FeynRules/Madgraph school on collider phenomenology, IMSc, Chennai

Searching for new physics with effective field theories

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Outline

Part 1: Intro to Effective Field Theory

- What is an EFT? Fermi theory as an example
- Dimensional analysis & renormalisability
- Matching exercise: tree-level and one loop → anomalous dimensions
- Using EFT to search for new physics

Part 2: The Standard Model as an Effective Field Theory

- The effective theory paradigm at the LHC
- Overview of SMEFT

Part 3: Application - SMEFT in the EW sector

- Impact of SMEFT: new interactions
- Tools for SMEFT - state of the art
- Latest results - Global fit and high energy top quark processes

References

These lectures are partly based on the following:

Reviews

[Burgess; Annu. Rev. Nucl. Part. Sci. 57 (2007) 329, arXiv:hep-th/0701053]

[Manohar; arXiv:1804.05683]

[Skiba; arXiv:1006.2142]

Lectures from previous MG5/FR schools

[Cen Zhang (2015)] https://www.physics.sjtu.edu.cn/madgraphschool/

[Eleni Vryonidou (2018)] https://indico.ihep.ac.cn/event/7822/

[Gauthier Durieux (2018)] https://indico.ihep.ac.cn/event/7822/

Part 1
Introduction to Effective Field Theory

Introduction

Nature contains an abundance of physical scales

- From the Hubble all the way to the Planck scale

In order to make sense of a given physical problem, we must first identify the ***relevant*** *scales*

- Do not need to know the composition of planets to calculate orbital motion
- Or the short distance properties of EW theory for Hydrogen energy levels

In High Energy Physics, we use Quantum Field Theory to compute scattering amplitudes

- Collider cross sections, Dark Matter annihilation/detection, decay rates etc…
- Relevant scales: particle masses, collider energies, momentum transfers,…

Scale separation

Computations can be challenging

- Especially when involving multiple, disparate scales
- **Not all scales may be relevant** at the energy of interest

QFT problems with this **scale separation** can be reorganised into an effective description

- Controlled by ***ratios*** of scales: E_1/E_2, $\log(E_1/E_2)$

Take a scattering amplitude at CM energy, E, that depends on two mass scales, m and M, with $E^2 \sim m^2 \ll M^2$

- Heavy physics should not have a big impact on 'low-energy' phenomena
- We can approximate the effects that depend on M
- Describe by an **expansion** in $(m/M)^n$, $(E/M)^n$ (power counting)

Decoupling

Decoupling theorem

[Appelquist & Carrazzone; PRD 11 (1975) 2856]

- Effects of heavy physics with mass M, 'decouple' at low momenta, p
- Result only in shifts of low energy renormalisation constants + O(p^2/M^2)

This is the basis of the viability of **Effective Field Theories**

- When applied at the right scale, EFTs can predict with arbitrary precision
- The world can be divided into successive EFT 'slices', each describing relevant physics at that scale

For QFT this means only including particles that are heavy enough to be produced at the energy of interest

- All other fields are 'integrated out' and appear indirectly via $(E/M)^n$ corrections

$$\mathcal{L}_{\text{full}}\left(\phi_{\text{light}}, \phi_{\text{heavy}}\right) \longrightarrow \mathcal{L}_{\text{full}}\left(\phi_{\text{light}}\right) + \mathcal{L}_{\text{eff.}}\left(\phi_{\text{light}}\right)$$

Different EFTs

Two main reasons for using EFT

A. If 'full theory' is **known**: greatly simplify calculations — 'Top Down'

B. If 'full theory' is **unknown**: universally parametrise UV effects — 'Bottom up'

Every EFT has a…

- **Power counting** expansion: appropriately small ratio of scales
- **Range of validity**: scales at which it reliably approximates full theory

Examples in HEP

A. Heavy quark effective theory, Soft collinear effective theory, Weak effective theory, non-relativistic QCD,…

B. Fermi theory, Chiral perturbation theory, EW chiral Lagrangian, **Standard model effective field theory**,…

A tower of EFTs

UV E

Full/UV theory — Unknown EFT? $\frac{M_{NP}}{M'_{NP}}$

$M_{\rm NP}$

Standard model EFT — $\frac{p}{M_{\rm NP}}$ $\frac{M_{\rm EW}}{M_{\rm NP}}$ — LHC

EW chiral Lagrangian

M_{EW} — LEP

Fermi Theory $\frac{m_\mu}{M_{\rm EW}}$ — Weak effective theory $\frac{p}{M_{\rm EW}}$

m_b, m_c — LHCb, Belle, BaBar

Heavy quark EFT $\frac{\Lambda_{\rm QCD}}{m_{b,c}}$

$\Lambda_{\rm QCD}$

Chiral perturbation theory $\frac{p}{\Lambda_{\rm QCD}}$

IR

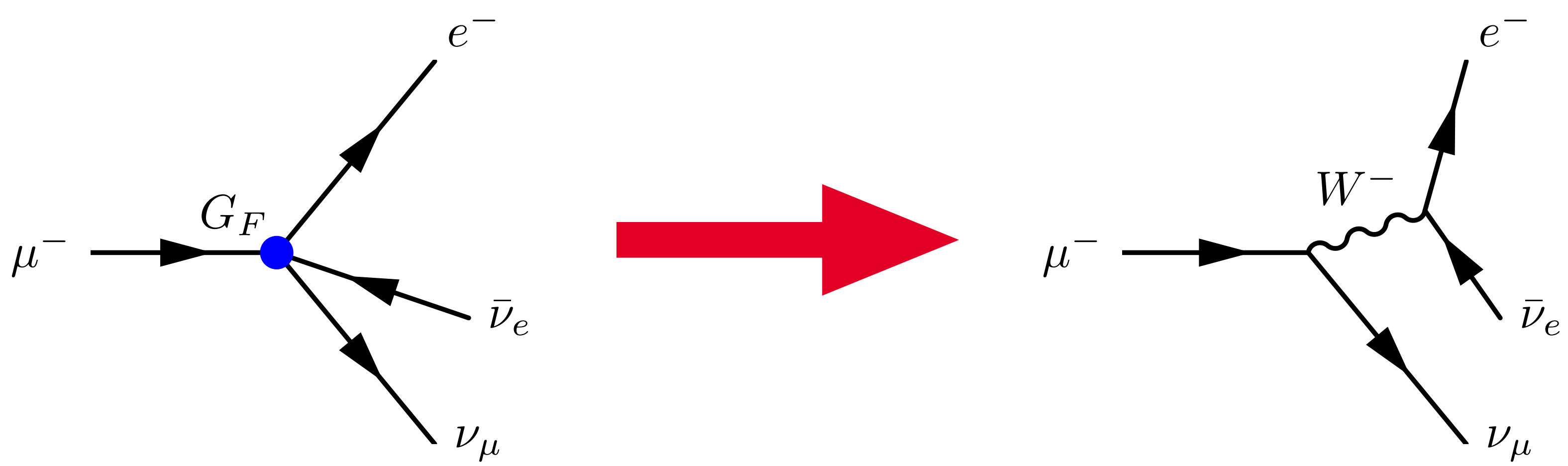
The most famous EFT

Fermi theory of weak interactions

- 30 years before the development of EW theory
- Describe beta decay, muon decay via a 4-fermion contact interaction

$$\mathcal{L}_{\rm Fermi} = -\frac{G_F}{\sqrt{2}}\,[\bar{\nu}_\mu\,\gamma^\mu(1-\gamma_5)\,\mu]\,[\bar{e}\,\gamma_\mu(1-\gamma_5)\,\nu_e]$$

$$G_F = 1.167\times 10^{-5}\ \mathrm{GeV}^{-2} \qquad\qquad [G_F] = -2$$



We know now that it is mediated by the W boson

- Scale separation: $(m_\mu/m_W)^2$

The most famous EFT

We can **match** EW theory to the Fermi Lagrangian

- Demand that the two descriptions give the same result at a given scale

Full theory:

$$\mathcal{M}_{EW} = \frac{ig^2}{8m_W^2}\left(\frac{m_W^2 g_{\mu\nu} - k_\mu k_\nu}{k^2 - m_W^2}\right)\left(\bar{u}(p_1)\gamma^\mu(1-\gamma^5)u(q_1)\right)\left(\bar{u}(p_2)\gamma^\nu(1-\gamma^5)v(q_3)\right)$$

$$k^2 \ll m_W^2 \to -\frac{ig^2}{8m_W^2}\left(\bar{u}(p_1)\gamma^\mu(1-\gamma^5)u(q_1)\right)\left(\bar{u}(p_2)\gamma_\mu(1-\gamma^5)v(q_3)\right)$$

EFT:

$$\mathcal{M}_{\text{Fermi}} = -\frac{iG_F}{\sqrt{2}}\left(\bar{u}(p_1)\gamma^\mu(1-\gamma^5)u(q_1)\right)\left(\bar{u}(p_2)\gamma_\mu(1-\gamma^5)v(q_3)\right)$$

