A NEW EVALUATION OF $a_{\mu}^{SM}$ TO BE DEVIATED FROM THE WORLD AVERAGED $a_{\mu}^{\text{exp}}$ BY 1.6$\sigma$ IS ACHIEVED BY NOVEL APPROACH

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Outline

1. MUON $g-2$ ANOMALY
2. RUNNING FINE STRUCTURE CONSTANT OF QED $\alpha(t)$
3. IMPROVED $a^{sm}_\mu$ BY THEORETICAL EVALUATION OF $\Delta\alpha_{had}^{(5)}(t(x))$
4. CONCLUSIONS
The **SM muon anomalous magnetic moment** consists in the following contributions

\[
a_{\mu}^{SM} = a_{\mu}^{QED} + a_{\mu}^{(LO)had} + a_{\mu}^{(NLO)had} + a_{\mu}^{(NNLO)had} + a_{\mu}^{(LbL)had} + a_{\mu}^{EW}
\]  

where dominant sources of its total uncertainties in theoretical predictions are given by the "Leading Order" hadronic contribution \(a_{\mu}^{(LO)had}\), represented by the vacuum-polarization diagram in Fig.1,

![Fig.1: The leading-order hadronic vacuum-polarization contribution to \(a_{\mu}\).](image)
The value of the latter can be found by a calculation of the sum of three dispersion integrals

\[
a_{\mu}^{(LO)\text{had}} = \frac{\alpha^2(0)}{3\pi^2} \left( \int_{m_{\pi}^2}^{s_{\text{cut}}} \frac{ds}{s} \frac{\sigma_{\text{tot}}(e^+e^- \rightarrow \text{had})}{\sigma_{\text{tot}}(e^+e^- \rightarrow \mu^+\mu^-)} K(s) + \int_{s_{\text{cut}}}^{s_{\text{pQCD}}} \frac{ds}{s} R^{\text{data}}(s) K(s) + \int_{s_{\text{pQCD}}}^{\infty} \frac{ds}{s} R^{\text{pQCD}}(s) K(s) \right),
\]

where \( s_{\text{cut}} \) is by various authors taken from the region 0.8-4 GeV\(^2\), in which highly fluctuating \( \sigma_{\text{tot}}(e^+e^- \rightarrow \text{had}) \), due to hadronic resonances and threshold effects, in the first integral is changed to smoother one in the second integral to be dependent on inclusive \( R^{\text{data}}(s) \) bare data.
The total cross section of $e^+e^-$ annihilation into two muons in the first integral is given by

$$\sigma_{tot}(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2(0)}{3s},$$

with $\alpha(0) = 1/137.036$.

The QED kernel function $K(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2+(1-x)s/m_{\mu}^2}$ can be calculated explicitly into more or less suitable forms, securing a better convergence of all three integrals.
The function $R^p_{QCD}(s)$ is calculated in the framework of the theory of strong interactions QCD and takes the form

$$R^p_{QCD}(s) = 3 \sum_f Q_f^2 \sqrt{1 - 4m_f^2/s}(1 + 2m_f^2/s)(1 + \frac{\alpha_s(s)}{\pi}) + c_1 \left(\frac{\alpha_s(s)}{\pi}\right)^2 + c_2 \left(\frac{\alpha_s(s)}{\pi}\right)^3 + ...,$$

where $Q_f$ and $m_f$ are the charge and mass of the quarks, respectively, $\alpha_s(s)$ is the running strong coupling constant of QCD and $c_1 = 1.9857 - 0.1153 N_f$, $c_2 = -6.6368 - 1.2002 N_f - 0.0052 N_f^2 - 1.2395 (\sum Q_f)^2/(3 \sum Q_f^2)$ are constants with the number of active flavors $N_f$. 
In the framework of the novel approach presented on eQCD’19 Conference in Schladming in the last year,

