Extraction of Transfer Map Including Self-Consistent Space-Charge Effects in a Symplectic Tracking Code

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Outline

- Introduction
- Self-consistent symplectic multi-particle tracking model
- Extraction of transfer map including self-consistent space-charge using differential algebra
- An illustrative example

Introduction: Transfer Map Provides a Tool for Analysis, Design and Tracking

- Use transfer map and normal form analysis to get amplitude dependent tune shift, nonlinear resonance strength, nonlinear chromaticity, including space-charge effects
- Use transfer map for dynamic aperture study including space-charge effects
- Use transfer map for long-term tracking without numerical noise
- Use transfer map to design compensation elements to mitigate the space-charge effects
- Use transfer map for high-order matching

A Symplectic Multi-Particle Tracking Model (1)

multi-particle Hamiltonian $H(\mathbf{r}_1, \mathbf{r}_2, \cdots, \mathbf{p}_1, \mathbf{p}_2, \cdots, s)$

$$H = \sum_i \mathbf{p}_i^2/2 + \frac{1}{2}\sum_i \sum_j q\phi(\mathbf{r}_i, \mathbf{r}_j) + \sum_i q\psi(\mathbf{r}_i)$$

space-charge
Coulomb potential

external focusing/acceleration

$$\frac{d\mathbf{r}_i}{ds} = \frac{\partial H}{\partial \mathbf{p}_i}$$

$$\frac{d\mathbf{p}_i}{ds} = -\frac{\partial H}{\partial \mathbf{r}_i}$$

$$\frac{d\zeta}{ds} = -[H, \zeta]$$

A formal single step solution

$$\zeta(\tau) = \exp(-\tau(:H:))\zeta(0)$$

$H = H_1 + H_2$

$$\begin{aligned}\zeta(\tau) &= \exp(-\tau(:H_1:+:H_2:))\zeta(0) \\ &= \exp(-\frac{1}{2}\tau:H_1:)\exp(-\tau:H_2:)\exp(-\frac{1}{2}\tau:H_1:)\zeta(0) + O(\tau^3)\end{aligned}$$

$$\boxed{\begin{aligned}\zeta(\tau) &= \mathcal{M}(\tau)\zeta(0) \\ &= \mathcal{M}_1(\tau/2)\mathcal{M}_2(\tau)\mathcal{M}_1(\tau/2)\zeta(0)\end{aligned}}$$

J. Qiang, Phys. Rev. ST Accel. Beams 20, 014203 (2017).

A Symplectic Multi-Particle Tracking Model (2)

- **The above integrator can be extended to higher order**
- **Each sub-map needs to be symplectic for the symplectic integrator**

higher order: $\mathcal{M}_{2n+2}(\tau) \;=\; \mathcal{M}_{2n}(z_0\tau)\mathcal{M}_{2n}(z_1\tau)\mathcal{M}_{2n}(z_0\tau)$

where $z_0 = 1/(2-2^{1/(2n+1)})$ and $z_1 = -2^{1/(2n+1)}/(2-2^{1/(2n+1)})$

Symplectic condition: $\boxed{M_i^T J M_i = J}$ **M is the Jacobi Matrix of $\mathcal{M}$**

where J denotes the $6N \times 6N$ matrix given by

$$J = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$$ and I is the $3N \times 3N$ identity matrix

H. Yoshida, Phys. Lett. A **150, p. 262, 1990.**

A Symplectic Multi-Particle Tracking Model (3)

$$H_1 = \sum_i \mathbf{p}_i^2/2 + \sum_i q\psi(\mathbf{r}_i) \longrightarrow \mathcal{M}_1$$

- symplectic map for H_1 can be found from charged particle optics method

$$H_2 = \frac{1}{2}\sum_i \sum_j q\phi(\mathbf{r}_i, \mathbf{r}_j) \longrightarrow \mathcal{M}_2$$

$$\mathbf{r}_i(\tau) = \mathbf{r}_i(0)$$

$$\mathbf{p}_i(\tau) = \mathbf{p}_i(0) - \frac{\partial H_2(\mathbf{r})}{\partial \mathbf{r}_i}\tau$$

$$M_2 = \begin{pmatrix} I & 0 \\ L & I \end{pmatrix}$$

To satisfy the symplectic condition: $\boxed{L = L^T}$

$$L_{ij} = \partial \mathbf{p}_i(\tau)/\partial \mathbf{r}_j = -\frac{\partial^2 H_2(\mathbf{r})}{\partial \mathbf{r}_i \partial \mathbf{r}_j}\tau$$

