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Holographic radial spectrum of mesons from higher dimensional QCD operators

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Based on: S.S. Afonin [arXiv:1905.13086]

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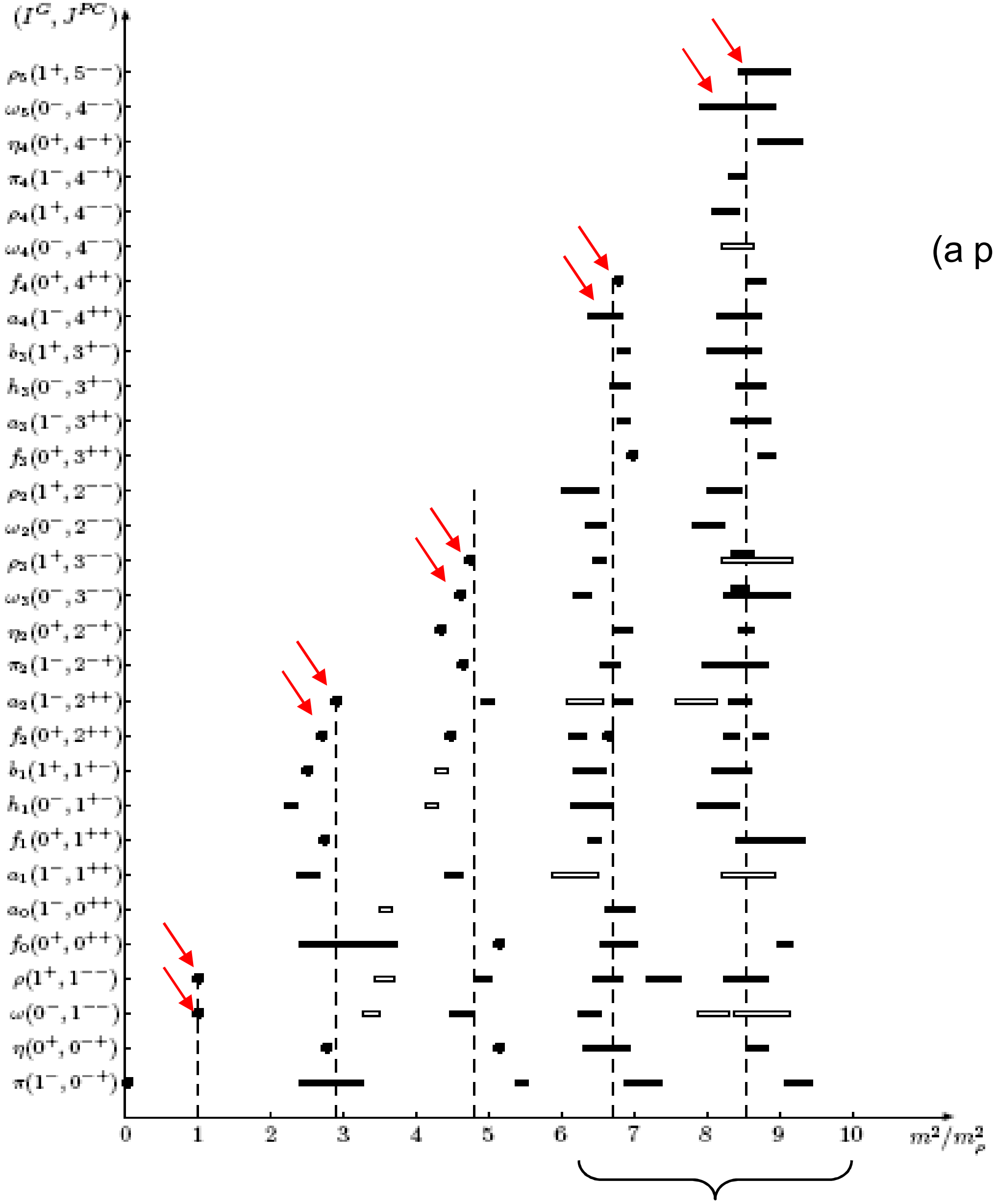
Experimental spectrum of light non-strange mesons

(a plot from S.S. Afonin, Eur. Phys. J. A 29 (2006) 327)



Many need confirmation

The major feature:
Spin-parity clustering

The spectrum of light non-strange mesons in units of M_ρ^2 . Experimental errors are indicated. Circles stay when errors are negligible. The dashed lines mark the mean mass squared in each cluster of states.
The arrows indicate the $J > 0$ mesons which have no chiral partners.

How to describe this spectrum from first principles?

The standard way is as follows:

Construct from quark and gluon fields an operator ***O(x)*** having quantum numbers of a hadron state $|n\rangle$. There is an infinite number of possibilities, one takes the operator with minimal canonical dimension (usually twist-2 operators). Then one calculates the two-point correlation function at large separation in Euclidean space,

$$\langle O(x)O(y)\rangle \sim e^{-m_n|x-y|}$$

$$\underline{|x-y|\to\infty}$$

This program is followed in lattice simulations and QCD sum rules.