Tree-level matching: $$\mathcal{M}_{EW} = \mathcal{M}_{\text{Fermi}} \longrightarrow G_F = \frac{g^2}{4\sqrt{2}m_W^2} = \frac{1}{\sqrt{2}v}$$

Both theories predict the same IR ($q^2 \ll m_W^2$) physics but differ in the UV ($q^2 \gtrsim m_W^2$) where on-shell W can occur

Dimensional analysis

Fermi interaction is a higher dimensional operator

4D QFT functional integral: $Z = \int \mathcal{D}\phi \, e^{iS[\phi]}, \quad S = \int d^4x \, \mathcal{L}[\phi(x)]$

Natural units, h=c=1: [Length] = Mass^{-1}

From kinetic terms

$$[\mathcal{L}] = 4: \quad [\phi] = 1, \; [\psi] = \frac{3}{2}, \; [D_\mu] = 1, \; [A_\mu] = 1, \; [g] = 0$$

Renormalisable interactions have couplings [c] ≥ 0

$$\mathcal{L}_{\text{int.}} = c\,\mathcal{O}, \quad [\mathcal{O}] \leq 4$$

- Renormalisable: need a **finite number** of counter-terms (CT) to absorb divergences in loop computations to **all orders** in perturbation theory

$$[\mathcal{O}] < 4, \; [c] > 0 \qquad [\mathcal{O}] = 4, \; [c] = 0 \qquad [\mathcal{O}] > 4, \; [c] < 0$$

'Relevant' 'Marginal' 'Irrelevant'

Renormalisability

Fermi interaction: ψ^4 = dimension-6

$$G_F \bar{\psi}\psi\bar{\psi}\psi \to \frac{c}{\Lambda_F^2}\bar{\psi}\psi\bar{\psi}\psi, \quad [c] = 0$$

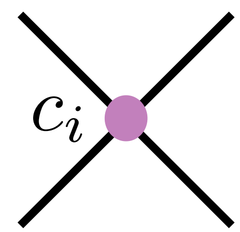
c = 'Wilson coefficient'

Λ_F = 'cutoff' (nothing to do with loop integration)

Inserting an operator once into a 2→2 amplitude:

$$\mathcal{L}_{\text{eff.}} = \sum_i \frac{c_i \mathcal{O}_i^d}{\Lambda^{4-d}}$$

c_i



$$[\mathcal{A}] = 0 \to \mathcal{A} \sim c_i \left(\frac{p}{\Lambda}\right)^{d-4}$$

Expect a power-like dependence on the external momenta

- **At most** equal to the power of Λ in the denominator
- Holds beyond tree-level: only physical (IR) scales result from loop integration
- Easily seen using dimensional regularisation, which discards power-like dependence on unphysical scales (unlike, e.g., cut-off regulator)

$$I_{\text{DR}} \sim f\left(\boxed{p^2,\, m^2}, \log\frac{\boxed{\mu^2}}{\boxed{p^2}}, \log\frac{\boxed{\mu^2}}{\boxed{m^2}}, \boxed{\frac{1}{\epsilon}}, \ldots\right)$$

physical

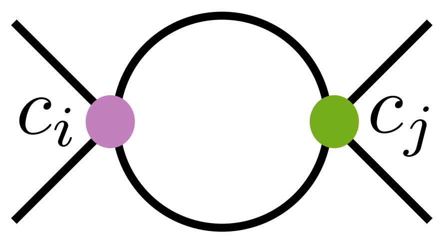
renormalisation

poles

Non-renormalisability?

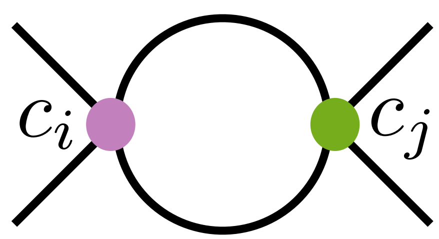
Consequence:

- Higher order corrections involving further d>4 operators lead to **higher power** momentum dependence



$$\longrightarrow \quad \mathcal{A} \sim \left(\frac{p}{\Lambda}\right)^{n_i+n_j}$$

- Renormalisation requires counter-terms from higher dimensional operator (d_i+d_j) to cancel divergent piece



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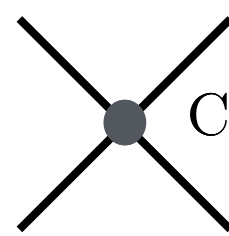


$\mathrm{CT}_{d_i+d_j} \quad = \quad$ finite

A. EFTs require an infinite number of CTs to cancel poles to all orders: formally **non-renormalisable**

B. Poles can be cancelled (renormalisable) **order-by-order** in Λ

Toy model

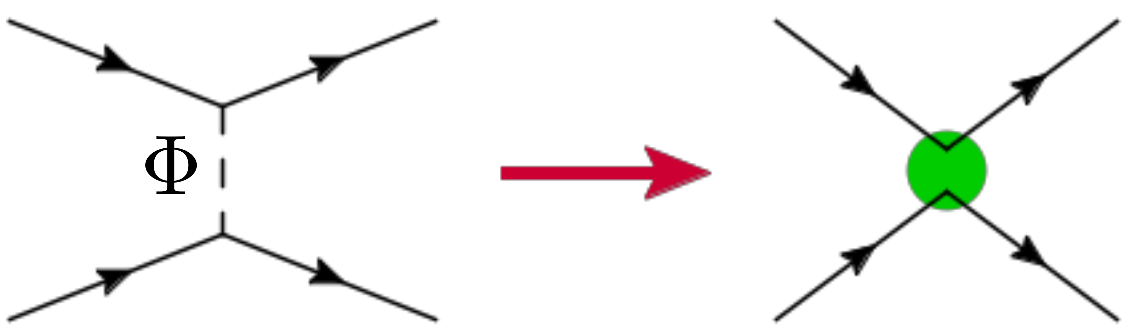
Let's work with a toy model to see this in action

- Yukawa theory of massless fermions, ψ, and a heavy scalar, ϕ

$$\mathcal{L}_{\text{Full}} = i\bar{\psi}\not{\partial}\psi + \frac{1}{2}\left(D_\mu\Phi\right)^2 - \frac{1}{2}\underbrace{\boxed{M^2}}_{\text{Heavy mass scale}}\Phi^2 - \underbrace{\boxed{\lambda\Phi\bar{\psi}\psi}}_{\text{Yukawa interaction}}$$

Find the EFT that describes the physics below the scale M

- Like Fermi theory, integrating out scalar leads to 4-fermion interaction



$$\mathcal{L}_{\text{EFT}} = i\bar{\psi}\not{\partial}\psi + \frac{c_S}{M^2}\frac{1}{2}(\bar{\psi}\psi)(\bar{\psi}\psi)$$

- Let's perform the matching

Tree-level matching

Full

EFT

Diagrammatic method: compare amplitude $\psi\psi \to \psi\psi$

Full:

$$\mathcal{M} = \bar{u}(p_3)(-i\lambda)u(p_1)\bar{u}(p_4)(-i\lambda)u(p_2)\left[\frac{i}{(p_1-p_3)^2-M^2}\right] - (3 \leftrightarrow 4)$$

$$= \mathcal{U}_S \frac{i\lambda^2}{M^2}\left[1+\mathcal{O}(q^2/M^2)+\cdots\right]$$

define: $\mathcal{U}_S = \bar{u}(p_3)u(p_1)\bar{u}(p_4)u(p_2) - (3 \leftrightarrow 4)$

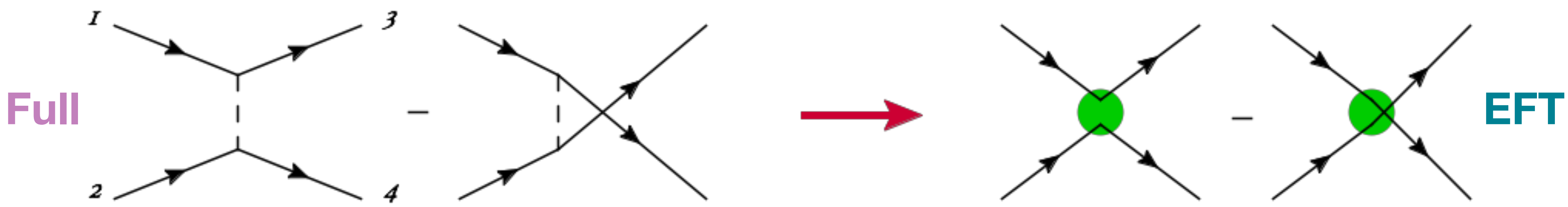
EFT: $\frac{c_S}{M^2}\frac{1}{2}(\bar{\psi}\psi)(\bar{\psi}\psi) \quad \longrightarrow \quad \mathcal{M} = \mathcal{U}_S \frac{ic_S}{M^2}$

Matching condition
$$\boxed{c_S = \lambda^2}$$

- After matching, dimension-6 EFT below M is:

$$\mathcal{L}_{\text{EFT}} = i\bar{\psi}\partial\!\!\!/\psi + \frac{\lambda^2}{2M^2}(\bar{\psi}\psi)(\bar{\psi}\psi)$$

