Magnetic field and ion-optical simulations for the Super-FRS

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Super-FRS commissioning and operation from ion-optics point of view: issues

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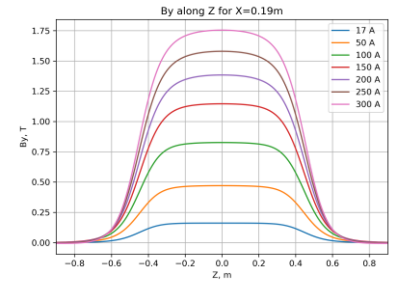
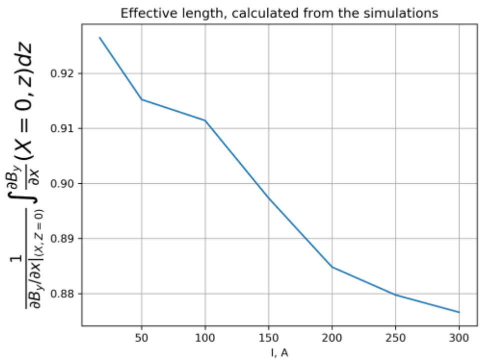
- Centering beam for arbitrary Bρ in range 2-20 Tm: L_{eff} changes non-linearly in all magnets! (up to 1.1% in dipoles, up to 5% in quads)
- 7 dipole stages, about 100 multipoles and 10 steerers can (and must) be tuned to achieve optimal resolution/transmission for any given experiment/rigidity.
- About 20 different types of elements are involved.
- One ramping period 4 min for dipoles, 2 min for quads.
- Central rigidity will be changed many times per day:
  - Manual tuning is very time consuming.

➢ ***Accurate and fast ion-optical model considering realistic magnetic fields from measurements or accurate 3D simulations is required for efficient operation of Super-FRS***





Quad type 3: 3D MS simulation by Dr. E.J. Cho

Step by step procedure to extract high order Taylor transfer maps

1. Measure/simulate the B-field
2. Set the reference trajectory (dipoles)
3. Obtain B-field as smooth functions of *X,Y,Z,I*
4. Generate transfer maps:

a) Integral harmonics + Fringe Fields (FF):

+ fast

0 achieved accuracy ≈ 2%

- don't really work for the SFRS dipoles with measured field

b) Numerical integration of ODEs of motion for reference particle in differential algebraic (DA*) framework:

+ very accurate (relative error down to 10^{-7} is possible)

0 in case of measurements requires Hall probing of (B_x,B_y,B_z)

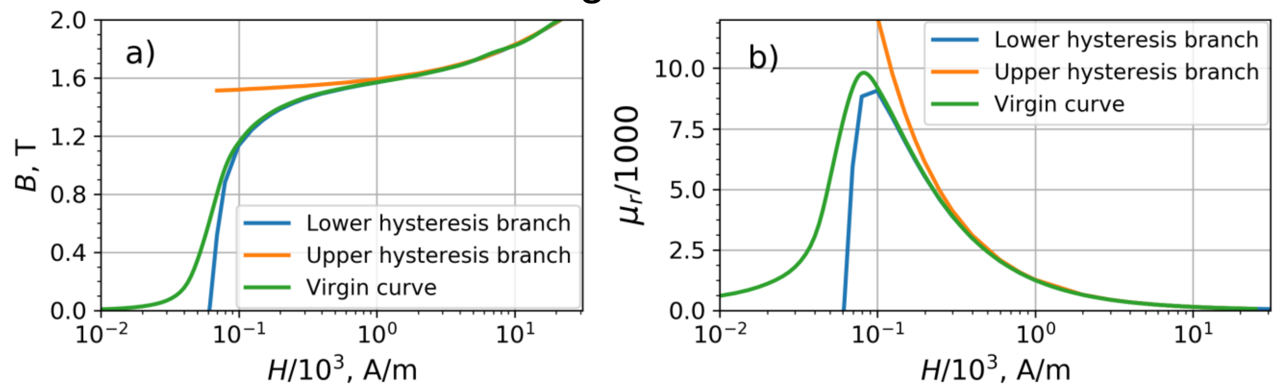
- rather complex method

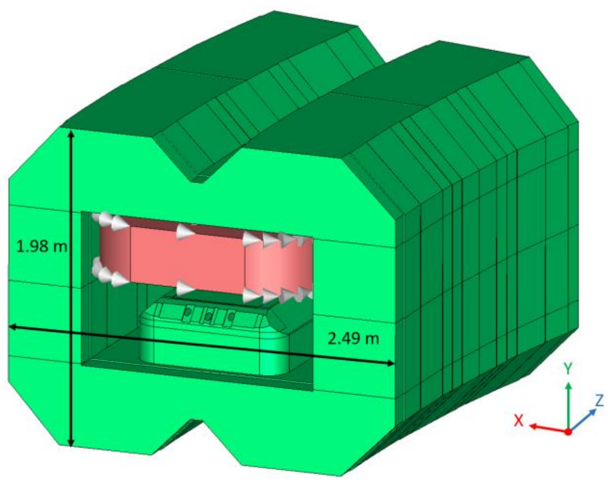
*TPSA + operations of derivation and antiderivation

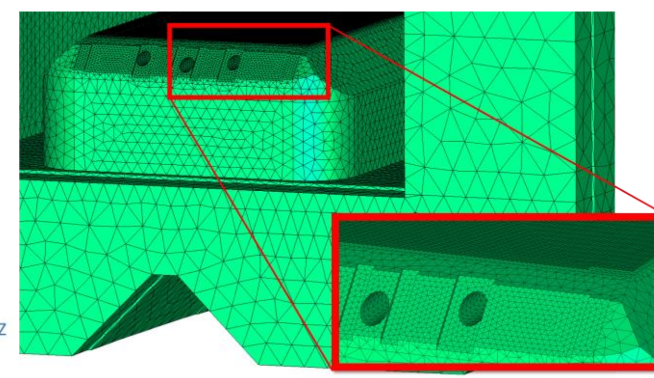
Example: NC dipole
FEM simulations

- CST EMS was used
- Accurate geometry
- Measured virgin curve:
    - No remanence in simulations!!!