But what about the higher radial excitations? They are exponentially suppressed in this approach.

An old assumption:
<u>They should couple to QCD operators of higher canonical dimensions.</u>

Difficult to realize in practice!

Some examples

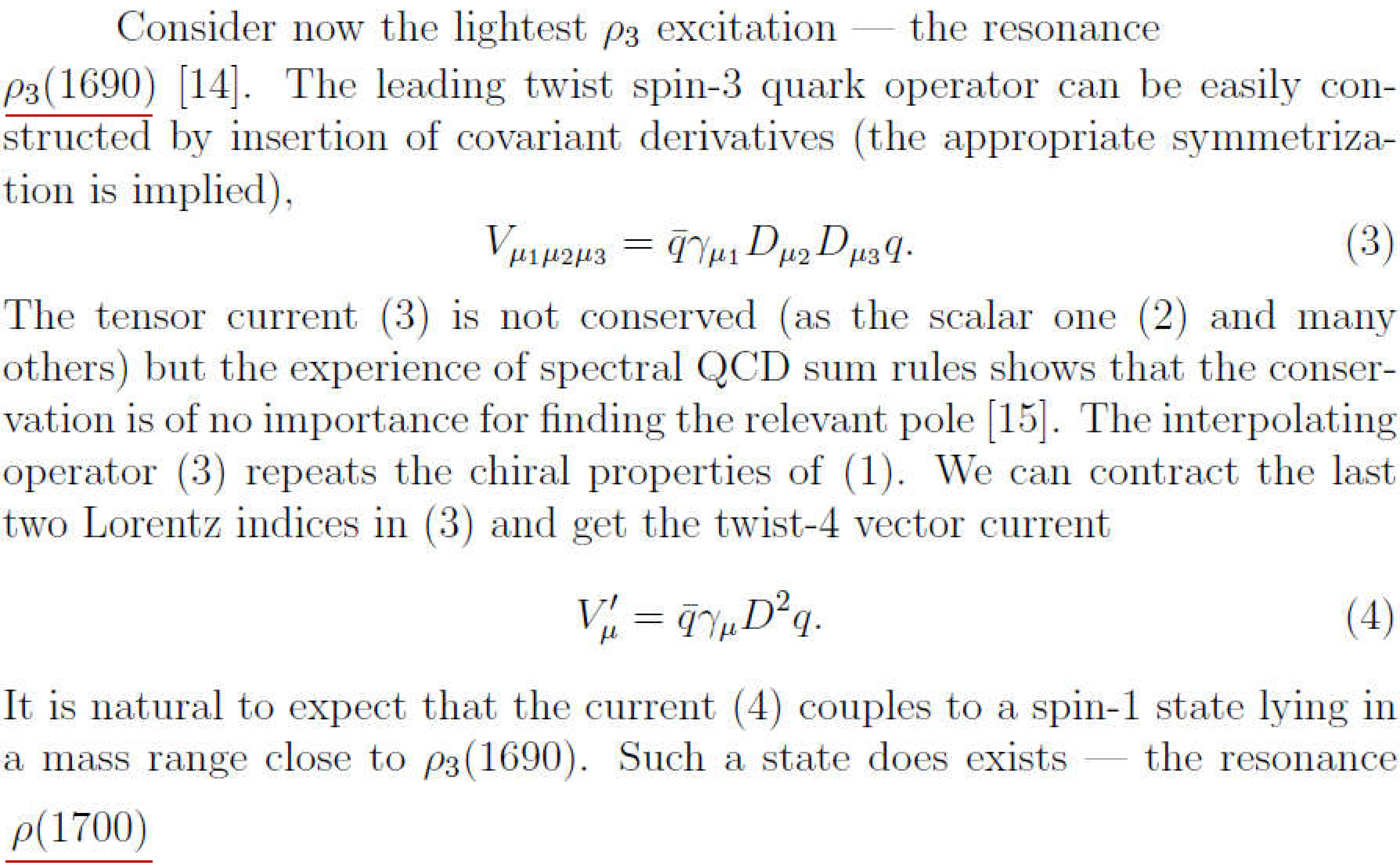
Consider two basic quark currents,

$$V_\mu = \bar{q}\gamma_\mu q = \bar{q}_{\mathrm{L}}\gamma_\mu q_{\mathrm{L}} + \bar{q}_{\mathrm{R}}\gamma_\mu q_{\mathrm{R}}, \tag{1}$$

$$S = \bar{q}q = \bar{q}_{\mathrm{L}}q_{\mathrm{R}} + \bar{q}_{\mathrm{R}}q_{\mathrm{L}}. \tag{2}$$

Here the Dirac spinor $q = q_{\mathrm{L}} + q_{\mathrm{R}}$ stays for u or d quark fields and $q_{\mathrm{L,R}} = \frac{1\mp\gamma_5}{2}q$. The isospin and γ_5 matrices can be also inserted in (1) and (2) but this is not essential for our discussions and will be dropped (but implicitly assumed where necessary). What is essential is the different chiral structure of twist-2 vector current (1) and twist-3 scalar current (2) — they transform differently under $\underline{SU_{\mathrm{L}}(2) \times SU_{\mathrm{R}}(2)}$ chiral transformations.

