Beyond the Standard Model physics made easy with FEYNRULES

Benjamin Fuks

LPTHE / Sorbonne Université

2019 MADGRAPH School

IMSc @ Chennai - 18 November 2019

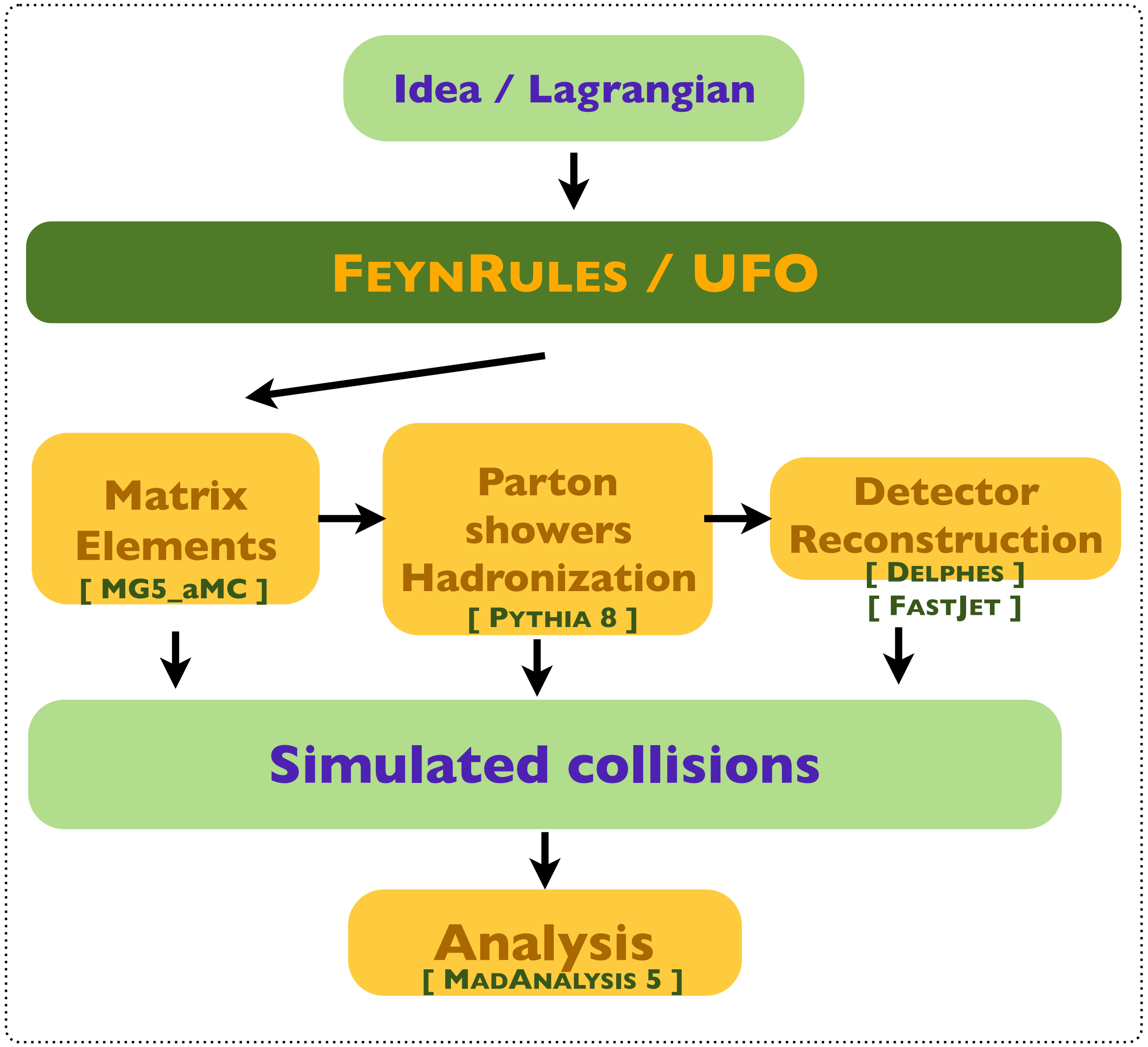
Outline

1. FEYNRULES in a nutshell
2. Implementing supersymmetric QCD in FEYNRULES
3. Using FEYNRULES with supersymmetric QCD model
4. Summary
5. Appendix: advanced model implementation techniques

A comprehensive approach to MC simulations

[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]

✦ Tools connecting an idea to simulated collisions



- Model building
- Hard scattering
  - ★ Feynman diagram and amplitude generation
  - ★ Monte Carlo integration
  - ★ Event generation
- QCD environment
  - ★ Parton showering
  - ★ Hadronization
  - ★ Underlying event
- Detector simulation
  - ★ Simulation of the detector response
  - ★ Object reconstruction
- Event analysis
  - ★ Signal/background analysis
  - ★ LHC recasting

FeynRules in a nutshell

✦ What is FeynRules?

- A framework to develop new physics models
- Automatic export to several Monte Carlo event generators
  - ★ Not entirely true anymore with the UFO (one format to rule them all)

➡ Facilitate phenomenological investigations of BSM models

➡ Facilitate the confrontation of BSM models to data

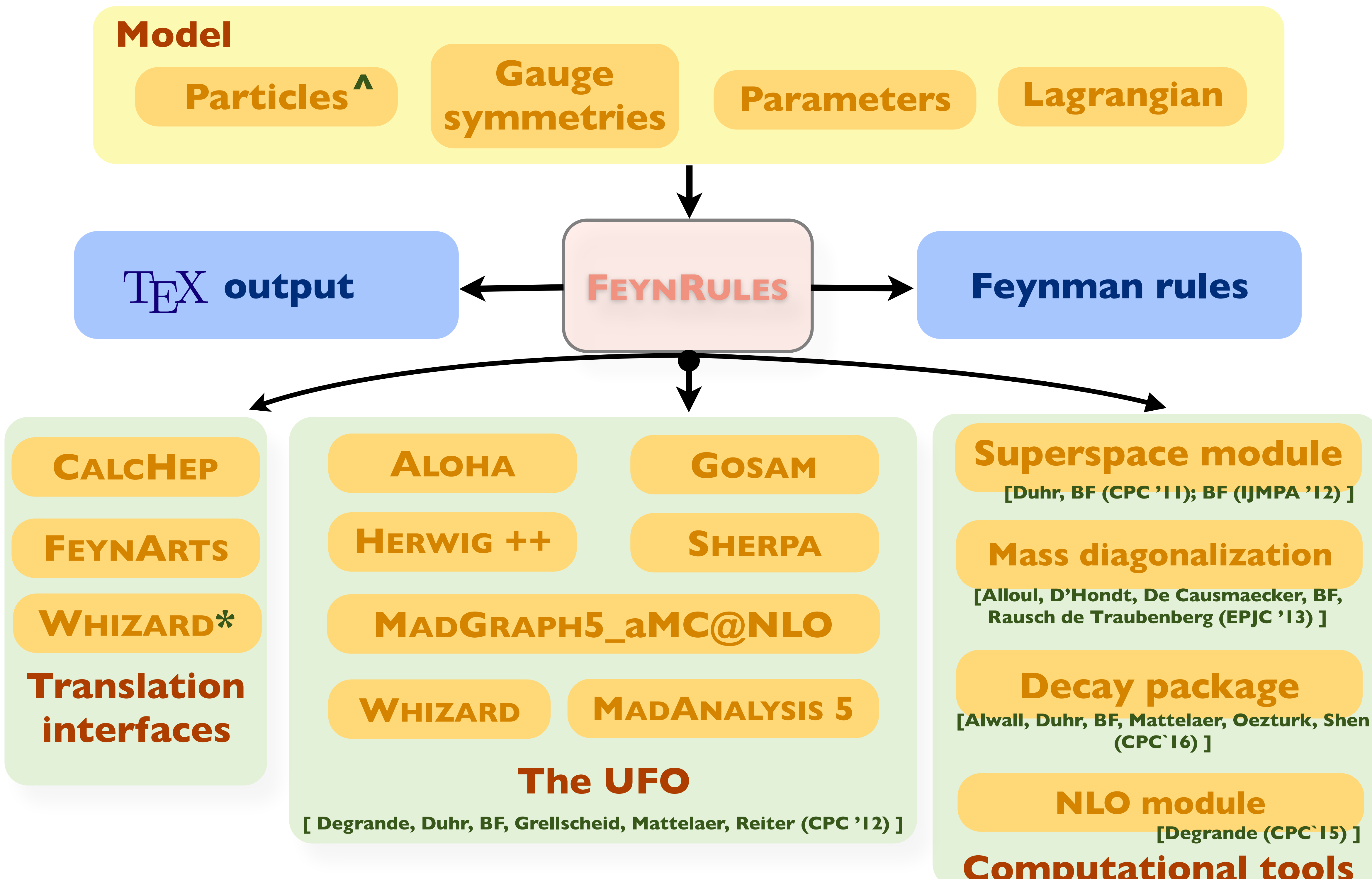
- Validation of an implementation using several Monte Carlo programs

✦ Main features

- Mathematica package
- Core function: derives Feynman rules from a Lagrangian
- Requirements: locality, Lorentz and gauge invariance
- Supported fields: scalar, (two- and four-component) fermion, vector (and ghost), spin-3/2, tensor, superfield

From FEYNRULES to Monte Carlo tools…

[Christensen, Duhr (CPC ’09); Alloul, Christensen, Degrande, Duhr, BF (CPC’14)]



* Whizard interface: Christensen, Duhr, BF, Reuter, Speckner (EPJC ’12)

^ Support for spin 3/2: Christensen, de Aquino, Deutschmann, Duhr, BF, Garcia-Cely, Mattelaer, Mawatari, Oexl, Takaesu (EPJC ’13)

Outline

Supersymmetric QCD: particle content

✦ Particle content

- ✤ Two matter supermultiplets in the fundamental representation of $SU(3)_c$
  - ★ One massive Dirac fermion: a quark
  - ★ Two massive scalar fields: a left-handed and a right-handed squark
- ✤ One ($SU(3)_c$) gauge supermultiplet
  - ★ One massive Majorana fermion: a gluino
  - ★ One massless gauge boson: the gluon

(Broken) supersymmetric QCD: the model

✦ The dynamics of the model is embedded in the Lagrangian

$$
\begin{aligned}
\mathcal{L} = & -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} + D_\mu \tilde{q}_L^\dagger D^\mu \tilde{q}_L + D_\mu \tilde{q}_R^\dagger D^\mu \tilde{q}_R + i \bar{q} \not{D} q \\
& - m^2_{\tilde{q}_L} \tilde{q}_L^\dagger \tilde{q}_L - m^2_{\tilde{q}_L} \tilde{q}_R^\dagger \tilde{q}_R - m_q \bar{q} q - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g} \\
& - \frac{g_s^2}{2} \Big[- \tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \Big] \Big[- \tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \Big] \\
& + \sqrt{2} g_s \Big[- \tilde{q}_L^\dagger T^a \big(\bar{\tilde{g}}^a P_L q \big) + \big(\bar{q} P_L \tilde{g}^a \big) T^a \tilde{q}_R + \text{h.c.} \Big]
\end{aligned}
$$

- ✤ Kinetic terms for all fields (first line)
- ✤ Mass terms for the squarks, quarks and gluino (second line)
- ✤ Supersymmetric gauge interactions for all fields (last two lines)

How to write a FeynRules model file?

