A NEW EVALUATION OF a_μ^{SM} TO BE DEVIATED FROM THE WORLD AVERAGED a_μ^{exp} BY 1.6σ IS ACHIEVED BY NOVEL APPROACH

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Outline

MUON g-2 ANOMALY

The **SM muon anomalous magnetic moment** consists in the following contributions

$$a_\mu^{SM} = a_\mu^{QED} + a_\mu^{(LO)had} + a_\mu^{(NLO)had} + a_\mu^{(NNLO)had} + a_\mu^{(LbL)had} + a_\mu^{EW} \quad (1)$$

where dominant sources of its **total uncertainties in theoretical predictions** are given by **the "Leading Order" hadronic contribution** $a_\mu^{(LO)had}$, represented by the vacuum-polarization diagram in Fig.1,

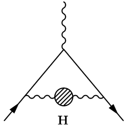

Fig.1: The leading-order hadronic vacuum-polarization contribution to a_μ.

MUON g-2 ANOMALY

The value of the latter can be found by a calculation of the sum of **three dispersion integrals**

$$a_\mu^{(LO)had} = \frac{\alpha^2(0)}{3\pi^2}\left(\int_{m_{\pi^0}^2}^{s_{cut}} \frac{ds}{s}\frac{\sigma_{tot}(e^+e^- \to had)}{\sigma_{tot}(e^+e^- \to \mu^+\mu^-)}K(s) + \right.$$
$$+\int_{s_{cut}}^{s_{pQCD}} \frac{ds}{s} R^{data}(s)K(s) + \qquad (2)$$
$$\left. + \int_{s_{pQCD}}^{\infty} \frac{ds}{s} R^{pQCD}(s)K(s)\right),$$

where s_{cut} is **by various authors taken from the region 0.8-4 GeV^2**, in which **highly fluctuating** $\sigma_{tot}(e^+e^- \to had)$, due to hadronic resonances and threshold effects, in the first integral is changed to **smoother one** in the second integral to be **dependent on inclusive** $R^{data}(s)$ **bare data**.

MUON g-2 ANOMALY

The **total cross section of** e^+e^- **annihilation into two muons** in the first integral is given by $\sigma_{tot}(e^+e^- \to \mu^+\mu^-) = \frac{4\pi\alpha^2(0)}{3s}$, with $\alpha(0) = 1/137.036$.

The QED kernel function $K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)\frac{s}{m_\mu^2}}$ can be **calculated explicitly into more or less suitable forms**, securing a better convergence of all three integrals.

MUON g-2 ANOMALY

The function $R^{pQCD}(s)$ is **calculated in the framework of the theory of strong interactions QCD** and takes the form
$R^{pQCD}(s) =$
$3\sum_f Q_f^2\sqrt{(1-4m_f^2/s)}(1+2m_f^2/s)(1+\frac{\alpha_s(s)}{\pi}+c_1(\frac{\alpha_s(s)}{\pi})^2+c_2(\frac{\alpha_s(s)}{\pi})^3+\ldots)$,
where Q_f and m_f are the **charge and mass of the quarks**, respectively, $\alpha_s(s)$ is the **running strong coupling constant of QCD** and $c_1 = 1.9857 - 01153N_f$,
$c_2 = -6.6368 - 1.2002N_f - 0.0052N_f^2 - 1.2395(\sum Q_f)^2/(3\sum Q_f^2)$
are constants with the number of active flavors N_f.

RUNNING FINE STRUCTURE CONSTANT OF QED

In the framework of the **novel approach** presented on eQCD'19 Conference in Schladming in the last year,

A.Z.Dubnickova et al
Acta Phys. Polonica B (Proc. Suppl.) Vol.13 No.1 (2020) 17

however, the **leading order of hadronic contribution** $a_\mu^{(LO)had}$ to a_μ^{SM} could be expressed through $\Delta\alpha_{had}^{(5)}(t(x))$ as follows

$$a_\mu^{(LO)had} = \frac{\alpha(0)}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}^{(5)}(t(x)) \qquad (3)$$

RUNNING FINE STRUCTURE CONSTANT OF QED

where $\Delta\alpha^{(5)}_{had}(t(x))$ represents a **contribution of the 5 light quarks** u, d, c, s, b **with the masses** $< 5GeV$ **into** $\alpha(t)$ **in the spacelike region** through the sum in $\Delta\alpha(t)$ of

$$\alpha(t) = \frac{\alpha(0)}{1 - \Delta\alpha(t)} \tag{4}$$

together with **contributions of leptons** (e, μ, τ) **and the** t**-quark** with the mass $\approx$ 175 GeV.