$\mathcal{M}_2$ will be ***symplectic*** if p_i is updated from H_2 ***analytically***

Self-Consistent Space-Charge Transfer Map (1)

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = -\frac{\rho}{\epsilon_0} \qquad \begin{aligned} \phi(x=0,y) &= 0 \\ \phi(x=a,y) &= 0 \\ \phi(x,y=0) &= 0 \\ \phi(x,y=b) &= 0 \end{aligned}$$

$$\rho(x,y) = \sum_{l=1}^{N_l} \sum_{m=1}^{N_m} \rho^{lm} \sin(\alpha_l x) \sin(\beta_m y)$$

$$\phi(x,y) = \sum_{l=1}^{N_l} \sum_{m=1}^{N_m} \phi^{lm} \sin(\alpha_l x) \sin(\beta_m y)$$

$$\rho^{lm} = \frac{4}{ab} \int_0^a \int_0^b \rho(x,y) \sin(\alpha_l x) \sin(\beta_m y)\, dx dy$$

$$\phi^{lm} = \frac{4}{ab} \int_0^a \int_0^b \phi(x,y) \sin(\alpha_l x) \sin(\beta_m y)\, dx dy$$

where $\alpha_l = l\pi/a$ and $\beta_m = m\pi/b$

$$\boxed{\phi^{lm} = \frac{\rho^{lm}}{\epsilon_0 \gamma_{lm}^2}} \qquad \text{where } \gamma_{lm}^2 = \alpha_l^2 + \beta_m^2$$

Self-Consistent Space-Charge Transfer Map (2)

$$\rho(x,y) = \frac{1}{N_p}\sum_{j=1}^{N_p} S(x-x_j)S(y-y_j)$$

$$\phi^{lm} = \frac{4\pi}{\gamma_{lm}^2}\frac{4}{ab}\frac{1}{N_p}\sum_{j=1}^{N_p}\int_0^a\int_0^b S(x-x_j)S(y-y_j)\sin(\alpha_l x)\sin(\beta_m y)dxdy$$

$$\phi(x,y) = 4\pi\frac{4}{ab}\frac{1}{N_p}\sum_{j=1}^{N_p}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\sin(\alpha_l x)\sin(\beta_m y)\int_0^a\int_0^b S(x-x_j)S(y-y_j)\sin(\alpha_l x)\sin(\beta_m y)dxdy$$

$$\phi(x_i,y_i) = \int_0^a\int_0^b \phi(x,y)S(x-x_i)S(y-y_i)dxdy$$

$$\varphi(x_i,y_i,x_j,y_j) = 4\pi\frac{4}{ab}\frac{1}{N_p}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\int_0^a\int_0^b S(x-x_j)S(y-y_j)\sin(\alpha_l x)\sin(\beta_m y)dxdy$$
$$\int_0^a\int_0^b S(x-x_i)S(y-y_i)\sin(\alpha_l x)\sin(\beta_m y)dxdy$$

Self-Consistent Space-Charge Transfer Map (3)

$$H_2 = 4\pi \frac{K}{2} \frac{4}{ab} \frac{1}{N_p} \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} \sum_{l=1}^{N_l} \sum_{m=1}^{N_m} \frac{1}{\gamma_{lm}^2}$$

$$\int_0^a \int_0^b S(x-x_j)S(y-y_j)\sin(\alpha_l x)\sin(\beta_m y)dxdy \int_0^a \int_0^b S(x-x_i)S(y-y_i)\sin(\alpha_l x)\sin(\beta_m y)dxdy$$

Symplectic Gridless Particle Model

$$\rho(x,y) = \sum_{j=1}^{N_p} w\delta(x-x_j)\delta(y-y_j)$$

w **is the particle weight factor**

$$H_2 = \frac{1}{2\epsilon_0}\frac{4}{ab}w\sum_i\sum_j\sum_l\sum_m\frac{1}{\gamma_{lm}^2}\sin(\alpha_l x_j)\sin(\beta_m y_j)\sin(\alpha_l x_i)\sin(\beta_m y_i)$$

