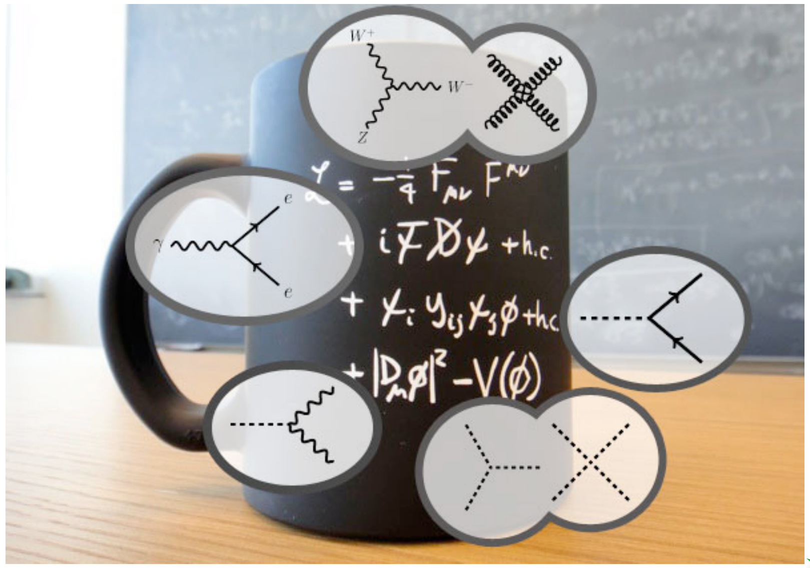
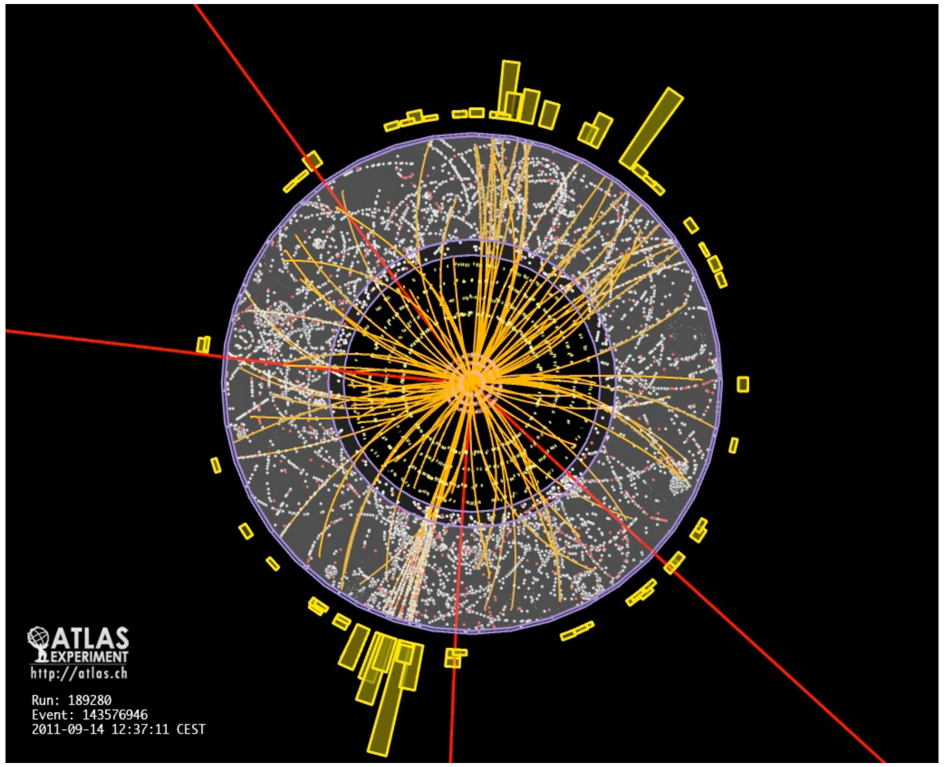
UCLouvain
Institut de recherche en mathématique et physique
Centre de Cosmologie, Physique des Particules et Phénoménologie

MadGraph5_aMC@NLO

Olivier Mattelaer

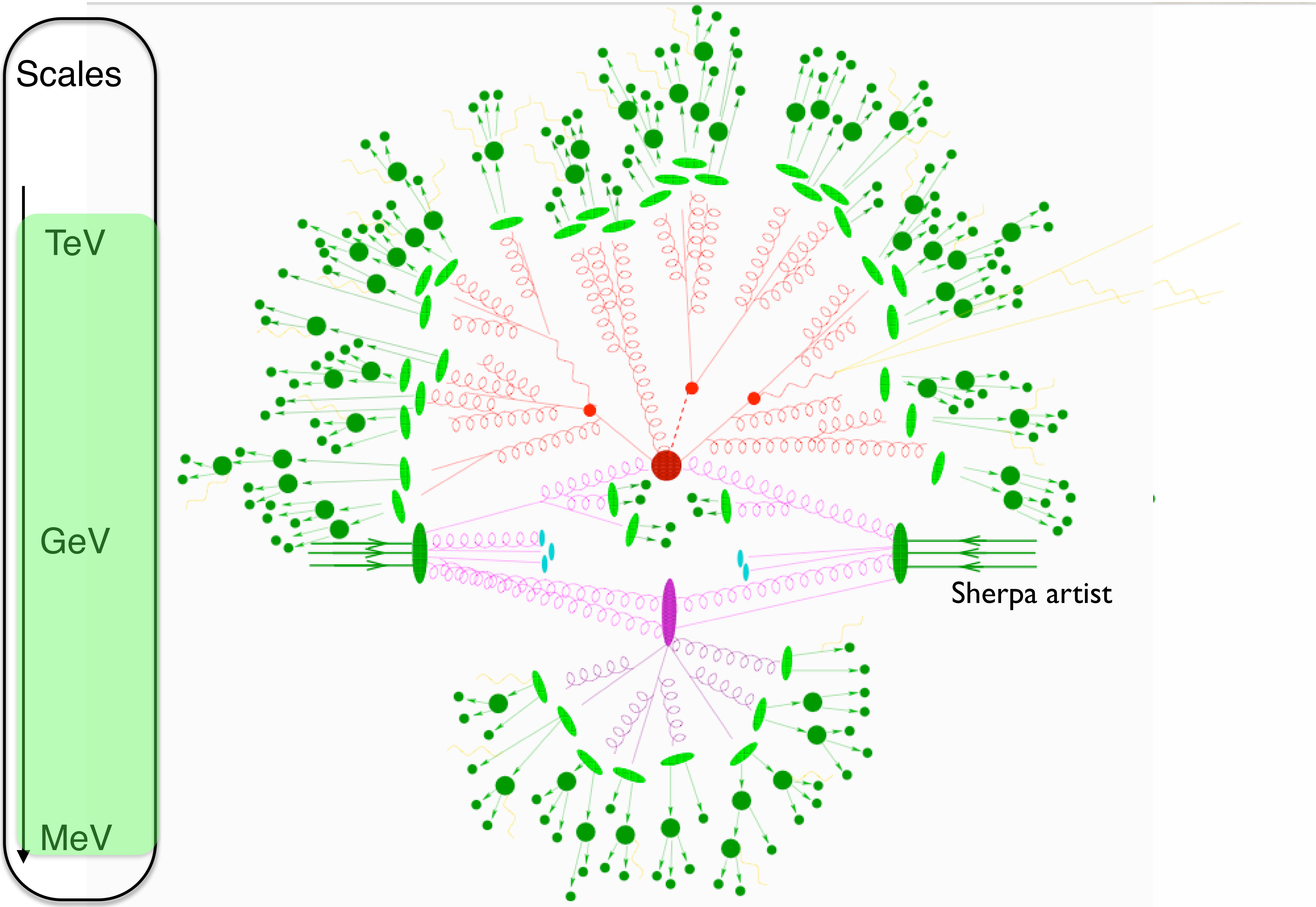


Monte-Carlo Physics



Simulation of collider events

What are the MC for?



What are the MC for?

Scales

TeV

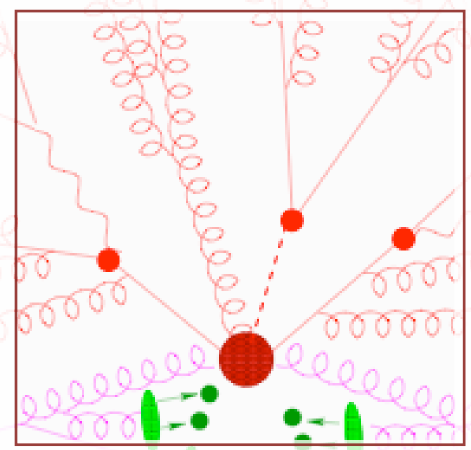
GeV

MeV

1. High-Q^2 Scattering

2. Parton Shower

☞ where BSM physics lies



☞ process dependent

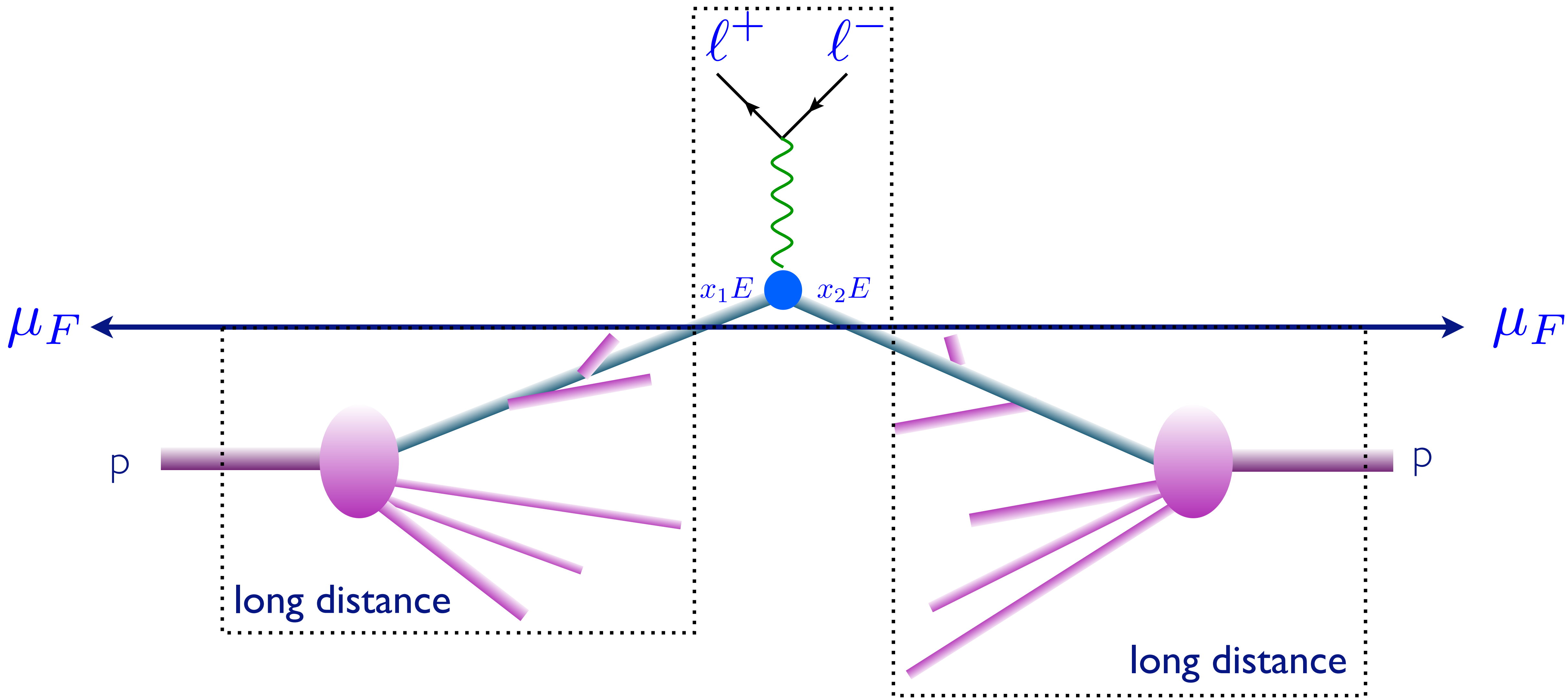
☞ first principles description

☞ it can be systematically improved

3. Hadronization

4. Underlying Event

Master formula for the LHC



$$\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$$

Parton-level cross section

Perturbative expansion

$d\hat{\sigma}_{ab\to X}(\hat{s}, \mu_F, \mu_R)$ **Parton-level cross section**

- The parton-level cross section can be computed as a series in perturbation theory, using the coupling constant as an expansion parameter, schematically:

$$\hat{\sigma} = \sigma^{\text{Born}}\left(1 + \frac{\alpha_s}{2\pi}\sigma^{(1)} + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma^{(2)} + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma^{(3)} + \ldots\right)$$