The twist-2 vector current (1) has been traditionally used for interpolation of ρ and ω mesons in QCD sum rules, lattice QCD and low-energy effective field theories.



Consider now the lightest ρ_3 excitation — the resonance $\rho_3(1690)$ [14]. The leading twist spin-3 quark operator can be easily constructed by insertion of covariant derivatives (the appropriate symmetrization is implied),

$$V_{\mu1\mu2\mu3} = \bar{q}\gamma_{\mu_1} D_{\mu_2} D_{\mu_3} q. \tag{3}$$

The tensor current (3) is not conserved (as the scalar one (2) and many others) but the experience of spectral QCD sum rules shows that the conservation is of no importance for finding the relevant pole [15]. The interpolating operator (3) repeats the chiral properties of (1). We can contract the last two Lorentz indices in (3) and get the twist-4 vector current

$$V'_\mu = \bar{q}\gamma_\mu D^2 q. \tag{4}$$

It is natural to expect that the current (4) couples to a spin-1 state lying in a mass range close to $\rho_3(1690)$. Such a state does exists — the resonance $\rho(1700)$

So the general principle: the higher is the canonical dimension of interpolating QCD operator the higher is the mass of corresponding hadron state.

On the other hand, a vector interpolating current can be constructed also by insertion of covariant derivative to the scalar current (2),

$$\tilde{V}_\mu = \bar{q} D_\mu q = \bar{q}_{\rm L} D_\mu q_{\rm R} + \bar{q}_{\rm R} D_\mu q_{\rm L}. \tag{5}$$

This operator inherits the chiral properties of the current (2) and its twist. In addition, the currents (1) and (5) look different from the point of view of Lorentz group since (5) can be represented on shell as [13]

$$\bar{q} D_\mu q \propto -\partial^\nu H_{\mu\nu}, \qquad H_{\mu\nu} = \bar{q}\sigma_{\mu\nu} q, \tag{6}$$

where $\sigma_{\mu\nu} = (\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)/2i$. The antisymmetric tensor current $H_{\mu\nu}$ transforms as $(1,0)+(0,1)$ while (1) has the Lorentz structure $\left(\frac{1}{2},\frac{1}{2}\right)$ [11–13]. One can notice that the vector current $\partial^\nu H_{\mu\nu}$ is trivially conserved but this conservation is topological, i.e. of different nature than the conservation of Noether current (1). What ρ-meson does the operator (5) interpolate? Since it has one covariant derivative, one can expect the corresponding state to lie between $\rho(770)$ and $\rho(1700)$. There exists one well-established ρ-meson in this range — the resonance $\rho(1450)$ [14]. One may expect also that the experimental study of this resonance is more difficult because its production should be suppressed in e^+e^--annihilation. The Particle Data [14] indeed makes caution that $\rho(1450)$ is the name for a broad resonance region rather than a definite resonance.

The higher spin Regge recurrences arise thus from two kinds of composite spin-J operators stemming from (1) and (2),

$$V_{\mu_1\mu_2\mu_3\dots\mu_J} = \bar{q}\gamma_{\mu_1}D_{\mu_2}D_{\mu_3}\dots D_{\mu_J}q, \tag{7}$$

$$\tilde{V}_{\mu_1\mu_2\mu_3\dots\mu_J} = \bar{q}D_{\mu_1}D_{\mu_2}D_{\mu_3}\dots D_{\mu_J}q. \tag{8}$$

The operators (7) interpolate the spin-J quark-antiquark states in the Lorentz representation $\left(\frac{J}{2},\frac{J}{2}\right)$ and have the chiral transformation properties of usual vector current (1) (and for this reason emerge naturally in QCD analysis of deep inelastic scattering via OPE), while the chiral and Lorentz properties of (8) are different. Contracting n times the Lorentz indices we get an interpolating operator for the n-th radial excitation of corresponding spin-$(J-2n)$ meson.

For high enough canonical dimensions, several covariant derivatives in (7) and (8) can be replaced by insertion of gluon field strength $G_{\mu\nu}$, the corresponding operators will interpolate hybrid states. For instance, the operator $\bar{q}\gamma^{\mu}G_{\mu\nu}D^{\nu}q$ couples to a scalar hybrid with the chiral properties of vector current (1). One can of course construct purely gluonic operators which are chiral singlets. The leading twist-2 operators of this sort have the structure

$$\tilde{G}_{\mu_1\mu_2\mu_3\dots\mu_J} = G^{\rho}_{\mu_1}D_{\mu_2}D_{\mu_3}\dots D_{\mu_{J-1}}G_{\mu_J\rho}. \tag{9}$$

How to relate the hadron mass and the canonical dimension of corresponding QCD operator within a working model?

The problem simplifies and becomes much better defined in the large-N_c limit of QCD (no multiquark states).

Such a connection appears naturally in the holographic QCD!