$\mathcal{M}_2$

$$\boxed{\begin{aligned} p_{xi}(\tau) &= p_{xi}(0) - \tau\frac{1}{\epsilon_0}\frac{4}{ab}w\sum_j\sum_l\sum_m\frac{\alpha_l}{\gamma_{lm}^2} \\ &\quad \sin(\alpha_l x_j)\sin(\beta_m y_j)\cos(\alpha_l x_i)\sin(\beta_m y_i) \\ p_{yi}(\tau) &= p_{yi}(0) - \tau\frac{1}{\epsilon_0}\frac{4}{ab}w\sum_j\sum_l\sum_m\frac{\beta_m}{\gamma_{lm}^2} \\ &\quad \sin(\alpha_l x_j)\sin(\beta_m y_j)\sin(\alpha_l x_i)\cos(\beta_m y_i)\end{aligned}}$$

Symplectic PIC Model

$$\phi(x_I, y_J) = \frac{4}{ab}\sum_{l=1}^{N_l}\sum_{m=1}^{N_m}\frac{1}{\gamma_{lm}^2}\sum_{I'}\sum_{J'}\rho(x_{I'}, y_{J'})\sin(\alpha_l x_{I'})\sin(\beta_m y_{J'})\sin(\alpha_l x_I)\sin(\beta_m y_J)$$

$\mathcal{M}_2$

$$\boxed{\begin{aligned} p_{xi}(\tau) &= p_{xi}(0) - \tau 4\pi K \sum_I \sum_J \frac{\partial S(x_I - x_i)}{\partial x_i} S(y_J - y_i)\phi(x_I, y_J) \\ p_{yi}(\tau) &= p_{yi}(0) - \tau 4\pi K \sum_I \sum_J S(x_I - x_i)\frac{\partial S(y_J - y_i)}{\partial y_i}\phi(x_I, y_J) \end{aligned}}$$

$$S(x_I - x_i) = \frac{1}{h}\begin{cases} \frac{3}{4} - (\frac{x_i - x_I}{h})^2, & |x_i - x_I| \le h/2 \\ \frac{1}{2}(\frac{3}{2} - \frac{|x_i - x_I|}{h})^2, & h/2 < |x_i - x_I| \le 3/2h \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial S(x_I - x_i)}{\partial x_i} = \begin{cases} -2(\frac{x_i - x_I}{h})/h, & |x_i - x_I| \le h/2 \\ (-\frac{3}{2} + \frac{(x_i - x_I)}{h})/h, & h/2 < |x_i - x_I| \le 3/2h, \quad x_i > x_I \\ (\frac{3}{2} + \frac{(x_i - x_I)}{h})/h, & h/2 < |x_i - x_I| \le 3/2h, \quad x_i \le x_I \\ 0 & \text{otherwise} \end{cases}$$

Solution of Hamilton Equation in Transfer Map

$$\frac{d\zeta}{ds} = -[H,\zeta]$$

$$\zeta_s = f(\zeta_0) = \sum_i^N \mathrm{M}_i \zeta_0{}^i$$

- f can be a very complicated function
- M_i is the i^{th} order transfer map, and is related to the i^{th} derivative of function f

How to attain M_i effectively?

Consider a one-dimensional Taylor approximation:

$$f(x) = f(x_0) + (x - x_0) f'(x_0) + \frac{1}{2!}(x - x_0)^2 f''(x_0) + \frac{1}{3!}(x - x_0)^3 f'''(x_0) + \cdots + \frac{1}{N!}(x - x_0)^N f^{(N)}(x_0)$$

To find the derivative, i.e. Taylor map, one can approximate the derivative numerically:

$$f'(x_0) \approx \frac{f(x_0 + \varepsilon) - f(x_0)}{\varepsilon}$$

$$f''(x_0) \approx \frac{f(x_0 + \varepsilon) - 2f(x_0) + f(x_0 - \varepsilon)}{\varepsilon^2}$$

loss of accuracy

Introduction to Truncated Power Series Algebra (TPSA)

Use symbolic calculation from package like Mathematica:

For example: $f(x)=\dfrac{1}{1+x+x^2}$ $\quad f^{'}(x)=\dfrac{-(1+2x)}{(1+x+x^2)^2}$ $\quad f^{''}(x)=\dfrac{6x+6x^2}{(1+x+x^2)^3}$

- very complicated for high order derivatives
- even impossible for some function without closed form (e.g. simulation)

Define a N-dimension function space with bases:

$$\boxed{\{1,(x-x_0),\frac{1}{2!}(x-x_0)^2,\frac{1}{3!}(x-x_0)^3,\cdots,\frac{1}{N!}(x-x_0)^N\}}$$

