



# Beyond the Standard Model physics From Lagrangians to events

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**LPTHE / Sorbonne Université**

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# Outline

1. A comprehensive approach for Monte Carlo simulations
2. Implementing models into Monte Carlo event generators
3. From models to hard-scattering events
4. Summary

# Monte Carlo simulations for new physics

## ◆ Path towards the characterisation of new physics

- ❖ Fitting and interpreting deviations
- ❖ Predictions of associated signatures/signals
  - Monte Carlo simulations play a key role

## ◆ Final words on any potential new physics at the LHC

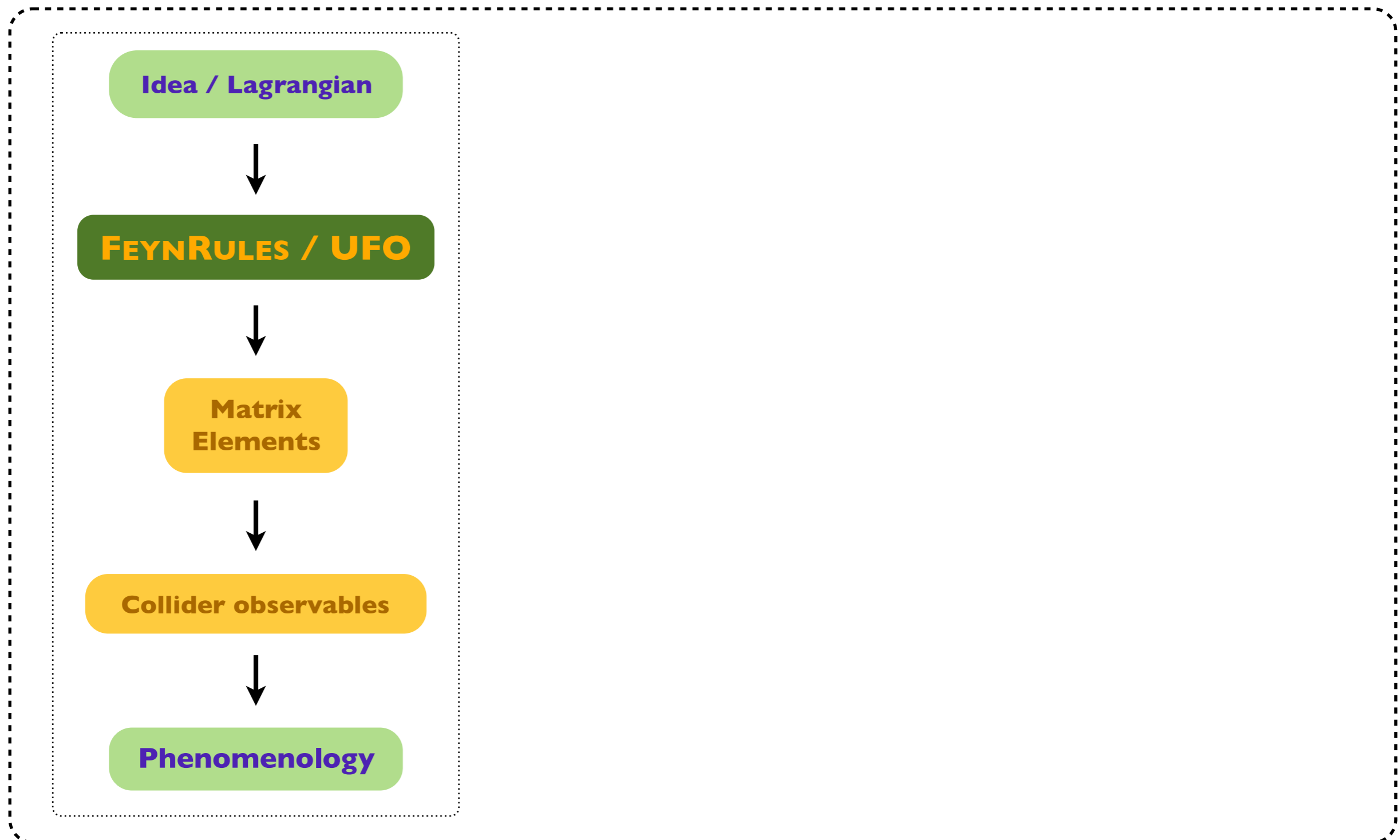
- ❖ Accurate measurements  $\oplus$  precision predictions (NLO QCD + PS)
  - Monte Carlo simulations play a key role

## ◆ New physics is standard in the simulation tools

- ❖ 20-25 years of developments → LO simulations are bread and butter
- ❖ Simulations at the NLO accuracy in QCD can be easily achieved
  - ★ For any model/process → the MADGRAPH5\_aMC@NLO framework

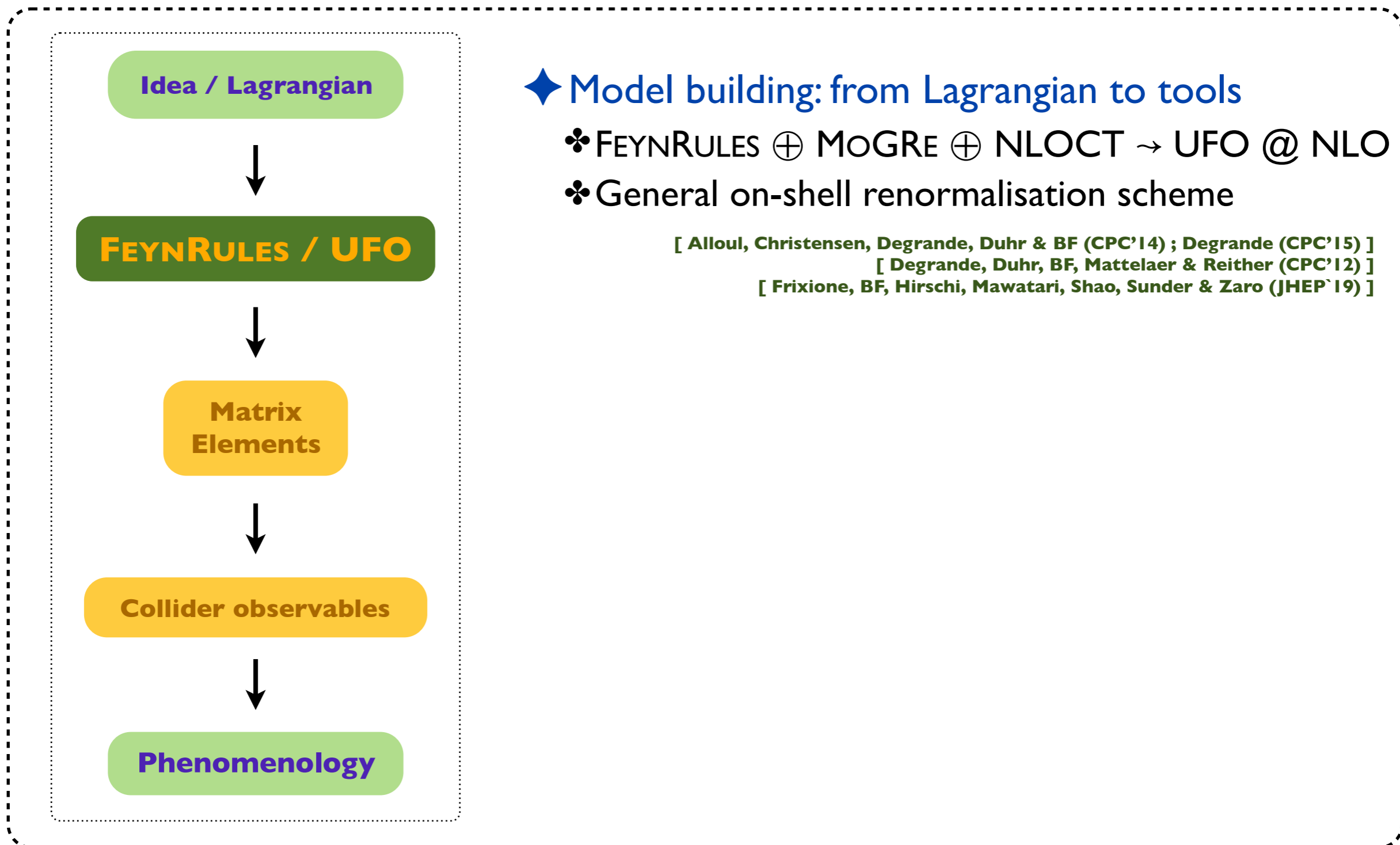
# A comprehensive approach to new physics calculations

[ Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC 11) ]



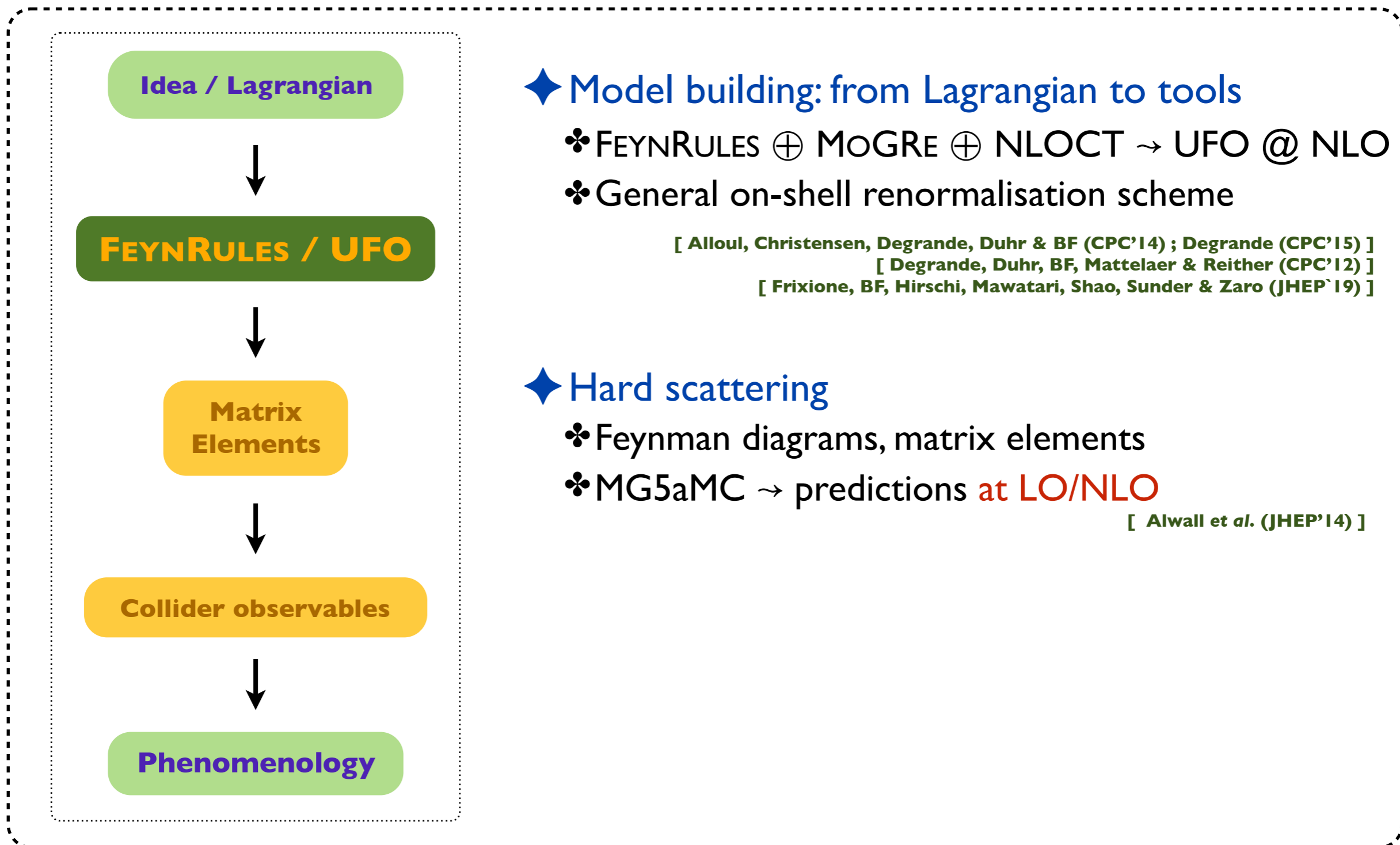
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## ◆ Model building: from Lagrangian to tools

- ♣ FEYNRULES  $\oplus$  MoGRE  $\oplus$  NLOCT  $\rightarrow$  UFO @ NLO
- ♣ General on-shell renormalisation scheme

[ Alloul, Christensen, Degrande, Duhr & BF (CPC'14) ; Degrande (CPC'15) ]  
 [ Degrande, Duhr, BF, Mattelaer & Reither (CPC'12) ]  
 [ Frixione, BF, Hirschi, Mawatari, Shao, Sunder & Zaro (JHEP'19) ]

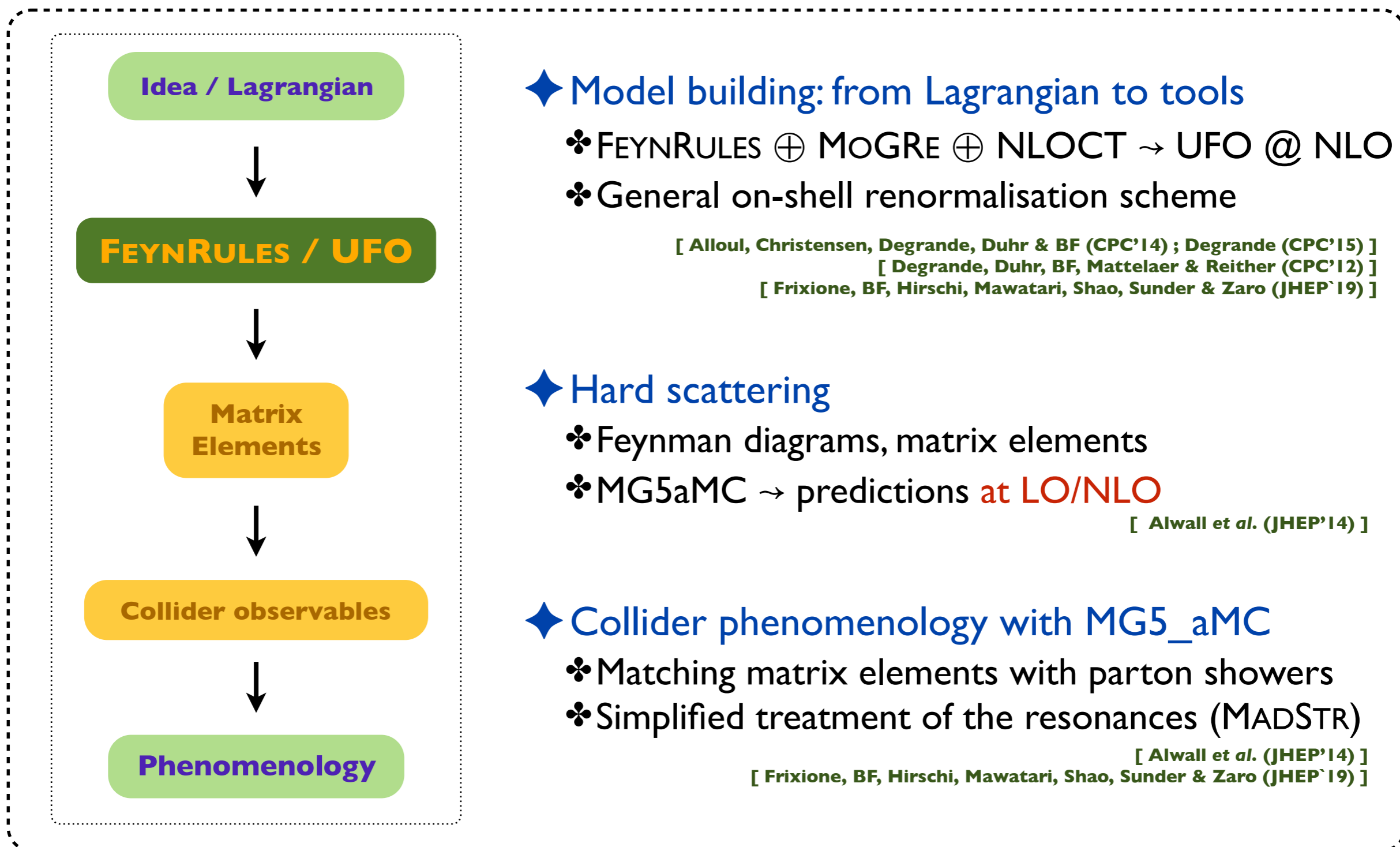
## ◆ Hard scattering

- ♣ Feynman diagrams, matrix elements
- ♣ MG5aMC  $\rightarrow$  predictions at **LO/NLO**