[Witten; Gubser, Polyakov, Klebanov (1998)]

The essence of the holographic method

$$\langle e^{\int d^d x J(x)\mathcal{O}(x)}\rangle_{\rm CFT} = \int \mathcal{D}\phi\; e^{-S[\phi\,,\,g]}\Big|_{\phi(x,\partial AdS)=J(x)}$$

generating functional

action of dual gravitational theory
evaluated on classical solutions

AdS boundary

$$\Pi_n \equiv \langle\, \mathcal{O}_{I_1}(x_1)\dots\mathcal{O}_{I_n}(x_n)\,\rangle = \frac{1}{\sqrt{g}}\frac{\delta}{\delta\phi^{I_1}(x_1)}\cdots\frac{1}{\sqrt{g}}\frac{\delta}{\delta\phi^{I_n}(x_n)}S[\phi\,,\,g]$$

The output of the holographic models: **Correlation functions**

Poles of the 2-point correlator → mass spectrum

Residues of the 2-point correlator → decay constants

Residues of the 3-point correlator → transition amplitudes

Alternative way for finding the mass spectrum is to solve e.o.m. $\phi(x_\mu, z) = e^{ixp}\phi(z)$

Hard wall model

(Erlich et al., PRL (2005); Da Rold and Pomarol, NPB (2005))

The AdS/CFT dictionary dictates: local symmetries in 5D → global symmetries in 4D

The chiral symmetry: $SU_L(2) \times SU_R(2)$

A typical model describing the chiral symmetry breaking and meson spectrum:

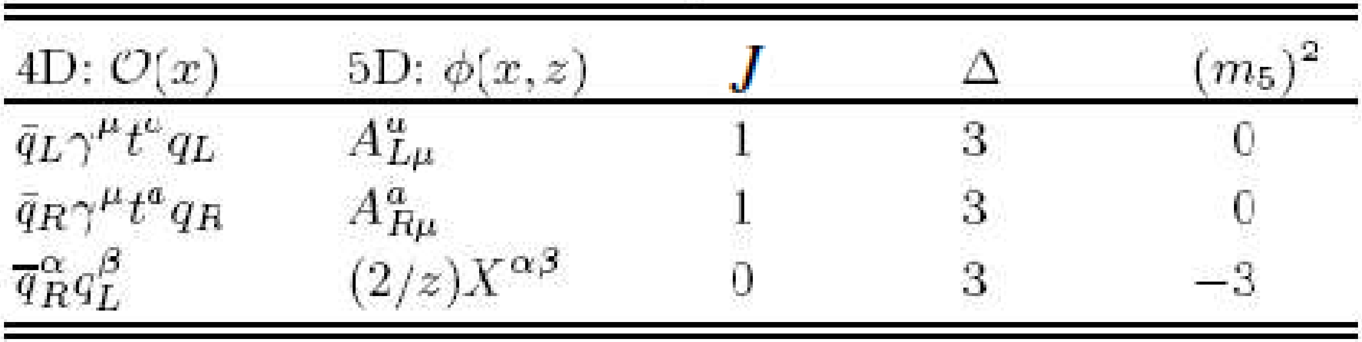
$$\boxed{S = \int d^5x \sqrt{g}\, \mathrm{Tr}\left\{|DX|^2 + 3|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right\}} \qquad 0 < z \leq z_m$$

$$D_\mu X = \partial_\mu X - iA_{L\mu}X + iXA_{R\mu},\ A_{L,R} = A^a_{L,R}t^a,\ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu].$$

The pions are introduced via $X = X_0 \exp(i2\pi^a t^a) \qquad t^a = \sigma^a/2$

$$V = (A_L + A_R)/2 \qquad A = (A_L - A_R)/2 \qquad m_5^2 R^2 = (\Delta - J)(\Delta + J - 4)$$

At $z = z_m$ one imposes certain gauge invariant boundary conditions on the fields.



4D: $\mathcal{O}(x)$	5D: $\phi(x,z)$	J	Δ	$(m_5)^2$
$\bar{q}_L\gamma^\mu t^a q_L$	$A^a_{L\mu}$	1	3	0
$\bar{q}_R\gamma^\mu t^a q_R$	$A^a_{R\mu}$	1	3	0
$\bar{q}^\alpha_R q^\beta_L$	$(2/z)X^{\alpha\beta}$	0	3	-3

Equation of motion for the scalar field

$$\frac{1}{z^5}3X = \frac{1}{z^3}\partial_\mu \partial^\mu X - \partial_z \frac{1}{z^3}\partial_z X$$