Improved matching



Can be improved by including higher dimension operators

- Keep more terms in the propagator expansion

Full

$$\mathcal{M} = \mathcal{U}'_S \frac{i\lambda^2}{M^2}\left[1 + \frac{(p_1 - p_3)^2}{M^2} + \cdots\right] - (3 \leftrightarrow 4)$$

$$\mathcal{L}_{\mathrm{EFT}} = i\bar{\psi}\partial\!\!\!/\psi + \frac{\lambda^2}{2M^2}(\bar{\psi}\psi)(\bar{\psi}\psi) + \boxed{\frac{c^{(8)}}{M^4}\left(\partial_\mu\bar{\psi}\partial^\mu\psi\right)\left(\bar{\psi}\psi\right)}$$

EFT

$$\mathcal{M} = \mathcal{U}'_S \frac{ic_S}{M^2} + \mathcal{U}'_S\left(\frac{-ic^{(8)}}{M^4}\right)(p_1 \cdot p_3 + p_2 \cdot p_4) - (3 \leftrightarrow 4)$$

Matching condition

$$c^{(8)} = \lambda^2$$

- EFTs can be systematically improved by higher dimension operators and higher order calculations

One-loop matching

Can also improve via H.O. corrections in low-energy couplings

Assume fermions couple to the photon with charge Q

- Include NLO QED corrections to the matching

$$\mathcal{L}_{\rm Full} = i\bar{\psi}\partial\!\!\!/\psi \boxed{- \sigma\bar{\psi}\psi} + \frac{1}{2}\left(D_\mu\Phi\right)^2 - \frac{1}{2}M^2\Phi^2 - \lambda\Phi\bar{\psi}\psi$$

$$\mathcal{L}_{\rm EFT} = i\bar{\psi}\partial\!\!\!/\psi \boxed{- \sigma\bar{\psi}\psi} + \frac{c_S}{2M^2}(\bar{\psi}\psi)(\bar{\psi}\psi)\,.$$

IR regulating mass term

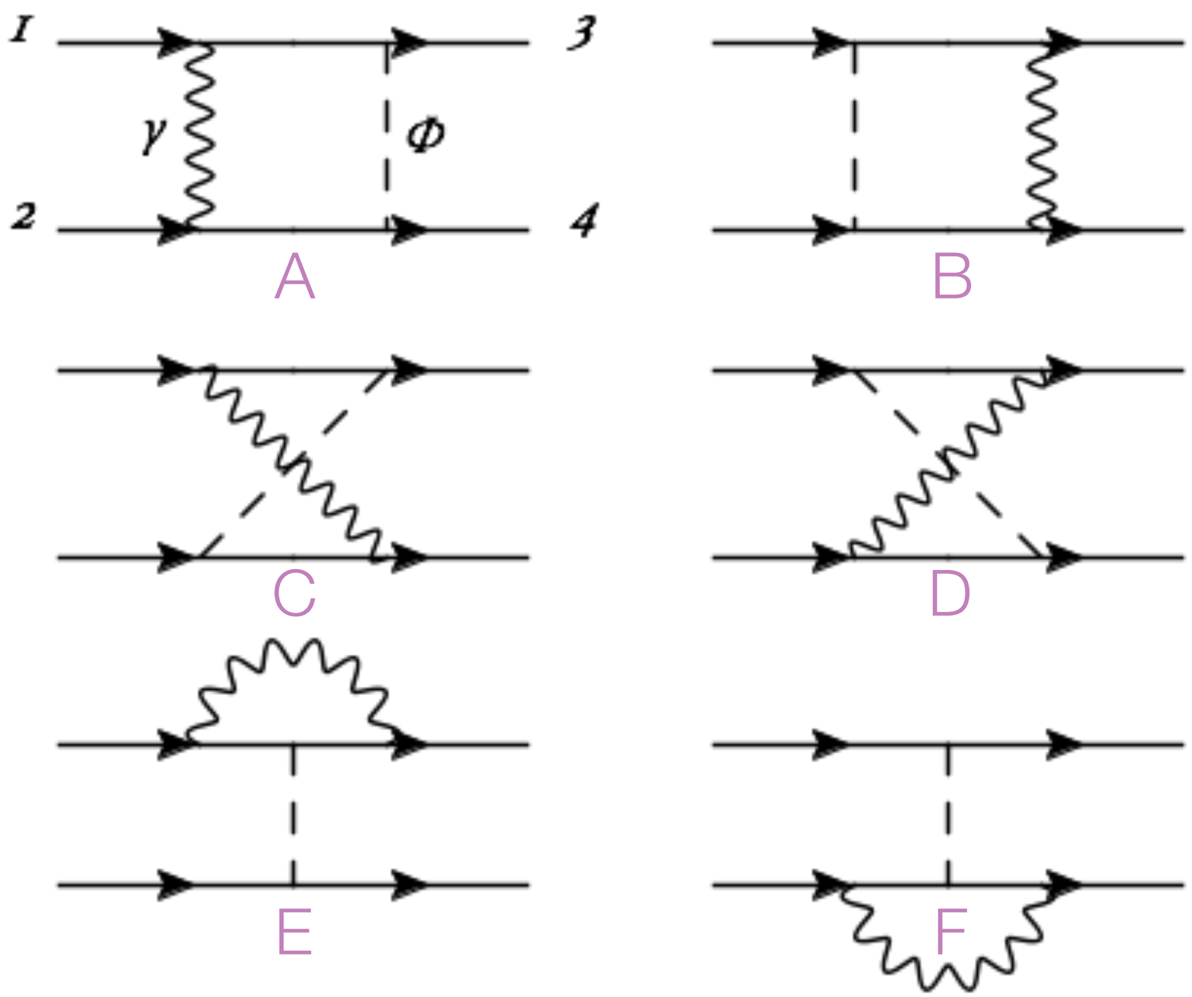
IR divergences between the full theory and EFT must **match & exactly cancel**

- Recall: both descriptions have equivalent IR behaviour, only differ in UV
- Very useful cross-check!

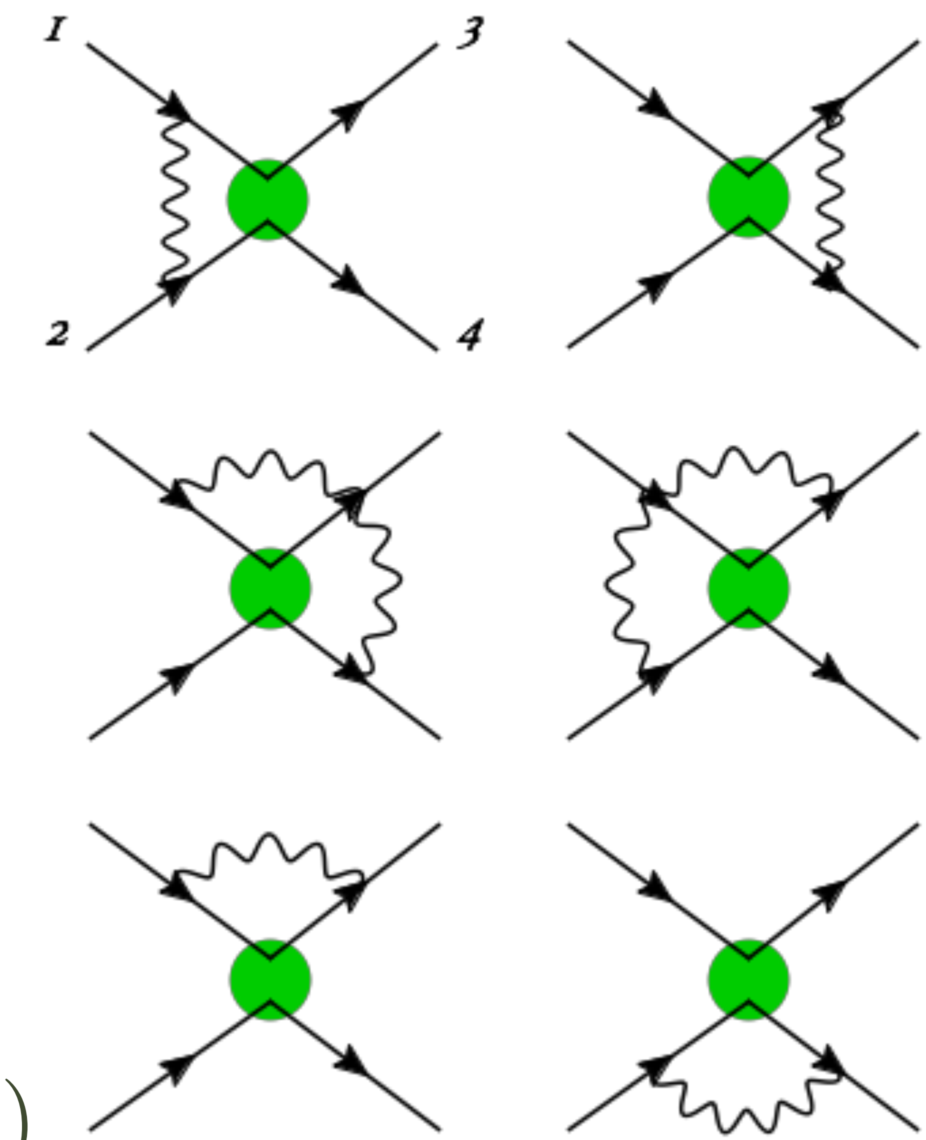
One-loop matching

Calculate corrections to same scattering process

Full



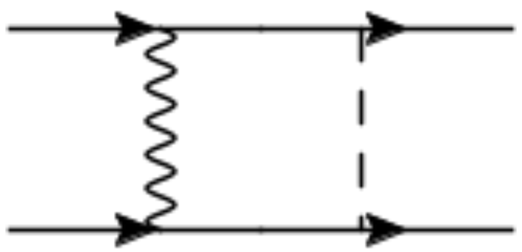
EFT



for dim-6, can set external momenta set to 0

$$-(3 \leftrightarrow 4)$$

One-loop: full theory



A

$$= \int \frac{d^4k}{(2\pi)^4} \bar{u}(p_3)(-i\lambda)\frac{i\not{k}}{k^2-\sigma^2}(ieQ)\gamma^\mu u(p_1)$$

