

The X(3872) as a mass distribution

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Excited QCD 2020

2-8 February 2020 Krynica Zdrój, Poland.

Based on: Chin.Phys. C43 (2019) no.12, 124107,
PoS Hadron2017 (2018) 236, Phys.Lett. B781 (2018) 678-683

The QCD spectrum

- Individual single state determination
- Collective state determination (thermodynamics, level density)
- Dashen and Kane (1975): Two types of hadrons depending on their contribution within the typical $SU(3)$ splitting.
- Impact on proliferation of X, Y, Z states
- X(3872) is a $J^{PC} = 1^{++}$ with mass $B_X = M_D + M_{\bar{D}^*} - M_X = 0.00(18)\text{MeV}$

To count or not to count

- The best way to look for missing states is to count them

1, 2, 3 . . .

- Thermodynamics is a way of counting
- The partition function of QCD counts

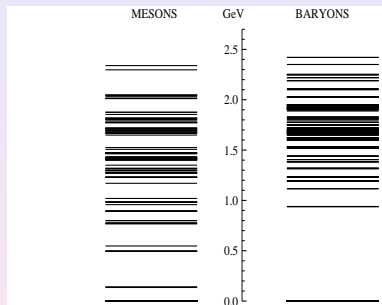
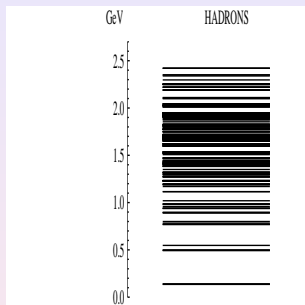
$$Z_{\text{QCD}} = \sum_n e^{-E_n/T} \quad H_{\text{QCD}}\psi_n = E_n\psi_n$$

- Spectrum of QCD \rightarrow Thermodynamics
- Colour singlet states (hadrons +???)
- Do we see quark-gluon substructure BELOW the “phase transition” ?
- Completeness relation in Hilbert space \mathcal{H}_{QCQ}

$$\mathbf{1} = \sum_n |\Psi_n\rangle\langle\Psi_n| \approx \underbrace{\sum_n |\bar{q}q; n\rangle\langle\bar{q}q; n|}_{\text{mesons}} + \underbrace{\sum_n |qqq; n\rangle\langle qqg; n|}_{\text{baryons}} + \underbrace{\sum_n |\bar{q}qg; n\rangle\langle\bar{q}qg; n|}_{\text{hybrids}} + \dots$$

- Given H, is there a sum rule involving ALL resonances ?.

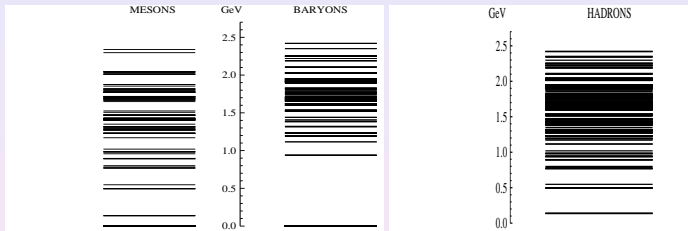
The states in the Particle Data Group (PDG) book



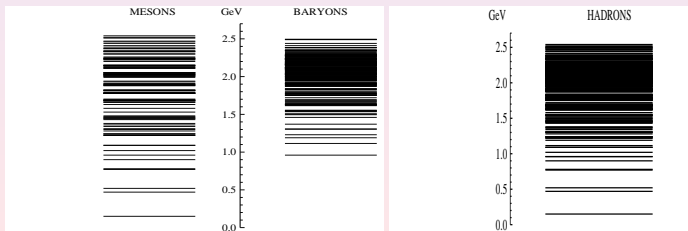
- No data (is a compilation)
- No particles (resonances)
- No book
- Which particles enter PDG ?