Practically, it can be extracted from the **QED running fine structure constant** $\alpha(t)$ **in space-like region** to be measured

G.Abbiendi at al, **Eur. Phys. J C (2017) 77:139**

in elastic scattering $\mu e \to \mu e$ data by the CERN North Area muon beam scattered on atomic electrons of Be and C.

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

Here an **improved evaluation of** $a_\mu^{(LO)had}$ **through a theoretical calculation of** $\Delta\alpha_{had}^{(5)}(t(x))$**, in spacelike region by pseudoscalar meson nonet** *Unitary&Analytic* **EM structure models**, avoiding uncertainties caused by the trapezoidal integration of scarce data in some region, see Fig.1

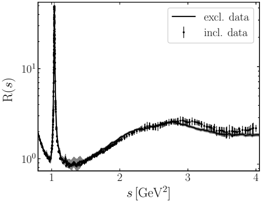

Fig.1 A comparison of the sum of bare total cross sections $e^+e^- \rightarrow had$ and the inclusive R-data.

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

and

also its **dependence on the chosen value of s_{cut} from the interval 0.8-4 GeV^2** is presented.

Unlike the lepton contributions to $\alpha(t(x))$ calculated in QED, $\Delta\alpha_{had}^{(5)}(t(x))$ **due to light masses of u, d, c, s, b-quarks can not be calculated in the framework of pQCD**.
Fortunately, as we have presented in Schladming in the last year,

A.Z.Dubnickova et al
Acta Phys. Polonica B (Proc. Suppl.) Vol.13 No.1 (2020) 17

one can evaluate it from $e^+e^- \to had$ data and $R^{pQCD}(s)$ calculated in pQCD through the following three dispersion integrals

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

$$\Delta\alpha_{had}^{(5)}(t(x)) = -\frac{\alpha(0)t(x)}{3\pi}\left(\int_{m_{\pi^0}^2}^{s_{cut}} \frac{ds'}{s'(s'-t(x))}\frac{\sigma_{tot}^0(e^+e^-\to had)}{\sigma_{tot}^0(e^+e^-\to\mu^+\mu^-)} + \right.$$
$$+\int_{s_{cut}}^{s_{pQCD}} \frac{ds'}{s'(s'-t(x))}R^{data}(s') + \quad (5)$$
$$\left. +\int_{s_{pQCD}}^{\infty} \frac{ds'}{s'(s'-t(x))}R^{pQCD}(s')\right),$$

to be similar to $a_\mu^{(LO)had}$ in (2), now **without the QED kernel** $K(s)$.

An evaluations of the first and the second integral **require the experimental data to be undressed of all "vacuum polarization" effects**.

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

In an identification of the **bare single total cross ections in the first integral** with sinking tendency of contributions beyond the process $e^+e^- \to \eta'\gamma$ in the sum