LO predictions

NLO corrections

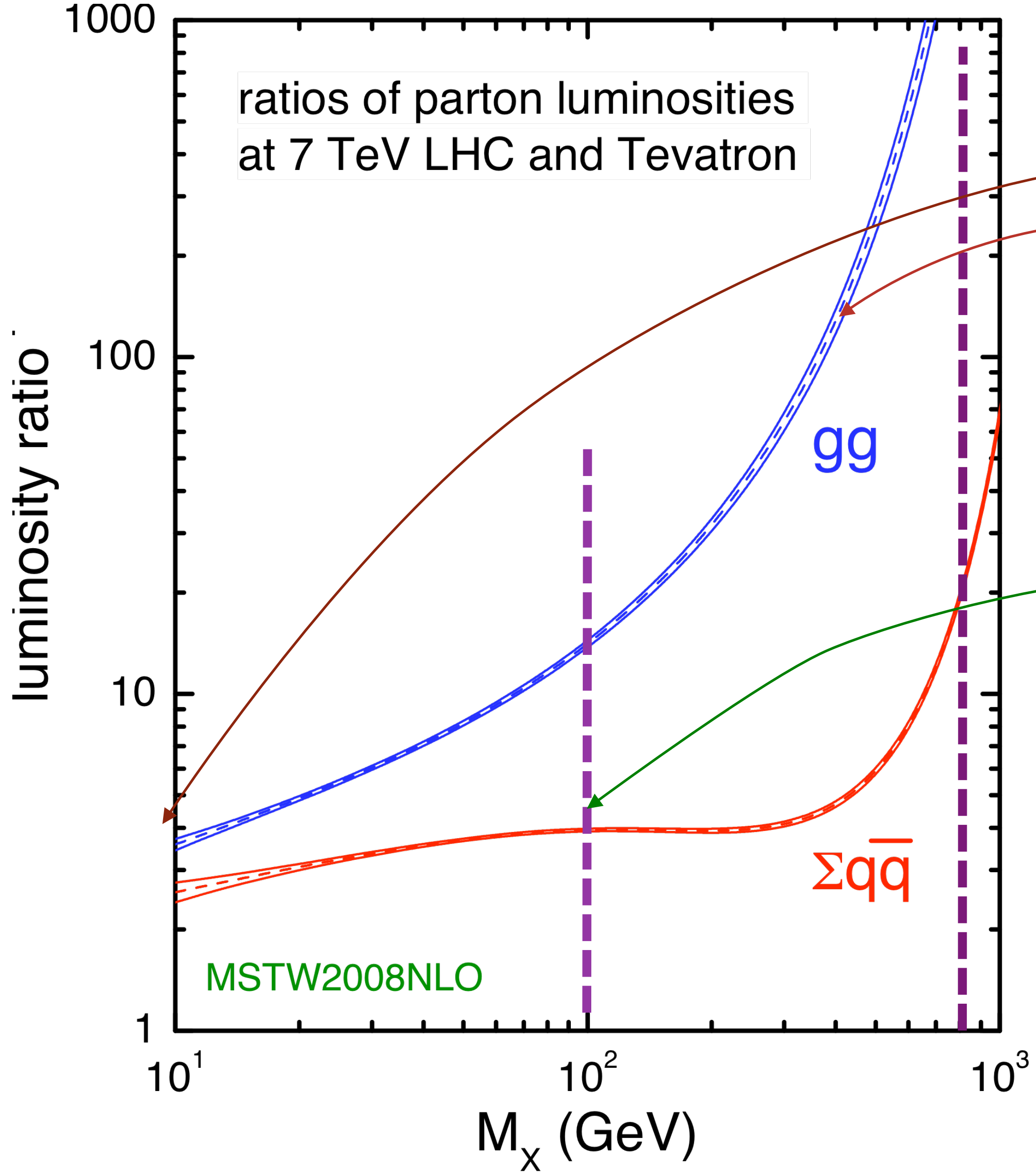
NNLO corrections

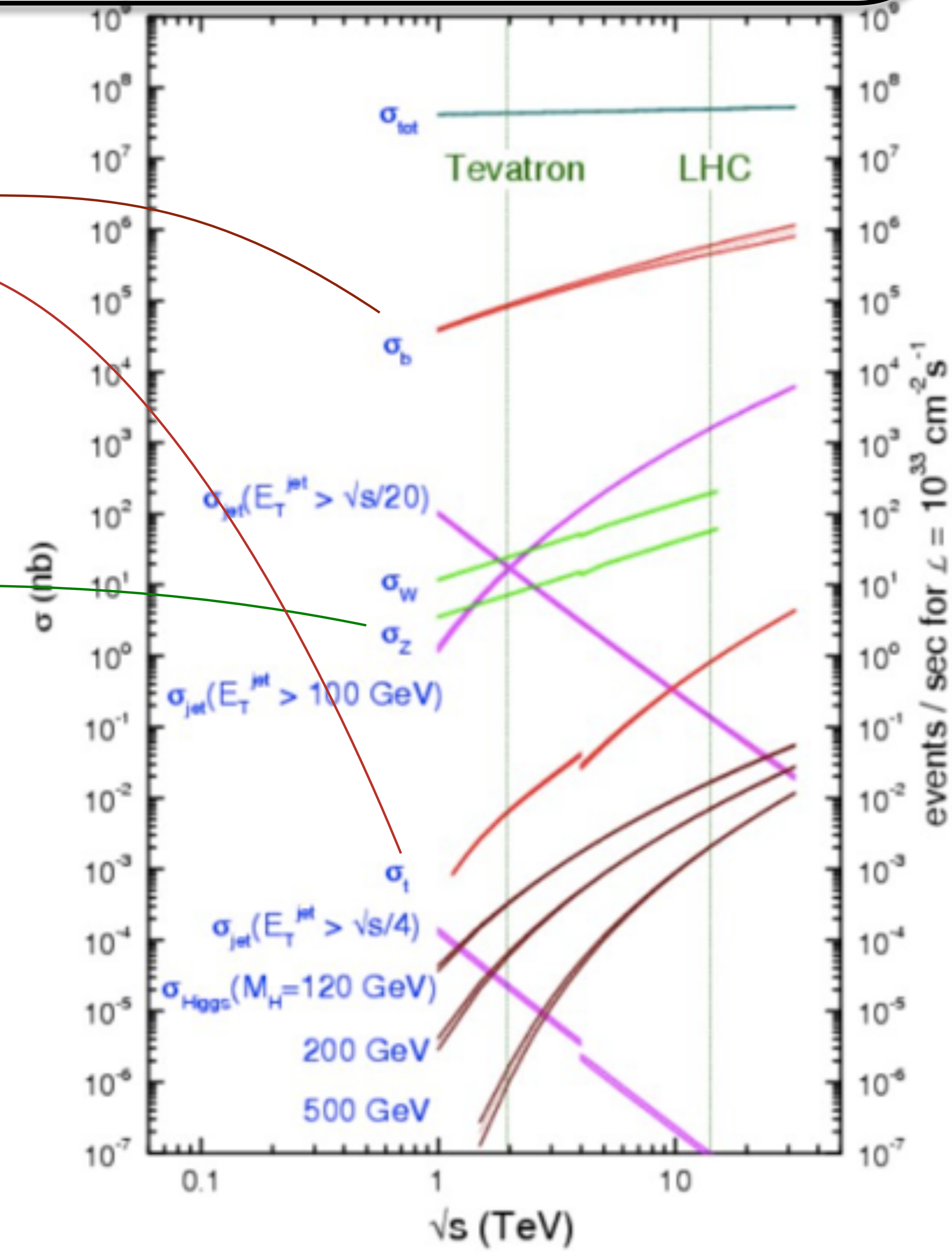
N3LO or NNNLO corrections

- Including higher corrections improves predictions and reduces theoretical uncertainties

Hadron colliders

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\mathrm{FS}}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$





To Remember

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat\sigma_{ab\to X}(\hat s, \mu_F, \mu_R)$$

Phase-space integral

Parton density functions

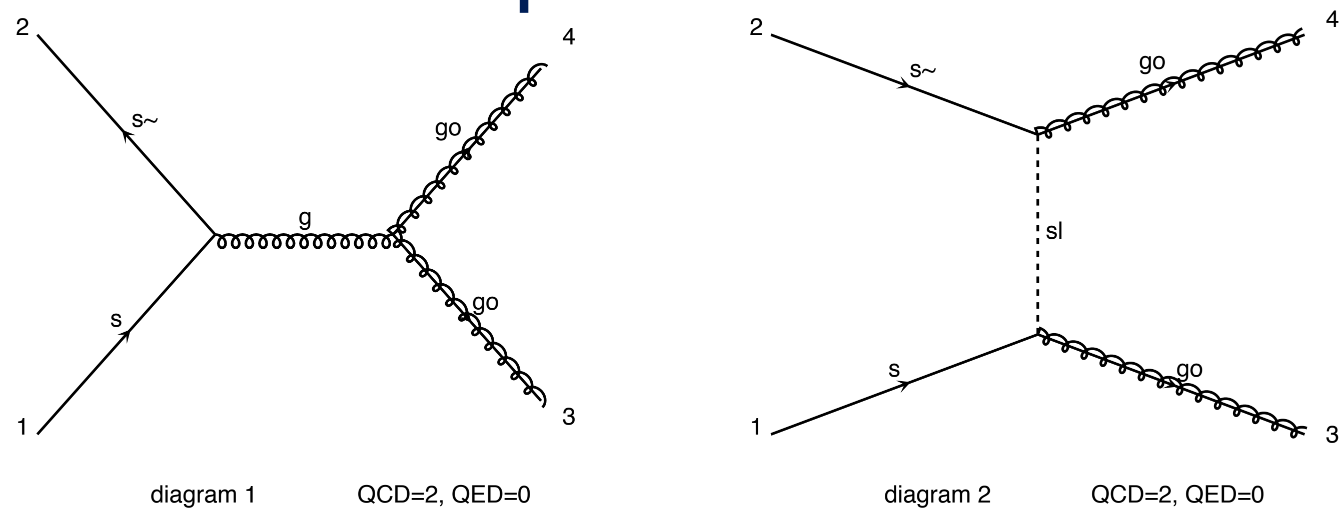
Parton-level cross section

- PDF: content of the proton
  - ➡ Define the physics/processes that will dominate on your accelerator
- NLO/NNLO: Reduce scale uncertainty

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

Hard

See Benj

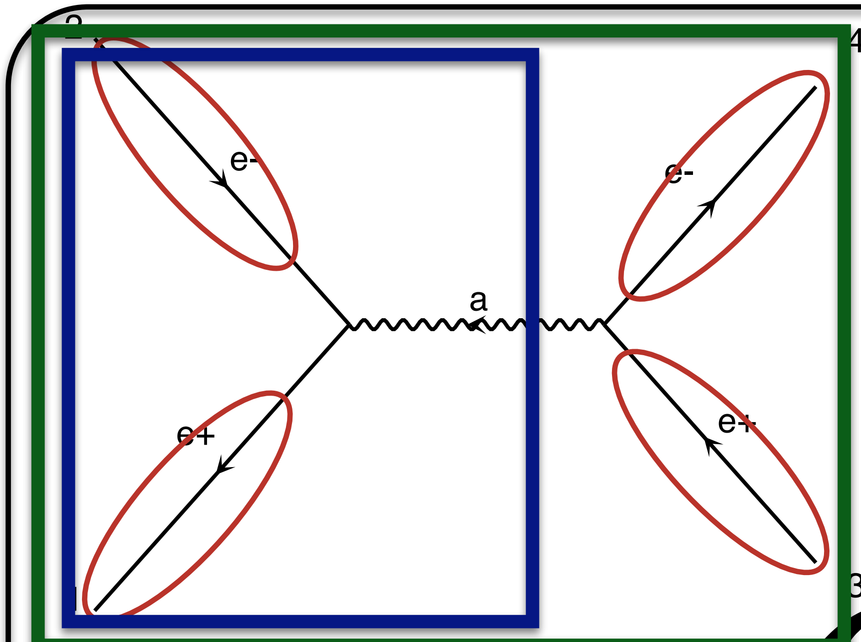
Very Hard (in general)

After

Helicity Amplitude

Idea

- Evaluate $\mathcal{M}$ for fixed helicity of external particles
  - ➡ Multiply $\mathcal{M}$ with $\mathcal{M}^*$ -> |$\mathcal{M}$|^2
  - ➡ Loop on Helicity and average the results



$$\mathcal{M} = ((\bar{u} e \gamma^\mu v)\frac{g_{\mu\nu}}{q^2})(\bar{v} e \gamma^\nu u))$$

Numbers for given helicity and momenta

Calculate propagator wavefunctions

Finally evaluate amplitude (c-number)

Lines present in the code.

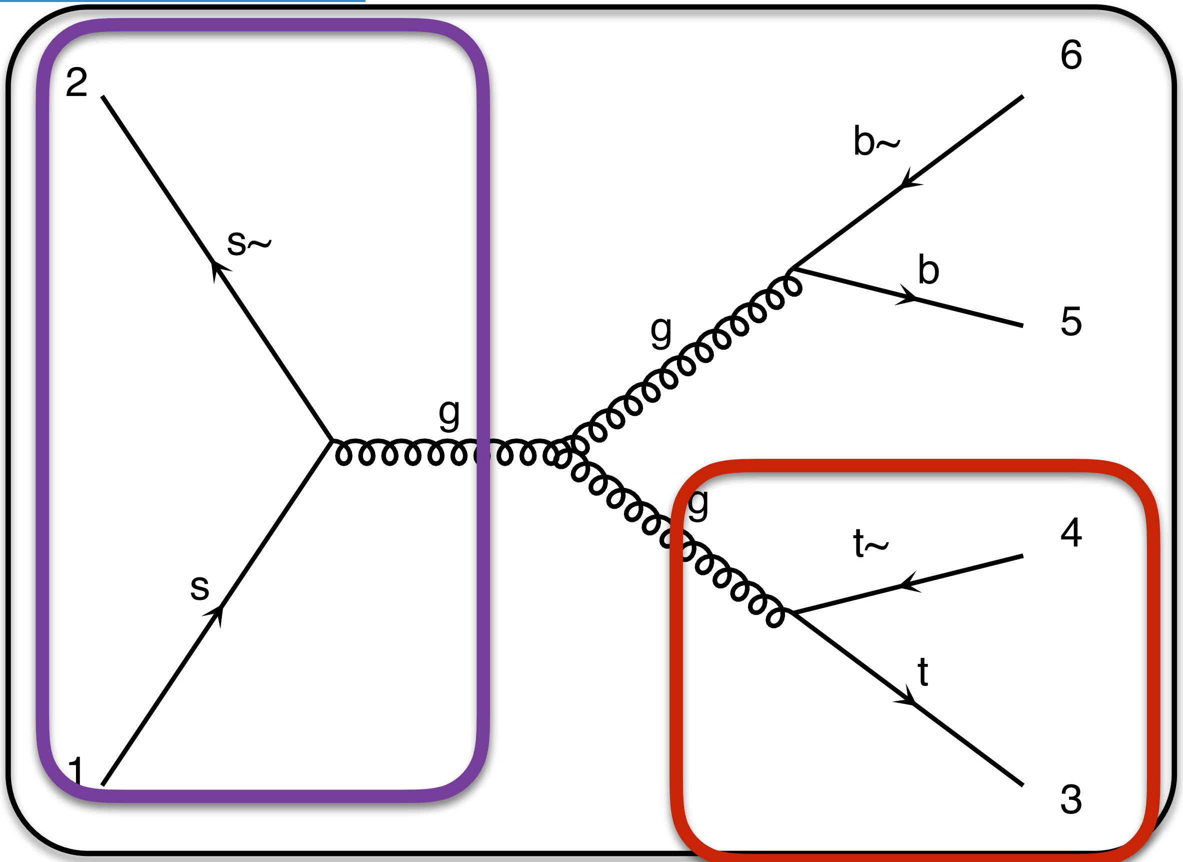
$$\bar{v}_1 = fct(\vec{p}_1, m_1)$$
$$u_2 = fct(\vec{p}_2, m_2)$$
$$v_3 = fct(\vec{p}_3, m_3)$$
$$\bar{u}_4 = fct(\vec{p}_4, m_4)$$
$$W_a = fct(\bar{v}_1, u_2, m_a, \Gamma_a)$$
$$\mathcal{M} = fct(v_3, \bar{u}_4, W_a)$$

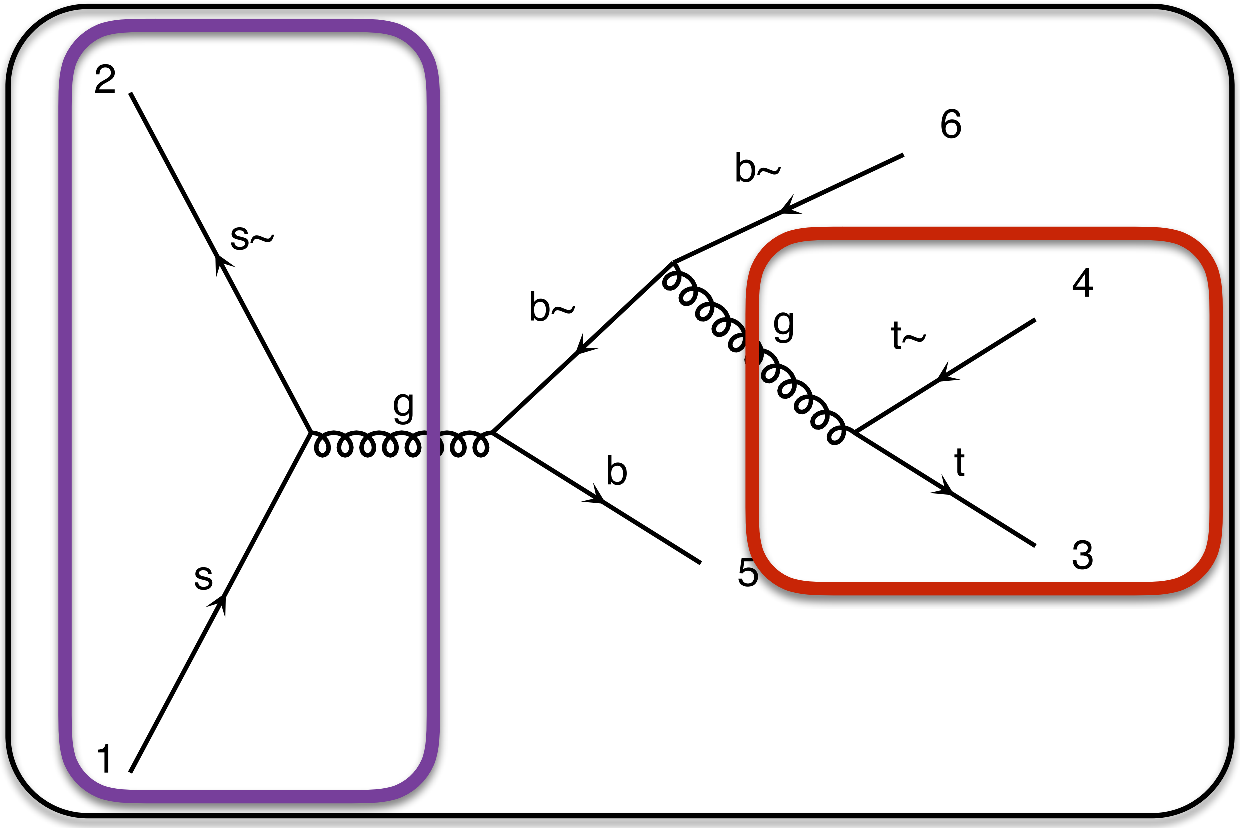
Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$