Solution independent of usual 4 space-time coordinates

$$X_0(z) = \frac{1}{2}Mz + \frac{1}{2}\Sigma z^3$$

quark condensate

current quark mass

As the holographic dictionary prescribes

$$\boxed{\Phi(x,z)_{z\to 0} = z^{4-\Delta}\Phi_0(x) + z^{\Delta}\frac{\langle O(x)\rangle}{2\Delta - 4}}$$

here $\Delta = 3$

Denoting $X_0(z) = \frac{1}{2}v(z)\mathbf{1}, \qquad v(z) = mz - \sigma z^3$

the equation of motion for the vector fields are (in the axial gauge *V_z=0*)

$$\left[\partial_z\left(\frac{1}{z}\partial_z V^a_\mu(q,z)\right) + \frac{q^2}{z}V^a_\mu(q,z)\right]_\perp = 0$$

where $V(q,z) = \int d^4x \;\; e^{iqx}V(x,z)$

due to the chiral symmetry breaking

$$\left[\partial_z\left(\frac{1}{z}\partial_z A^a_\mu\right) + \frac{q^2}{z}A^a_\mu - \frac{g_5^2 v^2}{z^3}A^a_\mu\right]_\perp = 0$$

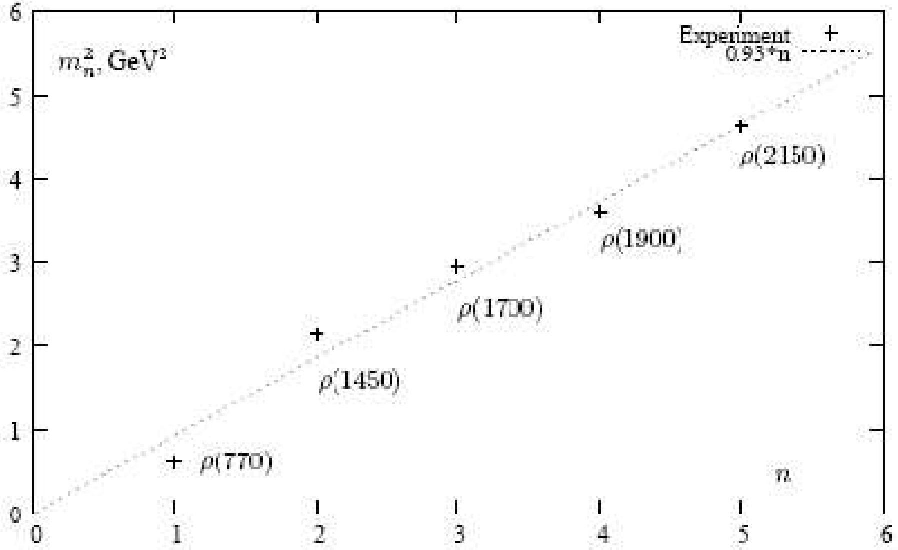
The GOR relation holds $m_\pi^2 f_\pi^2 = 2M\Sigma$

The spectrum of normalizable modes is given by (with Dirichlet boundary condition)

$$J_0(m_n z) = 0$$

thus the asymptotic behavior is $\boxed{m_n \sim n}$ (Rediscovery of 1979 Migdal’s result)

that is not Regge like $m_n^2 \sim n$



A. Karch, E. Katz, D. T. Son, M. A. Stephanov, PRD 74, 015005 (2006)

Soft-wall model:

$$S = -\frac{c^2}{4}\int d^4x\, dz \sqrt{g}\, e^{-az^2} F_{MN}F^{MN}$$

$$F_{MN} = \partial_M V_N - \partial_N V_M,\ M = 0, 1, 2, 3, 4$$

The IR boundary condition: the action is finite at $z = \infty$

Plane wave ansatz: $V_\mu(x, z) = \varepsilon_\mu e^{ipx} v(z)$ $p^2 = m^2$ Axial gauge $V_z = 0$

After the change of variables $v_n = \sqrt{z} e^{az^2/2} \psi_n$ the e.o.m. is reduced to:

$$\boxed{-\psi_n'' + U(z)\psi_n = m_n^2 \psi_n} \qquad U = a^2z^2 + \frac{3}{4z^2}$$

One has the radial Schroedinger equation for the harmonic oscillator with orbital momentum ***L=1***

$$-\psi'' + \left[z^2 + \frac{L^2 - 1/4}{z^2}\right]\psi = E\psi \qquad E = |a|m$$

The spectrum:

$$\boxed{m_n^2 = 4|a|(n+1)} \qquad n = 0, 1, 2, \ldots$$

The extension to massless higher-spin fields leads to (for ***a* > *0***)

$$\boxed{m_{n,J}^2 = 4a(n+J)}$$

Generalization to the arbitrary intercept: $m_n^2 = 4|a|(n+1+b)$

In the vector case: $e^{-az^2} \to \Gamma(1+b)U^2(b,0;az^2)e^{-az^2}$ (Afonin, PLB (2013))

Tricomi function

The scalar sector

$$S = \int d^4x\, dz \sqrt{g}\, e^{-az^2}(\partial_M\Phi\partial^M\Phi - m^2\Phi^2)$$

$$m^2R^2 = \Delta(\Delta - 4)$$

$$m_n^2 = 2|a|\left(2n + \Delta - 1 + \frac{a}{|a|}\right), \qquad n = 0, 1, 2, \ldots$$

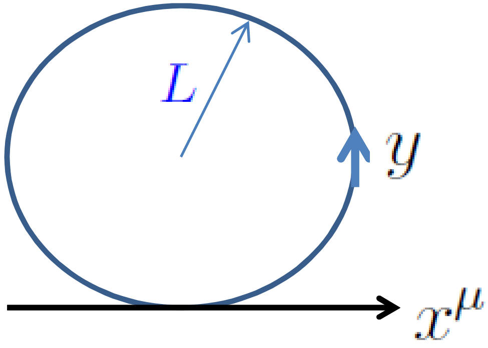
The essential property of holographic description: The spectrum obtained is the spectrum of Kaluza-Klein excitations!

