Lectures on Next-To-Leading Order Quantum Corrections

Hua-Sheng Shao

LPTHE
LABORATOIRE DE PHYSIQUE THEORIQUE ET HAUTES ENERGIES

SORBONNE UNIVERSITÉ
CRÉATEURS DE FUTURS DEPUIS 1257

MadGraph School 2019, Chennai, India
18-22 November 2019

Plan

- **Lecture 1:**
  - **Basics in NLO calculations**
- **Lecture 2:**
  - **Generics in NLO calculations**
- **Lecture 3:**
  - **Advanced NLO topics**

Lecture 1

NLO Basics

Lecture 1

NLO Basics

Introduction

PRECISION MEASUREMENTS AT THE LHC

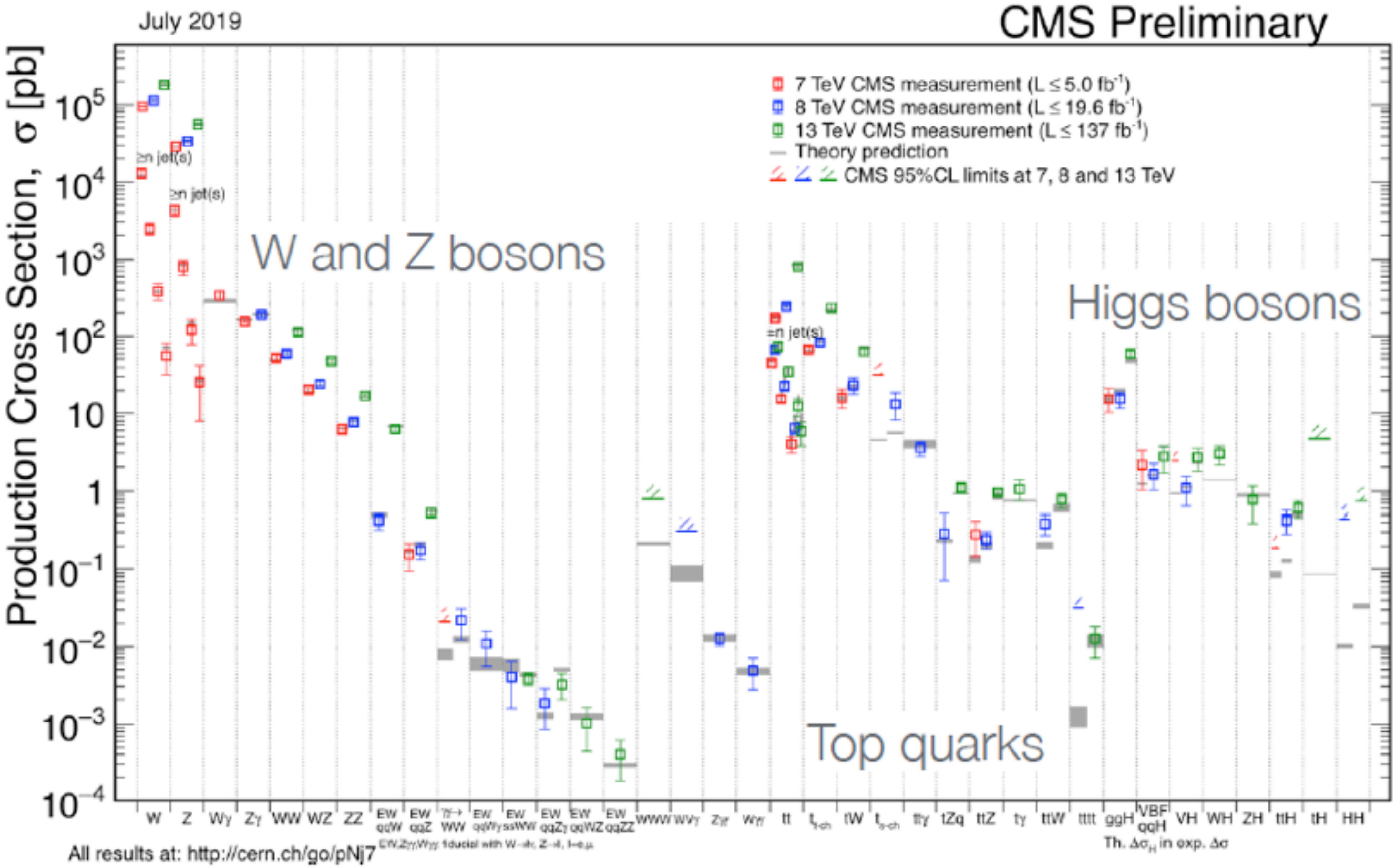
- **Huge data sample collected at the LHC run 2**