Comparison

	M diag	N particle
Analytical	M^2	$(N!)^2$
Helicity	M	$(N!)\,2^N$
Recycling	M	$(N-1)!\,2^{(N-1)}$





Comparison

	M diag	N particle	2 > 6
Analytical	M^2	$(N!)^2$	1.6e9
Helicity	M	$(N!)\,2^N$	1.0e7
Recycling	M	$(N-1)!\,2^{(N-1)}$	6.5e5
Recursion Relation	$log(M)$	$2^N\,2^{(N-1)}$	3.2e4

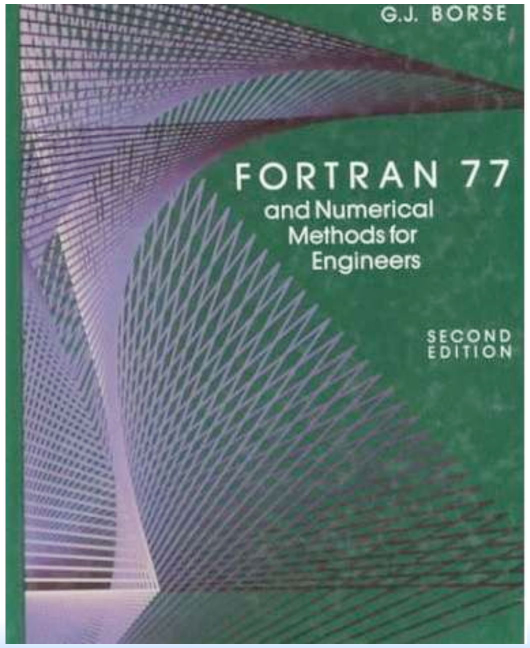
ALOHA

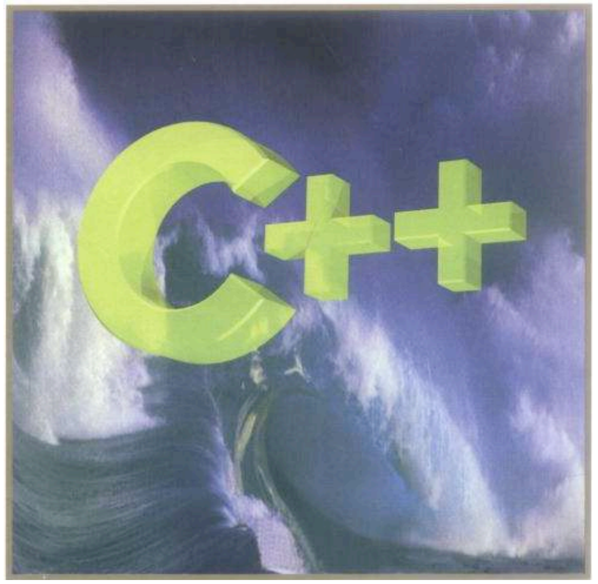
ALOHA ~~Google~~ translate

From: UFO To: Helicity Translate

Basically, any new operator can be handle by MG5/Pythia8 out of the box!

Type text or a website address or translate a document.





To Remember

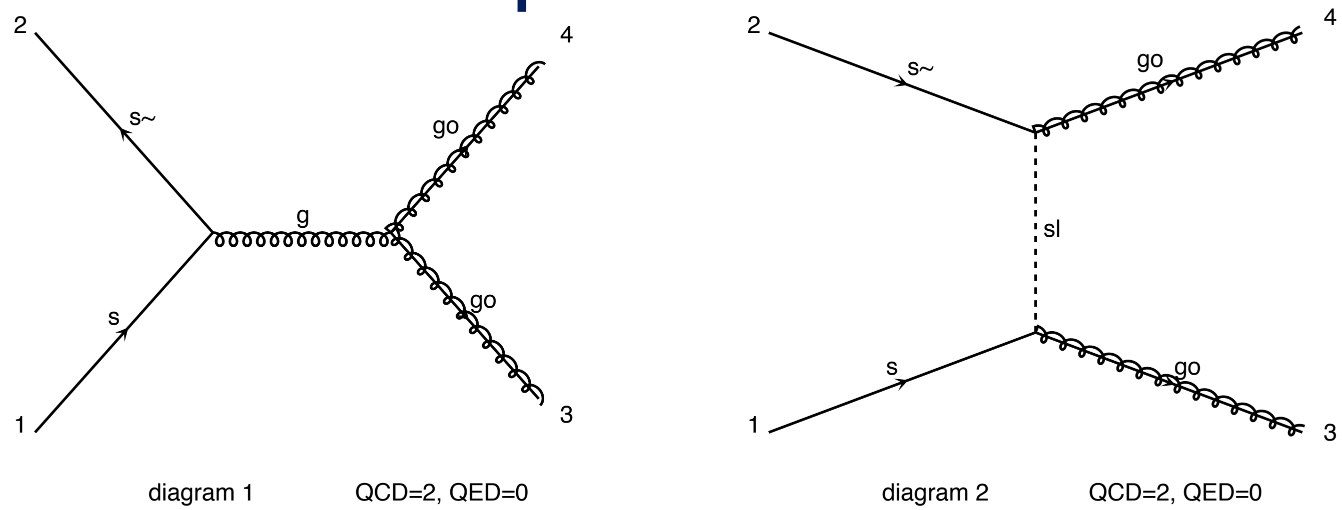
- Numerical computation faster than analytical computation
- We are able to compute matrix-elment
  - ➡ for large number of final state
  - ➡ for any BSM theory
  - ➡ actually also for loop

Monte Carlo Integration and Generation

Matrix-Element

Calculate a given process (e.g. gluino pair)

- Determine the production mechanism



- Evaluate the matrix-element

$|\mathcal{M}|^2$ ➡Need Feynman Rules!

- Phase-Space Integration

$$\sigma = \frac{1}{2s}\int |\mathcal{M}|^2 d\Phi(n)$$

Easy enough

Hard

Very Hard (in general)

Now

Monte Carlo Integration

Calculations of cross section or decay widths involve integrations over high-dimension phase space of very peaked functions:

$$\sigma = \frac{1}{2s} \int |\mathcal{M}|^2 d\Phi(n)$$

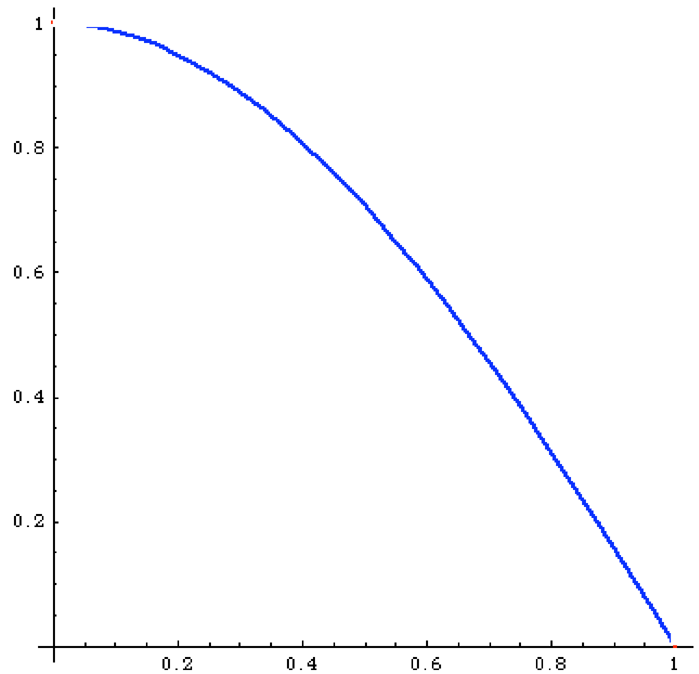
$Dim[\Phi(n)] \sim 3n$

General and flexible method is needed

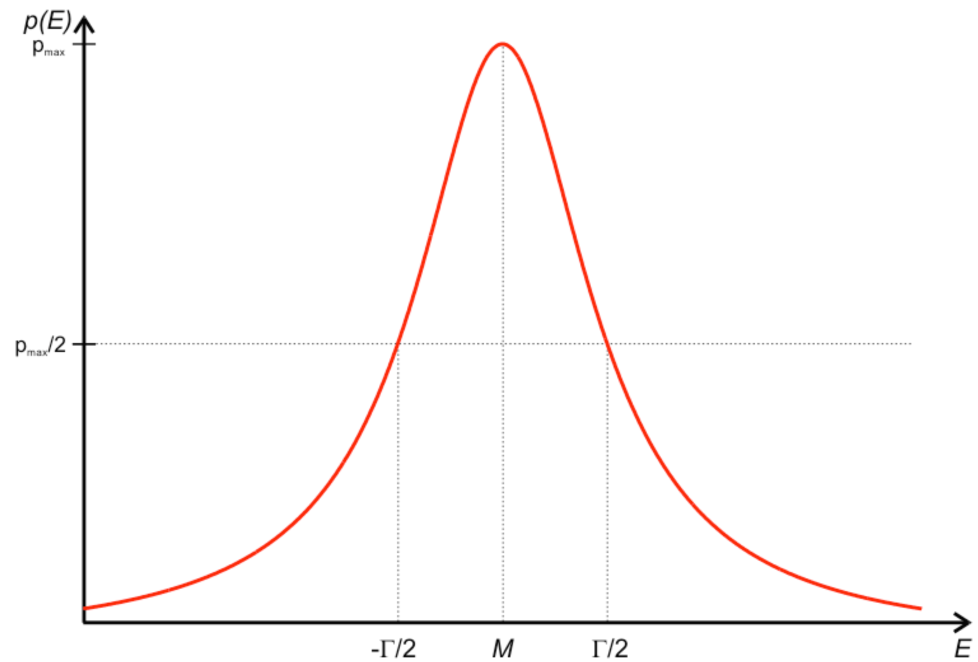
Not only integrating but also **generates events**

Integration

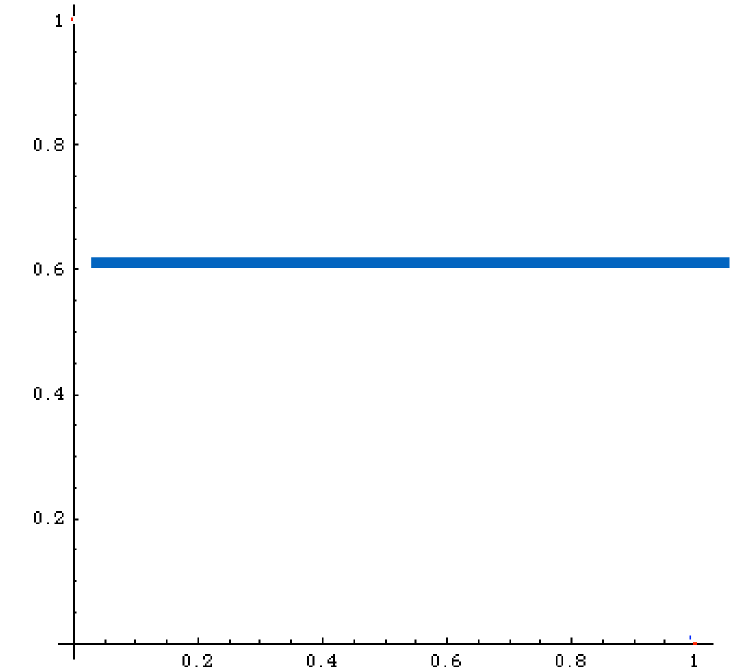
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$



$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$



$$\int dx\, C$$



	simpson	MC
3	0,638	0,3
5	0,6367	0,8
20	0,63662	0,6
100	0,636619	0,65
1000	0,636619	0,636

Method of evaluation

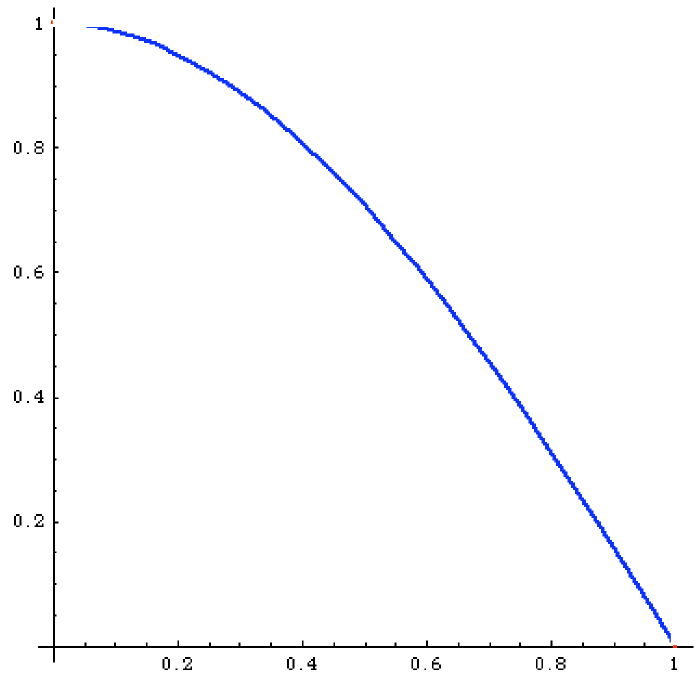
- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

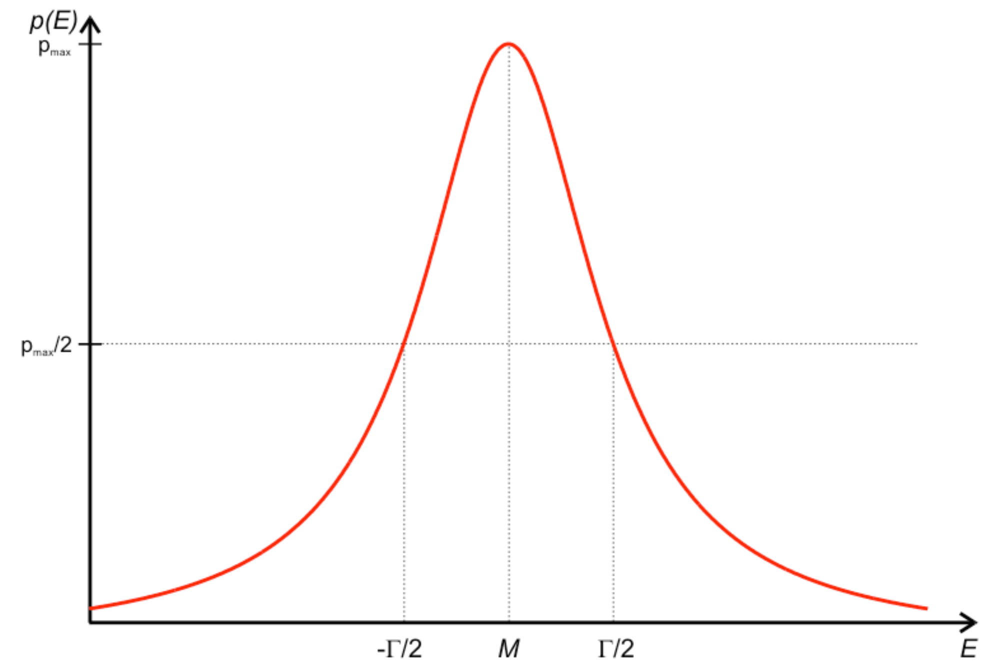
Integration

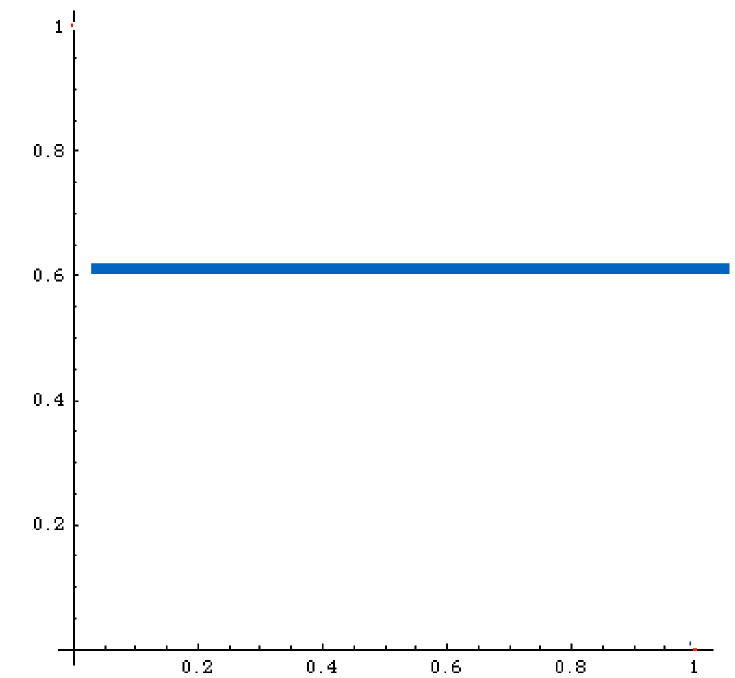
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$







