Emittance blow-up studies at the transfer from the PSB to the PS

M.A. Fraser, F. Antoniou, V. Forte, A. Guerrero, A. Huschauer, A. Oeftiger, S. Ogur, F. Roncarolo, F.M. Velotti
Outline

• Introduction
  • *A quick aside on systematic errors*

• Turn-by-turn measurements vs. simulation
  • Dispersion, chromaticity, beam size

• Emittance blow-up estimations
  • Analytic and tracking results
  • Measurements of filamented beam

• Conclusions
Introduction

- A **larger than expected** emittance blow-up is measured by comparing wire-scanner beam profiles measured in the PSB and PS
- Our understanding is that the dispersion mismatch should be the dominant source of blow-up:
  
  ...**but we are plagued by systematic errors**
- Relative beam size measurements in PS only saw a very small effect of re-matching the transfer line:
  - Operational (OP) transfer line optics was compared to a rematched (ReM) optics
  - Turn-by-turn beam profile measurements carried out to quantify the level of mismatch
Systematic errors

• Assuming a systematic error on beam size of ±100 μm:
  • Error of ±15% on $\varepsilon_x$ in PSB
  • Error of ±10% on $\varepsilon_x$ in PS

• In addition, we also have to consider other sources errors, for example:
  • Measured momentum spread (used to deconvolute)
  • Treatment of profile (fitting), e.g. typically Gaussian distributions are assumed
  • Measured optics functions
  • Even statistical - lots of shots needed f(intensity)

\[
\begin{array}{c|c|c|c|c|c}
\Delta \varepsilon_{x,n} & \Delta \varepsilon_{x,n} / \varepsilon_{x,n} \text{ [%]} \\
\hline
-20 & -10 & 0 & 10 & 20 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c}
\Delta \sigma_{x,PSB} \text{ [μm]} \\
\hline
-132 & -65 & 0 & 63 & 123 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c|c|c|c}
\text{RMS beam size in PSB (σ}_{x,PSB} \text{ [mm]} \\
1.80 & 1.86 & 1.93 & 1.99 & 2.05 \\
\hline
\end{array}
\]

\[
\begin{array}{c|c|c|c|c}
\text{RMS beam size in PS (σ}_{x} \text{ at PR.BWSH65 [mm]} \\
3.9 & 4.0 & 4.1 & 4.2 & 4.3 \\
\hline
\end{array}
\]

(1.4 GeV OP BCMS beam parameters)

See extra slides for similar plots with LIU beam parameters
Emittance blow-up

- No known source can explain such a large blow-up: systematic errors between different WS devices are important especially with the large dispersion component.

Detailed studies in 2018 showed ~25% transverse emittance in the horizontal plane.
- We expect ~10% from dispersion mismatch.

See F. Antoniou et al., at LIU Workshop 2019 [https://indico.cern.ch/event/774181](https://indico.cern.ch/event/774181)

M.A. Fraser, 4th ICFA Mini-Workshop on Space Charge, 4 - 6 November 2019
Turn-by-turn MD conditions

- Beam injected on BCMS cycle with low chroma, coupling corrected:
  - $Q_x \sim 6.21$, $Q_y \sim 6.23$
  - **RF OFF** and TFB ON
  - Single bunch from Ring 3 with $50 - 90 \times 10^{10}$ ppb
  - **Results for $\sim 65 \times 10^{10}$ ppp presented**
- Transfer line optics (**ReM**) designed and tested by V. Forte:
  - *See HB2018 paper for details*
  - **Significantly improved matching on Ring 3** used in PPM MD operation
- Beam profiles measured turn-by-turn using a wire-grid in SS52:
  - 30 turns acquired before beam removed by kickers (to protect grid)
- Limited beam time because inserting the BSG blocks the complex:
  - A couple of hours on the last day of dedicated proton MD
Turn-by-turn measurements

BSGH52:
Central 12 wires: 1 mm pitch
External 20 wires: 2.5 mm pitch

Multiple (redundant) kicker systems timed to remove the beam
D measurements (BSGH52) (1)

