Charmonium Spectrum from $N_f = 3 + 1$ Lattice QCD

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Motivation

Reasons for dynamical charm quarks

- investigate decoupling with light quarks, high precision charm physics, α_S in 4 flavor theory,...
- c.f. [Cali et al., arXiv:1905.12971], [Dalla Brida et al., arXiv:1912.06001]

Difficulties/Solutions

- lattice artefacts due to cutoff effects $\propto \mathcal{O}(am_c \approx 0.5)$
- [Fritzsch, Sommer, Stollenwerk and Wolff, arXiv:1805.01661]
- physical charm quark in a mass independent scheme gives improvement terms about an order of magnitude larger than strange contributions
- massive renormalization scheme with close to realistic charm mass m_c and $m_{uds} = \sum_{i=uds} m_i^{\text{phys}}/3$
- Symanzik improved 3+1 scheme for Wilson quarks

Symanzik improved 3+1 scheme for Wilson quarks

massive renormalization scheme and improvement

- massive renormalization and finite size scheme to maintain $\mathcal{O}(a)$ improvement

$$g_\mathrm{R}^2 = \tilde{Z}_g(g_0^2, a\mu, aM)g_0^2, \; m_{\mathrm{R},i} = \tilde{Z}_m^i(g_0^2, a\mu, aM)m_{\mathrm{q},i}$$

$$m_{\mathrm{q},i} = m_i - \tilde{m}_\mathrm{crit}(g_0^2, a\mathrm{tr}\,[M_\mathrm{q}])$$

- clover action term [Sheikholeslami and Wohlert (SW), 1985]

$$S_\mathrm{SW} = a^5 \tilde{c}_\mathrm{sw}(g_0^2, aM) \sum_x \bar{\psi}(x) \tfrac{i}{4}\sigma_{\mu\nu}\hat{F}_{\mu\nu}(x)\psi(x)$$

- non-perturbative fit formula for the clover coefficient $c_\mathrm{sw}(g_0^2)$ from LCP, cf. [Fritzsch et al., 2018]
- reduce the number of mass parameters via

$$M_\mathrm{q} = \mathrm{diag}(m_{\mathrm{q},l}, m_{\mathrm{q},l}, m_{\mathrm{q},l}, m_{\mathrm{q},c})$$

Preliminary Work

scale setting in CLS $N_\mathrm{f} = 2+1$ QCD

- relation betw. bare coupling and lattice spacing in fm
- dimensionless quantity $\sqrt{t_0^\star} m_\mathrm{had}$ in the continuum limit
- m_had experimentally accessible quantity
- $t_0^\star$ (mass dimension -2) flow scale [Lüscher, 2010]
- $\sqrt{8t_0^\star} = 0.413(5)(2)$fm [Bruno, Korzec, Schaefer, 2017]

non-perturbative decoupling of the charm quark

- scale $t_0^\star$ is the same in $N_\mathrm{f} = 3$ and $N_\mathrm{f} = 3+1$ theories, up to small corrections $O(1/m_\mathrm{charm}^2)$
- study of non-perturbative decoupling of the charm quark [Knechtli et al. 2017, Athenodorou et al. 2018, Cali et al. 2019]

Scale setting and tuning of $N_\mathrm{f} = 3+1$ QCD

- computation of $t_0^\star/a^2$ at the mass point $m_\text{up} = m_\text{down} = m_\text{strange}$ and

$$\phi_4 \equiv 8t_0\left(m_\mathrm{K}^2 + \frac{m_\pi^2}{2}\right) = 12t_0 m_\pi^2 = 1.11$$

$$\phi_5 \equiv \sqrt{8t_0}\,(m_{\mathrm{D_s}} + 2m_\mathrm{D}) = \sqrt{72t_0}\,m_\mathrm{D} = 11.94$$

- we use first tuning results from [Fritzsch et al., 2018]