Method of evaluation

- MonteCarlo $1/\sqrt{N}$
- Trapezium $1/N^2$
- Simpson $1/N^4$

More Dimension

$1/\sqrt{N}$

$1/N^{2/d}$

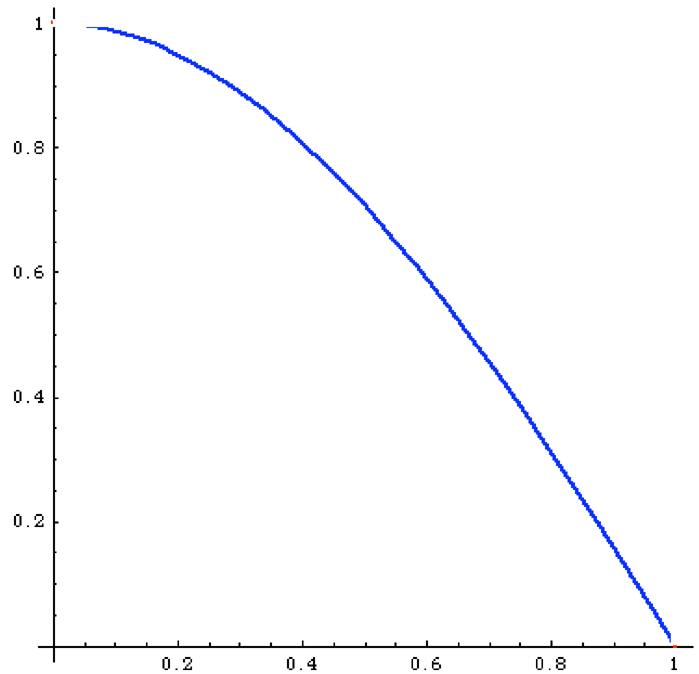
$1/N^{4/d}$

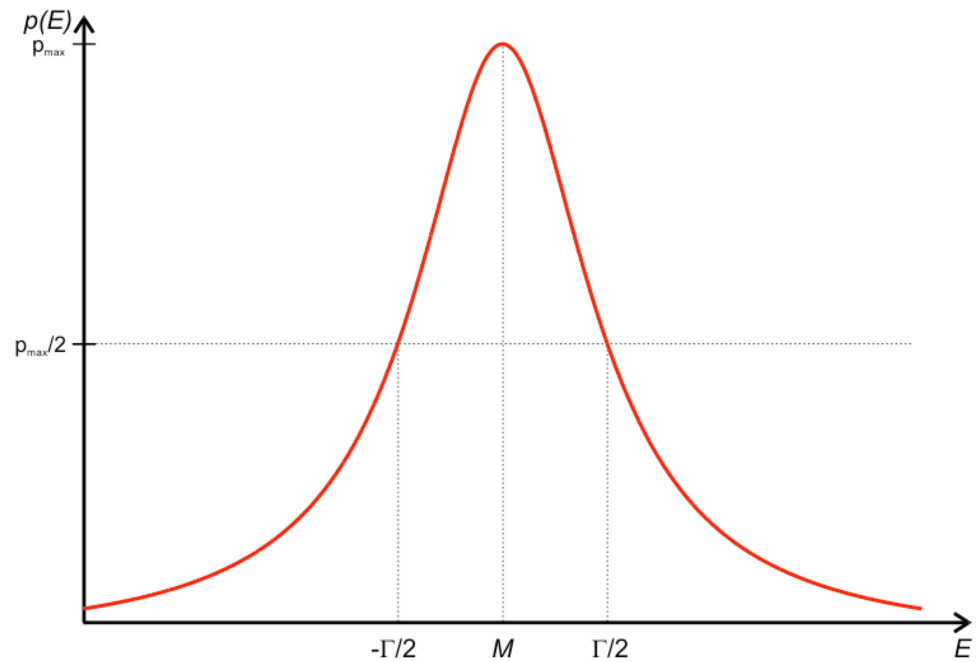
Integration

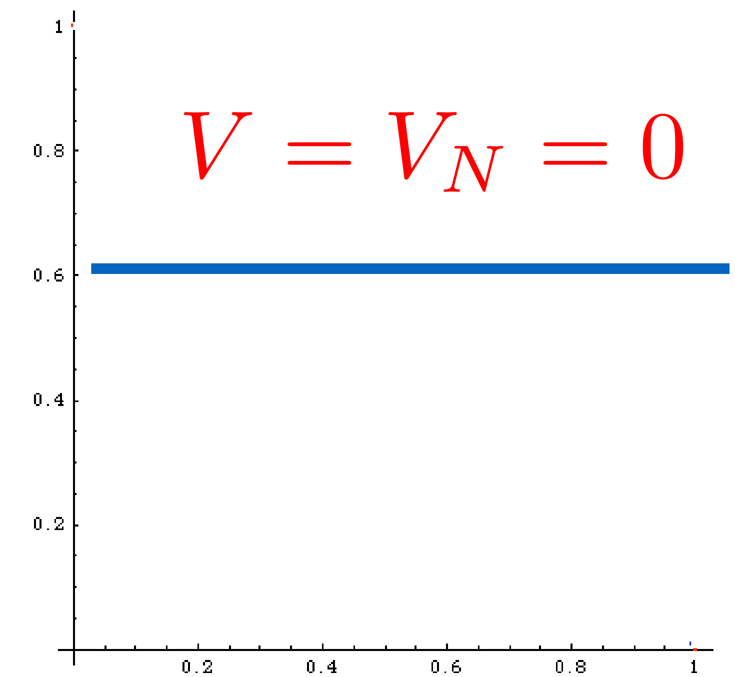
$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

$$\int dx\, C$$







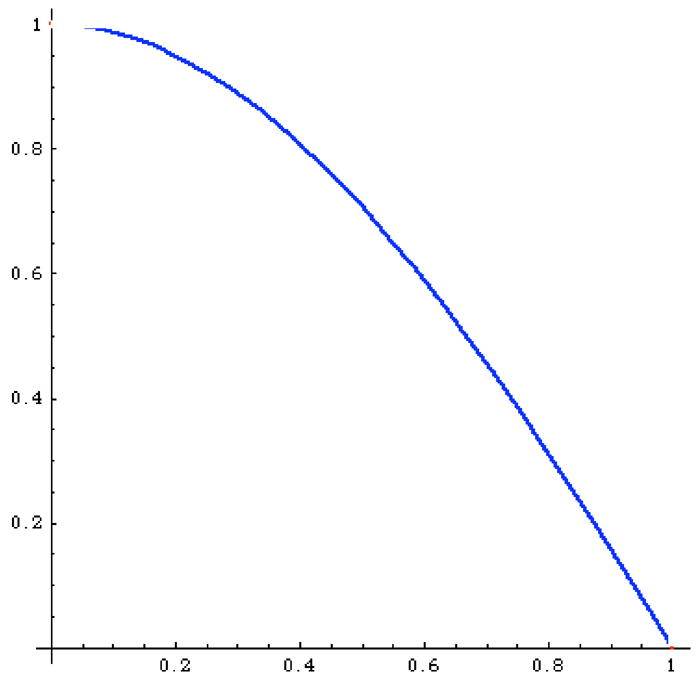
$$I = \int_{x_1}^{x_2} f(x)dx \quad \Rightarrow \quad I_N = (x_2 - x_1)\frac{1}{N}\sum_{i=1}^{N} f(x)$$

$$V = (x_2 - x_1)\int_{x_1}^{x_2} [f(x)]^2 dx - I^2 \quad \Rightarrow \quad V_N = (x_2 - x_1)^2 \frac{1}{N}\sum_{i=1}^{N}[f(x)]^2 - I_N^2$$

$$I = I_N \pm \sqrt{V_N/N}$$

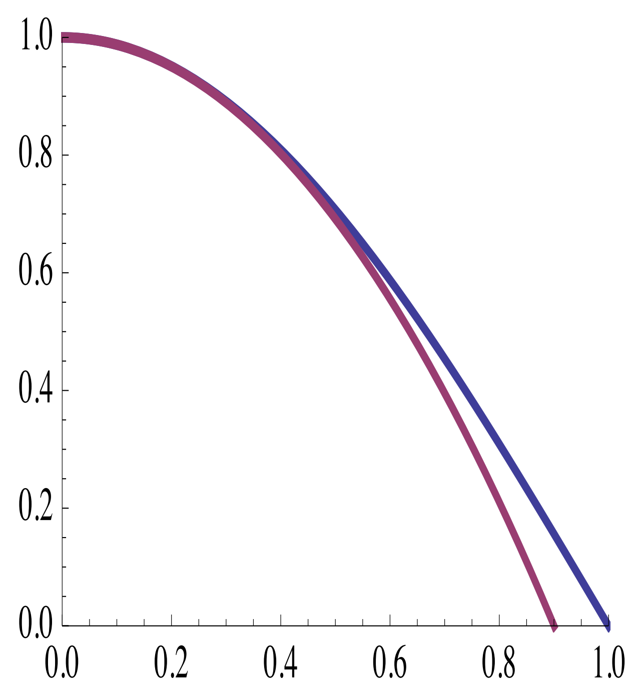
Can be minimized!

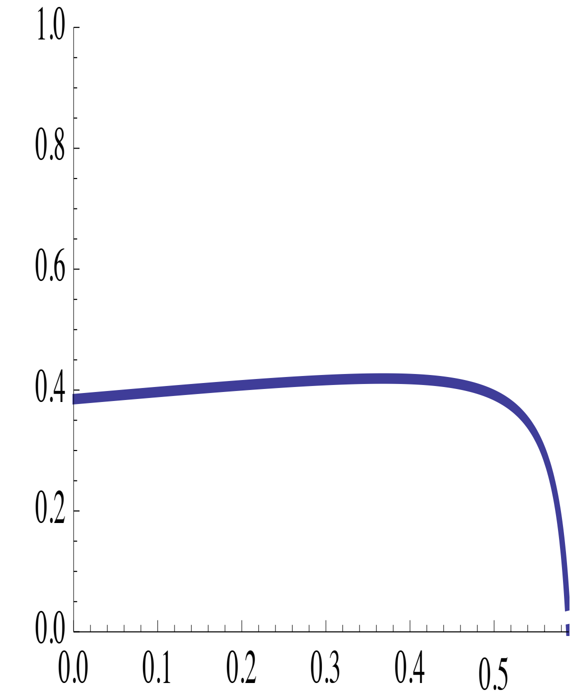
Importance Sampling



$$I = \int_0^1 dx \cos\frac{\pi}{2}x$$

$$I_N = 0.637 \pm 0.307/\sqrt{N}$$





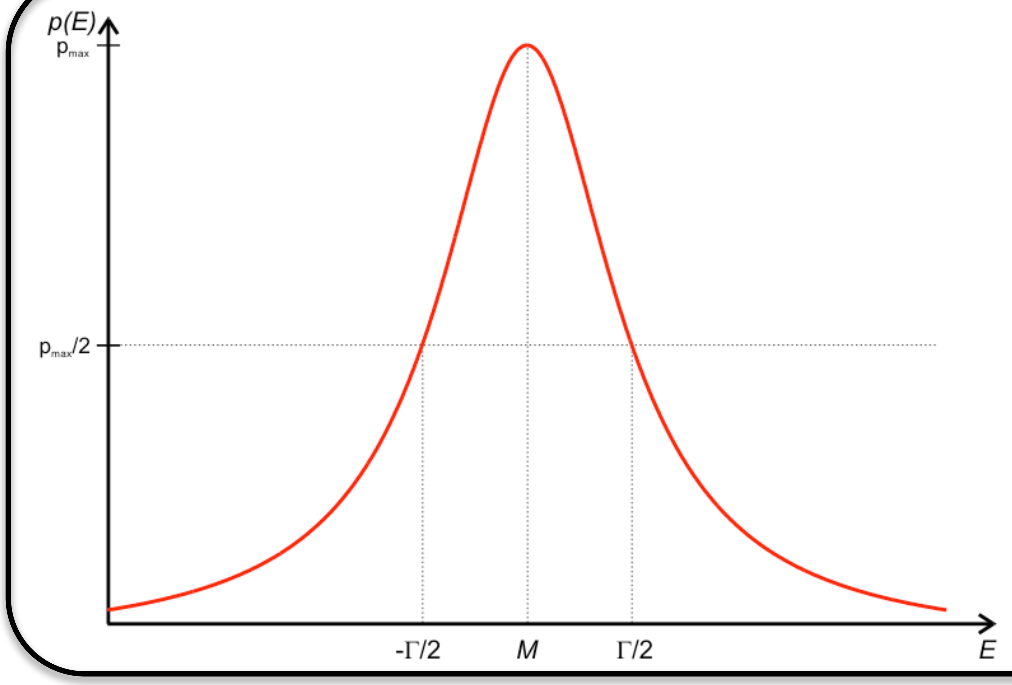
$$I = \int_0^1 dx(1-cx^2)\frac{\cos\left(\frac{\pi}{2}x\right)}{(1-cx^2)} = \int_{\xi_1}^{\xi_2} d\xi \frac{\cos\frac{\pi}{2}x[\xi]}{1-x[\xi]^2c}$$

$\simeq 1$

$$I_N = 0.637 \pm 0.031/\sqrt{N}$$

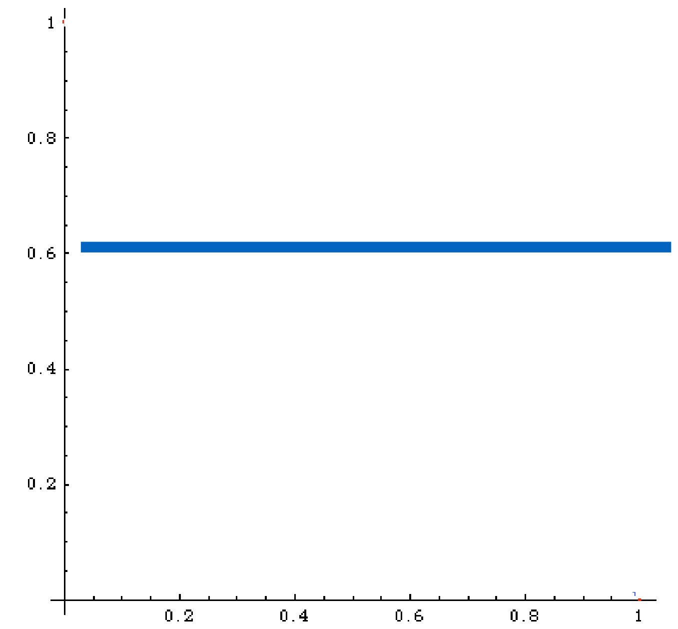
The Phase-Space parametrization is important to have an efficient computation!