- A FeynRules model file is compliant with the Mathematica syntax
- It is a .fr file containing:

A preamble
- ★ Author information
- ★ Model information
- ★ Index definitions

The declaration of the gauge group
- ★ Abelian or not
- ★ Representation matrices
- ★ Structure constants
- ★ Coupling constant
- ★ Gauge boson or vector superfield

The declaration of the fields
- ★ Names, spins, PDG codes
- ★ Indices, quantum numbers
- ★ Masses, widths
- ★ Classes and class members

The declaration of the parameters
- ★ External and internal
- ★ Scalar and tensor

A Lagrangian

The preamble of the model file: general information

✦ An electronic signature for the model implementation

- ✤ Important for traceability, documentation, contact with the authors, *etc*.
- ✤ Reference publications used can be added
- ✤ Webpage information can be added

```
M$ModelName = "SUSYQCD";

M$Information = {
  Authors      -> {"Benjamin Fuks"},
  Date         ->  "19.11.18”,
  Version      ->  "1.0",
  Institutions -> {"LPTHE / Sorbonne U."},
  Emails       -> {"fuks@lpthe.jussieu.fr"}
};
```

The preamble of the model file: indices

- ✦ The dimension of the indices must be declared
  - ✤ In our SUSY-QCD model:
    - ★ Fundamental $SU(3)_C$ indices for the squarks and quark: *Colour*, dimension 3
    - ★ Adjoint $SU(3)_C$ indices for the gluon and gluino: *Gluon*, dimension 8
    - ★ Lorentz and spin indices are automatically handled

```
IndexRange[Index[Gluon ]] = NoUnfold[Range[8]];
IndexRange[Index[Colour]] = NoUnfold[Range[3]];
```

  - ✤ QCD has a special role in MC event generators ➢ many special names
    - ★ *Colour, Sextet and Gluon* for the color indices
    - ★ *G* for the gluon field
    - ★ *T* for the fundamental representation matrices
    - ★ *f* and *d* for the structure constants
    - ★ G (gs will be used) and aS for the coupling constants

- ✦ The style of the indices can be specified
  - ✤ Fundamental indices starting with the letter *m*
  - ✤ Adjoint indices starting with the letter *a*

```
IndexStyle[Colour, m];
IndexStyle[Gluon,  a];
```

The declaration of the gauge group (I)

- Each element the group is declared in the *M$GaugeGroups* list
  - A declaration ≡ a set of MATHEMATICA replacement rules
  - In our SUSY-QCD model:
    - ★ We must only declare $SU(3)_C$: we choose the name *SU3C*

```
M$GaugeGroups = {
  SU3C == {
    Abelian              -> False,
    GaugeBoson           -> G,
    CouplingConstant     -> gs,
    StructureConstant    -> f,
    Representations      -> { {T,Colour} }
  }
};
```

- Each rule represents one group property (QCD: special names exist)
  - ★ *Abelian*: abelian or non-abelian group
  - ★ *GaugeBoson*: associated gauge boson
  - ★ *CouplingConstant*, *StructureConstant*: coupling and the structure constants
  - ★ *Representations*: list of 2-tuples linking indices to representation matrices

See the manual for more details on gauge groups

The declaration of the gauge group (2)

✦ Advantages of a proper gauge group declaration

- Render the writing of the Lagrangian easier:
  - ★ Covariant derivatives (*DC[field, Lorentz index]*)
  - ★ Field strength tensors (*FS[field, Lorentz index 1, Lorentz index 2]*)
  - ★ Useful for Lagrangian building in superspace (briefly covered in the last slides)
    - ➢ [Duhr & BF CPC 182 (2011) 2404; BF IJMPA 27 (2012) 1230007]

- Example: gluon and gluino kinetic terms

$$-\frac{1}{4}g_{\mu\nu}g^{\mu\nu} + \frac{i}{2}\bar{\tilde{g}}\not{D}\tilde{g}$$

```
-1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
  I/2 gobar.Ga[mu].DC[go, mu]
```

Declaring the gluon field

- ✦ Each field is an element of the *M$ClassesDescription* list
  - ✤ A declaration ≡ a set of MATHEMATICA replacement rules
  - ✤ In our SUSY-QCD model, we first declare the $SU(3)_C$ gauge boson: *G*

```
M$ClassesDescription = {
  V[1] == {
    ClassName        -> G,
    SelfConjugate    -> True,
    Indices          -> {Index[Gluon]},
    Mass             -> 0,
    Width            -> 0,
    PDG              -> 21
  },
  …
```

- ✤ Each rule represents a property of the field
  - ★ Vector field ➢ the label is V[1] (with V, and not F, S, R, T, *etc.*)
  - ★ ClassName: defines the symbol to use in the Lagrangian ➢ *G*
  - ★ Indices: the gluon lies in the adjoint representation of SU(3)c
    - ➢ The gluon has been previously set as the gauge boson of $SU(3)_c$
    - ➢ The index (*Gluon)* is internally linked to the adjoint representation
  - ★ Other: vanishing mass and width, PDG code set to 21, self-conjugate

See the manual for more options for field declarations

Declaring a Dirac fermion gluino field

✦ A second element in the *M$ClassesDescription* list

```
F[1] == {
  ClassName        -> go,
  SelfConjugate    -> True,
  Indices          -> {Index[Gluon]},
  Mass             -> {Mgo,500},
  Width            -> {Wgo,10},
  PDG              -> 1000021
},
```

✤ Differences with the gluon field declaration

- ★ Four-component fermionic field ➢ the label is F[1] (with an F)
- ★ Classname: defines two symbols to use in the Lagrangian ➢ *go* and *gobar*
- ★ Mass and width: two symbols and their associated numerical values

See the manual for more details on field declarations

Declaring the gluino field with Weyl fermions

✦ Second option: two-component spinors (Lagrangians sometimes easier)

```
W[1] == {
  ClassName      -> gow,
  Unphysical     -> True,
  Chirality      -> Left,
  SelfConjugate -> False,
  Indices        -> {Index[Gluon]},
},
```

```
F[1] == {
  ClassName          -> go,
  WeylComponents     -> gow,
  SelfConjugate      -> True,
  Indices            -> {Index[Gluon]},
  Mass               -> {Mgo,500},
  Width              -> {Wgo,10},
  PDG                -> 1000021
},
```

✤ Extra options are available

★ Two-component and four-component fermionic fields
  ➢ the labels are W[1] (with a W) and F[1]

★ Weyl fermions are unphysical and linked to four-component fermions
  ➢ WeylComponents

★ Several symbols are defined ➢ *go* and *gobar; gow* and *gowbar*

★ The chirality of the Weyl fermion can be specified

See the manual for more details on field declarations

Declaring the (top) quark field

✦ A third element in the *M$ClassesDescription* list

```
F[2] == {
  ClassName        -> q,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Mass             -> {Mq, 173},
  Width            -> {Wq, 1.50833649},
  PDG              -> 6
},
```

✤ Nothing special compared to the other fields
  ★ Fundamental QCD indices are specified

More on quarks: generation indices

✦ How to implement three generations of up-type quarks?

```
F[3] == {
  ClassName        -> uq,
  ClassMembers     -> {u, c, t},
  SelfConjugate    -> False,
  Indices          -> {Index[Gen], Index[Colour]},
  FlavorIndex      -> Gen,
  Mass             -> {Mu, {MU,2.55*^-3}, {MC,1.42}, {MT,173}},
  Width            -> {0, 0, {WT, 1.50833649}},
  PDG              -> {2, 4, 6}
},
```

✤ Slight differences for a bunch of options (+ new options)

- ★ ClassMembers: specify all the members of the class
- ★ We introduce a generation index: *Gen*
  - ➢ FlavorIndex defines which index is the flavor index
- ★ Mass, Width, PDG: one for each class member (plus a generic mass symbol)

Declaring squark fields

✦ Extra elements in the *M$ClassesDescription* list

```
S[3] == {
  ClassName        -> sq1,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Mass             -> {Msq1,300},
  Width            -> {Wsq1,10},
  PDG              -> 1000006
},
S[4] == {
  ClassName        -> sq2,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Mass             -> {Msq2,800},
  Width            -> {Wsq2,2},
  PDG              -> 2000006
}
```

✤ Nothing special

- ★ Scalar field ➢ the label is S[3]
- ★ Classname: defines pairs of symbols
  ➢ *sq1, sq2, sq1bar* and sq2*bar*
- ★ Mass, width, Indices, *etc.*: standard

Missing: squark mixing

Mixing squark fields

✦ Mixing implemented via physical and unphysical fields

```
S[1] == {
  ClassName        -> sqL,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Unphysical       -> True,
  Definitions      -> {sqL[c_] -> Cos[theta] sq1[c] - Sin[theta] sq2[c]}
},
S[2] == {
  ClassName        -> sqR,
  SelfConjugate    -> False,
  Indices          -> {Index[Colour]},
  Unphysical       -> True,
  Definitions      -> {sqR[c_] -> Sin[theta] sq1[c] + Cos[theta] sq2[c]}
},
```

✤ Squark fields mix as

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

★ Left and right-handed squarks unphysical

➢ removed from the Lagrangian and replaced

★ Definitions include the replacement rules (inversion of the above relation)

➢ The rotations will be performed automatically by FEYNRULES

➢ The Lagrangian can be written in the gauge basis (easier with sqL/sqR)

Parameters to implement

- The model requires to implement three parameters
  - Masses and widths are handled automatically

$$\mathcal{L} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} + D_\mu \tilde{q}_L^\dagger D^\mu \tilde{q}_L + D_\mu \tilde{q}_R^\dagger D^\mu \tilde{q}_R + i \bar{q} \not{D} q$$
$$- m_{\tilde{q}_L}^2 \tilde{q}_L^\dagger \tilde{q}_L - m_{\tilde{q}_L}^2 \tilde{q}_R^\dagger \tilde{q}_R - m_q \bar{q} q - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$
$$- \frac{g_s^2}{2} \Big[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \Big] \Big[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \Big]$$
$$+ \sqrt{2} g_s \Big[-\tilde{q}_L^\dagger T^a (\bar{\tilde{g}}^a P_L q) + (\bar{q} P_L \tilde{g}^a) T^a \tilde{q}_R + \text{h.c.} \Big]$$

  - The strong coupling appears explicitly and implicitly in the Lagrangian
    - ➢ g_s is needed
    - ➢ α_s is also needed (required by the Monte Carlo tools)
  - The squark mixing angle must be implemented θ