Hadron Spectrum (u,d,s)

- Particle Data Group (PDG) compilation 2016



- Relativized Quark Model (RQM) Isgur, Godfrey, Capstik, 1985

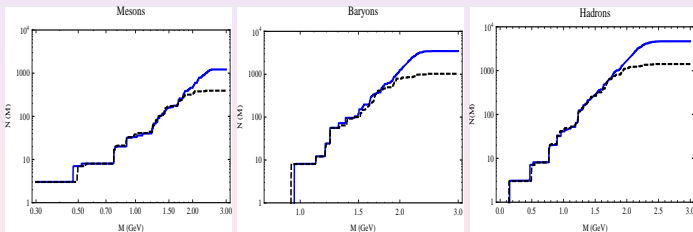


Cumulative number of states

- Compare H_{QCD} , H_{PDG} , H_{RQM} with staircase function

$$N(M) = \sum_n \theta(M - M_n)$$

- Which states count ?
- Is $N_{\text{QCD}}(M)$ accessible ?

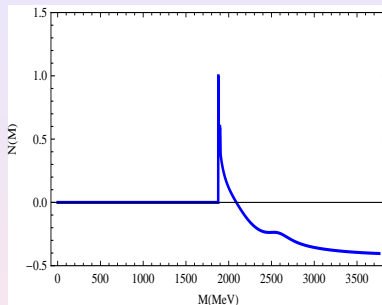


$$N_{q\bar{q}} \sim M^6 \quad N_{qq\bar{q}} \sim M^{12} \quad N_{qD} \sim M^6 \quad N_{qq\bar{q}q} \sim M^{18} \quad N_{\text{hadrons}} \sim e^{M/T_H}$$

$T_H \sim 150\text{MeV} = \text{Hagedorn temperature}$

Who counts ?

- Which are the complete set of states in the PDG ?
- Should X,Y,Z's or the deuteron or ^{208}Pb enter as multi-quark states ?



- The cumulative number in a given channel in the continuum with threshold M_{th}

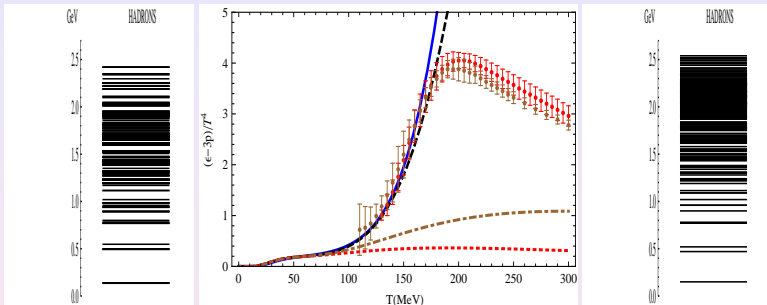
$$N(M) = \sum_n \theta(M - M_n) + [\delta(M) - \delta(M_{\text{th}})]/\pi$$

- Levinson's theorem

$$N(\infty) = n_B + [\delta(\infty) - \delta(M_{\text{th}})]/\pi = 0$$

- Deuteron doesn't count

QCD Spectrum and Trace anomaly



- PDG is thermodynamically equivalent to RQM !!

$$\mathcal{A}_{\text{HRG}}(T) \equiv \frac{\epsilon - 3P}{T^4} = \frac{1}{T^4} \sum_n \int \frac{d^3p}{(2\pi)^3} \frac{E_n(p) - \vec{p} \cdot \vec{\nabla}_p E_n(p)}{e^{E_n(p)/T} + \eta_n},$$

$$E_n(p) = \sqrt{p^2 + M_n^2} \quad \eta_n = \pm 1$$

- Non-interacting Hadron-Resonance Gas works for $T < 0.8T_c$
- Spectrum \rightarrow Thermodynamics