[ Alwall et al. (JHEP'14) ]

# A comprehensive approach to new physics calculations

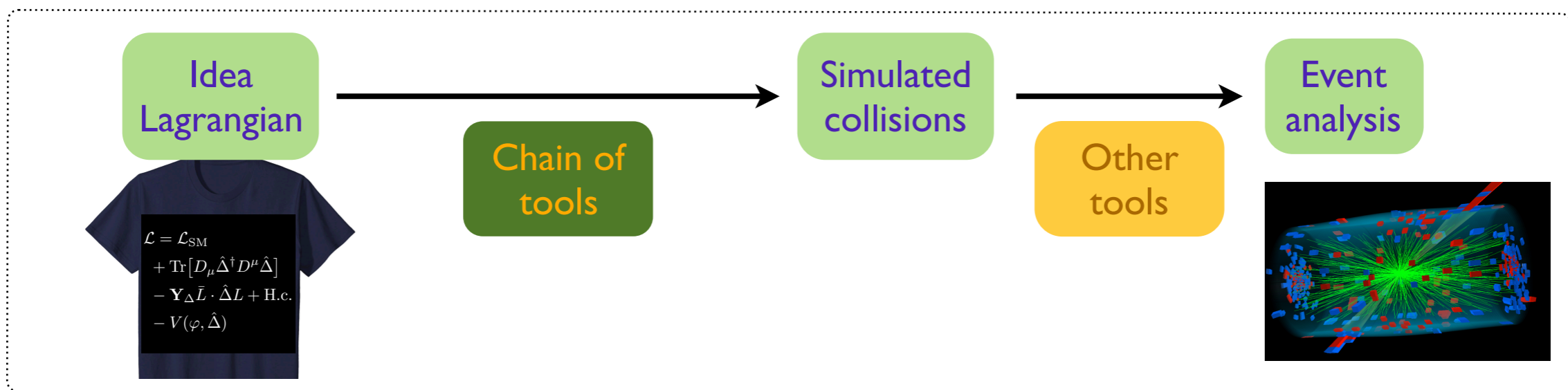
[ Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC'11) ]



# From Lagrangians to events

## ◆ Streamlining the connection of a physics models to events

- ♣ **Any** new physics model can be implemented
- ♣ Easy to **validate** and **maintain**

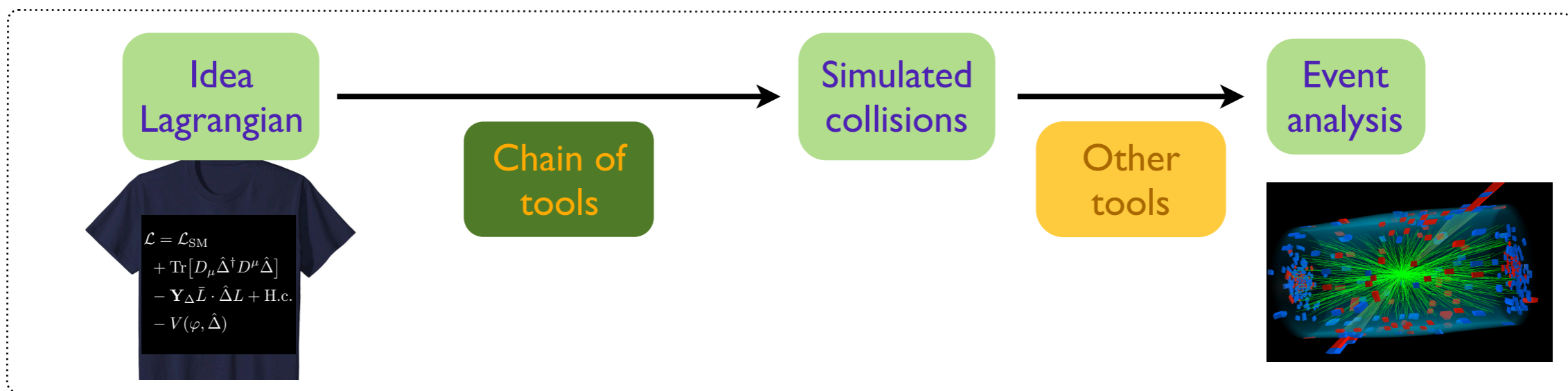




# From Lagrangians to events

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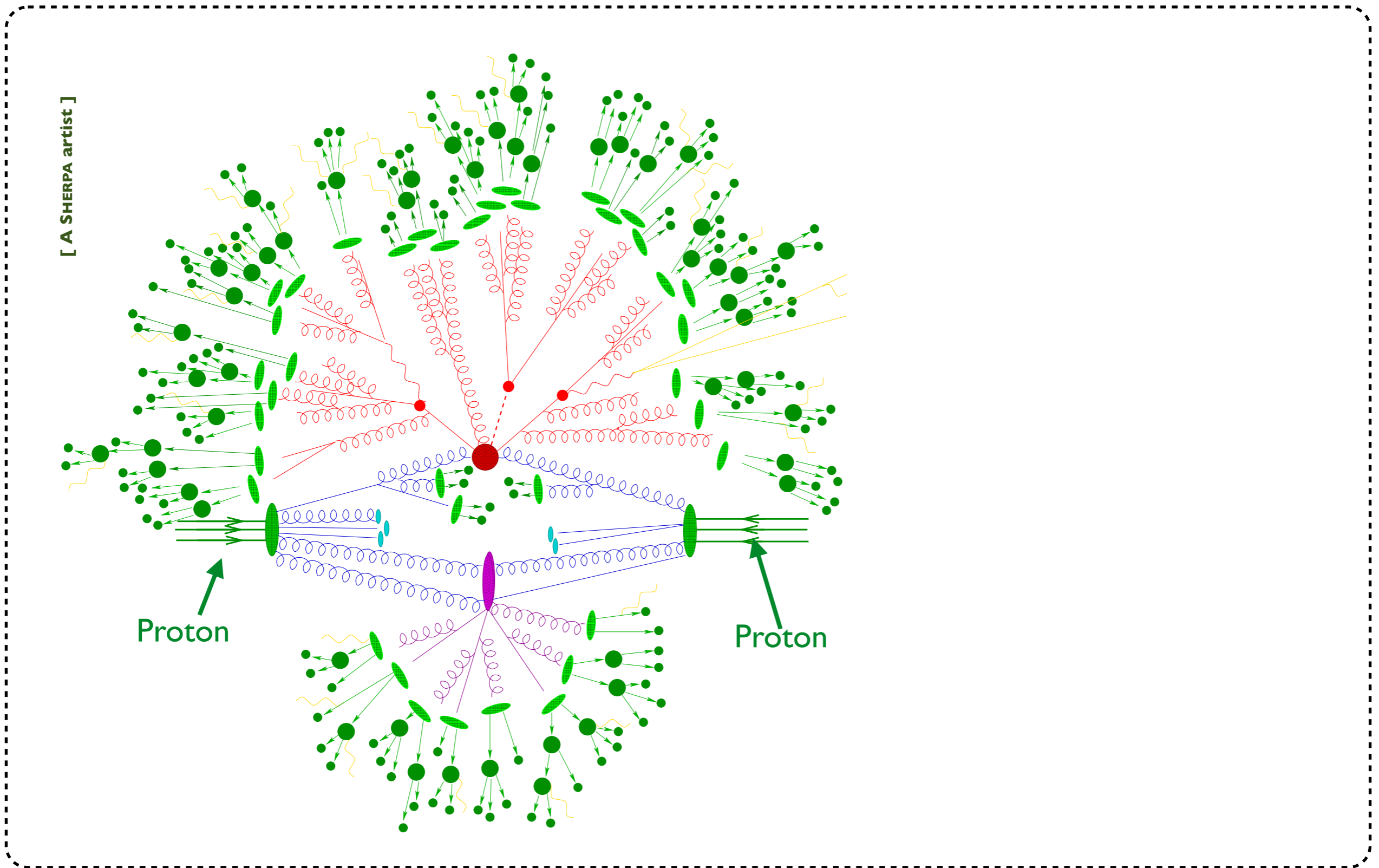
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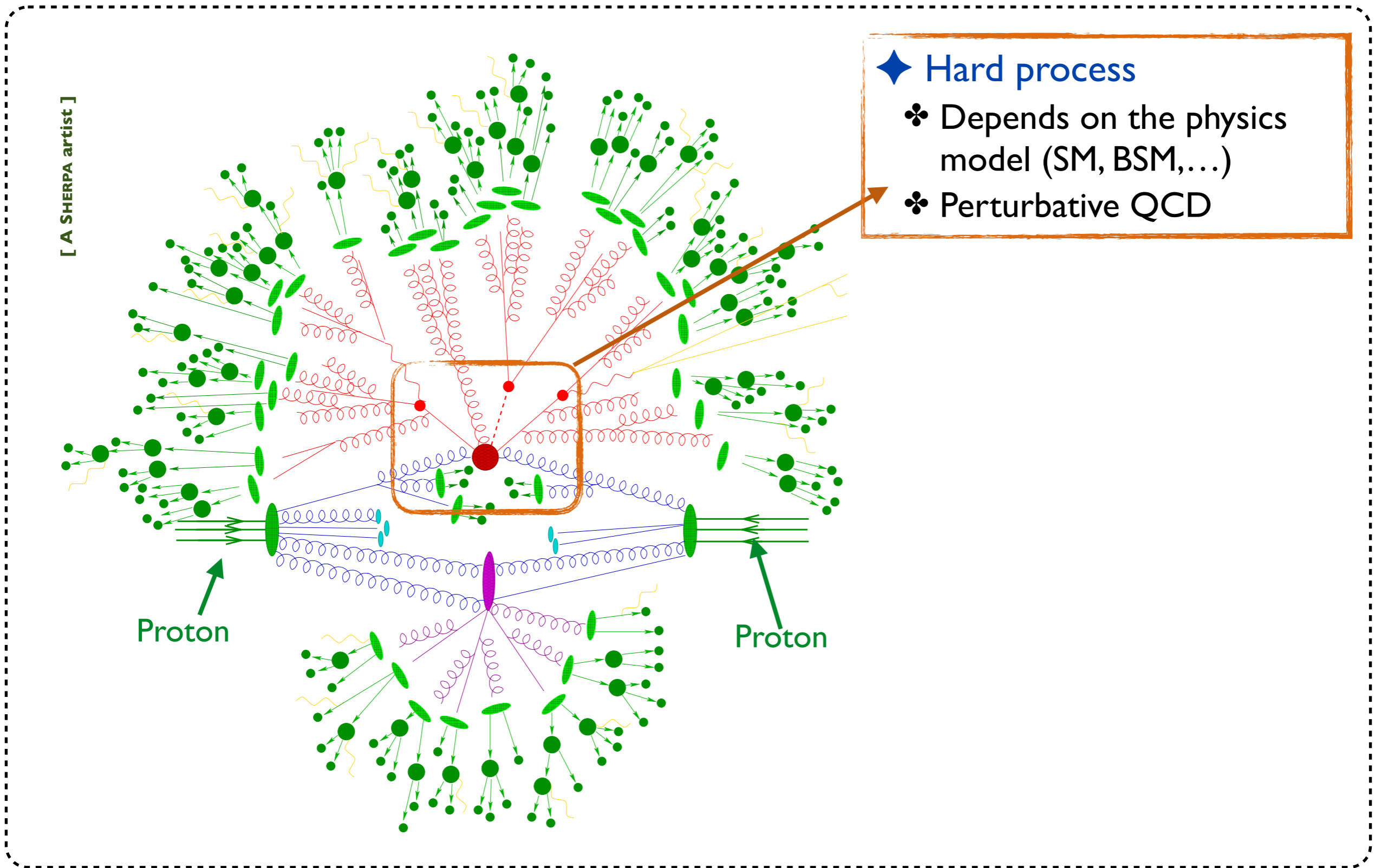
## ◆ Why a chain of several tools?

- ♣ Phenomena at colliders occur at different scales  $\rightarrow$  factorisation