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

Parameter declaration : strong couplings

- Parameters are declared as elements of the list *M$Parameters*
  - A declaration ≡ a set of MATHEMATICA replacement rules
  - The two strong coupling parameters α_s and g_s are not independent

```
M$Parameters = {
  aS    == {
    ParameterType    -> External,
    Value            -> 0.1184,
    InteractionOrder -> {QCD, 2}
  },
  gs == {
    ParameterType    -> Internal,
    Value            -> Sqrt[4 Pi aS],
    InteractionOrder -> {QCD, 1},
    ParameterName    -> G
  },
```

★ Internal and External parameters
- ➢ External ≡ free parameter
  ⇒ numerical value
- ➢ Internal ≡ dependent parameter
  ⇒ formula

★ InteractionOrder: specific to some MC
- ➢ more efficient diagram generation

★ ParameterName: QCD is special (in MC tools)

See the manual for more details on parameter declarations

Parameter declaration : squark mixing

✦ The squark mixing angle

✤ Nothing special (standard external parameter)

```
theta == {
  ParameterType     -> External,
  Value             -> Pi/4
}
```

✦ More options (assuming matrix-form mixing)

✤ Tensor parameters can be implemented too (Indices, Unitary)

✤ Complex parameters can be implemented too (ComplexParameter)

See the manual for more details on parameter declarations

Parameter organization

- ✦ Les Houches blocks can be specific to organize the external parameters
  - ✤ Use of a Les Houches parameter card by the MC programs
  - ✤ Implemented via BlockName and OrderBlock
  - ✤ If unspecified: everything goes into the 'FRBlock' block

- ✦ Complete α_s implementation as an example
  - ✤ New options are used (TeX, Description, BlockName, OrderBlock)

```
aS    == {
  ParameterType    -> External,
  BlockName        -> SMINPUTS,
  OrderBlock       -> 3,
  Value            -> 0.1184,
  InteractionOrder -> {QCD,2},
  TeX              -> Subscript[\[Alpha],s],
  Description      -> "Strong coupling constant at the Z pole"
},
```

Outline

Getting started

Starting a FeynRules Mathematica session

✦ Step1: loading FeynRules into Mathematica

- ✤ Setting up the FeynRules path
- ✤ Loading FeynRules itself

```
In[1]:= $FeynRulesPath = SetDirectory["~/Work/tools/FeynRules/branch/feynrules-current"];
        << FeynRules`

         - FeynRules -
        Version: 2.3.32 ( 12 March 2018).
        Authors: A. Alloul, N. Christensen, C. Degrande, C. Duhr, B. Fuks

        Please cite:
            - Comput.Phys.Commun.185:2250-2300,2014 (arXiv:1310.1921);
            - Comput.Phys.Commun.180:1614-1641,2009 (arXiv:0806.4194).

        http://feynrules.phys.ucl.ac.be

        The FeynRules palette can be opened using the command FRPalette[].
```

✦ The output

- ✤ Information on FeynRules, the authors, the version number, references, *etc.*

Loading the model implementation

- Step2: loading the SUSY-QCD implementation into FeynRules
  - Moving to the right directory
  - Loading the model itself

```
SetDirectory[NotebookDirectory[]];
LoadModel["susyqcd.fr"];


This model implementation was created by
Benjamin Fuks
Model Version: 1.0
For more information, type ModelInformation[].

   - Loading particle classes.
   - Loading gauge group classes.
   - Loading parameter classes.

Model SUSYQCD loaded.
```

- The output
  - Information encoded in the preamble of the model are printed to the screen
  - More information can be obtained by typing ModelInformation[ ]

The vector Lagrangian

The SUSYQCD vector Lagrangian

- The gauge sector of the theory : one gauge supermultiplet
  - One massive Majorana fermion: a gluino
  - One massless gauge boson: the gluon

- The dynamics of the model is embedded in the vector Lagrangian

$$\mathcal{L}_{\rm vector} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

  - Kinetic terms for all the gluino and gluon fields
  - Mass terms for the gluino
  - Gauge interactions for both fields (in the gauge-covariant objects)

Implementing the vector Lagrangian

✦ Textbook expression:

$$\mathcal{L}_{\rm vector} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

✦ Makes use of the strengths of a proper gauge-group implementation

- Existence of shortcut functions (DC, FS, *etc.*)
- Very compact implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
             I/2 gobar.Ga[mu].DC[go,mu] -
             1/2 Mgo gobar.go;
```

★ All non-Lorentz indices understood (FEYNRULES automatically handles them)

✦ Indices can be noted explicitly too (no differences)

```
LVector2 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
             I/2 Ga[mu,s1,s2] gobar[s1,a].DC[go[s2,a],mu] -
             1/2 Mgo gobar[s1,a].go[s1,a];
```

Check of the vector Lagrangian implementation

✦ Textbook expression

$$\mathcal{L}_{\rm vector} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

✦ FEYNRULES implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
        I/2 gobar.Ga[mu].DC[go,mu] -
        1/2 Mgo gobar.go;
```

✦ Printing the vector Lagrangian

In[6]:= **LVector1**

Out[6]= $-\frac{1}{2}\,\mathrm{Mgo}\,\overline{\mathrm{go}}.\mathrm{go} + \frac{1}{2}\,i\,\overline{\mathrm{go}}.\gamma^{\mathrm{mu}}.\left(\partial_{\mathrm{mu}}[\mathrm{go}] - i\,\mathrm{gs}\,\mathrm{FSU3C}^{\mathrm{a\$658}}.\mathrm{go}\,G_{\mathrm{mu,a\$658}}\right) - \frac{1}{4}\left(-\partial_{\mathrm{nu}}[G_{\mathrm{mu,a}}] + \partial_{\mathrm{mu}}[G_{\mathrm{nu,a}}] + \mathrm{gs}\,f_{\mathrm{a,bb\$656,cc\$656}}\,G_{\mathrm{mu,bb\$656}}\,G_{\mathrm{nu,cc\$656}}\right) \left(-\partial_{\mathrm{nu}}[G_{\mathrm{mu,a}}] + \partial_{\mathrm{mu}}[G_{\mathrm{nu,a}}] + \mathrm{gs}\,f_{\mathrm{a,bb\$657,cc\$657}}\,G_{\mathrm{mu,bb\$657}}\,G_{\mathrm{nu,cc\$657}}\right)$

✦ The output

- ✤ Covariant derivatives and field strength tensors have been automatically evaluated
- ✤ FSU3C denotes the structure constants of the SU3C gauge group
  - ➢ to be replaced in a second step by f_{abc}

Expanding the Lagrangian

✦ All indices can be restored automatically

```
In[7]:= ExpandIndices[LVector1]

Out[7]= -1/4 ∂nu[Gmu,a]^2 + 1/2 ∂nu[Gmu,a] ∂mu[Gnu,a] - 1/4 ∂mu[Gnu,a]^2 - 1/2 Mgo goi1$667,i2$667 . goi1$667,i2$667 +
  1/4 gs ∂nu[Gmu,a] fa,bb$662,cc$662 Gmu,bb$662 Gnu,cc$662 - 1/4 gs ∂mu[Gnu,a] fa,bb$662,cc$662 Gmu,bb$662 Gnu,cc$662 +
  1/4 gs ∂nu[Gmu,a] fa,bb$663,cc$663 Gmu,bb$663 Gnu,cc$663 - 1/4 gs ∂mu[Gnu,a] fa,bb$663,cc$663 Gmu,bb$663 Gnu,cc$663 -
  1/4 gs^2 fa,bb$662,cc$662 fa,bb$663,cc$663 Gmu,bb$662 Gmu,bb$663 Gnu,cc$662 Gnu,cc$663 +
  1/4 i goi$667,i1$669 . ∂mu[goj$667,i1$669] γi$667,j$667^mu -
  1/4 i gs goi$670,i$669 . goj$670,j$669 fa$664,i$669,j$669 Gmu,a$664 γi$670,j$670^mu -
  1/4 i ∂mu[goj$667,i1$669] . goi$667,i1$669 γj$667,i$667^mu + 1/4 i gs goj$670,j$669 . goi$670,i$669 fa$664,i$669,j$669 Gmu,a$664 γj$670,i$670^mu
```

- ✤ Kinetic terms (gluon and gluino)
- ✤ Mass terms (gluino)
- ✤ QCD interactions (with the proper structure constants)

Check of the implementation: hermiticity

✦ Textbook expression

$$\mathcal{L}_{\rm vector} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2} \bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

✦ FeynRules implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
         I/2 gobar.Ga[mu].DC[go,mu] -
         1/2 Mgo gobar.go;
```

✦ The Lagrangian must be Hermitian

- ✤ Calculation of the Feynman rules of the Lagrangian minus its Hermitian conjugate
  - ➢ $\mathcal{L}$ - $\mathcal{L}^\dagger$

```
In[8]:= CheckHermiticity[LVector1];
        Checking for hermiticity by calculating the Feynman rules contained in L-HC[L].
        If the lagrangian is hermitian, then the number of vertices should be zero.
        Starting Feynman rule calculation.
        Expanding the Lagrangian...
        No vertices found.
        0 vertices obtained.
        The lagrangian is hermitian.
```

Check of the implementation: normalization

✦ Textbook expression

$$\mathcal{L}_{\rm vector} = -\frac{1}{4} g_{\mu\nu} g^{\mu\nu} + \frac{i}{2}\bar{\tilde{g}} \not{D} \tilde{g} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g}$$

✦ FeynRules implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
        I/2 gobar.Ga[mu].DC[go,mu] -
        1/2 Mgo gobar.go;
```

✦ The kinetic and mass terms must be canonically normalized

- Normalization of the quadratic terms (kinetic and mass terms)
- Absence of non-diagonal quadratic terms

```
In[9]:= CheckKineticTermNormalisation[LVector1];

        Neglecting all terms with more than 2 particles.
        All kinetic terms are diagonal.
        All kinetic terms are correctly normalized.

In[11]:= CheckDiagonalMassTerms[LVector1];

        All mass terms are diagonal.
```

- Other methods: *CheckDiagonalQuadraticTerms*, *CheckDiagonalKineticTerms*

Check of the implementation: the mass spectrum

✦ FEYNRULES gluino implementation

```
F[1] == {
  ClassName        -> go,
  WeylComponents   -> gow,
  SelfConjugate    -> True,
  Indices          -> {Index[Gluon]},
  Mass             -> {Mgo,500},
  Width            -> {Wgo,10},
  PDG              -> 1000021
},
```

✦ FEYNRULES Lagrangian implementation

```
LVector1 := -1/4 FS[G,mu,nu,a] FS[G,mu,nu,a] +
             I/2 gobar.Ga[mu].DC[go,mu] -
             1/2 Mgo gobar.go;
```

✦ Checks that the mass information is consistent (numerically)

- Masses can be extracted from the Lagrangian
- Masses are fixed when particles are declared

```
In[12]:= CheckMassSpectrum[LVector1]
         Neglecting all terms with more than 2 particles.
         All mass terms are diagonal.
         Getting mass spectrum.
         Checking for less then 0.1% agreement with model file values.
Out[12]//TableForm=
```

Particle	Analytic value	Numerical value	Model-file value
go	Mgo	500.	500.