$$\beta = 3.24 \text{ (bare coupling)}$$

$$\kappa_\text{uds} = 0.134484 \text{ (light quark mass)}$$

$$\kappa_c = 0.12 \text{ (charm quark mass)}$$

$$c_0 = 5/3 \text{ (Lüscher–Weisz action)}$$

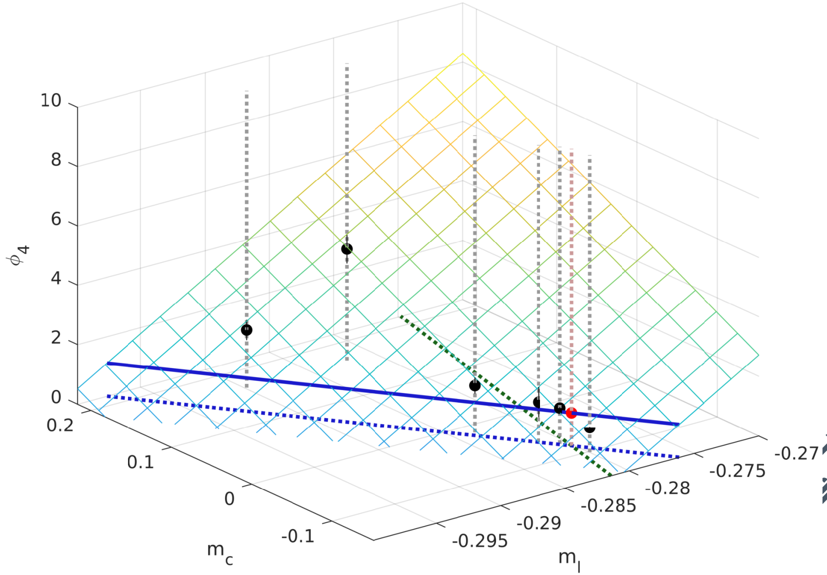
$$c_\text{sw} = 2.188591 \text{ (bulk improvement)}$$

$$c_F = c_G = 1.0 \text{ (boundary improvements)}$$

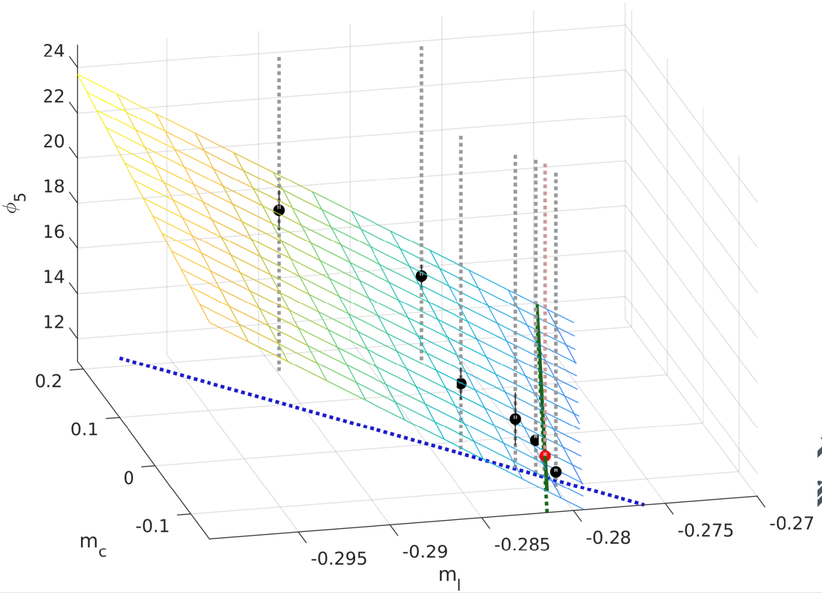
Simulations using openQCD-1.6 [Lüscher, Schaefer]

- start with algorithmic setup of CLS's H400 simulation [Bruno et al., 2015] and add a charm quark
- u/d quark doublet in terms of even-odd prec. $\hat{D}$ with weight $\propto \det[(\mathrm{D_{oo}})^2]\det[\hat{\mathrm{D}}^\dagger\hat{\mathrm{D}} + \mu^2]^2\det[\hat{\mathrm{D}}^\dagger\hat{\mathrm{D}} + 2\mu^2]^{-1}$
- strange and charm quarks are simulated with RHMC, Zolotarev rational functions have degrees 12 and 10
- both, doublet and rational parts need reweighting and are further factorized [Hasenbusch, 2001]
  $\det[D^2] = \det[D^\dagger D + \mu_0^2] \times \frac{\det[D^\dagger D+\mu_1^2]}{\det[D^\dagger D+\mu_0^2]} \times \ldots \times \frac{\det[D^\dagger D]}{\det[D^\dagger D+\mu_N^2]}$
- gauge + 13 pseudo-fermion fields on 3 different time scale integration levels: $N_0 = 2, N_1 = 1, N_2 = 8$
- 2nd and 4th order [Omelyan, Mryglod, Folk, 2003] integrators
- SAP preconditioning and low-mode-deflation based on local coherence [Lüscher, 2004, 2007, Frommer et al. 2013]