A. Hoecker's talk at EPS 2019

Particle	Produced in 139 fb^{-1} at $\sqrt{s}$ = 13 TeV	
Higgs boson	7.7 millions	
Top quark	275 millions	
Single top quark	50 millions	
Z boson	2.8 billions	290 millions leptonic
W boson	12 billions	3.7 billions leptonic
Bottom quark	~40 trillions	

PRECISION MEASUREMENTS AT THE LHC

- **Very impressive SM cross section measurements at the LHC**
  - many processes are at percent even subpercent level



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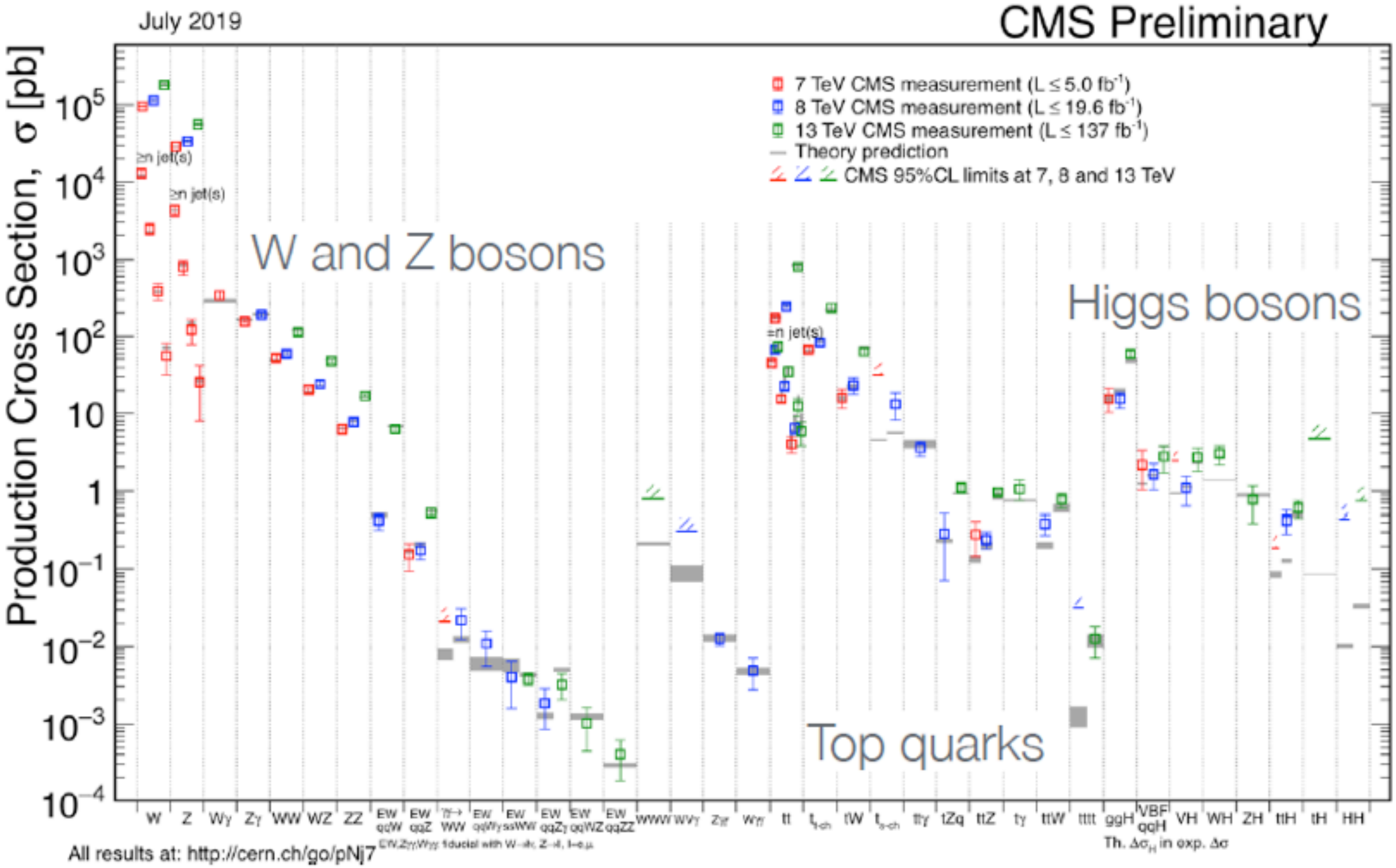
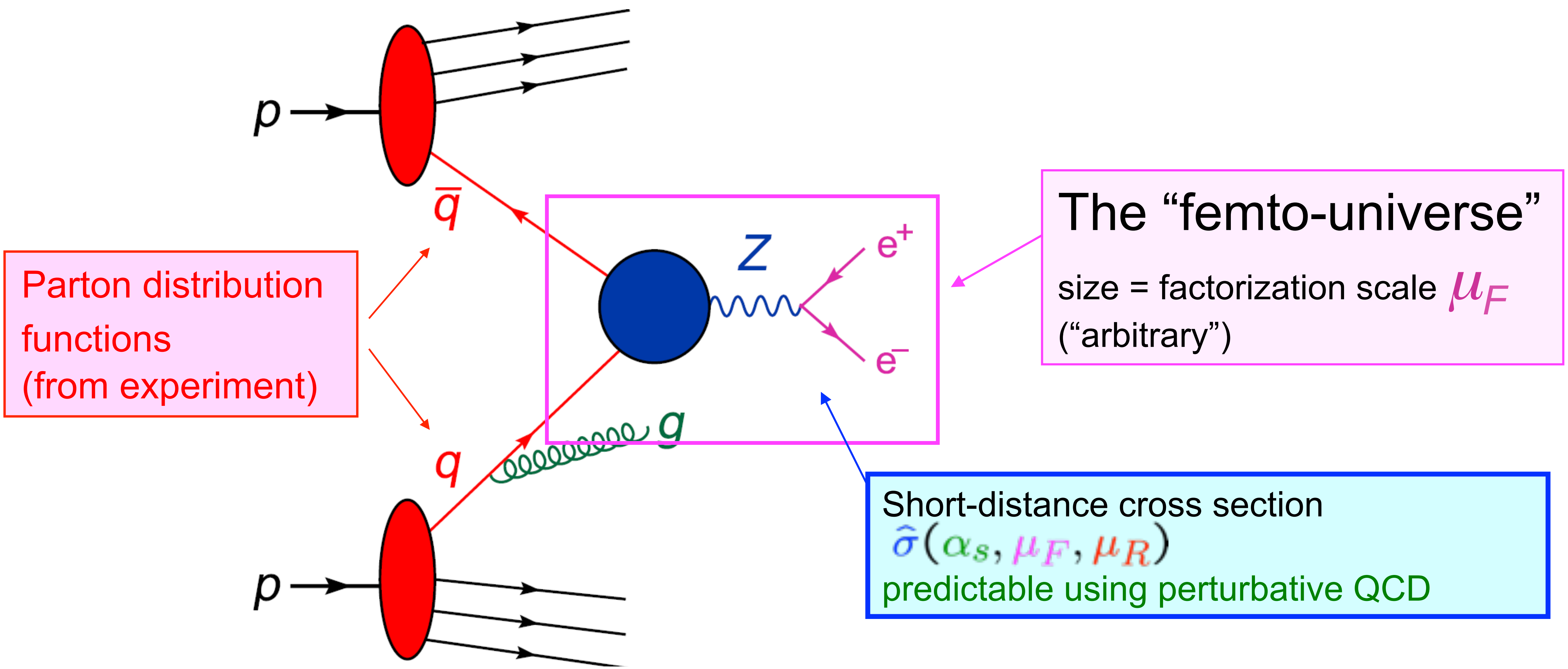