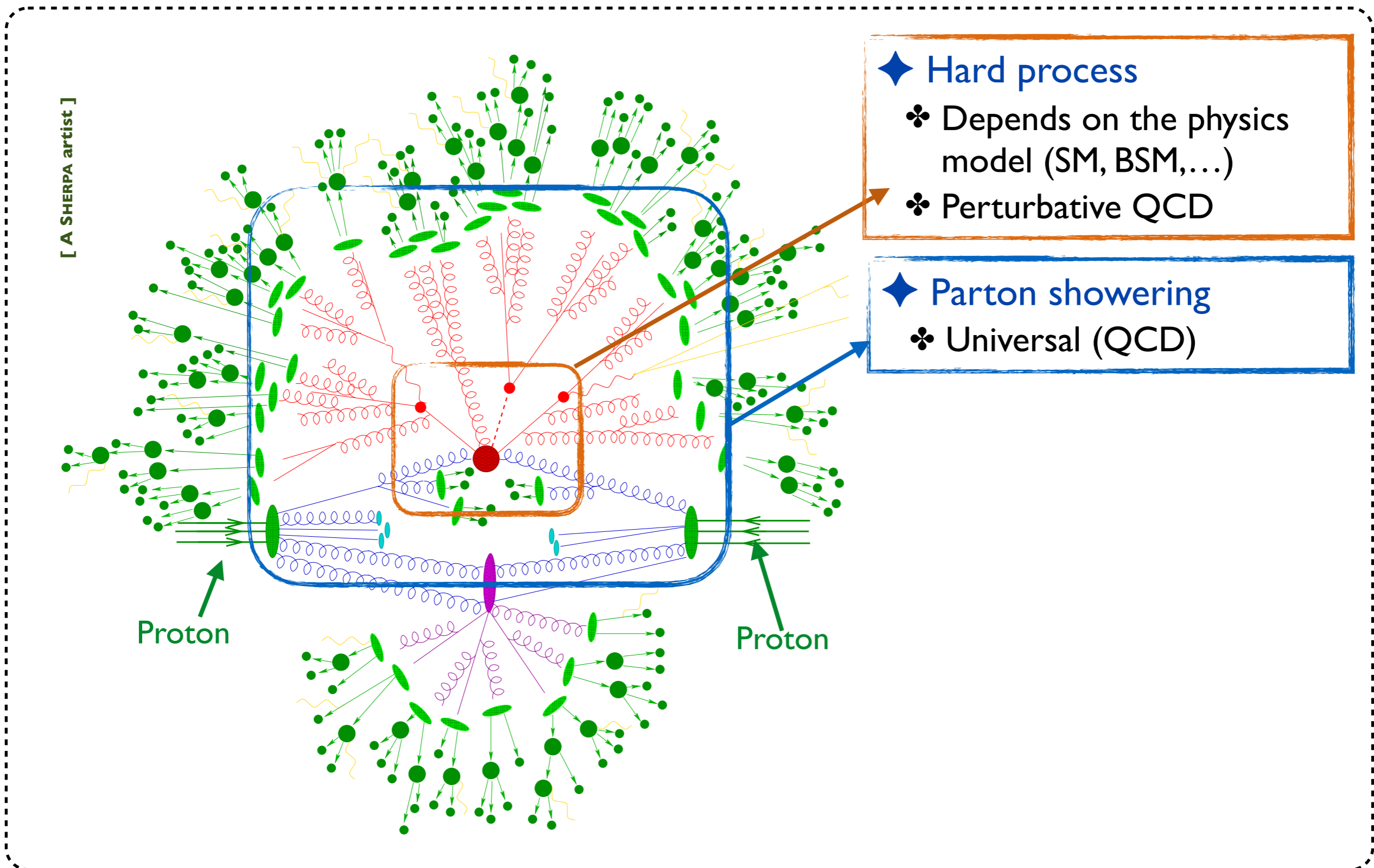
# Deciphering a proton-proton collision



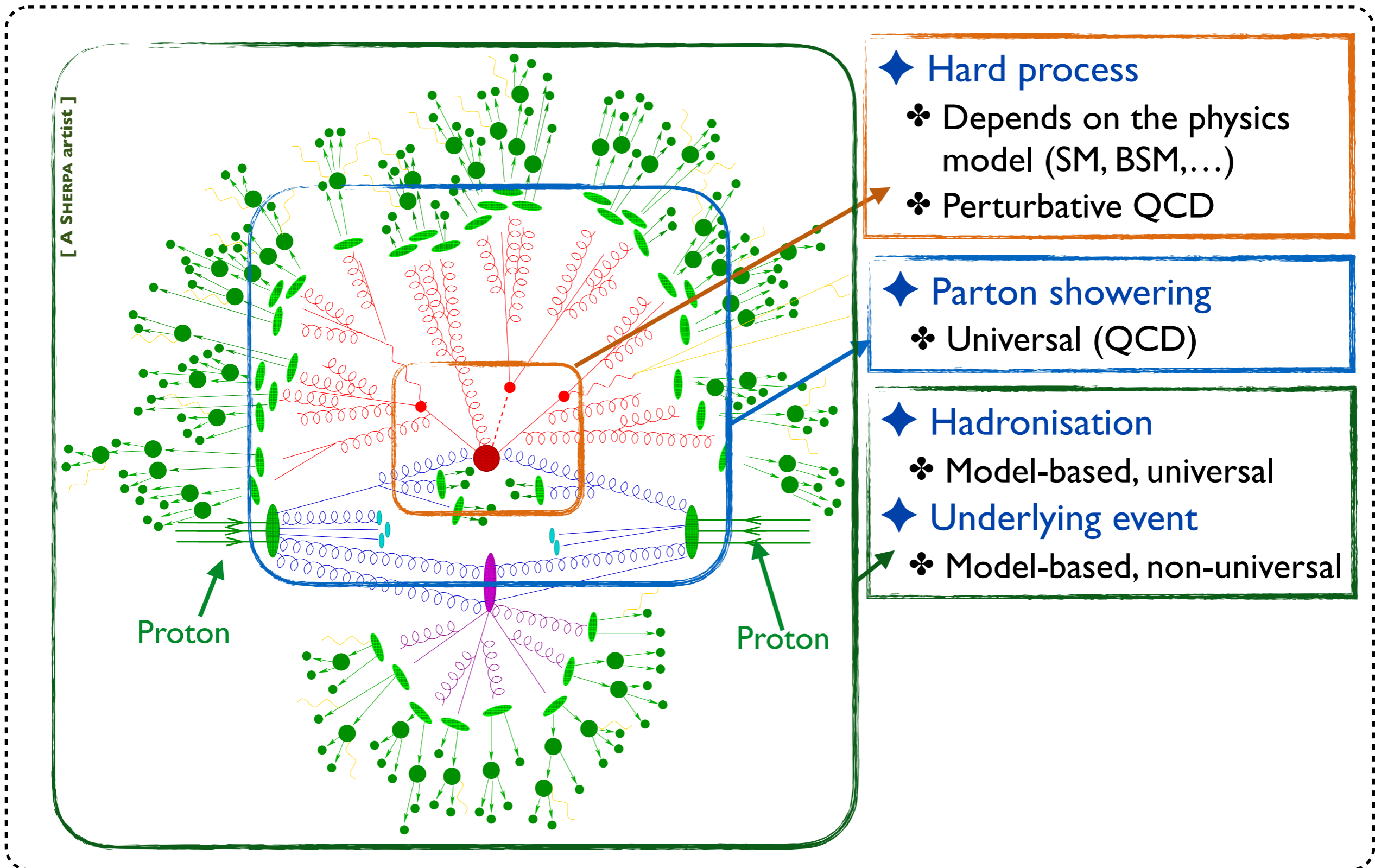
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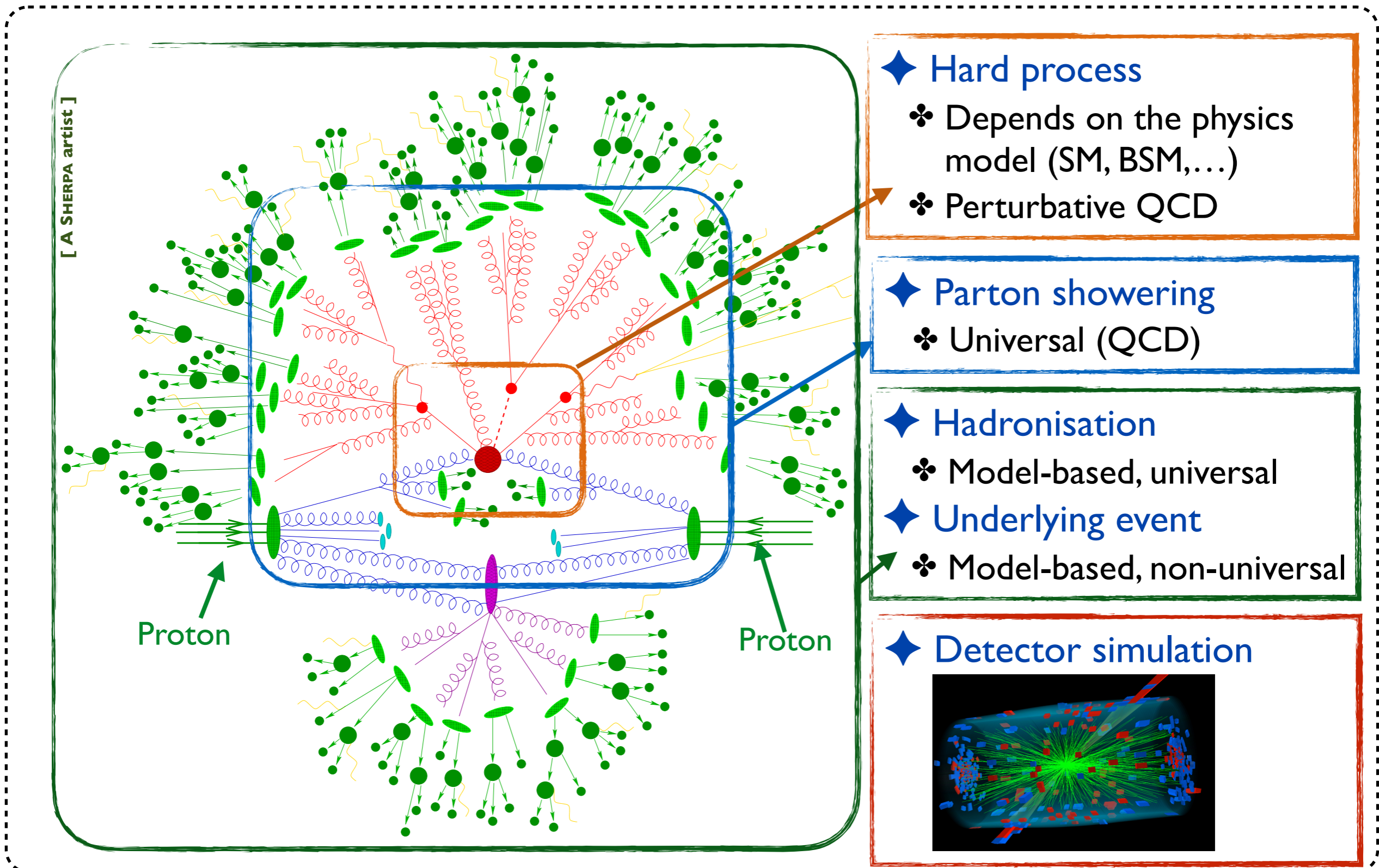
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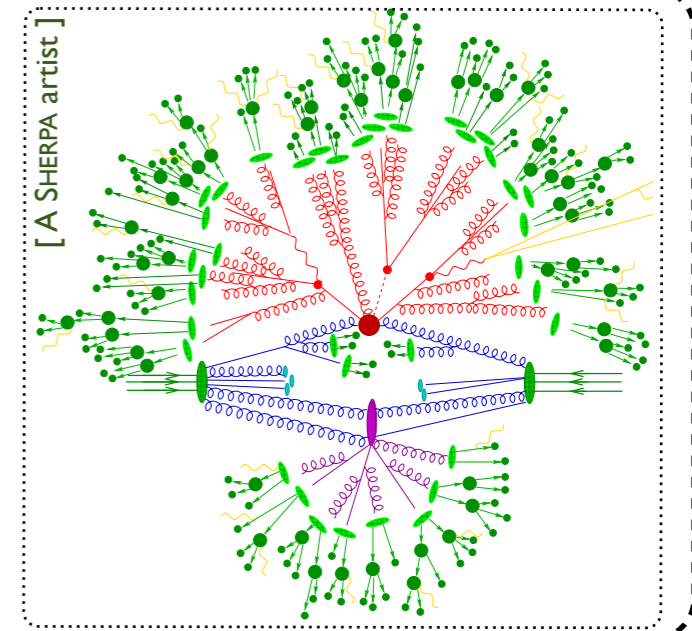


# Monte Carlo simulations for proton collisions

## ◆ Multi-scale problem $\rightarrow$ factorisation

- ♣ TeV scale: hard scattering (**new physics?**)
- ♣ Down to  $\Lambda_{\text{QCD}}$ : QCD environment Talk by L. Gellersen
- ♣ Down to sub-MeV: interactions with a detector Talk by BF

**Tools and methods for each step**

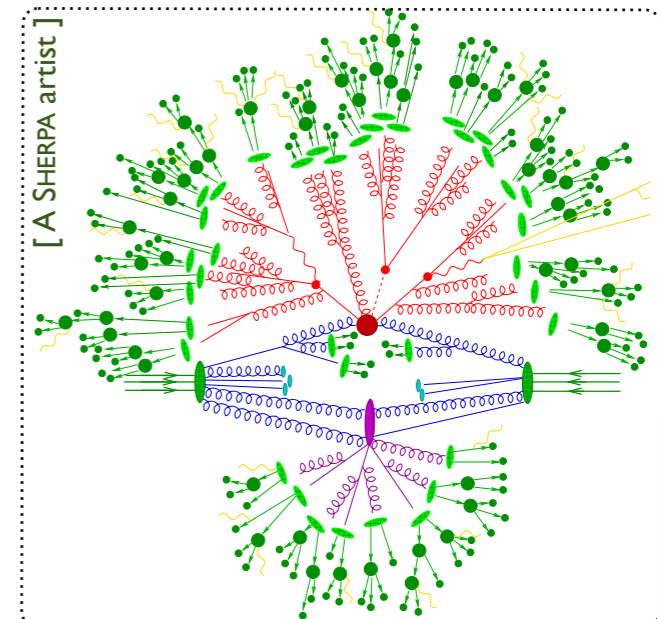


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**Tools and methods for each step**



## ◆ Tools development: a very intense activity

- ❖ Matrix-element generation (CALCHEP, HERWIG, MG5\_AMC, SHERPA, WHIZARD, etc.)
- ❖ Higher-order computation techniques (MC@NLO, POWHEG, NNLO)
- ❖ Parton showering / hadronisation (PYTHIA, HERWIG, SHERPA)
- ❖ Matrix element - parton shower matching
- ❖ Merging techniques (MLM, CKKW, FxFx, UNLOPS, etc.)
- ❖ Detector simulators (DELPHES, RIVET, MADANALYSIS 5)



# SM and BSM simulations: the status

- ◆ **Standard Model simulations under good control**
  - ♣ Relevant LHC processes: known with a very good precision
  - ♣ Further improvements expected in the next few years

Talk by A.  
Shivaji

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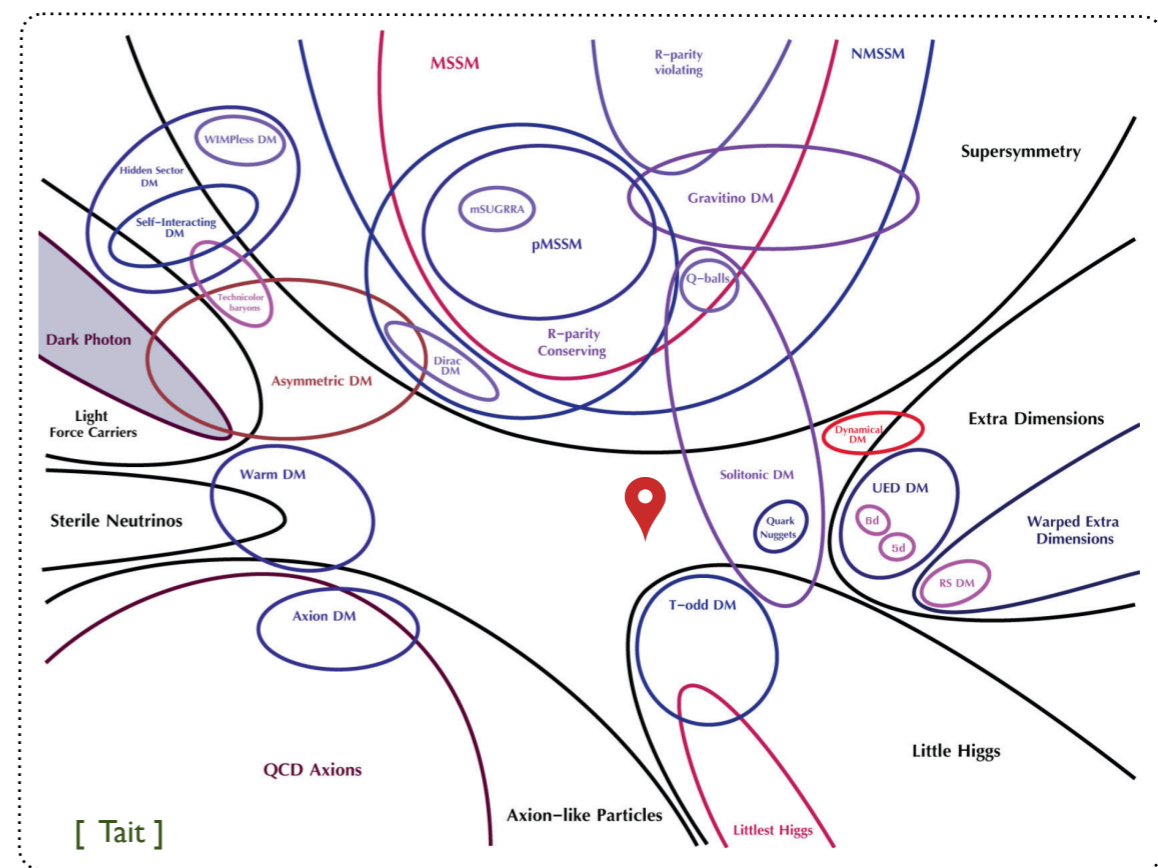
Talk by A. Shivaji

## ◆ Different challenges for new physics

- ♣ No sign of new physics
- ♣ SM-like measurements
  - no leading candidate theory
- ♣ Plethora of models to consider
  - many implementations in tools