Importance Sampling

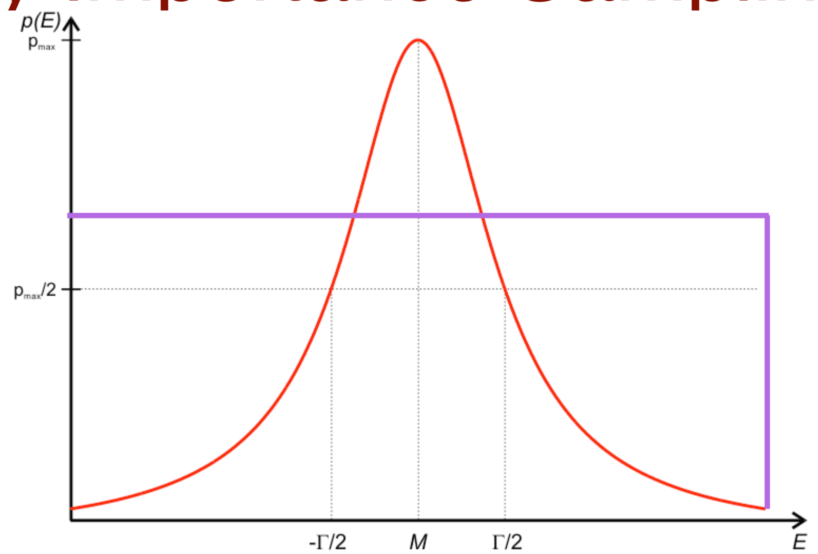


$$\int \frac{dq^2}{(q^2 - M^2 + iM\Gamma)^2}$$

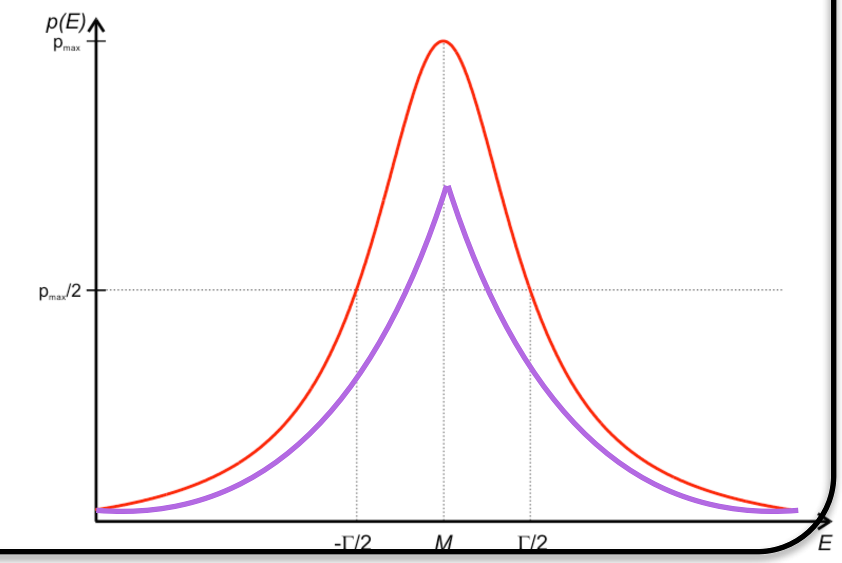
$$\xi = arctan\left(\frac{q^2 - M^2}{\Gamma\, M}\right)$$



Why Importance Sampling?



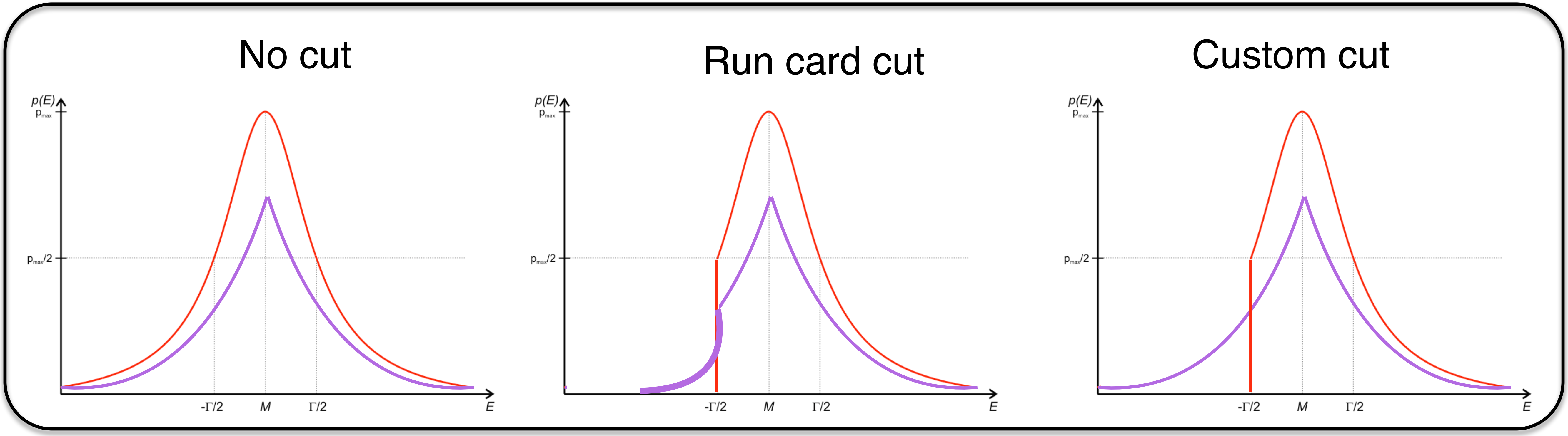
Probability of using that point p(x)



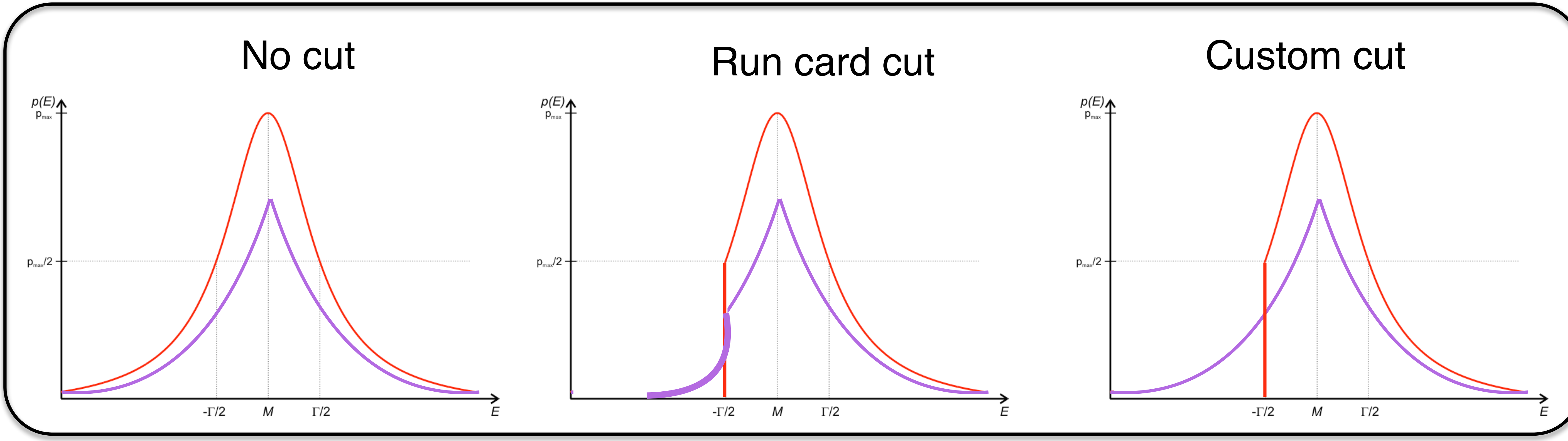
The change of variable ensure that the evaluation of the function is done where the function is the largest!

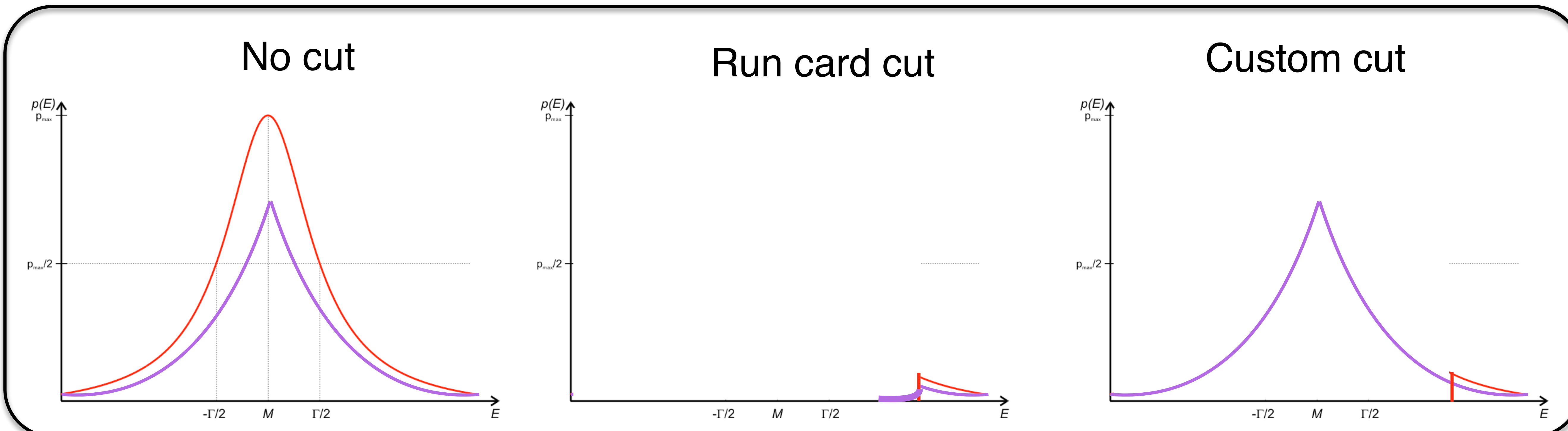
Cut Impact

- Events are generated according to our best knowledge of the function
  - ➡Basic cut include in this “best knowledge”
  - ➡Custom cut are ignored



Cut Impact





Might miss the contribution and think it is just zero.

Importance Sampling

Key Point

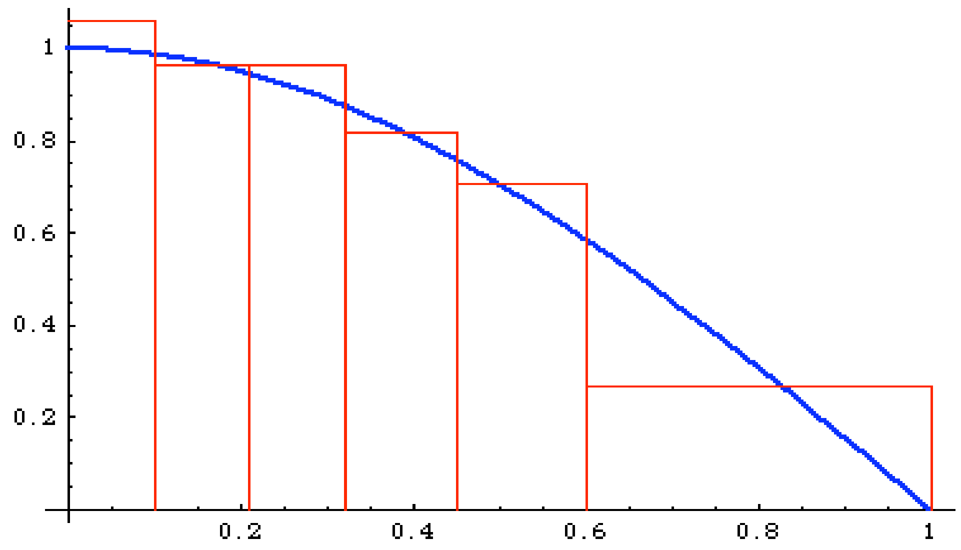
- Generate the random point in a distribution which is close to the function to integrate.
- This is a change of variable, such that the function is flatter in this new variable.
- Needs to know an approximate function.

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!

Adaptative Monte-Carlo

- Create an approximation of the function on the flight!



Algorithm

1. Creates bin such that each of them have the same contribution.

   ➡Many bins where the function is large

2. Use the approximate for the importance sampling method.

VEGAS

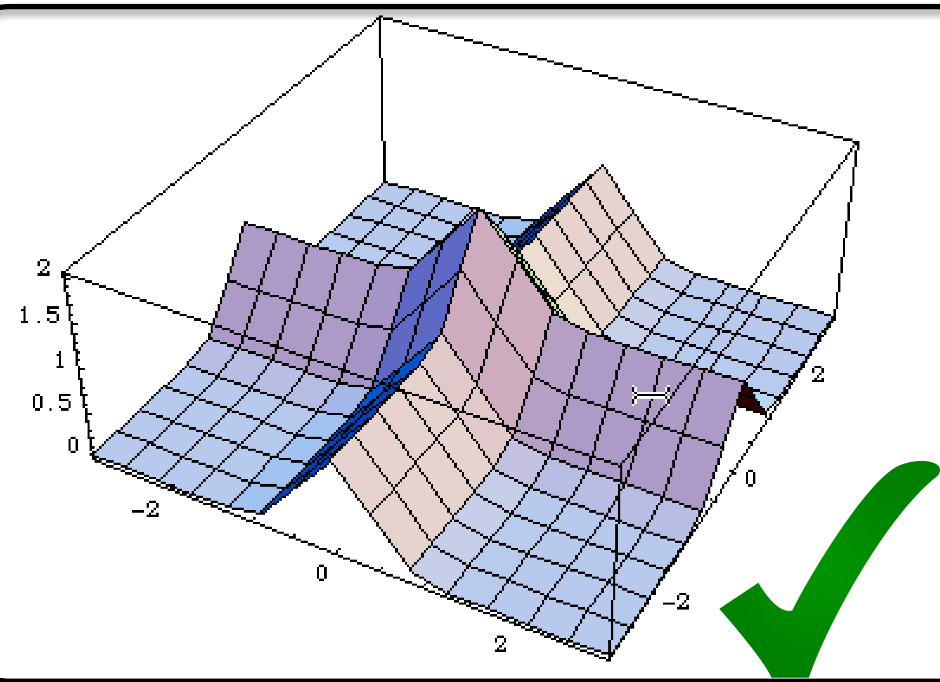
More than one Dimension

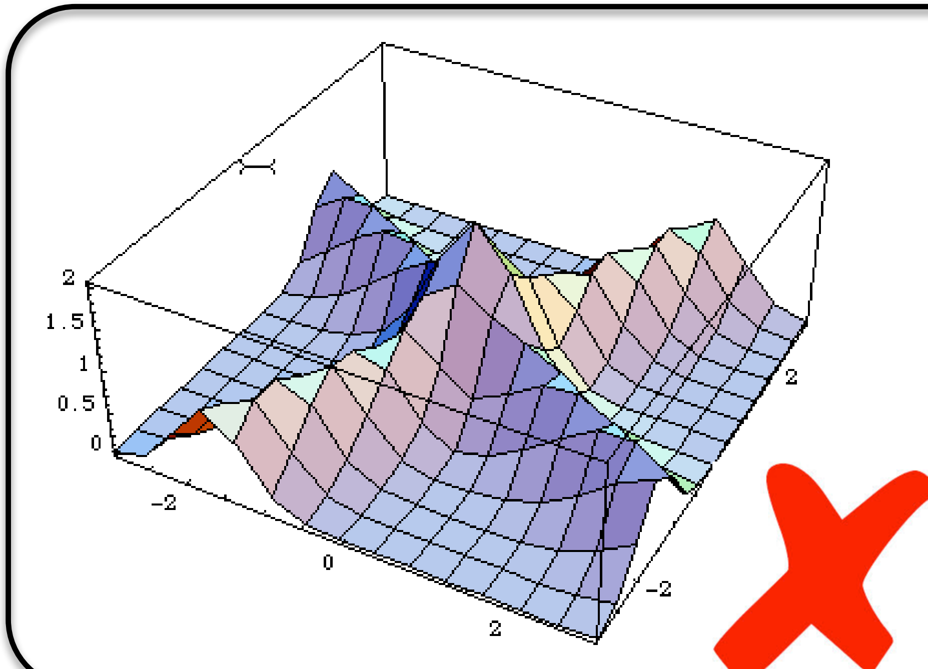
- VEGAS works only with 1(few) dimension
  - ➡memory problem