In order to fully exploit these data, theoretical calculations are crucial to keep pace !

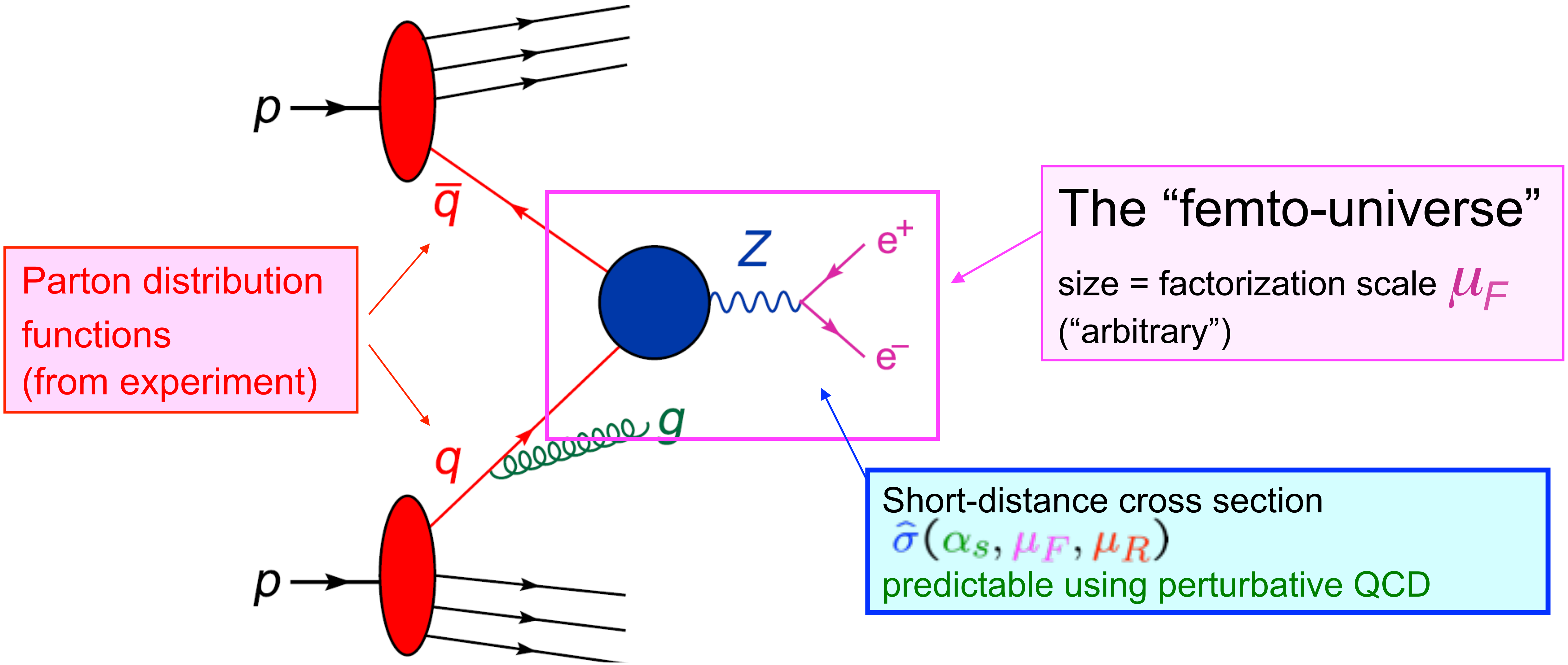
CROSS SECTION @ LHC



$$\sigma(pp \to Z + X) = \int dx_1 dx_2 f(x_1, \mu_F) f(x_2, \mu_F) \hat{\sigma}(\alpha_s, \mu_F, \mu_R)$$

