Basics of Parton Showers: From Inclusive to Exclusive Predictions

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Introduction: Why Monte Carlo Event Generators?

Theory

QFT: Lagrangian formulation of physics

- Standard Model: $\mathcal{L}_{\mathrm{SM}}$
- Beyond the Standard Model: $\mathcal{L}_{\mathrm{BSM}}$

Experiments

Collider experiments with complex detectors

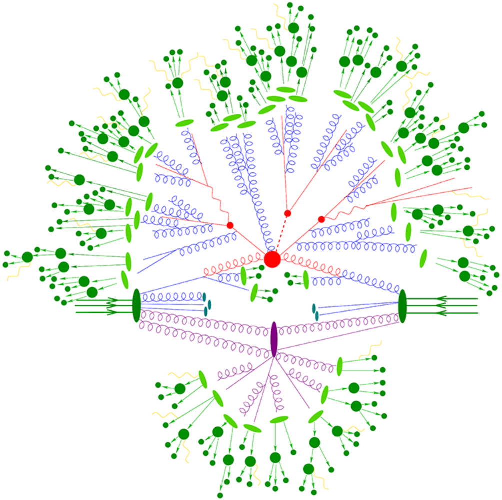
- LHC with ATLAS, CMS, ...
- Reconstruction of individual events
- Very advanced counting experiments

Simulation

Linking theory & experiment

- MC generators: Stochastic simulation of events
- Allow to compare theory and experiment
- Predict event count by integrating differential cross section over specific phase space regions

Hadron Collisions: QCD, QCD, QCD, . . .



Split the problem into many pieces

- Hard Process, resonant decays
- Parton Shower
- MPIs
- Hadronisation
- PDFs: Pick a parton from a hadron
- Hadron Decays
- Hadronic rescattering
- Beam Remnants/UE

Figure from Stefan Höche

Shower Monte Carlo Event Generators

Three commonly used general purpose Event Generators

Pythia (begun 1978)
- Originated from hadronization studies: Lund string model
- Pythia 6 virtuality shower, Pyhtia 8 $p_{\perp}$ shower with ME corrections
- Also DIRE and VINCIA dipole/antenna showers
- Interleaved multi parton interactions

Herwig (begun 1984)
- Originated from coherence studies: Angular ordered shower
- Also $p_{\perp}$ orderd CS dipole shower
- Cluster hadronization

SHERPA (begun ∼2000)
- Originated from Matrix Element/Parton-Shower matching/merging (CKKW(L))
- CS dipole shower and DIRE parton shower
- Own cluster hadronization

Outline

First Lecture: Parton Showers

- Initial and final state radiation (ISR, FSR)
- Differential (e.g., DGLAP) parton evolution equation
- Evolution in resolution scale from Q_{max} to $Q_{\text{had}} \sim 1\text{GeV}$

Second Lecture: Matching and Merging

- Combine benefits of matrix elements and parton showers
- Merging of multiple multi-jet matrix elements X, X + 1 jet, X + 2 jets, ...
- Matching of NLO matrix elements with parton showers

Suggested Reading

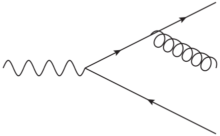
- Andy Buckley et al.
  General-purpose event generators for LHC physics
  Phys. Rept. 504 (2011) 145 arXiv:1101.2599 [hep-ph]
- Stefan Höche
  Introduction to parton-shower event generators arXiv:1411.4085 [hep-ph]
- John Campbell, Joey Huston, Frank Krauss
  The Black Book of Quantum Chromodynamics: A Primer for the LHC Era
  Oxford University Press, 2018
- Torbjörn Sjöstrand, Stephen Mrenna, Peter Skands
  PYTHIA 6.4 Physics and Manual
  JHEP 0605:026, 2006 arXiv:hep-ph/0603175

All references are clickable → download slides and follow links for further details

Parton Emission: An Example

- Example: Consider $e^+e^- \to q\bar{q}g$

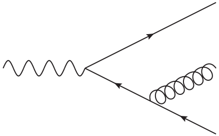
$$\frac{\mathrm{d}\sigma_{q\bar{q}g}}{\mathrm{d}\cos\theta\mathrm{d}z} \approx \sigma_{q\bar{q}} C_\mathrm{F} \frac{\alpha_\mathrm{s}}{2\pi} \frac{2}{\sin^2\theta} \frac{1+(1-z)^2}{z}$$



θ - angle between quark and gluon

z - energy fraction of gluon

- Divergent
  - Collinear limit $\theta \to 0, \pi$
  - Soft limit: $z \to 0$
- Separate into two independent collinear regions



$$\frac{2}{\sin^2\theta} = \frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta} \approx \frac{1}{1-\cos\theta} + \frac{1}{1-\cos\tilde{\theta}}$$