Tuning of $\phi_4 = 1.11$



Tuning of $\phi_5 = 11.94$

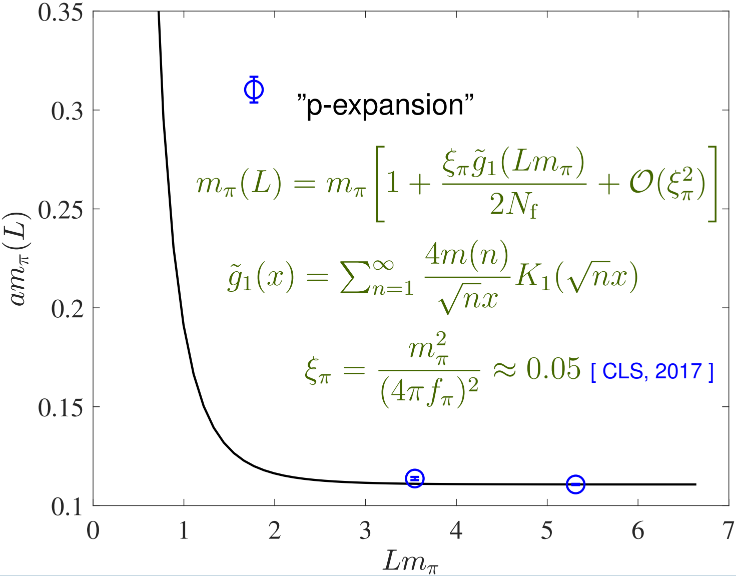


Tuning Results: $\kappa_l = 0.13440733$, $\kappa_c = 0.12784$

$\frac{T}{a} \times \frac{L^3}{a^3}$	$Lm_\pi^\star$	N_{ms}	t_0/a^2	$am_{\pi,K}$	am_{D,D_s}	ϕ_4	ϕ_5
96×16^3	1.7	700	8.8(2)	0.310(6)	0.614(17)	10.2(9)	15.5(4)
96×32^3	3.5	1954	7.43(4)	0.1138(8)	0.5251(7)	1.16(2)	12.17(4)
128×48^3	5.3	1934	7.36(3)	0.1108(4)	0.5236(4)	1.087(6)	12.06(2)

- The integrated autocorrelation time of t_0 is $\tau_{\text{int},t_0} \approx 20 \pm 10$ [4 MDU].
- Assuming decoupling, our value of $t_0/a^2 \approx 7.4$ corresponds to a lattice spacing $a \approx 0.054$ fm.
- The ratio of PCAC masses $m_{ud}/m_{cc'} \approx 0.026$ is very close to the experimental ratio $\frac{m_s/3}{m_c}$.
- In ϕ_4 and ϕ_5 the mass dependence of t_0 and the masses go in opposite directions.
- The sampling of the topology is sufficient.

Finite volume scaling effects for am_π [Colangelo, Dürr, Haefeli, 2005]

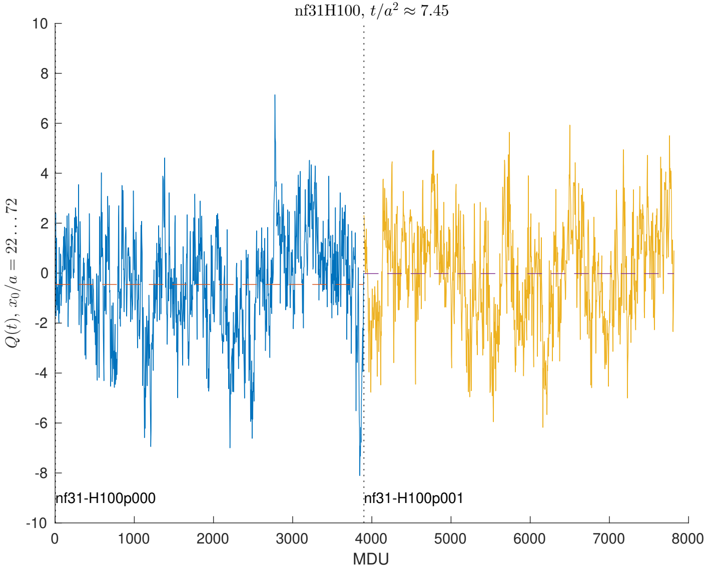


$$m_\pi(L) = m_\pi\left[1 + \frac{\xi_\pi \tilde{g}_1(Lm_\pi)}{2N_\mathrm{f}} + \mathcal{O}(\xi_\pi^2)\right]$$

