^R_{ij}q_{j,R}} + \mathrm{U^L_{ij}q_{j,L}}$$

$$U(3)_R \times U(3)_L = U(1)_{R+L} \times U(1)_{R-L} \times SU(3)_R \times SU(3)_L$$

baryon number **anomaly U(1)A** **SSB into SU(3)V**

In the chiral limit (mi=0) chiral symmetry is exact, but is **spontaneously broken** by the QCD vacuum

Axial anomaly: explicitely broken by quantum fluctuations

$$\boxed{\partial^\mu(\bar{q}^i\gamma_\mu\gamma_5 q^i) = \frac{3g^2}{16\pi^2}\varepsilon^{\mu\nu\rho\sigma}\mathrm{tr}(G_{\mu\nu}G_{\rho\sigma})}$$

Symmetries of QCD and breakings

$SU(3)_{color}$: exact. Confinement: you never see color, but only white states.

Dilatation invariance: holds only at a classical level and in the chiral limit. Broken by quantum fluctuations (**trace anomaly**) and by quark masses.

$SU(3)_R \times SU(3)_L$: holds in the chiral limit, but is broken by nonzero quark masses. Moreover, it is **spontaneously** broken to $U(3)_{V=R+L}$

$U(1)_{A=R-L}$: holds at a classical level, but is also broken by quantum fluctuations (**axial anomaly**)

Hadrons

The QCD Lagrangian contains ‘colored’ quarks and gluons. However, no ‚colored‘ state has been seen.

Confinement: physical states are white and are called hadrons.

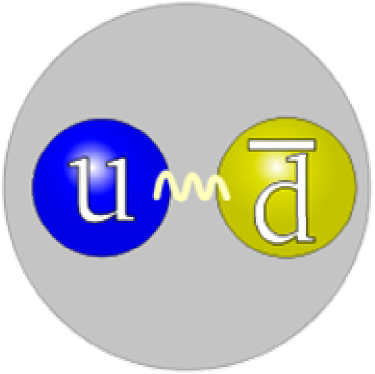
Hadrons can be:

Mesons: bosonic hadrons

Baryons: fermionic hadrons

A meson is **not necessarily** a quark-antiquark state.
A baryon is **not necessarily** a three-quark state.
Quark-antiquark and three-quark states are conventional mesons and baryons.

Example of conventional quark-antiquark states: the ρ and the π mesons



Rho-meson $$m_{\rho^+} = 775 \ \ \mathrm{MeV}$$

Pion $$m_{\pi^+} = 139 \ \ \mathrm{MeV}$$

$$m_u + m_d \approx 7 \ \mathrm{MeV}$$

Mass generation in QCD is a nonpert. feature based on SSB (mentioned previusly).

Non-conventional mesons: theoretical expectations

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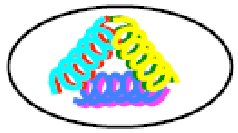
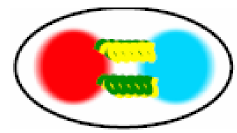
1) Glueballs

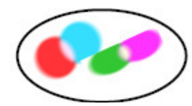


2) Hybrids

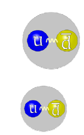


3) Four-quark states

Compact diquark-antidiquark states



Molecular states (a type of dynamical generation)



Companion poles (another type of dynamical generation)

Construction of a chiral model of QCD: the extended Linear Sigma Model (eLSM)

Francesco Giacosa

QCD not solvable in the domain of light mesons and baryons

• Lattice QCD: impressive improvements. However, some properties such as decays of resonances and fintie denisty still hard. Finite temperature: ☺ …but finite density tough (sign problem).

• Effective approaches with quarks dof: Bethe-Salpeter equations with, chirall models involving quarks (NJL and its extensions).

• Effective approaches involving hadrons: ChPT (taylor-made for pions and nucleons), effective models with linear relaization of chiral symmetry (->eLSM).

Motivation for the extended Linear Sigma Model (eLSM)

• Development of a a (chirally symmetric) linear sigma model for mesons and baryons including (axial-)vector d.o.f. and glueball(s)

• Study of the model for $T = \mu = 0$ (spectroscopy in vacuum)

(decays, scattering lengths,…)

Interrelation between these two aspects!

• Second goal: properties at nonzero T and μ

(Condensates and masses in thermal/matter medium,…)

Fields of the eLSM

• Quark-antiquark mesons: scalar, pseudoscalar, vector and axial-vector quarkonia.

• Additional mesons: The scalar and the pseudoscalar glueballs

• Baryons: nucleon doublet and its partner

(in the so-called mirror assignment)

We construct the Lagrangian of the so-called Extended Linear Sigma Model (ELSM) according to

chiral invariance and dilatation symmetry and their explicit breakings.

Fields of the eLSM/2

Field in eLSM	Assignment (predom.) [18]	Flavor content	I	J^{PC}
$\boldsymbol{a_0}$	$a_0(1450)$	$u\bar{d},\ (u\bar{u}-d\bar{d})/\sqrt{2},\ d\bar{u}$	1	
$\boldsymbol{K_0^{*[\pm,0]}}$	$K_0^*(1430)$	$u\bar{s},\ d\bar{s},\ \bar{d}s,\ \bar{u}s$	$\frac{1}{2}$	0^{++}
$\sigma_N,\ \sigma_S$	$f_0(1370),\ f_0(1500)$	$c_1(u\bar{u}+d\bar{d})+c_2(s\bar{s})$	0	
$\boldsymbol{\pi}$	$\{\pi^0,\pi^\pm\}$	$u\bar{d},\ (u\bar{u}-d\bar{d})/\sqrt{2},\ d\bar{u}$	1	
$\boldsymbol{K^{[\pm,0]}}$	$K^{[0,\pm]}$, $K(1460)$, $K(1630)$, $K(1830)$	$u\bar{s},\ d\bar{s},\ \bar{d}s,\ \bar{u}s$	$\frac{1}{2}$	0^{-+}
$\eta_N,\ \eta_S$	$\eta(547),\ \eta'(958)$, $\eta(1295)$, $\eta(1405)$, $\eta(1475)$	$c_1(u\bar{u}+d\bar{d})+c_2(s\bar{s})$	0	
$\boldsymbol{\rho^\mu}$	$\rho(770)$	$u\bar{d},\ (u\bar{u}-d\bar{d})/\sqrt{2},\ d\bar{u}$	1	
$\boldsymbol{K^{*\mu}},\ \boldsymbol{\bar{K}^{*\mu}}$	$K^*(892)$	$u\bar{s},\ d\bar{s},\ \bar{d}s,\ \bar{u}s$	$\frac{1}{2}$	0^{-+}
$\omega_N^\mu,\ \omega_S^\mu$ (small mixing angle)	$\omega(782),\ \phi(1020)$	$c_1(u\bar{u}+d\bar{d})+c_2(s\bar{s})$	0	
$\boldsymbol{a_1^\mu}$	$a_1(1260)$	$u\bar{d},\ (u\bar{u}-d\bar{d})/\sqrt{2},\ d\bar{u}$	1	
$\boldsymbol{K_1^\mu},\ \boldsymbol{\bar{K}_1^\mu}$	$K_{1,A}\equiv K_1(1270)$, $K_1(1400)$	$u\bar{s},\ d\bar{s},\ \bar{d}s,\ \bar{u}s$	$\frac{1}{2}$	0^{-+}
$f_{1N}^\mu,\ f_{1S}^\mu$ (small mixing angle)	$f_1(1285),\ f_1(1420)$	$c_1(u\bar{u}+d\bar{d})+c_2(s\bar{s})$	0	

and, in addition, the scalar/dilaton glueball G (plus evt other glueballs)

Meson phenomenology - literature

1) N_f = 2 (with frozen glueball): Parganlija FG DHR **PRD82 (2010) 054024**

2) Nf = 2 (with glueball): Janowski Parganlija FG DHR **PRD84 (2011) 054007**

3) N_f = 3 (with frozen glueball): Parganlija Kovacs Wolf FG DHR **PRD87 (2013) 014011**

4) Nf = 3 (with glueball): Janowski FG DHR **PRD90 (2014) 114005**

5) Pseudoscalar glueball: Eshraim Janowski FG DHR **PRD87 (2013) 054036** Eshraim Schramm **PRD95 (2017) 014028** Eshraim **PRD 100 (2019) no.9, 096007**

6) Nf =4: Eshraim FG DHR **EPJ.A51 (2015) no.9,** Eshraim Fischer **112 EPJ A54 (2018) 139**

7) Vector glueball: Sammet Janowski FG **PRD95 (2017) no.11, 114004**

8) Excited (pseudo)scalar mesons: Parganlija FG **Eur.Phys.J. C77 (2017) 450**

9) Consistency with ChPT: Divotgey Kovacs FG DHR **Eur.Phys.J. A54 (2018) 5**

10) $f_0(500)$ as a four-quark in the vacuum: Lakaschus Mauldin FG DHR **PRC 99 (2019) no.4, 045203**

Baryon phenomenology - literature

1) Baryonic eLSM for $N_f = 2$: Gallas FG DHR **PRD82 (2010) 014004,** Gallas FG **IJMP.A29 (2014) 1450098**

2) Nucleon-nucleon scattering: Teilab Deinet FG DHR **Phys.Rev. C94 (2016) 044001**

3) $N_f = 3$ (with four multiplets): Olbrich Zetenyi FG DHR **Phys.Rev. D93 (2016) 034021**

4) Nf = 3 and axial-anomaly for baryons: Olbrich Zetenyi FG DHR **Phys.Rev. D97 (2018) no.1, 014007**

5) Nuclear matter: Gallas Pagliara FG **Nucl.Phys. A872 (2011) 13-24**

6) Inhomogenous condensation in nuclear matter: Heinz FG DHR **Nucl.Phys. A933 (2015) 34-42**

7) Inclusion of delta for $N_f = 2$ and decuplets for $N_f = 3$: planned

8) $N_f = 3$ at nonzero density: planned

(Pseudo)scalar sector

$$P = P_a \lambda^a = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_N}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_N}{\sqrt{2}} & K^0 \\ K^- & \overline{K}^0 & \eta_S \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix}$$

$$\Gamma = i\gamma^5$$
$$J^{PC} = 0^{-+}$$

$$S = S_a \lambda^a = \begin{pmatrix} \frac{a_0^0}{\sqrt{2}} + \frac{\sigma_N}{\sqrt{2}} & a_0^+ & K_S^+ \\ a_0^- & -\frac{a_0^0}{\sqrt{2}} + \frac{\sigma_N}{\sqrt{2}} & K_S^0 \\ K_S^- & \overline{K}_S^0 & \sigma_S \end{pmatrix} \equiv \begin{pmatrix} \bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u \\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d \\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s \end{pmatrix}$$

$$\Gamma = 1$$
$$J^{PC} = 0^{++}$$

$a_0^+ = a_0(1450) \equiv u\bar{d}$ and not $a_0(980)$!!!

