Analytical approach to granular target cooling

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Outline of the talk

- Analytical model of gas flow through a pebble-bed target
- Results for axial helium flow
- Results for transverse gas flow
- Conclusions

Finite element results and a comparison between the analytical model and FEA will be given in the talk by L. Lacny

Main assumptions

- The analytical model is one-dimensional (changes only along the axial- or in the transverse direction are taken into account)
- Heat exchange takes place only at the interface of the spheres and the flowing helium
- The heat flux from the spheres to the cooling medium is proportional to the temperature difference between the sphere surface and that of the cooling medium. The constant of proporionality is the heat transfer coefficient α
- Steady-state condition is being considered
- Uniform energy deposition is assumed in the present study (the results are now being updated using the real energy deposition map)

Temperature of the spheres

Balance of heat in the spheres:

$$dm \cdot c_k \cdot dT_k = -(T_k - T) \cdot dS \cdot \alpha \cdot dt$$

Equation that describes the temperature of the spheres:

$$\tau_0 \frac{dT_k}{dt} + T_k = T \qquad \tau_0 = \frac{\rho_k c_k r}{3\alpha}$$

Temperature of the spheres:

$$T_k(x,t) = T(x) + \left(T_{\max}(x) - T(x)\right)\exp\left(-t/\tau_0\right)$$

Temperature of the spheres

Condition of steady-state operation of the heat exchanger:

$$(T_k)_{\max} - T_k(t_c) = \Delta T_k \qquad \Delta T_k = \frac{q}{c_k}$$

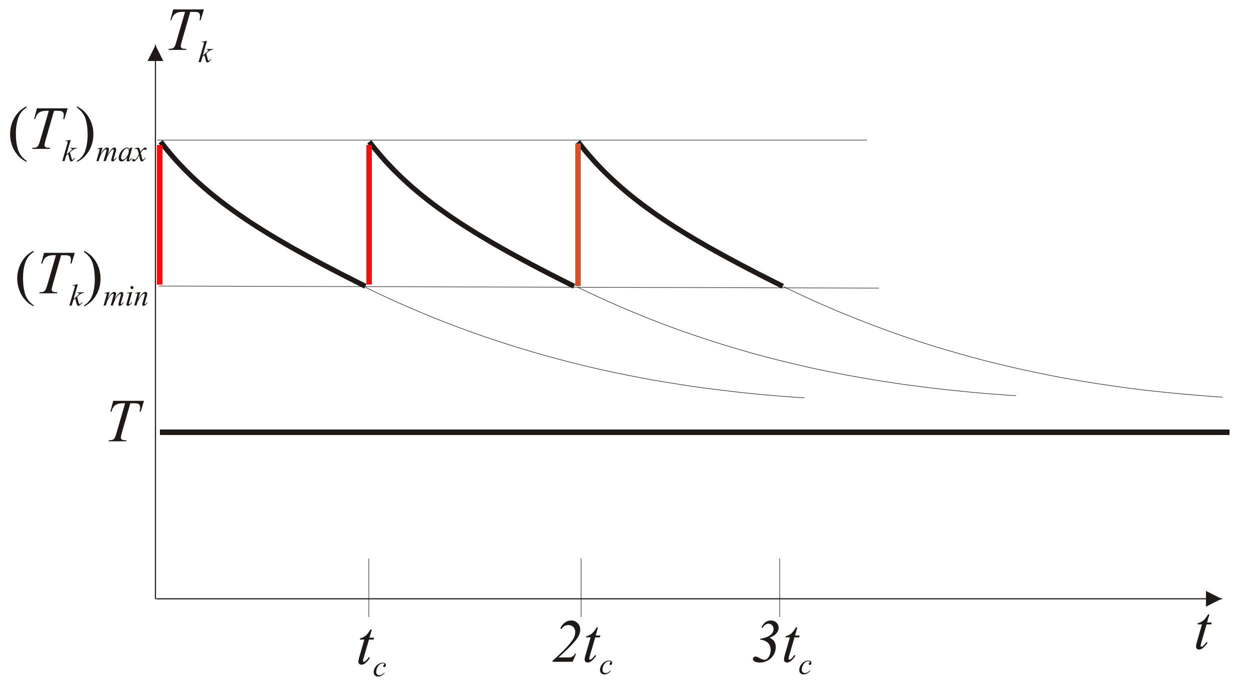
Combining this equation with the equation giving the previous temperature of the spheres vs. time one can find the minimum and maximum temperature of the spheres:

$$(T_k)_{\min}(x) = T(x) + \frac{\exp(-t_c/\tau_0)}{1-\exp(-t_c/\tau_0)}\Delta T_k$$

$$(T_k)_{\max}(x) = T(x) + \frac{1}{1-\exp(-t_c/\tau_0)}\Delta T_k$$

Temperature of the spheres

Character of temperature change of the spheres, at a given distance from the target beam-incoming end, during cycles of heat exchange (in red – heating, in black- cooling)