Measured magnetization curves







Example: NC dipole
Benchmarking of simulations with the measurements

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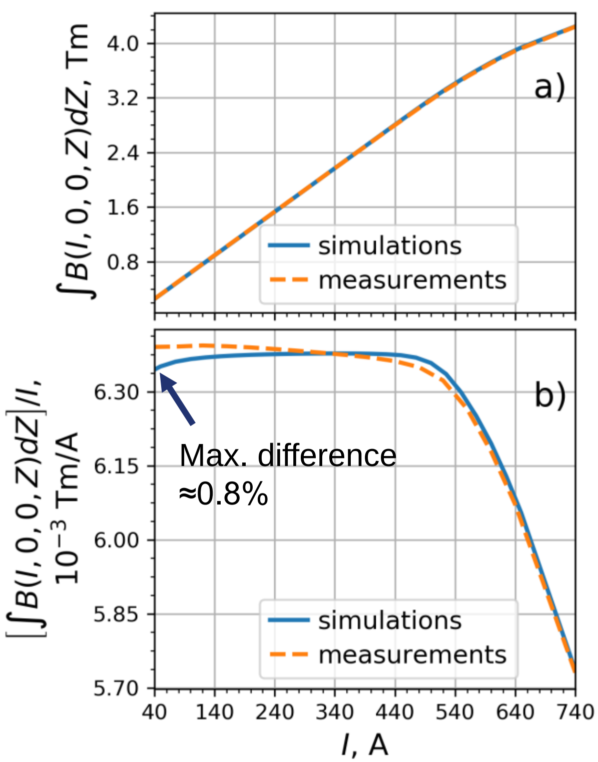
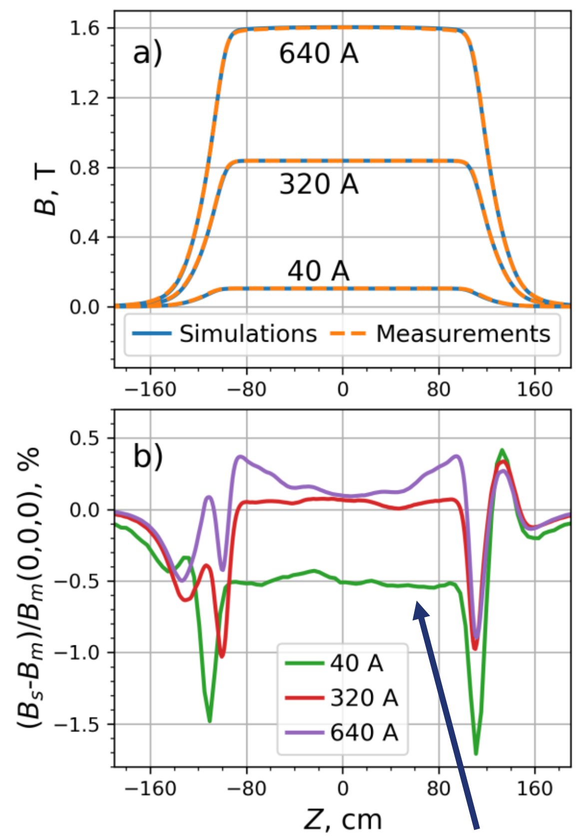
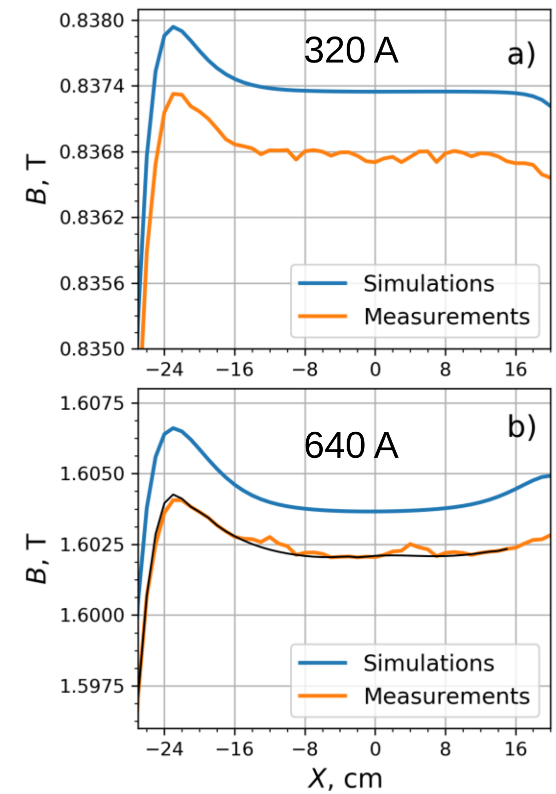
Excitation curve



B_Y along Z axis



B_Y along X axis

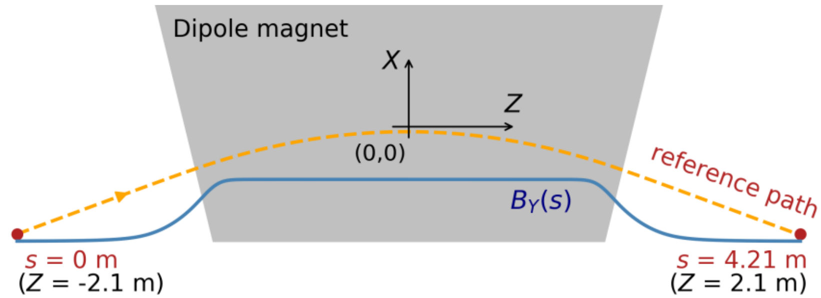


"Missing" remanence in FEM model

Criteria to set a reference trajectory in a dipole

- Deflecting angle in the dipole must be fixed for all rigidities from 2 to 20 Tm