Solution

- Use projection on the axis

$$\vec{p}(x) = p(x) \cdot p(y) \cdot p(z) \ldots$$

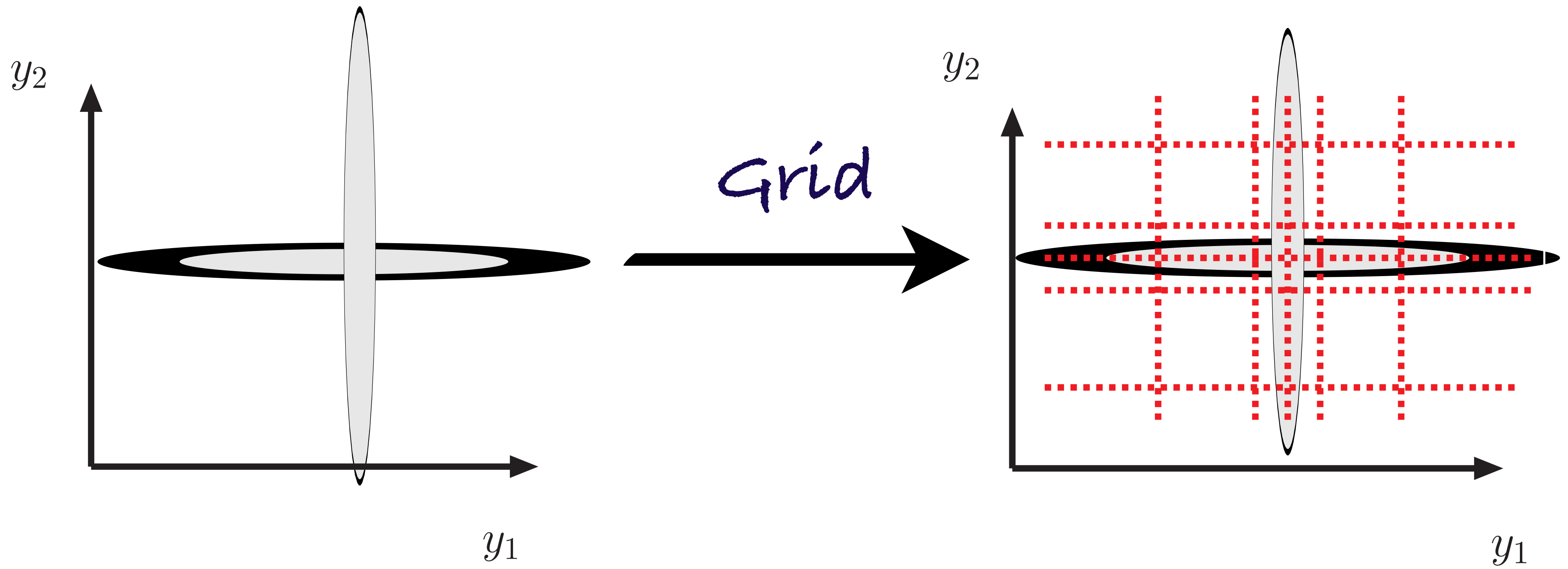




- We need to ensure the factorization !
  - ➡Additional change of variable

Monte-Carlo Integration

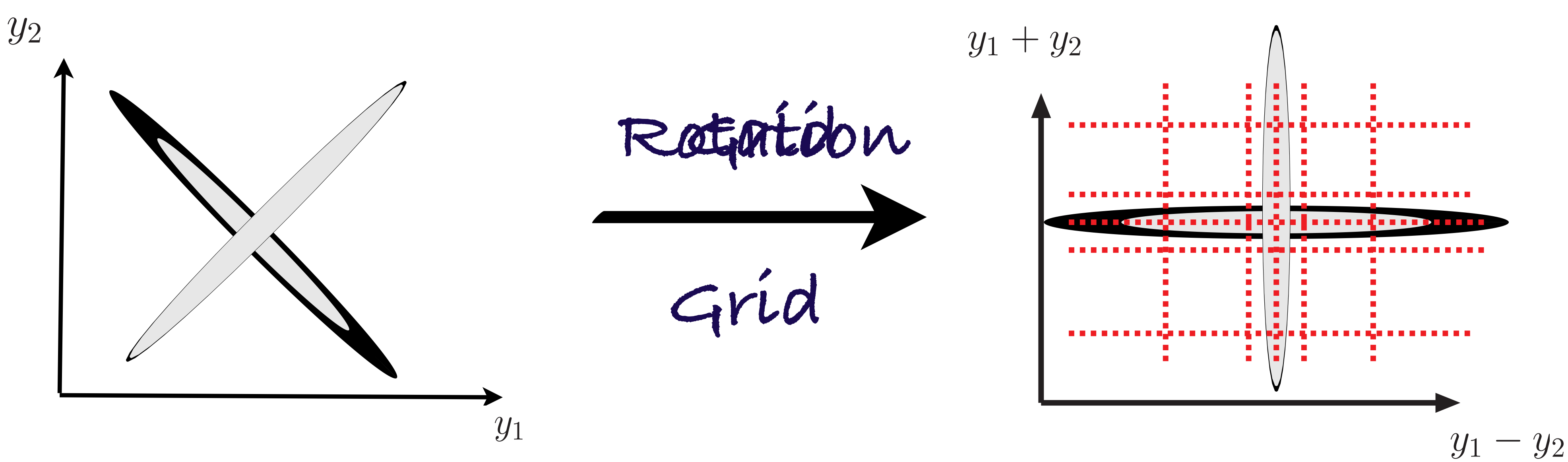
- The choice of the parameterisation has a strong impact on the efficiency



- The adaptive Monte-Carlo Technique picks point in interesting areas
  → The technique is efficient

Monte-Carlo Integration

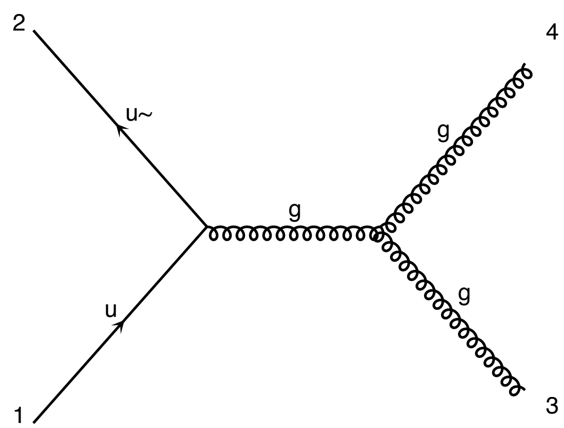
- The choice of the parametrization has a strong impact on the efficiency

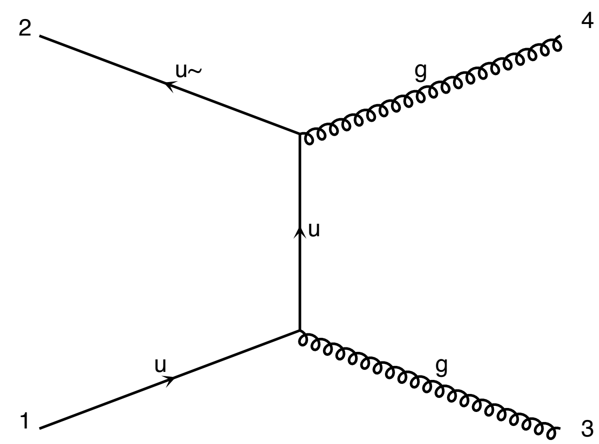


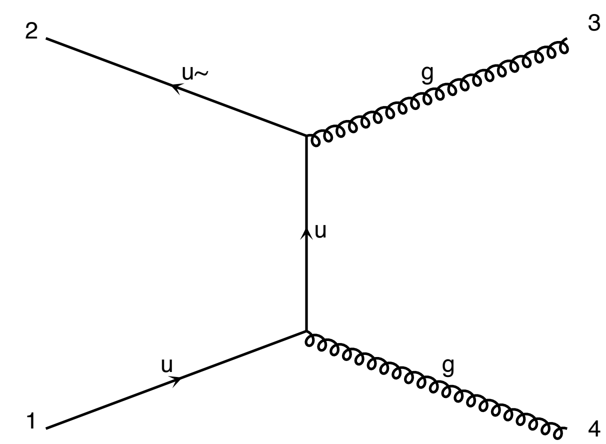
- The adaptive Monte-Carlo Techniques picks point interesting areas

→ The technique is efficient

Example: QCD 2 → 2







$$\propto \frac{1}{\hat{s}} = \frac{1}{(p_1 + p_2)^2} \qquad \propto \frac{1}{\hat{t}} = \frac{1}{(p_1 - p_3)^2} \qquad \propto \frac{1}{\hat{u}} = \frac{1}{(p_1 - p_4)^2}$$

Three very different pole structures contributing to the same matrix element.

Multi-channel based on single diagrams*

*Method used in MadGraph

Trick in MadEvent: Split the complexity

$$\int |M_{tot}|^2 = \int \frac{\sum_i |M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2 = \sum_i \int \frac{|M_i|^2}{\sum_j |M_j|^2} |M_{tot}|^2$$

$$\approx 1$$

Key Idea

- Any single diagram is "easy" to integrate (pole structures/ suitable integration variables known from the propagators)
- Divide integration into pieces, based on diagrams
- All other peaks taken care of by denominator sum

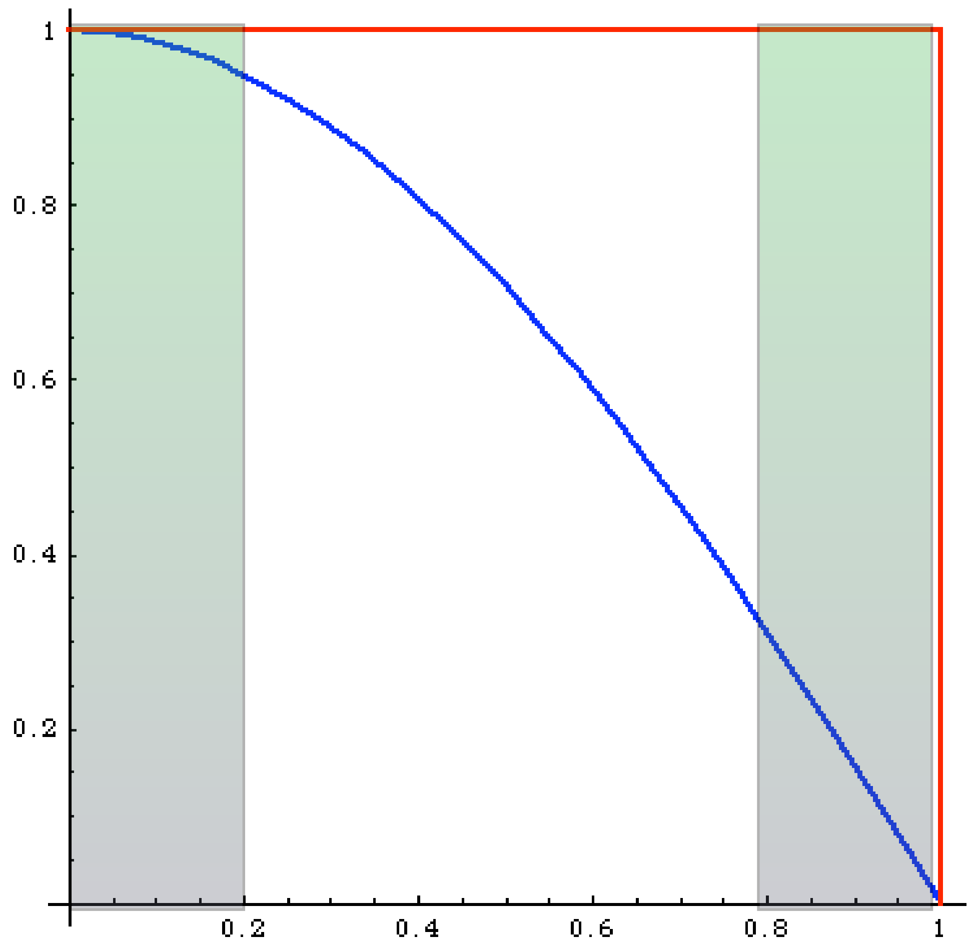
N Integral

- Errors add in quadrature so no extra cost
- "Weight" functions already calculated during $|M|^2$ calculation
- Parallel in nature

To Remember

- Phase-Space integration are difficult
- We need to know the function
  - ➡ Be careful with cuts
- MadGraph split the integral in different contribution linked to the Feynman Diagram
  - ➡Those are not the contribution of a given diagram

Event generation

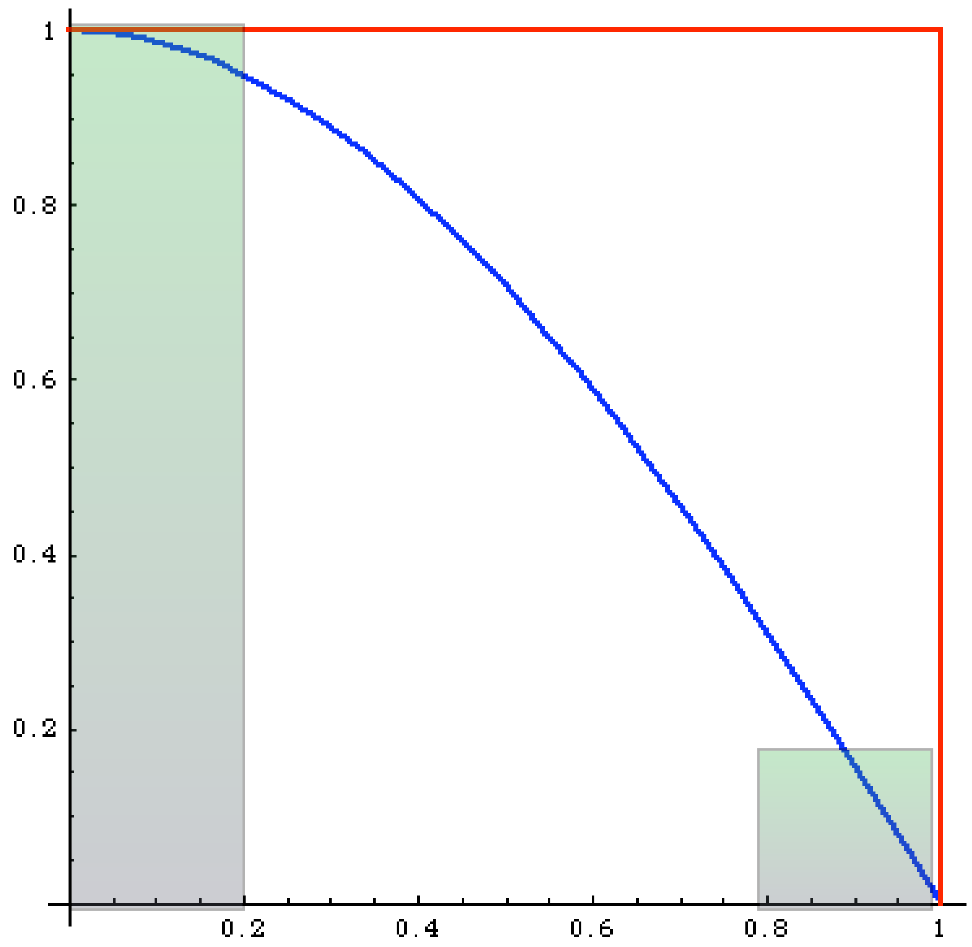


What's the difference between weighted and unweighted?