T – steady-state temperature of the cooling medium (avaraged over a cycle)

Temperature of the cooling gas (ideal compressible gas under steady-state condition)

Balance of mass: $\rho_1 A_{efek} u_1 = \rho_2 A_{efek} u_2$

Balance of momentum for a porous medium (to be discussed in more detail in the talk by L. Lacny):

$$\frac{dp}{dx} = -\rho u \frac{du}{dx} - \left(\frac{\mu}{\alpha_{pm}} + \frac{1}{2} \rho C_{pm} \left| u_{sef} \right| \right) u_{sef}$$

where: $u_{sef} = u\,(1-\beta_A)$ – superficial velocity

State equations for ideal gass:
$$p_1 = \rho_1 R T_1$$
$$p_2 = \rho_2 R T_2$$

Balance of enthalpy:

$$c_p \left(T_{02} - T_{01} \right) = \frac{\dot{Q}}{\dot{m}} \qquad T_0 = T \left(1 + \frac{\kappa - 1}{2} M^2 \right) \qquad M = \frac{u}{\sqrt{\kappa R T}}$$

T_0 - stagnation temperature, M – Mach number, $\kappa = c_p / c_v$ - specific heat ratio, for helium taken to be 5/3 (as for a monatomic ideal gas)

Data used in simulations of longitudinal flow

- Diameter of the target Φ_w=30 mm , volume packing β_V=0.66, avarage effective area coefficient β_A=0.7 (on avarage, 0.3 of the total target cross section is available for the gas flow)
- Titanium spheres of radius r =1.5 mm (mass density ρ_k =4500 kg/m^3 , specific heat c_k=600 J/(kgK))
- Cooling medium – helium with parameters: the individual gas constant R=2079 J/(kgK), specific heat c_p=5193 J/(kgK), specific heat ratio $\kappa=c_p/c_v$=5/3, viscosity $\mu=1.99\times10^{-5}$ kg/(ms)
- Beam parameters: frequency 14 Hz (period of one cycle t_c=0.07143 s), deposited beam energy density per 1 kg of the sphere material q = 9400 J/kg
- Parameters of the porous medium (permeability $\alpha_{pm} = 5.4\times10^{-9}$ m^2 , hence $1/\alpha_{pm} = 1.85\times10^{8}$ 1/m^2, inertial resistance factor $C_{pm} = 1.96\times10^{4}$ 1/m)

Results for longitudinal flow

A pebble bed of length 5 cm is considered for test purposes. The cooling of a target 78 cm long by axial flow is not feasable.

Assumed gas helium conditions on entering the target - pressure p_1=10^6 Pa, temperature T_1=293 K, density ρ_1=1.64 kg/m^3, velocity u_1=29 m/s (Mach number M_1=0.029, mass flow rate m_p=0.01 kg/s, superficial velocity $(u_1)_{sef}$ =8.7 m/s)