$$\bar{u}(p_4)(-i\lambda)\frac{-i\not{k}}{k^2-\sigma^2}(ieQ)\gamma^\mu u(p_2)\frac{i}{k^2-M^2}\frac{-i}{k^2-\sigma^2}$$

$$= \frac{i\alpha Q^2}{16\pi}\frac{\lambda^2}{\boxed{M^2}}\left[\boxed{\log\frac{\sigma^2}{M^2}}+1\right][\bar{u}(p_3)\gamma^\mu\gamma^\nu u(p_1)\bar{u}(p_4)\gamma_\mu\gamma_\nu u(p_2)]$$

required M scaling regulated IR divergence

Diagrams B, C, D permute gamma matrices: $\gamma^\mu\gamma^\nu \to [\gamma^\mu,\gamma^\nu] \equiv -2i\sigma^{\mu\nu}$

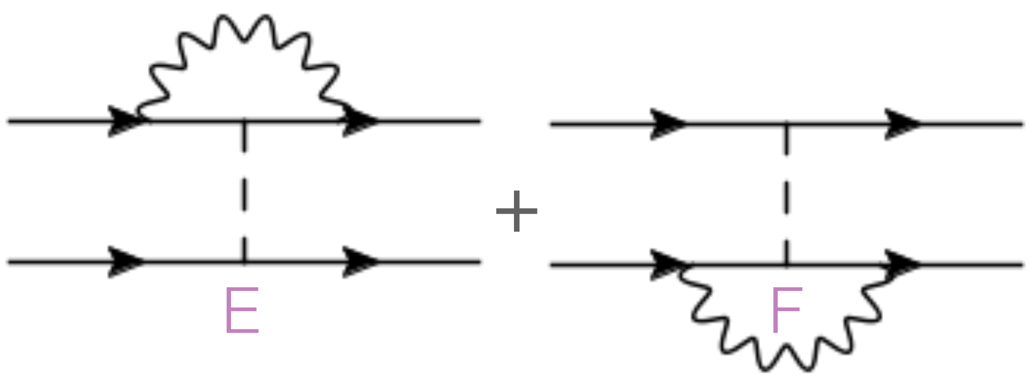
$$M_a + M_b + M_c + M_d = \left(i\frac{\lambda^2}{M^2}\right)\left(\frac{-\alpha Q^2}{4\pi}\right)\mathcal{U}_T\left[\log\frac{\sigma^2}{M^2}+1\right]$$

with: $\mathcal{U}_T = \bar{u}(p_3)\sigma^{\mu\nu}u(p_1)\bar{u}(p_4)\sigma_{\mu\nu}u(p_2) - (3\leftrightarrow 4)$

Lorentz structure does not match c_S: new operator @ 1-loop

$$\mathcal{L}_{\rm EFT} = i\bar{\psi}\not{\partial}\psi - \sigma\bar{\psi}\psi + \frac{c_S}{2M^2}(\bar{\psi}\psi)(\bar{\psi}\psi) + \boxed{\frac{c_T}{2M^2}(\bar{\psi}\sigma^{\mu\nu}\psi)(\bar{\psi}\sigma_{\mu\nu}\psi)}$$

One-loop: full theory



$$= \boxed{\mathcal{U}_S} \left(\frac{i\lambda^2}{M^2}\right)\left(\frac{\alpha Q^2}{\pi}\right)\left[\boxed{\frac{2}{\epsilon}} - 2\log\frac{\sigma^2}{\mu^2} - 1\right]$$

c_S Lorentz structure

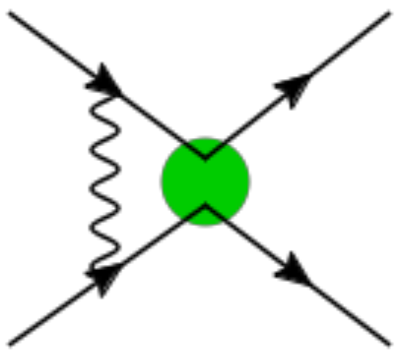
UV divergence related to QED renormalisation of Yukawa, λ

Final result in full theory:

$$\lambda \Phi \bar{\psi}\psi$$

$$M_{\text{full}} = \mathcal{U}_S \left(\frac{i\lambda^2}{M^2}\right)\left(\frac{\alpha Q^2}{\pi}\right)\left[\frac{2}{\epsilon} - 2\log\frac{\sigma^2}{\mu^2} - 1\right]$$
$$+\mathcal{U}_T \left(\frac{i\lambda^2}{M^2}\right)\left(\frac{-\alpha Q^2}{4\pi}\right)\left[\log\frac{\sigma^2}{M^2} + 1\right]$$

One-loop: EFT



$$= \int \frac{d^4k}{(2\pi)^4}\frac{ic_S}{M^2}\bar{u}(p_3)\frac{i\not{k}}{k^2-\sigma^2}(ieQ)\gamma^\mu u(p_1)\cdot\bar{u}(p_4)\frac{-i\not{k}}{k^2-\sigma^2}(ieQ)\gamma_\mu u(p_2)\frac{-i}{k^2-\sigma^2}$$

$$= \frac{i\alpha Q^2}{16\pi}\frac{c_S}{M^2}\left[\boxed{-\frac{1}{\epsilon}}+\boxed{\log\frac{\sigma^2}{\mu^2}}-\frac{1}{2}\right]\left[\bar{u}(p_3)\gamma^\mu\gamma^\nu u(p_1)\bar{u}(p_4)\gamma_\mu\gamma_\nu u(p_2)\right]$$

regulated IR divergence

- Heavy propagator replaced by $1/M^2$
- UV pole proportional to c_S: renormalisation of EFT coefficients

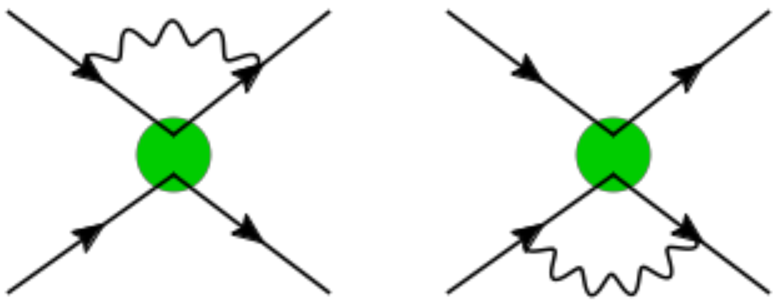
Diagrams B, C, D permute gamma matrices: $\gamma^\mu\gamma^\nu \to [\gamma^\mu,\gamma^\nu] \equiv -2i\sigma^{\mu\nu}$

$$M_A + M_B + M_C + M_D = \left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\mathcal{U}_T\left[\frac{2}{\epsilon} - 2\log\frac{\sigma^2}{\mu^2} + 1\right]$$

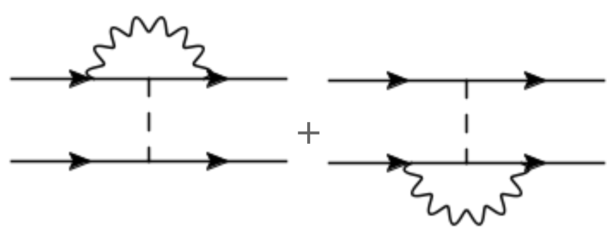
UV divergence is proportional to c_T structure!