$\sigma_N \equiv \sqrt{1/2}(u\bar{u} + d\bar{d}) \approx f_0(1370)$ and not $f_0(500)$!!!

Chiral transformation of (pseudo)scalar mesons

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$$q_i = q_{i,R} + q_{i,L} \rightarrow (U_R)_{ij} q_{j,R} + (U_L)_{ij} q_{j,L} \qquad U_R,\ U_L \subset U(3)$$

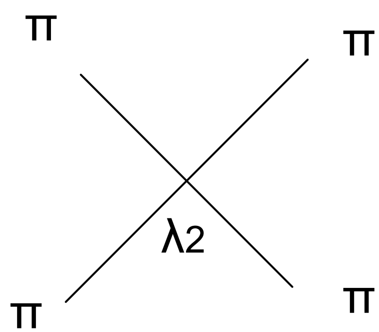
$$\Phi = S + iP$$

$$\Phi_{ij} = \bar{q}_j q_i + i\bar{q}_j i\gamma^5 q_i = \sqrt{2}\,\bar{q}_{R,j} q_{L,i}$$

$$\boxed{\Phi \rightarrow U_L \Phi U_R^+}$$

Example of an invariant term

$$\Phi \rightarrow U_L \Phi U_R^+ \qquad U_R,\ U_L \subset SU(3)$$



$$\lambda_2 Tr\left[\Phi^+\Phi\Phi^+\Phi\right] \rightarrow$$

$$\lambda_2 Tr\left[U_R\Phi^+U_L^+U_L\Phi U_R^+U_R\Phi^+U_L^+U_L\Phi U_R^+\right] = \lambda_2 Tr\left[\Phi^+\Phi\Phi^+\Phi\right]$$

$$U_L^+U_L = 1\ ,\ U_R^+U_R = 1$$

(Axial-)Vector sector

$$V^{\mu}=V^{\mu}{}_{a}\lambda^{a}=\begin{pmatrix}\frac{\rho^{0}}{\sqrt{2}}+\frac{\omega_{N}}{\sqrt{2}} & \rho^{+} & K_{*}(892)^{+}\\ \rho^{-} & -\frac{\rho^{0}}{\sqrt{2}}+\frac{\omega_{N}}{\sqrt{2}} & K_{*}(892)^{0}\\ K_{*}(892)^{-} & \overline{K}_{*}(892)^{0} & \phi_{S}\end{pmatrix}\equiv\begin{pmatrix}\bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u\\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d\\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s\end{pmatrix}$$

$$\Gamma=\gamma^{\mu}$$

$$J^{PC}=1^{--}$$

$$A^{\mu}=A^{\mu}{}_{a}\lambda^{a}=\begin{pmatrix}\frac{a_{1}^{0}}{\sqrt{2}}+\frac{f_{1,N}}{\sqrt{2}} & a_{1}^{+} & K_{1}^{+}\\ a_{1}^{-} & -\frac{a_{1}^{0}}{\sqrt{2}}+\frac{\omega_{N}}{\sqrt{2}} & K_{1}^{0}\\ K_{1}^{-} & \overline{K}_{1}^{0} & f_{1,S}\end{pmatrix}\equiv\begin{pmatrix}\bar{u}\Gamma u & \bar{d}\Gamma u & \bar{s}\Gamma u\\ \bar{u}\Gamma d & \bar{d}\Gamma d & \bar{s}\Gamma d\\ \bar{u}\Gamma s & \bar{d}\Gamma s & \bar{s}\Gamma s\end{pmatrix}$$

$$\Gamma=\gamma^{\mu}\gamma^{5}$$

$$J^{PC}=1^{++}$$

$$a_{1}^{+}=a_{1}^{+}(1260)\equiv u\bar{d}$$

$$K_{1}^{+}=K_{1}^{+}(1270)\equiv u\bar{s}$$

$$\boxed{\begin{aligned}L^{\mu}&=V^{\mu}+A^{\mu} & L^{\mu}&\rightarrow U_{L}L^{\mu}U_{L}^{+}\\ R^{\mu}&=V^{\mu}-A^{\mu} & R^{\mu}&\rightarrow U_{R}R^{\mu}U_{R}^{+}\end{aligned}}$$

Model of QCD – eLSM

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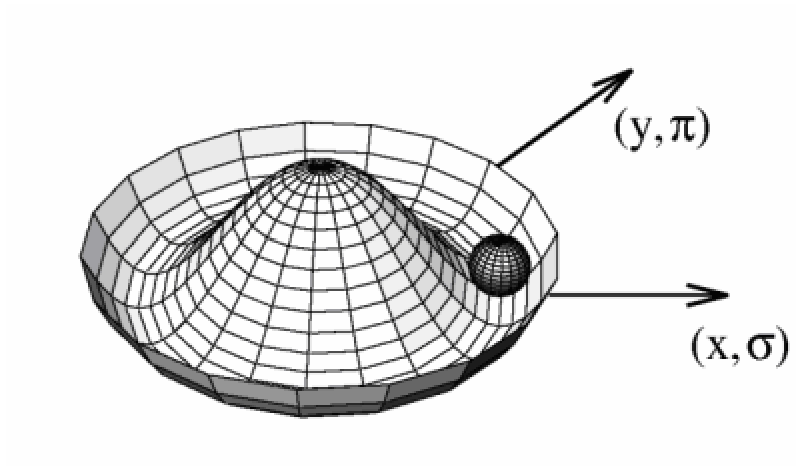
$$\mathcal{L}_{eLSM} = \mathcal{L}_{dil} + \mathrm{Tr}[(D_\mu\Phi)^\dagger(D^\mu\Phi)] - m_0^2\left(\frac{G}{G_0}\right)^2 \mathrm{Tr}(\Phi^\dagger\Phi) - \lambda_1[\mathrm{Tr}(\Phi^\dagger\Phi)]^2 - \lambda_2\mathrm{Tr}(\Phi^\dagger\Phi)^2$$
$$-\frac{1}{4}\mathrm{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \mathrm{Tr}\left[\left(\frac{m_1^2}{2}\left(\frac{G}{G_0}\right)^2 + \Delta\right)(L_\mu^2 + R_\mu^2)\right] + \mathrm{Tr}[H(\Phi+\Phi^\dagger)]$$
$$+ c_1(\det\Phi - \det\Phi^\dagger)^2 + i\frac{g_2}{2}\{\mathrm{Tr}(L_{\mu\nu}[L^\mu, L^\nu]) + \mathrm{Tr}(R_{\mu\nu}[R^\mu, R^\nu])\}$$
$$+ \frac{h_1}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}\left(L_\mu^2 + R_\mu^2\right) + h_2\mathrm{Tr}[|L_\mu\Phi|^2 + |\Phi R_\mu|^2]$$
$$+ 2h_3\mathrm{Tr}(L_\mu\Phi R^\mu\Phi^\dagger) + \mathcal{L}_{eLSM}^{\tilde{\Phi}}\cdots,$$

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_0^{\star+} + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_0^{\star 0} + iK^0 \\ K_0^{\star-} + iK^- & \bar{K}_0^{\star 0} + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

$$L^\mu, R^\mu = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\omega_N \pm \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \pm a_1^0}{\sqrt{2}} & \rho^+ \pm a_1^+ & K^{\star+} \pm K_1^+ \\ \rho^- \pm a_1^- & \frac{\omega_N \mp \rho^0}{\sqrt{2}} \pm \frac{f_{1N} \mp a_1^0}{\sqrt{2}} & K^{\star 0} \pm K_1^0 \\ K^{\star-} \pm K_1^- & \bar{K}^{\star 0} \pm i\bar{K}_1^0 & \omega_S \pm f_{1S} \end{pmatrix}$$

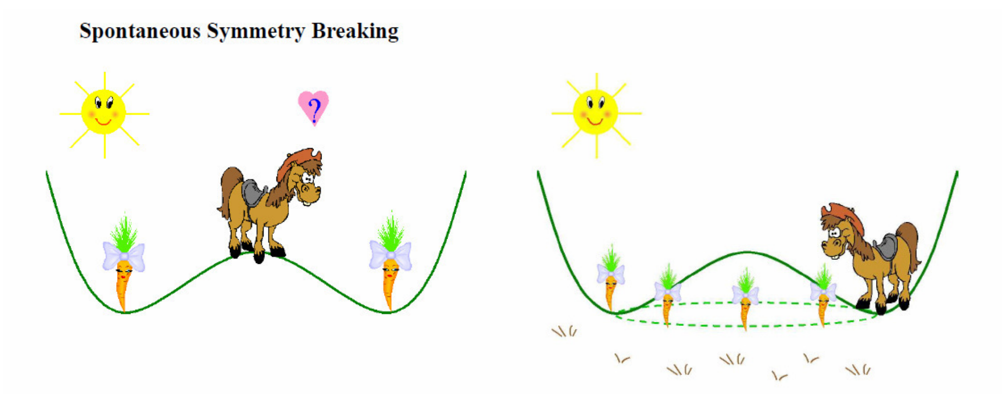
S. Janowski, D. Parganlija, F. Giacosa, D. H. Rischke, **Phys. Rev. D84, 054007 (2011**)
D. Parganlija, P. Kovacs, G. Wolf , F. Giacosa, D. H. Rischke, **Phys.Rev. D87 (2013) 014011**

SSB and the donkey of Buridan



$$\boxed{\sigma_N \to \sigma_N + \phi}$$

Jean Buridan (in Latin, *Johannes Buridanus*) (ca. 1300 – after 1358)



Results of the fit
(11 parameters, 21 exp. quantities)