$$\theta = \int_S \frac{B_y(I)}{B\rho} ds$$

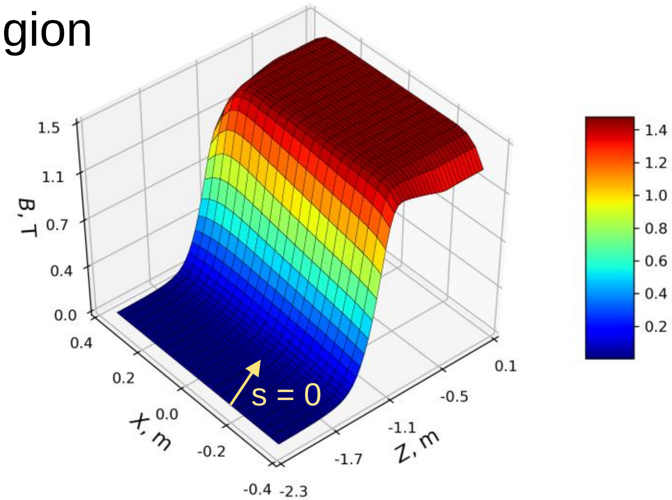


➢ Need to deduce* $I_{opt}(B\rho)$: $\theta(I_{opt})$=const

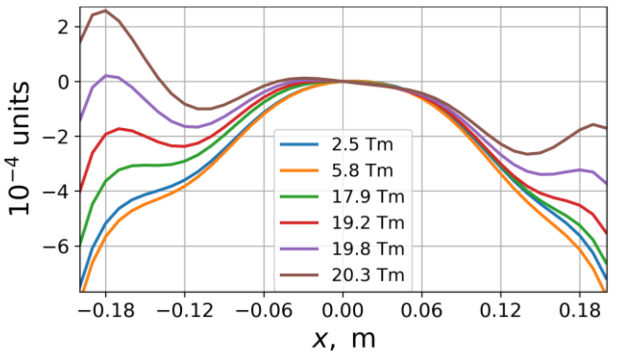
*have to **measure** the **working function** along the reference path

- Reference trajectory must be centered in a good field region

B in *Y*=0
for *I* = 575 A



Integral field non-uniformity for different rigidities



usable aperture

Obtaining accurate *B(X,Y,Z,I)* polynomials

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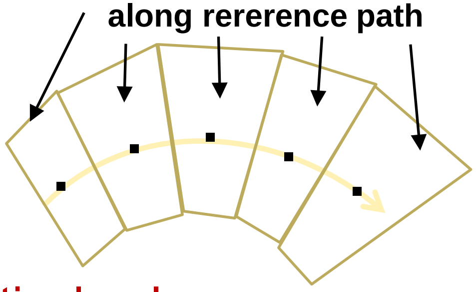
- Using superposition principle and least squares fit, $B_{X,Y,Z}(I)$ can be decomposed in

$$B_\alpha(I) \approx b_0^\alpha + b_1^\alpha(I - I_0) + b_2^\alpha(I - I_0)^2 + ... + b_n^\alpha(I - I_0)^n$$

where $\alpha \in \{X, Y, Z\}$



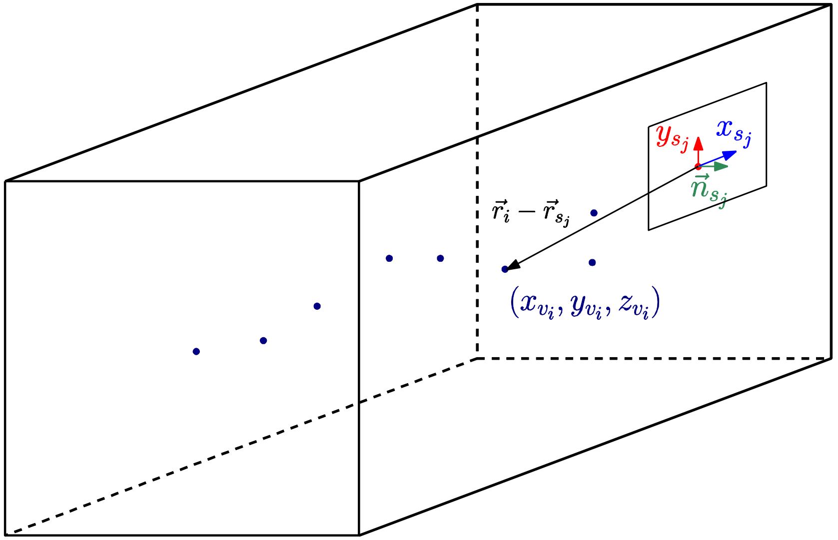
- DA SIHM can be applied to b_i^α.
  - In dipoles radius of convergence of resulting DA vectors ≤ vertical pole gap
- Solution approach (is not needed for quads or sextupoles):
  - SIHM calculation of low (2nd) order polynomials of $b_i^\alpha(X,Z)$ in 2D array of points
  - least squares fit of higher orders polynomials in midplane.
  - DA fixed point theorem reconstruction of harmonic off-plane field.

Surface Integration Helmholtz Method (SIHM)

$$\vec{B}(\vec{r}) = \vec{\nabla} \cdot \varphi(\vec{r}) + \vec{\nabla} \times \vec{A}(\vec{r})$$

$$\varphi(\vec{r}) = \frac{1}{4\pi} \int_{\partial\Omega} \frac{\vec{n}(\vec{r}_s) \cdot \vec{B}(\vec{r}_s)}{|\vec{r} - \vec{r}_s|} ds,$$

$$\vec{A}(\vec{r}) = -\frac{1}{4\pi} \int_{\partial\Omega} \frac{\vec{n}(\vec{r}_s) \times \vec{B}(\vec{r}_s)}{|\vec{r} - \vec{r}_s|} ds.$$