The derivative up to Nth order can be regarded as a point in that space and represented as a vector:

$$\boxed{Df_{x_0}=[f(x_0),f^{'}(x_0),f^{''}(x_0),f^{'''}(x_0),\cdots,f^{(N)}(x_0)]}$$

For example, a constant c, its representation as $Dc=[c,0,0,0,\cdots,0]$

a variable x as, $Dx=[x,1,0,0,\cdots,0]$

$$\boxed{x\Rightarrow y=f(x)}$$

A point x in number space maps to another point y=f(x) in number space

$$\boxed{Dx\Rightarrow Df_x=f(Dx)}$$

A point Dx in DA vector space maps to another point Df_x in DA vector space

Basic Operations for the TPSA vector

➢ A complicated function can be broken down as the operations of **addition** and **multiplication**

❖ Rule of addition:

$$Df_{x_0} = [f(x_0), f'(x_0), f''(x_0), f'''(x_0), \cdots, f^{(N)}(x_0)] = [a_0, a_1, a_2, a_3, \cdots a_N]$$

$$Df_{x_1} = [f(x_1), f'(x_1), f''(x_1), f'''(x_1), \cdots, f^{(N)}(x_1)] = [b_0, b_1, b_2, b_3, \cdots b_N]$$

$$Df_{x_0} + Df_{x_1} = [f(x_0) + f(x_1), f'(x_0) + f'(x_1), f''(x_0) + f''(x_1), f'''(x_0) + f'''(x_1), \cdots, f^{(N)}(x_0) + f^{(N)}(x_1)]$$

$$Df_{x_0} + Df_{x_1} = [a_0 + b_0, a_1 + b_1, a_2 + b_2, a_3 + b_3, \cdots, a_N + b_N]$$

❖ Rule of multiplication:

$$Df_{x_0} \times Df_{x_1} = ?$$

$$Df_{x_0} \times Df_{x_1} \neq [f(x_0) \times f(x_1), f'(x_0) \times f'(x_1), f''(x_0) \times f''(x_1), f'''(x_0) \times f'''(x_1), \cdots, f^{(N)}(x_0) \times f^{(N)}(x_1)]$$

Basic Operations for the TPSA vector

❖ Rule of multiplication:

$$(g(x)\times h(x))' = g(x)h'(x) + g'(x)h(x)$$

$$(g(x)\times h(x))'' = g(x)h''(x) + 2g'(x)h'(x) + g''(x)h(x)$$

$$\cdots$$

$$(g(x)\times h(x))^{(N)} = \sum_{k=0}^{N} \frac{N!}{k!(N-k)!} g^{(k)}(x)h^{(N-k)}(x)$$

$$Df_{x_0} \times Df_{x_1} = [f(x_0)f(x_1), f(x_0)f'(x_1) + f'(x_0)f(x_1), f(x_0)f''(x_1) + 2f'(x_0)f'(x_1) + f''(x_0)f(x_1), \cdots]$$

$$Df_{x_0} \times Df_{x_1} = [a_0 b_0, a_0 b_1 + a_1 b_0, a_0 b_2 + 2a_1 b_1 + a_2 b_0, \cdots, c_N]$$

$$c_N = \sum_{k=0}^{N} \frac{N!}{k!(N-k)!} a_k b_{N-k}$$

- Operation of DA vector in a complicated function can be calculated using the rules of addition and multiplication

Example of Calculation of Derivatives Using TPSA

For example, inverse of DA vector $[a_0, a_1, a_2, a_3, \cdots a_N]^{-1} = [x_0, x_1, x_2, x_3, \cdots x_N]$

$$[a_0, a_1, a_2, a_3, \cdots a_N] \times [x_0, x_1, x_2, x_3, \cdots x_N] = [1,0,0,0,\cdots 0]$$

$$[a_0, a_1, a_2, a_3, \cdots a_N]^{-1} = [\frac{1}{a_0}, -\frac{a_1}{a_0^2}, \frac{2a_1^2}{a_0^3} - \frac{a_2}{a_0^2}, \cdots]$$