Weighted:

Same # of events in areas of phase space with very different probabilities: events must have different weights

Event generation



What's the difference between weighted and unweighted?

Unweighted:

events is proportional to the probability of areas of phase space:
events have all the same weight ("unweighted")

Events distributed as in nature

Event generation

$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$

Number between 0 and 1 (assuming positive function)
-> re-interpret as the probability to keep the events

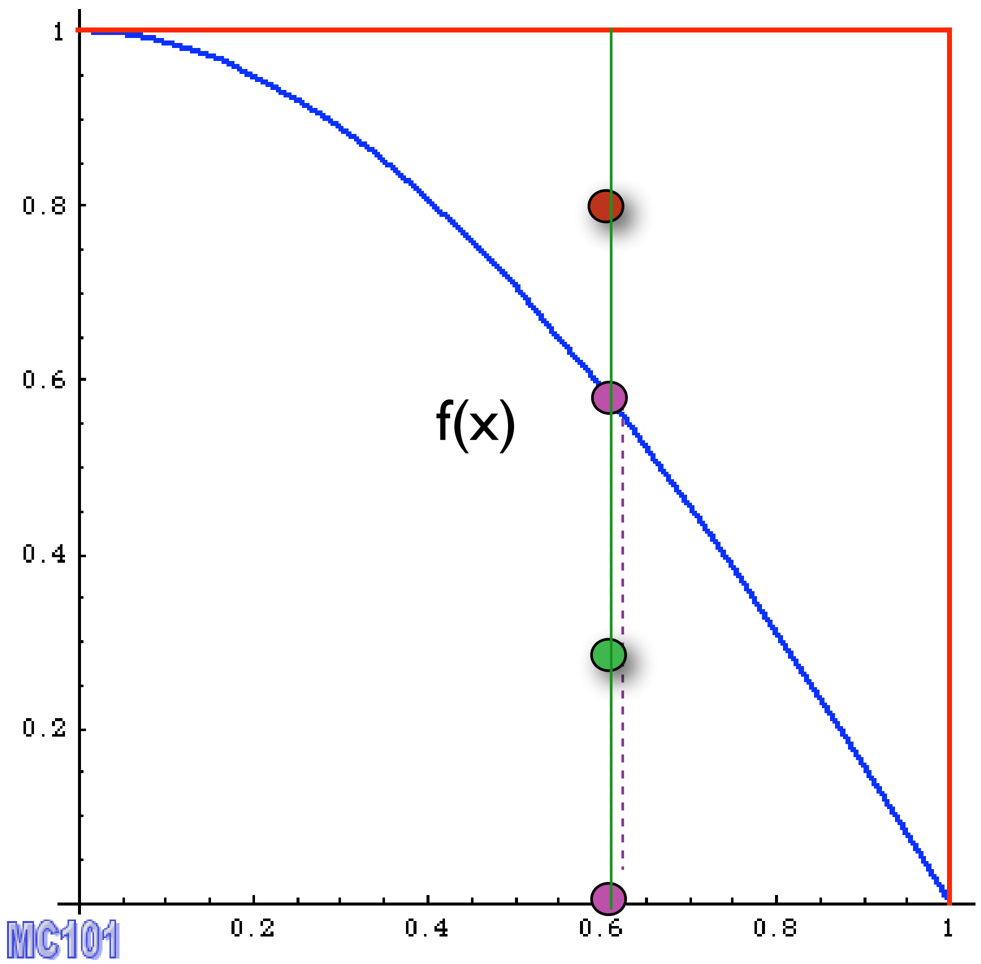
$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i) = \frac{1}{N}\sum_{i=1}^{N} P(x_i)max(f)$$

Let's reduce the sample size by playing the lottery.
For each events throw the dice and see if we keep or reject the events

$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i) = \frac{1}{N}\sum_{i=1}^{N} P(x_i)max(f) \simeq \frac{max(f)}{N}\sum_{i=1}^{n} 1$$

Event generation

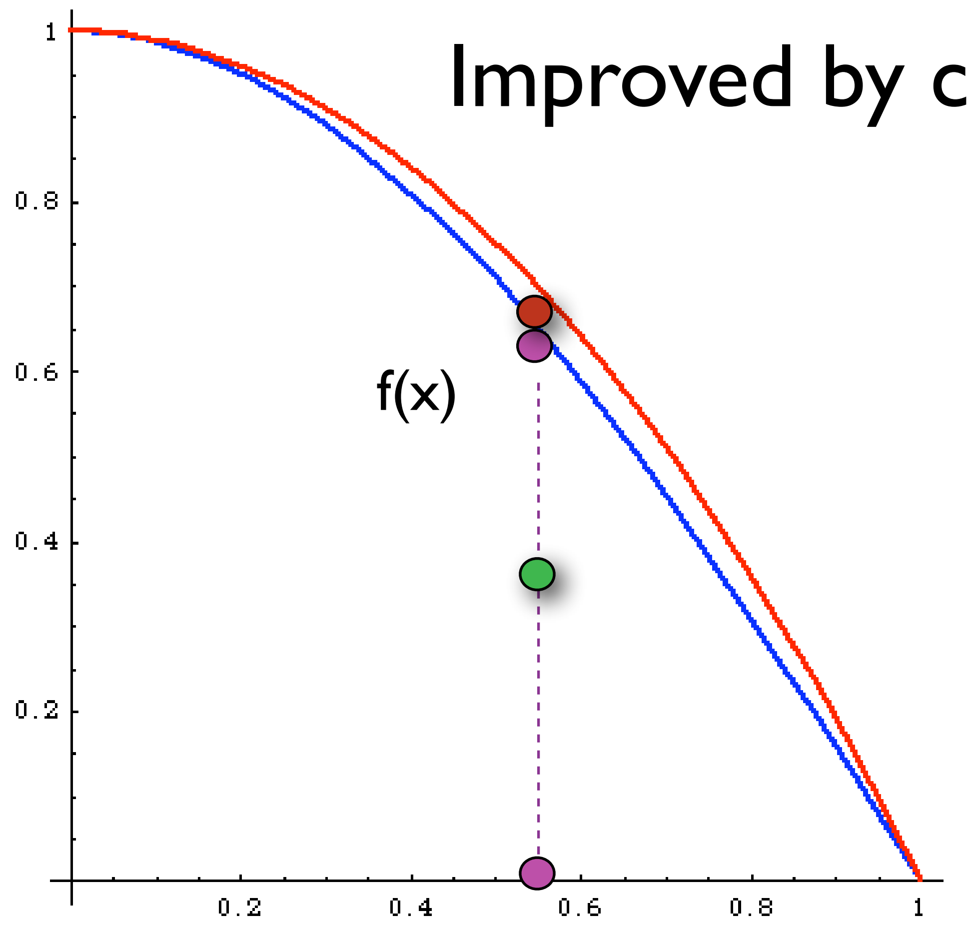
$$\int f(x)dx = \frac{1}{N}\sum_{i=1}^{N} f(x_i) = \frac{1}{N}\sum_{i=1}^{N} \frac{f(x_i)}{max(f)} max(f)$$



1. pick x_i
2. calculate $f(x_i)$
3. pick $y \in [0, max(f)]$
4. Compare:
   if $y < f(x_i)$ accept event,

   else reject it.

Event generation



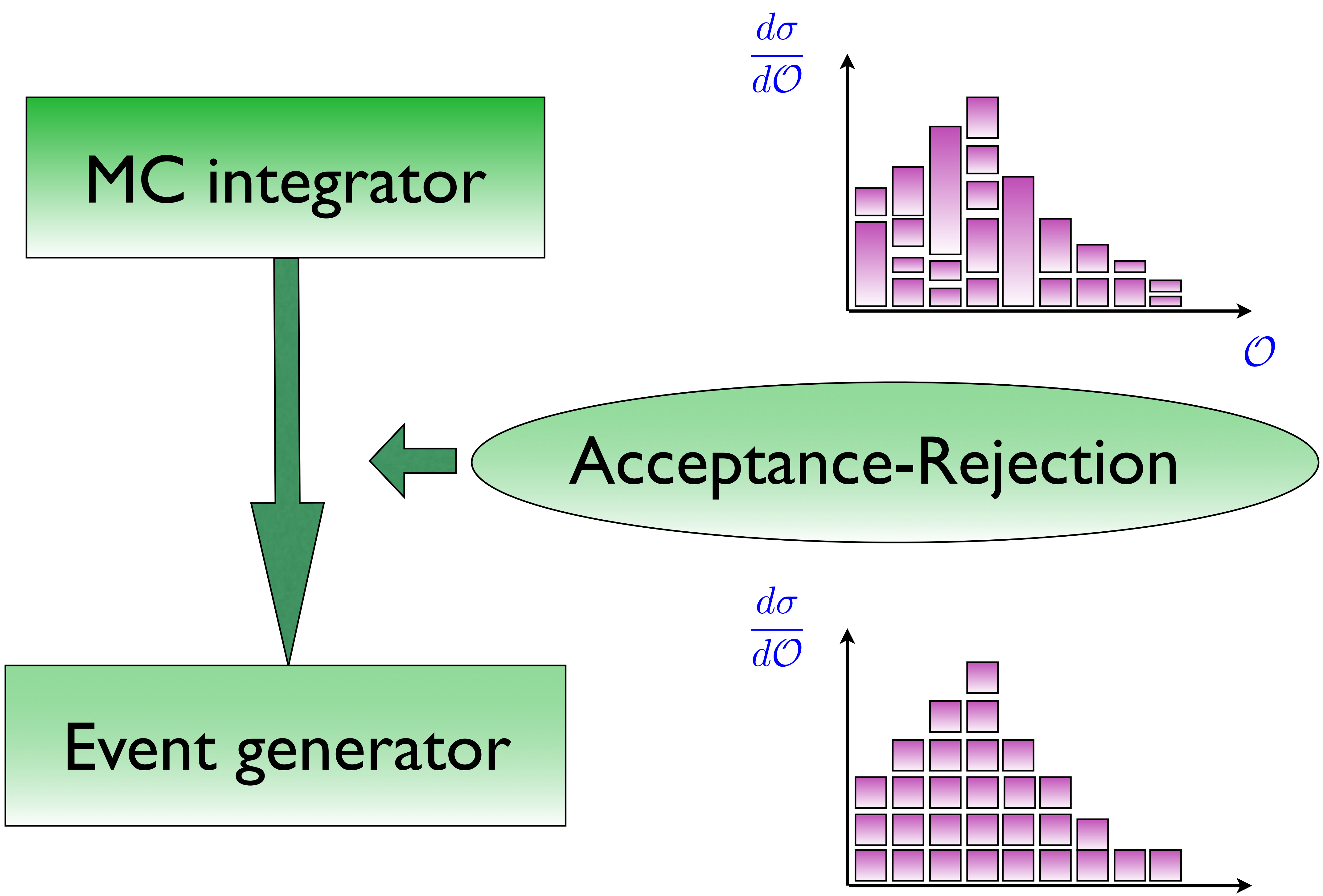
Improved by combining with importance sampling:

1. pick x distributed as p(x)
2. calculate f(x) and p(x)
3. pick 0<y<1
4. Compare:
   if f(x)>y p(x) accept event,

else reject it.

much better efficiency!!!

Event generation



☞ **This is possible only if f(x)<∞ AND has definite sign!**

Monte-Carlo Summary

Bad Point

- Slow Convergence (especially in low number of Dimension)
- Need to know the function
  - Impact on cut

Good Point

- Complex area of Integration
- Easy error estimate
- quick estimation of the integral
- Possibility to have unweighted events

What have we learned!

$$\sum_{a,b} \int dx_1 dx_2 d\Phi_{\rm FS}\, f_a(x_1,\mu_F) f_b(x_2,\mu_F)\, \hat{\sigma}_{ab\to X}(\hat{s},\mu_F,\mu_R)$$

Phase-space integral

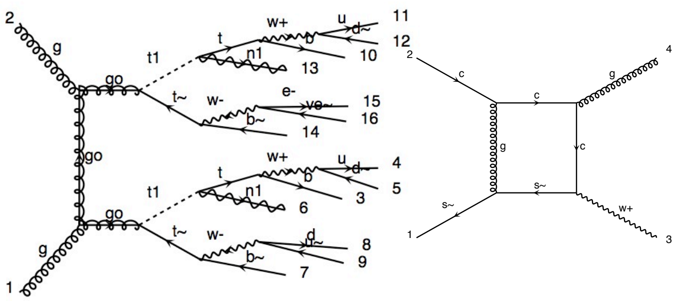
Parton density functions

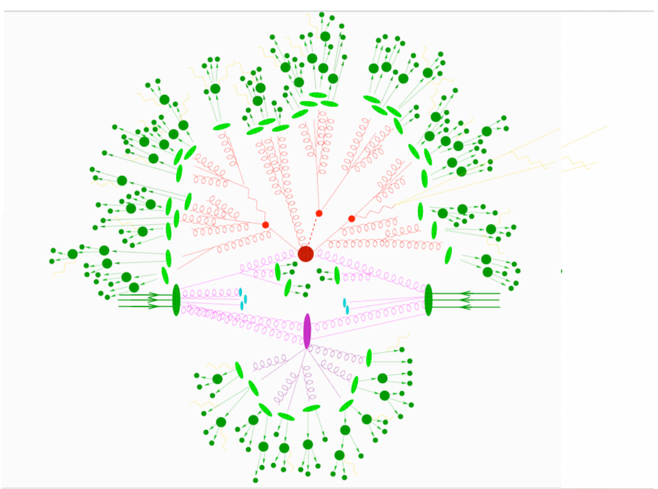
Parton-level cross section

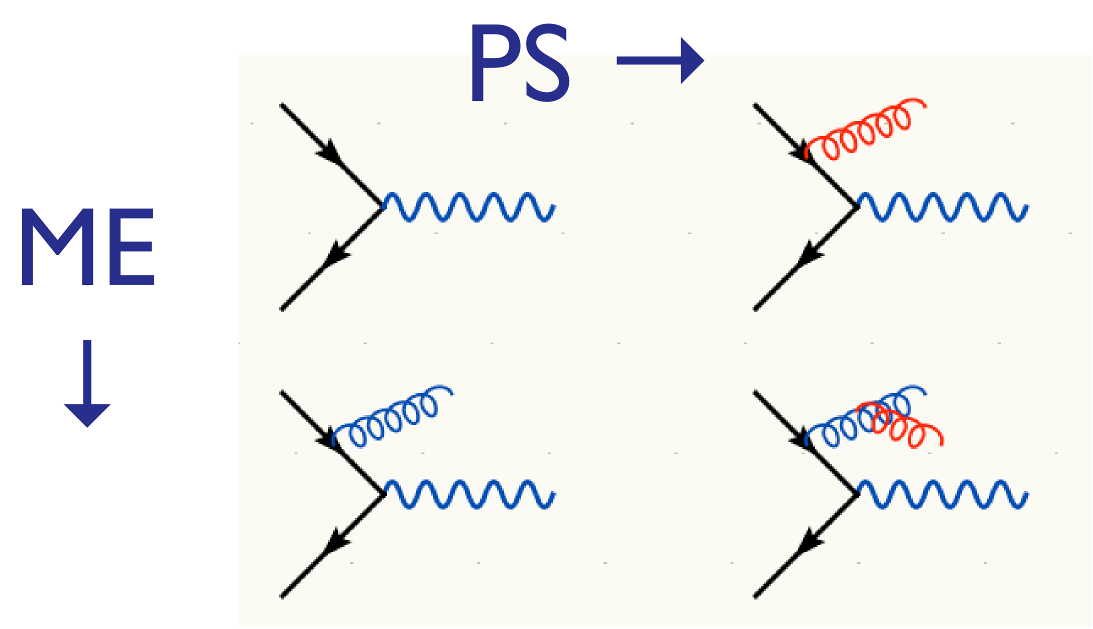
- The Importance of PDF
  - ➡Defines the physics
- Evaluation of Matrix Element
  - ➡ Numerical method faster than analytical formula
  - ➡ cross-section prediction needs NLO
- Phase Space Integration
  - ➡ Need to know in advance what we integrate. Be careful with strong cuts!