- Closed dispersion at BSGH52 differs from the model by 10%:

Model $D_x = 2.35 \text{ m}$, measurements $D_x = 2.60 \text{ m (OP)}$ and $2.55 \text{ m (ReM)}$

See extra slides for dispersion vs. model at injection for all BPM’s
D measurements (BSGH52) (2)

\[ \bar{D}(n) = \bar{D}_0(n) + M_D \cos(\theta + 2\pi(n - 1)q_s) \]

Amplitude of mismatch

Normalisation made with model \( \beta_x = 12.04 \) m, measurements (P. Skowronski) \( \beta_x = 12.02 - 12.07 \) m

OP transfer line optics

ReM transfer line optics
Non-linear component (BSGH52)

\[ \bar{D}(n) = \bar{D}_0(n) + \bar{D}_1(n)\delta_p \]

\[ \bar{D}_1, x = D_1, x / \sqrt{\beta_x} \text{ [m}^{1/2}] \]

OP transfer line optics

ReM transfer line optics
Dispersion mismatch summary

- Identical behaviour observed on all ring BPM’s:

\[
\bar{D}(n) = \bar{D}_0(n) + M_D \cos(\theta + 2\pi(n - 1)q_x)
\]

See extra slides for example on BPM00
Tracking simulations

- Latest optics model of machine:
  - acc-models by A. Huschauer
  - https://gitlab.cern.ch/ahuschau/acc-models

- Maps computed using MADX-PTC and exported from tracking using python:
  - maptrack by F. Velotti https://gitlab.cern.ch/fvelotti/maptrack
  - Used and tested extensively for SX studies

- Conditions: no space-charge, injection bump on but static (no collapse), 5D (RF off as in measurements)

- Transverse distribution is Gaussian and longitudinal distribution is parabolic:
  \[ \propto (1 - \Delta p^2) \text{ where } \Delta p_{\text{max}} = \sqrt{5}\Delta p_{\text{rms}} \]
Dispersion: OP optics vs model

- Relative mismatch fitted in simulation by adjusting the unknown $D'$:

\[ M_D = 0.350 \]

$D_{x,0} = 2.633$

$D'_{x,0} = -0.104$
Dispersion: ReM optics vs model

- Relative mismatch fitted in simulation by adjusting the unknown $D'$:

\[ M_D = 0.107 \]

\[ D_{x,0} = 2.683 \]
\[ D'_{x,0} = -0.021 \]
Chromaticity (BPM data)

- During first 30 turns $Q'$ measured x2 larger than expected:
  - FFT (Hanning window) vs. simple harmonic fit gives same result
  - Measurements using Q-meter a few ms later $Q'_x \sim 0.8$ and $Q'_y \sim -2.9$

Horizontal:
- $q_x = 0.216$
- $Q'_x = 2.01$

Vertical:
- $Q'_y = -5.29$
- $q_y = 0.2326$
H chromaticity vs. model

- During first 30 turns $Q'$ measured $x2$ larger than expected:
  - Simulating the measurement gives the nominal chromaticity $Q'_{x} \sim 0.8$

![Measurement graph](image1)

- $Q'_{x} = 2.01$
- $q_{x} = 0.216$

![Simulation graph](image2)

- $Q'_{x} = 0.77$
- $q_{x} = 0.218$
V chromaticity vs. model

- During first 30 turns $Q'$ measured **x2 larger** than expected:
  - Simulating the measurement gives the nominal chromaticity $Q'_y \sim -2.9$

Measurement:

\[
Q'_y = -5.29
\]

\[
q_y = 0.232
\]

Simulation:

\[
Q'_y = -2.84
\]

\[
q_y = 0.235
\]
Beam profile measurements - H (1)

- Fitting with a 5-parameter Gaussian with linear baseline:

\[
\frac{\sigma_x^2(n)}{\beta_x} = \frac{\sigma_B^2(n)}{\beta_x} + \frac{\sigma_D^2(n)}{\beta_x} + \frac{\sigma_{BSG}(n)}{\beta_x}
\]