Another example: evaluate f'(1) and f''(1) for the following function:

$$f(x) = \frac{1}{1+x+x^2}$$

Analytical function method:

$$f'(x) = \frac{-(1+2x)}{(1+x+x^2)^2} \qquad f''(x) = \frac{6x+6x^2}{(1+x+x^2)^3}$$

$$f'(1) = -\frac{1}{3} \qquad f''(1) = \frac{4}{9}$$

TPSA method:

$$x = 1 \qquad D1 = [1,1,0]$$

$$Df_1 = f(D1) = \frac{1}{1+[1,1,0]+[1,1,0]^2} = \frac{1}{[1,0,0]+[1,1,0]+[1,2,2]} = \frac{1}{[3,3,2]} = [\frac{1}{3}, -\frac{3}{9}, \frac{18-6}{27}] = [\frac{1}{3}, -\frac{1}{3}, \frac{4}{9}]$$

Special Functions of TPSA Vector

- How about special functions such as sin(X), exp(X), log(X), etc

➢ Answer: use Taylor expansion:

$$f(x) = f(x_0) + (x - x_0) f^{'}(x_0) + \frac{1}{2!}(x - x_0)^2 f^{''}(x_0) + \frac{1}{3!}(x - x_0)^3 f^{'''}(x_0) + \cdots + \frac{1}{N!}(x - x_0)^N f^{(N)}(x_0)$$

$$X = [x_0, x_1, x_2, x_3, \cdots x_N] = [x_0, 0, 0, 0, \cdots 0] + [0, x_1, x_2, x_3, \cdots x_N]$$

$$([x_0, x_1, x_2, x_3, \cdots x_N] - [x_0, 0, 0, 0, \cdots 0])^m = [0, x_1, x_2, x_3, \cdots x_N]^m = [\underbrace{0, 0, 0, 0, \cdots 0}_{\text{leading } m \text{ zeros}}, ?, ?]$$

➢ This means *[0,x₁,x₂,x₃,…x_N]* raised to *(*N+1)$_{th}$ power is exactly zero in TPSA.

$$f(X) = f([x_0, x_1, x_2, x_3, \cdots x_N]) = f([x_0, 0, 0, 0, \cdots 0]) + \sum_{m=1}^{N} \frac{[0, x_1, x_2, x_3, \cdots x_N]^m f^{(m)}([0, x_1, x_2, x_3, \cdots x_N])}{m!}$$

Some Special Functions of TPSA Vector

$$
\begin{aligned}
e^{(a_0,a_1,a_2,\dots\, a_\Omega)} &= e^{a_0}\sum_{k=0}^{\Omega}\frac{1}{k!}(0,a_1,a_2,\dots\, a_\Omega)^k \\
\ln(a_0,a_1,a_2,\dots\, a_\Omega) &= (\ln a_0, 0, 0, 0\dots, 0) \\
&\quad + \sum_{k=1}^{\Omega}(-1)^{k+1}\frac{1}{k}(0,\frac{a_1}{a_0},\frac{a_2}{a_0},\dots,\frac{a_\Omega}{a_0})^k \\
\sqrt{(a_0,a_1,a_2,\dots\, a_\Omega)} &= \sqrt{a_0}\left[(1,0,0,0\dots 0)+\frac{1}{2}(0,\frac{a_1}{a_0},\frac{a_2}{a_0},\dots,\frac{a_\Omega}{a_0})\right. \\
&\quad \left. + \sum_{k=2}^{\Omega}(-1)^k\frac{(2k-3)!!}{(2k)!!}(0,\frac{a_1}{a_0},\frac{a_2}{a_0},\dots,\frac{a_\Omega}{a_0})^k\right] \\
\sin(a_0,a_1,a_2,\dots\, a_\Omega) &= \sin a_0\sum_{k=0}\frac{(-1)^k}{(2k)!}(0,a_1,a_2,\dots,a_\Omega)^{2k} \\
&\quad + \cos a_0\sum_{k=0}\frac{(-1)^k}{(2k+1)!}(0,a_1,a_2,\dots,a_\Omega)^{2k+1} \\
\cos(a_0,a_1,a_2,\dots\, a_\Omega) &= \cos a_0\sum_{k=0}\frac{(-1)^k}{(2k)!}(0,a_1,a_2,\dots,a_\Omega)^{2k} \\
&\quad - \sin a_0\sum_{k=0}\frac{(-1)^k}{(2k+1)!}(0,a_1,a_2,\dots,a_\Omega)^{2k+1}
\end{aligned}
$$

Ref: A. Chao,2010.