$\tilde{\theta}$ - angle between gluon and antiquark

Universal Parton Evolution

- Independent emission distribution

$$\mathrm{d}\sigma_{q\bar{q}g} \approx \sigma_{q\bar{q}} \sum_{\text{partons}} C_F \frac{\alpha_s}{2\pi} \frac{\mathrm{d}\theta^2}{\theta^2} \mathrm{d}z \frac{1 + (1-z)^2}{z}$$

- Same equation for any "evolution variable" $\rho \propto \theta^2$
  - $t = q^2 = z(1-z)\theta^2 E^2$ virtuality of off-shell propagator
  - $p_\perp^2 = z^2(1-z)^2\theta^2 E^2$ gluons transverse momentuma w.r.t. parent quark

$$\frac{\mathrm{d}\theta^2}{\theta^2} = \frac{\mathrm{d}q^2}{q^2} = \frac{\mathrm{d}p_\perp^2}{p_\perp^2}$$

- Structure completely general (for all hard processes with partons i)

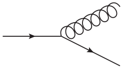
$$\mathrm{d}\sigma_{n+1} \approx \sigma_n \sum_{\text{partons i}} \frac{\alpha_s}{2\pi} \frac{\mathrm{d}\rho}{\rho} \mathrm{d}z P_{ij}(z, \phi) \mathrm{d}\phi$$

- P_{ij} set of universal, flavour-dependent functions, depend on azimuthal angle of emission ϕ

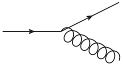
Universal Splitting Functions

$$\mathcal{P}_{\text{incl}} = \int_{\rho_{\min}}^{\rho_{\max}} \frac{\mathrm{d}\rho}{\rho} \frac{\alpha_s}{2\pi} \int_{z_{\min}}^{z_{\max}} \mathrm{d}z P_{ij}(z)$$

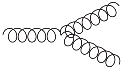
Spin-averaged $i \to j$ splitting functions P_{ij}:



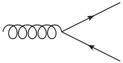
$$= P_{qg} = C_F \frac{1+(1-z)^2}{z}$$



$$= P_{qq} = C_F \frac{1+z^2}{1-z}$$



$$= P_{gg} = 2C_A \frac{(1-z(1-z))^2}{z(1-z)}$$



$$= P_{gq} = T_R(z^2(1-z)^2)$$

with $C_F = \frac{N_c^2-1}{2N_c}$, $C_A = N_c$ and $T_R = \frac{1}{2}$

Emission Probabilities

$$\mathcal{P}_{\mathrm{incl}} = \int_{p_{\perp,\mathrm{min}}}^{p_{\perp,\mathrm{max}}} \frac{\mathrm{d}p_\perp^2}{p_\perp^2} \frac{\alpha_\mathrm{s}}{2\pi} \int_{z_{\mathrm{min}}=\frac{p_\perp}{E}}^{z_{\mathrm{max}}=1-\frac{p_\perp}{E}} \mathrm{d}z P_{ij}(z)$$

Probabilistic interpretation: Probability to emit a parton with $p_\perp^2 \in [p_{\perp,\mathrm{min}}, p_{\perp,\mathrm{max}}]$ and energy fraction $z \in [z_{\mathrm{min}}, z_{\mathrm{max}}]$. For gluon emission, successive integration over z and $p_\perp^2$ gives

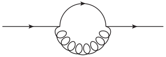
$$\mathcal{P}_{\mathrm{incl}} \propto \alpha_\mathrm{s} \ln^2\left(\frac{Q^2}{p_{\perp,\mathrm{min}}^2}\right) \qquad \text{“double log”}$$

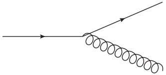
Where $Q^2 = \mathcal{O}(p_{\perp,\mathrm{max}}^2)$ and $p_{\perp,\mathrm{min}} = \mathcal{O}(\Lambda_{\mathrm{QCD}}) \approx \mathrm{GeV}$. In general, with $L = \ln(Q^2/p_{\perp,\mathrm{min}}^2)$

$$\mathrm{d}\sigma(X + ng) = \mathrm{d}\sigma(X) \otimes \alpha_\mathrm{s}^n \left(c_{2n}L^{2n} + c_{2n-1}L^{2n-1} + \cdots + c_0\right)$$

Multiplied splitting kernels approximate multi-parton cross section, dominated by soft and collinear splittings

No-emission Probabilities

- Introduce resolution criterion, e.g. $\rho > \rho_{\min}$
-  $+$  $=$ virtual $+$ unresolved $=$ finite