Betatronic component

Dispersive component

blow-up from wire grid

See extra slides for V plane measurements
Beam profile measurements - H (1)

- Fitting with a 5-parameter Gaussian with linear baseline:
  
  \[
  \frac{\sigma_x^2(n)}{\beta_x} = \frac{\varepsilon_x}{2} \left( \left(M_g + \frac{1}{M_g}\right) + \left(M_g - \frac{1}{M_g}\right) \cos(\phi + 4\pi(n-1)q_x) \right) + \left(\frac{\sigma_p}{p} \overline{D_{0,x}} + M_{D,x} \cos(\theta + 2\pi(n-1)q_x) \right) + \frac{\sigma_{BSG}^2}{\beta_x}
  \]

  - Betatronic mismatch: \( M_g > 1 \)
  - Dispersion mismatch: \( M_D \neq 0 \)
  - Envelope oscillating at 2\( q_x \)
  - Envelope oscillation at \( q_x \) for \( M_D \ll 1 \)
  - Blow-up from interaction with grid:

  \[
  \frac{\sigma_{BSG}^2}{\beta_x} = \beta_x \Delta x_{\text{rms}}^2 \left( \frac{n-1/2}{2} - \frac{\sin(2\pi(2n-1)q_x)}{4\sin(2\pi q_x)} \right)
  \]

- Gaussian works well as a quick and easy fitting method:
  - Aim here is not to deconvolute, yet… it’s really tricky
  - Blow-up from wire grid

See extra slides for V plane measurements
Beam profile measurements - H (2)

- Beam size computed from profiles at BSGH52 by fitting a 5-parameter Gaussian (with linear baseline)

Don’t forget that the grid is scattering and blowing up the beam!

M.A. Fraser, 4th ICFA Mini-Workshop on Space Charge, 4 - 6 November 2019
Oscillation frequency of beam size is **clearly depressed** and depends on the mismatch

<table>
<thead>
<tr>
<th>Free parameters</th>
<th>OP optics</th>
<th>ReM optics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_x$</td>
<td>0.188</td>
<td>0.204</td>
</tr>
<tr>
<td>$\varepsilon$ [mm mrad]</td>
<td>0.69</td>
<td>0.74</td>
</tr>
<tr>
<td>$\sigma_p/p [10^{-3}]$</td>
<td>1.09</td>
<td>1.09</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.38</td>
<td>-0.29</td>
</tr>
<tr>
<td>$M_g$</td>
<td>1.09</td>
<td>1.05</td>
</tr>
<tr>
<td>$\beta_x \Delta x_{\text{rms}}^2 [\mu m]$</td>
<td>0.027</td>
<td>0.038</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Constrained parameters</th>
<th>OP optics</th>
<th>ReM optics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_D$</td>
<td>0.38</td>
<td>0.12</td>
</tr>
<tr>
<td>$\bar{D}_0 [m^{1/2}]$</td>
<td>0.72</td>
<td>0.74</td>
</tr>
<tr>
<td>$\theta$</td>
<td>1.27</td>
<td>0.59</td>
</tr>
</tbody>
</table>
Beam size simulations

- Gaussian fitting plays a role where D is large (parabolic distribution)
Beam size simulations

- Gaussian fitting plays a role in systematic errors where \( D \) is large (parabolic distribution):

\[
\sigma_x^2/\beta_x \quad [\mu m]
\]

Turn 5
12% error

Turn 2
4% error

OP transfer line optics

\[ \Delta p/p = 0.9 \times 10^{-3} \]
Beam size simulations vs model

• Detuning large and obvious:
  • One can simply count the different number of oscillations!