Multi-Variable TPSA/DA Package

- For a multi-variable function, one can define a similar TPSA/DA vector that contains 0^{th}, 1^{st}, 2^{nd} , …, N^{th} derivatives of the function
- The addition rule is still valid
- The multiplication rule has more complicated form
- TPSA/DA library packages were developed to handle the operations of DA vector
- A good TPSA/DA library should
  - accept arbitrary order and dimension
  - optimize the memory usage
  - optimize the calculation
  - has fast speed

Space-Charge Kick for a TPSA Particle

Define a TPSA particle (X, Px, Y, Py)

$$X(\tau/2) = X(0) + \frac{\tau}{2} Px(0)$$

$$Y(\tau/2) = Y(0) + \frac{\tau}{2} Py(0)$$

$$Px(\tau) = Px(0) - \tau K \sum_{l=1}^{L} \sum_{m=1}^{M} \phi^{lm} \alpha_l \cos(\alpha_l X) \sin(\beta_m Y)$$

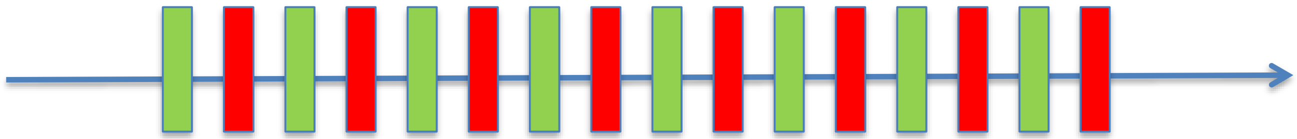
$$Py(\tau) = Py(0) - \tau K \sum_{l=1}^{L} \sum_{m=1}^{M} \phi^{lm} \beta_m \sin(\alpha_l X) \cos(\beta_m Y)$$

$$X(\tau) = X(\tau/2) + \frac{\tau}{2} Px(\tau)$$

$$Y(\tau) = Y(\tau/2) + \frac{\tau}{2} Py(\tau)$$

where ϕ is the self-consistent space-charge potential from the multi-particle tracking simulation

An Illustrative Example



- 1 GeV proton beam
- Each turn contains 10 FODO lattice
- 0 current tunes (1.895, 2.452)
- Initial 4D Gaussian distribution
- 40mA

An Illustrative Example: One-Turn Transfer Map Including Space-Charge Effects

X

Order 0 :

0	0	0	0	2.601664386891709E-006

Order 1 :

1	0	0	0	-9.108190796569636E-002
0	1	0	0	-0.536293186565298
0	0	1	0	9.796542703230136E-003
0	0	0	1	3.674801772135906E-003

Order 2 :

2	0	0	0	-29.0754583613708
1	1	0	0	20.9156435118371
1	0	1	0	-39.8073175869492
1	0	0	1	-6.49867594749186
0	2	0	0	-4.02520351183274
0	1	1	0	17.0989556914644
0	1	0	1	1.88215553577798
0	0	2	0	4.69419419442297
0	0	1	1	2.65108049848306
0	0	0	2	0.923545456871374

Order 3 :

3	0	0	0	4671274.96668114
2	1	0	0	-6863330.61159331
2	0	1	0	-39455.0543527188
2	0	0	1	-14968.9416320131
1	2	0	0	3795062.85249869
1	1	1	0	30506.6883046430
1	1	0	1	8615.71444431781

Y

Order 0 :

0	0	0	0	-2.234110257806358E-006

Order 1 :