- Need a c_T operator counter-term to cancel it
- This will lead to operator mixing under renormalisation group evolution

One-loop: EFT



$$= \mathcal{U}_S \left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{\pi}\right)\left[\boxed{\frac{2}{\epsilon}} - 2\log\frac{\sigma^2}{\mu^2} - 1\right]$$

- Same as 
- UV pole represents QED corrections → renormalisation of c_S

Final result in EFT:

$$M_{\mathrm{EFT}} = \mathcal{U}_S \left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{\pi}\right)\left[\frac{2}{\epsilon} - 2\log\frac{\sigma^2}{\mu^2} - 1\right]$$
$$+\,\mathcal{U}_T \left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left[\frac{2}{\epsilon} - 2\log\frac{\sigma^2}{\mu^2} + 1\right]$$

Matching

$$M_{\text{full}} = \mathcal{U}_S \left(\frac{i\lambda^2}{M^2}\right)\left(\frac{\alpha Q^2}{\pi}\right)\left[\frac{2}{\epsilon} - 2\log\frac{\sigma^2}{\mu^2} - 1\right]$$
$$+\,\mathcal{U}_T \left(\frac{i\lambda^2}{M^2}\right)\left(\frac{-\alpha Q^2}{4\pi}\right)\left[\log\frac{\sigma^2}{M^2} + 1\right]$$

$$M_{\text{EFT}} = \mathcal{U}_S \left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{\pi}\right)\left[\frac{2}{\epsilon} - 2\log\frac{\sigma^2}{\mu^2} - 1\right]$$
$$+\,\mathcal{U}_T \left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left[\frac{2}{\epsilon} - 2\log\frac{\sigma^2}{\mu^2} + 1\right]$$

Matching

Tree Loop Tree Loop

$$\text{EFT}: \ \mathcal{U}_S \left(\frac{ic_S}{M^2}\right)\left[1+\left(\frac{\alpha Q^2}{\pi}\right)\left(\frac{2}{\epsilon}-2\log\frac{\boxed{\sigma^2}}{\mu^2}-1\right)\right]+\mathcal{U}_T\left[\left(\frac{ic_T}{M^2}\right)+\left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(\frac{2}{\epsilon}-2\log\frac{\boxed{\sigma^2}}{\mu^2}+1\right)\right]$$

$$\text{Full}: \ \mathcal{U}_S \left(\frac{i\lambda^2}{M^2}\right)\left[1+\left(\frac{\alpha Q^2}{\pi}\right)\left(\frac{2}{\epsilon}-2\log\frac{\boxed{\sigma^2}}{\mu^2}-1\right)\right]+\mathcal{U}_T\left(\frac{i\lambda^2}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(-2\log\frac{\boxed{\sigma^2}}{M^2}-2\right)$$

At leading order:

$$c_S = \lambda^2 + \mathcal{O}(\alpha)$$ Tree-level plus correction

$$c_T = \mathcal{O}(\alpha)$$ One-loop only

Substitute LO matching result:

- As promised: IR divergences cancel exactly!
- Both theories predict same IR behaviour
- **All** dependence on low scale variables (masses, momenta) will cancel

Matching

Tree Loop Tree Loop

$$\mathrm{EFT}: \; \mathcal{U}_S \left(\frac{ic_S}{M^2}\right)\left[1+\left(\frac{\alpha Q^2}{\pi}\right)\left(\boxed{\frac{2}{\epsilon}} - 2\log\frac{\sigma^2}{\mu^2} - 1\right)\right] + \mathcal{U}_T\left[\left(\frac{ic_T}{M^2}\right) + \left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(\boxed{\frac{2}{\epsilon}} - 2\log\frac{\sigma^2}{\mu^2} + 1\right)\right]$$

$$\mathrm{Full}: \; \mathcal{U}_S \left(\frac{i\lambda^2}{M^2}\right)\left[1+\left(\frac{\alpha Q^2}{\pi}\right)\left(\boxed{\frac{2}{\epsilon}} - 2\log\frac{\sigma^2}{\mu^2} - 1\right)\right] + \mathcal{U}_T\left(\frac{i\lambda^2}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(-2\log\frac{\sigma^2}{M^2} - 2\right)$$

At leading order:

$$c_S = \lambda^2 + \mathcal{O}(\alpha)$$ Tree-level plus correction

$$c_T = \mathcal{O}(\alpha)$$ One-loop only

Substitute LO matching result:

- UV poles do not match → that's OK
- Theories are not the same in the UV
- Use a subtraction scheme (e.g. MS) to perform necessary renormalisation

Matching

$$\begin{array}{l} \text{EFT}: \; \mathcal{U}_S \left(\frac{ic_S}{M^2}\right)\left[1 + \left(\frac{\alpha Q^2}{\pi}\right)\left(\frac{2}{\epsilon} - 2\log\frac{\sigma^2}{\boxed{\mu^2}} - 1\right)\right] + \mathcal{U}_T\left[\left(\frac{ic_T}{M^2}\right) + \left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(\frac{2}{\epsilon} - 2\log\frac{\sigma^2}{\boxed{\mu^2}} + 1\right)\right] \end{array}$$

Tree — Loop — Tree — Loop

$$\text{Full}: \; \mathcal{U}_S \left(\frac{i\lambda^2}{M^2}\right)\left[1 + \left(\frac{\alpha Q^2}{\pi}\right)\left(\frac{2}{\epsilon} - 2\log\frac{\sigma^2}{\boxed{\mu^2}} - 1\right)\right] + \mathcal{U}_T\left(\frac{i\lambda^2}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(-2\log\frac{\sigma^2}{\boxed{M^2}} - 2\right)$$

At leading order:

$$c_S = \lambda^2 + \mathcal{O}(\alpha)$$ Tree-level plus correction

$$c_T = \mathcal{O}(\alpha)$$ One-loop only

Substitute LO matching result:

- log μ^2 (renormalisation) and log M^2 (heavy mass)
- Set μ=M to cancel these terms
- Physically, we have performed matching at the scale M
- Obtaining $c_S(M)$, $c_T(M)$

Matching

Tree Loop Tree Loop

$$\text{EFT}:\ \mathcal{U}_S\left(\frac{ic_S}{M^2}\right)\left[1+\left(\frac{\alpha Q^2}{\pi}\right)\left(\cancel{\frac{2}{\epsilon}}-2\log\cancel{\frac{\sigma^2}{\mu^2}}-\cancel{1}\right)\right]+\mathcal{U}_T\left[\left(\frac{ic_T}{M^2}\right)+\left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(\cancel{\frac{2}{\epsilon}}-2\log\cancel{\frac{\sigma^2}{\mu^2}}+1\right)\right]$$

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At leading order:

$$c_S = \lambda^2 + \mathcal{O}(\alpha)$$ Tree-level plus correction

$$c_T = \mathcal{O}(\alpha)$$ One-loop only

One-loop matching result:

$$c_S = \lambda^2 + \mathcal{O}(\alpha^2)$$ No one loop correction

$$c_T = -\frac{3\alpha Q^2}{8\pi}\lambda^2 + \mathcal{O}(\alpha^2)$$ Leading order one-loop

Matching

Tree Loo… …ee Loop

$$\text{EFT}:\ \mathcal{U}_S\left(\frac{ic_S}{M^2}\right)\left[1+\left(\frac{\alpha Q^2}{\pi}\right)\left(\cdots - 2\,\mathrm{l}\cdots\right.\right. \cdots \left.\frac{c_T}{M^2}\right) \cdots \left.\left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(\frac{2}{\cancel{\epsilon}} - 2\log\frac{\sigma^2}{\cancel{\mu^2}} + 1\right)\right]$$

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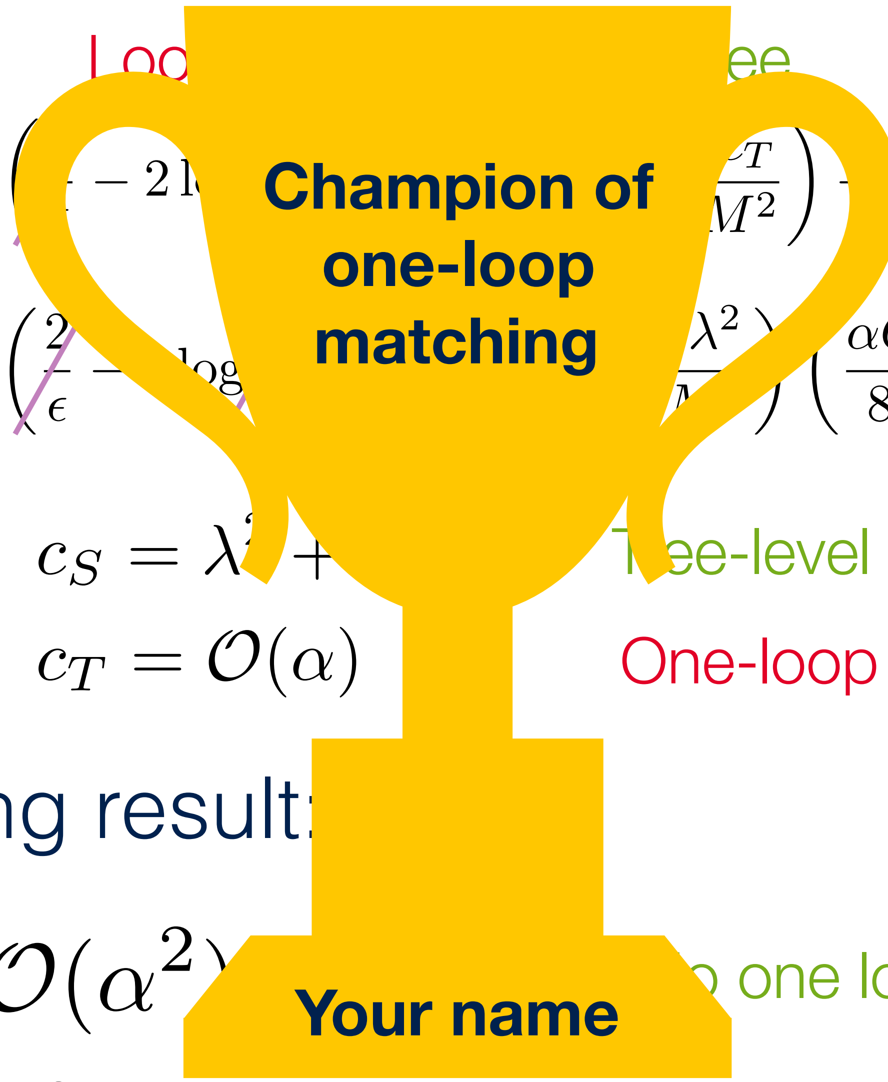
At leading order:

$$c_S = \lambda^2 + \cdots$$ …ee-level plus correction

$$c_T = \mathcal{O}(\alpha)$$ One-loop only



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$$c_T = -\frac{3\alpha Q^2}{8\pi}\lambda^2 + \mathcal{O}(\alpha^2)$$ Leading order one-loop

Matching

There are also functional methods to integrate out heavy d.o.f .and obtain an effective action
See e.g. 'Universal one-loop effective action'
[Henning et al.; JHEP 01 (2016) 023]
[Drozd et al.; JHEP 03 (2016) 80]

Tree Loop ee Loop

$$\text{EFT}: \ \mathcal{U}_S\left(\frac{ic_S}{M^2}\right)\left[1+\left(\frac{\alpha Q^2}{\pi}\right)\left(\ldots - 2\,\mathrm{l} \ldots \frac{c_T}{M^2}\right) \ldots \left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(\cancel{\frac{2}{\epsilon}} - 2\log\cancel{\frac{\sigma^2}{\mu^2}} + 1\right)\right]$$

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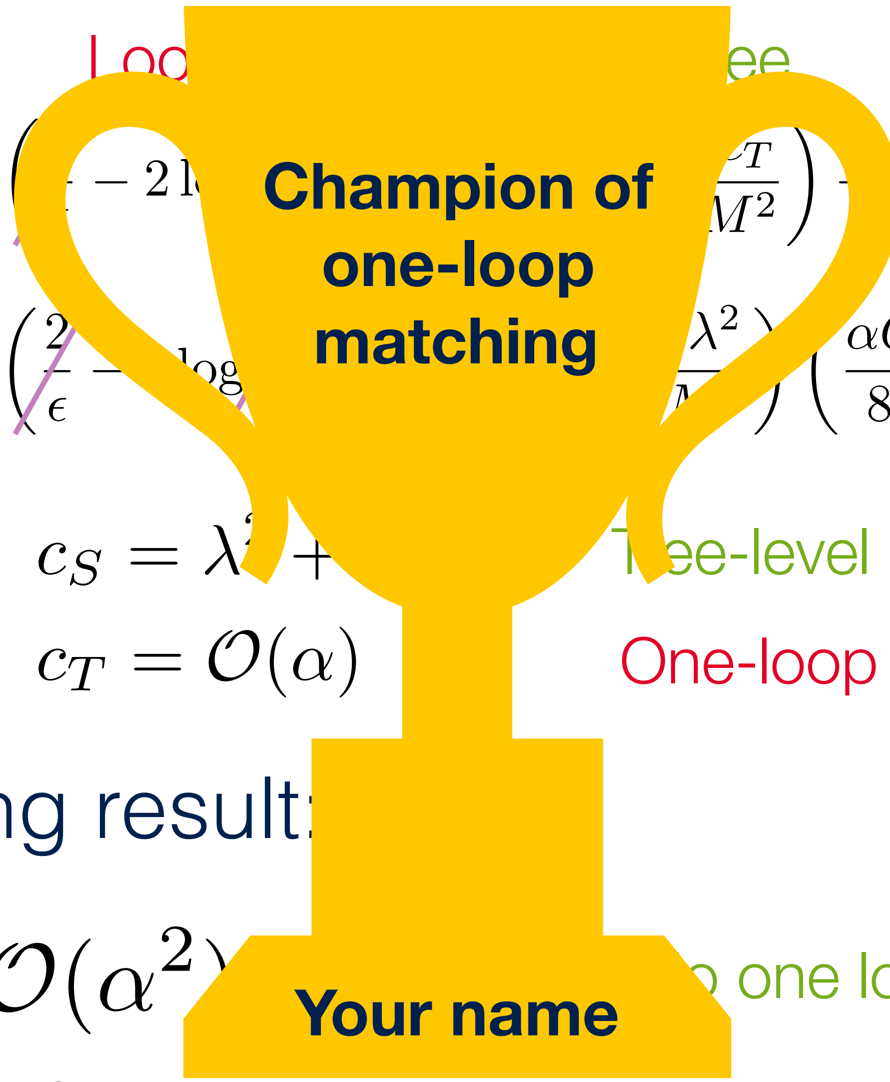
Tree-level plus correction

One-loop only



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Leading order one-loop

Running & mixing

$$\text{EFT}:\ \mathcal{U}_S\left(\frac{ic_S}{M^2}\right)\left[1+\left(\frac{\alpha Q^2}{\pi}\right)\left(\frac{2}{\epsilon}-2\log\frac{\sigma^2}{\mu^2}-1\right)\right]+\mathcal{U}_T\left[\left(\frac{ic_T}{M^2}\right)+\left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(\frac{2}{\epsilon}-2\log\frac{\sigma^2}{\mu^2}+1\right)\right]$$

Recall UV poles in EFT prediction

- Contributions to both c_S and c_T Lorentz structures
- Combined with wavefunction renormalisations:

Counter-terms $$\delta c_S = -\frac{3}{2}\frac{\alpha Q^2}{\pi}\frac{1}{\epsilon}c_S, \quad \delta c_T = -\frac{1}{4}\frac{\alpha Q^2}{\pi}\frac{1}{\epsilon}c_S$$

Defines 'anomalous dimension' matrix

$$\gamma = \frac{2Q^2\alpha}{\pi}\begin{array}{c} \begin{array}{cc} c_S & c_T \end{array} \\ \begin{pmatrix} -3/2 & ? \\ -1/4 & ? \end{pmatrix} \end{array}\begin{array}{c} c_S \\ c_T \end{array} \qquad \frac{d}{d\ln\mu}c_i = \gamma_{ij}c_j$$

Coefficients run & mix

$$\frac{dc_S(\mu)}{d\ln\mu} = -3\frac{Q^2\alpha}{\pi}c_S(\mu) - 24\frac{Q^2\alpha}{\pi}c_T(\mu)$$
$$\frac{dc_T(\mu)}{d\ln\mu} = -\frac{1}{2}\frac{Q^2\alpha}{\pi}c_S(\mu) + \frac{Q^2\alpha}{\pi}c_T(\mu)$$

Solve RGE to find c(μ) from c(M)

Running & mixing

$$\text{EFT}: \; \mathcal{U}_S\left(\frac{ic_S}{M^2}\right)\left[1+\left(\frac{\alpha Q^2}{\pi}\right)\left(\frac{2}{\epsilon}-2\log\frac{\sigma^2}{\mu^2}-1\right)\right]+\mathcal{U}_T\left[\left(\frac{ic_T}{M^2}\right)+\left(\frac{ic_S}{M^2}\right)\left(\frac{\alpha Q^2}{8\pi}\right)\left(\frac{2}{\epsilon}-2\log\frac{\sigma^2}{\mu^2}+1\right)\right]$$

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$$\gamma = \frac{2Q^2\alpha}{\pi}\begin{array}{c} \begin{array}{cc} c_S & c_T \end{array} \\ \begin{pmatrix} -3/2 & -12 \\ -1/4 & 1/2 \end{pmatrix} \end{array}\begin{array}{c} c_S \\ c_T \end{array} \qquad \frac{d}{d\ln\mu}c_i = \gamma_{ij}c_j$$

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Mini summary

EFT **approximate** physical systems with **scale separation**

- Only include relevant low energy degrees of freedom
- Supplement low energy theory with an operator expansion

$$\mathcal{L}_{\text{eff.}} = \sum_i \frac{c_i \mathcal{O}_i^d}{\Lambda^{4-d}}$$

Order-by-order renormalisable QFT

Well-defined **matching** procedure to predict **Wilson coefficients**

- Systematically improvable through higher order terms in the power counting and perturbation theory

Higher dimension operators have **power-like** contributions to amplitudes as a function of external momenta

- Cross-sections that **grow with energy** below the new physics scale!

