Classical and Quantum Integrable Systems

- **Session 1 [2 hours]**

  Classical integrability part 1: Liouville theorem, examples of classically integrable dynamical systems, superintegrability (1 hour) [1];

  Lax pair, monodromy and transfer matrix, example of the Non-Linear Schrödinger Equation (NLS) (1 hour) [1, 2].

  Prerequisites: Classical Mechanics

- **Session 2 [2 hours]**

  Classical integrability part 2: Classical r-matrices, Belavin-Drinfeld theorems, non-local charges, example of the Principal Chiral Model (1 hour) [1, 3];

  Classical inverse scattering method, solitons (1 hour) [1, 3].

  Prerequisites: Lie algebras

- **Session 3 [2 hours]**

  Quantum integrability part 1: Hopf algebras and universal R-matrix, quantum groups, example of $U_q(\mathfrak{su}(2))$ (1 hour) [4];

  RTT relations, Algebraic Bethe Ansatz, example of the NLS (1 hour) [5–7].

  Prerequisites: Quantum Mechanics

- **Session 4 [2 hours]**

  Quantum integrability part 2: Exact S-matrices, bound states, perturbation theory of the NLS (1 hour) [8, 9];

  Coordinate Bethe ansatz and Thermodynamic Bethe ansatz (1 hour) [10].

  Prerequisites: Classical and Quantum Statistical Mechanics

- **Session 5 [2 hours]**

  Quantum integrability part 3: massless scattering and massless flows, example of the Tricritical to Critical Ising Model (1 hour) [11];

  Quantisation of the Kadomtsev-Petviashvili equation (1 hour) [12];

  Prerequisites: Quantum Field Theory

[1] Alessandro Torrielli. Lectures on Classical Integrability. *J. Phys.*, A49(32):323001, 2016.

[2] E. K. Sklyanin. Quantum version of the method of inverse scattering problem. *J. Sov. Math.*, 19:1546–1596, 1982. [Zap. Nauchn. Semin.95,55(1980)].

[3] Olivier Babelon, Denis Bernard, and Michel Talon. *Introduction to Classical Integrable Systems*. Cambridge Monographs on Mathematical Physics. Cambridge University Press, 2003.

[4] C. Kassel. *Quantum groups*. 1995.

[5] L. D. Faddeev. How algebraic Bethe ansatz works for integrable model. In *Relativistic gravitation and gravitational radiation. Proceedings, School of Physics, Les Houches, France, September 26-October 6, 1995*, pages pp. 149–219, 1996.

[6] Fedor Levkovich-Maslyuk. The Bethe ansatz. *J. Phys.*, A49(32):323004, 2016.

[7] H. B. Thacker. Exact Integrability in Quantum Field Theory. In *4th Workshop on Current Problems in High-Energy Particle Theory Bad Honnef, Germany, June 2-4, 1980*, page 0179, 1980.

[8] P. Dorey. Exact S matrices. In *Conformal field theories and integrable models. Proceedings, Eotvos Graduate Course, Budapest, Hungary, August 13-18, 1996*, pages 85–125, 1996.

[9] Diego Bombardelli. S-matrices and integrability. *J. Phys.*, A49(32):323003, 2016.

[10] Stijn J. van Tongeren. Introduction to the thermodynamic Bethe ansatz. 2016. [J. Phys.A49,no.32,323005(2016)].

[11] P. Fendley and H. Saleur. Massless integrable quantum field theories and massless scattering in (1+1)-dimensions. In *Proceedings, Summer School in High-energy physics and cosmology: Trieste, Italy, June 14-July 30, 1993*, pages 301–332, 1993. [,87(1993)].

[12] Karol K Kozlowski, Evgeny Sklyanin, and Alessandro Torrielli. Quantization of the KadomtsevPetviashvili equation. *Theor. Math. Phys.*, 192(2):1162–1183, 2017.