The Feynman rules calculation

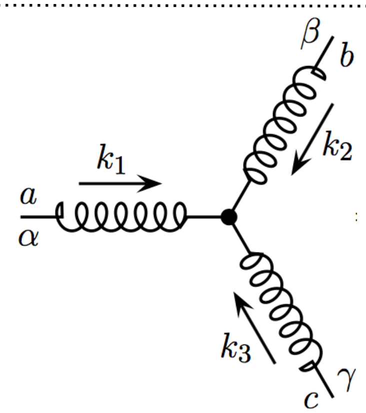
✦ Extract all N-point interactions from the Lagrangian (with N>2)

```
In[14]:= FeynmanRules[LVector1, ScreenOutput → True];

Starting Feynman rule calculation.

Expanding the Lagrangian...

Collecting the different structures that enter the vertex.

3 possible non-zero vertices have been found -> starting the computation: 3 / 3.

3 vertices obtained.

(* * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *)
```

- ✤ Three vertices are a priori found form the Lagrangian
- ✤ When terms are gathered together, three vertices are finally left
  - ★ No vanishing vertex (possible when different contributions to it)
    - ➢ One triple and quartic gluon interactions
    - ➢ One gluino-gluon interaction

The triple gluon vertex

✦ Literature



$$g_s f_{abc}\Big[(k_1-k_2)^\gamma\eta^{\alpha\beta}+(k_3-k_1)^\beta\eta^{\gamma\alpha}+(k_2-k_3)^\alpha\eta^{\beta\gamma}\Big]$$

[Incoming momenta]

✦ FEYNRULES

```
Vertex 1
Particle 1 : Vector , G
Particle 2 : Vector , G
Particle 3 : Vector , G
Vertex:
```

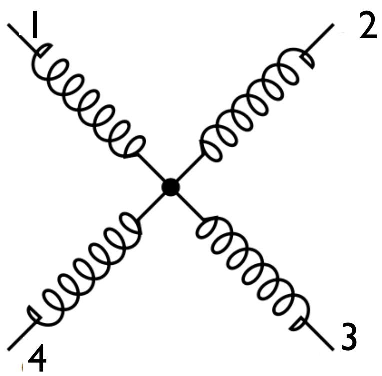
$$\mathrm{gs}\, f_{a_1,a_2,a_3}\, p_1^{\mu_3}\,\eta_{\mu_1,\mu_2} - \mathrm{gs}\, f_{a_1,a_2,a_3}\, p_2^{\mu_3}\,\eta_{\mu_1,\mu_2} - \mathrm{gs}\, f_{a_1,a_2,a_3}\, p_1^{\mu_2}\,\eta_{\mu_1,\mu_3} + \mathrm{gs}\, f_{a_1,a_2,a_3}\, p_3^{\mu_2}\,\eta_{\mu_1,\mu_3} + \mathrm{gs}\, f_{a_1,a_2,a_3}\, p_2^{\mu_1}\,\eta_{\mu_2,\mu_3} - \mathrm{gs}\, f_{a_1,a_2,a_3}\, p_3^{\mu_1}\,\eta_{\mu_2,\mu_3}$$

✤ Color: the index a_i is related to the i^{th} particle
(*a* is the index style for the adjoint $SU(3)_C$ indices)

✤ Spin: the index μ_i is the Lorentz index of the i^{th} (vector) particle

The quartic gluon vertex

✦ Literature



$$i g_s^2 \, f^{a_1 a_2 b} f^{b a_3 a_4} \left(\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}\right) + i g_s^2 \, f^{a_1 a_3 b} f^{b a_2 a_4} \left(\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}\right) + i g_s^2 \, f^{a_1 a_4 b} f^{b a_2 a_3} \left(\eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}\right)$$

✦ FEYNRULES

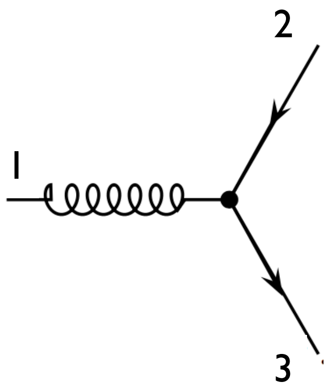
```
Vertex 2
Particle 1 : Vector , G
Particle 2 : Vector , G
Particle 3 : Vector , G
Particle 4 : Vector , G
Vertex:
```

$$\mathrm{i}\, gs^2 \left(f_{a_1,a_3,\mathrm{Gluon\$1}} f_{a_2,a_4,\mathrm{Gluon\$1}} \eta_{\mu_1,\mu_4} \eta_{\mu_2,\mu_3} + f_{a_1,a_2,\mathrm{Gluon\$1}} f_{a_3,a_4,\mathrm{Gluon\$1}} \eta_{\mu_1,\mu_4} \eta_{\mu_2,\mu_3} + f_{a_1,a_4,\mathrm{Gluon\$1}} f_{a_2,a_3,\mathrm{Gluon\$1}} \eta_{\mu_1,\mu_3} \eta_{\mu_2,\mu_4} - f_{a_1,a_2,\mathrm{Gluon\$1}} f_{a_3,a_4,\mathrm{Gluon\$1}} \eta_{\mu_1,\mu_3} \eta_{\mu_2,\mu_4} - f_{a_1,a_4,\mathrm{Gluon\$1}} f_{a_2,a_3,\mathrm{Gluon\$1}} \eta_{\mu_1,\mu_2} \eta_{\mu_3,\mu_4} - f_{a_1,a_3,\mathrm{Gluon\$1}} f_{a_2,a_4,\mathrm{Gluon\$1}} \eta_{\mu_1,\mu_2} \eta_{\mu_3,\mu_4} \right)$$

- Color: a_i is related to the i^{th} particle; the index *Gluon$1* is summed over
- Spin: μ_i is related to the i^{th} (vector) particle

Gluon and gluino interactions

✦ Literature



$$g_s f_{a_1 a_2 a_3} \gamma^{\mu_1}$$

✦ FEYNRULES

```
Vertex 3
Particle 1 : Majorana , go
Particle 2 : Majorana , go
Particle 3 : Vector , G
Vertex:
```

$$\text{gs}\ f_{a_1,a_2,a_3}\ \gamma_{s_1,s_2}{}^{\mu_3}$$

- ✤ Color: a_i is related to the i^{th} particle
- ✤ Spin: μ_i is related to the i^{th} (vector) particle; s_i is related to the i^{th} fermion

The FeynmanRules function

- The function FeynmanRules has many options
  - Restriction on the interactions to display
    - MaxParticles, MaxCanonicalDimension, *etc*.
  - Selection of specific particles
    - Free, Contains, *etc*.
  - ScreenOutput: displaying the vertices to the screen or not
  - FlavorExpand: perform a flavor expansion (otherwise, classes are used)

See the manual for more details on the function *FeynmanRules*

The matter Lagrangian

The SUSYQCD matter sector

✦ Matter fields

✤ Two matter supermultiplets in the fundamental representation of $SU(3)_c$

- ★ One massive Dirac fermion: a quark
- ★ Two mixing massive scalar fields: two squark
- ★ Gauge coupling to the $SU(3)_c$ gauge supermultiplet

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

✦ The model dynamics is embedded in the Lagrangian

$$\begin{aligned}\mathcal{L}_{\rm matter} = &\ D_\mu \tilde{q}_L^\dagger D^\mu \tilde{q}_L + D_\mu \tilde{q}_R^\dagger D^\mu \tilde{q}_R + i\bar{q}\not{D}q - m_{\tilde{q}_i}^2 \tilde{q}_i^\dagger \tilde{q}_i - m_q \bar{q} q \\ & - \frac{g_s^2}{2}\Big[- \tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \Big]\Big[- \tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R \Big] \\ & + \sqrt{2} g_s \Big[- \tilde{q}_L^\dagger T^a \big(\bar{\tilde{g}}^a P_L q\big) + \big(\bar{q} P_L \tilde{g}^a\big) T^a \tilde{q}_R \Big] + \text{h.c.}\end{aligned}$$

✤ Mixed use of the mass / gauge bases (makes things easier to implement)

✤ Kinetic terms (first three terms) and D-terms (second line) in the gauge basis

✤ Mass terms (end of first line) in the mass basis

✤ SUSY gauge quark-squark-gluino interactions (fourth line) in the gauge basis

Kinetic and gauge interactions

✦ Kinetic and gauge interactions, as well as mass terms

$$\mathcal{L}_{\rm matter} = D_\mu \tilde{q}_L^\dagger D^\mu \tilde{q}_L + D_\mu \tilde{q}_R^\dagger D^\mu \tilde{q}_R + i\bar{q} \not{D} q - m_{\tilde{q}_i}^2 \tilde{q}_i^\dagger \tilde{q}_i - m_q \bar{q} q$$

✦ The implementation in FEYNRULES follows the textbook expression

- Existence of shortcut functions (DC)
- Very compact implementation
- Repeated indices are summed

```
Lkin := DC[sqLbar[cc],mu] DC[sqL[cc],mu] +
        DC[sqRbar[cc],mu] DC[sqR[cc],mu] +
        I qbar.Ga[mu].DC[q,mu] -
        Mq qbar.q -
        Msq1^2 sq1bar[cc] sq1[cc] -
        Msq2^2 sq2bar[cc] sq2[cc];
```

Gauge eigenstates are used

Mass eigenstates are used

Checking the Feynman rules

✦ We can compute the Feynman rules

```
In[18]:= Simplify[FeynmanRules[Lkin]] // MatrixForm
         Starting Feynman rule calculation.
         Expanding the Lagrangian...
         Collecting the different structures that enter the vertex.
         9 possible non-zero vertices have been found -> starting the computation: 9 / 9.
         5 vertices obtained.
Out[18]//MatrixForm=
```

{{G, 1}, {sq1, 2}, {sq1†, 3}}	$i\, gs \left(p_2^{\mu 1} - p_3^{\mu 1}\right) T^{a1}_{m_3,m_2}$
{{G, 1}, {sq2, 2}, {sq2†, 3}}	$i\, gs \left(p_2^{\mu 1} - p_3^{\mu 1}\right) T^{a1}_{m_3,m_2}$
{{G, 1}, {G, 2}, {sq1, 3}, {sq1†, 4}}	$i\, gs^2\, \eta_{\mu 1,\mu 2} \left(T^{a1}_{m_4,Colour\$1} T^{a2}_{Colour\$1,m_3} + T^{a1}_{Colour\$1,m_3} T^{a2}_{m_4,Colour\$1}\right)$
{{G, 1}, {G, 2}, {sq2, 3}, {sq2†, 4}}	$i\, gs^2\, \eta_{\mu 1,\mu 2} \left(T^{a1}_{m_4,Colour\$1} T^{a2}_{Colour\$1,m_3} + T^{a1}_{Colour\$1,m_3} T^{a2}_{m_4,Colour\$1}\right)$
{{q̄, 1}, {q, 2}, {G, 3}}	$i\, gs\, \gamma_{s1,s2}{}^{\mu 3}\, T^{a3}_{m_1,m_2}$