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however, the leading order of hadronic contribution $a_{\mu}^{(LO)\text{had}}$ to $a_{\mu}^{SM}$ could be expressed through $\Delta\alpha_{\text{had}}^{(5)}(t(x))$ as follows

$$a_{\mu}^{(LO)\text{had}} = \frac{\alpha(0)}{\pi} \int_{0}^{1} dx (1 - x) \Delta\alpha_{\text{had}}^{(5)}(t(x))$$  \hspace{1cm} (3)
where $\Delta \alpha_{\text{had}}^{(5)}(t(x))$ represents a **contribution** of the 5 light quarks $u, d, c, s, b$ with the masses $< 5\, \text{GeV}$ into $\alpha(t)$ in the spacelike region through the sum in $\Delta \alpha(t)$ of

$$\alpha(t) = \frac{\alpha(0)}{1 - \Delta \alpha(t)} \quad (4)$$

**Contributions** of leptons ($e, \mu, \tau$) and the $t$-quark with the mass $\approx 175$ GeV. Practically, it can be extracted from the **QED running fine structure constant** $\alpha(t)$ in space-like region to be measured in elastic scattering $\mu e \rightarrow \mu e$ data by the CERN North Area muon beam scattered on atomic electrons of Be and C.

Here an improved evaluation of \( a_{\mu}^{(LO)\text{had}} \) through a theoretical calculation of \( \Delta \alpha_{\text{had}}^{(5)}(t(x)) \), in spacelike region by pseudoscalar meson nonet **Unitary&Analytic EM structure models**, avoiding uncertainties caused by the trapezoidal integration of scarce data in some region, see Fig.1

![Graph](image)

**Fig.1** A comparison of the sum of bare total cross sections \( e^+e^- \rightarrow \text{had} \) and the inclusive R-data.
THEORETICAL EVALUATION OF \( \Delta \alpha_{\text{had}}^{(5)}(t(x)) \)

and

also its dependence on the chosen value of \( s_{\text{cut}} \) from the interval 0.8-4 GeV\(^2\) is presented.

Unlike the lepton contributions to \( \alpha(t(x)) \) calculated in QED, \( \Delta \alpha_{\text{had}}^{(5)}(t(x)) \) due to light masses of u, d, c, s, b-quarks can not be calculated in the framework of pQCD.

Fortunately, as we have presented in Schladming in the last year, A.Z.Dubnickova et al


one can evaluate it from \( e^+e^- \rightarrow \text{had data} \) and \( R^{pQCD}(s) \) calculated in pQCD through the following three dispersion integrals
\[
\Delta \alpha^{(5)}_{\text{had}}(t(x)) = -\frac{\alpha(0) t(x)}{3\pi} \left( \int_{m^2_{\pi}}^{s_{\text{cut}}} \frac{ds'}{s'(s' - t(x))} \frac{\sigma^0_{\text{tot}}(e^+ e^- \rightarrow \text{had})}{\sigma^0_{\text{tot}}(e^+ e^- \rightarrow \mu^+ \mu^-)} + \right.
\]

\[
\left. + \int_{s_{\text{cut}}}^{s_{pQCD}} \frac{ds'}{s'(s' - t(x))} R^\text{data}(s') + \int_{s_{pQCD}}^{\infty} \frac{ds'}{s'(s' - t(x))} R^pQCD(s') \right),
\]

to be similar to \( a^{(LO)\text{had}}_\mu \) in (2), now without the QED kernel \( K(s) \).

An evaluations of the first and the second integral require the experimental data to be undressed of all ”vacuum polarization” effects.

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A NEW EVALUATION OF \( a^{SM}_\mu \) TO BE DEVIATED FROM THE
THEORETICAL EVALUATION OF $\Delta \alpha_{had}^{(5)}(t(x))$