Despite of this, new physics is standard today

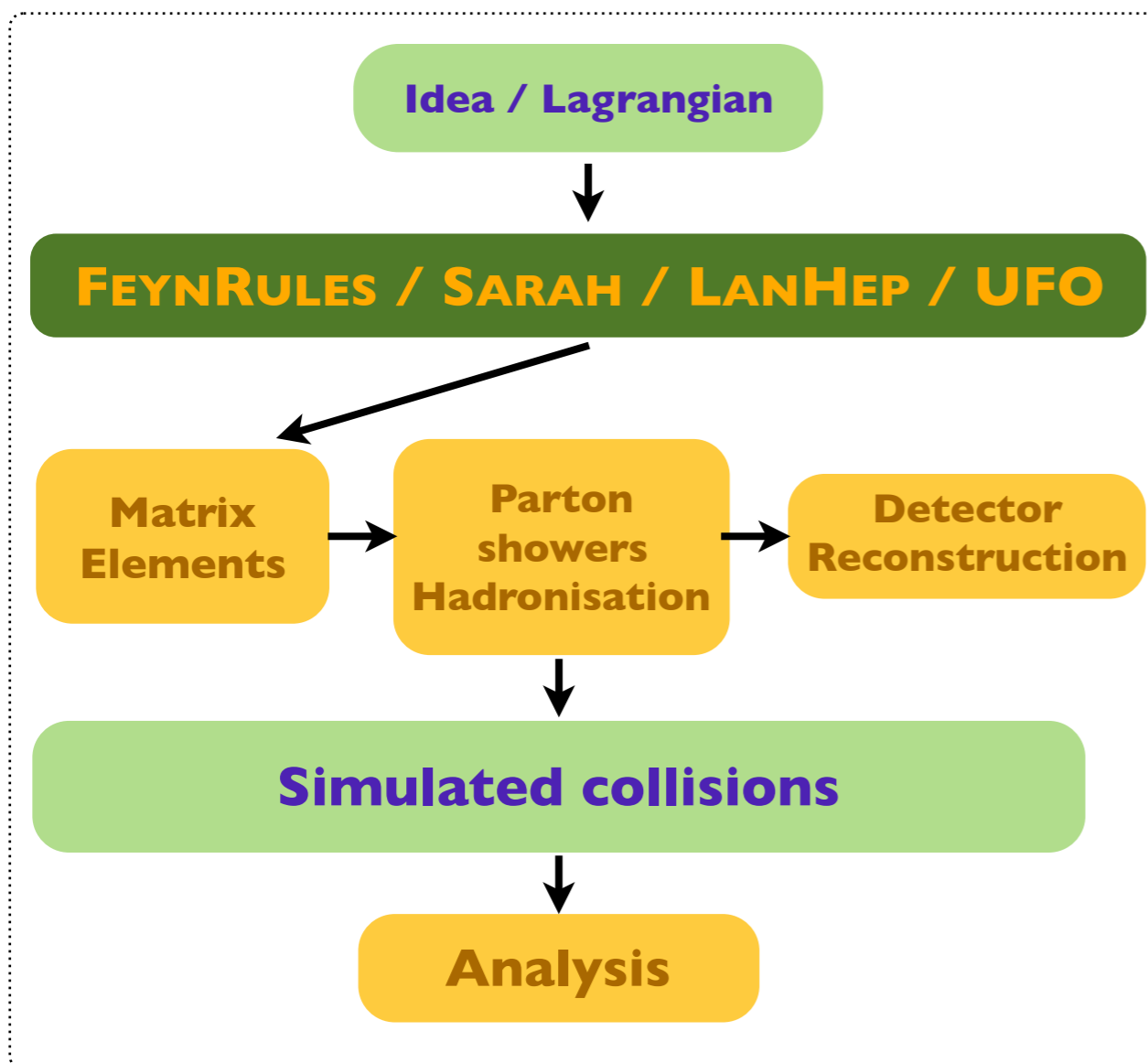
Talks by K. Mawatari, K. Mimasu & R. Ruiz



# Making new physics a standard

[ Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC 11) ]

## ◆ Tools connecting an idea to simulated collisions

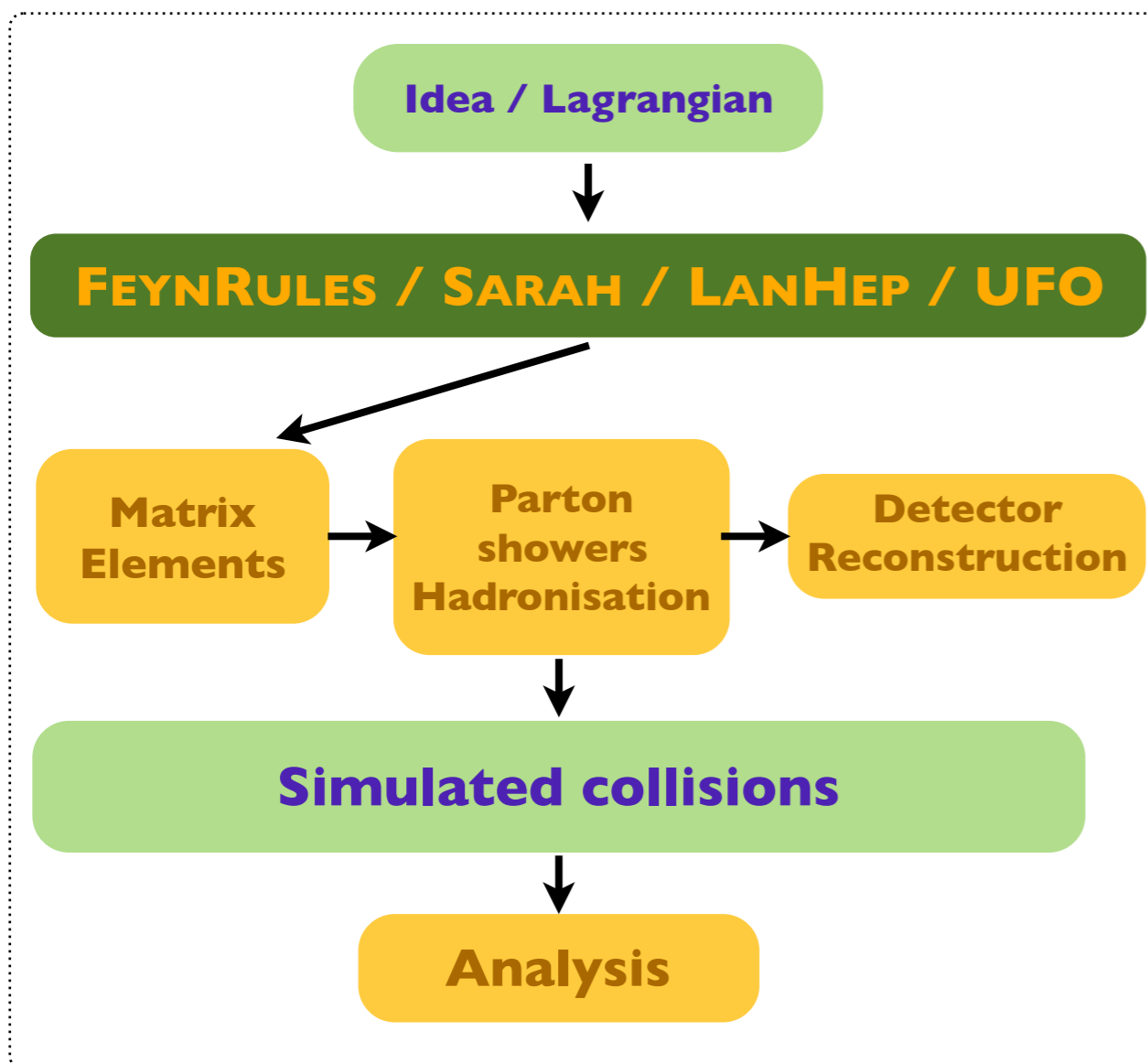


- ❖ Model building
  - ★ FEYNRULES, LANHEP, SARAH
  - ★ UFO
- ❖ Matrix element generation
  - ★ CALCHEP, HERWIG++, MG5\_AMC, SHERPA, WHIZARD, ...
- ❖ QCD environment
  - ★ HERWIG, PYTHIA, SHERPA
- ❖ Detector simulation
  - ★ DELPHES / PGS
  - ★ RIVET / MADANALYSIS 5
- ❖ Event analysis
  - ★ RIVET / MADANALYSIS 5

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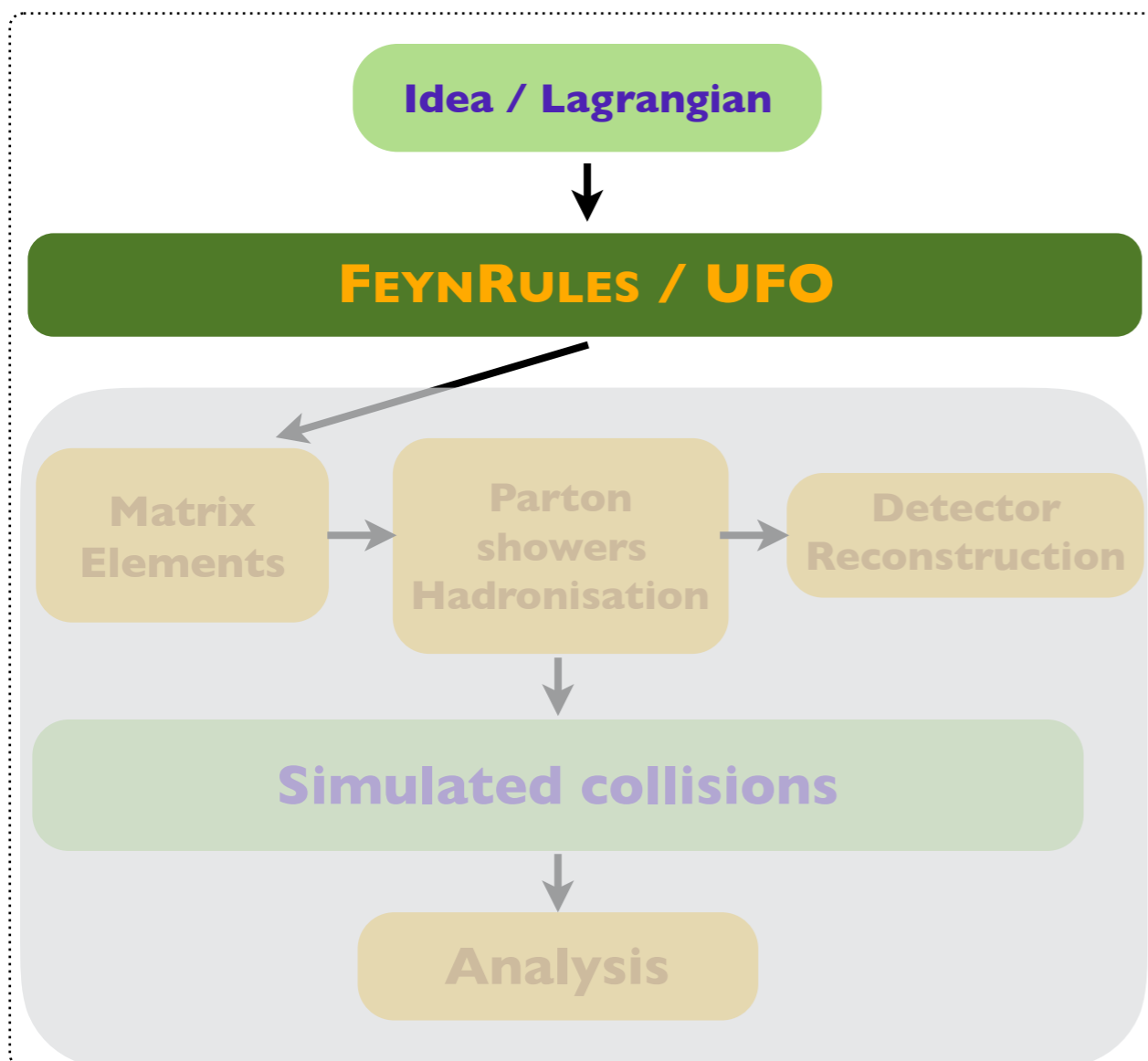
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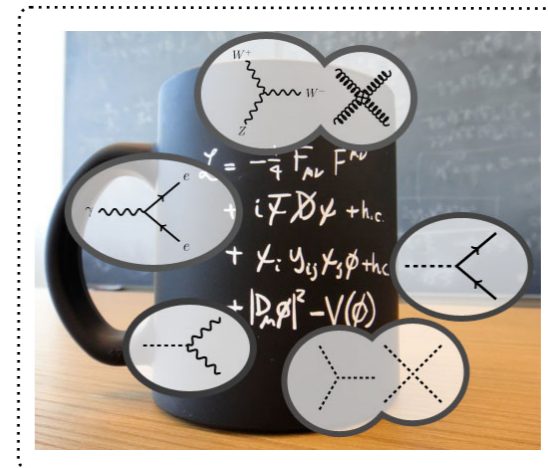
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# New physics simulations: the 'how-to'

## ◆ How to implement a new physics model in a Monte Carlo program?

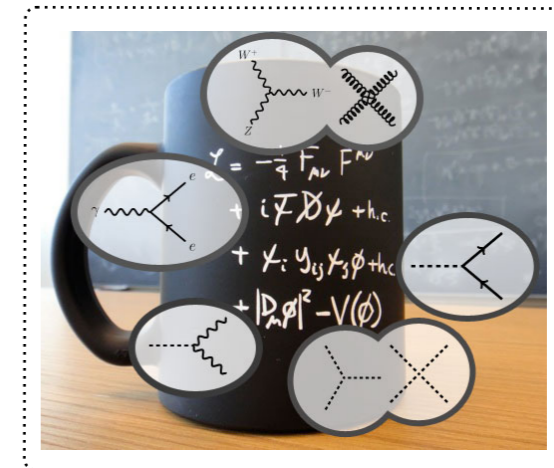
- ★ Definition: particles, parameters & vertices ( $\equiv$  Lagrangian)  
→ translated in some programming language
- ★ Tedious, time-consuming, error prone
- ★ Beware of restrictions/conventions



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- ★ **Highly redundant** (each tool, each model)
- ★ **No-brainer tasks** (from Feynman rules to codes)

**Systematisation  
Automation**

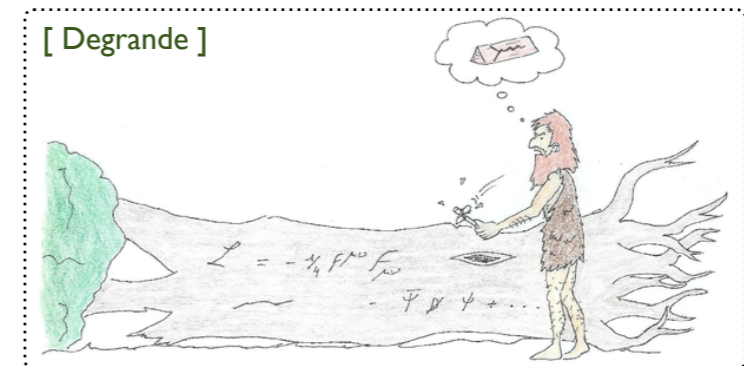


# The FEYNRULES package

[ Christensen & Duhr (CPC '09); Alloul, Christensen, Degrande, Duhr & BF (CPC'14) ]

## ◆ The FEYNRULES platform (since 2009)

- ❖ From Lagrangians to files in a programming language
  - ★ **Automatic**
- ❖ Very few limitations
  - ★ Higher-dimensional operators all supported
  - ★ Spins: up to 2
  - ★ Colour structures: **1, 3, 6, 8**



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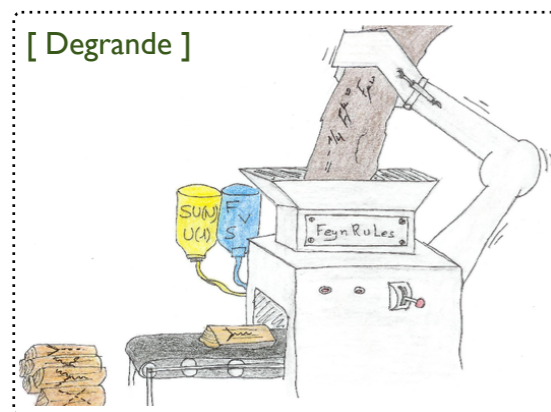
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♣ Working environment: **MATHEMATICA**

★ Flexibility, symbolic manipulations, design of new methods, etc.

★ **Many built-in methods** (superspace, spectrum, decays, NLO, etc.)