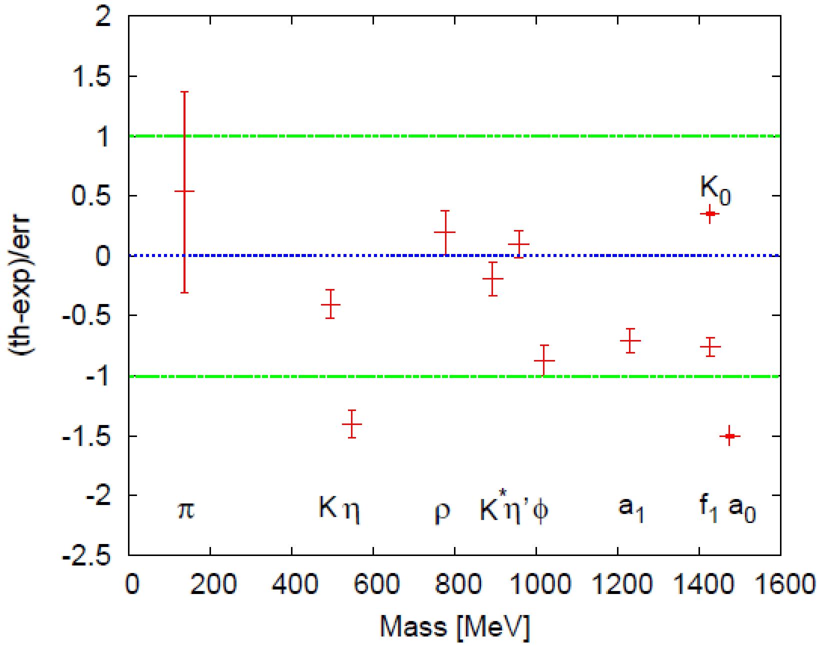
Observable	Fit [MeV]	Experiment [MeV]
f_π	96.3 ± 0.7	92.2 ± 4.6
f_K	106.9 ± 0.6	110.4 ± 5.5
m_π	141.0 ± 5.8	137.3 ± 6.9
m_K	485.6 ± 3.0	495.6 ± 24.8
m_η	509.4 ± 3.0	547.9 ± 27.4
$m_{\eta'}$	962.5 ± 5.6	957.8 ± 47.9
m_ρ	783.1 ± 7.0	775.5 ± 38.8
$m_{K^\star}$	885.1 ± 6.3	893.8 ± 44.7
m_ϕ	975.1 ± 6.4	1019.5 ± 51.0
m_{a_1}	1186 ± 6	1230 ± 62
$m_{f_1(1420)}$	1372.5 ± 5.3	1426.4 ± 71.3
m_{a_0}	1363 ± 1	1474 ± 74
$m_{K_0^\star}$	1450 ± 1	1425 ± 71
$\Gamma_{\rho\to\pi\pi}$	160.9 ± 4.4	149.1 ± 7.4
$\Gamma_{K^\star\to K\pi}$	44.6 ± 1.9	46.2 ± 2.3
$\Gamma_{\phi\to\bar{K}K}$	3.34 ± 0.14	3.54 ± 0.18
$\Gamma_{a_1\to\rho\pi}$	549 ± 43	425 ± 175
$\Gamma_{a_1\to\pi\gamma}$	0.66 ± 0.01	0.64 ± 0.25
$\Gamma_{f_1(1420)\to K^\star K}$	44.6 ± 39.9	43.9 ± 2.2
Γ_{a_0}	266 ± 12	265 ± 13
$\Gamma_{K_0^\star\to K\pi}$	285 ± 12	270 ± 80

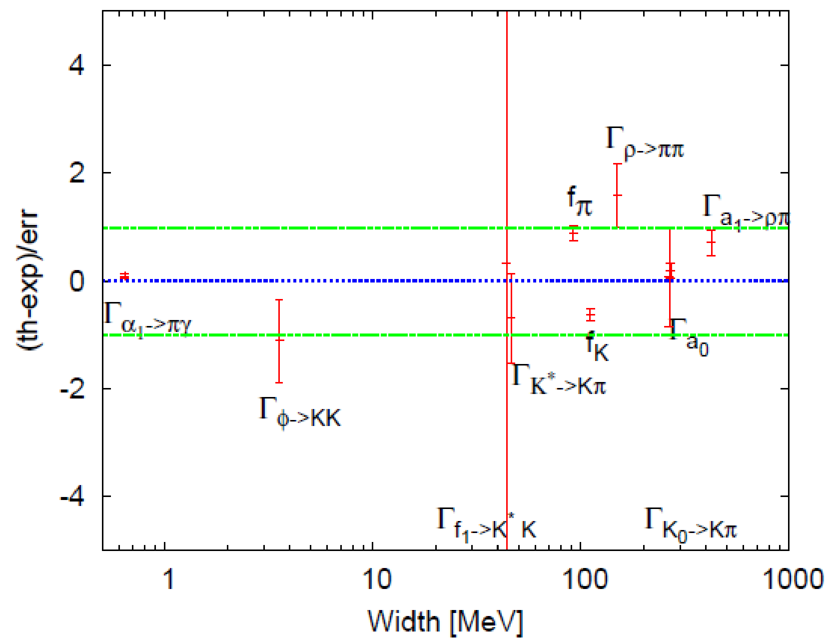
Error from PDG or 5%.
Scalar-isoscalar sector not included.

$$\chi^2_{red} = 1.2$$

arXiv:1208.0585

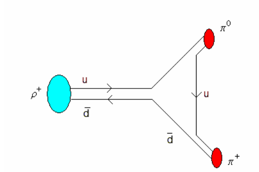
Results of the fit/2



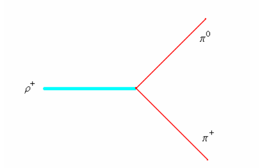


arXiv:1208.0585

Microscpic



eLSM



Overall phenomenology is good. Further quantities calculated afterwards.

Scalar mesons $a_0(1450)$ and $K_0(1430)$ above 1 GeV and are quark-antiquark states. The chiral partner of the pion (the σ) is $f_0(1370)$.

Importance of the (axial-)vector mesons

Glueballs in the eLSM (brief!)

Scalar glueball: mixing pattern

Above 1 GeV one has two quark-antiquark states and a bare glueball.

$$\sqrt{\tfrac{1}{2}}(\bar{\mathrm{u}}\mathrm{u}+\bar{\mathrm{d}}\mathrm{d})$$

$$\bar{\mathrm{s}}\mathrm{s}$$

$$\text{Glueball: gg}$$

They mix to form the 3 resonances on the right.

Note:

$a_0(980)$ $k(800)$ $f_0(980)$ $f_0(500)$

are regarded as non-quarkonium objects

$\boxed{f_0(1370)}$ $I^G(J^{PC}) = 0^+(0^{++})$

See also the mini-reviews on scalar mesons under $f_0(500)$ (see the index for the page number) and on non-$q\bar{q}$ candidates in PDG 06, Journal of Physics **G33** 1 (2006).

$f_0(1370)$ T-MATRIX POLE POSITION

Note that $\Gamma \approx 2\,\mathrm{Im}(\sqrt{s_{\mathrm{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
(1200–1500)−*i*(150–250) OUR ESTIMATE			

$\boxed{f_0(1500)}$ $I^G(J^{PC}) = 0^+(0^{++})$

See also the mini-reviews on scalar mesons under $f_0(500)$ (see the index for the page number) and on non-$q\bar{q}$ candidates in PDG 06, Journal of Physics **G33** 1 (2006).

$f_0(1500)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
1504± 6 OUR AVERAGE	Error includes scale factor of 1.3. See the ideogram below.			

$f_0(1500)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
109± 7 OUR AVERAGE				

$\boxed{f_0(1710)}$ $I^G(J^{PC}) = 0^+(0^{++})$

See our mini-review in the 2004 edition of this *Review*, Physics Letters **B592** 1 (2004). See also the mini-review on scalar mesons under $f_0(500)$ (see the index for the page number).

$f_0(1710)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
1723^{+6}_{-5} OUR AVERAGE	Error includes scale factor of 1.6. See the ideogram below.			

$f_0(1710)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
139 ± 8 OUR AVERAGE	Error includes scale factor of 1.1.			

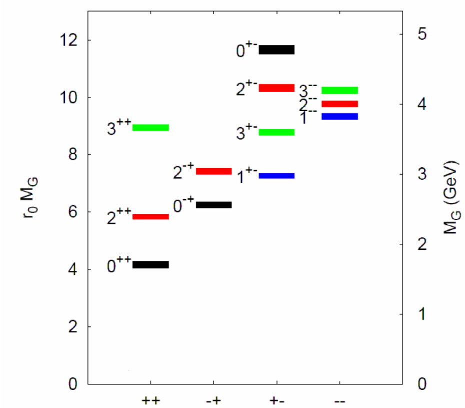
The scalar glueball in the eLSM

The calculation of the full mixing problem in the I=J=0 sector shows that:

$$\begin{pmatrix} \mathbf{f}_0(\mathbf{1370}) \\ \mathbf{f}_0(\mathbf{1500}) \\ \mathbf{f}_0(\mathbf{1710}) \end{pmatrix} = \begin{pmatrix} \mathbf{0.91} & \mathbf{-0.24} & \mathbf{0.33} \\ \mathbf{0.30} & \mathbf{0.94} & \mathbf{-0.17} \\ \mathbf{-0.27} & \mathbf{0.26} & \boxed{\mathbf{0.93}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{1}{2}}(\bar{\mathbf{u}}\mathbf{u} + \bar{\mathbf{d}}\mathbf{d}) \\ \bar{\mathbf{s}}\mathbf{s} \\ \mathbf{Glueball:\ gg} \end{pmatrix}$$

Ergo: $f_0(1710)$ is predominantly a glueball!
…and $f_0(1370)$ is the chiral partner of the pion

In BESIII, CLAS, COMPASS, and in the future in PANDA:
production processes with these states.