Type of generation

	Tree (B)SM	NLO (QCD) (SM)	NLO (QCD) (BSM)	NLO (EW) (SM)	Loop Induced (B)SM
Fix Order	✓	✓	✓	✓	✓





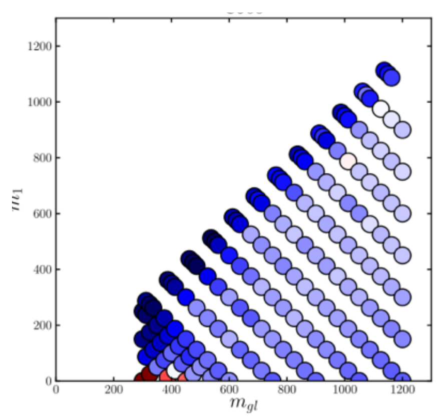


LO Feature

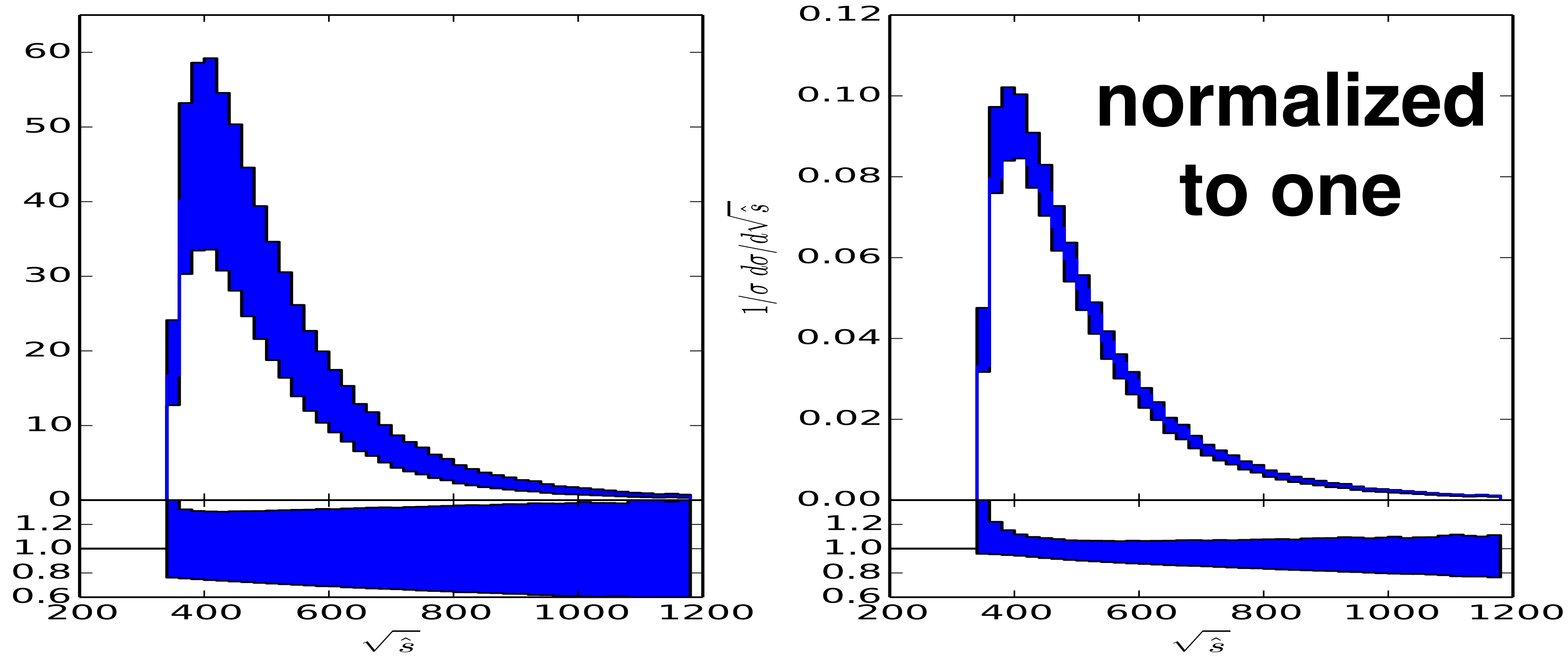
Auto-Width

$$\Gamma = ?$$

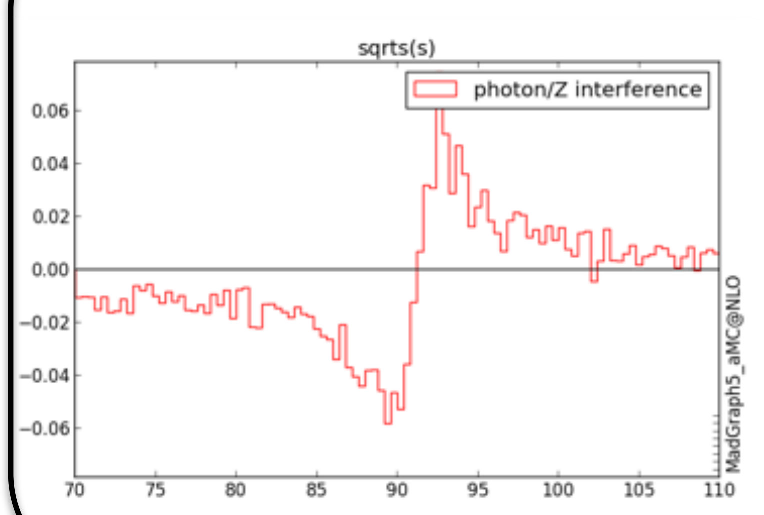
Parameter scan



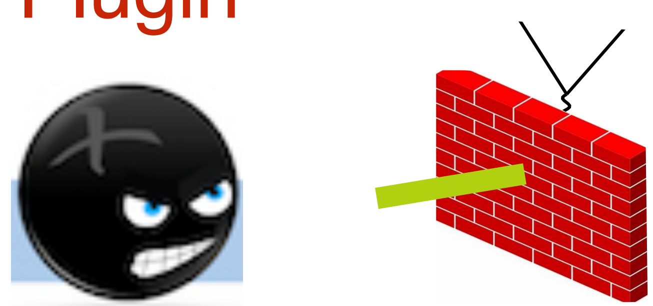
Systematics



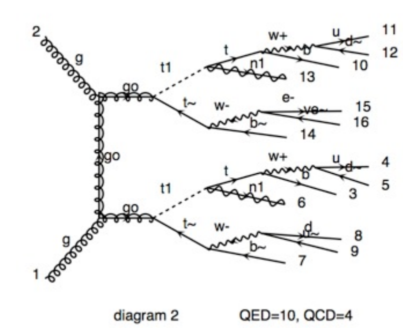
Interference



Plugin



Narrow-width



Interface

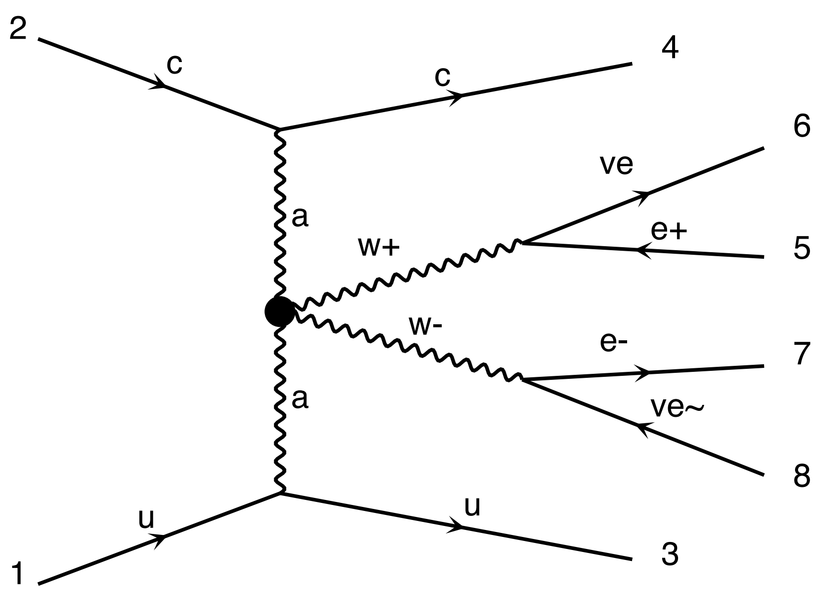
MadAnalysis 5

BSM re-weighting

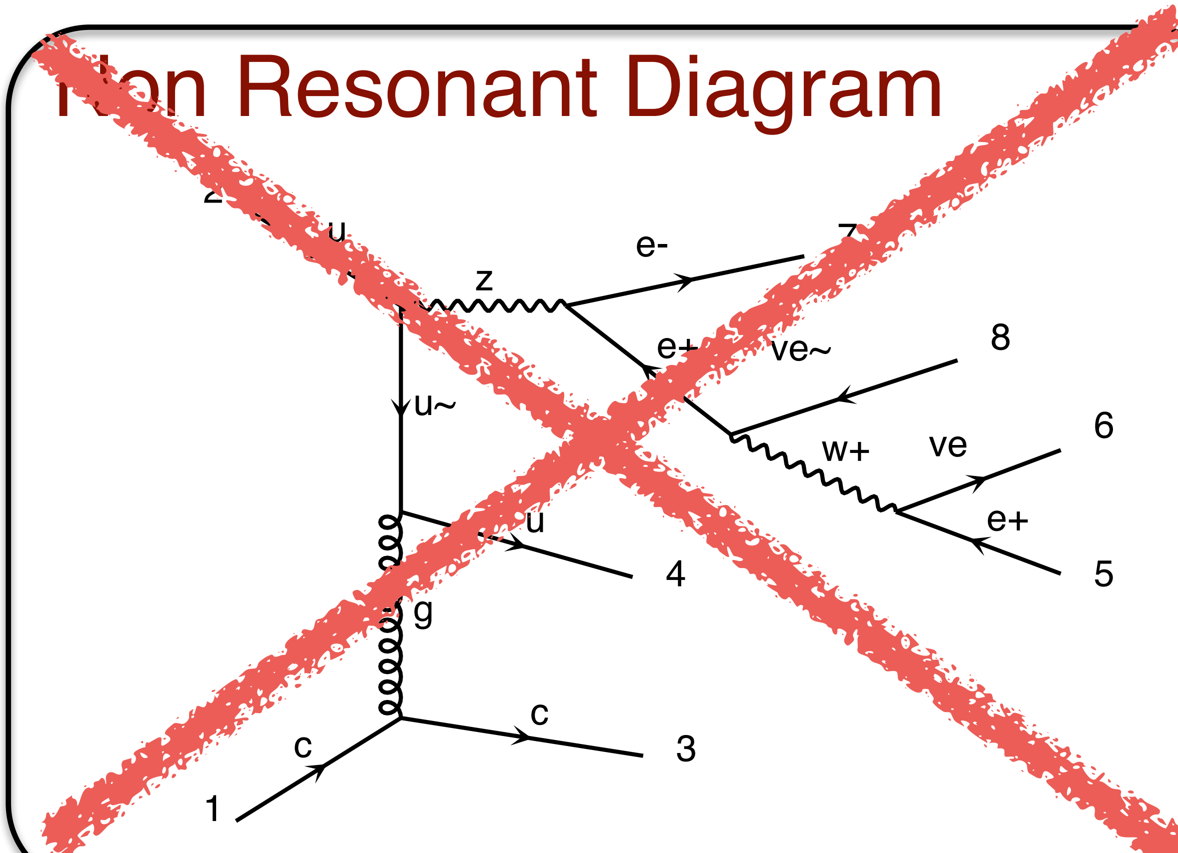
$$|M_{new}|^2 / |M_{old}|^2$$

Decay

Resonant Diagram



Non Resonant Diagram



Problem

- Process complicated to have the full process

  ➡Including off-shell contribution

Solution

- Only keep on-shell contribution

Narrow-Width Approx.

Theory

$$\int dq^2 \left| \frac{1}{q^2 - M^2 - iM\Gamma} \right|^2 \approx \frac{\pi}{M\Gamma} \, \delta(q^2 - M^2)$$

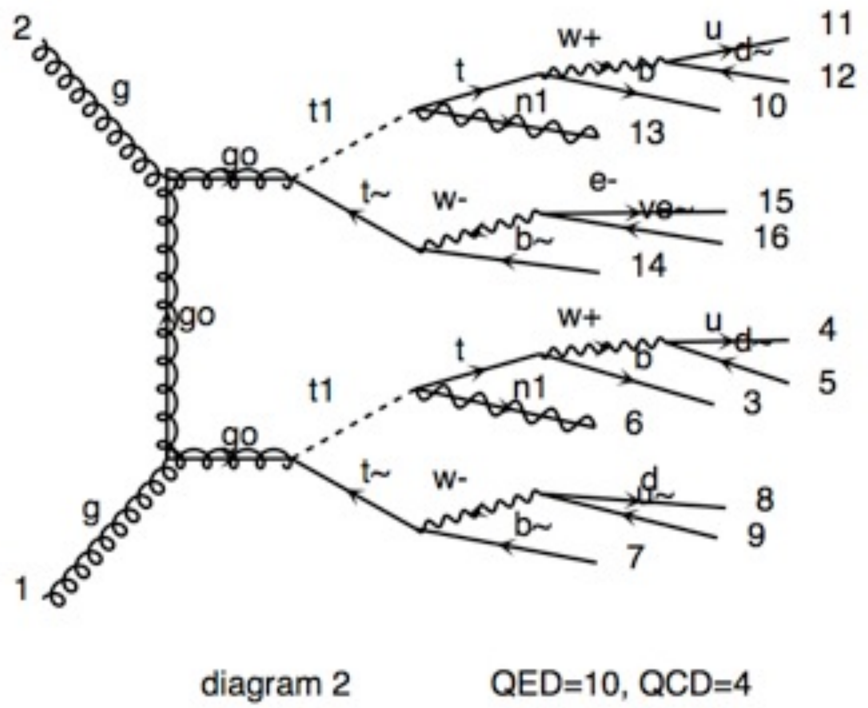
$$\sigma_{full} = \sigma_{prod} * (BR + \mathcal{O}(\frac{\Gamma}{M}))$$

Comment

- This is an Approximation!
- This force the particle to be on-shell!
  - Recover by re-introducing the Breit-wigner up-to a cut-off

Decay chain

- p p > t t~ w+, (t > w+ b, w+ > l+ vl), \
  (t~ > w- b~, w- > j j), \
  w+ > l+ vl



very long
decay chains possible to simulate
directly in MadGraph!

- Full spin-correlation
- Off-shell effects (up to cut-off)
- NWA not used for the cross-section

What to remember

- Analytical computation can be slower than numerical method
- Any BSM model are supported (at LO)
- Phase Space integration are slow
    - need knowledge of the function
    - cuts can be problematic
- Event generation are from free.
- All this are automated in MG5_aMC@NLO
- Important to know the physical hypothesis