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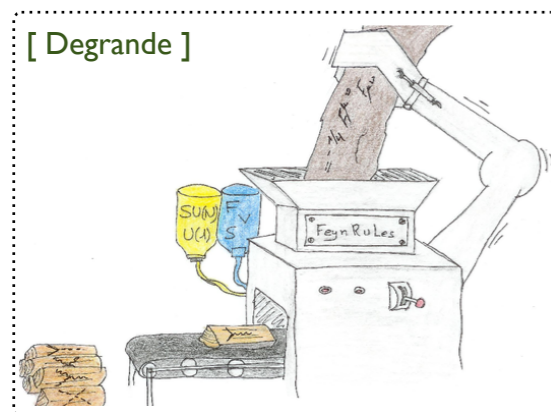
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♣ Working environment: **MATHEMATICA**

★ Flexibility, symbolic manipulations, design of new methods, etc.

★ **Many built-in methods** (superspace, spectrum, decays, NLO, etc.)

♣ Interfaced to many Monte Carlo tools

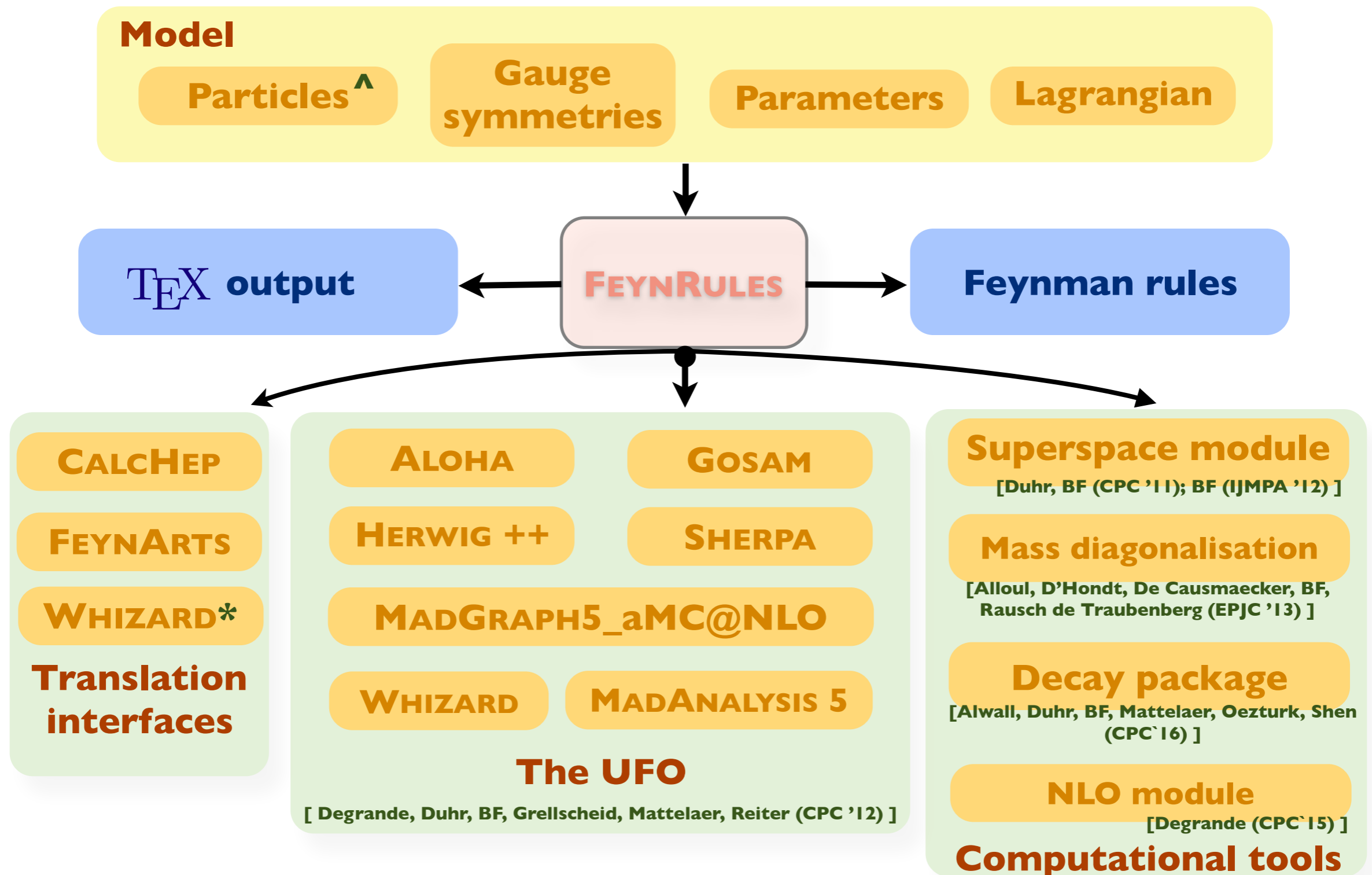
★ CALCHEP, FEYNARTS, WHIZARD (more previously)

★ More UFOs (HERWIG++, MG5AMC, SHERPA, WHIZARD, ...)



# From FEYNRULES to Monte Carlo tools...

[ Christensen & Duhr (CPC '09); Alloul, Christensen, Degrande, Duhr & BF (CPC'14) ]



\* Whizard interface: Christensen, Duhr, BF, Reuter, Speckner (EPJC '12)

<sup>^</sup> Support for spin 3/2: Christensen, de Aquino, Deutschmann, Duhr, BF, Garcia-Cely, Mattelaer, Mawatari, Oexl, Takaesu (EPJC '13)

# More about interfaces

- ◆ Each interface dedicated to a given tool is specific
  - ❖ Removal of vertices not compliant with the tool
    - ★ Colour structures
    - ★ Lorentz structures
  - ❖ Translation to a specific format and programming language

→ not efficient

→ a unique translation and the tools parse it

# More about interfaces

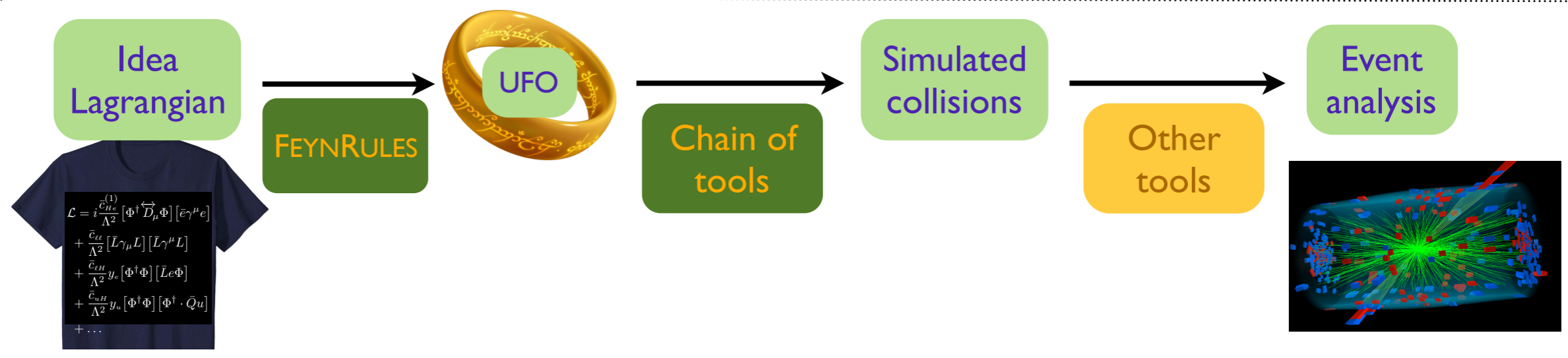
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## ◆ One format to rule them all!



# A step further: the Universal FEYNRULES Output

## ◆ The UFO in a nutshell

[ Degrande, Duhr, BF, Grellscheid, Mattelaer, Reiter (CPC '12) ]  
[ Degrande, Duhr, BF, Hirschi, Mattelaer, Shao (in prep.) ]

- ❖ UFO  $\equiv$  Universal FEYNRULES output  $\rightarrow$  **Universal Feynman Output**
  - ★ **Universal** as not tied to any specific Monte Carlo program
- ❖ Consists of a set of **PYTHON files** to be linked to any code
- ❖ This module contains **all the model information**
  - ★ Generic colour and Lorentz structures
- ❖ Can be employed for next-to-leading order calculations

## ◆ The UFO is now a standard

ALOHA

GOSAM

HERWIG ++

MADANALYSIS 5

SHERPA

MADGRAPH5\_aMC@NLO

WHIZARD

LANHEP

SARAH

# The UFO in practice

## ◆ The UFO is a set of PYTHON files

- ❖ Factorisation of the information: particles, interactions, parameters, NLO, etc.

## ◆ Example

```
[fuchs@Benjamins-MacBook-Pro-3 ~/Work/tools/FeynRules/trunk/models/SUSYQCD_UFO$] ls
CT_couplings.py      SUSYQCD_UFO.log      couplings.py          object_library.py     propagators.py
CT_parameters.py     __init__.py          function_library.py   parameters.py          vertices.py
CT_vertices.py       coupling_orders.py   lorentz.py            particles.py           write_param_card.py
[fuchs@Benjamins-MacBook-Pro-3 ~/Work/tools/FeynRules/trunk/models/SUSYQCD_UFO$]
```



# Examples: particles & parameters

## ◆ Particles $\equiv$ instances of the particle class

- ❖ Attributes: spin, colour representation, mass, width, etc.
- ❖ Antiparticles automatically derived

## ◆ Parameters $\equiv$ instances of the parameter class

- ❖ External parameters: Les Houches-like structure
- ❖ PYTHON-compliant formula for the internal parameters

```
aS = Parameter(name = 'aS',
               nature = 'external',
               type = 'real',
               value = 0.1184,
               texname = '\\alpha_s',
               lhablock = 'SMINPUTS',
               lhacode = [ 3 ])

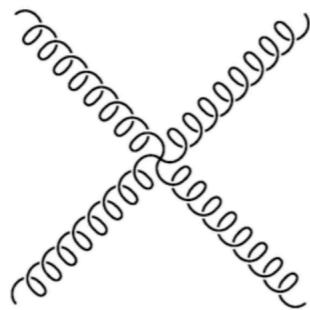
G = Parameter(name = 'G',
              nature = 'internal',
              type = 'real',
              value = '2*cmath.sqrt(aS)*cmath.sqrt(cmath.pi)',
              texname = 'G')
```

```
go = Particle(pdg_code = 1000021,
              name = 'go',
              antiname = 'go',
              spin = 2,
              color = 8,
              mass = Param.Mgo,
              width = Param.Wgo,
              texname = 'go',
              antitexname = 'go',
              charge = 0)
```

# Interactions: the key strategy

## ◆ Decomposition in a **spin x colour** basis (coupling strengths $\equiv$ coordinates)

### ♣ Example: the quartic gluon vertex

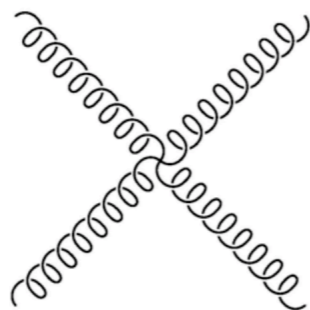


$$\begin{aligned}
 & ig_s^2 f^{a_1 a_2 b} f^{b a_3 a_4} (\eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4}) \\
 & + ig_s^2 f^{a_1 a_3 b} f^{b a_2 a_4} (\eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4}) \\
 & + ig_s^2 f^{a_1 a_4 b} f^{b a_2 a_3} (\eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4})
 \end{aligned}$$

# Interactions: the key strategy

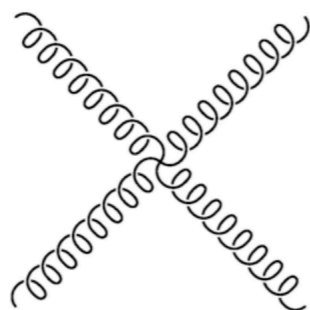
## ◆ Decomposition in a **spin x colour** basis (coupling strengths $\equiv$ coordinates)

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$$\begin{aligned}
 & ig_s^2 f^{a_1 a_2 b} f^{b a_3 a_4} (\eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4}) \\
 & + ig_s^2 f^{a_1 a_3 b} f^{b a_2 a_4} (\eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4}) \\
 & + ig_s^2 f^{a_1 a_4 b} f^{b a_2 a_3} (\eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4})
 \end{aligned}$$

### ❖ UFO version



$$\begin{aligned}
 & (f^{a_1 a_2 b} f^{b a_3 a_4}, f^{a_1 a_3 b} f^{b a_2 a_4}, f^{a_1 a_4 b} f^{b a_2 a_3}) \\
 & \times \begin{pmatrix} ig_s^2 & 0 & 0 \\ 0 & ig_s^2 & 0 \\ 0 & 0 & ig_s^2 \end{pmatrix} \begin{pmatrix} \eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} \\ \eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4} \\ \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} - \eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4} \end{pmatrix}
 \end{aligned}$$

- ★ 3 elements for the colour basis
- ★ 3 elements for the spin (Lorentz structure) basis
- ★ 9 coordinates (6 are zero)

Information encoded  
in different files

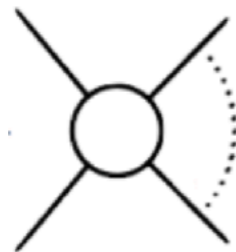
# NLO cross sections

## ◆ Contributions to an NLO result in QCD

♣ Three ingredients: the Born, virtual loop and real emission contributions

$$\sigma_{NLO} = \int d^4\Phi_n \mathcal{B} + \int d^4\Phi_n \int_{\text{loop}} d^d\ell \mathcal{V} + \int d^4\Phi_{n+1} \mathcal{R}$$

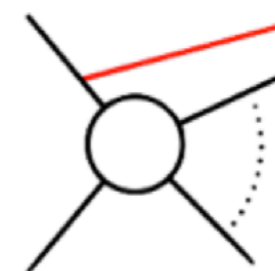
Born



Virtuals: one extra power of  $\alpha_s$  and divergent



Reals: one extra power of  $\alpha_s$  and divergent



**Extra information is needed**

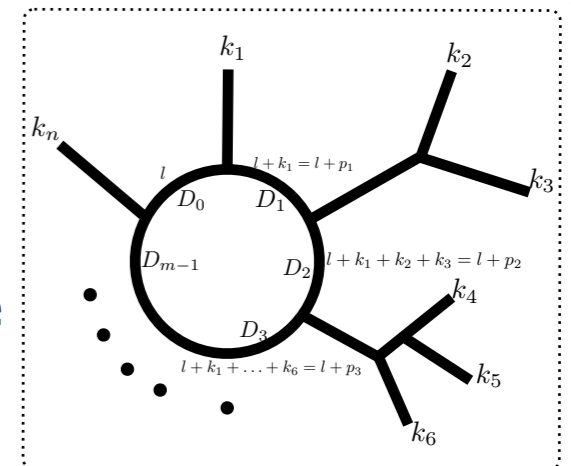
Talks by HS. Shao

# Loop calculations

- ◆ Dimensional regularisation: calculations in  $d = 4 - 2\varepsilon$ 
  - ♣ Divergences explicit ( $1/\varepsilon^2$ ,  $1/\varepsilon$ )
  - ♣ Reduction of tensor loop-integrals to scalar integrals
- ◆ The reduction must be performed in a  $d$ -dimensional space
  - ♣ Numerical methods work in **4 dimensions**  $\rightarrow$   $R_1$  and  $R_2$  terms

$$\int d^d \ell \frac{N(\ell, \tilde{\ell})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \quad \text{with } \bar{\ell} = \ell + \tilde{\ell}$$

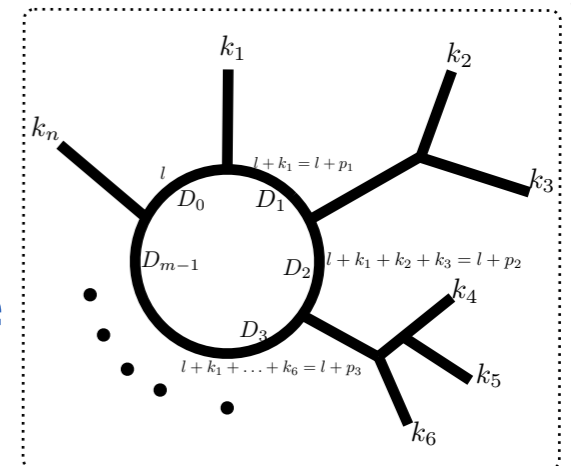
D-dim
4-dim
(-2ε)-dim



[ Ossala, Papadopoulos, Pittau (NPB'07; JHEP'08) ]