- ✤ Field rotations have been performed automatically
  - ★ No more gauge eigenstates (sqL, sqR)
- ✤ All the QCD interactions of the squarks and quarks are derived
  - ★ The 3-point qqg interaction
  - ★ Two 3-point squark-gluon interactions (one for each squark species)
  - ★ Two 4-point squark-gluon interactions (one for each squark species)

D-terms

✦ D-terms (in the gauge eigenbasis)

$$\mathcal{L}_{\rm matter} = -\frac{g_s^2}{2}\Big[-\tilde q_L^\dagger T^a \tilde q_L + \tilde q_R^\dagger T^a \tilde q_R\Big]\Big[-\tilde q_L^\dagger T^a \tilde q_L + \tilde q_R^\dagger T^a \tilde q_R\Big]$$

✦ The FEYNRULES implementation (almost) follows the textbook expression

```
LD := -1/2 gs^2 *
       (sqRbar[cc1] T[a,cc1,cc2] sqR[cc2] - sqLbar[cc1] T[a,cc1,cc2] sqL[cc2]) *
       (sqRbar[cc3] T[a,cc3,cc4] sqR[cc4] - sqLbar[cc3] T[a,cc3,cc4] sqL[cc4]);
```

- Repeated indices (*cc1, cc2, cc3, cc4*) are summed
- A single index can only be used twice

Feynman rules from the D-terms

✦ D-terms (in the gauge eigenbasis)

$$\mathcal{L}_{\rm matter} = -\frac{g_s^2}{2}\Big[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R\Big]\Big[-\tilde{q}_L^\dagger T^a \tilde{q}_L + \tilde{q}_R^\dagger T^a \tilde{q}_R\Big]$$

✦ Feynman rules computation

```
Simplify[FeynmanRules[LD]] // MatrixForm
```

{{sq1, 1}, {sq1, 2}, {sq1†, 3}, {sq1†, 4}}	$-i\, gs^2 \, \mathrm{Cos}[2\,\mathrm{theta}]^2 \left(T^{Gluon\$1}_{m_3,m_2} T^{Gluon\$1}_{m_4,m_1} + T^{Gluon\$1}_{m_3,m_1} T^{Gluon\$1}_{m_4,m_2}\right)$
{{sq1, 1}, {sq1†, 2}, {sq1†, 3}, {sq2, 4}}	$\frac{1}{2}\, i\, gs^2 \, \mathrm{Sin}[4\,\mathrm{theta}] \left(T^{Gluon\$1}_{m_2,m_4} T^{Gluon\$1}_{m_3,m_1} + T^{Gluon\$1}_{m_2,m_1} T^{Gluon\$1}_{m_3,m_4}\right)$
{{sq1†, 1}, {sq1†, 2}, {sq2, 3}, {sq2, 4}}	$-i\, gs^2 \, \mathrm{Sin}[2\,\mathrm{theta}]^2 \left(T^{Gluon\$1}_{m_1,m_4} T^{Gluon\$1}_{m_2,m_3} + T^{Gluon\$1}_{m_1,m_3} T^{Gluon\$1}_{m_2,m_4}\right)$
{{sq1, 1}, {sq1, 2}, {sq1†, 3}, {sq2†, 4}}	$\frac{1}{2}\, i\, gs^2 \, \mathrm{Sin}[4\,\mathrm{theta}] \left(T^{Gluon\$1}_{m_3,m_2} T^{Gluon\$1}_{m_4,m_1} + T^{Gluon\$1}_{m_3,m_1} T^{Gluon\$1}_{m_4,m_2}\right)$
{{sq1, 1}, {sq1†, 2}, {sq2, 3}, {sq2†, 4}}	$-i\, gs^2 \left(\mathrm{Sin}[2\,\mathrm{theta}]^2 \, T^{Gluon\$1}_{m_2,m_3} T^{Gluon\$1}_{m_4,m_1} - \mathrm{Cos}[2\,\mathrm{theta}]^2 \, T^{Gluon\$1}_{m_2,m_1} T^{Gluon\$1}_{m_4,m_3}\right)$
{{sq1†, 1}, {sq2, 2}, {sq2, 3}, {sq2†, 4}}	$-\frac{1}{2}\, i\, gs^2 \, \mathrm{Sin}[4\,\mathrm{theta}] \left(T^{Gluon\$1}_{m_1,m_3} T^{Gluon\$1}_{m_4,m_2} + T^{Gluon\$1}_{m_1,m_2} T^{Gluon\$1}_{m_4,m_3}\right)$
{{sq1, 1}, {sq1, 2}, {sq2†, 3}, {sq2†, 4}}	$-i\, gs^2 \, \mathrm{Sin}[2\,\mathrm{theta}]^2 \left(T^{Gluon\$1}_{m_3,m_2} T^{Gluon\$1}_{m_4,m_1} + T^{Gluon\$1}_{m_3,m_1} T^{Gluon\$1}_{m_4,m_2}\right)$
{{sq1, 1}, {sq2, 2}, {sq2†, 3}, {sq2†, 4}}	$-\frac{1}{2}\, i\, gs^2 \, \mathrm{Sin}[4\,\mathrm{theta}] \left(T^{Gluon\$1}_{m_3,m_2} T^{Gluon\$1}_{m_4,m_1} + T^{Gluon\$1}_{m_3,m_1} T^{Gluon\$1}_{m_4,m_2}\right)$
{{sq2, 1}, {sq2, 2}, {sq2†, 3}, {sq2†, 4}}	$-i\, gs^2 \, \mathrm{Cos}[2\,\mathrm{theta}]^2 \left(T^{Gluon\$1}_{m_3,m_2} T^{Gluon\$1}_{m_4,m_1} + T^{Gluon\$1}_{m_3,m_1} T^{Gluon\$1}_{m_4,m_2}\right)$

✤ All nine vertices automatically derived from the (very compact) Lagrangian

SUSY-gauge squark-quark-gluino couplings

✦ The SUSY gauge gluino-quark-squark interactions (in the gauge basis)

$$\mathcal{L}_{\rm matter} = \sqrt{2} g_s \Big[- \tilde{q}_L^\dagger T^a \big(\bar{\tilde{g}}^a P_L q\big) + \big(\bar{q} P_L \tilde{g}^a\big) T^a \tilde{q}_R \Big] + {\rm h.c.}$$

✦ The FEYNRULES is again following the textbook expression

```
Lgosqq :=  Sqrt[2] gs ProjM[s1,s2] * (
    - sqLbar[cc1] T[a,cc1,cc2] gobar[s1,a].q[s2,cc2] +
      qbar[s1,cc1].go[s2,a] T[a,cc1,cc2] sqR[cc2]);
```

✤ All indices must be explicit (scalars cannot be included in a fermion chain)

✤ *ProjM* ia the left-handed chirality projector (*ProjP* is the right-handed one)

✤ The dot is used to connect the different elements of a fermion chain
  ★ Remark: ProjM(s1,s2) is a scalar object

Feynman rules from the D-terms

✦ The SUSY gauge gluino-quark-squark interactions (in the gauge basis)

$$\mathcal{L}_{\rm matter} = \sqrt{2} g_s \Big[- \tilde{q}^\dagger_L T^a \big(\bar{\tilde{g}}^a P_L q\big) + \big(\bar{q} P_L \tilde{g}^a\big) T^a \tilde{q}_R \Big] + {\rm h.c.}$$

✦ Feynman rules computation

- ✤ Including the Hermitian conjugate pieces with the HC method

```
In[19]:= Simplify[FeynmanRules[Lgosqq + HC[Lgosqq]]] // MatrixForm
Starting Feynman rule calculation.
Expanding the Lagrangian...
Collecting the different structures that enter the vertex.
4 possible non-zero vertices have been found -> starting the computation: 4 / 4.
4 vertices obtained.
Out[19]//MatrixForm=
```

$$\begin{pmatrix} \{\{\bar{q},1\},\{go,2\},\{sq1,3\}\} & -i\sqrt{2}\,gs\,\big(\mathrm{Cos}[theta]\,P_{+s_1,s_2} - P_{-s_1,s_2}\,\mathrm{Sin}[theta]\big)\,T^{a2}_{m_1,m_3} \\ \{\{go,1\},\{q,2\},\{sq1^\dagger,3\}\} & -i\sqrt{2}\,gs\,\big(\mathrm{Cos}[theta]\,P_{-s_1,s_2} - P_{+s_1,s_2}\,\mathrm{Sin}[theta]\big)\,T^{a1}_{m_3,m_2} \\ \{\{\bar{q},1\},\{go,2\},\{sq2,3\}\} & i\sqrt{2}\,gs\,\big(\mathrm{Cos}[theta]\,P_{-s_1,s_2} + P_{+s_1,s_2}\,\mathrm{Sin}[theta]\big)\,T^{a2}_{m_1,m_3} \\ \{\{go,1\},\{q,2\},\{sq2^\dagger,3\}\} & i\sqrt{2}\,gs\,\big(\mathrm{Cos}[theta]\,P_{+s_1,s_2} + P_{-s_1,s_2}\,\mathrm{Sin}[theta]\big)\,T^{a1}_{m_3,m_2} \end{pmatrix}$$

To phenomenology

From FeynRules to phenomenology

✦ The SUSY-QCD model has been implemented into FeynRules

```
LMatter := Lkin + LD + Lgosqq + HC[Lgosqq];

LSUSYQCD := LVector1 + LMatter;
```

✦ The Feynman rules have been extracted (and checked)

✦ We are now ready to export the model to one or several MC tool(s)

- CalcHep / CompHep
- FeynArts / FormCalc
- UFO ➢ MadGraph5_aMC@NLO / Sherpa / Herwig++ / Whizard

```
WriteCHOutput[{LVector1, LMatter}];

WriteFeynArtsOutput[{LVector1, LMatter}];

WriteUFO[{LVector1, LMatter}];
```

Limitations and fineprints

- Particle / parameter names
  - The strong interaction has a special role
    - ★ Name for the gluon field, the coupling constant, *etc.*
    - ★ Which parameter is internal/external
    - ★ The numerical value of α_s at the Z-pole
  - For some generators, the electroweak interaction has also a special role
    - ★ Name for the Fermi coupling, the Z-boson mass
    - ★ Which parameter is external/internal
    - ★ At which scale must the numerical values be given

- Color structures: not supported in full generality
  - The interfaces discard the non-supported vertices
  - Representations handled by the FeynRules interfaces

- Lorentz structures and spins: not supported in full generality
  - The interfaces discard the non-supported vertices
  - Representations handled by the FeynRules interfaces:

Outline

Advanced techniques for FEYNRULES implementation

- ✦ More about the UFO
- ✦ Extension / restriction of existing models
- ✦ The superspace module of FEYNRULES
- ✦ Mass diagonalization
- ✦ Two-body decays
- ✦ Next-to-leading order module

Summary

✦ The quest for new physics at the LHC has started

- Relies on Monte Carlo event generators for background and signal modeling
- FeynRules facilitates the implementation of new physics models in those tools

✦ FeynRules: http://feynrules.irmp.ucl.ac.be

- Straightforward implementation of new physics model in Monte Carlo tools
  - ★ Interfaces to many programs
- FeynRules is shipped with its own computational modules
  - ★ A superspace module
  - ★ A decay package
  - ★ A mass diagonalization module (ASperGe)
  - ★ A brand new NLO module

Try it on with your favorite model!