   $=$ resolved $=$ finite

  [Kinoshita (1962)] [Lee, Nauenberg (1964)]
- Unitarity / probability conservation: $\mathcal{P}_{\mathrm{no-em}} = 1 - \mathcal{P}_{\mathrm{incl}}$
- Multiplicative in evolution: $\mathcal{P}_{\mathrm{no-em}}(\rho_{\max} > \rho > \rho_{\min})$

$$= \mathcal{P}_{\mathrm{no-em}}(\rho_{\max} > \rho > \rho_1) \cdot \mathcal{P}_{\mathrm{no-em}}(\rho_1 > \rho > \rho_{\min})$$

$$= \lim_{n\to\infty} \prod_{i=0}^{n-1} \mathcal{P}_{\mathrm{no-em}}(\rho_i > \rho > \rho_{i+1}) = \lim_{n\to\infty} \prod_{i=0}^{n-1} (1 - \mathcal{P}_{\mathrm{incl}}(\rho_i > \rho > \rho_{i+1}))$$

$$= \lim_{n\to\infty} \exp\left(- \sum_{i=0}^{n-1} \mathcal{P}_{\mathrm{incl}}(\rho_i > \rho > \rho_{i+1}) \right) = \exp\left(- \int_{\rho_{\min}}^{\rho_{\max}} \mathrm{d}\mathcal{P}_{\mathrm{incl}}(\rho) \right)$$

Sudakov Form Factor

- Must implement emission and no-emission probability

$$\mathrm{d}\mathcal{P}_{\mathrm{incl}}(\rho) = \frac{\mathrm{d}\rho}{\rho}\frac{\alpha_{\mathrm{s}}}{2\pi}\int_{z_{\min}}^{z_{\max}} \mathrm{d}z P_{ij}(z)$$

$$\mathcal{P}_{\mathrm{no-em}}(\rho_1, \rho_2) = \exp\left(-\int_{\rho_2}^{\rho_1}\frac{\mathrm{d}\rho}{\rho}\frac{\alpha_{\mathrm{s}}}{2\pi}\int_{z_{\min}}^{z_{\max}} \mathrm{d}z P_{ij}(z)\right)$$

- Probability of parton i to have hardest splitting ρ follows Poission statistics:

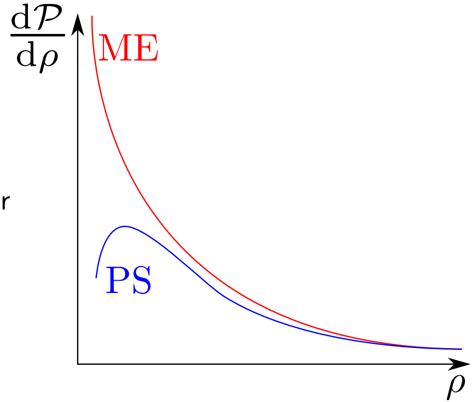
$$\mathrm{d}\mathcal{P}_{\mathrm{first}}(\rho) = \mathrm{d}\mathcal{P}_{\mathrm{incl}}(\rho)\cdot\mathcal{P}_{\mathrm{no-em}}(\rho_{\max}, \rho)$$

- Call $\Delta(\rho_1, \rho_2) := \mathcal{P}_{\mathrm{no-em}}(\rho_1, \rho_2)$ Sudakov from factor

First and Repeated Emissions

ME: Emission rate derived from matrix element
PS: Scale of hardest emission as given by parton shower

- Sudakov regulates singularity for hardest emission
- Repeated soft emission $q \to qg$ lead to ME ρ emission spectrum
- Naively: Divergent ME spectrum $\Leftrightarrow$ infinite number of PS emissions



Complications in reality:

- Energy and momentum conservation in splittings
- Additional $g \to gg$ splittings lead to accelerated multiplication of partons

Sudakov Factor is All Order Expression

Expanding Sudakov factor in orders of strong coupling gives

$$\begin{aligned}\Delta(\rho_0, \rho_1) &= \exp\left(-\int_{\rho_1}^{\rho_0} \frac{\mathrm{d}\rho}{\rho}\frac{\alpha_\mathrm{s}}{2\pi}\int_{z_\mathrm{min}}^{z_\mathrm{max}} \mathrm{d}z P_{ij}(z)\right) \\ &= 1 - \int_{\rho_1}^{\rho_0} \frac{\mathrm{d}\rho}{\rho}\frac{\alpha_\mathrm{s}}{2\pi}\int_{z_\mathrm{min}}^{z_\mathrm{max}} \mathrm{d}z P_{ij}(z) \\ &\quad + \frac{1}{2}\left(\int_{\rho_1}^{\rho_0} \frac{\mathrm{d}\rho}{\rho}\frac{\alpha_\mathrm{s}}{2\pi}\int_{z_\mathrm{min}}^{z_\mathrm{max}} \mathrm{d}z P_{ij}(z)\right)^2 + \cdots\end{aligned}$$

No emission probability → no change in state → virtual correction
Sudakov contains divergent terms of first order virtual correction, second order virtual correction, . . . all orders!