Fluctuations of conserved charges

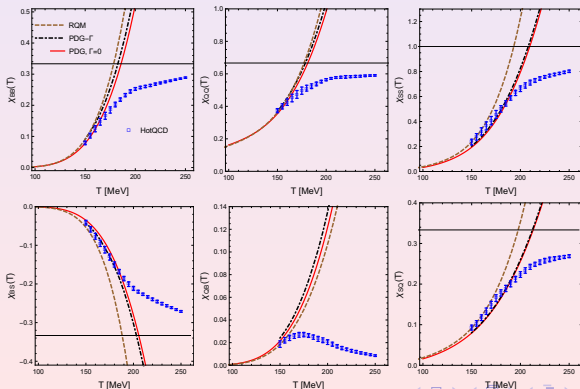
- Conserved charges

$$\langle N_B \rangle_T = 0 \quad \langle N_Q \rangle_T = 0 \quad \langle N_S \rangle_T = 0$$

- Vacuum Fluctuations

$$\chi_{BB}(T) = \langle N_B^2 \rangle_T \rightarrow \frac{1}{N_c} \quad \chi_{QQ}(T) = \langle N_Q^2 \rangle_T \rightarrow \sum_{i=1}^{N_f} q_i^2$$

$$\chi_{SS}(T) = \langle N_S^2 \rangle_T \rightarrow 1 \quad C_{BS}(T) = -3 \frac{\langle N_S N_B \rangle_T}{\langle N_S^2 \rangle_T} \rightarrow 1$$



Occupation number in the continuum

- Quantum virial expansion
- Single mass state

$$\bar{n} = \frac{\langle N \rangle_T}{V} = \int \frac{d^3k}{(2\pi)^3} \frac{g}{e^{\sqrt{k^2+m^2}/T} + \eta} \quad \eta = \pm \quad (\text{Bosons/Fermions})$$

- Effects of interactions (scattering phase shifts and resonance)

$$n(T) = \int \frac{d^3p}{(2\pi)^3} dm \frac{g}{e^{\sqrt{p^2+m^2}/T} + \eta} \rho(m),$$

- Level density in the continuum

$$\rho(m) = \sum_n \delta(m - m_n) + \frac{1}{\pi} \frac{d\delta}{dm}, \quad m = \sqrt{s} \text{ (CM energy)}$$

- Example Resonance: $\rho \rightarrow \pi\pi$ (contribution)

$$\delta(m) = \tan^{-1} \left[\frac{m - m_R}{\Gamma_R} \right] \rightarrow \frac{1}{\pi} \delta'(m) = \frac{1}{\pi} \frac{\Gamma_R}{((m - m_R)^2 + \Gamma_R^2)}$$

- Example Weakly bound state: $d \rightarrow pn$ (no-contribution)

$$p \cot \delta = -\frac{1}{\alpha_0} + \frac{1}{2} r_0 p^2 + \dots \quad m = \sqrt{p^2 + M^2}$$

X(3872) Bound state in a continuum

The Salamanca model

$$|\Psi\rangle = |c\bar{c}\rangle + |D\bar{D}^*\rangle = \sum_{\alpha} c_{\alpha} |\psi_{\alpha}\rangle + \sum_{\beta} \chi_{\beta}(P) |\phi_A \phi_{B\beta}\rangle,$$

This model assumes that the transition operator is

$$\begin{aligned} T &= -3\sqrt{2}\gamma' \sum_{\mu} \int d^3p d^3p' \delta^{(3)}(\mathbf{p} + \mathbf{p}') \times \\ &\times \left[\mathcal{Y}_1\left(\frac{\mathbf{p} - \mathbf{p}'}{2}\right) b_{\mu}^{\dagger}(\mathbf{p}) d_{\nu}^{\dagger}(\mathbf{p}') \right]^{C=1, l=0, S=1, J=0}, \end{aligned}$$

Matrix elements

$$\langle \phi_A \phi_{B\beta} | T | \psi_{\alpha} \rangle = P h_{\beta\alpha}(P) \delta^{(3)}(\vec{P}_{\text{cm}}).$$

$$\sum_{\beta} \int (H_{\beta' \beta}(P', P) + V_{\beta' \beta}^{\text{eff}}(P', P)) \times \chi_{\beta}(P) P^2 dP = E \chi_{\beta'}(P'), \quad (1)$$

where $H_{\beta' \beta}$ is the Resonating Group Method (RGM) Hamiltonian for the two-meson states obtained from the $q\bar{q}$ interaction. The effective potential $V_{\beta' \beta}^{\text{eff}}$ encodes the coupling with the $c\bar{c}$ bare spectrum, and can be written as

$$V_{\beta' \beta}^{\text{eff}}(P', P; E) = \sum_{\alpha} \frac{h_{\beta' \alpha}(P') h_{\alpha \beta}(P)}{E - M_{\alpha}}, \quad (2)$$