Simulation:

Measurement:

Scattering from grid not included in simulation
Beam size oscillation frequency

- FFT used to compute beam size oscillation frequency:
  - Envelope oscillation depressed from $q_x$
  - Dispersion dominated mismatch: *effect of optics rematch is clear*
  - Frequency depends on the amount of mismatch
  - Limited resolution from number of turns: parabolic interpolation of the spectral peaks employed
  - Higher frequency mode also appears depressed from $2q_x$

![Graph showing FFT results](image)

Measurement: $(q_x = 0.216)$

**Hanning window function:**

- OP: $f_\sigma = 0.19$
- ReM: $f_\sigma = 0.205$

See extra slides for explanation of error propagation (extracted from FFT on analytic model)
Beam size oscillation freq. vs. model

- As expected, no detuning in simulation without space-charge:

Measurement:
\( q_x = 0.216 \)
Hanning window function:

Simulation:
\( q_x = 0.216 \)
Hanning window function:

See extra slides for explanation of error propagation (extracted from FFT on analytic model)
Beam size oscillation freq. vs. model

- Beam size oscillation frequency depends on momentum:
  - Hanning window applied to simulation and measurements
  - Larger than can be explained by chromaticity (but errors in FFT play a significant role)

Measurement:

![Graph showing the beam size oscillation frequency as a function of momentum offset.

Simulation:

![Graph showing the simulation of beam size oscillation frequency as a function of momentum offset.

See extra slides for explanation of error propagation (extracted from FFT on analytic model)
Emittance blow-up estimates

- Blow-up from dispersion mismatch can be written analytically as:
  \[ \Delta \varepsilon = \frac{M_D^2}{2} \left( \frac{\sigma_p}{p} \right)^2 (\beta \gamma)_{\text{rel}} \sim 0.11 \text{ mm mrad} \]

- From dispersion mismatch alone we would expect a factor 10 difference in the measured blow-up:
  \[ \frac{\Delta \varepsilon_{\text{OP}}}{\Delta \varepsilon_{\text{Rem}}} = \left( \frac{M_{D,\text{OP}}}{M_{D,\text{Rem}}} \right)^2 \sim 10 \]

- Analytic estimates have been confirmed with tracking studies (independent of distribution using RMS)

See extra slides for comparison to analytic estimates and associated errors introduced by assuming Gaussian distributions
Tracking studies: $E_\parallel = 0.9$ eVs

RMS horizontal beam size:

- **Comments:**
  - Beam is still filamenting at 2000 turns
  - $\Delta \varepsilon/\varepsilon \sim 11\%$ corresponds to 0.1 mm in the beam size
  - Challenging to measure!
  - Analytic results verified

$\Delta p/p = 0.9 \times 10^{-3}$

$\sqrt{\langle x^2 \rangle} = \sqrt{\frac{\langle (x-\langle x \rangle)^2 \rangle}{x_{\text{rms}}}}$

$\frac{\Delta x_{\text{rms}}}{x_{\text{rms}}} \sim \text{few } \%$

$\frac{\Delta \varepsilon_{\text{OP}}}{\Delta \varepsilon_{\text{ReM}}} \sim 10 \checkmark$
Tracking studies: $E_l = 0.9$ eVs

- Horizontal profiles compared at turn 2000:
  - Such small differences in RMS ($\sqrt{\langle x^2 \rangle}$) will be very hard to discern using the wire-scanner
  - Fitting a Gaussian function ($\sigma$) reduces the difference

---

**OP optics:**

- $\Delta p/p = 0.9 \times 10^{-3}$

<table>
<thead>
<tr>
<th>Frequency [mm]</th>
<th>$x$ [mm]</th>
</tr>
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<tbody>
<tr>
<td>120</td>
<td>-15</td>
</tr>
<tr>
<td>100</td>
<td>-10</td>
</tr>
<tr>
<td>80</td>
<td>-5</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
</tr>
<tr>
<td>40</td>
<td>5</td>
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<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>15</td>
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**ReM optics**

- $\Delta p/p = 0.9 \times 10^{-3}$

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<td>10</td>
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<tr>
<td>0</td>
<td>15</td>
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</table>

Turn 2000

Binning 100 $\mu$m
Emittance measurements

- Prototype LIU WS used and transfer line optics rematched:

- Large dataset (> 500 shots) observed a small beam size reduction of ~30 um at 65 × 10^{10} ppp

- Details of analysis now being checked by BI for fit quality / tail population
Conclusions