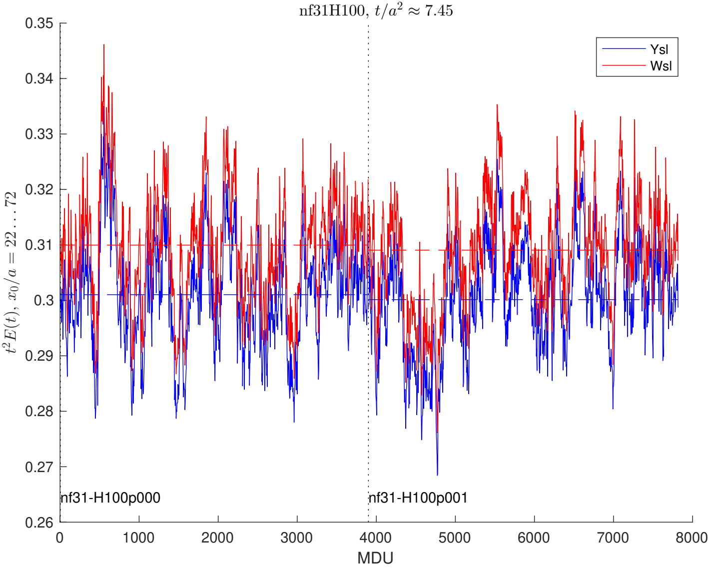
$$\tilde{g}_1(x) = \sum_{n=1}^{\infty} \frac{4m(n)}{\sqrt{n}x} K_1(\sqrt{n}x)$$

$$\xi_\pi = \frac{m_\pi^2}{(4\pi f_\pi)^2} \approx 0.05$$ [CLS, 2017]

History of the topological charge $Q(t \approx t_0)$



History of $t^2 E(t)$ where $E(t) = \frac{1}{4} G^a_{\mu\nu}(t) G^a_{\mu\nu}(t)$



Tuning of ensemble B and mis-tuning corrections

$\frac{T}{a} \times \frac{L^3}{a^3}$	β	a[fm]	$Lm_\pi^\star$	N_{ms}	τ_{exp}
128×48^3	3.24	0.0536(11)	5.354(13)	1934	25
144×48^3	3.43	0.0428(7)	4.282(14)	2000	40

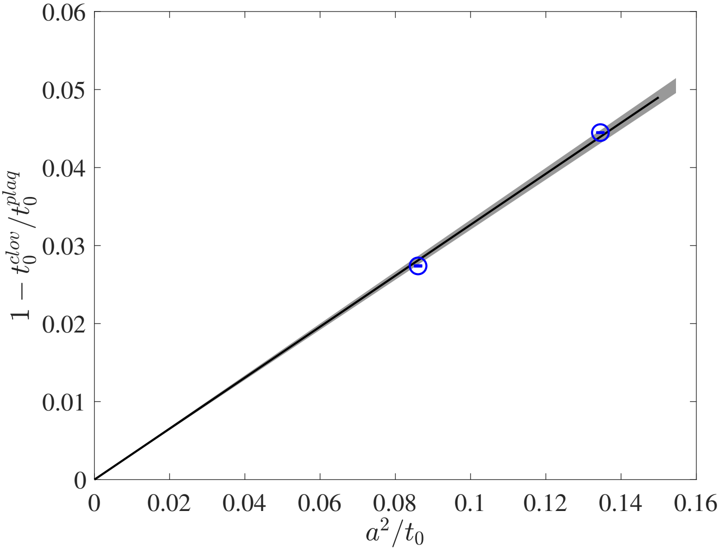
$$\frac{d\langle \mathcal{O}_i \rangle}{dm} = \left\langle \frac{\partial \mathcal{O}_i}{\partial m} \right\rangle - \left\langle \mathcal{O}_i \frac{\partial S}{\partial m} \right\rangle + \langle \mathcal{O}_i \rangle_{QCD} \left\langle \frac{\partial S}{\partial m} \right\rangle$$

$$1.11 = \phi_4 + \left(\frac{d\phi_4}{dm_u} + \frac{d\phi_4}{dm_d} + \frac{d\phi_4}{dm_s} \right) \Delta m_l + \frac{d\phi_4}{dm_c} \Delta m_c$$