Searching for new physics

In our toy model, new physics was a heavy scalar, ϕ

- Now we build a $\psi\psi$-collider to study $\psi\psi$-scattering
- Measure total, differential cross sections → exploit energy-growth
- Use data to constrain/measure c_S, c_T → indirectly probe λ,M

Why is this useful if we already know the full theory?

- We can now compute **any** observable using the EFT, should be easier
- Less obvious for this very simple model

The most useful is if we don't know the full theory

- The EFT is generic and model-independent, applies to all heavy NP
- Get limits on c_S, c_T **once and for all** and indirectly constrain **many models**

Searching for new physics

EFT serves as an interface between UV physics and 'low energy' phenomena

- It is the ultimate bottom up theory for arbitrary, heavy new physics
- *That reproduces your low-energy theory in the IR

Provided it is used 'properly'
i.e. there are rules:

- Golden rule: new physics scale sufficiently higher than all mass scales in your calculation (m, p_{ext})

➡ 'EFT validity'

- Second rule: only **global** analyses

➡ Include all possible (independent) operators consistent with symmetry assumptions

Searching for new physics

UV model

Predictions

Data (IR)

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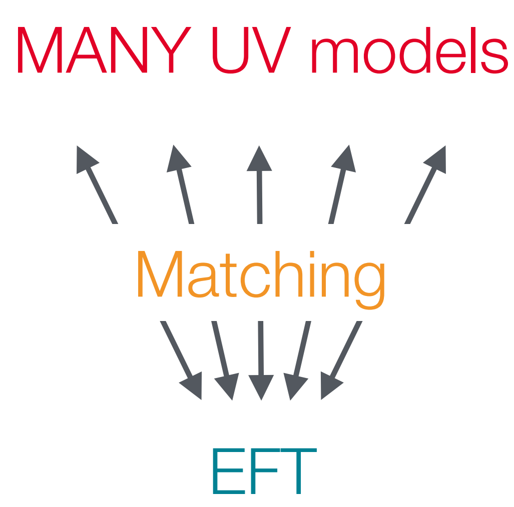

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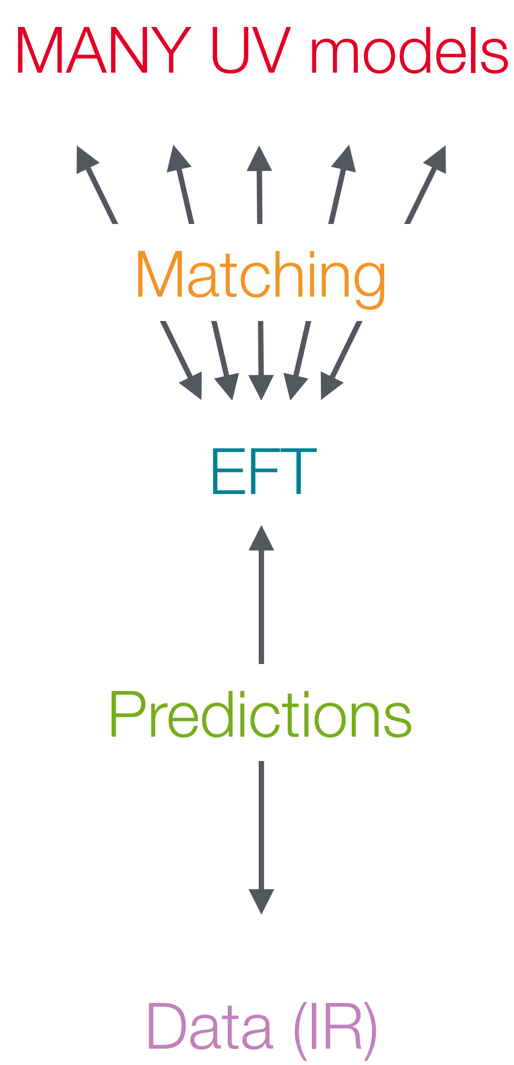

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Valid interpretations

From now assume we start with the bottom-up approach

- Try to constrain operators without thinking about their origin (yet)

parameter space: $\left(\frac{c_S}{\Lambda^2}, \ \frac{c_T}{\Lambda^2}\right)$ unknowns: $(c_S, \ c_T, \ \Lambda)$

Λ is a fictitious scale, just to make c dimensionless

- Not to be directly identified with the true mass scale of new physics, M
- We can only measure c/Λ, inference on M is a matter of **interpretation**
- Depends on the specific predictions for c (model dependent)

What is important is to not probe an EFT at energies above the new physics mass scale

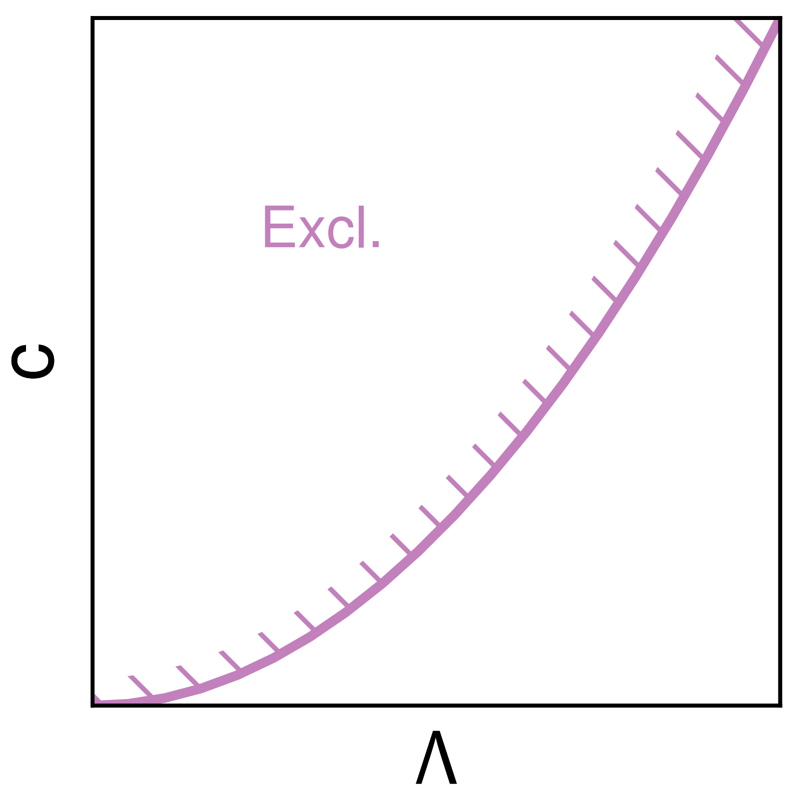
- Expansion breaks down & all orders are important → resonant production of new particles at mass M

Valid interpretations

Testing of validity criterion is an **a posteriori** exercise

Suppose we measure ψψ production at our collider

- Use invariant mass distribution to set a limit on c_S: $$\frac{c_S}{\Lambda^2} < 1\,\mathrm{TeV}^{-2}$$



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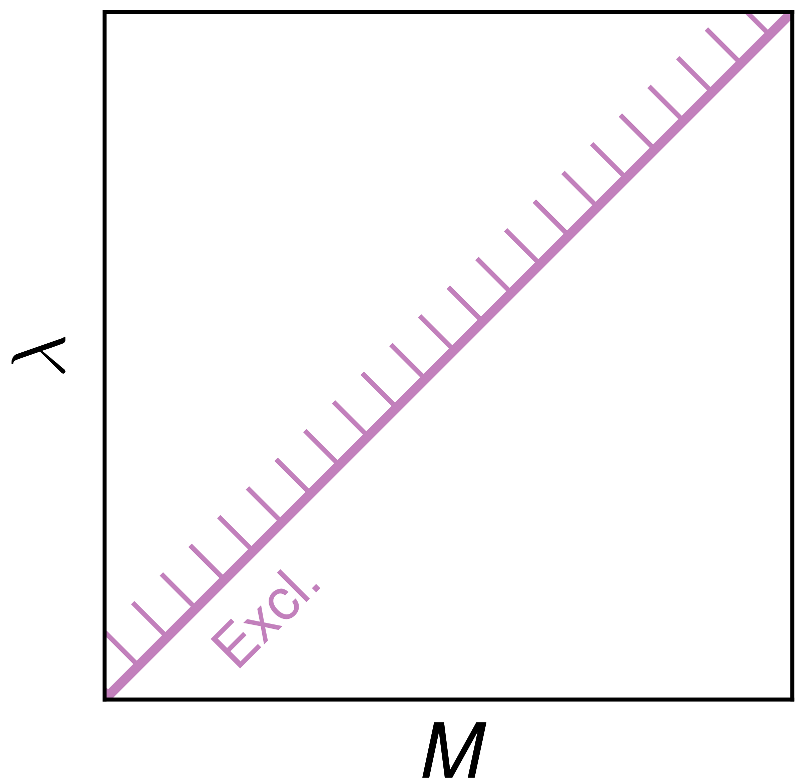