1	0	0	0	-1.293829543281796E-003
0	1	0	0	1.251785206450719E-003
0	0	1	0	1.74042218321237
0	0	0	1	0.735319968989055

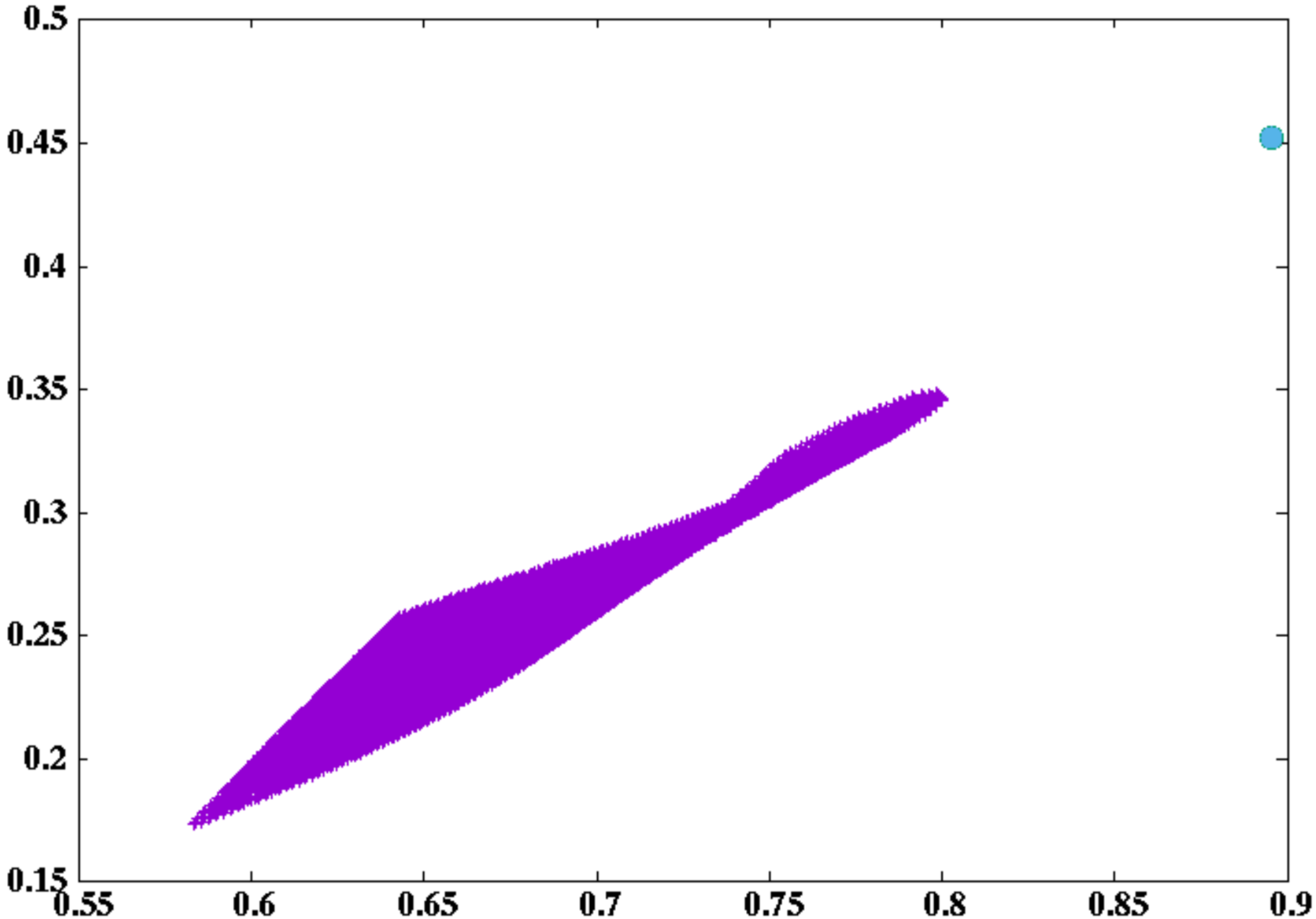
Order 2 :

2	0	0	0	15.4274814200973
1	1	0	0	-15.6451906136942
1	0	1	0	-3.60954422551933
1	0	0	1	2.12752799255609
0	2	0	0	4.90906282911728
0	1	1	0	2.19440121436049
0	1	0	1	-2.09233253697833
0	0	2	0	6.11980971336471
0	0	1	1	-8.28258251619088
0	0	0	2	-4.64265451263100

Order 3 :

3	0	0	0	-7342.06826387678
2	1	0	0	5983.61446885548
2	0	1	0	-316937.047797246
2	0	0	1	544481.036607568
1	2	0	0	253.327413657724
1	1	1	0	211371.302318552
1	1	0	1	-560190.236859571
1	0	2	0	-4461.08962278009

Amplitude-Dependent Tune-Shift from the One-Turn Map



Summary

- Symplectic integration in long-term space-charge simulation can be achieved by using gridless or PIC symplectic space-charge integrator.

- Self-consistent space-charge effects can be included and extracted in Taylor map by integrating a DA particle the multi-particle tracking code.

- Useful information such as amplitude dependent tune shift and resonance strengths can be obtained based on the one-turn Taylor map.

- Space-charge compensation could be done using the Taylor map that includes the self-consistent space-charge effects.

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Thank You for Your Attention!