In an identification of the **bare single total cross sections in the first integral** with sinking tendency of contributions beyond the process $e^+ e^- \rightarrow \eta' \gamma$ in the sum

$$
\sigma^0_{tot}(e^+ e^- \rightarrow \text{had}) = \sigma^0_{tot}(e^+ e^- \rightarrow \pi^+ \pi^-) + \sigma^0_{tot}(e^+ e^- \rightarrow K^+ K^-) + \sigma^0_{tot}(e^+ e^- \rightarrow K^0 \overline{K}^0) + \sigma^0_{tot}(e^+ e^- \rightarrow \pi^0 \gamma) + \sigma^0_{tot}(e^+ e^- \rightarrow \eta \gamma) + \sigma^0_{tot}(e^+ e^- \rightarrow \eta' \gamma) + \sigma^0_{tot}(e^+ e^- \rightarrow \pi^+ \pi^- 2\pi^0) + \sigma^0_{tot}(e^+ e^- \rightarrow 2\pi^+ 2\pi^-) + \sigma^0_{tot}(e^+ e^- \rightarrow \pi^+ K^- K^0) + \sigma^0_{tot}(e^+ e^- \rightarrow \pi^- K^+ K^0) + \sigma^0_{tot}(e^+ e^- \rightarrow \pi^+ \pi^- K^+ K^-) + \sigma^0_{tot}(e^+ e^- \rightarrow \pi^0 \pi^- K^+ K^0) + \sigma^0_{tot}(e^+ e^- \rightarrow 2\pi^+ 2\pi^- 2\pi^0) + \sigma^0_{tot}(e^+ e^- \rightarrow \pi^+ K^+ K^0) + \sigma^0_{tot}(e^+ e^- \rightarrow \pi^- K^+ K^0) + \sigma^0_{tot}(e^+ e^- \rightarrow \pi^+ \pi^- 0K^0 K^-) + \sigma^0_{tot}(e^+ e^- \rightarrow \pi^- \pi^0 K^+ K^0) + \sigma^0_{tot}(e^+ e^- \rightarrow 2\pi^+ 2\pi^- 2\pi^0) + \sigma^0_{tot}(e^+ e^- \rightarrow \eta \pi^+ \pi^-) + \sigma^0_{tot}(e^+ e^- \rightarrow 2\pi^+ 2\pi^- \pi^0) + \sigma^0_{tot}(e^+ e^- \rightarrow \pi^0 \omega) + \sigma^0_{tot}(e^+ e^- \rightarrow \eta \pi^+ \pi^-) + \sigma^0_{tot}(e^+ e^- \rightarrow \eta \phi) + \sigma^0_{tot}(e^+ e^- \rightarrow \eta \omega \pi^0) + \sigma^0_{tot}(e^+ e^- \rightarrow \eta 2\pi^+ 2\pi^-) + \sigma^0_{tot}(e^+ e^- \rightarrow p \overline{p}) + \sigma^0_{tot}(e^+ e^- \rightarrow n \overline{n}),
$$
the following recent papers

M. Davier, A. Hoecker, B. Malaescu, Z. Zhang:
A. Keshavarzi, D. Nomura, Th. Teubner:

plaid very useful role.

However, while in these papers all data on the total cross sections (besides the missing $\sigma^{0}_{\text{tot}}(e^{+}e^{-} \rightarrow \eta'\gamma)$) are integrated in (2) using only the trapezoidal rule, in our case the first six total cross sections in the sum above are first expressed through the corresponding electromagnetic (EM) form factors (FFs) squared and
then the free parameters of these form factors are found in a comparison with all existing bare FF data in space-like and time-like regions simultaneously.

Contributions to $\Delta \alpha_{had}^{(5)}(t(x))$ in (5) are obtained by integration of the resultant curve of the corresponding total cross sections.
In this approach, the pseudoscalar EM FFs are first split into \textit{isoscalar} and \textit{isovector} parts as follows

\begin{align*}
F_{\pi^\pm}(s) &= F_{\pi}^{I=1}[W(s)] \\
F_{K^\pm}(s) &= F_{K}^{I=0}[V(s)] + F_{K}^{I=1}[W(s)] \\
F_{K^0}(s) &= F_{K}^{I=0}[V(s)] - F_{K}^{I=1}[W(s)] \\
F_{\pi^0\gamma}(s) &= F_{\pi^0\gamma}^{I=0}[V(s)] + F_{\pi^0\gamma}^{I=1}[W(s)] \\
F_{\eta\gamma}(s) &= F_{\eta\gamma}^{I=0}[V(s)] + F_{\eta\gamma}^{I=1}[W(s)] \\
F_{\eta'\gamma}(s) &= F_{\eta'\gamma}^{I=0}[V(s)] + F_{\eta'\gamma}^{I=1}[W(s)]
\end{align*}