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^n \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots\right]$$

CROSS SECTION @ LHC



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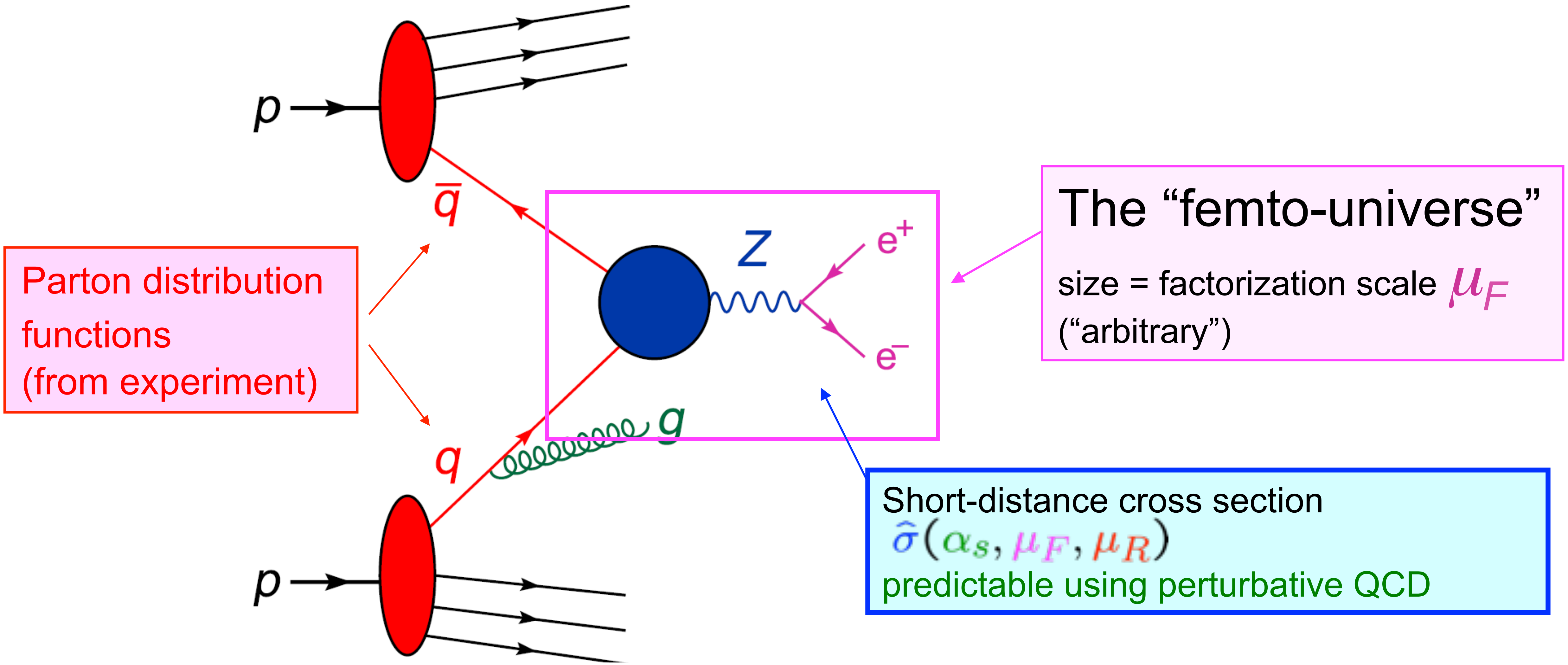
LO