$$
\begin{aligned}
&\sigma^0_{tot}(e^+e^- \to had) = \sigma^0_{tot}(e^+e^- \to \pi^+\pi^-) + \sigma^0_{tot}(e^+e^- \to K^+K^-) + \sigma^0_{tot}(e^+e^- \to K^0\bar{K}^0)\\
&+\sigma^0_{tot}(e^+e^- \to \pi^0\gamma) + \sigma^0_{tot}(e^+e^- \to \eta\gamma) + \sigma^0_{tot}(e^+e^- \to \eta'\gamma)\\
&+\sigma^0_{tot}(e^+e^- \to \pi^+\pi^-\pi^0) + \sigma^0_{tot}(e^+e^- \to \pi^+\pi^-2\pi^0) + \sigma^0_{tot}(e^+e^- \to 2\pi^+2\pi^-)\\
&+\sigma^0_{tot}(e^+e^- \to \pi^0K^+K^-) + \sigma^0_{tot}(e^+e^- \to \pi^0K^0\bar{K}^0) + \sigma^0_{tot}(e^+e^- \to \pi^+K^-K^0)\\
&+\sigma^0_{tot}(e^+e^- \to \pi^-K^+K^0) + \sigma^0_{tot}(e^+e^- \to \pi^+\pi^-K^+K^-) + \sigma^0_{tot}(e^+e^- \to \pi^0\pi^-K^+K^0)\\
&+\sigma^0_{tot}(e^+e^- \to \pi^+\pi^0K^0K^-) + \sigma^0_{tot}(e^+e^- \to \pi^0\pi^0K^0K^0) + \sigma^0_{tot}(e^+e^- \to 2\pi^+2\pi^-2\pi^0)\\
&+\sigma^0_{tot}(e^+e^- \to \pi^-K^+K^0) + \sigma^0_{tot}(e^+e^- \to \pi^0\pi^0K^+K^-) + \sigma^0_{tot}(e^+e^- \to \pi^0\pi^0K^0\bar{K}^0)\\
&+\sigma^0_{tot}(e^+e^- \to \pi^+\pi^0K^-K^0) + \sigma^0_{tot}(e^+e^- \to \pi^-\pi^0K^+K^0) + \sigma^0_{tot}(e^+e^- \to 2\pi^+2\pi^-2\pi^0)\\
&+\sigma^0_{tot}(e^+e^- \to \eta\pi^+\pi^-) + \sigma^0_{tot}(e^+e^- \to 2\pi^+2\pi^-\pi^0) + \sigma^0_{tot}(e^+e^- \to \pi^0\omega)\\
&+\sigma^0_{tot}(e^+e^- \to \eta\pi^+\pi^-\pi^0) + \sigma^0_{tot}(e^+e^- \to \eta\phi) + \sigma^0_{tot}(e^+e^- \to \eta\omega\pi^0)\\
&+\sigma^0_{tot}(e^+e^- \to \eta\omega) + \sigma^0_{tot}(e^+e^- \to 3\pi^+3\pi^-) + \sigma^0_{tot}(e^+e^- \to \eta 2\pi^+2\pi^-)\\
&+\sigma^0_{tot}(e^+e^- \to p\bar{p}) + \sigma^0_{tot}(e^+e^- \to n\bar{n}),
\end{aligned}
$$

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

the following recent papers

M.Davier, A.Hoecker, B.Malaescu, Z.Zhang:
Eur. Phys. J. C (2017) 77:827
A.Keshavarzi, D.Nomura, Th.Teubner:
Phys. Rev. D97 (2018) 114025.

plaid very useful role.

However, while in these papers all data on the total cross sections (besides the missing $\sigma^0_{tot}(e^+e^- \to \eta'\gamma)$) are integrated in (2) **using only the trapezoidal rule**, in our case the **first six total cross sections in the sum above** are first expressed through the corresponding electromagnetic (EM) form factors (FFs) squared and

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

then the **free parameters of these form factors are found in a comparison with all existing bare FF data in space-like and time-like regions simultaneously**.

Contributions to $\Delta\alpha_{had}^{(5)}(t(x))$ in (5) are **obtained by integration of the resultant curve of the corresponding total cross sections**.

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

In this approach, the pseudoscalar EM FFs are first split into **isoscalar** and **isovector** parts as follows

$$\begin{aligned}
F_{\pi^\pm}(s) &= F_\pi^{I=1}[W(s)] \\
F_{K^\pm}(s) &= F_K^{I=0}[V(s)] + F_K^{I=1}[W(s)] \\
F_{K^0}(s) &= F_K^{I=0}[V(s)] - F_K^{I=1}[W(s)] \\
F_{\pi^0\gamma}(s) &= F_{\pi^0\gamma}^{I=0}[V(s)] + F_{\pi^0\gamma}^{I=1}[W(s)] \\
F_{\eta\gamma}(s) &= F_{\eta\gamma}^{I=0}[V(s)] + F_{\eta\gamma}^{I=1}[W(s)] \\
F_{\eta'\gamma}(s) &= F_{\eta'\gamma}^{I=0}[V(s)] + F_{\eta'\gamma}^{I=1}[W(s)]
\end{aligned} \tag{6}$$

then all **theoretical form factor properties**

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

- normalization of FFs
- asymtotic behaviour as predicted by the quark model
- analytic properties of FFs
- unitarity conditions of FFs
- reality conditions of FFs
- experimental fact of a creation of vector mesons in $e^+e^- \rightarrow had$ process
- $F^{I=1}(s)$ are **saturated** by ρ, ρ', ρ'' and $F^{I=0}(s)$ by $\omega, \phi, \omega', \phi', \omega'', \phi''$
  are **incorporated into these pseudoscalar EM FFs**. Then every $F^{I=1}[W(s)]$ and $F^{I=0}[V(s)]$ represents **one analytic function in the whole complex s-plane**, besides two square root cuts on the positive real axis.