Details in S. Janowski, F.G, D. H. Rischke,

Phys.Rev. D90 (2014) 11, 114005, arXiv: 1408.4921

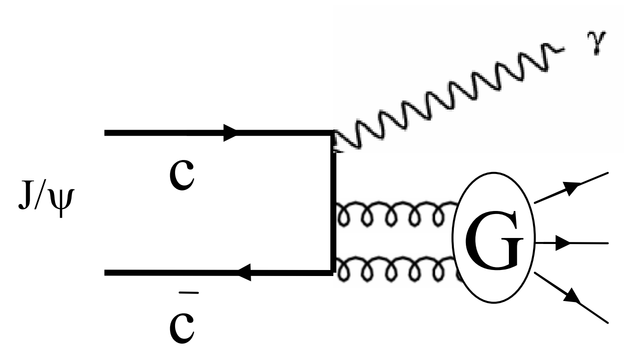
Lattice result on J/Psi decay into glueball

PRL 110, 021601 (2013) PHYSICAL REVIEW LETTERS week ending 11 JANUARY 2013

Scalar Glueball in Radiative J/ψ Decay on the Lattice

Long-Cheng Gui,[1,2] Ying Chen,[1,2,*] Gang Li,[3] Chuan Liu,[4] Yu-Bin Liu,[5] Jian-Ping Ma,[6] Yi-Bo Yang,[1,2] and Jian-Bo Zhang[7]

(CLQCD Collaboration)



From the PDG (decay of the j/ψ): the radiative decays into $f_0(1710)$ are larger than into $f_0(1500)$.

$\gamma f_0(1710) \to \gamma K\overline{K}$	$(8.5\ {}^{+1.2}_{-0.9}) \times 10^{-4}$
$\gamma f_0(1710) \to \gamma\pi\pi$	$(4.0 \pm 1.0) \times 10^{-4}$
$\gamma f_0(1710) \to \gamma\omega\omega$	$(3.1 \pm 1.0) \times 10^{-4}$
$\gamma f_0(1710) \to \gamma\eta\eta$	$(2.4\ {}^{+1.2}_{-0.7}) \times 10^{-4}$

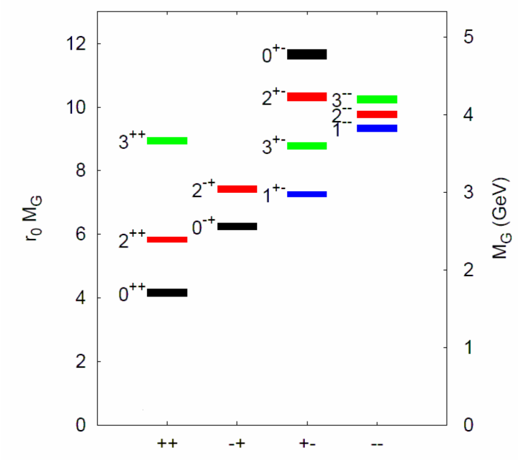
$\gamma f_0(1500) \to \gamma\pi\pi$	$(1.01 \pm 0.32) \times 10^{-4}$
$\gamma f_0(1500) \to \gamma\eta\eta$	$(1.7\ {}^{+0.6}_{-1.4}) \times 10^{-5}$

The pseudoscalar glueball

$$\mathcal{L}^{int}_{\tilde{G}\text{-mesons}} = i c_{\tilde{G}\Phi} \tilde{G} \left(\det\Phi - \det\Phi^{\dagger} \right)$$

Quantity	Value
$\Gamma_{\tilde{G}\to KK\eta}/\Gamma^{tot}_{\tilde{G}}$	0.049
$\Gamma_{\tilde{G}\to KK\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.019
$\Gamma_{\tilde{G}\to \eta\eta\eta}/\Gamma^{tot}_{\tilde{G}}$	0.016
$\Gamma_{\tilde{G}\to \eta\eta\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.0017
$\Gamma_{\tilde{G}\to \eta\eta'\eta'}/\Gamma^{tot}_{\tilde{G}}$	0.00013
$\Gamma_{\tilde{G}\to KK\pi}/\Gamma^{tot}_{\tilde{G}}$	0.46
$\Gamma_{\tilde{G}\to \eta\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.16
$\Gamma_{\tilde{G}\to \eta'\pi\pi}/\Gamma^{tot}_{\tilde{G}}$	0.094

Quantity	Value
$\Gamma_{\tilde{G}\to KK_S}/\Gamma^{tot}_{\tilde{G}}$	0.059
$\Gamma_{\tilde{G}\to a_0\pi}/\Gamma^{tot}_{\tilde{G}}$	0.083
$\Gamma_{\tilde{G}\to \eta\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.028
$\Gamma_{\tilde{G}\to \eta\sigma_S}/\Gamma^{tot}_{\tilde{G}}$	0.012
$\Gamma_{\tilde{G}\to \eta'\sigma_N}/\Gamma^{tot}_{\tilde{G}}$	0.019



$$\boxed{\Gamma_{\tilde{G}\to\pi\pi\pi} = 0}$$

PANDA will produce a pseudoscalar glueball (if existent).

Details in:

W. Eshraim, S. Janowski, F.G., D. Rischke, **Phys.Rev. D87 (2013) 054036. arxiv: 1208.6474** .
W. Eschraim, S. Janowski, K. Neuschwander, A. Peters, F.G., **Acta Phys. Pol. B**, Prc. Suppl. 5/4, **arxiv: 1209.3976**

Vector glueball

From arXiv:1607.03640 [hep-ph],
Decays of the vector glueball by F.G, J. Sammet, S. Janowski, PRD 95 (2017) 114004

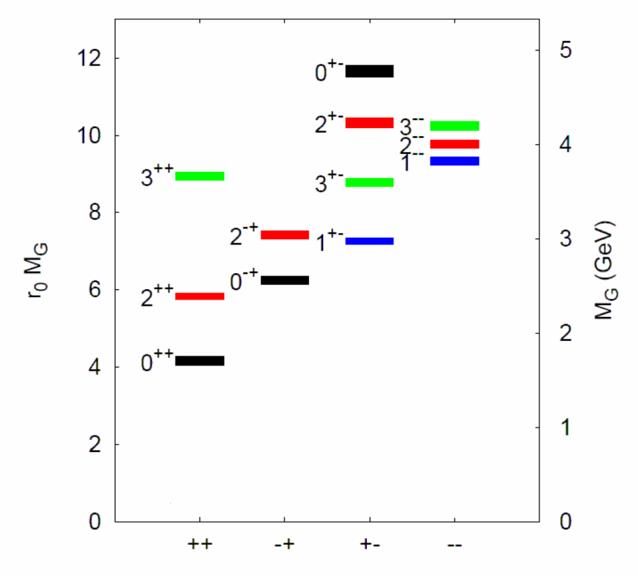
We predict that the vector glueball decays mostly into:

$$\mathcal{O} \to b_1\pi \to \omega\pi\pi \qquad \mathcal{O} \to \omega\pi\pi \qquad \mathcal{O} \to \pi K K^*(892)$$

The lattice mass of 3.8 GeV has been used;
Tables of ratios are in the preprintpaper

Planned studies:

tensor glueball, pseudovector glueball.



Hybrid mesons in the eLSM

arXiv:2001.061

Francesco Giacosa

Candidate/1

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018) and 2019 update

$\boxed{\pi_1(1600)}$ $\quad I^G(J^{PC}) = 1^-(1^{-+})$

$\pi_1(1600)$ MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
1660^{+15}_{-11} OUR AVERAGE		Error includes scale factor of 1.2.		

$\pi_1(1600)$ WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*COMMENT*
257 ± 60 OUR AVERAGE		Error includes scale factor of 1.9. See the ideogram below.		

$\pi_1(1600)$ DECAY MODES

	Mode	Fraction (Γ_i/Γ)
Γ_1	$\pi\pi\pi$	seen
Γ_2	$\rho^0\pi^-$	seen
Γ_3	$f_2(1270)\pi^-$	not seen
Γ_4	$b_1(1235)\pi$	seen
Γ_5	$\eta'(958)\pi^-$	seen
Γ_6	$f_1(1285)\pi$	seen

Candidate/2

Citation: M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**, 030001 (2018) and 2019 update

$\boxed{\pi_1(1400)}$ $\quad I^G(J^{PC}) = 1^-(1^{-+})$

See also the mini-review under non-$q\overline{q}$ candidates in PDG 06, Journal of Physics **G33** 1 (2006).

π_1(1400) MASS

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
1354 ±25 OUR AVERAGE		Error includes scale factor of 1.8. See the ideogram below.			

π_1(1400) WIDTH

VALUE (MeV)	*EVTS*	*DOCUMENT ID*	*TECN*	*CHG*	*COMMENT*
330 ±35 OUR AVERAGE					

π_1(1400) DECAY MODES

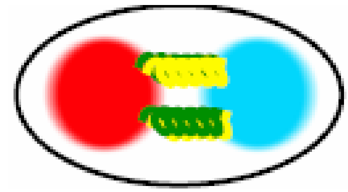
	Mode	Fraction (Γ_i/Γ)
Γ_1	$\eta\pi^0$	seen
Γ_2	$\eta\pi^-$	seen
Γ_3	$\eta'\pi$	
Γ_4	$\rho(770)\pi$	not seen

Only one pole

Determination of the pole position of the lightest hybrid meson candidate
JPAC Collaboration (A. Rodas (Madrid U.) *et al.*).
Phys.Rev.Lett. 122 (2019) no.4, 042002

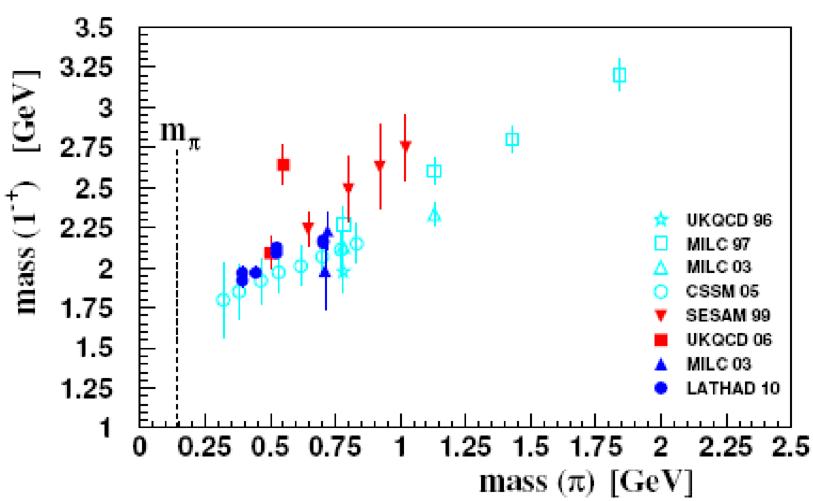
"We provide a robust extraction of a single exotic π1 resonant pole, with mass and width 1564±24±86 MeV and 492±54±102 MeV, which couples to both η (0)π channels. We find no evidence for a second exotic state."