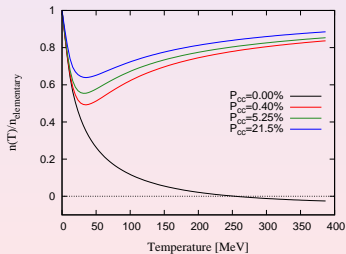
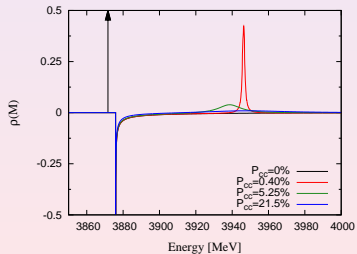
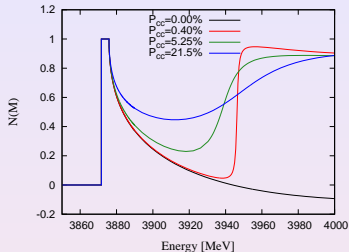
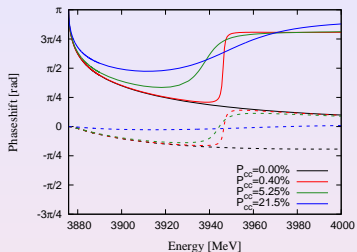
where M_{α} are the masses of the bare $c\bar{c}$ mesons.

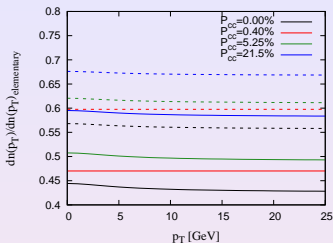
$$S^{J1} = \begin{pmatrix} \cos \epsilon_j & -\sin \epsilon_j \\ \sin \epsilon_j & \cos \epsilon_j \end{pmatrix} \begin{pmatrix} e^{2i\delta_j^{1j}-1} & 0 \\ 0 & e^{2i\delta_j^{1j}+1} \end{pmatrix} \times \begin{pmatrix} \cos \epsilon_j & -\sin \epsilon_j \\ \sin \epsilon_j & \cos \epsilon_j \end{pmatrix}. \quad (3)$$

From here we define the T-matrix

$$S^{JS} = 1 - 2iKT^{JS}, \quad (4)$$

Level density and occupation number



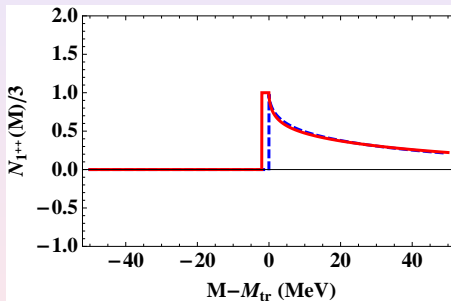


$\gamma(^3P_0)$	$\mathcal{P}_{c\bar{c}}$ [%]	α_0 [fm]	r_0 [fm]	M [MeV]	Γ [MeV]
0.00	0.00	3.14	1.21	3947.43	0.00
0.05	0.40	3.14	1.20	3946.29	1.38
0.10	1.82	3.11	1.17	3943.06	5.88
0.16	5.25	3.05	1.10	3938.56	15.18
0.20	14.25	2.88	0.85	3937.09	37.93
0.23	21.50	2.73	0.63	3947.05	56.03

Table: PDG values are $M = 3942(9)\text{MeV}$ and $\Gamma = 37_{-17}^{+27}\text{MeV}$.)