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

They are **defined on the four-sheeted Riemann surface** with vector-meson resonances as poles in complex region of the unphysical sheets.

These form factors **depend on only physically interpretable parameters like coupling constant ratios** (f_{VMM}/f_V) **and effective inelastic thresholds** t_{in} to be evaluated numerically with errors by a comparison of their models with existing experimental data.

The *Unitary&Analytic* model can be used also for estimation of the contributions of the last two total cross sections, $\sigma^0_{tot}(e^+e^- \to p\bar{p})$ and $\sigma^0_{tot}(e^+e^- \to n\bar{n})$, to $\Delta\alpha_{had}^{(5)}(t(x))$, if $s_{cut} > 4m_n^2$.

Because s_{cut} - used from one author to another to be different, **the calculations are carried out one after the other** with:

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

s_{cut}= 0.8 GeV^2 J.F.de Troconiz and F.J.Ynduraín, **Phys. Rev. D71 (2005) 073008**

s_{cut}=1.96 GeV^2 S. Eidelman and F. Jegerlehner, **Z.Phys. C67 (1995) 585**

s_{cut}=2.0449 GeV^2 K.Hagivara, A.D.Martin, D.Nomura, T.Teubner, **Phys. Lett. B649 (2007) 173**

s_{cut}=3.0 GeV^2 A.Z.Dubnickova, S.Dubnicka, A.Liptaj, **Acta. Phys. Polonica B(Proc. Suppl.) 9 (2016) 407**

s_{cut}=3.24 GeV^2 M.Davier, A.Hoecker, B.Malaescu, Z.Zhang, **Eur. Phys. J. C (2017) 77:827**

s_{cut}=3.75 GeV^2 A.Keshavarzi, D.Nomura, Th.Teubner, **Phys. Rev. D97 (2018) 114025**

s_{cut}=4.0 GeV^2 K.Hagivara, R.Liao, A.D.Martin, D.Nomura, T.Teubner, **J. Phys. G38 (2011) 085003**.

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

Every contribution to $\Delta\alpha_{had}^{(5)}(t(x))$, calculated in the first integral, **including also contributions from the second and third integral** in (5), is represented as the **sum of all of them** with total error in Fig.2.

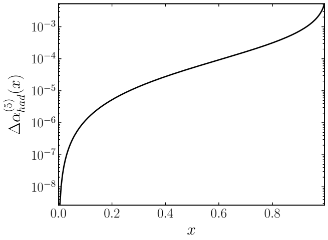

Fig.2 Sum of all predicted curves in the logarithmic scale with error.

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

And though the **total error has been drawn by upper and lower dashed lines** with regard to the full line, **due to its very small value one is unable to observe them in the Fig. visually.**

The resultant full curve can be **specified in the measurement of the running QED fine structure constant in the space-like region by Bhabha $\mu e \to \mu e$ scattering at CERN**.
However, its **comparison with our result in Fig.2 will not be straightforward**.

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

Therefore **we integrate the obtained curve by means of the relation**

$$a_\mu^{(LO)had} = \frac{\alpha(0)}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}^{(5)}(t(x)) \qquad (7)$$

however, substituting always result for $\Delta\alpha_{had}^{(5)}(t(x))$ to be obtained by changing the first upper integration border $s_{cut} =$ $0.8\text{GeV}^2, 1.96\text{GeV}^2, 2.0449\text{GeV}^2, 3.0\text{GeV}^2, 3.24\text{GeV}^2, 3.752\text{GeV}^2, 4.0\text{GeV}^2$ as discussed above.