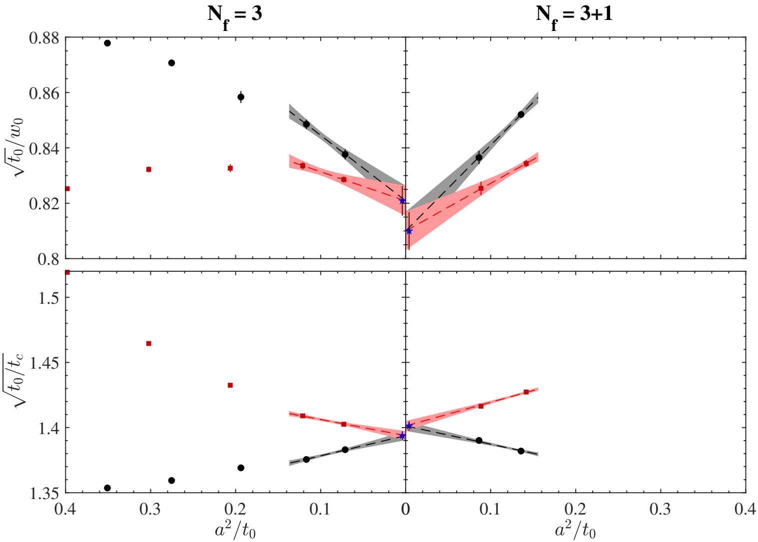
$$11.94 = \phi_5 + \left(\frac{d\phi_5}{dm_u} + \frac{d\phi_5}{dm_d} + \frac{d\phi_5}{dm_s} \right) \Delta m_l + \frac{d\phi_5}{dm_c} \Delta m_c$$

$$f_s = f + \left(\frac{df}{dm_u} + \frac{df}{dm_d} + \frac{df}{dm_s} \right) \Delta m_l + \frac{df}{dm_c} \Delta m_c$$

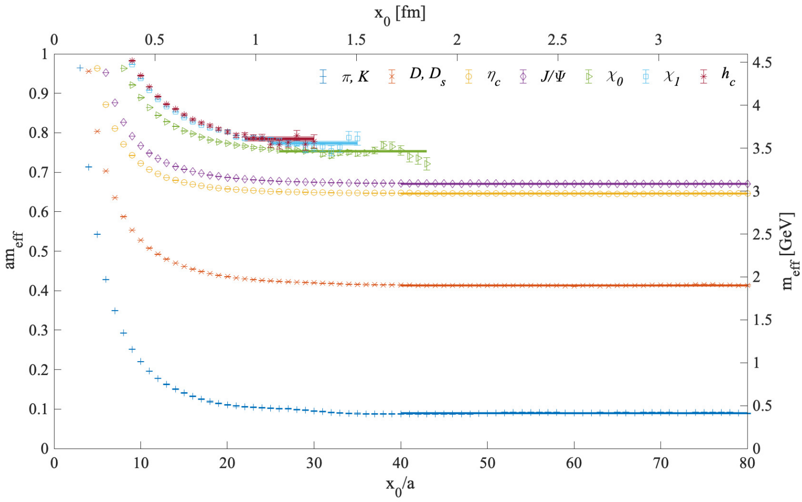
Continuum limit, 1.7 permille non-a^2 lattice artifacts



Decoupling of charm quark



Meson spectrum



Charmonium Spectrum

	η_c	J/ψ	χ_{c0}	χ_{c1}	h_c
m_{eff} [GeV]	2.989(7)	3.101(7)	3.434(31)	3.543(46)	3.581(60)
PDG [GeV]	2.9834(5)	3.096900(6)	3.4148(3)	3.51066(7)	3.52538(11)

charm. hyperfine splitting $(m_{J/\psi} - m_{\eta_c})/m_{\eta_c} = 0.0380(3)$!!!

- sum of the light quark masses has physical value
- no light quarks in the valence sector, hence the derivatives $dm_x/dm_{\text{up}} = dm_x/dm_{\text{down}} = dm_x/dm_{\text{strange}}$
- $m_x^{\text{phys}} = m_x + (\Delta_{\text{up}} + \Delta_{\text{down}} + \Delta_{\text{strange}})\frac{dm_{\eta_c}}{dm_u} + O(\Delta^2)$
- linear term vanishes, because ϕ_4 is chosen such that $\Delta_{\text{up}} = \Delta_{\text{down}} = -0.5\Delta_{\text{strange}}$ ($m_{uds} = \sum_{i=uds} m_i^{\text{phys}}/3$)
- derivatives of correlation functions with respect to bare quark masses allow only small shifts

Conclusions & Outlook

Conclusions

- scale setting and tuning of $N_\mathrm{f} = 3 + 1$ QCD
- massive renormalization scheme with a non-perturbatively determined clover coefficient
- two ensembles with $a = 0.054$ and $a = 0.043$fm
- charmonium spectrum and hyperfine splitting

Outlook

- further states, smearing, distillation, disconnected
- continuum limit
- Λ, α_S in $N_\mathrm{f} = 4$

ens.	$\frac{T}{a} \times \frac{L^3}{a^3}$	a [fm]	$Lm_\pi^\star$
A1	96×32^3	0.054	3.5
A2	128×48^3	0.054	5.3
B	144×48^3	0.043	4.3
C ?	192×64^3	0.032	4.2

Acknowledgements

THANK YOU

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