# Loop calculations

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$$\int d^d \ell \frac{N(\ell, \tilde{\ell})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \quad \text{with } \bar{\ell} = \ell + \tilde{\ell}$$

D-dim
4-dim
(-2\varepsilon)-dim

[ Ossala, Papadopoulos, Pittau (NPB'07; JHEP'08) ]

- ◆ **The  $R_1$  terms originate from the denominators**
  - ♣ Connected to the internal propagators
- ◆ **The  $R_2$  terms originate from the numerator**
  - ♣ Process-dependent contributions proportional to  $\tilde{\ell}^2$
  - ♣ Renormalisable theory: finite number of  $R_2$ 's
    - ★ Seen as extra diagrams with special Feynman rules ( $\rightarrow R_2$  Feynman rules)
    - ★ Connected to the UV structure of the integrals (like the UV counterterms)
    - ★ Can be derived from the bare Lagrangian  $\rightarrow$  NLOCT

Talk from O. Mattelaer

**UFO @ NLO**

# Automated NLO simulations with MG5\_AMC

Idea / Lagrangian



FEYNRULES / UFO



Matrix  
Elements



Collider observables



Phenomenology

## ◆ Model building: from Lagrangian to tools

- ♣ FEYNRULES  $\oplus$  MOGRE  $\oplus$  NLOCT  $\leadsto$  UFO @ NLO
- ♣ General on-shell renormalisation scheme

[ Alloul, Christensen, Degrande, Duhr & BF (CPC'14) ; Degrande (CPC'15) ]  
 [ Degrande, Duhr, BF, Mattelaer & Reither (CPC'12) ]  
 [ Frixione, BF, Hirschi, Mawatari, Shao, Sunder & Zaro (JHEP'19) ]

## ◆ Hard scattering

- ♣ Feynman diagram, matrix elements
- ♣ MG5aMC  $\leadsto$  predictions at LO/NLO

[ Alwall et al. (JHEP'14) ]

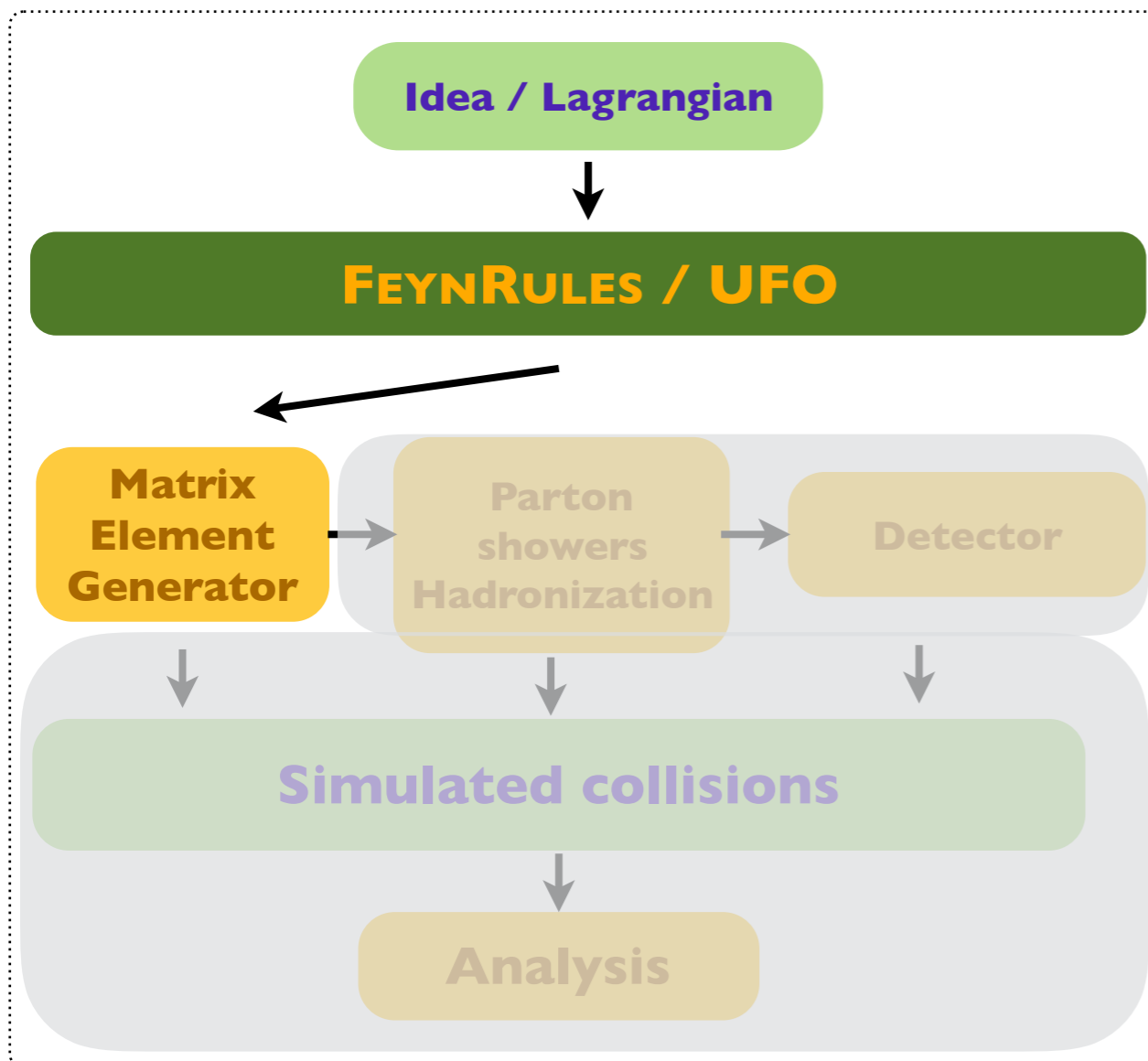
# Outline

1. A comprehensive approach for Monte Carlo simulations
2. Implementing models into Monte Carlo event generators
- 3. From models to hard-scattering events**
4. Summary



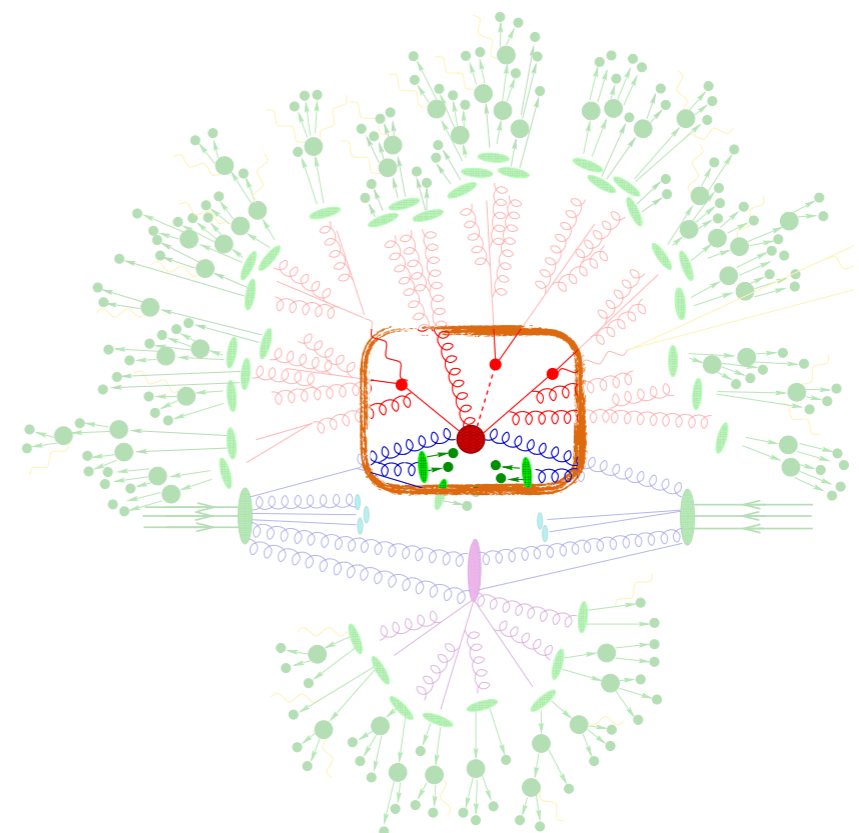
# Back to the simulation chain

## ◆ Tools connecting an idea to simulated collisions



## ✿ Hard scattering process

- ★ Feynman diagram and amplitude generation
- ★ Monte Carlo integration
- ★ Event generation



# QCD 101: predictions at the LHC

## ◆ Distribution of an observable $\omega$ : the QCD factorisation theorem

$$\frac{d\sigma}{d\omega} = \sum_{ab} \int dx_a dx_b f_{a/p_1}(x_a; \mu_F) f_{b/p_2}(x_b; \mu_F) \frac{d\sigma_{ab}}{d\omega}(\dots, \mu_F)$$

- ❖ Long distance physics: **the parton densities**
- ❖ Short distance physics: the differential parton cross section  **$d\sigma_{ab}$**
- ❖ **Separation of both regimes through the factorisation scale  $\mu_F$** 
  - ★ Choice of the scale  $\triangleright$  theoretical uncertainties

Talk by F. Maltoni

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Talk by F. Maltoni

## ◆ Short distance physics: the partonic cross section

- ❖ Calculated **order by order in perturbative QCD**:  $d\sigma = d\sigma^{(0)} + \alpha_s d\sigma^{(1)} + \dots$ 
  - ★ The more orders included, the more precise the predictions
  - ★ Truncation of the series and  $\alpha_s \triangleright$  theoretical uncertainties

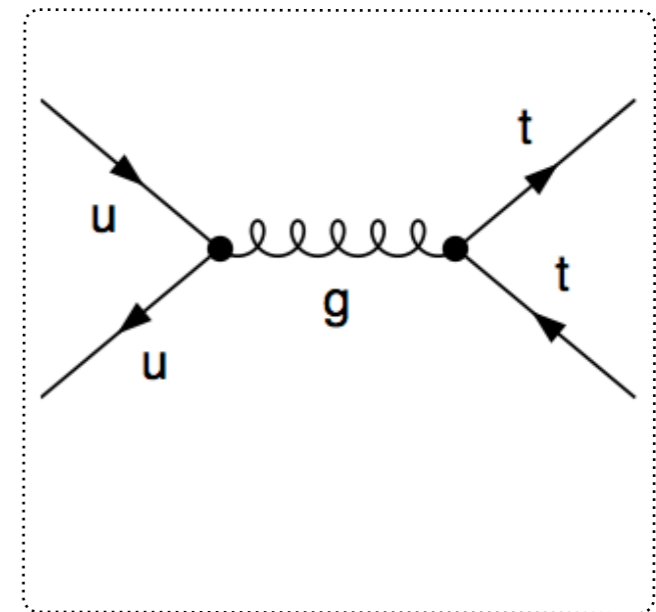
**Feynman diagrams  
(from UFOs)**

# Feynman diagram calculations

## ◆ Direct squared matrix element computations

- ✿ Extraction of the amplitude from the Feynman rules

$$i\mathcal{M} = ig_s^2 \left[ \bar{v}_2 \gamma^\mu u_1 \right] \frac{\eta_{\mu\nu}}{s} \left[ \bar{u}_3 \gamma^\nu v_4 \right] T_{c_2 c_1}^a T_{c_3 c_4}^a$$

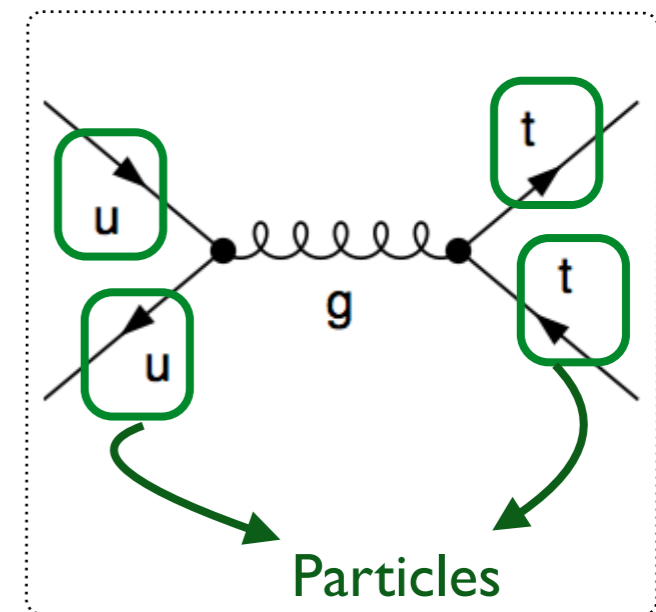


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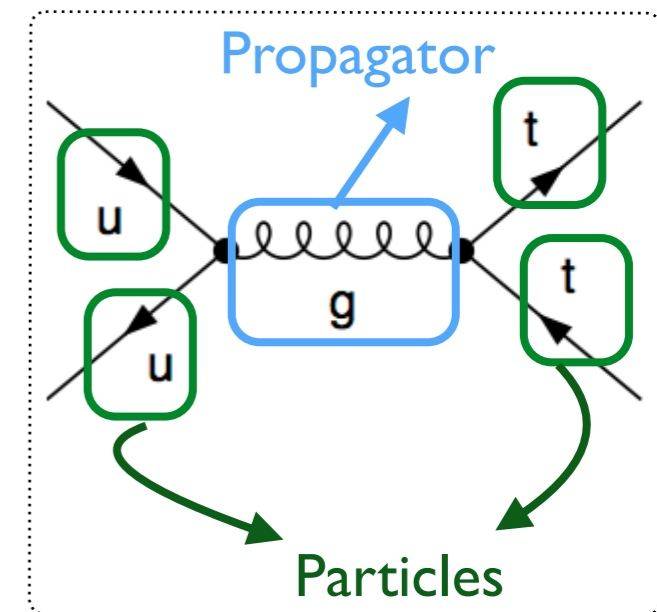


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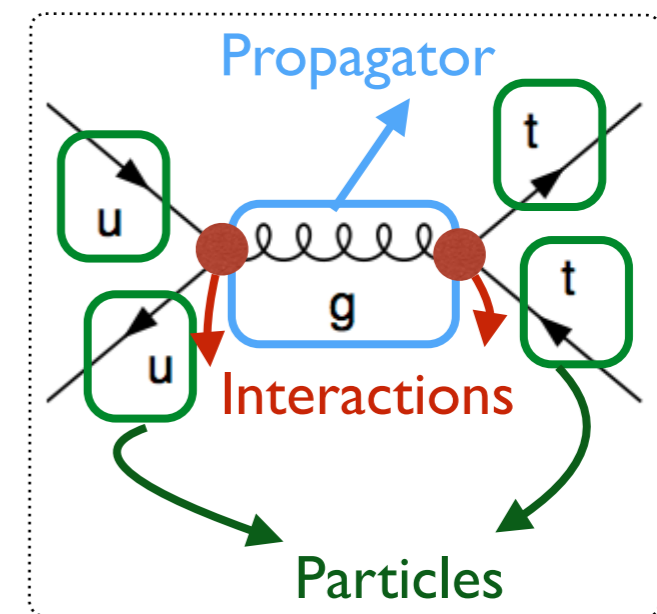


# Feynman diagram calculations

## ◆ Direct squared matrix element computations

- ✿ Extraction of the amplitude from the Feynman rules

$$i\mathcal{M} = ig_s^2 \underbrace{[\bar{v}_2 \gamma^\mu u_1]}_{\text{Particles}} \underbrace{\left[ \frac{\eta_{\mu\nu}}{s} \right]}_{\text{Propagator}} \underbrace{[\bar{u}_3 \gamma^\nu v_4]}_{\text{Particles}} \underbrace{T_{c_2 c_1}^a T_{c_3 c_4}^a}_{\text{Interactions}}$$