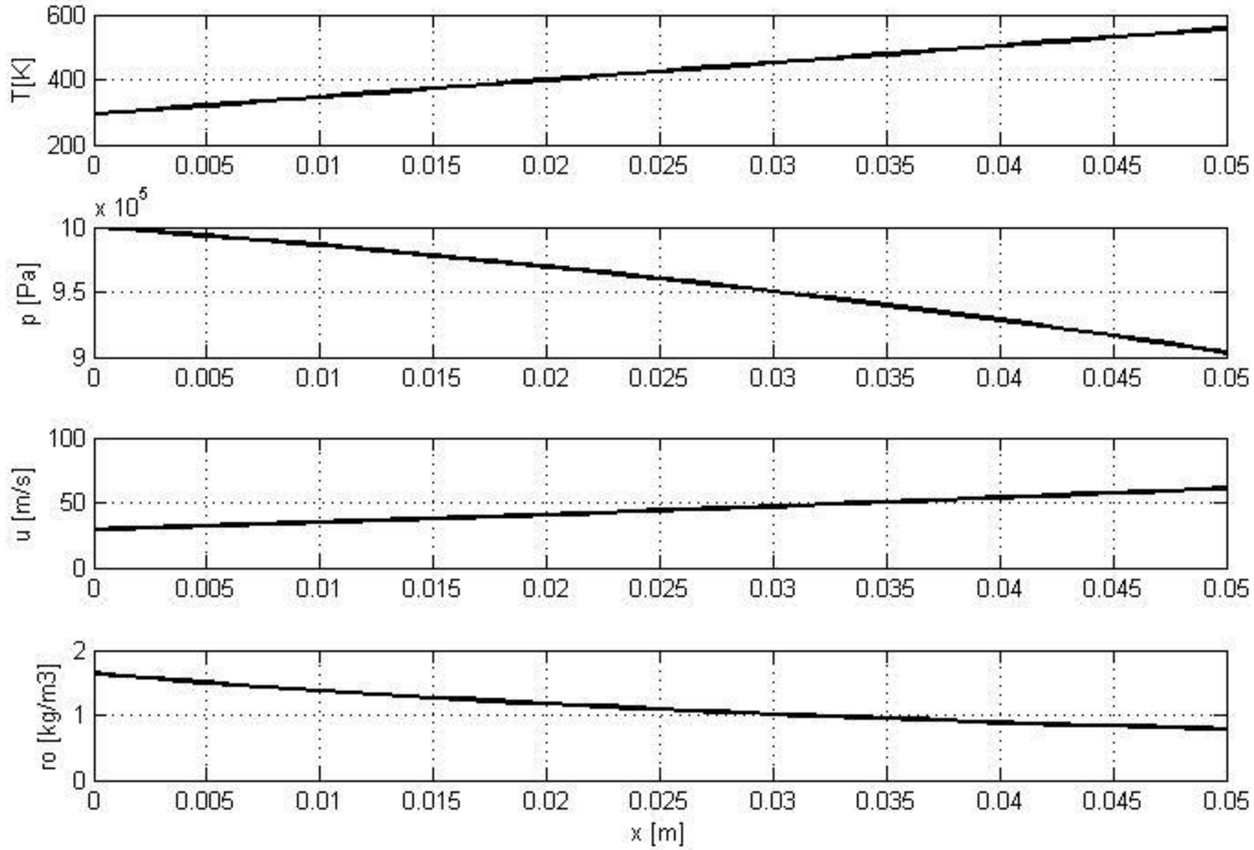
Results for a 5 cm pebble bed in axial flow:

Temperature increase of the spheres due to each beam pulse is ΔT_k = 16 K

Gas parameters on leaving the target: pressure p_2=0.903×10^6 Pa, temperature T_2=556 K, density ρ_2=0.78 kg/m^3, velocity u_2=60.7 m/s (superficial velocity $(u_2)_{sef}$ =18.3 m/s)

Results for longitudinal flow

Temperature, pressure, velocity and density distribution along length of the pebble bed



Results for transverse flow

In the case of transverse flow higher mass flow rates can be achieved than for axial flow. The value m_p= 0.1 kg/s has been assumed (comparable to that used by T. Davenne for the Euronu calculations)

Assumed gas conditions on entering the target - pressure p_1=10^6 Pa, temperature T_1=293 K, density ρ_1=1.64 kg/m^3, velocity u_1= 66 m/s (Mach number M_1= 0.066, mass flow rate m_p= 0.1 kg/s, superficial velocity $(u_1)_{sef}$ =19.8 m/s).

Results for a transverse flow across a target of diameter 3 cm, 78 cm long (the avarage power deposited in the target used was eqal to 215 kW)