Integration on each surface element using DA ∂^{-1} operation

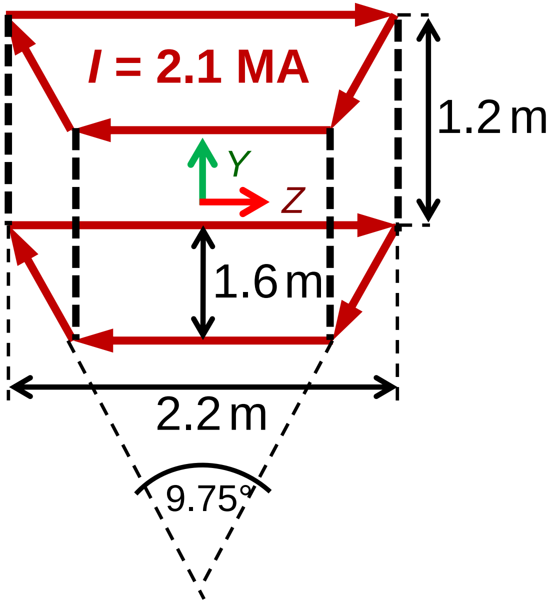
Integrands expanded in surface and volume coordinates

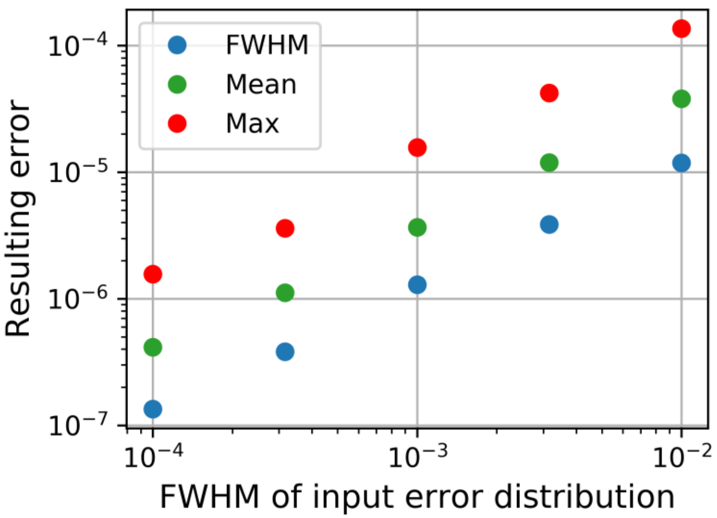
Resulting *B* is harmonic

S. Manikonda, High Order Finite Element Methods to Compute Taylor Transfer Maps, PhD Dissertation, 2006;
M. Berz, Modern Map Methods in Particle Beam Physics, 1999

Robustness of SIHM: analytic example

- Biot-Savart magnetic field from a current wire configuration.





Beam physics ODEs

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- $\vec{z}(s) = \vec{f}(s, \vec{z}(s_0)) = (x, a, y, b, l, \delta_K)|_s$
- ODEs for flat reference trajectory and conserved energy:

$$x' = a(1+hx)\frac{p_0}{p_s},$$

$$y' = b(1+hx)\frac{p_0}{p_s},$$

$$l' = \left((1+hx)\frac{1+\eta}{1+\eta_0}\frac{p_0}{p_s} - 1\right)\frac{\eta_0+1}{\eta_0+2},$$

$$a' = \left(b\frac{B_s}{B\rho_0}\frac{p_0}{p_s} - \frac{B_y}{B\rho_0}\right)(1+hx) + h\frac{p_0}{p_s},$$

$$b' = \left(\frac{B_x}{B\rho_0} - a\frac{B_s}{B\rho_0}\frac{p_0}{p_s}\right),$$

$$\delta' = 0$$

$h = B\rho/B_y(x,s)$ – curvature, $\eta = K/E_0 = \gamma - 1$

Bρ can be added as an extra parameter into the variables, dependent on it: *η*, *h*, p_0, p_s, $B_{x,y,s}$ to obtain *Bρ*-dependent transfer maps

DA Transfer Maps

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x	a	y	b	Order	Exponents			
$\partial x_f/\partial x_i$	$\partial a_f/\partial x_i$	$\partial y_f/\partial x_i$	$\partial b_f/\partial x_i$	1	1	0	0	0
$\partial x_f/\partial a_i$	$\partial a_f/\partial a_i$	$\partial y_f/\partial a_i$	$\partial b_f/\partial a_i$	1	0	1	0	0
…								
$2\partial^2 x_f/\partial x_i^2$	$2\partial^2 a_f/\partial x_i^2$	$2\partial^2 y_f/\partial x_i^2$	$2\partial^2 b_f/\partial x_i^2$	2	2	0	0	0
…								
$j!k!l!m!\ \partial^{j+k+l+m} x_f/ \partial x_i^j \partial a_i^k \partial y_i^l \partial b_i^m$	$j!k!l!m!\ \partial^{j+k+l+m} a_f/ \partial x_i^j \partial a_i^k \partial y_i^l \partial b_i^m$	$j!k!l!m!\ \partial^{j+k+l+ml} y_f/ \partial x_i^j \partial a_i^k \partial y_i^l \partial b_i^m$	$j!k!l!m!\ \partial^{j+k+l+m} b_f/ \partial x_i^j \partial a_i^k \partial y_i^l \partial b_i^m$	j+k+l+m	j	k	l	m

Table 1 Example of the Taylor Transfer Map Structure for 4 variables: *x, a, y, b*