Kaluza-Klein reduction (the simplest case)

Consider a free massless scalar in $D+1$ dimensions, satisfying $\hat{\Box}\hat{\phi}(x^\mu, y) = 0$. Fourier expanding on the circle $0 \leq y \leq 2\pi L$, i.e. $\hat{\phi}(x,y) = \sum_n \phi_n(x) e^{iny/L}$, implies the D-dimensional fields ϕ_n have masses $m^2 = n^2/L^2$:

$$\Box\phi_n - \frac{n^2}{L^2}\phi_n = 0\,.$$

One massless field ϕ_0 plus an infinite tower of massive fields.

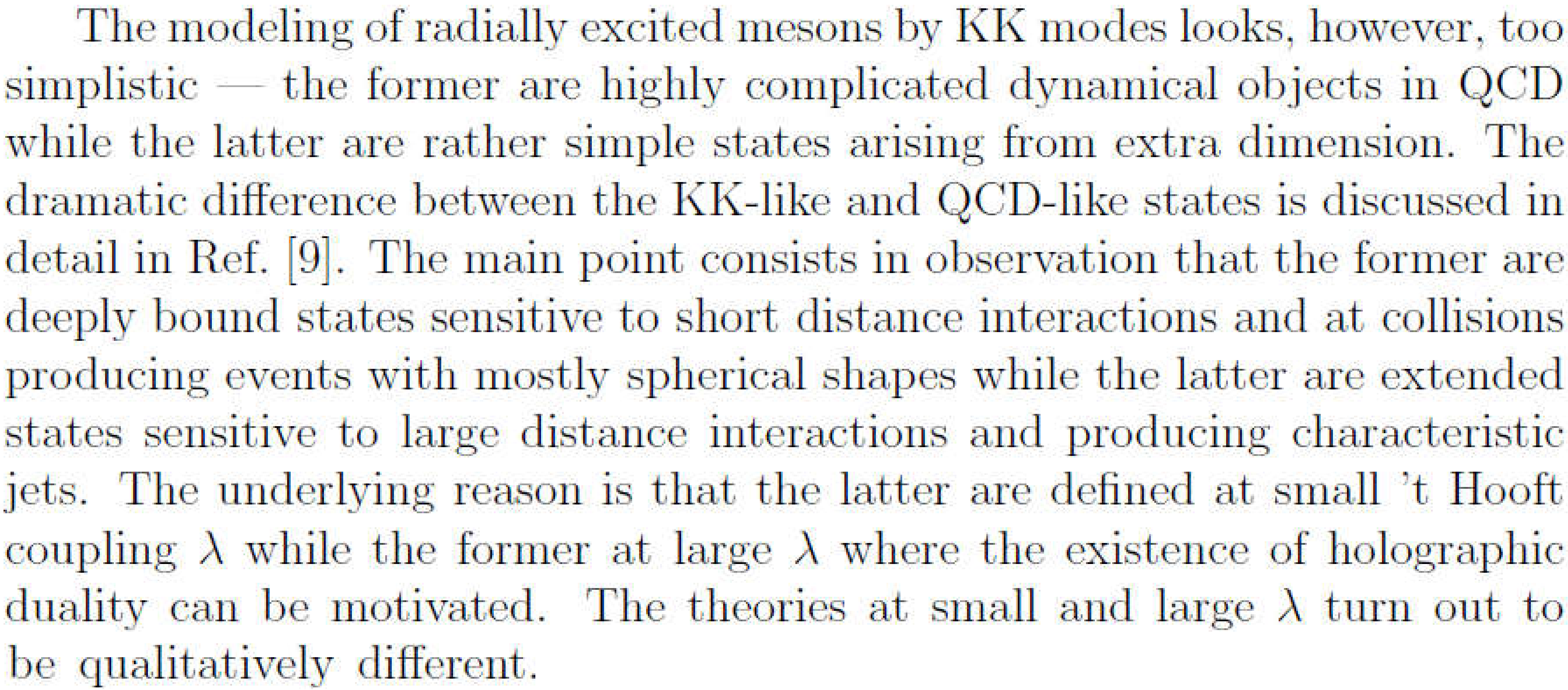


These Kaluza-Klein states are identified with the infinite tower of “radial excitations” expected in the large-*N*c limit of QCD,

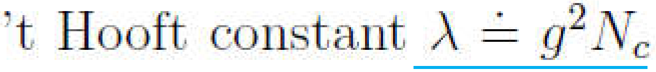
$$\boxed{\langle J(q)J(-q)\rangle = \sum_n \frac{F_n^2}{q^2 - M_n^2}}$$

$$M_n = \mathcal{O}(1) \qquad F_n^2 = \langle 0|J|n\rangle^2 = \mathcal{O}(N_c) \qquad \Gamma = \mathcal{O}(1/N_c)$$

The poblem which is usually ignored: The narrow mesons in QCD and Kaluza-Klein states are drastically different in their physical properties!