(6)

then all \textit{theoretical form factor properties}
normalization of FFs
asymptotic behaviour as predicted by the quark model
analytic properties of FFs
unitarity conditions of FFs
reality conditions of FFs
experimental fact of a creation of vector mesons in $e^+ e^- \rightarrow \text{had}$ process
$F^{l=1}(s)$ are saturated by $\rho, \rho', \rho''$ and $F^{l=0}(s)$ by $\omega, \phi, \phi', \phi'', \omega'', \phi''$ are incorporated into these pseudoscalar EM FFs. Then every $F^{l=1}[W(s)]$ and $F^{l=0}[V(s)]$ represents one analytic function in the whole complex $s$-plane, besides two square root cuts on the positive real axis.
They are defined on the four-sheeted Riemann surface with vector-meson resonances as poles in complex region of the unphysical sheets. These form factors depend on only physically interpretable parameters like coupling constant ratios \((f_{VMM}/f_V)\) and effective inelastic thresholds \(t_{in}\) to be evaluated numerically with errors by a comparison of their models with existing experimental data.

The Unitary & Analytic model can be used also for estimation of the contributions of the last two total cross sections, \(\sigma^0_{tot}(e^+e^- \to p\bar{p})\) and \(\sigma^0_{tot}(e^+e^- \to n\bar{n})\), to \(\Delta\alpha^{(5)}_{had}(t(x))\), if \(s_{cut} > 4m_n^2\).

Because \(s_{cut}\) - used from one author to another to be different, the calculations are carried out one after the other with:
THEORETICAL EVALUATION OF $\Delta \alpha_{\text{had}}^{(5)}(t(x))$

$s_{\text{cut}} = 0.8 \text{ GeV}^2$  J.F. de Troconiz and F.J. Yndurain, *Phys. Rev. D71* (2005) 073008


THEORETICAL EVALUATION OF $\Delta \alpha_{\text{had}}^{(5)}(t(x))$

Every contribution to $\Delta \alpha_{\text{had}}^{(5)}(t(x))$, calculated in the first integral, including also contributions from the second and third integral in (5), is represented as the sum of all of them with total error in Fig.2.

Fig.2 Sum of all predicted curves in the logarithmic scale with error.
And though the total error has been drawn by upper and lower dashed lines with regard to the full line, due to its very small value one is unable to observe them in the Fig. visually.

The resultant full curve can be specified in the measurement of the running QED fine structure constant in the space-like region by Bhabha $\mu e \rightarrow \mu e$ scattering at CERN. However, its comparison with our result in Fig.2 will not be straightforward.
Therefore we integrate the obtained curve by means of the relation

\[ a^{(LO)\text{had}}_\mu = \frac{\alpha(0)}{\pi} \int_0^1 dx (1 - x) \Delta \alpha^{(5)}_{\text{had}}(t(x)) \] (7)

however, substituting always result for \( \Delta \alpha^{(5)}_{\text{had}}(t(x)) \) to be obtained by changing the first upper integration border \( s_{\text{cut}} = 0.8\text{GeV}^2, 1.96\text{GeV}^2, 2.0449\text{GeV}^2, 3.0\text{GeV}^2, 3.24\text{GeV}^2, 3.752\text{GeV}^2, 4.0\text{GeV}^2 \) as discussed above.
In this way it is demonstrated that the resultant value of $a_{\mu}^{(LO)had}$, as it is seen from Table