Unitarity of Parton Shower

Parton Shower derived from unitarity: Probability of splitting and non-splitting add to 1 $\Rightarrow$ does not change inclusive cross section, but changes shape.
Probability that hardest emission is somewhere:

$$\int_{\rho_{\min}}^{\rho_0} \frac{\mathrm{d}\rho}{\rho} \int_{z_{\min}}^{z_{\max}} \mathrm{d}z \frac{\alpha_s}{2\pi} P(z) \Delta(\rho_0, \rho) = \int_{\rho_{\min}}^{\rho_0} \mathrm{d}\rho \frac{\mathrm{d}}{\mathrm{d}\rho} \exp\left(- \int_{\rho}^{\rho_0} \frac{\mathrm{d}\rho'}{\rho'} \int_{z_{\min}}^{z_{\max}} \mathrm{d}z \frac{\alpha_s}{2\pi} P(z) \right)$$
$$= \exp\left(- \int_{\rho_0}^{\rho_0} \frac{\mathrm{d}\rho}{\rho} \cdots \right) - \exp\left(- \int_{\rho_{\min}}^{\rho_0} \frac{\mathrm{d}\rho}{\rho} \cdots \right)$$
$$\xrightarrow{\rho_{\min} \to 0} \exp(-0) - \exp(-\infty) = 1$$

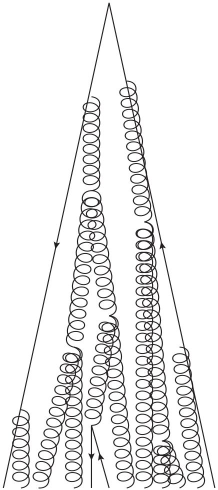
But we are interested in observable stat with $\rho_{\min} > 0$. Parton shower reinterprets a part of the 0 jet cross section as $+1, +2, \cdots$ parton cross section

An MC Algorithm for Parton Showers: The Sudakov Veto Algorithm

- Start with n partons at scale ρ_1, evolve simultaneously
- Sudakov factors factorize:

$$\Delta(\rho_1, \rho) = \prod_{i=1}^{n} \Delta_i(\rho_1, \rho), \qquad \Delta_i(\rho_1, \rho) = \prod_{j=q,g} \Delta_{i\to j}(\rho_1, \rho)$$

- Use veto algorithm to find scales of subsequent emissions
  - Propose ρ using MC based on overestimate $P_{ij}^{\max}(z)$
  - Determine "winner" parton i and new flavor j
  - Select splitting variable z according to overestimate $P_{ij}^{\max}(z)$
  - Accept splitting with probability $P_{ij}(z)/P_{im}^{\max}(z)$, else continue sampling from present scale
- Construct full splitting kinematics and color configuration
- Iterate until reaching cutoff $\rho_{\min} \sim 1\text{GeV}$



see e.g. [Lönnblad (2013)] for more details

Collinear Factorization of QCD Cross Sections

Hadronic cross section for scattering $ab \to n$ given by

$$\sigma = \sum_{a,b} \int_0^1 \frac{\mathrm{d}x_a}{x_a} \frac{\mathrm{d}x_b}{x_b} \int x_a f_a^{h_1}(x_a, \mu_\mathrm{F}) x_b f_b^{h_2}(x_b, \mu_\mathrm{F}) \mathrm{d}\hat{\sigma}_{ab\to n}(\mu_\mathrm{F}, \mu_\mathrm{R})$$

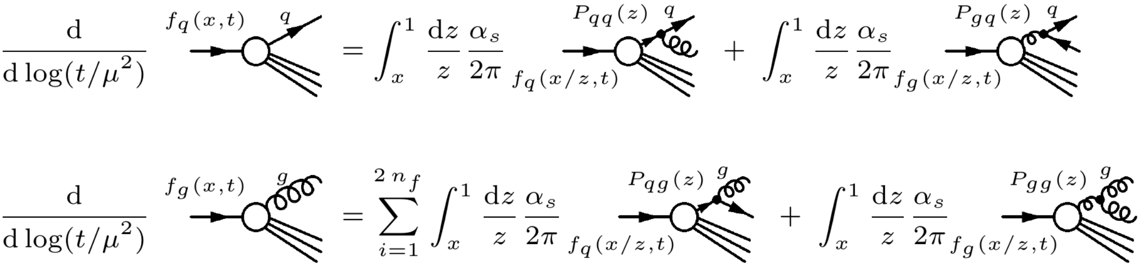
- $\hat{\sigma}$ Partonic cross section
- $f_a^h(x_a, \mu_\mathrm{F})$ parton distribution functions (PDFs)
- x_a light cone momentum fraction $\to$ $x_a f_a$ momentum flux of parton a at x_a
- μ_F factorization scale

Need to take PDFs into account in initial state radiation (ISR), since they change flux

See [Collins, Soper, Sterman (1989)] for factorization theorems in QCD

DGLAP Equations

[Dokshitzer (1977)] [Gribov, Lipatov (1972)] [Altarelli, Parisi (1977)]



borrowed from S. Höche

- Coupled differential equations describing the parton flux of a hadron at different resolution scales

Initial State Radiation and PDFs

- Modify emission and no-emission probabilities to include PDFs: $x_{\mathrm{new}} = x/z$:

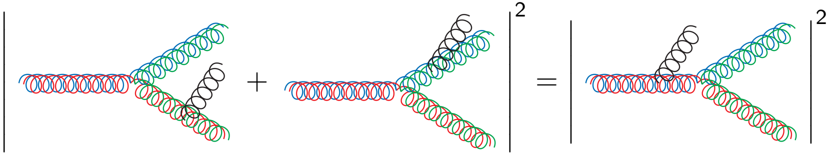
$$\mathrm{d}\mathcal{P}_{\mathrm{emission}}(\rho) = \frac{\mathrm{d}f_j}{f_j} = \frac{\mathrm{d}\rho}{\rho}\frac{\alpha_{\mathrm{s}}}{2\pi}\int_{z_{\mathrm{min}}}^{z_{\mathrm{max}}} \mathrm{d}z P_{ij}(z)\frac{\frac{x}{z}f_i(\frac{x}{z},\rho)}{xf_j(x,\rho)}$$

$$\mathcal{P}_{\mathrm{no-em}}(\rho_1,\rho_2) = \exp\left(-\int_{\rho_2}^{\rho_1}\frac{\mathrm{d}\rho}{\rho}\frac{\alpha_{\mathrm{s}}}{2\pi}\int_{z_{\mathrm{min}}}^{z_{\mathrm{max}}} \mathrm{d}z P_{ij}(z)\frac{\frac{x}{z}f_i(\frac{x}{z},\rho)}{xf_j(x,\rho)}\right) := \Pi(\rho_1,\rho_2)$$

- Initial state shower reproduces DGLAP
- DGLAP evolution from small to large scale
- ISR usually uses "backwards evolution": from large to small scales
  $\Rightarrow$ makes sure we can start from partonic process of interest at high scale [Sjöstrand (1985)]

Soft Gluons: Physical Picture of Color Coherence

- Soft gluons (large wavelength) not able to resolve charges of emitting color dipole individually



- Emission with combined color charge of mother parton
  ⇒ destructive interference outside cone with opening angle defined by dipole
- Can be solved by
  - Angular ordering (Herwig) [Marchesini, Webber (1988)]
  - Additional ordering constraint (approximately)
  - Dipole showers with transverse momentum ordering

Evidence for Color Coherence in 3-jet Events

Pseudorapidity of third jet [CDF (1994)]

- Very old Pythia: purely virtual ordered: too much radiation in central region
- Very old Pythia+: additional phase space constraint on initial-final dipole (angular veto) ok
- Herwig angular ordered ok

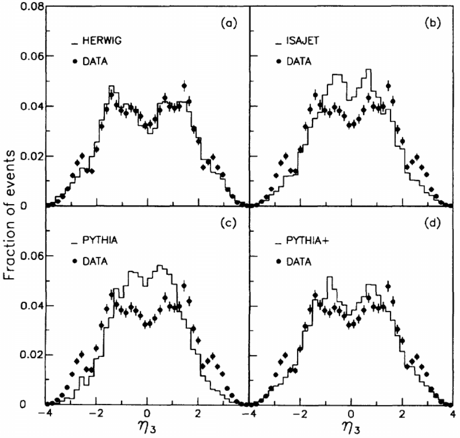


FIG. 13. Observed η_3 distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

Soft Factorization: QCD "Antenna" and "Dipole"

- Consider $e^+e^- \to q\bar{q}g$. For soft gluon:

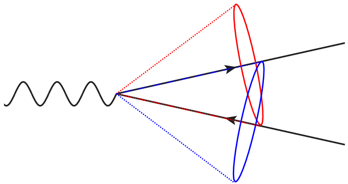
$$\mathrm{d}\sigma_{q\bar{q}g} = \mathrm{d}\sigma_{q\bar{q}} \frac{\mathrm{d}\omega}{\omega} \frac{\mathrm{d}\Omega}{2\pi} \frac{\alpha_\mathrm{s}}{2\pi} C_\mathrm{F} W_{q\bar{q}} \qquad \text{with} \qquad W_{q\bar{q}} = \frac{1-\cos\theta_{q\bar{q}}}{(1-\cos\theta_{qg})(1-\cos\theta_{\bar{q}g})}$$

- Split "Antenna" radiation term $W_{q\bar{q}}$ into "Dipole" terms $W_{q\bar{q}}^{(q)}$ and $W_{q\bar{q}}^{(\bar{q})}$, divergent only if g collinear to q or $\bar{q}$:

$$W_{q\bar{q}} = W_{q\bar{q}}^{(q)} + W_{q\bar{q}}^{(\bar{q})}, \qquad \text{where} \qquad W_{q\bar{q}}^{(q)} = \frac{1}{2}\left(W_{q\bar{q}} + \frac{1}{1-\cos\theta_{qg}} - \frac{1}{1-\cos\theta_{\bar{q}g}}\right)$$