The temperature increase of the sphere due to each beam pulse is ΔT_k = 16 K

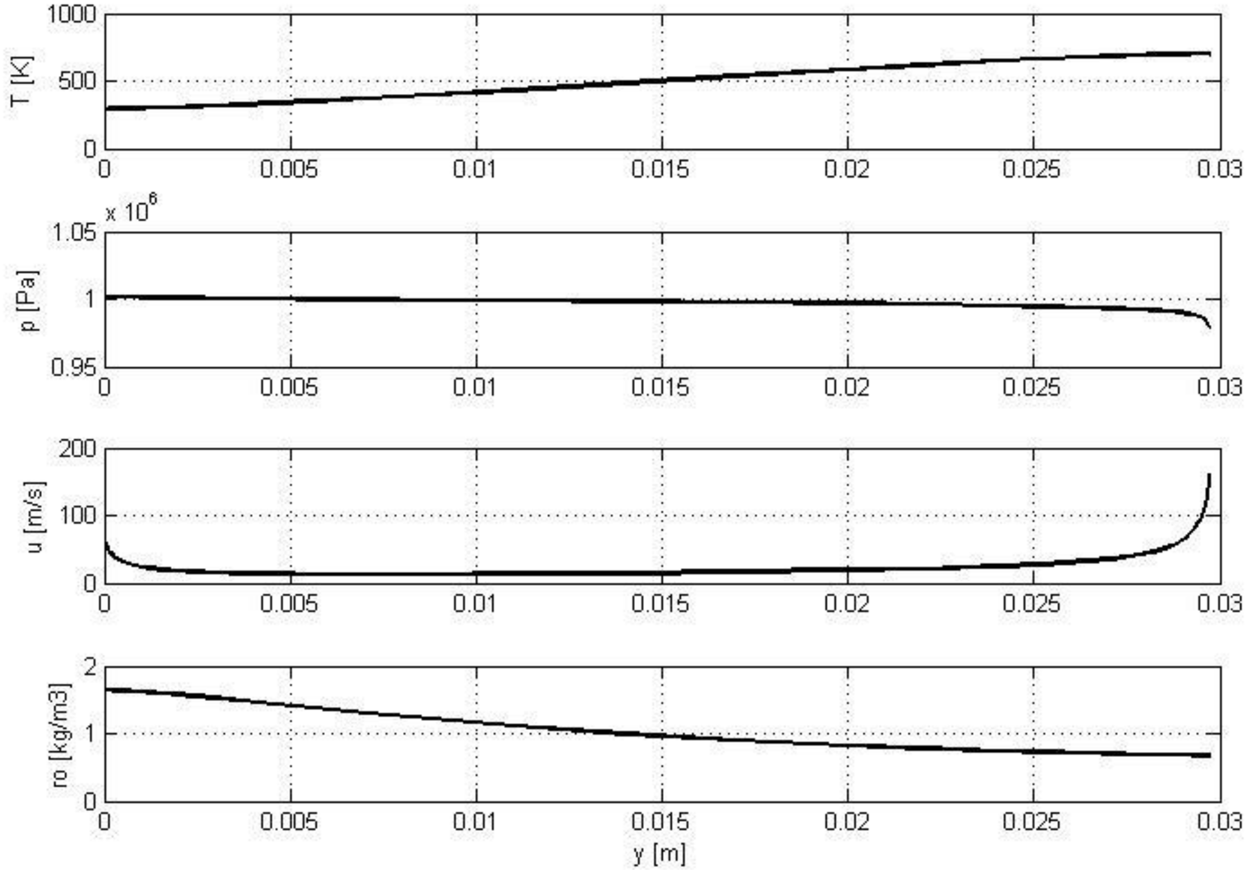
Results for transverse flow

Helium parameters on leaving the target: pressure $p_2 = 0.978\times10^6$ Pa, temperature $T_2 = 700$ K, density $\rho_2 = 0.673$ kg/m^3, velocity $u_2 = 160$ m/s (superficial velocity $(u_2)_{sef} = 48.2$ m/s)

Minimum and maximum temperature of the spheres during a cycle of heat exchange: $(T_k)_{\mathrm{min}}(y)=T(y)+267$ K , $(T_k)_{\mathrm{max}}(y)=T(y)+283$ K

Results for transverse flow

Temperature, pressure, velocity and density distribution along length of the pebble bed for the case of transverse flow



Conclusions

- The analytical model is of much use in understanding helium gas flow through granular targets and the cooling efficiency
- More accurate results of the finite element calculations confirm the validity of this model
- Cooling by longitudinal flow seems not feasible. Transverse flow is much more efficient. The present results are comparable with the earlier numerical studies made at RAL
- The present results considered the case of uniform power deposition in the target. These results are now being updated using the power deposition map calculated at CNRS.

Conclusions

- A technical note giving the details of the analytical and finite element approaches to granular target cooling is under preparation

Thank you for your attention