Super-FRS preseparator optics with realistic dipole transfer maps

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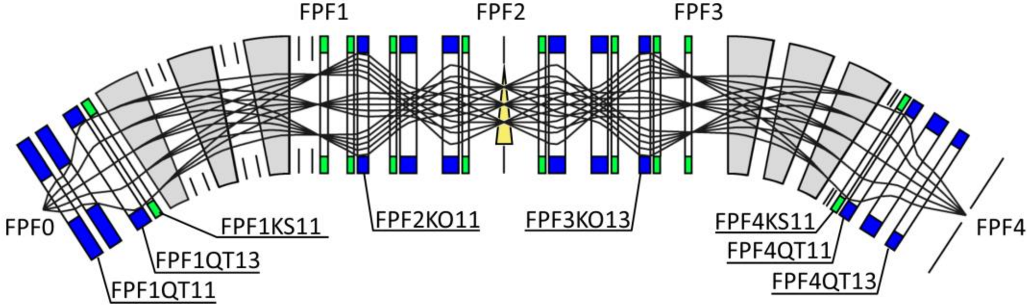
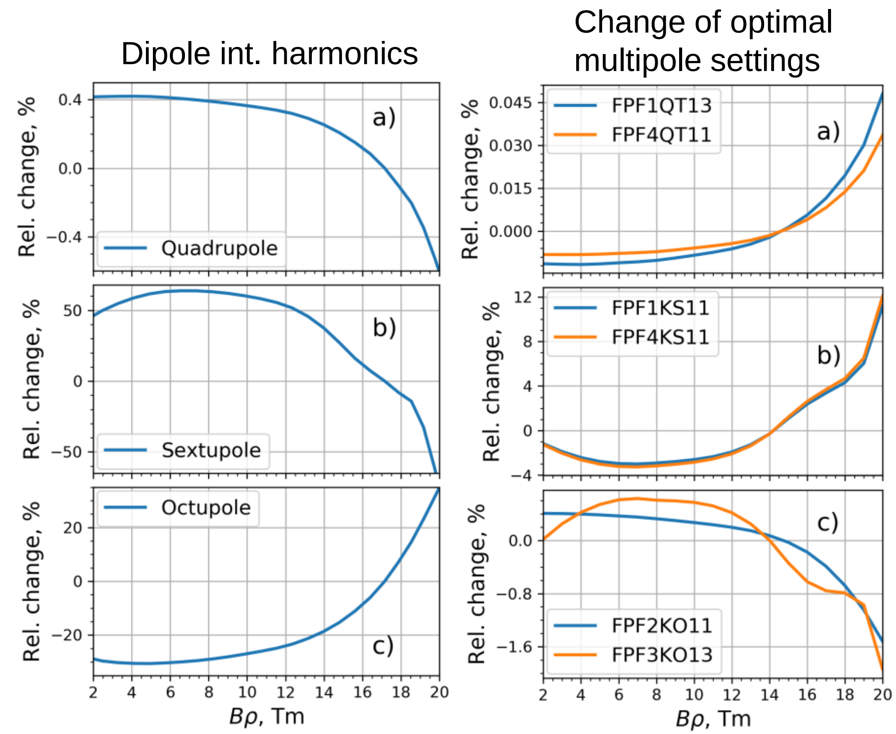
- One *Bρ-ΔE-Bρ* stage
- 3 NC and 3 SC dipoles
- Standard COSY transfer maps were used for multioles
- Optics is optimized for design *Bρ* range