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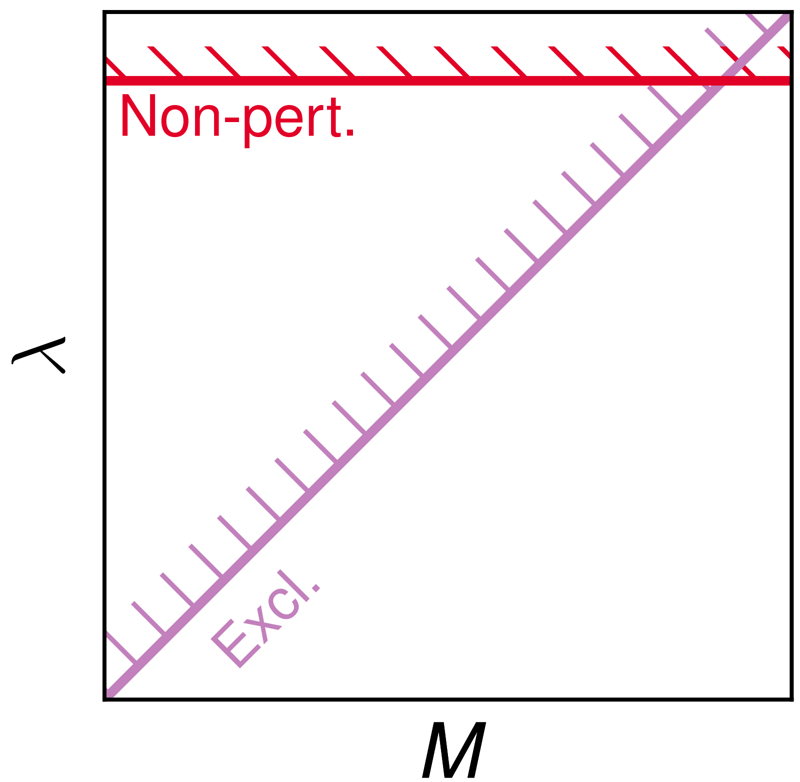

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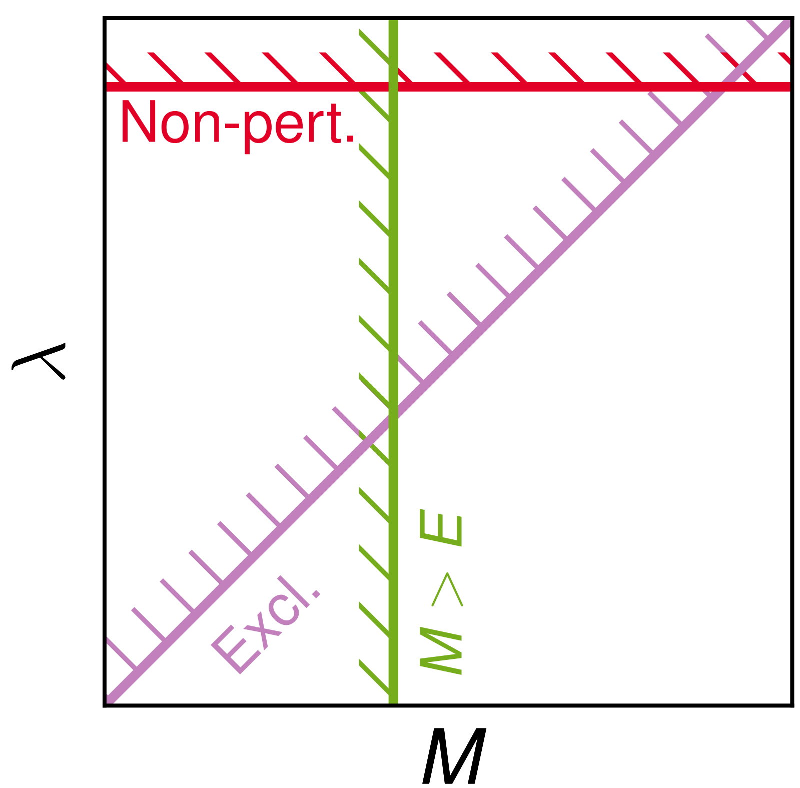

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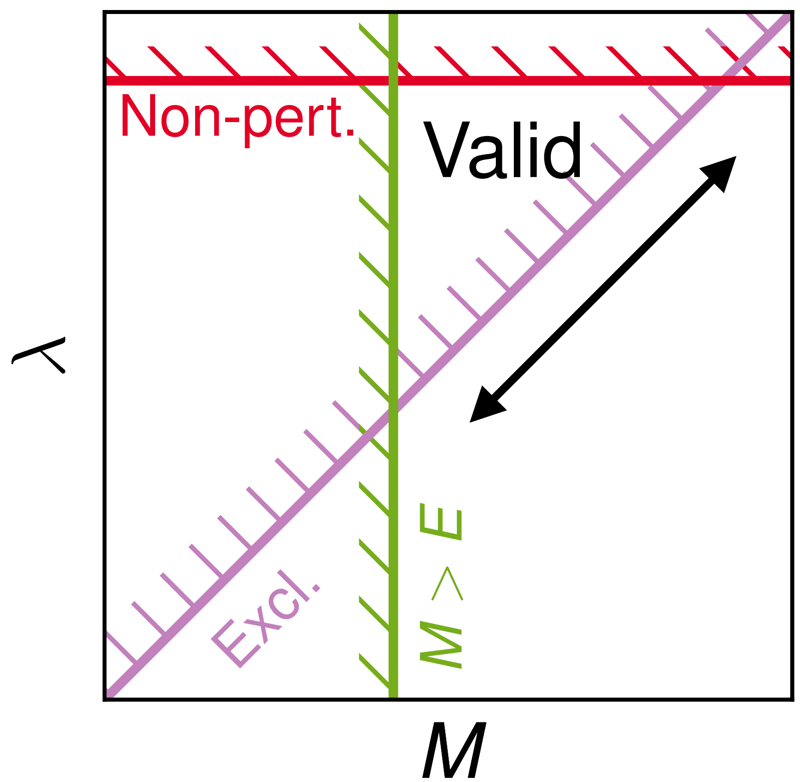

- Beyond some value (≲ 4π) λ is non perturbative
  - ➡ Theory not predictive, can't match
- We measure at some energy/mass bin, E: must be less than M
  - ➡ EFT expansion breakdown
- Valid region only known after measurement
  - ➡ Model dependent (tree/loop, weak/strong)

Global interpretations

We must consider all possible operators of a given dimension

- In general, we do not know which operators UV physics could generate
- As we saw, RG evolution mixes operators together
- Only **symmetries** can protect you and forbid certain operators
- Setting c_i=0 is only possible **at one scale**

Ignoring operators can lead to over-optimistic bounds

- Operators could cancel each other in observables $$(c_S,\, c_T) = \left(\lambda^2,\, -\frac{3\alpha Q^2}{8\pi}\lambda^2\right)$$
- 'One-at-a-time' operator constraints are instructive but not robust
- A robust statistical analysis will account for simultaneous variations of all relevant coefficients
- This will impact the derived confidence intervals on individual coefficients

Marginalisation/Profiling

Operator basis

All possible operators…

- The space of operators at a given dimension is technically infinite!
- Thankfully there are redundancies, only a finite number are **independent**
- i.e. independent contributions to **on-shell S-matrix elements**

Operators can be related by simple identities

- Integration by parts (operators with derivatives)
- Fierz identities in Dirac, gauge group algebras

Field redefinitions and/or equations of motion

- Equivalence theorem states that **S-matrix is unchanged** by field redefinitions

[Chisholm; Nucl. Phys. 26 (1961) 469]

- Can be used to eliminate operators in favour of others
- Non-redundant set of operators = **basis**

Eliminating redundancies

Small change in a field induces variation of action

- c.f. Euler-Lagrange equation

$$\delta S[\phi] = \int d^4x\, \mathcal{L}[\phi + \delta\phi] - \mathcal{L}[\phi] = \int d^4x \left(\frac{\partial \mathcal{L}}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \right) \delta\phi + \mathcal{O}(\delta\phi^2)$$

For an effective action:

$$\mathcal{L} = \mathcal{L}_0 + \frac{1}{\Lambda^2}\mathcal{L}_1; \quad \delta\phi = \frac{a}{\Lambda^2}F[\phi]$$

same quantum numbers as ϕ

$$\delta S[\phi] = \int d^4x\, \frac{a}{\Lambda^2} \left(\frac{\partial \mathcal{L}_0}{\partial \phi} - \partial_\mu \frac{\partial \mathcal{L}_0}{\partial(\partial_\mu \phi)} \right) F[\phi] + \mathcal{O}\left(\frac{1}{\Lambda^4} \right)$$

0^{th} order EOM

Dimension 8 terms

Choose *a, F[ϕ]* to eliminate an operator $c_i F[\phi] G[\phi] \subset \mathcal{L}_1,\ G[\phi] \subset \mathrm{EOM}_0$

$$a = -c_i \qquad\qquad \mathcal{L} \to \mathcal{L}_0 + \frac{1}{\Lambda^2}\mathcal{L}_1' + \mathcal{O}\left(\frac{1}{\Lambda^4} \right)$$

- New, equivalent action **without O_i** and **shifted coefficients for O_j's**

Bottom-up EFT roadmap