Developments in Incoherent vs. Coherent Resonances
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Overview

- Intro: Classification of transverse resonances with space charge
- 90 degree stopband as a „test-bed“
- Higher order coherent resonance effects – role of Landau damping?
- Comparison with SIS18 experiment
- Conclusions

Acknowledgment: O. Boine-Frankenheim, G. Franchetti, A. Oeftiger
Collective – coherent – incoherent
in beams with space charge

Any beam in an external potential with space charge behaves collectively!

Collective effect
- may be on amplitudes of particles (re-arrangement) such that self-field counteracts external field – in synchotrons „weak“
  - example: „Debye shielding“, or „profile flattening“
  - influencing incoherent resonances
- may be on phases of particles: coherent effects (modes)
  - coherent resonances
  - instabilities
  - shifted frequencies compared with single particles

FSM*: Frozen Space Charge Model

FSM*: „good enough“

FSM*: fails
Coherent modes

- Coherent impedance driven instabilities
  - rigid dipolar motion

- Quadrupolar deformations

- Higher (3\textsuperscript{rd} or 4\textsuperscript{th}) order coherent modes
  - driven by resonant effects
  - not by impedances!
  - do they exist?

this talk
Overview: 3 resonance situations

- **Incoherent**
  \[ l v_x + m v_y = n \]

- **Coherent**
  \[ l v_x + m v_y + \Delta \omega_{coh,l,m} = n \]
  weak coherence

- **Coherent parametric**
  \[ l v_x + m v_y + \Delta \omega_{coh,l,m} = n/2 \]
  „strong“ coherence

**Resonant excitation**
- driven by magnets and/or self-fields
- single particle „incoherent“ resonances
- also „coherent resonances“?
- 2nd order example in rings:
  - gradient error resonance – coherent.

**Parametric resonance**
- driven by self-field only
- instability with exponential growth
- example in linacs: envelope instability – not measured - avoided
Simulation in **linear** FODO lattice: short ellipsoidal 3d bunch showing the two types of resonance in the 90 degree stopband

\[ 2k_x + \Delta k_{coh,2} = \frac{360^0}{2} \]

\[ 4k_x = 360^0 \]

\[ k_{0x,y} = 120^0 \]
\[ k_{xy} = 73^0 \]

6d Gauss/3\(\sigma\)

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I.Hofmann, O. Boine-Frankenheim, PRAB 2017
Scan over full regime of resonant response
- use $90^0$ band as „test bed“ for higher order and externally driven -

- Need an **integrated** picture over all regimes
  - where incoherent, where coherent? transitions etc.
  - compare with experimental procedures in SIS18 – CERN

- This talk: Elongated 3d Gaussian bunch on $90^0$ stopband
  - periodic linear focusing (straight FODO + RF lattice)
  - $T_{\text{synch}} \approx 3T_{\beta}$
  - fully self-consistent Poisson solver (TRACEWIN)
  - use a (mechanical) aperture at $7\sigma$ to „model“ halo loss regimes
Early (300 cells) response on rms emittance 6d $G$ on $90^0$ stopband - $T_{syn}=3T_{betatron}$

$4k_{xy} \approx 360^0$

$2k_{xy} + C_2\Delta k_{xy} = 360^0/2$

coh. frequency on resonance

k$_{xy}$/rms on resonance

$k_{0xy}$

k$_{xy}$/rms on coherent fr.

coherent shift $C_1=1/2$

Gaussian 6d

Waterbag 6d
Longer term loss case (6d G - $T_{syn}=3 \times T_{betatron}$)

mechanical aperture at 7σ (4.5 mm)

SIS 18 - sextupolar bunched beam – theory FSM

FSM:
- coherent effects absent!
- emittance peaks not coherent, but long-term incoherent

coherent shift $C_2=1/2$
Tail incoherent resonance regime
by periodic crossing of 90° resonance during synchrotron motion


$k_{0x,y} = 92^0$
$k_{xy} = 80^0$
$k_z = 29^0$

5000 cell simulation (3D)
no aperture!

$k_{x,y} = 92^0$
$k_{xy} = 80^0$
$k_z = 29^0$

(6d $G \sim T_{syn} \sim 3xT_{betatron}$)

It can be assumed that it
- is an entirely incoherent resonance
- scattering by multiple kicks
- „frozen-in“ sp.ch. initially ok
- requires self-consistent treatment if high deviations from initial

Selfconsistent effect
not only a shift!