Hybrids from lattice



Hybrid mesons: lattice predictions for 1^-+ hybrids at about 1.7 GeV

See for instance the review:
C. Meyer and E. Swanson, Hybrid Mesons,
Prog. Part. Nucl. Phys. **82** (2015) 21 [arXiv:1502.07276 [hep-ph]].



Note, 1^-+ is an exotic combination impossible for a quark-antiquark pair

New quark-antiquark nonets in the eLSM are needed

Nonet	L	S	J^{PC}	Current	Assignment
P	0	0	0^{-+}	$P_{ij}=\frac{1}{\sqrt{2}}\bar{q}_j i\gamma^5 q_i$	π, K, η, η'
S	1	1	0^{++}	$S_{ij}=\frac{1}{\sqrt{2}}\bar{q}_j q_i$	$a_0(1450),\ K_0^*(1430),\ f_0(1370),\ f_0(1510)$
V^μ	0	1	1^{--}	$V_{ij}^\mu=\frac{1}{\sqrt{2}}\bar{q}_j\gamma^\mu q_i$	$\rho(770),\ K^*(892),\ \omega(785),\ \phi(1024)$
A^μ	1	1	1^{++}	$A_{ij}^\mu=\frac{1}{\sqrt{2}}\bar{q}_j\gamma^5\gamma^\mu q_i$	$a_1(1230),\ K_{1,A},\ f_1(1285),\ f_1(1420)$
B^μ	1	0	1^{+-}	$B_{ij}^\mu=\frac{1}{\sqrt{2}}\bar{q}_j\gamma^5 \overleftrightarrow{\partial}^\mu q_i$	$b_1(1230),\ K_{1,B},\ h_1(1170),\ h_1(1380)$
E_{ang}^μ	2	1	1^{--}	$E_{\text{ang},ij}^\mu=\frac{1}{\sqrt{2}}\bar{q}_j i\overleftrightarrow{\partial}^\mu q_i$	$\rho(1700),\ K^*(1680),\ \omega(1650),\ \phi(???)$

For excited vectors
see Piotrowska et al
PRD 96 (2017) no.5, 054033

Chiral multiplet	Current	$U_R(3)\times U_L(3)$	P	C
$\Phi=S+iP$	$\sqrt{2}\bar{q}_{R,j}q_{L,i}$	$U_L\Phi U_R^\dagger$	$\Phi^\dagger$	Φ^t
$R^\mu=V^\mu-A^\mu$	$\sqrt{2}\bar{q}_{R,j}\gamma^\mu q_{R,i}$	$U_R R^\mu U_R^\dagger$	L_μ	$L^{t\mu}$
$L^\mu=V^\mu+A^\mu$	$\sqrt{2}\bar{q}_{L,j}\gamma^\mu q_{L,i}$	$U_L R^\mu U_L^\dagger$	R_μ	$R^{t\mu}$
$\tilde{\Phi}^\mu=E_{\text{ang}}^\mu-iB^\mu$	$\sqrt{2}\bar{q}_{R,j}i\overleftrightarrow{\partial}^\mu q_{L,i}$	$U_L\tilde{\Phi}^\mu U_R^\dagger$	$\tilde{\Phi}^{\dagger\mu}$	$-\tilde{\Phi}^{t\mu}$

ArXiv: 1607.03640

New quark-antiquark nonets/2

$$B^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{h_{1,N}+b_1^0}{\sqrt{2}} & b_1^+ & K_{1,B}^{\star+} \\ b_1^- & \frac{h_{1,N}+b_1^0}{\sqrt{2}} & K_{1,B}^{\star0} \\ K_{1,B}^{\star-} & \bar{K}_{1,B}^{\star0} & h_{1,S} \end{pmatrix}^{\mu} \qquad V_E^{\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\omega_{E,N}+\rho_E^0}{\sqrt{2}} & \rho_E^+ & K_E^{\star+} \\ \rho_E^- & \frac{\omega_{E,N}-\rho_E^0}{\sqrt{2}} & K_E^{\star0} \\ K_E^{\star-} & \bar{K}_E^{\star0} & \omega_{E,S} \end{pmatrix}^{\mu}$$

$$\tilde{\Phi}^{\mu} = V_E^{\mu} - iB^{\mu}$$

$$\boxed{\tilde{\Phi}^{\mu} \to U_L \tilde{\Phi}^{\mu} U_R^{\dagger}}$$

Hybrid nonets in the eLSM

$$\Pi_{ij}^{hyb,\mu} = \frac{1}{\sqrt{2}} \bar{q}_j G^{\mu\nu} \gamma_\nu q_i$$

Exotic quantum numbers: 1^-+

$$\Pi^{hyb,\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\eta_{1,N}^{hyb}+\pi_1^0}{\sqrt{2}} & \pi_1^{hyb+} & K_1^{hyb+} \\ \pi_1^{hyb-} & \frac{\eta_{1,N}^{hyb}-\pi_1^0}{\sqrt{2}} & K_1^{hyb0} \\ K_1^{hyb-} & \bar{K}_1^{hyb0} & \eta_{1,S}^{hyb} \end{pmatrix}^{\mu}$$

$$B_{ij}^{hyb,\mu} = \frac{1}{\sqrt{2}} \bar{q}_j G^{\mu\nu} \gamma^5 \gamma_\nu q_i$$

$$B^{hyb,\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{h_{1N,B}^{hyb}+b_1^{hyb,0}}{\sqrt{2}} & b_1^{hyb,+} & K_{1,B}^{hyb+} \\ b_1^{hyb,+} & \frac{h_{1N,B}^{hyb}-b_1^{hyb,0}}{\sqrt{2}} & K_{1,B}^{hyb0} \\ K_{1,B}^{hyb-} & \bar{K}_{1,B}^{hyb0} & h_{1S,B}^{hyb} \end{pmatrix}^{\mu}$$

Quantum numbers 1^+-

Hybrid nonets/2

Nonet	J^{PC}	Current	Assignment	P	C
$\Pi^{hyb,\mu}$	1^{-+}	$\Pi_{ij}^{hyb,\mu}=\frac{1}{\sqrt{2}}\bar{q}_j G^{\mu\nu}\gamma_\nu q_i$	$\pi_1(1600), K_1(?), \eta_1(?), \eta_1(?)$	$\Pi_\mu^{hyb}(t,-\mathbf{x})$	$\Pi^{hyb,\mu,t}$
$B^{hyb,\mu}$	1^{+-}	$B_{ij}^{hyb,\mu}=\frac{1}{\sqrt{2}}\bar{q}_j G^{\mu\nu}\gamma_\nu\gamma^5 q_i$	$b_1(2000?)$, $K_{1,B}(?)$, $h_1(?)$, $h_1(?)$	$-B_\mu^{hyb}(t,-\mathbf{x})$	$-B^{hyb,\mu,t}$

Chiral multiplet	Current	$U_R(3)\times U_L(3)$	P	C
$R^{hyb,\mu}=\Pi^{hyb,\mu}-B^{hyb,\mu}$	$\sqrt{2}\bar{q}_{R,j}G^{\mu\nu}\gamma_\nu q_{R,i}$	$U_R R^{hyb,\mu} U_R^\dagger$	L_μ^{hyb}	$(L^{hyb,\mu})^t$
$L^{hyb,\mu}=\Pi^{hyb,\mu}+B^{hyb,\mu}$	$\sqrt{2}\bar{q}_{L,j}G^{\mu\nu}\gamma_\nu q_{L,i}$	$U_L R^{hyb,\mu} U_L^\dagger$	R_μ^{hyb}	$(R^{hyb,\mu})^t$

Inclusion of hybrids into the Lagrangian of the eLSM

$$\mathcal{L}_{eLSM}^{\text{enlarged}} = \mathcal{L}_{eLSM} + \mathcal{L}_{eLSM}^{\text{hybrid}}$$

$$\mathcal{L}_{eLSM}^{\text{hybrid}} = \mathcal{L}_{eLSM}^{\text{hybrid-quadratic}} + \mathcal{L}_{eLSM}^{\text{hybrid-linear}}$$

$$\mathcal{L}_{eLSM}^{\text{hybrid-quadratic}} = \mathcal{L}_{eLSM}^{\text{hybrid-kin}} + \mathcal{L}_{eLSM}^{\text{hybrid-mass}}$$

Masses of hybrids

Uniwersytet Jana Kochanowskiego w Kielcach

$$\mathcal{L}_{eLSM}^{\text{hybrid-mass}} = m_1^{hyb,2}\frac{G^2}{G_0^2}\mathrm{Tr}\left(L_\mu^{hyb,2}+R_\mu^{hyb,2}\right)+\mathrm{Tr}\left(\Delta^{hyb}\left(L_\mu^{hyb,2}+R_\mu^{hyb,2}\right)\right)$$
$$+\frac{h_1^{hyb}}{2}\mathrm{Tr}(\Phi^\dagger\Phi)\mathrm{Tr}\left(L_\mu^{hyb,2}+R_\mu^{hyb,2}\right)+h_2^{hyb}\mathrm{Tr}\left[\left|L_\mu^{hyb}\Phi\right|^2+\left|\Phi R_\mu^{hyb}\right|^2\right]+2h_3^{nyb}\mathrm{Tr}\left(L_\mu^{hyb}\Phi R^{hyb,\mu}\Phi^\dagger\right)$$

$$m^2_{\pi_1^{hyb}} = m_1^{hyb,2}+\frac{1}{2}(h_1^{hyb}+h_2^{hyb}+h_3^{hyb})\phi_N^2+\frac{h_1^{hyb}}{2}\phi_S^2+2\delta_N^{hyb}\,,$$
$$m^2_{K_1^{hyb}} = m_1^{hyb,2}+\frac{1}{4}\left(2h_1^{hyb}+h_2^{hyb}\right)\phi_N^2+\frac{1}{\sqrt{2}}\phi_N\phi_S h_3^{hyb}+\frac{1}{2}(h_1^{hyb}+h_2^{hyb})\phi_S^2+\delta_N^{hyb}+\delta_S^{hyb}\,,$$
$$m^2_{\eta_{1,N}^{hyb}} = m^2_{\pi_1}\,,$$
$$m^2_{\eta_{1,S}^{hyb}} = m_1^{hyb,2}+\frac{h_1^{hyb}}{2}\phi_N^2+\left(\frac{h_1^{hyb}}{2}+h_2^{hyb}+h_3^{hyb}\right)\phi_S^2+2\delta_S^{hyb}\,,$$

$$m^2_{b_1^{hyb}} = m_1^{hyb,2}+\frac{1}{2}(h_1^{hyb}+h_2^{hyb}-h_3^{hyb})\phi_N^2+\frac{h_1^{hyb}}{2}\phi_S^2+2\delta_N^{hyb}\,,$$
$$m^2_{K_{1,B}^{hyb}} = m_1^{hyb,2}+\frac{1}{4}\left(2h_1^{hyb}+h_2^{hyb}\right)\phi_N^2-\frac{1}{\sqrt{2}}\phi_N\phi_S h_3^{hyb}+\frac{1}{2}\left(h_1^{hyb}+h_2^{hyb}\right)\phi_S^2+\delta_N^{hyb}+\delta_S^{hyb}$$
$$m^2_{h_{1N}^{hyb}} = m^2_{b_1^{hyb}}\,,$$
$$m^2_{h_{1S}^{hyb}} = m_1^{hyb2}+\frac{h_1^{hyb}}{2}\phi_N^2+\left(\frac{h_1^{hyb}}{2}+h_2^{hyb}-h_3^{hyb}\right)\phi_S^2+2\delta_S^{hyb}\,.$$