Outline

Advanced techniques for FEYNRULES implementation

- More about the UFO
- Extension / restriction of existing models
- The superspace module of FEYNRULES
- Mass diagonalization
- Two-body decays
- Next-to-leading order module

The UFO

A step further: the Universal FEYNRULES Output

✦ The UFO in a nutshell

[Degrande, Duhr, BF, Grellscheid, Mattelaer, Reiter (CPC '12)]
[Degrande, Duhr, BF, Hirschi, Mattelaer, Shao *et al.* (*in prep.*)]

- ✤ UFO ≡ Universal FEYNRULES output
  - ★ Universal as not tied to any specific Monte Carlo program
- ✤ Consists of a set of PYTHON files to be linked to any code
- ✤ It contains all the model information
  - ★ Allows the models to contain generic color and Lorentz structures
- ✤ Can be employed for next-to-leading order calculations

✦ The UFO is now a standard and used by many other programs

ALOHA | GOSAM | HERWIG ++ | MADANALYSIS 5 | SHERPA

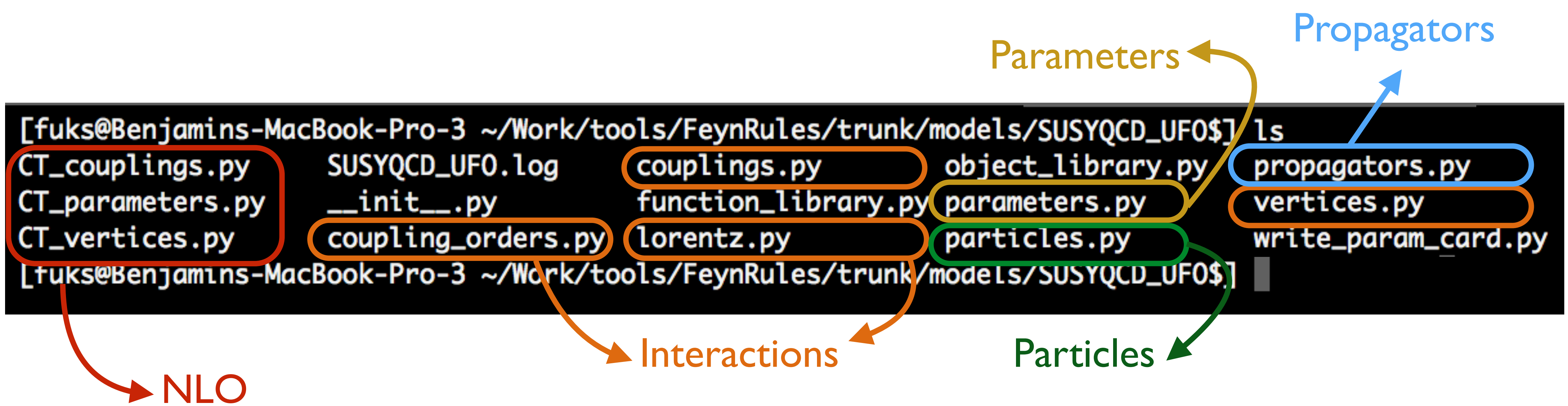
MADGRAPH5_aMC@NLO | WHIZARD | LANHEP | SARAH

The UFO in practice

✦ The UFO is a set of PYTHON files

✤ Factorization of the information: particles, interactions, propagation, parameters, NLO, *etc.*

✦ Example



Particles

- Particles are stored in the particles.py file
  - Instances of the particle class
  - Attributes: particle spin, color representation, mass, width, PDG code, *etc.*
  - Antiparticles automatically derived

```
G = Particle(pdg_code = 21,
             name = 'G',
             antiname = 'G',
             spin = 3,
             color = 8,
             mass = Param.ZERO,
             width = Param.ZERO,
             texname = 'G',
             antitexname = 'G',
             charge = 0)

go = Particle(pdg_code = 1000021,
              name = 'go',
              antiname = 'go',
              spin = 2,
              color = 8,
              mass = Param.Mgo,
              width = Param.Wgo,
              texname = 'go',
              antitexname = 'go',
              charge = 0)
```

```
sq1 = Particle(pdg_code = 1000006,
               name = 'sq1',
               antiname = 'sq1~',
               spin = 1,
               color = 3,
               mass = Param.Msq1,
               width = Param.Wsq1,
               texname = 'sq1',
               antitexname = 'sq1~',
               charge = 0)

sq1__tilde__ = sq1.anti()

sq2 = Particle(pdg_code = 2000006,
               name = 'sq2',
               antiname = 'sq2~',
               spin = 1,
               color = 3,
               mass = Param.Msq2,
               width = Param.Wsq2,
               texname = 'sq2',
               antitexname = 'sq2~',
               charge = 0)

sq2__tilde__ = sq2.anti()
```

```
q = Particle(pdg_code = 6,
             name = 'q',
             antiname = 'q~',
             spin = 2,
             color = 3,
             mass = Param.Mq,
             width = Param.Wq,
             texname = 'q',
             antitexname = 'q~',
             charge = 0)

q__tilde__ = q.anti()
```

Parameters

- Parameters are stored in the parameters.py file
  - Instances of the parameter class
  - External parameters are organized following a Les Houches-like structure (blocks and counters)
  - PYTHON-compliant formula for the internal parameters

```
aS = Parameter(name = 'aS',
               nature = 'external',
               type = 'real',
               value = 0.1184,
               texname = '\\alpha _s',
               lhablock = 'SMINPUTS',
               lhacode = [ 3 ])

G = Parameter(name = 'G',
              nature = 'internal',
              type = 'real',
              value = '2*cmath.sqrt(aS)*cmath.sqrt(cmath.pi)',
              texname = 'G')
```

```
Mgo = Parameter(name = 'Mgo',
                nature = 'external',
                type = 'real',
                value = 500,
                texname = '\\text{Mgo}',
                lhablock = 'MASS',
                lhacode = [ 1000021 ])

Wq = Parameter(name = 'Wq',
               nature = 'external',
               type = 'real',
               value = 1.50833649,
               texname = '\\text{Wq}',
               lhablock = 'DECAY',
               lhacode = [ 6 ])
```

Interactions: generalities

✦ Vertices decomposed in a spin x color basis (coupling strengths ≡ coordinates)

✤ Example: the quartic gluon vertex can be written as

$$\begin{aligned} & i g_s^2\, f^{a_1 a_2 b} f^{b a_3 a_4}\, (\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}) \\ & \quad + i g_s^2\, f^{a_1 a_3 b} f^{b a_2 a_4}\, (\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}) \\ & \quad + i g_s^2\, f^{a_1 a_4 b} f^{b a_2 a_3}\, (\eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}) \end{aligned} \Rightarrow \begin{aligned} & \left(f^{a_1 a_2 b} f^{b a_3 a_4},\ f^{a_1 a_3 b} f^{b a_2 a_4},\ f^{a_1 a_4 b} f^{b a_2 a_3}\right) \\ & \times \begin{pmatrix} i g_s^2 & 0 & 0 \\ 0 & i g_s^2 & 0 \\ 0 & 0 & i g_s^2 \end{pmatrix} \begin{pmatrix} \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} \\ \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \\ \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \end{pmatrix} \end{aligned}$$

- ★ 3 elements for the color basis
- ★ 3 elements for the spin (Lorentz structure) basis
- ★ 9 coordinates (6 are zero)

✤ Several files are used for the storage of the information

Example: the quartic gluon vertex

✦ General information in vertex.py

```
V_2 = Vertex(name = 'V_2',
             particles = [ P.G, P.G, P.G, P.G ],
             color = [ 'f(-1,1,2)*f(3,4,-1)', 'f(-1,1,3)*f(2,4,-1)', 'f(-1,1,4)*f(2,3,-1)' ],
             lorentz = [ L.VVVV1, L.VVVV2, L.VVVV3 ],
             couplings = {(1,1):C.GC_4,(0,0):C.GC_4,(2,2):C.GC_4})
```

- ★ lorentz ≡ spin basis (in lorentz.py; common to all vertices)
- ★ color ≡ color basis
- ★ couplings ≡ coordinates (in couplings.py; common to all vertices)

$$\left(f^{a_1a_2b}f^{ba_3a_4},\ f^{a_1a_3b}f^{ba_2a_4},\ f^{a_1a_4b}f^{ba_2a_3}\right) \times \begin{pmatrix} ig_s^2 & 0 & 0 \\ 0 & ig_s^2 & 0 \\ 0 & 0 & ig_s^2 \end{pmatrix} \begin{pmatrix} \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} \\ \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \\ \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \end{pmatrix}$$

✦ Lorentz structures: straightforward implementations in lorentz.py

```
VVVV1 = Lorentz(name = 'VVVV1',
                spins = [ 3, 3, 3, 3 ],
                structure = 'Metric(1,4)*Metric(2,3) - Metric(1,3)*Metric(2,4)')
```

✦ Couplings: straightforward implementations in couplings.py

```
GC_4 = Coupling(name = 'GC_4',
                value = 'complex(0,1)*G**2',
                order = {'QCD':2})
```

Coupling orders: for selecting diagrams

Extending models

Merging and extending models (I)

✦ Many BSM models of interest are simple extensions of another model

✤ FEYNRULES allows one to start from a given model

★ Add new particles, parameters, Lagrangian terms

★ Modify existing particles, parameters, Lagrangian terms

★ Remove some particles, parameters, Lagrangian terms

✦ Simplified models

✤ Special cases very relevant for LHC physics: Simplified Model Spectra

★ The Standard Model + one or two new particles

★ Often inspired by the MSSM or dark matter models

★ *Ex.*: the SM + lightest stop and neutralino + relevant subset of MSSM interactions

Merging and extending models (2)