CROSS SECTION @ LHC



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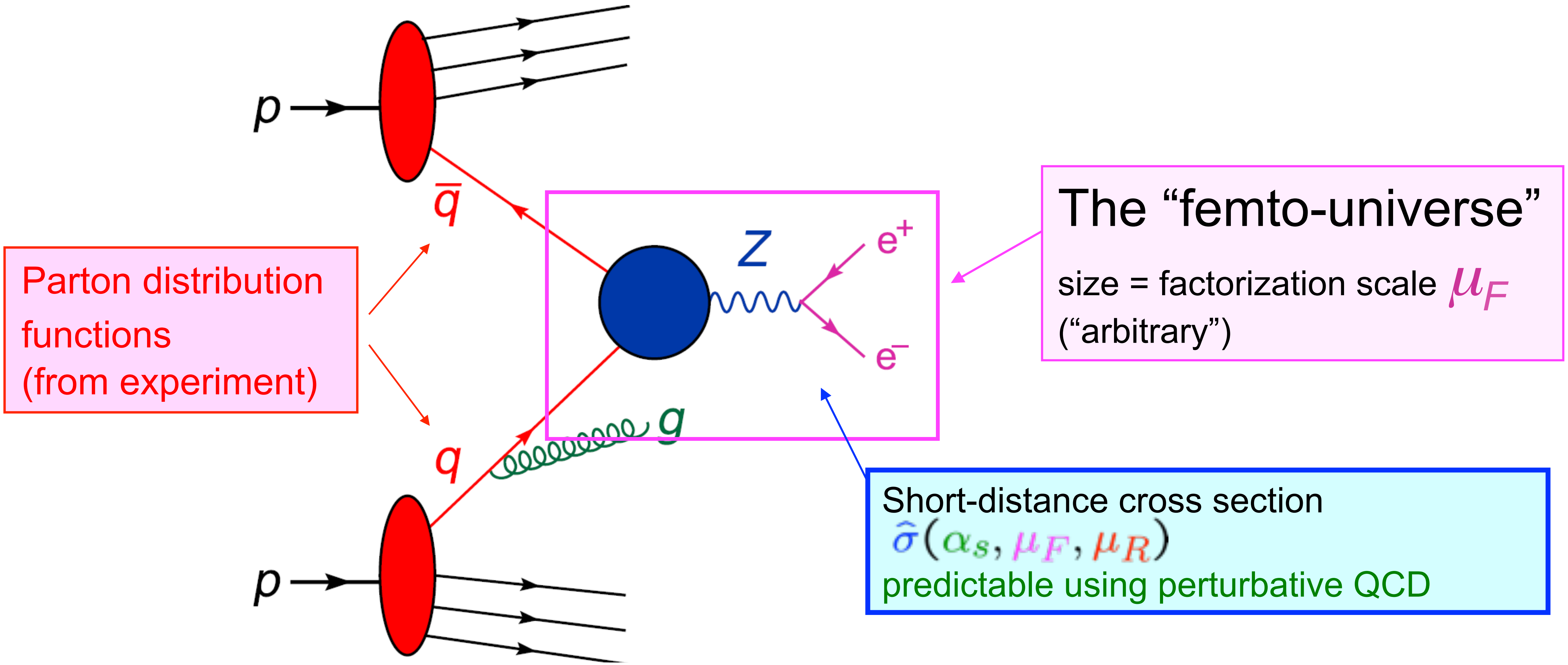
NLO