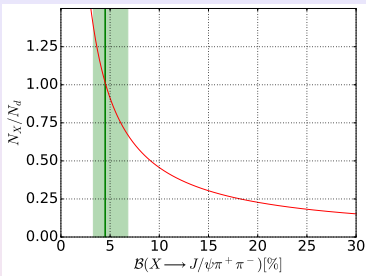
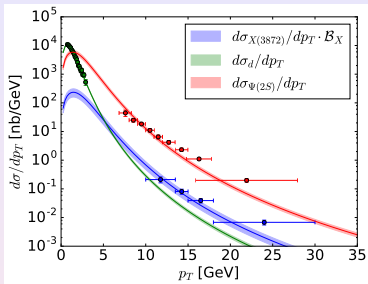
X(3872) vs deuteron

- deuteron is detected by its track *directly*
- $X(3872) \rightarrow \rho J/\psi, \omega J/\psi$ decay channels (Resolution is relevant)



X(3872) and deuteron production in pp collisions

Large tail in X production (compact object)



Tsallis distribution for a pure mass state

$$\frac{d^3N}{d^3p} = \frac{gV}{(2\pi)^3} \left(1 + (q-1) \frac{E(p)}{T} \right)^{-\frac{q}{q-1}} \xrightarrow{q \rightarrow 1} \frac{gV}{(2\pi)^3} e^{-\frac{E(p)}{T}}, \quad E(p) = \sqrt{m^2 + p^2}$$

Production cross section $E(p_T, y) = \sqrt{p_T^2 + m^2} \cosh y$, $d^3N/(d^2p_T dy) \equiv E_p d^3N/d^3p$ with $y = \tanh^{-1}(E_p/p_z)$

$$\frac{1}{2\pi p_T} \frac{d\sigma(m)}{dp_T} = \mathcal{N} \int dy E(p_T, y) \left[1 + \frac{q-1}{T} E(p_T, y) \right]^{\frac{q}{1-q}}$$

Finite Resolution Δm

- $X(3872) \rightarrow \rho J/\psi, \omega J/\psi$ decay channels
- Any state within the resolution $\pm \Delta m/2$ with $J^{PC} = 1^{++}$
- Given $O(m)$ we get

$$O_{\Delta m} \equiv \int_{m-\Delta m/2}^{m+\Delta m/2} dM \rho(M) O(M).$$

where $\rho(M)$ is the density of states, defined as

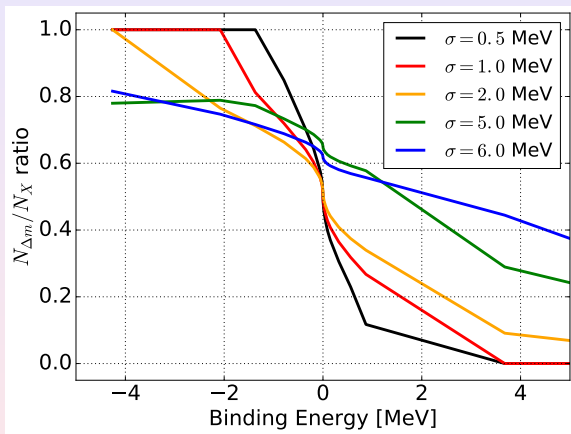
$$\rho(M) = \frac{dN(M)}{dM} = \sum_i \delta(M - M_i^B) + \frac{1}{\pi} \sum_{\alpha=1}^n \delta'_\alpha(M), \quad (5)$$

- Case $\Delta m \gg |B| \equiv |M_B - M_{tr}|$

$$O|_{MB \pm \Delta m} = O(M^B) + \frac{1}{\pi} \int_{M_{tr}}^{M_{tr} + \Delta m/2} dM \delta'_\alpha(M) O(M).$$

Production ratio

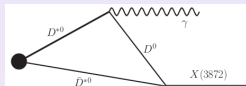
Common Tsallis distribution for $X(3872)$ and deuteron



The relative occupation number *with* and *without* finite resolution $\Delta m = 2\sigma$ as a function of the $X(3872)$ binding energy