• Non-linear behaviour of PS is well characterised by the MADX-PTC model:
  • “filamentation” of dispersion well described
• Chromaticity measured a factor 2 higher than expected during first 30 turns
• Significant detuning of beam size (envelope) oscillation frequency observed:
  • Shift of -0.026 for OP transfer line optics
  • Frequency is a function of injection mismatch
Next steps…

• Comparison with simulations including space-charge in PyORBIT:
  • H. Rafique to implement dispersion mismatch
• **We cannot rely on beam size measurements** after filamentation for…
  1. absolute measurements of betatronic emittance
  2. injection matching optimization
• Transfer line matching will be significantly improved with LIU upgrade:
  • However, we will need to develop an optimizer working to **minimise the turn-by-turn beating** after injection
Extra slides
Systematic errors (1.4 GeV, 1.5 eVs)

\[ \Delta \varepsilon_{x,n}/\varepsilon_{x,n} \text{ [%]} \]

\[ \Delta \sigma_{x,PSB} \text{ [\mu m]} \]

RMS beam size in PSB (\( \sigma_{x,PSB} \) [mm])

\( E_k = 1.4 \text{ GeV} \) (\( \varepsilon_l = 1.5 \text{ eVs} \))

PR.BWSH65: \( \beta_x = 22.1 \), \( D_x = 3.1 \text{ m} \)

PSB WS: \( \beta_x = 5.6 \), \( D_x = -1.3 \text{ m} \)
Systematic errors (LIU STD)

\[
\frac{\Delta \varepsilon_{x,n}}{\varepsilon_{x,n}} [\%] \\
-11 \quad -6 \quad 0 \quad 6 \quad 11
\]

\[
\Delta \sigma_{x,PSB} [\mu m] \\
-73 \quad -36 \quad 0 \quad 36 \quad 71
\]

RMS beam size in PSB ($\sigma_{x,PSB}$) [mm]

\[
2.56 \quad 2.59 \quad 2.63 \quad 2.66 \quad 2.70
\]

\[
E_k = 2.0 \text{ GeV} \quad (\varepsilon_l = 3.0 \text{ eVs})
\]

RMS beam size in PS at PR.BWSH65 [mm]

PR.BWSH65: $\beta_x = 22.1$, $D_x = 3.1$ m
PSB WS: $\beta_x = 5.6$, $D_x = -1.3$ m
Systematic errors (LIU BCMS)

\[ \Delta \varepsilon_x, n / \varepsilon_x, n \% \]

\[ \Delta \sigma_{x, PSB} [\mu m] \]

RMS beam size in PSB (\( \sigma_{x, PSB} \)) [mm]

2.04 2.08 2.13 2.17 2.22

\( E_k = 2.0 \text{ GeV} \) (\( \varepsilon_I = 1.48 \text{ eVs} \))

PR.BWSH65: \( \beta_x = 22.1 \), \( D_x = 3.1 \text{ m} \)
PSB WS: \( \beta_x = 5.6 \), \( D_x = -1.3 \text{ m} \)
D in first turns (BPM) (1)
D vs. model in first turns (BPM) (2)
D measurements (BPM)

OP transfer line optics

ReM transfer line optics
Beam profile measurements - V

- Fitting with a 5-parameter Gaussian with linear baseline:

\[
\frac{\sigma_y^2(n)}{\beta_y} = \frac{\varepsilon_y}{2} \left( \left( M_g + \frac{1}{M_g} \right) + \left( M_g - \frac{1}{M_g} \right) \cos(\phi + 4\pi(n - 1)q_x) \right) + \frac{\sigma_{BSG}^2(n)}{\beta_y}
\]

- Care to be taken with fit parameters:
  - Only a few useful turns
  - Fit only made on later turns!