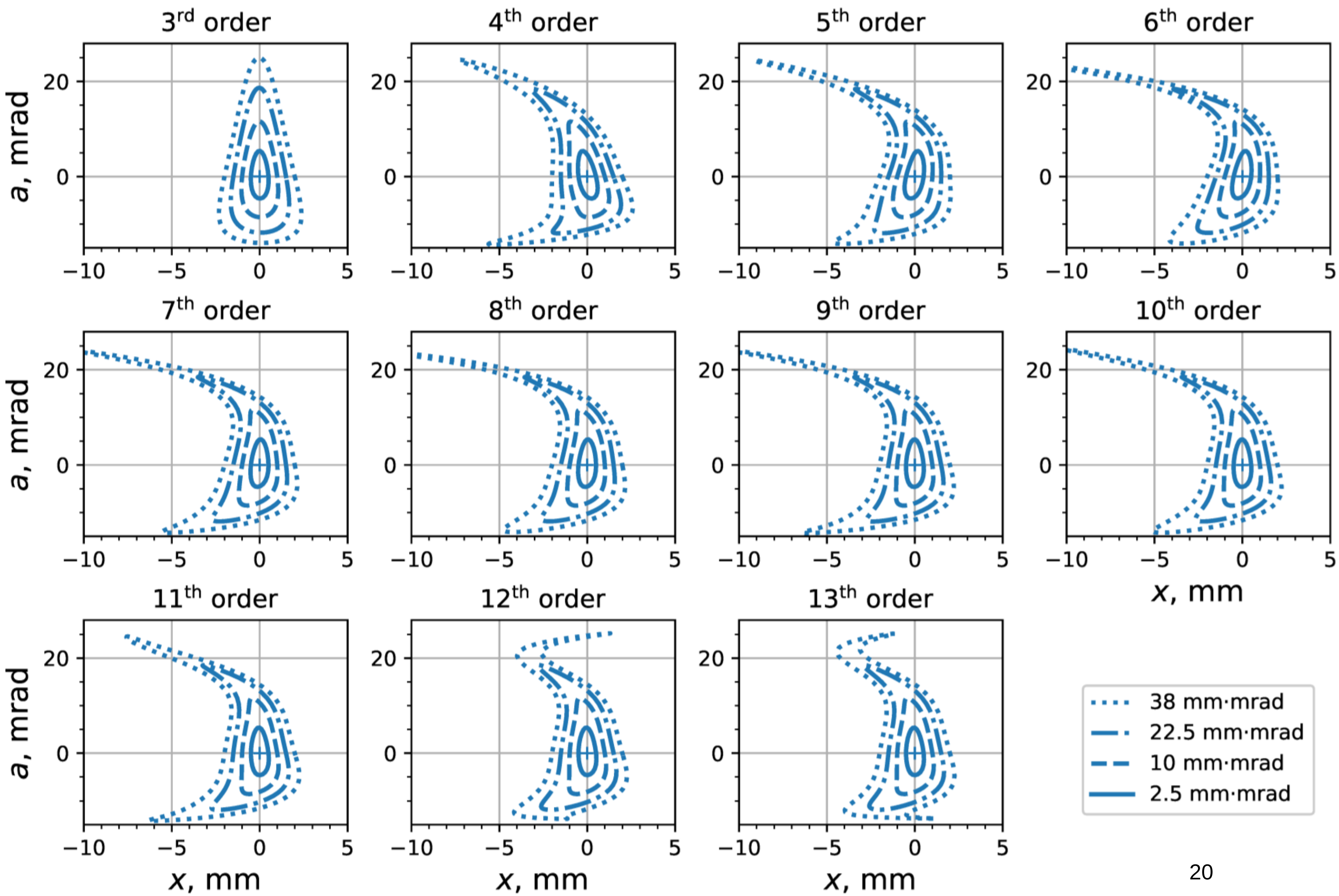


In optimal case the multipoles “compensate” the dipole inhomogeneity



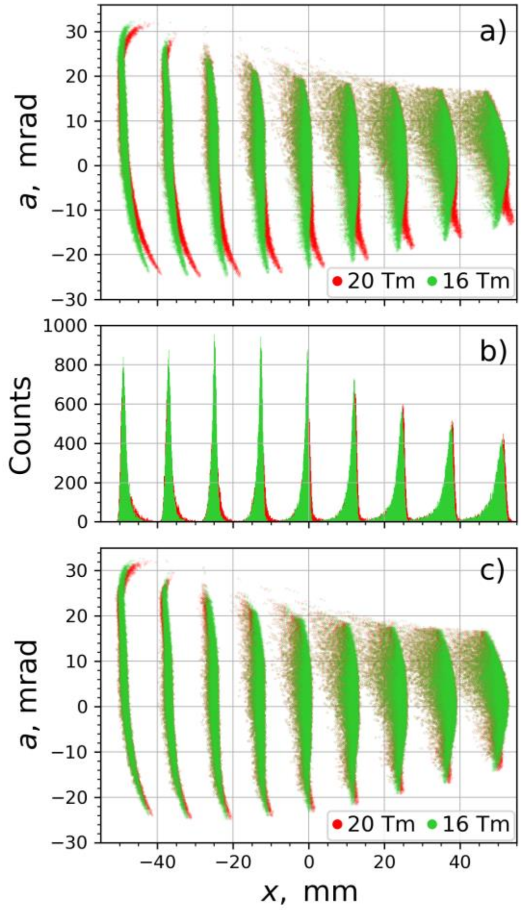
More details in E. Kazantseva et al., accepted in Nucl. Instr. Meth. A.

Maps of different orders: *xa* at FPF4



Spectrometer mode example

- Dispersion of many stages added
- 1st stage of Super-FRS was repeated 4 times to imitate the effect of the whole Super-FRS
- Separation of monoenergetic slices with $\Delta p/p = 1.2 \times 10^{-3}$.
- If the optics is optimized for one *Bρ* (16Tm, a)), light changes are visible
- If the optics is optimized for each Bρ, the difference can be well compensated.



Outlook

- Testing the SIHM approach with measured magnetic field (in collaboration with CERN)
- Taking the remanence into account (modelling in ONELAB or FEniCS).

  Steps:

  1. Measure the hysteresis properties, set up a model (Preisach model)
  2. Using FEM solve $\nabla \times (\nu \nabla \times \vec{A}) = \vec{J}$ with virgin curve at max. current => hysteresis turning points for each discrete element
  3. Finding $\vec{B}_{\mathrm{rem}}$ using Preisach model obtained with measurements data
  4. Using FEM solve $\nabla \times (\nu \nabla \times \vec{A}) = \vec{J} - \nabla \times (\nu \vec{B}_{\mathrm{rem}})$
- Computing the transfer maps for all types of the Super-FRS magnets
- Studying fringe field overlapping effects

THANK YOU FOR THE ATTENTION!

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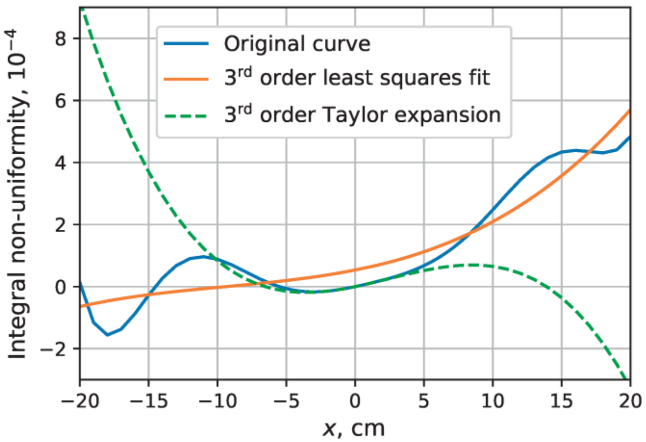
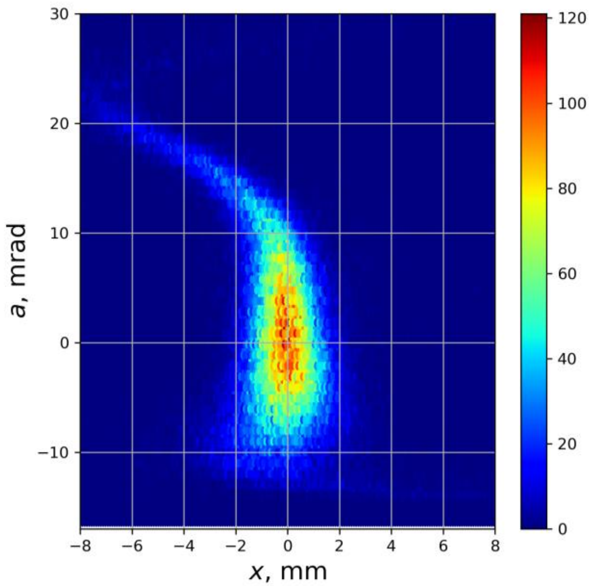
Correction by tuning available multipoles