Mass differences and approximated expressions

$$m^2_{b_1^{hyb}} - m^2_{\pi_1^{hyb}} = -2h_3^{hyb}\phi_N^2\,,$$
$$m^2_{K_{1,B}^{hyb}} - m^2_{K_1^{hyb}} = -\sqrt{2}\phi_N\phi_S h_3^{hyb}$$
$$m^2_{h_{1S}^{hyb}} - m^2_{\eta_{1,S}^{hyb}} = -h_3^{hyb}\phi_S^2\,.$$

Mass splitting caused by the chiral condensate

$$m^2_{K_1^{hyb}} \simeq m^2_{\pi_1^{hyb}} + \delta_S^{hyb}\,,$$
$$m^2_{\eta_{1,N}^{hyb}} \simeq m^2_{\pi_1^{hyb}}\,,$$
$$m^2_{\eta_{1,S}^{hyb}} \simeq m^2_{\pi_1^{hyb}} + 2\delta_S^{hyb}\,,$$
$$m^2_{b_1^{hyb}} \simeq m^2_{\pi_1^{hyb}} - 2h_3^{hyb}\phi_N^2\,,$$
$$m^2_{K_{1,B}^{hyb}} \simeq m^2_{K_1^{hyb}} - \sqrt{2}\phi_N\phi_S h_3^{hyb}$$
$$m^2_{h_{1S}^{hyb}} \simeq m^2_{\eta_{1,S}^{hyb}} - h_3^{hyb}\phi_S^2\,.$$

Masses: results

Resonance	Mass [MeV]
π_1^{hyb}	1660 [**input** using $\pi_1(1600)$ [7]]
$\eta_{1,N}^{hyb}$	1660
$\eta_{1,S}^{hyb}$	1751
K_1^{hyb}	1707
b_1^{hyb}	2000 [**input** set as an estimate]
$h_{1N,B}^{hyb}$	2000
$K_{1,B}^{hyb}$	2063
$h_{1S,B}^{hyb}$	2126

If the π1(1600) is indeed an hybrid mesons, we should find all the others…

Lagrangian for decays of hybrids

$$\begin{aligned}\mathcal{L}_{eLSM}^{\text{hybrid-linear}} =& i\lambda_1^{hyb} G \mathrm{Tr}\left[L_\mu^{hyb}(\tilde{\Phi}^\mu \Phi^\dagger - \Phi\tilde{\Phi}^{\dagger\mu}) + R_\mu^{hyb}(\tilde{\Phi}^{\mu\dagger}\Phi - \Phi^\dagger\tilde{\Phi}^\mu)\right] \\ &+ i\lambda_2^{hyb}\mathrm{Tr}([L_\mu^{hyb}, L^\mu]\Phi\Phi^\dagger + [R_\mu^{hyb}, R^\mu]\Phi^\dagger\Phi) \\ &+ \alpha^{hyb}\mathrm{Tr}(\tilde{L}_{\mu\nu}^{hyb}\Phi R^{\mu\nu}\Phi^\dagger - \tilde{R}_{\mu\nu}^{hyb}\Phi^\dagger L^{\mu\nu}\Phi) \\ &+ \beta_A^{hyb}(\det\Phi - \det\Phi^\dagger)\mathrm{Tr}(L_\mu^{hyb}(\partial^\mu\Phi\cdot\Phi^\dagger - \Phi\cdot\partial^\mu\Phi^\dagger) - R_\mu^{hyb}(\partial^\mu\Phi^\dagger\cdot\Phi - \Phi^\dagger\cdot\partial^\mu\Phi))\end{aligned}$$

Chiral symmetry fulfilled in all terms.

First and second term: dilatation invariance

Third term: breaks dilatation invariance but involves Levi-Civita

Fourth term: axial anomaly

First decay term

$$\mathcal{L}_{eLSM,1}^{\text{hybrid-linear}} = i2\lambda_1^{hyb} G \left\{ \mathrm{Tr}\left[\Pi_\mu^{hyb}\left[P, B^\mu\right]\right] + \mathrm{Tr}\left[\Pi_\mu^{hyb}\left[V_E^\mu, S\right]\right] \right\} + 2\lambda_1^{hyb} G \left\{ \mathrm{Tr}\left[B_\mu^{hyb}\left\{P, V_E^\mu\right\}\right] + \mathrm{Tr}\left[B_\mu^{hyb}\left\{B^\mu, S\right\}\right] \right\}$$

$\Pi^{hyb} \to BP$

$\boxed{\pi_1 \to b_1(1230)\pi}$

Ratio	Value
$\Gamma_{K_1^{hyb} \to K h_1(1170)} / \Gamma_{\pi_1^{hyb} \to \pi b_1}$	0.050
$\Gamma_{b_1^{hyb} \to \pi\omega(1650)} / \Gamma_{\pi_1^{hyb} \to \pi b_1}$	0.065
$\Gamma_{K_{1B}^{hyb} \to \pi K^*(1680)} / \Gamma_{\pi_1^{hyb} \to \pi b_1}$	0.19
$\Gamma_{h_{1,N}^{hyb} \to \pi\rho(1700)} / \Gamma_{\pi_1^{hyb} \to \pi b_1}$	0.16

Second decay term

$$\mathcal{L}_{eLSM,2}^{\text{hybrid-linear}} = 2i\lambda_2^{hyb}\mathrm{Tr}\left[\left([\Pi_\mu^{hyb}, V^\mu] + [B_\mu^{hyb}, A^\mu]\right)\left(S^2 + P^2\right)\right] - 2\lambda_2^{hyb}\mathrm{Tr}\left[\left([\Pi_\mu^{hyb}, A^\mu] + [B_\mu^{hyb}, V^\mu]\right)[P, S]\right]$$

$\Pi^{hyb} \to VPP$ $\quad\Pi^{hyb} \to A^\mu PS$ $\quad B_\mu^{hyb} \to A^\mu PP$ $\quad B_\mu^{hyb} \to PP\,P$

The decays $\pi_1 \to \eta\pi$ and $\pi_1 \to \eta'\pi$, however, do not follow from this term.

Ratio	Value
$\Gamma_{\pi_1^{0hyb} \to K^0\overline{K}^0} / \Gamma_{b_1^{0hyb} \to \pi^+\pi^-\eta}$	0.0080
$\Gamma_{\eta_{1N}^{hyb} \to K^0\overline{K}^0} / \Gamma_{b_1^{0hyb} \to \pi^+\pi^-\eta}$	0.0080
$\Gamma_{\eta_{1S}^{hyb} \to K^0\overline{K}^0} / \Gamma_{b_1^{0hyb} \to \pi^+\pi^-\eta}$	0.017
$\Gamma_{K_1^{0hyb} \to K^-\pi^+} / \Gamma_{b_1^{0hyb} \to \pi^+\pi^-\eta}$	0.0041
$\Gamma_{K_1^{0hyb} \to \overline{K}^0\eta} / \Gamma_{b_1^{0hyb} \to \pi^+\pi^-\eta}$	0.0022
$\Gamma_{K_1^{0hyb} \to \overline{K}^0\eta'} / \Gamma_{b_1^{0hyb} \to \pi^+\pi^-\eta}$	0.0026
$\Gamma_{b_1^{0hyb} \to \pi^+ a_0^-} / \Gamma_{b_1^{0hyb} \to \pi^+\pi^-\eta}$	0.24

$$\boxed{b_1^{hyb} \to \pi\pi\eta}$$

Ratio	Value
$\Gamma_{\pi_1^{0hyb} \to K^{*0}\overline{K}^0\pi^0} / \Gamma_{b_1^{0hyb} \to \pi^+\pi^-\eta}$	0.0046
$\Gamma_{\pi_1^{+hyb} \to \pi^0\rho^+\eta} / \Gamma_{b_1^{0hyb} \to \pi^+\pi^-\eta}$	0.1832
$\Gamma_{\eta_{1N}^{hyb} \to K^{*0}\overline{K}^0\pi^0} / \Gamma_{b_1^{0hyb} \to \pi^+\pi^-\eta}$	0.0046

(much more decays and details in the paper)

Third decay term

$$\mathcal{L}_{eLSM,3}^{\text{hybrid-linear}} = i\alpha^{hyb}\phi_N \left\{ \mathrm{Tr}(\tilde{\Pi}_{\mu\nu}^{hyb}[P,V^{\mu\nu}]) - \mathrm{Tr}(\tilde{B}_{\mu\nu}^{hyb}([P,A^{\mu\nu}]) \right\} + \ldots$$

$$\boxed{\pi_1^{hyb} \rightarrow \rho\pi \text{ and } \pi_1^{hyb} \rightarrow K^*K}$$

Ratio	Value
$\Gamma_{\pi_1^{0hyb}\rightarrow\overline{K}^0 K^{*0}}/\Gamma_{\pi_1^{-hyb}\rightarrow\rho^0\pi^-}$	0.61
$\Gamma_{\eta_{1N}^{hyb}\rightarrow\overline{K}^0 K^{*0}}/\Gamma_{\pi_1^{-hyb}\rightarrow\rho^0\pi^-}$	0.61
$\Gamma_{\eta_{1S}^{hyb}\rightarrow\overline{K}^0 K^{*0}}/\Gamma_{\pi_1^{-hyb}\rightarrow\rho^0\pi^-}$	1.6
$\Gamma_{K_1^{0hyb}\rightarrow K^0\omega_S}/\Gamma_{\pi_1^{-hyb}\rightarrow\rho^0\pi^-}$	0.00022
$\Gamma_{K_1^{0hyb}\rightarrow\overline{K}^{*0}\eta}/\Gamma_{\pi_1^{-hyb}\rightarrow\rho^0\pi^-}$	0.0011
$\Gamma_{K_1^{0hyb}\rightarrow K^{*0}\pi^0}/\Gamma_{\pi_1^{-hyb}\rightarrow\rho^0\pi^-}$	0.00022
$\Gamma_{K_1^{0hyb}\rightarrow\overline{K}^0\rho^0}/\Gamma_{\pi_1^{-hyb}\rightarrow\rho^0\pi^-}$	0.0011

Fourth decay term

$$\mathcal{L}_{eLSM,4}^{\text{hybrid-linear}} = -\beta_A^{hyb} Z_\pi \sqrt{\frac{3}{2}} \phi_N^3 \eta_0 \mathrm{Tr}(\Pi_\mu^{hyb} \partial^\mu P) + \dots$$

$$\boxed{\pi_1^{hyb} \to \eta\pi \text{ and } \pi_1^{hyb} \to \eta'\pi}$$

Note: η’π is dominant!