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<td>(q_x)</td>
<td>0.23</td>
<td>0.204</td>
</tr>
<tr>
<td>(\varepsilon) [mm mrad]</td>
<td>0.62</td>
<td>0.58</td>
</tr>
<tr>
<td>(\phi)</td>
<td>4.65</td>
<td>-0.48</td>
</tr>
<tr>
<td>(M_g)</td>
<td>1.02</td>
<td>1.00</td>
</tr>
<tr>
<td>(\beta_y) (\Delta y'^2_{rms}) [(\mu m)]</td>
<td>0.019</td>
<td>0.020</td>
</tr>
</tbody>
</table>
Dispersion beating after injection

• If D-mismatch is small, \( M_D/D_0 << 1 \):
  • Envelope will oscillate with \( q_x \):

\[
\frac{D_0^{-2}}{} \left( 1 + \frac{M_D}{D_0} \cos(\theta + 2\pi(n-1)q_x) \right)^2 \approx \frac{D_0^{-2}}{} + 2\frac{D_0}{D_0}M_D \cos(\theta + 2\pi(n-1)q_x) + O(M_D^2)
\]

• If D-mismatch is large, \( M_D/D_0 \sim 1 \):
  • Envelope will oscillate with a component at \( 2q_x \):

\[
O(M_D^2) = M_D^2 \cos^2(\theta + 2\pi(n-1)q_x) = \frac{M_D^2}{2} (1 + \cos2(\theta + 2\pi(n-1)q_x))
\]
Benchmarking maptrack

- Maps exported from PTC at 5th order and a few particles tracked for 300 turns:
Benchmarking maptrack

- Turn-by-turn dispersion to 5\textsuperscript{th} order:

See extra slides for convergence
Resolution of DFT – no window

- Analytic expression for T-by-T beam size to check DFT:
  - Sample the analytic function every turn, for different numbers of turns, perform DFT without windowing
  - python numpy implementation using fft.rfft() and parabolic interpolation of the spectrum

- \( f_\sigma \) is accurate to better than \( \pm 5\% \) for 29 turns
- Spectral leakage seen:
  - Non-periodic sampling
  - Resolution improves with number of samples
  - Little benefit in resolution above 30 turns
Resolution of FFT - Hanning

- Analytic expression for T-by-T beam size to check DFT:
  - Sample the analytic function every turn, for different numbers of turns, perform DFT without windowing
  - Python numpy implementation using fft.rfft() and parabolic interpolation of the spectrum

- With Hanning window $f_\sigma$ is accurate to better than $\pm 1.5\%$ for 29 turns
- Spectral leakage seen:
  - Non-periodic sampling
  - Resolution improves with number of samples
  - Little benefit in resolution above 30 turns
Comparison to analytic results (1)

- Dispersion mismatch (OP transfer line optics):

\[ \Delta \varepsilon = \frac{M_D^2}{2} \left( \frac{\Delta \rho}{\rho} \right)^2 \]

\[ M_D = 0.350 \]
Comparison to analytic results (2)

- Injection mis-steering (optics matched):

\[ \Delta \varepsilon = \frac{\Delta x^2 + (\alpha \Delta x)^2}{2\beta} \]

\[ \frac{\Delta p}{p} = 0.9 \times 10^{-3} \]
Comparison to analytic results (3)

- **Betatronic mismatch:**
  \[
  \varepsilon = \varepsilon_0 \frac{1}{2} \left[ \frac{\beta}{\beta_0} + \frac{\beta_0}{\beta} + \beta \beta_0 \alpha_0^2 \left( \frac{1}{\beta} - \frac{1}{\beta_0} \right)^2 \right]
  \]

\[\Delta \frac{p}{p} = 0.9 \times 10^{-3}\]
Tracking studies: $E_I = 0.9$ eVs

Average beam position:

- **Comments:**
  - Average beam position oscillates as filamentation ensues, (see skewness)
  - Even matched optics is perturbed: asymmetry of non-linear dispersion
Tracking studies: $E_l = 0.9$ eVs

Skewness:

- Comments:
  - Skewness indicator of filamentation
  - Asymmetric non-linear dispersion induces asymmetry across beam
  - Driving oscillation of average position
Tracking studies: $E_1 = 1.5$ eVs

RMS horizontal beam size:

- Comments:
  - RMS beam size is still shaking at 2000 turns
  - Analytic results verified

\[
\Delta p/p = 1.4 \times 10^{-3}
\]

\[
\sqrt{\langle x^2 \rangle} = \sqrt{\langle (x - \langle x \rangle)^2 \rangle} \text{ (rms)}
\]

\[
\frac{\Delta x_{\text{rms}}}{x_{\text{rms}}} \sim \text{few %}
\]

\[
\frac{\Delta \varepsilon_{\text{OP}}}{\Delta \varepsilon_{\text{ReM}}} \sim 10 \checkmark
\]
Tracking studies: $E_\parallel = 1.5$ eVs

RMS horizontal beam size:

- Comments:
  - Check made to 4000 turns...
  - Jitter of $\sim 80$ um on the beam size remains
  - Due to non-linear dispersion

\[
\frac{\Delta \varepsilon}{\varepsilon} \text{OP} \approx 10
\]

\[
\frac{\Delta x_{\text{rms}}}{x_{\text{rms}}} \sim \text{few } \%
\]

\[
\frac{\Delta \varepsilon_{\text{OP}}}{\Delta \varepsilon_{\text{ReM}}} \approx 10
\]
Tracking studies: $E_T = 1.5 \text{ eVs}$

**Skewness:**

- **Comments:**
  - Skewness indicator of filamentation
  - Asymmetric non-linear dispersion induces asymmetry across beam
  - Driving oscillation of average position
Tracking studies: $E_1 = 1.5 \text{ eVs}$

- No discernable difference between the two transfer line optics:
  - The effect of the tails generated during filamentation has a significant effect when fitted with a Gaussian function ($\sigma$)
  - No surprise that beam size measurements were inconclusive
Tracking studies: $E_\parallel = 1.5$ eVs

- No discernable difference between the two transfer line optics:
  - The effect of the tails generated during filamentation has a significant effect when fitted with a Gaussian function ($\sigma$)
  - No surprise that beam size measurements were inconclusive
Tracking studies: matched case

- Fitting a Gaussian to the beam profile over-estimates the RMS beam size:
- Exaggerated by the larger momentum spread:

\[ \Delta p/p = 0.9 \times 10^{-3} \]

\[ \Delta p/p = 1.4 \times 10^{-3} \]

0.9 eVs

1.5 eVs

Turn 2000
Binning 100 μm
Tracking studies: vertical plane

RMS vertical beam size:

\[ \Delta p/p = 0.9 \times 10^{-3} \]

\[ \Delta p/p = 1.4 \times 10^{-3} \]

\[ \sqrt{\langle (y - \langle y \rangle)^2 \rangle_{\text{rms}}} \]

- OP: \( \Delta \epsilon/\epsilon = 0.0\% \)
- ReM: \( \Delta \epsilon/\epsilon = 0.0\% \)
- M: \( \Delta \epsilon/\epsilon = 0.0\% \)
A quick aside… emittance

- Emittance was computed in two ways as the beam size is still beating after 2000 turns (at 1 – 2 %):
  1. **Average RMS beam size taken in the last 10 turns**
     - Emittance computed using the closed dispersion and betatron functions removed in quadrature:
       \[ \varepsilon = \sqrt{\langle x^2 \rangle_{10 \text{ turns}} - \left( \frac{D \Delta p}{p} \right)^2 / \beta} \]
     - **Mathematically correct: no assumptions made on distribution**
  2. **Last turn statistical emittance:**
     \[ \varepsilon = \sqrt{\langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2} \]
     - **Dispersion computed from the particle distribution itself (statistical) and removed particle-by-particle**
A quick aside… convergence

- Both approaches are suitable and analytic results verified:

![Graph showing difference from analytic result in percentage against number of particles. The graph includes two lines: one blue and one red, with labels on the lines indicating different methods of calculation. The y-axis represents the difference from the analytic result in percentage, while the x-axis shows the number of particles. The graph indicates an analytic blow-up estimate of 11.3%. The blue line is labeled 'From particle distribution on turn = 2000', and the red line is labeled 'From last 10-turn average of $\sqrt{\langle x^2 \rangle}$'. The graph also highlights $N_p = 20000$.]

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