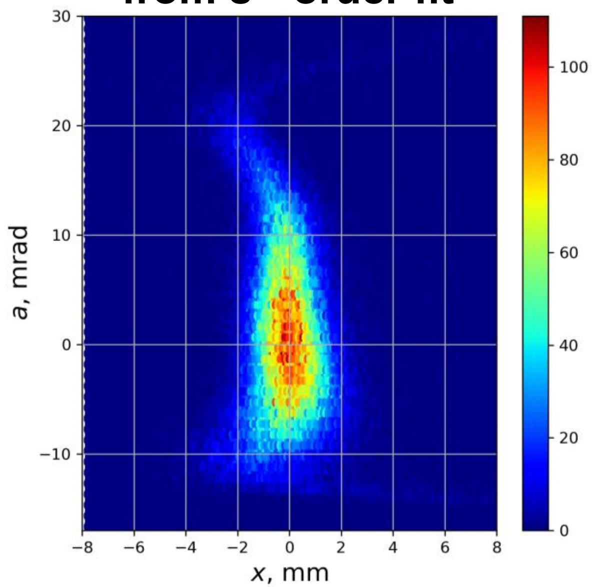
- Maximum correction order = 3
  - 3rd order transfer maps with least squares fitted integral multipoles can be used for fitting optics
  - Obtained optimal settings can be used in higher order calculations



13th order, optimal fit



13 order, coefficients from 3rd order fit



Horizontal phase space at FPF4 for initial Gaussian distribution with FWHM equal to design acceptance

Analytical test model of ironless inhomogeneous dipole magnet

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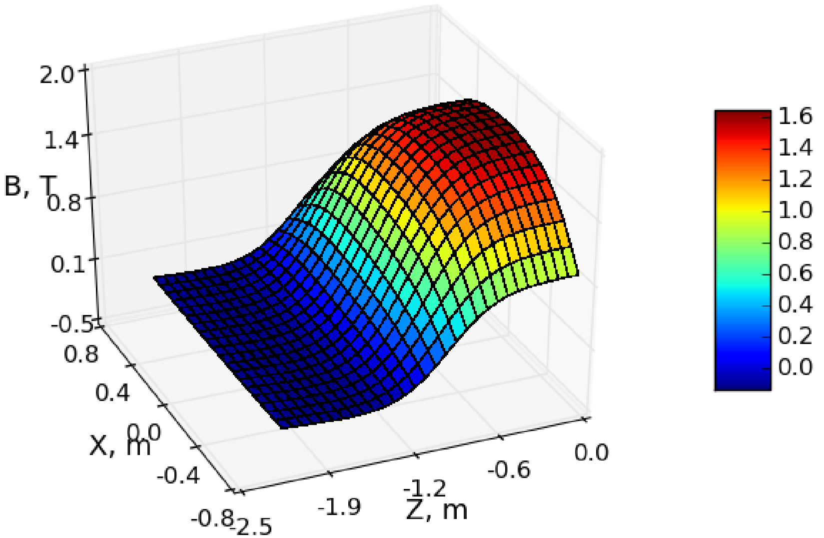
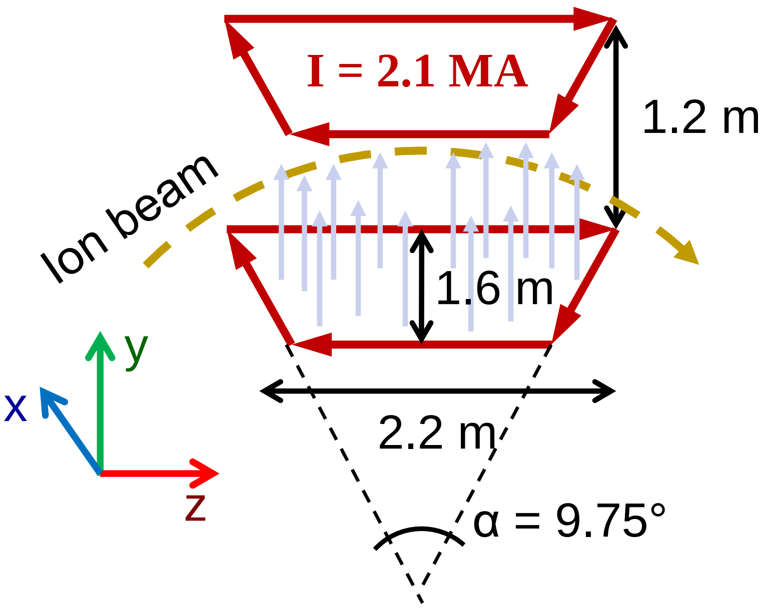
- Analytical magnetic field from Biot-Savart law:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_C \frac{Id\mathbf{l}\times\mathbf{r'}}{|\mathbf{r'}|^3}$$



Straight wires configuration:



- Inhomogeneous field
- Symmetry like in usual accelerator dipole magnet: In Y=0, $B_x = B_z = 0$

Helmholtz theorem in 3D case

Theorem: *A general continuous three-vector field defined in a Euclidean 3-space, that along with its first derivatives vanishes sufficiently rapidly at infinity, may be uniquely represented as a sum of an irrotational part and a solenoidal part.*

$$\vec{B}(\vec{r}) = -\vec{\nabla}\phi(\vec{r}) + \vec{\nabla} \times \vec{A}(\vec{r}), \text{ where}$$

$$\varphi(\vec{r}) = \frac{1}{4\pi} \int\limits_{\partial\Omega} \frac{\vec{n}(\vec{r}_s)\cdot\vec{B}(\vec{r}_s)}{|\vec{r}-\vec{r}_s|} ds - \frac{1}{4\pi} \int\limits_{\Omega} \frac{\vec{\nabla}\cdot\vec{B}(\vec{r}_v)}{|\vec{r}-\vec{r}_v|} dV, \text{ and}$$

0 for magnetic field

$$\vec{A}(\vec{r}) = -\frac{1}{4\pi} \int\limits_{\partial\Omega} \frac{\vec{n}(\vec{r}_s)\times\vec{B}(\vec{r}_s)}{|\vec{r}-\vec{r}_s|} ds + \frac{1}{4\pi} \int\limits_{\Omega} \frac{\vec{\nabla}\times\vec{B}(\vec{r}_v)}{|\vec{r}-\vec{r}_s|} dV.$$