- Azimuthal integration gives angular ordering

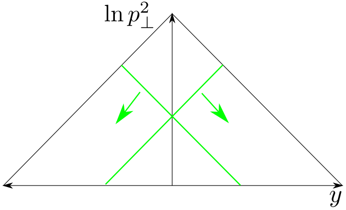
$$\int_0^{2\pi} \frac{\mathrm{d}\phi_{qg}}{2\pi} W_{q\bar{q}}^{(q)} = \begin{cases} \frac{1}{1-\cos\theta_{qg}} & \text{if } \theta_{qg} < \theta_{q\bar{q}} \\ 0 & \text{else} \end{cases}$$



Choosing an Ordering Variable: Hardness or Angle

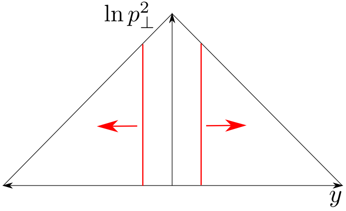
Hardness inspired by ISR (virtuality, transverse momentum), angular ordering by soft limit

Virtuality $\mathrm{d} \ln q^2$



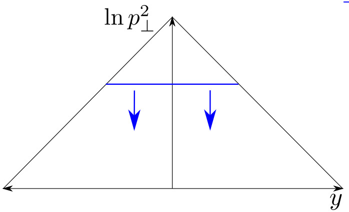
- Defines hardness, as necessary by ISR
- No coherence, need vetoes

Angle $\mathrm{d} \ln \theta^2 E^2$



- Does not define hardness, vetoes necessary
- Coherence by construction

Transverse momentum $\mathrm{d} \ln p_\perp^2$

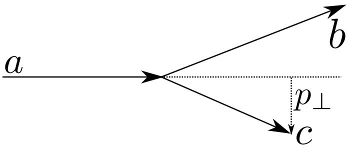


- Defines hardness, as necessary by ISR
- Coherence in FSR

Get hardness ordering and color coherence $\Rightarrow$ Dipole & Antenna showers (emission with W_{ij})

Time- and Space-like Splittings

Consider 4-momentum conservation in branching $a \to bc$



$$\vec{p}_{\perp a} = 0 \Rightarrow \vec{p}_{\perp c} = -\vec{p}_{\perp b}$$
$$p_+ = E + p_L \Rightarrow p_{+a} = p_{+b} + p_{+c}$$
$$p_- = E - p_L \Rightarrow p_{-a} = p_{-b} + p_{-c}$$

Define $p_{+b} = zp_{+a}$, $p_{+c} = (1-z)p_{+a}$
Use $p_+p_- = E^2 - p_L^2 = m^2 + p_\perp^2$

$$\frac{m_a^2 + p_{\perp a}^2}{p_{+a}} = \frac{m_b^2 + p_{\perp b}^2}{zp_{+a}} + \frac{m_c^2 + p_{\perp c}^2}{(1-z)p_{+a}}$$
$$m_a^2 = \frac{m_b^2 + p_\perp^2}{z} + \frac{m_c^2 + p_\perp^2}{1-z} = \frac{m_b^2}{z} + \frac{m_c^2}{1-z} + \frac{p_\perp^2}{z(1-z)}$$

Final-state shower:
$m_b = m_c = 0$
$\Rightarrow m_a^2 = \frac{p_\perp^2}{z(1-z)} > 0$
$\Rightarrow$ timelike

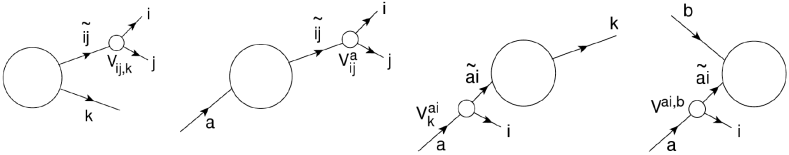
Initial-state shower:
$m_a = m_c = 0$
$\Rightarrow m_b^2 = -\frac{p_\perp^2}{(1-z)} < 0$
$\Rightarrow$ spacelike

Energy and Momentum Conservation

- For $1 \to 2$ branching with non-vanishing $p_\perp$: need other partons to absorb "recoil"
- Different choices: global vs. local recoil
- Can choose colour connected third parton to absorb "recoil" (Dipole)
- Equivalent: don't distinguish emitter and spectator, do $2 \to 3$ splitting instead (Antenna)
- Momentum conservation in each step is advantage compared to analytical tool
  - Allows for fully exclusive predictions
  - Makes systematic replacements with matrix elements possible (matching & merging)
- Ambiguities lead to uncertainty $\Rightarrow$ differences between different implementations

Dipole and Antenna Showers

Dipole/Antenna parton showers can be constructed based on NLO subtraction of fixed order cross section $\Rightarrow$ provide $2 \to 3$ phase-space mapping and coherent splitting probabilities



final-final final-initial initial-final initial-initial

[Catani,Seymour (1997)]

Ordered in transverse momentum, but ambiguities in exact implementation evolution variable

Note: Dipole showers employ dipole recoil, but dipole recoil also possible in DGLAP showers!