Ratio	Value
$\Gamma_{\pi_1^{hyb} \to \pi\eta'} / \Gamma_{\pi_1^{hyb} \to \pi\eta}$	12.7
$\Gamma_{K_1^{hyb} \to K\eta} / \Gamma_{\pi_1^{hyb} \to \pi\eta}$	0.69
$\Gamma_{K_1^{hyb} \to K\eta'} / \Gamma_{\pi_1^{hyb} \to \pi\eta}$	5.3
$\Gamma_{\eta_{1,N}^{hyb} \to \eta\eta} / \Gamma_{\pi_1^{hyb} \to \pi\eta}$	0.62
$\Gamma_{\eta_{1,N}^{hyb} \to \eta\eta'} / \Gamma_{\pi_1^{hyb} \to \pi\eta}$	2.2
$\Gamma_{\eta_{1,S}^{hyb} \to \eta\eta} / \Gamma_{\pi_1^{hyb} \to \pi\eta}$	0.58
$\Gamma_{\eta_{1,S}^{hyb} \to \eta\eta'} / \Gamma_{\pi_1^{hyb} \to \pi\eta}$	1.57

Conclusions

- eLSM: an effective model of QCD for mesons (and baryons). In the mesonic sector (pseudo)scalar and (axial-)vector mesons included.

- Glueballs: the scalar glueball from the very beginning (trace anomaly), pseudoscalar (axial anomaly) and vector have been coupled to the eLSM

- Hybrids: inclusion of the **nonets** of 1^-+ hybrids and their chiral partners. Moreover, inclusions of (pseudo)vector and orbitally excited vector mesons.

- Mass relations obtained. Decay ratio obtained.

$$\boxed{\pi_1(1600) \to \pi b_1,\ \pi_1(1600) \to \rho\pi\eta,\ \pi_1(1600) \to \rho\pi\ ,\ \pi_1(1600) \to \eta'\pi,\ \pi_1(1600) \to \eta\pi}$$

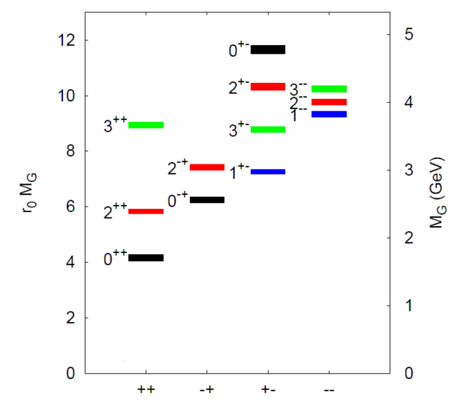
Thank You!

Dilaton-Scalar glueball/1

We start from the scalar glueball.

The lightest glueball is included as part of the chiral Lagrangian in order to reproduce at a composite level the breaking of dilatation invariance.

Development of a dilaton field and a dilaton potential.



Exotic: 2^+- and 0^+-
They are heavy!

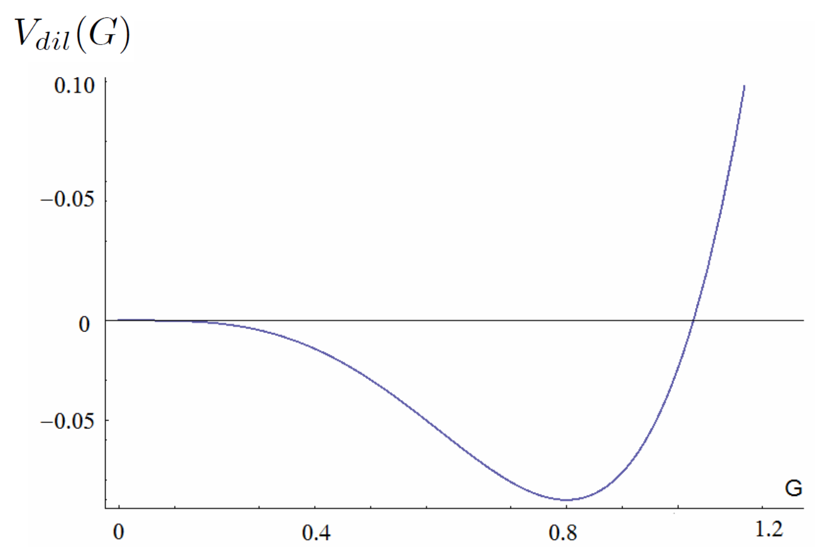
Dilaton - Scalar glueball/2

At the hadronic level, we describe these properties as:

$$G^4 \sim G^a_{\mu\nu} G^{a,\mu\nu}$$

$$\mathcal{L}_{dil} = \frac{1}{2}(\partial_\mu G)^2 - V_{dil}(G)$$

$$V_{dil}(G) = \frac{1}{4}\frac{m_G^2}{\Lambda_G^2}\left[G^4 \ln\left(\frac{G}{\Lambda_G}\right) - \frac{G^4}{4}\right]$$



Λ_G dimensionful param that breaks dilatation inv!

$$\langle G \rangle = G_0 = \Lambda_G \propto \Lambda_{YM}$$

$$\boxed{\partial_\mu J^\mu = T^\mu_\mu = -\frac{1}{4}\frac{m_G^2}{\Lambda_G^2} G^4}$$

In Yang-Mills (QCD wo quarks) it is:

$$\partial_\mu J^\mu = T^\mu_\mu = \frac{\beta(g)}{4g} G^a_{\mu\nu} G^{a,\mu\nu} \neq 0$$

J. Schechter et al,
Phys. Rev. D 24, 2545 (1981)

M. Migdal and Shifman,
Phys. Lett. 114B, 445 (1982)

Side remark: The light scalar mesons below 1 GeV: what are they? They are -at first- not part of the eLSM

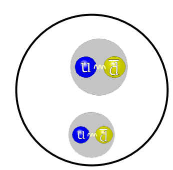
$a_0(980)$ $f_0(980)$ $K_0^*(700)$ $\boxed{f_0(500)}$

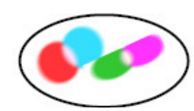
$$J^{PC} = 0^{++}$$

Various studies show that these states are **not** quark-antiquark states.

They can be meson-meson molecules and/or diquark-antidiquark states.
For instance, a0(980) and K0*(700) may emerge as companion poles.

In both cases we have **four-quark** objects.





$f_0(500)$ is the lighest scalar states: important in nuclear interaction and in studies of chiral symmetry restorations.

Ellis-Lanik 'warning' (1984)

Volume 150B, number 4 PHYSICS LETTERS

IS SCALAR GLUONIUM OBSERVABLE?

John ELLIS
CERN, Geneva, Switzerland

and

Jozef LÁNIK
JINR, Dubna, USSR

Received 26 October 1984

Physics Letters 150 B, 1984

Dilaton Lagrangian which mimics the trace anomaly: very large glueball is found.
Decay into pion reads:

$$\Gamma = 0.6(M_G/1\,\mathrm{GeV})^5 \;\; \mathrm{GeV}$$

For a glueball of about 1.5 GeV in mass, one gets a width of about 4.5 GeV!

Disagreement with the large-Nc expectation

M. Migdal and Shifman,
Phys. Lett. 114B, 445 (1982)

J. Schechter et al,
Phys. Rev. D 24, 2545 (1981)

There are many consequences of the fit. Example: a0(1450)

Theory

$$\frac{\Gamma_{a_0\rightarrow\eta'\pi}}{\Gamma_{a_0\rightarrow\eta\pi}} = 0.19 \pm 0.02\ , \quad \frac{\Gamma_{a_0\rightarrow KK}}{\Gamma_{a_0\rightarrow\eta\pi}} = 1.12 \pm 0.07$$

Exp (PDG)

$$\frac{\Gamma_{a_0(1450)\rightarrow\eta'\pi}}{\Gamma_{a_0(1450)\rightarrow\eta\pi}} = 0.35 \pm 0.16\ , \quad \frac{\Gamma_{a_0(1450)\rightarrow KK}}{\Gamma_{a_0(1450)\rightarrow\eta\pi}} = 0.88 \pm 0.23\ .$$

Wave function of the scalar dibaryon (dimeron)

Recall:

$$|\text{Deuteron}\rangle = |\text{space:ground-state}\rangle\, |\uparrow\uparrow\rangle\, |np - pn\rangle \quad J^P = 1^+ \quad I = 0$$

We now switch spin and isospin and get the isotriplet:

$$|\text{Dimeron-np}\rangle = |\text{space:ground-state}\rangle\, |\uparrow\downarrow - \downarrow\uparrow\rangle\, |np + pn\rangle$$

$$|\text{Dimeron-nn}\rangle = |\text{space:ground-state}\rangle\, |\uparrow\downarrow - \downarrow\uparrow\rangle\, |nn\rangle \qquad I = 1$$

$$|\text{Dimeron-pp}\rangle = |\text{space:ground-state}\rangle\, |\uparrow\downarrow - \downarrow\uparrow\rangle\, |pp\rangle$$

$$J^P = 0^+$$

Existence and pole position of f0(500)

Complicated PDG history. Existence through the position of the pole.
Now: established.