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Resonant particles: maximum synchr. amplitudes
→ multiple kicks (G. Franchetti et al., PRSTAB 2010)
Loss curve with assumed mechanical aperture at $7\sigma$

loss on an aperture at 4.5 mm: (approximately through fixed points)
Higher order stopbands? – (6d Gaussian – $T_{\text{syn}} \sim 3xT_{\text{betatron}}$)

Test with slow **tune ramp**:  
- $k_{0xy}: 142 \rightarrow 135$  
- $k_{xy}: 136 \rightarrow 132$

**Incoherent:**  
$$8 \ k_{xy} = 3 \cdot 360^0$$  
$$8 \ Q_{xy} = m \cdot N$$

**Coherent – „theoretical“:**  
$$4k_{xy} + \Delta k_{coh,4} = 3 \cdot 360^0 / 2$$  
$$4Q_{xy} + \Delta Q_{coh,4} = mN / 2$$

Incoherent resonance driven by 3rd harmonic of basic FODO cell

Compare with CERN PS: $8 \ Q_{xy} = 50$

Loss from resonance (too fast scan)

Theoretically expected (by theory ignoring LD)  
Half-integer parametric resonance
Detailed scan of this 8\textsuperscript{th} order stopband (6d G – T\textsubscript{syn} ~ 3xT\textsubscript{betatron})

\[ 8 \ k_{xy} = 3 \cdot 360^0 \]

\[ \rightarrow k_{xy} = 135^0 \]

incoherent rms tune on resonance!

✓ No indication of coherent parametric resonance!
  – should show during first 300 cells (exponential growth!)
✓ rms emittance growth entirely incoherent

\[ \text{„theoretically“:} \quad 4 \ k_{xy} + \Delta k_{coh,4} = 3 \cdot 360^0 / 2 \]

assumed again 7 σ mechanical aperture
Coherent resonant frequencies and Landau damping

\[ \omega = k(v_x + C_k\Delta v_x) \]

- only second order mode outside spectrum
  - found unstable in 90 deg stopband
- 3rd and 4th within spectrum of G and WB
  - consistent with absence in bunch simulations
  - in 2d found for WB!
Confirmed $3^{\text{rd}}$ and $4^{\text{th}}$ order parametric resonances in WB (!) coasting beam

- $60\%$ rms emittance growth in WB
  
  $3(\nu_x + C_3 \Delta \nu_x) = 1/2$

- $10\%$ rms emittance growth in WB
  
  $4(\nu_x + C_4 \Delta \nu_x) = 1/2$

Not found for Gaussian
- $< 1\%$ emittance growth
Recent suggestion by Hiroshima-group (2019): Employ *only coherent* resonance conditions in resonance diagrams (Kojima, Okamoto et al. / S-POD experiment)

\[
k(v_{0x} - C_m \Delta v_x) + l (v_{0y} - C_m \Delta v_y) = n
\]

\[
k(v_{0x} - C_m \Delta v_x) + l (v_{0y} - C_m \Delta v_y) = n' / 2
\]

\(n' / 2\): for even \(n'\) parametric instability would nearly double # of lines

**FIG. 4.** Tune diagram obtained from 2D WARP simulations with a fixed beam intensity and fixed rms emittances at injection. The
SIS18 high intensity measurements:

\[ \Delta \nu_{x/y} = -0.04/-0.045 \]

- No indication of coherent shifts
- Dominated by long-term emittance growth and loss
- If any: measurement peaks should be more to the left

\[ 3(\nu_x + \xi \Delta \nu_x) = 13 \]

- Quantitative loss with "frozen" space charge underestimates long-term loss
  - Could be due to lack of self-consistency

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G. Franchetti et al., PR-STAB 13 (2010)
Conclusions

✔ Distinguish incoherent – coherent – parametric
  – latter would increase # of resonance lines – no evidence > 2\textsuperscript{nd} order
    (except for WB – coasting beam)

✔ 90 degree „test-bed“ describing 3 regimes:
  – core + coherent (half-integer) + tail

✔ 135 degree stopband – free of 4\textsuperscript{th} order coherent half-integer!!
  – rms emittance growth only from 8\textsuperscript{th} order

✔ Coherent part Landau damped for Gaussian and higher order

✔ No evidence of „coherent“ in SIS18 experiments (+ CERN)

✔ Future work should carefully consider LD and possibly loss of LD