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

In this way it is demonstrated that the resultant value of $a_\mu^{(LO)had}$, as it is seen from Table

0.80	GeV^2	............$(700.083 \pm 2.866) \times 10^{-10}$
1.96	GeV^2	............$(706.666 \pm 4.018) \times 10^{-10}$
2.0449	GeV^2	............$(707.005 \pm 3.531) \times 10^{-10}$
3.0	GeV^2	............$(708.095 \pm 4.616) \times 10^{-10}$
3.24	GeV^2	............$(707.591 \pm 4.124) \times 10^{-10}$
3.752	GeV^2	............$(707.215 \pm 4.226) \times 10^{-10}$
4.0	GeV^2	............$(706.820 \pm 3.741) \times 10^{-10}$

does not depend on the choice of s_{cut}**,** besides the first one, by various authors. The **first central value in TABLE can be explained by the fact** that $s_{cut} = 0.8$ GeV^2 corresponds to 0.894 GeV, in this case the **first integral in (5), evaluated prevailingly by means of the** *Unitary&Analytic* **models of the corresponding FFs, does not cover the contribution of the** ϕ**-meson peak** from the Fig.3

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

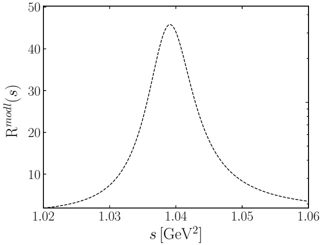

Fig.3 ϕ-peak contribution by $U\&A$-models.

It is **taken into account by means of the trapezoidal integration in the second integral** of (5) through the data in Fig.4 from $R^{data}(s')$ and these **data are little bit sparse**.

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

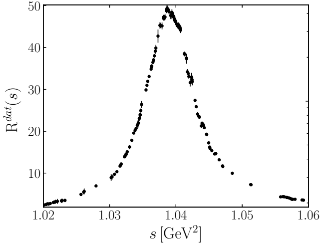

Fig.4 ϕ-peak contribution by inclusive R.

In all other evaluations with higher values of s_{cut} to be beyond the ϕ peak, the **contribution of the ϕ-meson peak** is taken into account in the first integral of (5) by means of the *Unitary&Analytic* models of the corresponding FFs given in Fig.3.

THEORETICAL EVALUATION OF $\Delta\alpha^{(5)}_{had}(t(x))$

The values of $a_\mu^{(LO)had}$ $(693.1 \pm 3.4) \times 10^{-10}$ in
M.Davier, A.Hoecker, B.Malaescu, Z.Zhang:
Eur. Phys. J. C (2017) 77:827
and $(693.26 \pm 2.46) \times 10^{-10}$ in
A.Keshavarzi, D.Nomura, Th.Teubner:
Phys. Rev. D97 (2018) 114025.
are lower in comparison with our values in TABLE and this effects can be **explained by a similar arguments as presented above** as evaluations of integrals through existing experimental data there have been carried out from the lowest threshold $s = m_\pi^2$ up to the s_{pQCD} **by the trapezoidal method**.

THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$

Adding to our averaged value (the first value from TABLE is not included) of the LO hadronic contribution $\bar{a}_\mu^{(LO)had} = (707.232 \pm 4.043) \times 10^{-10}$ the contributions from **higher order hadronic loops**, -9.87 ± 0.09 (NLO) and 1.24 ± 0.01 (NNLO), the **hadronic light-by-light scattering** 10.5 ± 2.6, as well as **QED** 11658471.895 ± 0.008 and **electroweak effects** 15.36 ± 0.10, **one obtains the complete SM prediction** to be

$$a_\mu^{SM} = (11659196.35 \pm 4.81) \times 10^{-10} \tag{8}$$

This **result deviates from the world average experimental value** $a_\mu^{exp} = (11659209 \pm 6) \times 10^{-10}$ [?] by $12.65 \pm 7.69 (1.6\sigma)$.

Conclusions

We have presented, a **novel approach to determine the LO of hadronic contribution to muon g-2 anomaly** $a_\mu^{(LO)had}$ to be expressed through the hadronic contribution to the QED running fine structure constant $\Delta\alpha_{had}^{(5)}(t(x))$ in the space-like region.

Then **it was demonstrated how the elaborated Unitary and Analytic models of electromagnetic structure of pseudoscalar meson nonet** can be helpful in favor of a more precise theoretical prediction of $\Delta\alpha_{had}^{(5)}(t)$ behavior.

Conclusions

Finally, **adding to our averaged value of the LO hadronic contribution**
the contributions **from higher order hadronic loops,**
the **hadronic light-by-light scattering contribution,**
as well as **value from QED**
and **electroweak effects,**
the obtained result deviates from the world average experimental value by (1.6σ)

Thank you for your attention.