The modeling of radially excited mesons by KK modes looks, however, too simplistic — the former are highly complicated dynamical objects in QCD while the latter are rather simple states arising from extra dimension. The dramatic difference between the KK-like and QCD-like states is discussed in detail in Ref. [9]. The main point consists in observation that the former are deeply bound states sensitive to short distance interactions and at collisions producing events with mostly spherical shapes while the latter are extended states sensitive to large distance interactions and producing characteristic jets. The underlying reason is that the latter are defined at small 't Hooft coupling λ while the former at large λ where the existence of holographic duality can be motivated. The theories at small and large λ turn out to be qualitatively different.



't Hooft constant $\lambda \doteq g^2 N_c$

[9] C. Csaki, M. Reece and J. Terning, JHEP **0905**, 067 (2009).

Way out?

Our proposal: Let us assume that higher KK states cannot be excited for some reasons.

This is strongly supported by a recent finding of ref.

S. Fichet, Phys. Rev. D **100**, 095002 (2019)

that in Anti-de Sitter space, gravity dresses free propagators of particles on the quantum level leading to exponential suppresion in the infrared region,

$$\Delta_p(z) \propto e^{-\alpha p z},$$

where $p = \sqrt{p^\mu p_\mu}$ and positive constant $\alpha \sim 1/(R M_{\rm Pl})^2$ arises from one-loop gravitational corrections. The one-loop corrections due to interactions with other fields in the bulk (including self-interaction) also lead to the suppression

This expelling resembles Meissner effect for magnetic field in superconductor!

But what about excited states in holographic models?

Let us do a one-to-one mapping of each of them to some QCD operator.
The subsequent strategy: choose a model, calculate the discrete spectrum as a function of dimension of operator for some set of quantum numbers, remove the dependence on discrete number and substitute instead the dependence on operator dimension.

Example: the Soft-Wall model for vector mesons

$$S = c^2 \int d^4x\, dz \sqrt{g}\, e^{-az^2} \left(-\frac{1}{4} F_{MN} F^{MN} + \frac{1}{2} m_5^2 V_M V^M \right)$$

where $g = |\mathrm{det} g_{MN}|$, $F_{MN} = \partial_M V_N - \partial_N V_M$, $M, N = 0, 1, 2, 3, 4$, c is a normalization constant for the vector field V_M, and the background space represents the Poincaré patch of the AdS$_5$ space with the metric

$$g_{MN} dx^M dx^N = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \qquad z > 0.$$

The holographic prescription for 5D mass will be $m_5^2 R^2 = (\Delta - 1)(\Delta - 3)$

The spectrum is $\boxed{m_n^2 = 2|a|\,(2n + \Delta - 1)}$ $\quad n = 0, 1, 2, ...$

There is an infinite number of spin-1 QCD operators with identical chiral and Lorentz properties. Their canonical dimensions grow as

$$\Delta = 3 + 2k, \qquad k = 0, 1, 2, \ldots .$$

Substituting this to the spectrum we obtain

$$m_n^2 = 4|a|\,(n + k + 1)\,, \qquad n, k = 0, 1, 2, \ldots .$$

Now we delete "*n*" (i.e. we set *n=0*) – the higher KK modes are not excited!

We see that our prescription does not change the spectrum in this particular model!

The Soft-Wall model for arbitrary spins

$$I = (-1)^J \frac{1}{2} \int d^4x \, dz \sqrt{g} \, e^{-az^2} \left(\nabla_N \Phi_J \nabla^N \Phi^J - m_J^2 \Phi_J \Phi^J \right)$$

$\Phi_J \doteq \Phi_{M_1 M_2 \ldots M_J}, \; M_i = 0, 1, 2, 3, 4,$ $\quad \partial^\mu \Phi_{\mu \ldots} = 0$ $\quad \Phi_{z \ldots} = 0$

Using again the 4D plane-wave ansatz $\Phi_J(x_\mu, z) = e^{ipx} \phi^{(J)}(z) \epsilon_J$

we get the equation of motion for the profile function $\phi^{(J)}(z)$

$$-\partial_z \left(e^{-az^2} z^{2J-3} \partial_z \phi_n^{(J)} \right) + m_J^2 R^2 e^{-az^2} z^{2J-5} \phi_n^{(J)} = m_n^2 e^{-az^2} z^{2J-3} \phi_n^{(J)}.$$

The discrete spectrum

$$m_{n,J}^2 = 4|a|(n + k + J), \qquad n, k = 0, 1, 2, \ldots, \quad J > 0.$$

Here we again should set *n=0* – the form of spectrum is not changed.