# Feynman diagram calculations

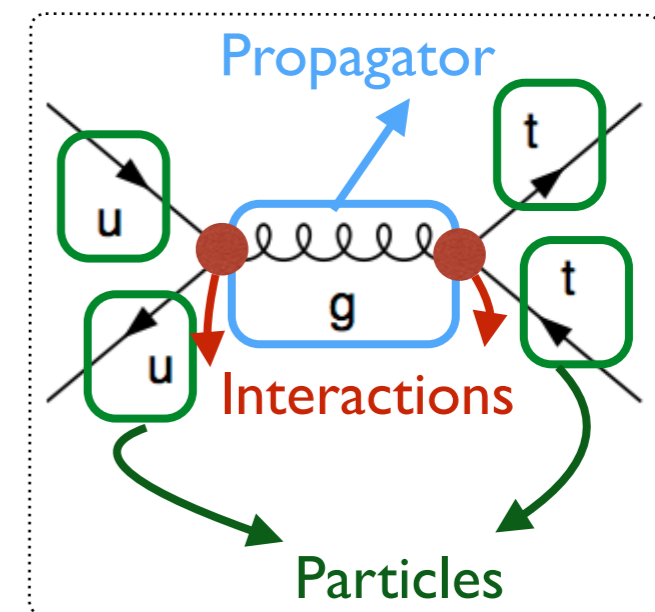
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- ❖ Squaring with the conjugate amplitude
- ❖ Algebraic calculation (colour and Lorentz structures)
- ❖ Sum/average over the external states

$$\begin{aligned} |\overline{\mathcal{M}}|^2 &= \frac{1}{36} \frac{2g_s^4}{s^2} \text{Tr} \left[ \not{p}_1 \gamma^\mu \not{p}_2 \gamma^\nu \right] \left[ \not{p}_3 \gamma_\mu \not{p}_4 \gamma_\nu \right] \\ &= \frac{16g_s^4}{9s^2} \left[ (p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) \right] \end{aligned}$$





# Feynman diagram calculations

## ◆ Direct squared matrix element computations

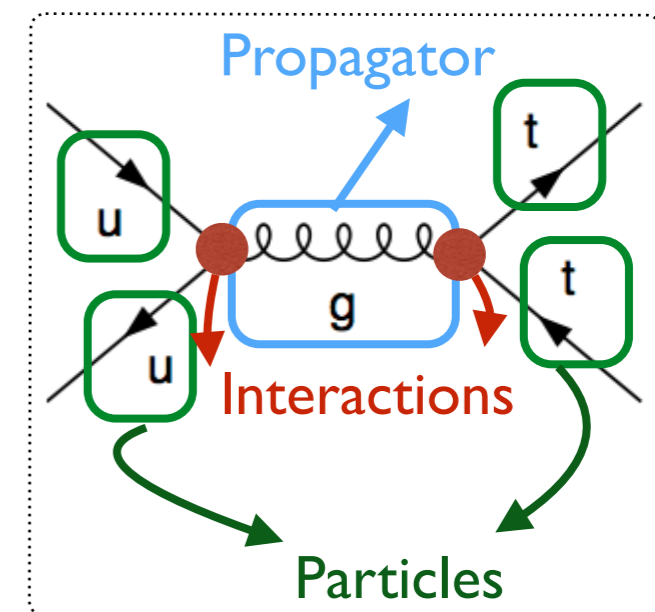
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- ❖ The squared matrix element needs to be integrated



Talk by O. Mattelaer

# Feynman diagram calculations

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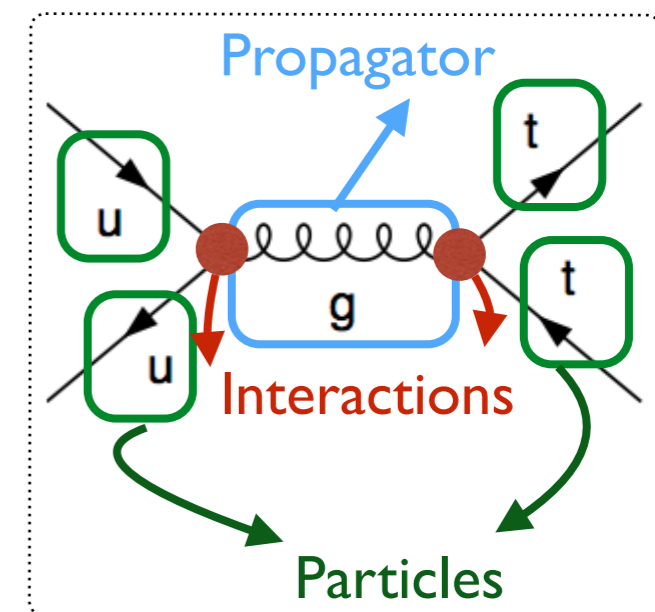
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Talk by O. Mattelaer



## ◆ The number of diagrams increases with the number of final-state particles

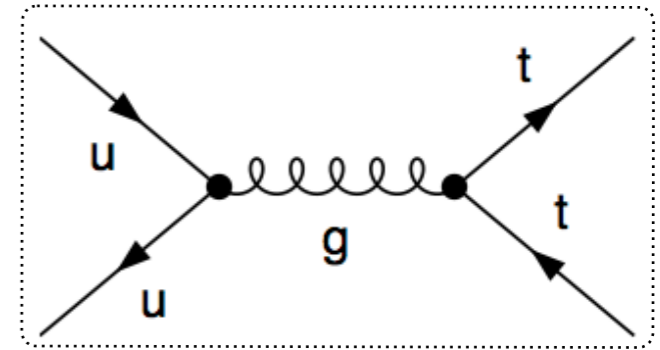
- ❖ The complexity rises as  $N^2$
- ❖ Any calculation beyond 2-to-3 becomes a problem

➤ Helicity amplitudes

# Helicity amplitudes

## ◆ Principle

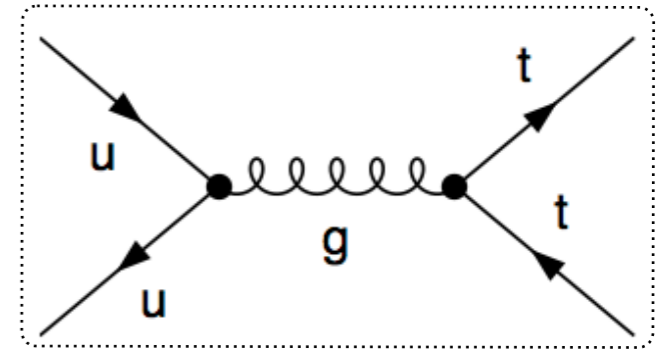
- ❖ Evaluation of the amplitude for fixed external helicities
- ❖ Add all amplitudes (we get complex numbers)
- ❖ Squaring
- ❖ Sum/average over the external states



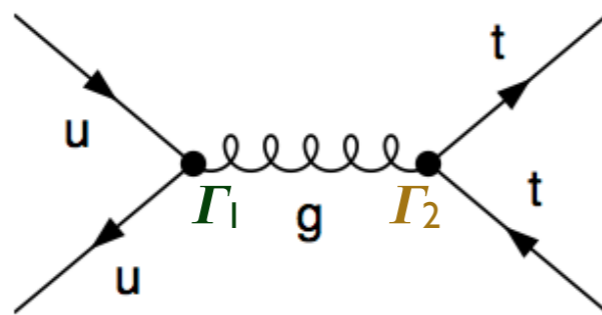
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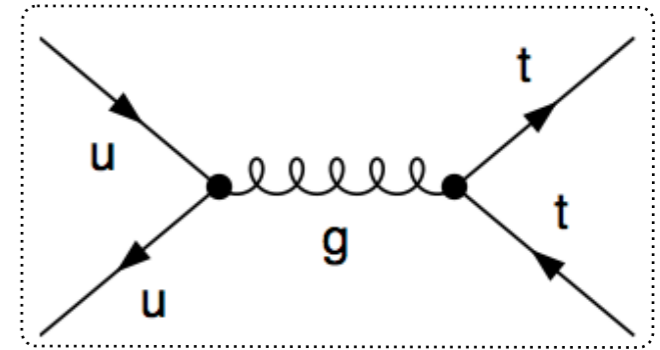
## ◆ Practical example



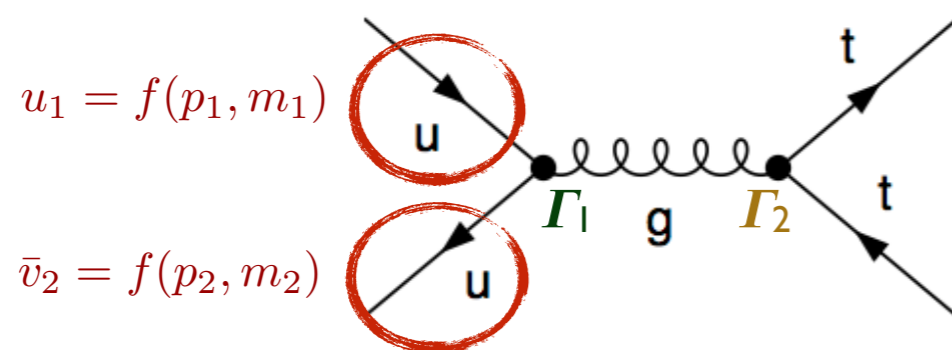
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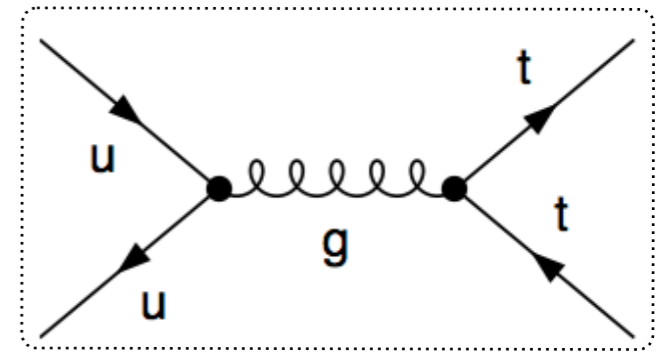
### I. External incoming particles (numbers)

★ For fixed helicity and momentum

# Helicity amplitudes

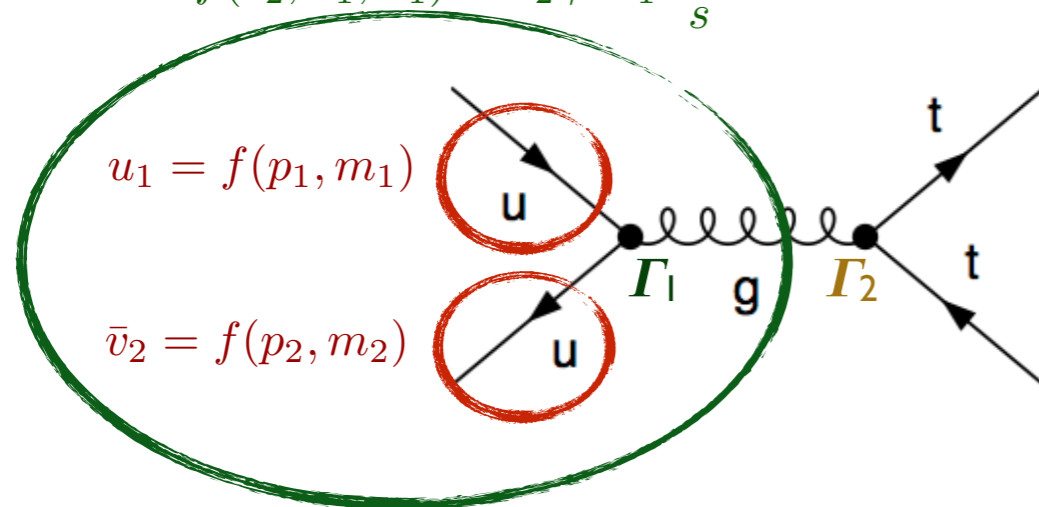
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## ◆ Practical example

$$W = f(\bar{v}_2, u_1, \Gamma_1) \propto \bar{v}_2 \gamma^\mu u_1 \frac{\eta_{\mu\nu}}{s}$$

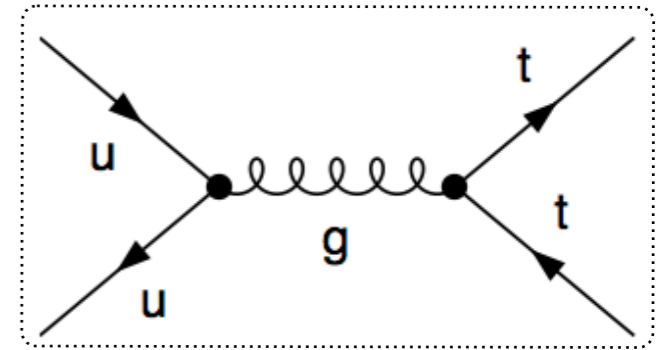


1. External incoming particles (numbers)
  - ★ For fixed helicity and momentum
2. Wave function of the gluon propagator

# Helicity amplitudes

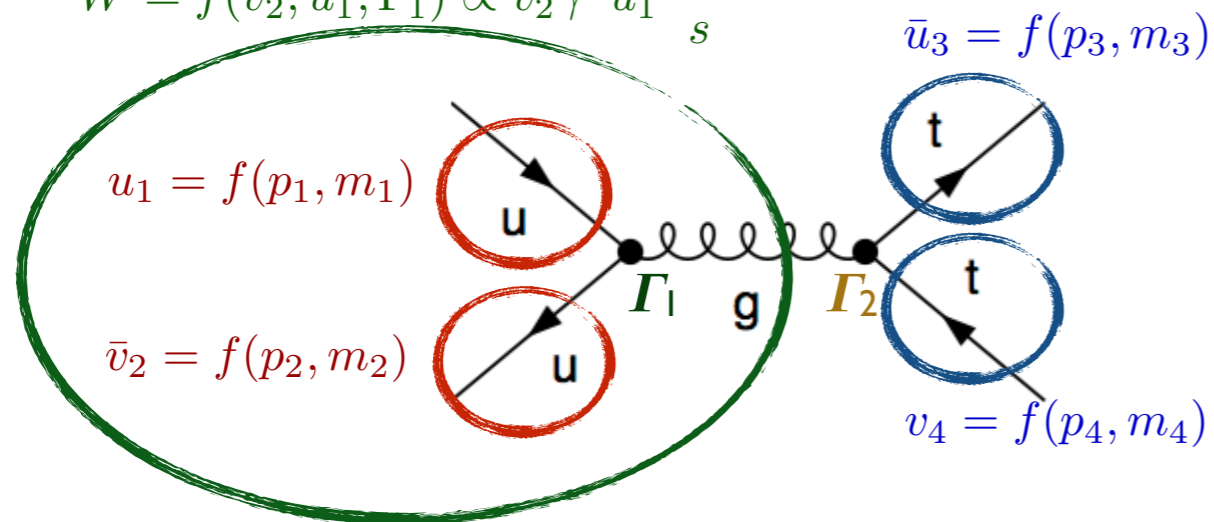
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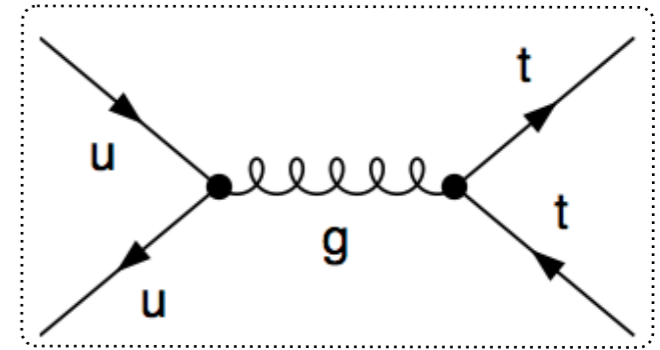