0 for harmonic field

$$\Rightarrow \quad \varphi(\vec{r}) = \frac{1}{4\pi} \int\limits_{\partial\Omega} \frac{\vec{n}(\vec{r}_s)\cdot\vec{B}(\vec{r}_s)}{|\vec{r}-\vec{r}_s|} ds, \quad \vec{A}(\vec{r}) = -\frac{1}{4\pi} \int\limits_{\partial\Omega} \frac{\vec{n}(\vec{r}_s)\times\vec{B}(\vec{r}_s)}{|\vec{r}-\vec{r}_s|} ds.$$

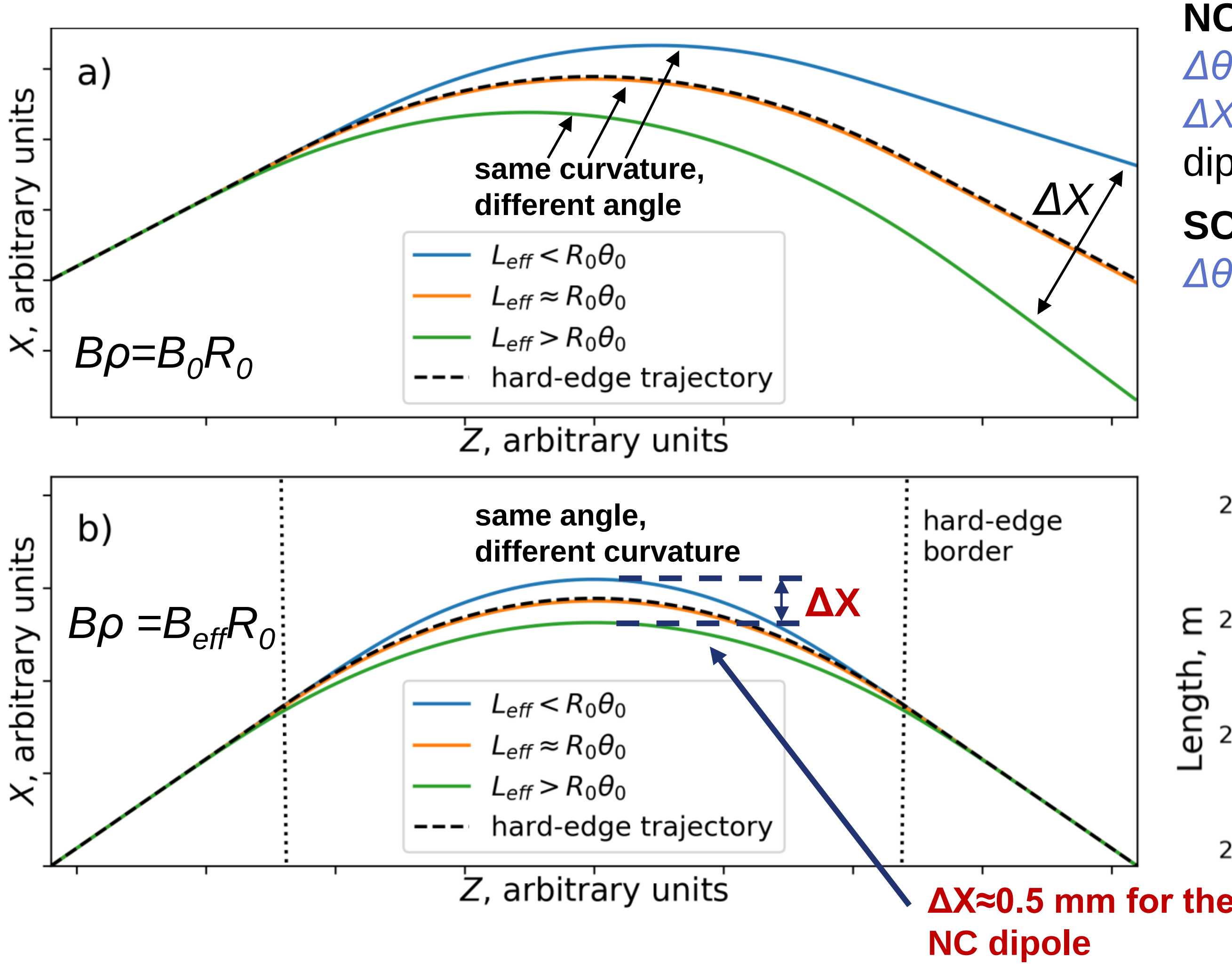
Fixing R_0 versus fixing θ_0

- Effective length $L_{\text{eff}}(I) := \dfrac{1}{B_0(I)} \int\limits_S B_y(I) ds$ is equal $R_0\theta$ if $B\rho = B_0 R_0$

Actual deflecting angle

- θ drops with L_{eff}
- If $B\rho \neq B_0 R_0$ then L_{eff} is not informative
- It is possible to fix θ_0 by varying $B\rho(I)$ or I
- Using $B_{\text{eff}} = B\rho/R_0$ one gets $L_{\text{eq}}(I) := \dfrac{1}{B_{\text{eff}}(I)} \int\limits_S B_y(I) ds = R_0\theta(I)$
- In optimal case $L_{\text{eq}} = R_0\theta_0$

Fixing R_0 versus fixing θ_0



NC SFRS dipole:

$\Delta\theta$(2-20 Tm) = *1 mrad* (*0.56%*θ_0)

ΔX = 1.2 cm after stage of 3 dipoles.

SC (9.75º) dipole:

$\Delta\theta$(2-20 Tm) = *1.9 mrad* (*1.1%*θ_0)

L_{eff} and L_{eq} for NC SFRS dipole. $L_{eq}(I)$ was adjusted for each $B\rho$ *to* result in $\theta_0 R_0$