Triangle singularities and $X(3872)$ mass

- Recent proposals to make accurate measurements of mass (Guo, Braaten;2019)
 $e^+e^- \rightarrow X(3872)\gamma$



- Level density in $J^{PC} = 1^{++}$ channel

$$\rho(m) = \delta(m - M_X) + \frac{1}{\pi} \delta'(m). \quad (6)$$

- Detector efficiency

$$R_\sigma(m, M) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(m-M)^2}{2\sigma^2}} \quad (7)$$

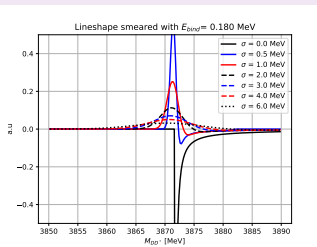
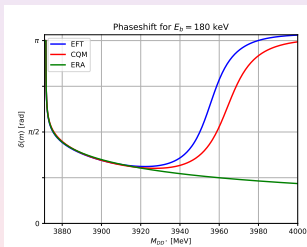
Channel	σ	ΔM	Reference
$J/\psi\pi^+\pi^-$	1.14 ± 0.07	20	Ablikim:2013dyn
$J/\psi\pi^+\pi^-$	3.33 ± 0.08	$6\sigma \approx 20$	Aaij:2011sn
$J/\psi\pi^+\pi^-$	1.2?	18	Choi:2011fc

Smearing of level density

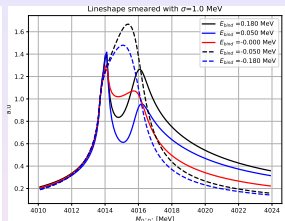
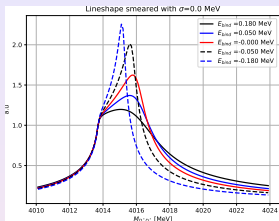
- Effective range expansion

$$k \cot \delta = -\frac{1}{\alpha_0} + \frac{1}{2} r_0 k^2$$

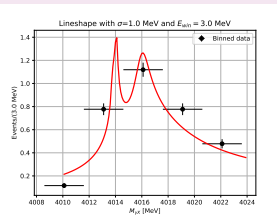
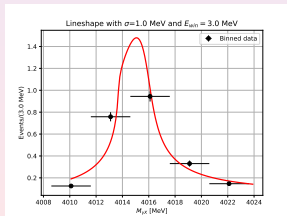
- Comparison between ERA and two coupled-channels models
- $r_0 = 1 \text{ fm}$ $\alpha_0 = \frac{1}{\sqrt{2\mu E_b}} = 10.58 \text{ fm}$



Smearing of production



MonteCarlo simulation $N = 1000$ runs between $E \in [4010, 4020]$



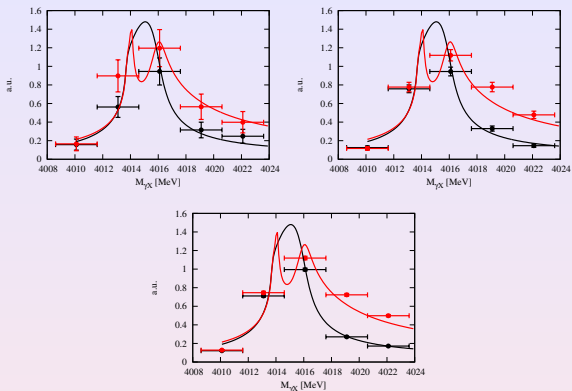


Figure: Binned smeared lineshapes of states for $\sigma = 1$ MeV, $E_w = 3$ MeV and $\Delta M = 20$ MeV for $E_b = 180$ keV (black points) and $E_b = -180$ keV (red points). Full lines show the no-binned smeared lineshape (same color code). The results are done with $N = 100$ (upper left), $N = 1000$ (upper right) and $N = 10000$ (lower left).

Conclusions

- Is the $X(3872)$ a bound state ?
- If not , should it appear in the PDG ?
- Can its mass be determined accurately ?
- Does $X(3872)$ count ?