- ✦ Merged FEYNRULES model contains two .fr files
  - ✤ The parent model implementation
  - ✤ One extra file with the modifications
  - ✤ They must be loaded together (the parent model first)

```
LoadModel["SM.fr", "stops.fr"];
```

  - ★ No need to re-implement what is common (gauge groups, *etc.*)

- ✦ One can start from the models available on the FEYNRULES database
  http://feynrules.irmp.ucl.ac.be

The FeynRules model database

- O(100) models are available online
  - Simple extensions of the Standard Model
    - ★ Simplified model spectra
    - ★ Four generation models
    - ★ Vector-like quarks
    - ★ Two-Higgs-Doublet Models, Hidden Abelian Higgs
    - ★ *etc.*
  - Supersymmetric models
    - ★ MSSM with and without R-parity
    - ★ The NMSSM
    - ★ R-symmetric supersymmetric models
    - ★ Left-right supersymmetric models
  - Extra-dimensional models
    - ★ Universal extra-dimensions
    - ★ Large extra-dimensions
    - ★ Heidi, Minimal Higgsless models
    - ★ Randall-Sundrum
  - Strongly coupled and effective field theories
    - ★ Technicolor
    - ★ Models with dimension-six and dimension-eight operators
    - ★ *etc.*

Restricting model implementations

- Many BSM models of interest are subset of other models
  - Equivalent to the parent model, with some parameters set to 0 or 1
    - ★ Example 1: the massless version of a model (massless light quarks in the SM)
    - ★ Example 2: a mixing matrix set to the identity (no-CKM matrix in the SM)
  - FEYNRULES allows one to start from a given model
    - ★ Write the restrictions under the form of a list of MATHEMATICA replacement rules
      - ➢ M$Restrictions
    - ★ Read them into FEYNRULES
    - ★ Apply them before the computation of any Feynman rule

- The output Feynman rules (and thus Monte Carlo model files)
  - Are free from the restricted parameters
  - Smaller files, more efficiency at the MC level
    - ★ *Ex.*: the MSSM has more than 10000 vertices; its flavor-conserving version ~1000

Example of restrictions

✦ One practical example: the Standard Model without CKM-mixing

```
M$Restrictions = {
            CKM[i_,i_] -> 1,
            CKM[i_?NumericQ, j_?NumericQ] :> 0 /; (i =!= j),
            cabi -> 0
}
```

```
LoadRestriction["DiagonalCKM.rst"]
```

Superspace module

See the manual for more details on the superspace module

Superfield declaration

- A module dedicated to calculations in superspace
  - Superfield declaration and links to the component fields

A left-handed squark superfield

```
CSF[1] == {
  ClassName       -> QL,
  Chirality       -> Left,
  Weyl            -> qLw,
  Scalar          -> sqL,
  Indices         -> {Index[Colour]}
}
```

★ Component fields to be declared too

★ Vector superfields can be linked to gauge groups

Supersymmetric model implementation

- Supersymmetric model implementation
  - Declaration of the model gauge group
  - Declaration of all fields and superfields
  - Declaration of all model parameters
  - Writing the Lagrangian (in a simplified way)

- Supersymmetric Lagrangian in superspace are very compact

$$\mathcal{L} = \Phi^\dagger e^{-2gV}\Phi_{|_{\theta^2\bar\theta^2}} + \frac{1}{16g^2\tau_\mathcal{R}}\mathrm{Tr}(W^\alpha W_\alpha)_{|_{\theta^2}} + \frac{1}{16g^2\tau_\mathcal{R}}\mathrm{Tr}(\bar W_{\dot\alpha}\bar W^{\dot\alpha})_{|_{\bar\theta^2}}$$
$$+ W(\Phi)_{|_{\theta^2}} + W^\star(\Phi^\dagger)_{|_{\bar\theta^2}} + \mathcal{L}_{\rm soft}$$

  - First line: kinetic and gauge interaction terms for all fields
    - ➢ Model independent ⇒ can be automated
  - Second line: superpotential and supersymmetry breaking Lagrangian
    - ➢ Model dependent ⇒ to be provided
  - Series expansion in terms of component fields
  - Automatic derivation of supersymmetric Lagrangians
  - Solving the equations of motion of the unphysical fields

Implementing supersymmetric Lagrangians

✦ Supersymmetric Lagrangian in superspace are very compact

$$\mathcal{L} = \Phi^\dagger e^{-2gV}\Phi_{|_{\theta^2\bar\theta^2}} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{Tr}(W^\alpha W_\alpha)_{|_{\theta^2}} + \frac{1}{16g^2\tau_{\mathcal{R}}}\mathrm{Tr}(\bar W_{\dot\alpha}\bar W^{\dot\alpha})_{|_{\bar\theta^2}}$$
$$+ W(\Phi)_{|_{\theta^2}} + W^\star(\Phi^\dagger)_{|_{\bar\theta^2}} + \mathcal{L}_{\rm soft}$$

✤ First line: kinetic and gauge interaction terms for all fields
- ★ Model independent ⇒ can be automated (CSFKineticTerms / VSFKineticTerms)
- ★ Dedicated methods to access the components of a superfield (Theta2Component, *etc.*)

✤ Second line: superpotential and supersymmetry-breaking terms
- ★ Model dependent ⇒ to be provided by the user (*SuperW* and *LSoft*)

```
Lag = Theta2Thetabar2Component[ CSFKineticTerms[] ] +
    Theta2Component[ VSFKineticTerms[] ] +
    Thetabar2Component[ VSFKineticTerms[] ];
```

```
Lag2 = LSoft + Theta2Component[ SuperW ] +  Thetabar2Component[ SuperW ];
```

Implementing supersymmetric Lagrangians

✦ The implemented Lagrangian above must be post-processed

```
Lag = Theta2Thetabar2Component[ CSFKineticTerms[] ] +
   Theta2Component[ VSFKineticTerms[] ] +
   Thetabar2Component[ VSFKineticTerms[] ];
```

```
Lag2 = LSoft + Theta2Component[ SuperW ] +  Thetabar2Component[ SuperW ];
```

- Solving the equation of motion for the auxiliary fields
- Inserting the solution back into the Lagrangian
  - ★ Automated (SolveEqMotionF and SolveEqMotionD)
- Replacement of Weyl spinors in terms of Majorana and Dirac spinors
  - ★Automated (WeylToDirac)
- Rotation to the mass basis
  - ★Standard FeynRules function (ExpandIndices)

Mass matrix diagonalization

See the manual for more details on the ASperGe module

Mass matrices and their diagonalization

✦ The problematics of the mass matrices

- ✤ Lagrangians are usually easily written in the gauge basis
- ✤ The included mass matrices are thus in general non-diagonal
  - ➢ diagonalization required
- ✤ The gauge basis must be rotated to the mass basis where the mass matrices are diagonal
- ✤ This diagonalization cannot in general be achieved analytically

✦ The ASPERGE package of FEYNRULES

- ✤ A module allowing one to extract the mass matrices from the Lagrangian
- ✤ A generator of C++ code ➢ numerical diagonalization of all mass matrices (the generated code can be used in a standalone way)
- ✤ See: Alloul, D'Hondt, De Causmaecker, BF, Rausch de Traubenberg [EPJC 73 (2013) 2325]

Example of the Z-boson/photon mixing

- Example: the Z-boson and photon in the Standard Model
  - Each mixing is declared as a set of replacement rules (in *M$MixingsDescription)*
  - Each rule represent a property of the mixing relation

$$\begin{pmatrix} A_\mu \\ Z_\mu \end{pmatrix} = U_w \begin{pmatrix} B_\mu \\ W^3_\mu \end{pmatrix} \Rightarrow$$

```
Mix["AZ"] == {
  MassBasis     -> {A, Z},
  GaugeBasis    -> {B, Wi[3]},
  MixingMatrix -> UW,
  BlockName     -> WEAKMIX
}
```

  - ASPERGE can compute the mass matrices:
  - ASPERGE can generate its standalone C++ version

```
ComputeMassMatrix[Lag];
WriteASperGe[Lag];
```

Decays

See the manual for more details on the decay module

Two-body decays

✦ The problematics of the decay widths and branching ratios

- ✤ Some MC tools need decay tables (widths and branching ratios) to decay particles
- ✤ Widths and branching ratios are not independent quantities
  - ➢ need to be calculated
- ✤ Some Monte Carlo tools compute these quantities on the fly
  - ➢ the procedure is repeated each time it is needed
- ✤ FEYNRULES offers a way to include analytical information on the two-body decay

Two-body decays

✦ Two-body decays in general

✤ Two-body decays can be directly read from three-point vertices ($\mathcal{V}$)

$$\Gamma_{1\to 2} = \frac{1}{2|M|S}\int \mathrm{d}\Phi_N\,|\mathcal{M}_{1\to 2}|^2 = \frac{\sqrt{\lambda(M^2,m_1^2,m_2^2)}}{16\,\pi\,S\,|M|^3}\mathcal{V}^{a_1a_2a_3}_{\ell_1\ell_2\ell_3}\,\mathcal{P}_1^{\ell_1\ell_1'}\,\mathcal{P}_2^{\ell_2\ell_2'}\,\mathcal{P}_3^{\ell_3\ell_3'}\,(\mathcal{V}^*)^{a_1a_2a_3}_{\ell_1'\ell_2'\ell_3'}$$

★ Partial width for the decay of a particle of mass M to two particles of masses m_1 and m_2

★ Includes a symmetry factor S and $\mathcal{P}$ denotes the polarization tensor of each particle

✦ The decay module of FEYNRULES

✤ FEYNRULES makes use of MATHEMATICA to compute all partial widths of the model

★ Ignores open and closed channels ➢ benchmark independent

★ The information is exported to the UFO (used, e.g, by MADWIDTH)

Running the decay module of FEYNRULES

✦ Automatic decay width computations

- ✤ All two-body decay widths can be easily computed from the Lagrangian

```
verts      = FeynmanRules[Lag];
vertsexp   = FlavorExpansion[verts];
results    = ComputeWidths[vertsexp];
```

- ✤ Many functions available for analytical calculations
  - ➢ PartialWidth, TotWidth, BranchingRatio
- ✤ The numerical value provided for the particle widths can be updated accordingly
  - ➢ UpdateWidths
- ✤ See: Alwall, Duhr, BF, Mattelaer, Öztürk, Shen [CPC (2015)]