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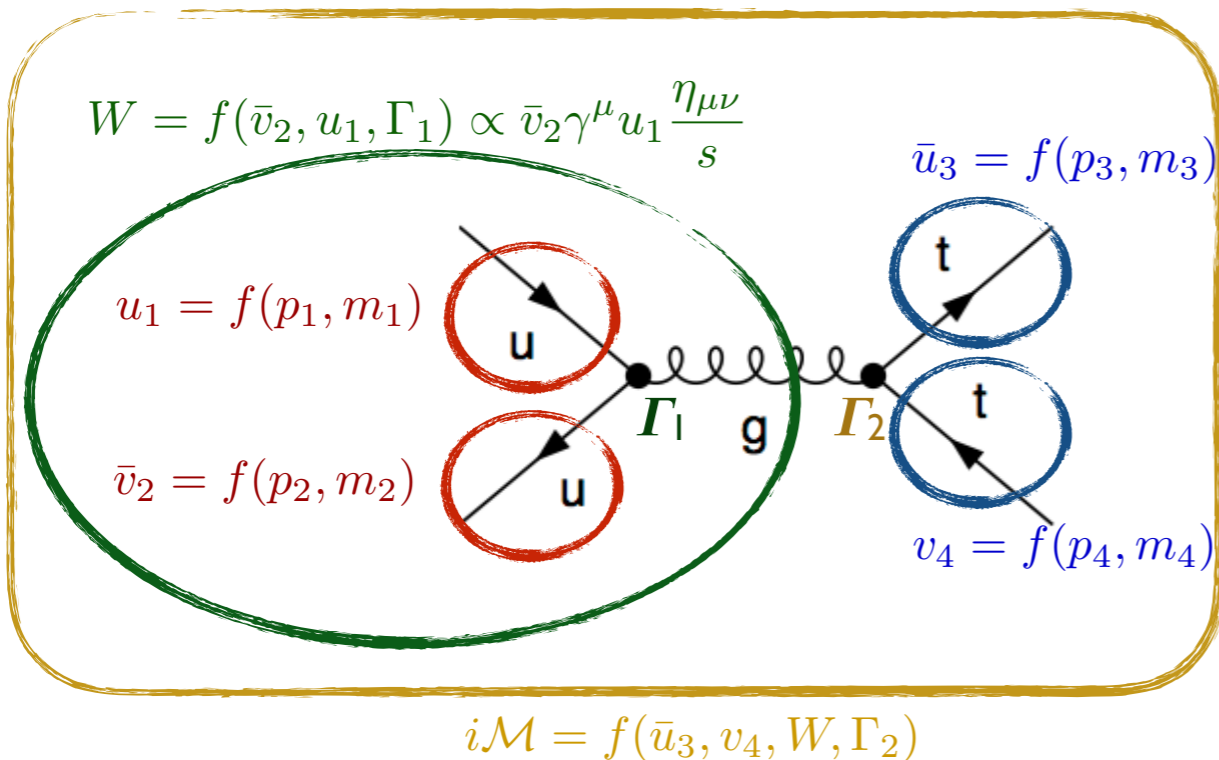
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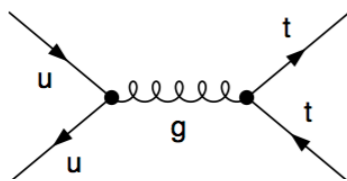


1. External incoming particles (numbers)
  - ★ For fixed helicity and momentum
2. Wave function of the gluon propagator
3. External outgoing particles
4. Full amplitude (complex number)



# HELAS

◆ The building blocks of the amplitude are the so-called HELAS functions



$$u_1 = f(p_1, m_1)$$

$$\bar{v}_2 = f(p_2, m_2)$$

$$\bar{u}_3 = f(p_3, m_3)$$

$$v_4 = f(p_4, m_4)$$

$$W = f(\bar{v}_2, u_1, \Gamma_1)$$

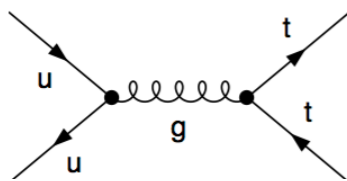
$$i\mathcal{M} = f(\bar{u}_3, v_4, W, \Gamma_2)$$

- ❖ HELAS  $\equiv$  HELicity Amplitude Subroutine
- ❖ One specific routine for each Lorentz structure ( $\Gamma_i$ )
- ❖ Not generic for any model
  - ★ SM [ Murayama, Watanabe & Hagiwara (KEK-91-11) ]
  - ★ MSSM [ Cho, Hagiwara, Kanzaki, Plehn, Rainwater & Stelzer (PRD`06) ]
  - ★ HEFT [ Frederix (2007) ]
  - ★ Spin 2 [ Hagiwara, Kanzaki, Li & Mawatari (EPJC`08) ]
  - ★ Spin 3/2 [ Mawatari & Takaesu (EPJC`11) ]

**Sufficient for many models**

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**Sufficient for many models**

## ◆ Generalisation: ALOHA

[ de Aquino, Link, Maltoni, Mattelaer & Stelzer (CPC`12) ]

- ❖ Translation of any vertex present in a UFO into a HELAS subroutine
- ❖ **Any model** supported in MG5\_aMC@NLO
- ◆ Recycling: reusing pieces from one diagram to another

# Comparison

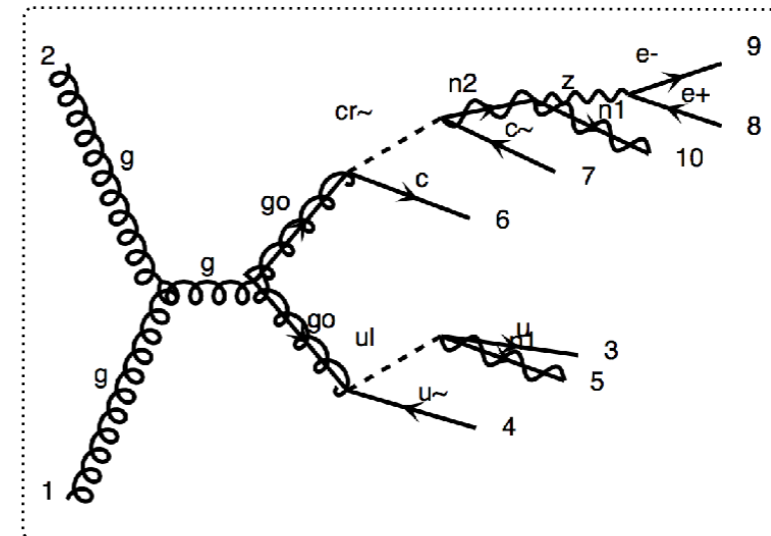
	For $M$ diags	For $N$ particles	$2 \rightarrow 6$ example
Analytical	$M^2$	$(N!)^2$	$10^9$
Helicity	$M$	$N! 2^N$	$10^7$
Recycling	$M$	$(N-1)! 2^{N-1}$	$5 \times 10^5$

# Heavy particle decays

## ◆ Concrete BSM models

- ❖ **Many additional new states**
  - ★ Usually pair-produced
  - ★ Cascade-decaying into each other
- ❖ The lightest new state often stable (cf. dark matter)

Is the simulation of 2-to- $N$  processes (with a large  $N$ ) a problem?





# Making decays easy: the key principle

## ◆ Production and decay processes are factorised

- ❖ Propagators can be seen as sums of products of external wave functions
- ❖ Example for a vector resonance

$$\mathcal{M} = j_1^\mu \left[ \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right] j_2^\nu = \sum_\lambda \underbrace{j_1^\mu \varepsilon_\mu^*(\lambda)}_{\text{Production of the resonance}} \underbrace{j_2^\nu \varepsilon_\nu(\lambda)}_{\text{Decay of the resonance}}$$

Propagation

# Making decays easy: the key principle

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The diagram shows a Feynman diagram for the production and decay of a vector resonance. An incoming quark  $q$  and antiquark  $\bar{q}$  meet at a vertex, producing a resonance  $Z$ . The resonance  $Z$  then decays into a quark  $\bar{q}$  and a lepton  $l$ . The production part is highlighted in an orange box, and the decay part is highlighted in a green box. The propagator is highlighted in a blue box.

- ❖ **Off-shell effects** are lost (as a result of the factorisation)

★ Resonance mass smearing: partial recovery [ Frixione, Laenen, Motylinski, Webber (JHEP '07) ]

# Practical implementations of decays

## ◆ Case I: loss of spin correlations

♣ Helicity sums performed independently (production  $\oplus$  decays)

$$\mathcal{M} = j_1^\mu \left[ \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right] j_2^\nu = \sum_\lambda j_1^\mu \varepsilon_\mu^*(\lambda) j_2^\nu \varepsilon_\nu(\lambda) \approx \underbrace{\sum_\lambda j_1^\mu \varepsilon_\mu^*(\lambda)}_{\text{Production}} \underbrace{\sum_{\lambda'} j_2^\nu \varepsilon_\nu(\lambda')}_{\text{Decay}}$$



# Practical implementations of decays

## ◆ Case 1: loss of spin correlations

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$$\mathcal{M} = j_1^\mu \left[ \eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right] j_2^\nu = \sum_\lambda j_1^\mu \varepsilon_\mu^*(\lambda) j_2^\nu \varepsilon_\nu(\lambda) \simeq \underbrace{\sum_\lambda j_1^\mu \varepsilon_\mu^*(\lambda)}_{\text{Production}} \underbrace{\sum_{\lambda'} j_2^\nu \varepsilon_\nu(\lambda')}_{\text{Decay}}$$

## ◆ Case 2: including spin correlations

- ♣ Keeping track of helicities

$$\mathcal{M} = \sum_\lambda j_1^\mu \varepsilon_\mu^*(\lambda) j_2^\nu \varepsilon_\nu(\lambda)$$

- ★ Reweighting according to decay matrix element (e.g. MADSPIN)

[Artoisenet et al. (JHEP '13)]

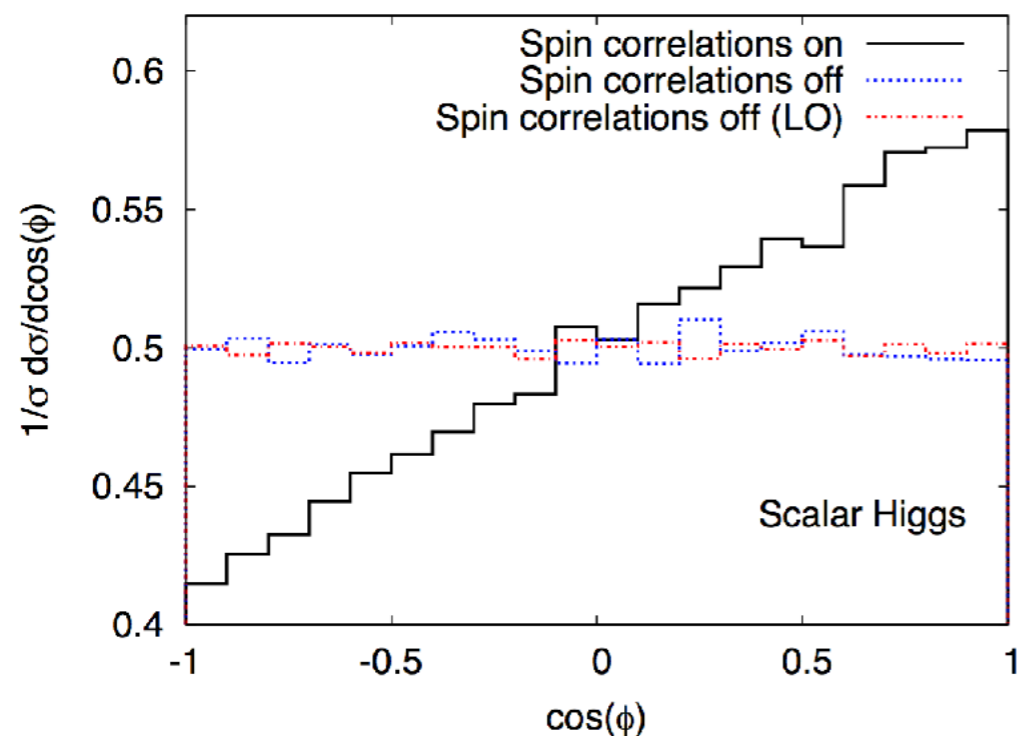
- ★ Using spin density matrices (e.g. HERWIG, SHERPA).

[Richardson (JHEP '01); Höche et al. (EPJC '15)]

# Importance of correctly-handled decays

## ◆ Two examples (dependent of the observable)

Angle between the leptons in the respective mother top rest frames

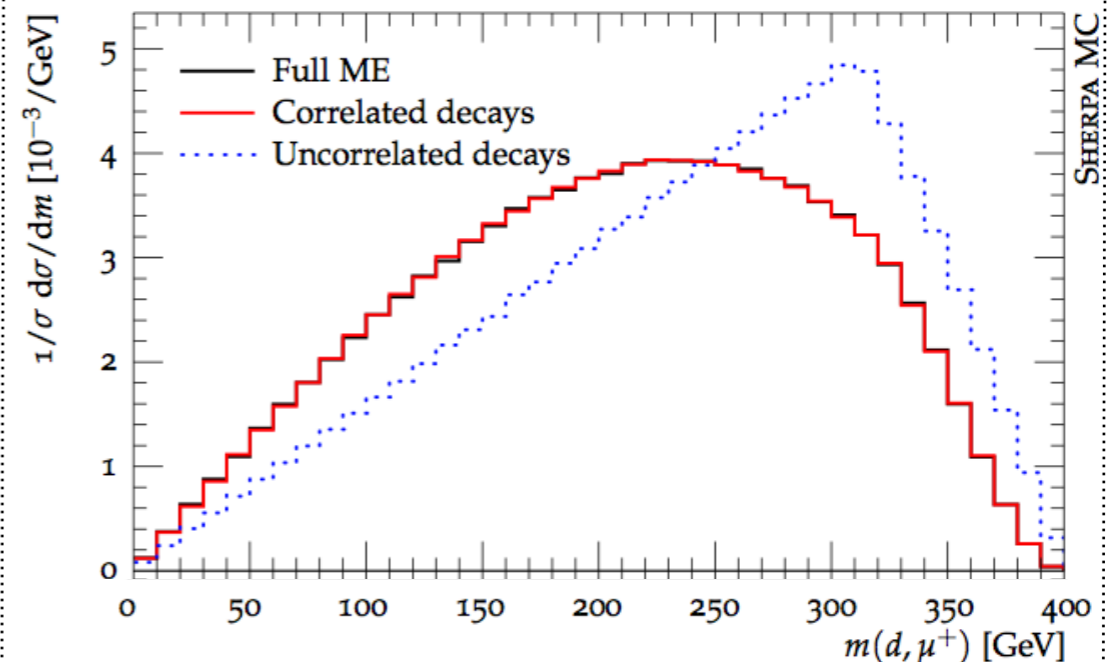


MADSPIN

$t\bar{t}H$  production @ (N)LOQCD  
[ LHC8, dileptonic  $t\bar{t}$  decay]

[ Artoisenet, Frederix, Mattelaer & Rietkerk (JHEP'13) ]

Invariant mass between decay products originating from different cascade steps



SHERPA @ LO [ LHC8 ]

$$pp \rightarrow \tilde{u}\tilde{u}^\dagger$$

$$\tilde{u} \rightarrow d\tilde{\chi}_1^+ \rightarrow d\chi_1^0 W^+ \rightarrow d\chi_1^0 \mu^+ \nu_\mu$$

$$\tilde{u}^\dagger \rightarrow \dots \rightarrow \bar{u}e^+e^-\tilde{\chi}_1^0$$

[ Höche, Kuttimalai, Schumann & Siebert (EPJC'15) ]

# Outline

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2. Implementing models into Monte Carlo event generators
3. From models to hard-scattering events
- 4. Summary**

# Summary

Idea / Lagrangian



FEYNRULES / UFO



Matrix  
Elements



Collider observables



Phenomenology

- ◆ Event simulation is a complex process
  - ♣ Nature allows us to factorise it into **pieces**
  - ♣ Event simulation is performed **step-by-step**

- ◆ This talk: 1<sup>st</sup> parts of the simulation chain
  - ♣ Connecting **models** (Lagrangians) to tools
  - ♣ Generating squared **matrix elements**
  - ♣ Including the **decays** of heavy particles