Running Coupling

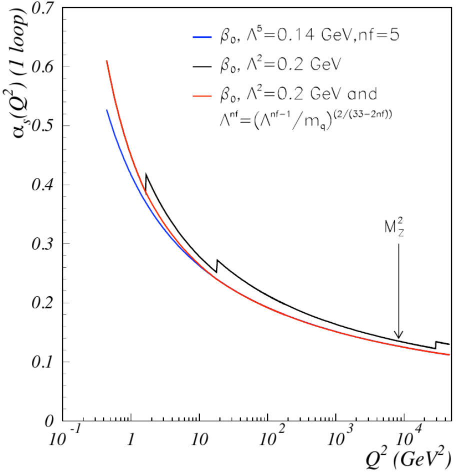
We found so far:

- ISR requires PDF evaluation at dynamic scales

Other important ingredient: Evaluation of α_s at dynamic scales

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu_R)}{1 + \alpha_s(\mu_R) b_0 \log \frac{Q^2}{\mu_R^2}}, \quad b_0 = \frac{11 C_A - 2 n_f}{12\pi}$$

- Many more soft emissions
- PS must avoid Landau pole, e.g. $\rho_{\min} > \Lambda \Rightarrow$ Cut-off has physical relevance
- With α_s running: "double log" → "leading log"



See review [Deur, Brodksy, Teramond (2016)]

Perturbative Ambiguities

Final states generated by parton shower depend on

- Choice of evolution variable: t, $p_\perp$, $\theta^2 E^2$. Ordering and scales affected
- Choice of phase space mapping $\mathrm{d}\phi_i \to \mathrm{d}\phi_{i+1}$, e.g. recoil
- Choice of radiation functions, i.e. DGLAP vs. dipole/antenna
- Choice of renormalization scale μ_R
- Choice of starting and ending scales, i.e. phase space constraints, hadronization scale
- Handling of azimuthal correlations and colour configuration

$\Rightarrow$ Can estimate uncertainties based on above choices.
(Ambiguities can be reduced by additional pQCD input $\to$ matching/merging)

Publicly Available Parton Showers

	Evolution variable	Splitting variable	Coherence
Ariadne	dipole $p_\perp^2$	Rapidity	$2 \to 3$ kernel
Herwig	$E^2\theta^2$	Energy fraction	Ang. ord.
Herwig++ / H7	$(t - m^2)/(z(1-z))$	LC mom. frac.	Ang. ord.
	dipole $p_\perp^{2\prime}$	LC mom. frac.	$2 \to 3$ kernel
Pythia 6	t	Energy fraction	Enforced
Pythia 8	$p_\perp^2$	Energy fraction	Enforced
Sherpa 1.1	t	Energy fraction	Enforced
Sherpa ≥1.2	dipole-$p_\perp^{2\prime\prime}$	LC mom. frac.	$2 \to 3$ kernel
Vincia	dipole-$p_\perp^{2\prime\prime\prime}$	LC mom. frac.	$2 \to 3$ kernel
Dire	dipole-$p_\perp^{2\prime\prime\prime\prime}$	LC mom. frac.	$2 \to 3$ kernel
...			

$p_\perp$ depends on reference direction

Developments

Improvements on seed cross sections:

- pQCD corrections for hardest emissions $\rightarrow$ Merging
- Combining higher order ME and PS $\rightarrow$ Matching

Improvements of parton shower evolution:

- Treatment of subleading color terms $1/N_\mathrm{C}^2$ [Plätzer, Sjödahl, Thorén (2018)] [Isaacson, Prestel (2019)]
- Including higher order splitting functions to get NLL correct shower: Dire [Höche, Krauss, Prestel (2017)] and Vincia [Li, Skands (2017)]
  - Need $1 \rightarrow 3$ splitting functions, e.g. $q \rightarrow qq\bar{q}$, $q \rightarrow qgg$, ...
  - Need $\mathcal{O}(\alpha_\mathrm{s}^2)$ splitting functions for $1 \rightarrow 2$
  - Avoid double counting between $1 \rightarrow 3$ and iterated $1 \rightarrow 2$
- Azimuthal correlations of emissions [Richardson, Webster (2018)]

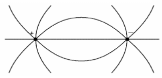
Color Neutralization: Lund String Hadronization

Quark-Antiquark Potential

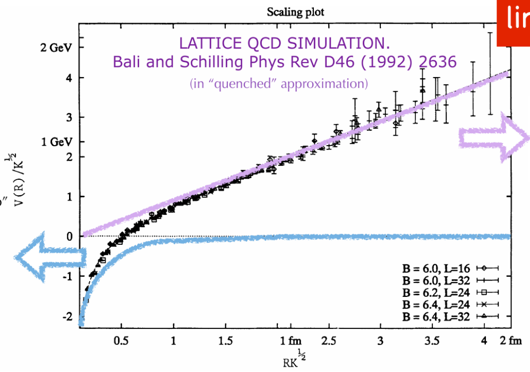
As function of separation distance

What physical system has a linear potential?