Citation: K.A. Olive *et al.* (Particle Data Group), Chin. Phys. C, **38**, 090001 (2014) and 2015 update

$f_0(500)$ or σ [g]
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

Mass $m = (400\text{–}550)$ MeV
Full width $\Gamma = (400\text{–}700)$ MeV

$f_0(500)$ DECAY MODES	Fraction (Γ_i/Γ)	p (MeV/c)
$\pi\pi$	dominant	–
$\gamma\gamma$	seen	–

Citation: K.A. Olive *et al.* (Particle Data Group), Chin. Phys. C, **38**, 090001 (2014) and 2015 update

$f_0(500)$ or σ
was $f_0(600)$

$I^G(J^{PC}) = 0^+(0^{++})$

A REVIEW GOES HERE – Check our WWW List of Reviews

$$\sqrt{s_{pole}} = M - i\frac{\Gamma}{2}$$

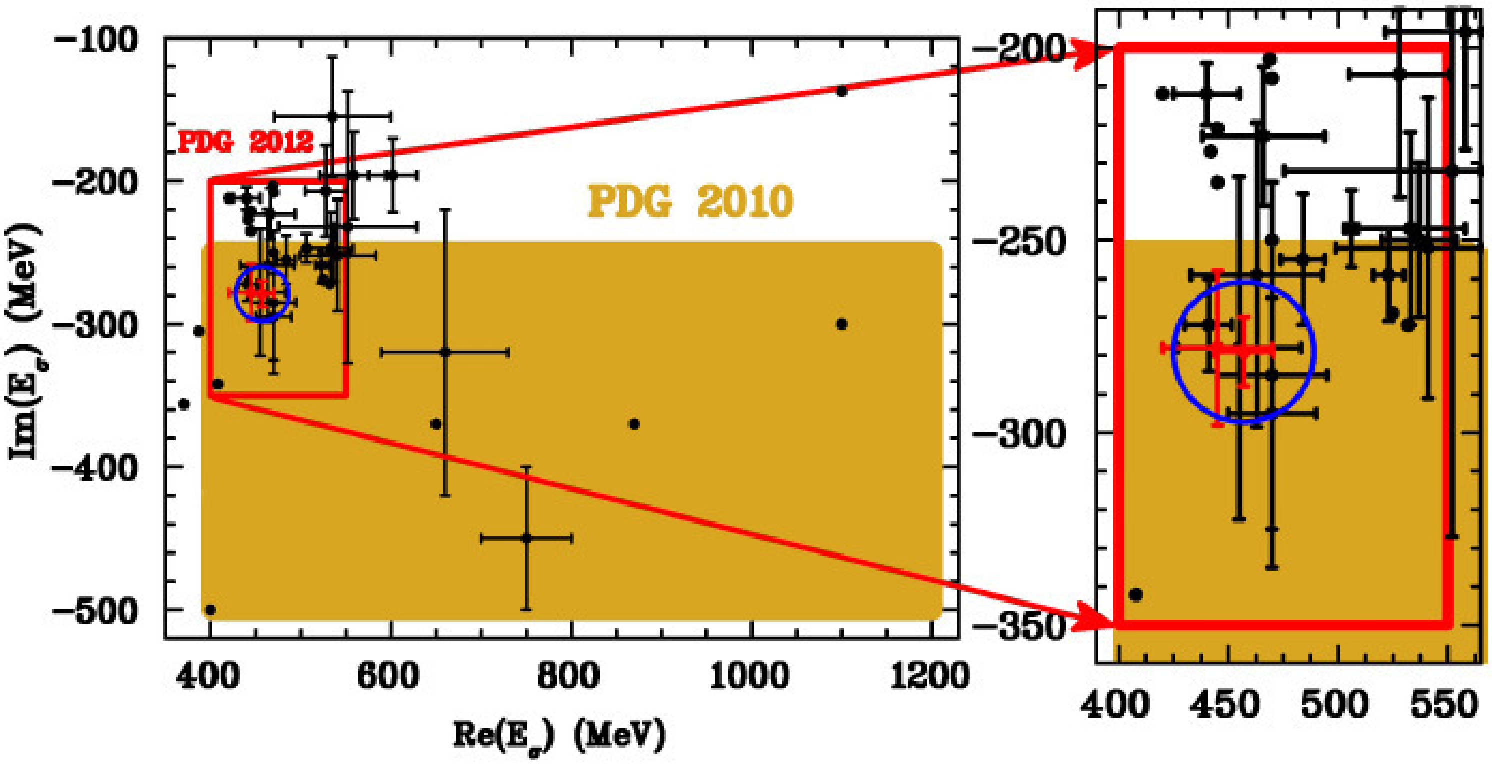
$f_0(500)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\,\mathrm{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*	*TECN*	*COMMENT*
(400–550)−i(200–350) OUR ESTIMATE			

Existence and pole position of f0(500)

From 2010 to 2012: update



See the review of J.R. Pelaez (Madrid U.), e-Print: **arXiv:1510.00653**
A review on the status of the non-ordinary f0(500) **resonance**

Vector glueball into BP, PPV, VP

$$\mathcal{L}_1 = \lambda_{\mathcal{O},1} G \mathcal{O}_\mu Tr\left[\Phi^\dagger \tilde{\Phi}^\mu + \tilde{\Phi}^{\mu\dagger}\Phi\right]$$

Quantity	Value
$\frac{\mathcal{O}\to\eta h_1(1170)}{\mathcal{O}\to b_1\pi}$	0.17
$\frac{\mathcal{O}\to\eta h_1(1380)}{\mathcal{O}\to b_1\pi}$	0.11
$\frac{\mathcal{O}\to\eta' h_1(1170)}{\mathcal{O}\to b_1\pi}$	0.15
$\frac{\mathcal{O}\to\eta' h_1(1380)}{\mathcal{O}\to b_1\pi}$	0.10
$\frac{\mathcal{O}\to K K_1(1270)}{\mathcal{O}\to b_1\pi}$	0.75
$\frac{\mathcal{O}\to K K_1(1400)}{\mathcal{O}\to b_1\pi}$	0.30
$\frac{\mathcal{O}\to K_0^*(1430) K^*(1680)}{\mathcal{O}\to b_1\pi}$	0.20
$\frac{\mathcal{O}\to a_0(1450)\rho(1700)}{\mathcal{O}\to b_1\pi}$	0.14
$\frac{\mathcal{O}\to f_0(1370)\omega(1650)}{\mathcal{O}\to b_1\pi}$	0.034

$$\boxed{\mathcal{O}\to b_1\pi\to\omega\pi\pi}$$

ArXiv: 1607.03640

$$\mathcal{L}_2 = \lambda_{\mathcal{O},2}\mathcal{O}_\mu Tr\left[L^\mu\Phi\Phi^\dagger + R^\mu\Phi^\dagger\Phi\right]$$

Quantity	Value
$\frac{\mathcal{O}\to KK\rho}{\mathcal{O}\to\omega\pi\pi}$	0.50
$\frac{\mathcal{O}\to KK\omega}{\mathcal{O}\to\omega\pi\pi}$	0.17
$\frac{\mathcal{O}\to KK\phi}{\mathcal{O}\to\omega\pi\pi}$	0.21
$\frac{\mathcal{O}\to\pi K K^*(892)}{\mathcal{O}\to\omega\pi\pi}$	1.2
$\frac{\mathcal{O}\to\eta\eta\omega}{\mathcal{O}\to\omega\pi\pi}$	0.064
$\frac{\mathcal{O}\to\eta\eta'\omega}{\mathcal{O}\to\omega\pi\pi}$	0.019
$\frac{\mathcal{O}\to\eta'\eta'\omega}{\mathcal{O}\to\omega\pi\pi}$	0.019
$\frac{\mathcal{O}\to\eta\eta\phi}{\mathcal{O}\to\omega\pi\pi}$	0.039
$\frac{\mathcal{O}\to\eta\eta'\phi}{\mathcal{O}\to\omega\pi\pi}$	0.011
$\frac{\mathcal{O}\to\eta'\eta'\phi}{\mathcal{O}\to\omega\pi\pi}$	0.011
$\frac{\mathcal{O}\to a_0(1450)a_0(1450)\omega}{\mathcal{O}\to\omega\pi\pi}$	0.00029

Quantity	Value
$\frac{\mathcal{O}\to a_0(1450)\rho}{\mathcal{O}\to\omega\pi\pi}$	0.47
$\frac{\mathcal{O}\to f_0(1370)\omega}{\mathcal{O}\to\omega\pi\pi}$	0.15
$\frac{\mathcal{O}\to K_0^*(1430)K^*(892)}{\mathcal{O}\to\omega\pi\pi}$	0.30
$\frac{\mathcal{O}\to KK}{\mathcal{O}\to\omega\pi\pi}$	0.018

$$\boxed{\mathcal{O}\to\omega\pi\pi \qquad \mathcal{O}\to K^*(892)}$$

$$\mathcal{L}_3 = \alpha\varepsilon_{\mu\nu\rho\sigma}\partial^\rho\mathcal{O}^\sigma Tr\left[L^\mu\Phi R^\nu\Phi^\dagger\right]$$

Quantity	Value
$\frac{\mathcal{O}\to KK^*(892)}{\mathcal{O}\to\rho\pi}$	1.3
$\frac{\mathcal{O}\to\eta\omega}{\mathcal{O}\to\rho\pi}$	0.16
$\frac{\mathcal{O}\to\eta'\omega}{\mathcal{O}\to\rho\pi}$	0.13
$\frac{\mathcal{O}\to\eta\phi}{\mathcal{O}\to\rho\pi}$	0.21
$\frac{\mathcal{O}\to\eta'\phi}{\mathcal{O}\to\rho\pi}$	0.18
$\frac{\mathcal{O}\to\rho a_1(1230)}{\mathcal{O}\to\rho\pi}$	1.8
$\frac{\mathcal{O}\to\omega f_1(1285)}{\mathcal{O}\to\rho\pi}$	0.55
$\frac{\mathcal{O}\to\omega f_1(1420)}{\mathcal{O}\to\rho\pi}$	0.82

$$\boxed{\rho\pi,\ KK^*(892),\ \text{and}\ \rho a_1(1230)}$$