We can get a further insight from consideration of normalized eigenfunctions corresponding to the discrete spectrum (35),

$$\phi_n^{(J)} = \sqrt{\frac{2n!}{(J+2k+n)!}}\, e^{-|a|z^2} \left(|a|z^2\right)^{1+k} L_n^{J+2k}\left(|a|z^2\right), \qquad (36)$$

where $L_n^{\alpha}(x)$ are associated Laguerre polynomials. It is seen that the numbers n and k are not completely interchangeable in the radial wave function: While the large z asymptotics depends on the sum $n+k$ (because $L_n^{\alpha}(x) \sim x^n$ at large x), the number of zeros is controlled by n only (as the polynomial $L_n^{\alpha}(x)$ has n zeros). By setting $n=0$, i.e. by keeping the zero KK mode only, we thus choose the wave function without zeros in holographic coordinate. This wave function is the least "entangled" with the 5th holographic dimension and thereby is the least sensitive to deviations from the AdS structure.

The Hard-Wall model (briefly)

In the standard approach, the spectrum of vector states is determined from the Dirichlet boundary condition $\partial_z \phi(m_n z_m) = 0 \Longrightarrow J_0(m_\rho z_m) = 0$

Normalizing the first zero of Bessel function to the rho-meson mass we get the IR-cutoff

$$z_m^{-1} \approx 323 \text{ MeV}$$

The radial spectrum is then given by further zeros, $\boxed{m_{\rho,n} \approx \{776, 1777, 2810, 3811, \dots\}}$

In our approach, the spectrum for arbitrary spins is determined by the condition

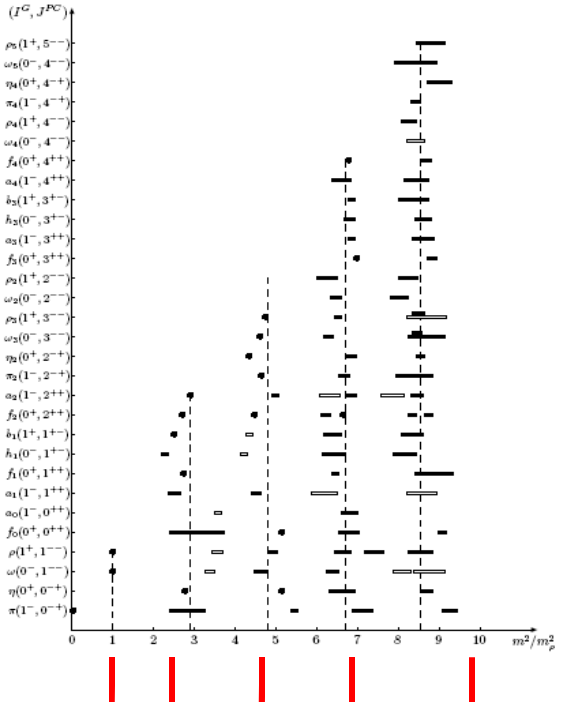
$$J_{\Delta-3}(m_n z_m) = 0 \qquad \Delta = 3, 4, 5, \dots$$

The radial spectrum numerically (in MeV)

$$\boxed{m_n \approx \{776, 1234, 1653, 2056, 2452, \dots\}}$$

The phenomenology becomes much better: We get 5 rho-mesons below 2.5 GeV (as the experimental data seem to suggest) instead of 2 in the usual HW model.

It is curious to note that this spectrum interpolates with a relatively good precision the averaged positions of meson clusters in the light non-strange sector.



CONCLUSIONS

- A new holographic description of excited hadrons is proposed
- The method is free of some conceptual drawbacks related with the Kaluza-Klein states
- It agrees better with the experimental spectroscopic data
- It is more related with real QCD than previous holographic models

* On anomalous dimensions

The real QCD operators have anomalous dimensions and this represents a notorious problem for the whole bottom-up holographic approach. One makes reference to asymptotic freedom at best, any serious discussion of this problem is usually avoided. We will not give a real physical justification but make an observation. Within our considerations, the account for the anomalous dimension of operators is tantamount to replacement $\underline{k \to k + \varepsilon(k, J)}$ in the spectrum (35). Then $2\varepsilon(k, J)$ (see Eq. (13)) reflects contribution to the canonical dimension Δ from the anomalous part. The systematic form of $\varepsilon(k, J)$ is unknown but it is naturally expected that $\varepsilon(k, J)$ is a growing function of both arguments. However, the spectrum (35) more or less meets the existing phenomenology [13, 16, 22]. This should mean that $\varepsilon(k, J)$ is either suppressed in the large-N_c limit (perhaps the size of $\varepsilon(k, J)$ can be then systematically estimated by a phenomenological analysis of deviations from the relation (35)) or by itself is an approximate linear function of its arguments (hence, the effects of anomalous dimensions are then effectively absorbed by the phenomenological values of parameters in (35)). The both possibilities could constitute an interesting prediction of the SW holographic approach.