Snippet of the UFO output

- The information is (by default) employed by the UFO interface
  - Can be turned of: (AddDecays → False)
  - The UFO contains an extra file *decays.py*
  - This file can be used by MC codes
  - Example of the Standard Model UFO: the top quark

```
Decay_t = Decay(name = 'Decay_t',
                particle = P.t,
                partial_widths = {(P.W__plus__,P.d):'((MT**2 - MW**2)*((3*CKM3x1*ee**2*MT**2*complexconjugate(CKM3x1))/(2.*sw**2) +
(3*CKM3x1*ee**2*MT**4*complexconjugate(CKM3x1))/(2.*MW**2*sw**2) - (3*CKM3x1*ee**2*MW**2*complexconjugate(CKM3x1))/sw**2))/
(96.*cmath.pi*abs(MT)**3)',
                                  (P.W__plus__,P.s):'((MT**2 - MW**2)*((3*CKM3x2*ee**2*MT**2*complexconjugate(CKM3x2))/(2.*sw**2) +
(3*CKM3x2*ee**2*MT**4*complexconjugate(CKM3x2))/(2.*MW**2*sw**2) - (3*CKM3x2*ee**2*MW**2*complexconjugate(CKM3x2))/sw**2))/
(96.*cmath.pi*abs(MT)**3)',
                                  (P.W__plus__,P.b):'(((3*CKM3x3*ee**2*MB**2*complexconjugate(CKM3x3))/(2.*sw**2) +
(3*CKM3x3*ee**2*MT**2*complexconjugate(CKM3x3))/(2.*sw**2) + (3*CKM3x3*ee**2*MB**4*complexconjugate(CKM3x3))/(2.*MW**2*sw**2) -
(3*CKM3x3*ee**2*MB**2*MT**2*complexconjugate(CKM3x3))/(MW**2*sw**2) + (3*CKM3x3*ee**2*MT**4*complexconjugate(CKM3x3))/(2.*MW**2*sw**2) -
(3*CKM3x3*ee**2*MW**2*complexconjugate(CKM3x3))/sw**2)*cmath.sqrt(MB**4 - 2*MB**2*MT**2 + MT**4 - 2*MB**2*MW**2 - 2*MT**2*MW**2 +
MW**4))/(96.*cmath.pi*abs(MT)**3)'})
```

NLO

Higher-order corrections (in QCD)

- ✦ NLO calculations matched to parton shower (for BSM) are automated
  - ✤ Model-dependent parts of calculations (on top of the tree-level information)
    - ★ Counterterms
    - ★ Finite pieces of the loop-integrals
  - ✤ Model independent contributions
    - ★ Subtraction of the divergences
    - ★ Matching to the parton showers

UFO @ NLO

[Degrande, Duhr, BF, Hirschi, Mattelaer, Shao *et al.* (*in prep.*)]

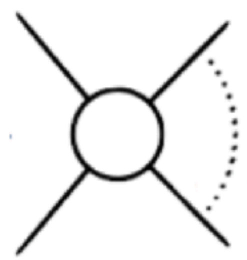
Recap' on NLO calculations

✦ Contributions to an NLO result in QCD

✤ Three ingredients: the Born, virtual loop and real emission contributions

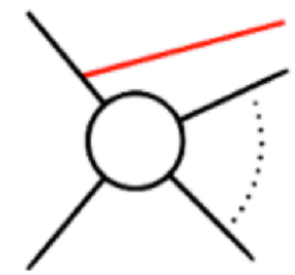
$$\sigma_{NLO} = \int \mathrm{d}^4\Phi_n \mathcal{B} \quad + \int \mathrm{d}^4\Phi_n \int_{\mathrm{loop}} \mathrm{d}^d\ell \; \mathcal{V} \quad + \int \mathrm{d}^4\Phi_{n+1} \; \mathcal{R}$$

Born



Virtuals: one extra power of α_s and divergent

Reals: one extra power of α_s and divergent



Extra information is needed

Virtual contributions

✦ Loop diagram calculations

✤ Calculations to be done in $d=4-2\varepsilon$ dimensions

★ Divergences made explicit ($1/\varepsilon^2$, $1/\varepsilon$)

✤ Rewriting loop integrals with scalar integrals

$$\int \mathrm{d}^d\ell \frac{N(\ell)}{D_0 D_1 \cdots D_{m-1}} = \sum a_i \int \mathrm{d}^d\ell \frac{1}{D_{i_0} D_{i_1} \cdots}$$

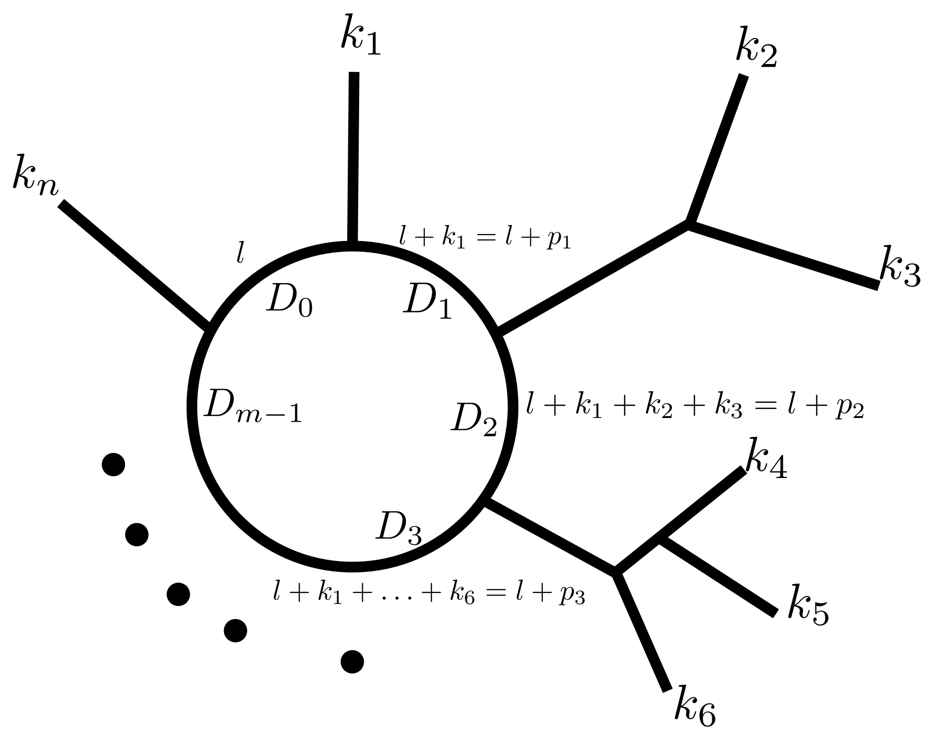
★ Involves integrals with up to four denominators

➢ The decomposition basis is finite

➢ Can be computed once and for all

★ The reduction is the process-dependent part

m-point diagram with *n* external momenta



The rational terms (R_1 and R_2)

- The loop momentum lives in a *d*-dimensional space
  - Reduction to be done in *d* dimensions

$$\int \mathrm{d}^d\ell \frac{N(\ell,\tilde{\ell})}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{m-1}} \quad \text{with } \bar{\ell} = \ell + \tilde{\ell}$$

D-dim 4-dim (-2ε)-dim

  - Numerical methods works in 4 dimensions: need to be compensated!

- The R_1 terms originates from the denominators
  - Connected to the internal propagators
- The R_2 terms originates from the numerator
  - Can be seen as extra diagrams with special Feynman rules

R₂ terms

✦ The R_2 terms originates from the denominators

$$\frac{1}{\bar{D}} = \frac{1}{D}\left(1 - \frac{\tilde{\ell}^2}{\bar{D}}\right)$$

✤ These extra pieces can be calculated generically (3 integrals in total)

$$\int \mathrm{d}^d\bar{\ell}\,\frac{\tilde{\ell}^2}{\bar{D}_i\bar{D}_j} = -\frac{i\pi^2}{2}\left[m_i^2 + m_j^2 - \frac{p_i - p_j)^2}{2}\right] + \mathcal{O}(\varepsilon)$$

$$\int \mathrm{d}^d\bar{\ell}\,\frac{\tilde{\ell}^2}{\bar{D}_i\bar{D}_j\bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\varepsilon)$$

$$\int \mathrm{d}^d\bar{\ell}\,\frac{\tilde{\ell}^2}{\bar{D}_i\bar{D}_j\bar{D}_k\bar{D}_l} = -\frac{i\pi^2}{6} + \mathcal{O}(\varepsilon)$$

✤ The denominator structure is already known at the reduction time

✤ The R_2 coefficients are extracted during the reduction

R_2 terms

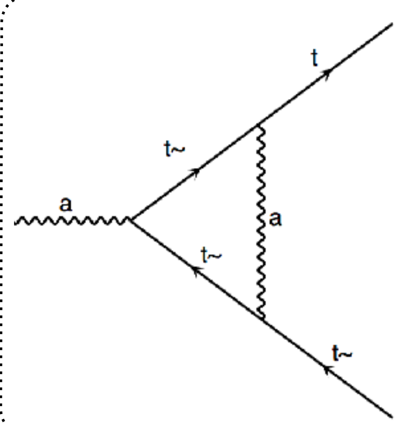
- The R_2 terms originates from the numerator

$$\bar{N}(\bar{\ell}) = N(\ell) + \tilde{N}(\tilde{\ell}, \ell, \varepsilon) \quad \Rightarrow \quad R_2 \equiv \lim_{\varepsilon \to 0} \frac{1}{(2\pi)^4} \int \mathrm{d}^d \bar{\ell} \frac{\tilde{N}(\tilde{\ell}, \ell, \varepsilon)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

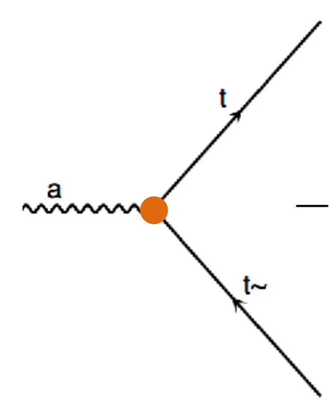
($\bar{N}(\bar{\ell})$: D-dim; $N(\ell)$: 4-dim; $\tilde{N}(\tilde{\ell}, \ell, \varepsilon)$: (-2$\varepsilon$)-dim)

  - Practically, we isolate the epsilon part
  - There is only a finite set of loops for which it does not vanish

- They can be re-expressed in terms of R_2 Feynman rules



$$\propto \int \mathrm{d}^d \bar{\ell} \frac{\tilde{\ell}^2}{\bar{D}_i \bar{D}_j \bar{D}_k} = -\frac{i\pi^2}{2} + \mathcal{O}(\varepsilon) \quad \Rightarrow$$



$$-i\frac{\alpha}{2\pi} e\gamma^\mu$$

R_2 Feynman rules

✦ The R_2 are process dependent and model-dependent (like Feynman rules)

✤ In a renormalizable theory, there is a finite number of them

✤ They can be derived from the sole knowledge of the bare Lagrangian

[Ossala, Papadopoulos, Pittau (JHEP'08)]

✦ The R_2 calculation can be automated and performed once and for all

✤ Development of the NLOCT package (extension of FeynRules)

✤ Computation, for any model, of all R_2 and UV counterterms

★ In the on-shell and MSbar schemes

✤ Inclusion of the output in the UFO

[Degrande (CPC'15)]

Automated NLO simulations with MG5_aMC

✦ A comprehensive approach to Monte Carlo simulations at the NLO in QCD