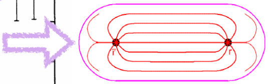
Short Distances ~ "Coulomb"



"Free" Partons



Long Distances ~ Linear Potential



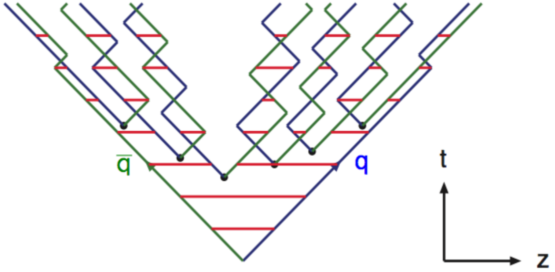
"Confined" Partons
(a.k.a. Hadrons)

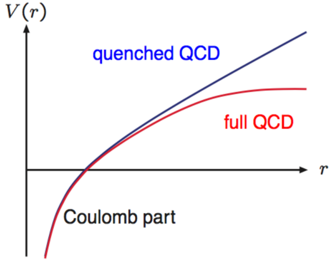
$$F(r) \approx \text{const} = \kappa \approx 1 \text{ GeV/fm} \iff V(r) \approx \kappa r$$

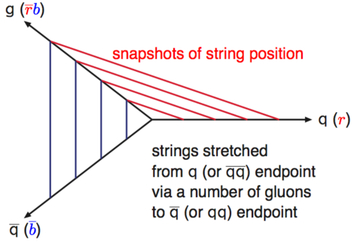
borrowed from Peter Skands

Lund String Hadronization [Andersson, Gustafson, Ingelman, Sjöstrand (1983)]

- Unquenched QCD: Non-perturbative string breaks → e.g. new $q\bar{q}$ pair
- Expanding string breaks into hadrons, the yo-yo modes
- Baryons modeled by quark-diquark pairs
- Collinear save matching to parton shower, soft/collinear gluons irrelevant



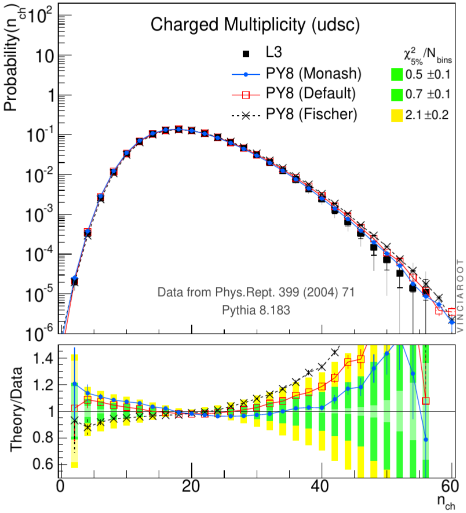




borrowed from Torbjörn Sjörstrand

Tuning Parton Showers

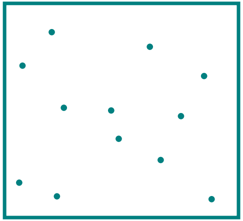
- Perturbative parton shower only few parameters, α_s and ρ_min
- Non-perturbative hadronization has many parameters
- Optimize parameters based on well-measured data

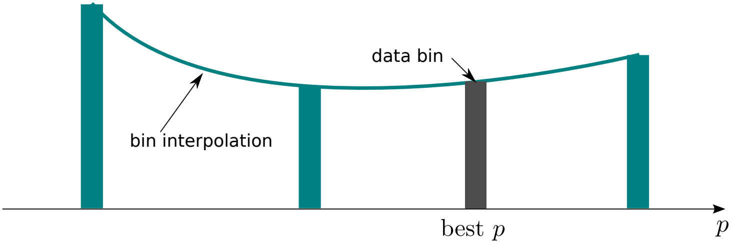


[Skands, Carrazza, Rojo (2014)]

How to Tune

- Generate MC pseudodata $f_b(\vec{p})$, compare to experimental data bin $\mathcal{R}_b$
- Iterative MC event generation slow $\rightarrow$ Use bin-wise parametrization of MC generator response





- Minimize $\chi^2(\vec{p}) = \sum_b w_b \frac{(f^{(b)}(\vec{p}) - \mathcal{R}_b)^2}{\Delta_b^2}$, with data uncertainty Δ_b, bin weights w_b
- PROFESSOR: Python package for MC tuning, highly automated, includes validation tools [Buckley, Hoeth, Lacker, Schulz, von Seggern (2010)]

Parton Shower Summary

- QCD cross section factorizes in soft/collinear limit
- Divergent terms of splitting probabilities are universal
- Parton showers based on emission and no-emission probabilities: inclusive $\rightarrow$ exclusive
- Different ordering criteria possible, e.g. virtuality, angle
- Modern showers based on antenna/dipole
- Improved by momentum conservation, running scales
- Some effects beyond parton shower approximation, but systematic improvements possible $\rightarrow$ matching/merging

Further ingredients for full event generation:

- Hadronization: Convert partons to color-neutral final state
- Multi-parton interactions, beam remnants, hadron decays, rescattering, ...