<table>
<thead>
<tr>
<th>$s_{cut}$ (GeV)</th>
<th>$a_{\mu}^{(5)_{had}}$ (GeV$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.80</td>
<td>$(700.083 \pm 2.866) \times 10^{-10}$</td>
</tr>
<tr>
<td>1.96</td>
<td>$(706.666 \pm 4.018) \times 10^{-10}$</td>
</tr>
<tr>
<td>2.0449</td>
<td>$(707.005 \pm 3.531) \times 10^{-10}$</td>
</tr>
<tr>
<td>3.0</td>
<td>$(708.095 \pm 4.616) \times 10^{-10}$</td>
</tr>
<tr>
<td>3.24</td>
<td>$(707.591 \pm 4.124) \times 10^{-10}$</td>
</tr>
<tr>
<td>3.752</td>
<td>$(707.215 \pm 4.226) \times 10^{-10}$</td>
</tr>
<tr>
<td>4.0</td>
<td>$(706.820 \pm 3.741) \times 10^{-10}$</td>
</tr>
</tbody>
</table>

does not depend on the choice of $s_{cut}$, besides the first one, by various authors. The **first central value in TABLE** can be explained by the fact that $s_{cut} = 0.8$ GeV$^2$ corresponds to 0.894 GeV, in this case the **first integral in (5)**, evaluated prevailing by means of the **Unitary&Analytic models of the corresponding FFs**, does not cover the contribution of the **$\phi$-meson peak** from the Fig.3
MUON g-2 ANOMALY  
RUNNING FINE STRUCTURE CONSTANT OF QED $\alpha(t)$  
IMPROVED $a_{\mu}^{\text{sm}}$ BY THEORETICAL EVALUATION OF $\Delta \alpha_{\text{had}}^{(5)}(t(x))$

THEORETICAL EVALUATION OF $\Delta \alpha_{\text{had}}^{(5)}(t(x))$

Fig.3 $\phi$-peak contribution by $U&A$-models.

It is taken into account by means of the trapezoidal integration in the second integral of (5) through the data in Fig.4 from $R^{\text{data}}(s')$ and these data are little bit sparse.
In all other evaluations with higher values of $s_{cut}$ to be beyond the $\phi$ peak, the contribution of the $\phi$-meson peak is taken into account in the first integral of (5) by means of the Unitary & Analytic models of the corresponding FFs given in Fig.3.
The values of $a_{\mu}^{(LO)had} (693.1 \pm 3.4) \times 10^{-10}$ in
M. Davier, A. Hoecker, B. Malaescu, Z. Zhang:
and $(693.26 \pm 2.46) \times 10^{-10}$ in
A. Keshavarzi, D. Nomura, Th. Teubner:

are lower in comparison with our values in TABLE and this effects can be explained by a similar arguments as presented above as evaluations of integrals through existing experimental data there have been carried out from the lowest threshold $s = m_{\pi}^2$ up to the $s_{pQCD}$ by the trapezoidal method.
Adding to our averaged value (the first value from TABLE is not included) of the LO hadronic contribution
\[ \bar{a}_\mu^{(\text{LO})\,\text{had}} = (707.232 \pm 4.043) \times 10^{-10} \]
the contributions from higher order hadronic loops, \(-9.87 \pm 0.09\) (NLO) and \(1.24 \pm 0.01\) (NNLO), the hadronic light-by-light scattering \(10.5 \pm 2.6\), as well as QED \(11658471.895 \pm 0.008\) and electroweak effects \(15.36 \pm 0.10\), one obtains the complete SM prediction to be
\[ a_\mu^{\text{SM}} = (11659196.35 \pm 4.81) \times 10^{-10} \] (8)

This result deviates from the world average experimental value \[ a_\mu^{\exp} = (11659209 \pm 6) \times 10^{-10} \] [?] by \(12.65 \pm 7.69\)(1.6\(\sigma\)).
We have presented, a novel approach to determine the LO of hadronic contribution to muon g-2 anomaly $a_{\mu}^{(LO)had}$ to be expressed through the hadronic contribution to the QED running fine structure constant $\Delta \alpha_{had}^{(5)}(t(x))$ in the space-like region.

Then it was demonstrated how the elaborated Unitary and Analytic models of electromagnetic structure of pseudoscalar meson nonet can be helpful in favor of a more precise theoretical prediction of $\Delta \alpha_{had}^{(5)}(t)$ behavior.
Conclusions

Finally, adding to our averaged value of the LO hadronic contribution the contributions from higher order hadronic loops, the hadronic light-by-light scattering contribution, as well as value from QED and electroweak effects, the obtained result deviates from the world average experimental value by (1.6σ)
Thank you for your attention.