CROSS SECTION @ LHC



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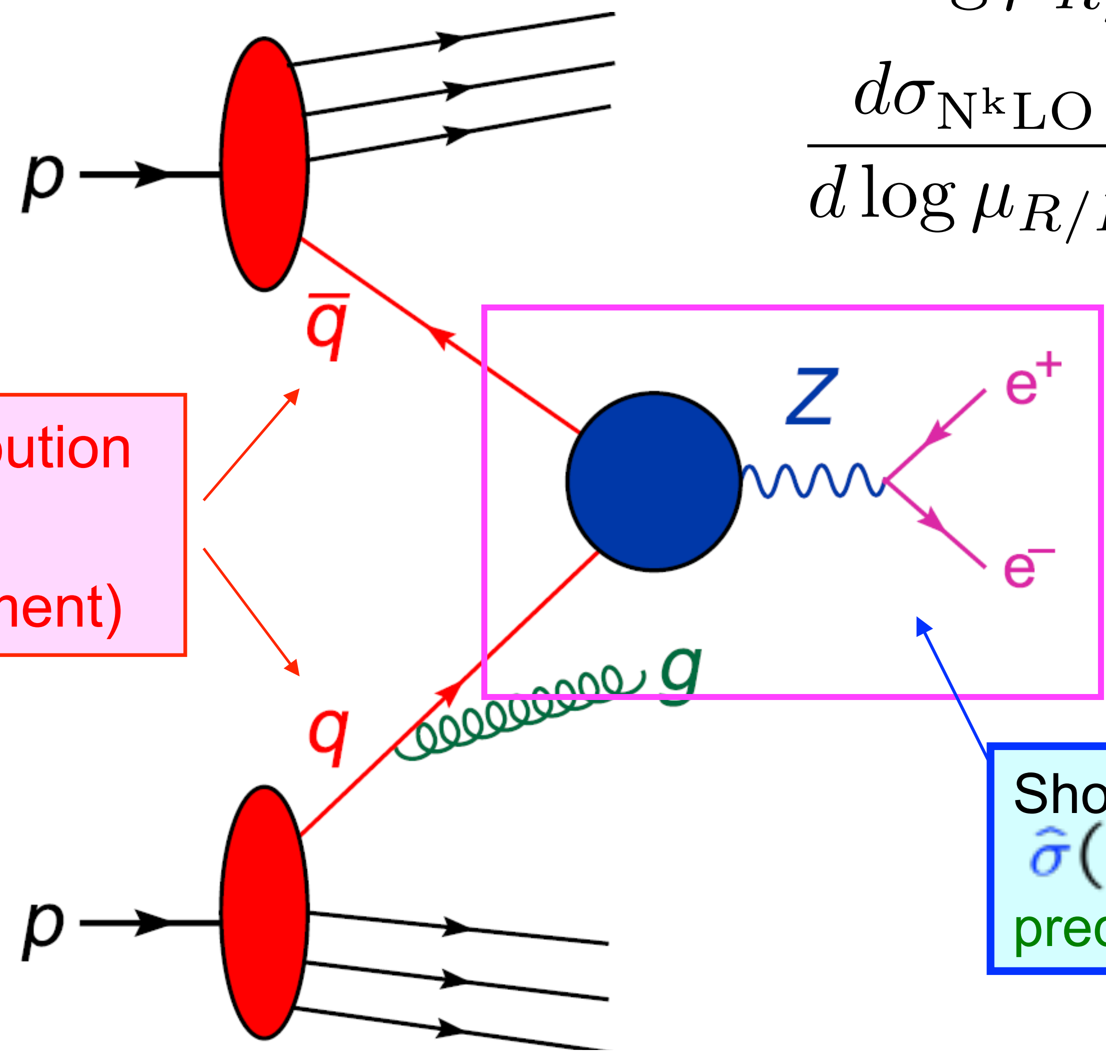
NNLO

CROSS SECTION @ LHC

$$\frac{d\sigma_{\text{all-orders}}}{d\log\mu_{R/F}} = 0$$

$$\frac{d\sigma_{\text{N}^k\text{LO}}}{d\log\mu_{R/F}} \sim \mathcal{O}(\alpha_s^{n+k+1})$$

remaining scale uncertainty



Parton distribution functions (from experiment)

The "femto-universe"

size = factorization scale μ_F ("arbitrary")

Short-distance cross section
$\hat{\sigma}(\alpha_s, \mu_F, \mu_R)$
predictable using perturbative QCD

$$\sigma(pp \to Z + X) = \int dx_1 dx_2 f(x_1, \mu_F) f(x_2, \mu_F) \hat{\sigma}(\alpha_s, \mu_F, \mu_R)$$

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^n \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots\right]$$

NNLO

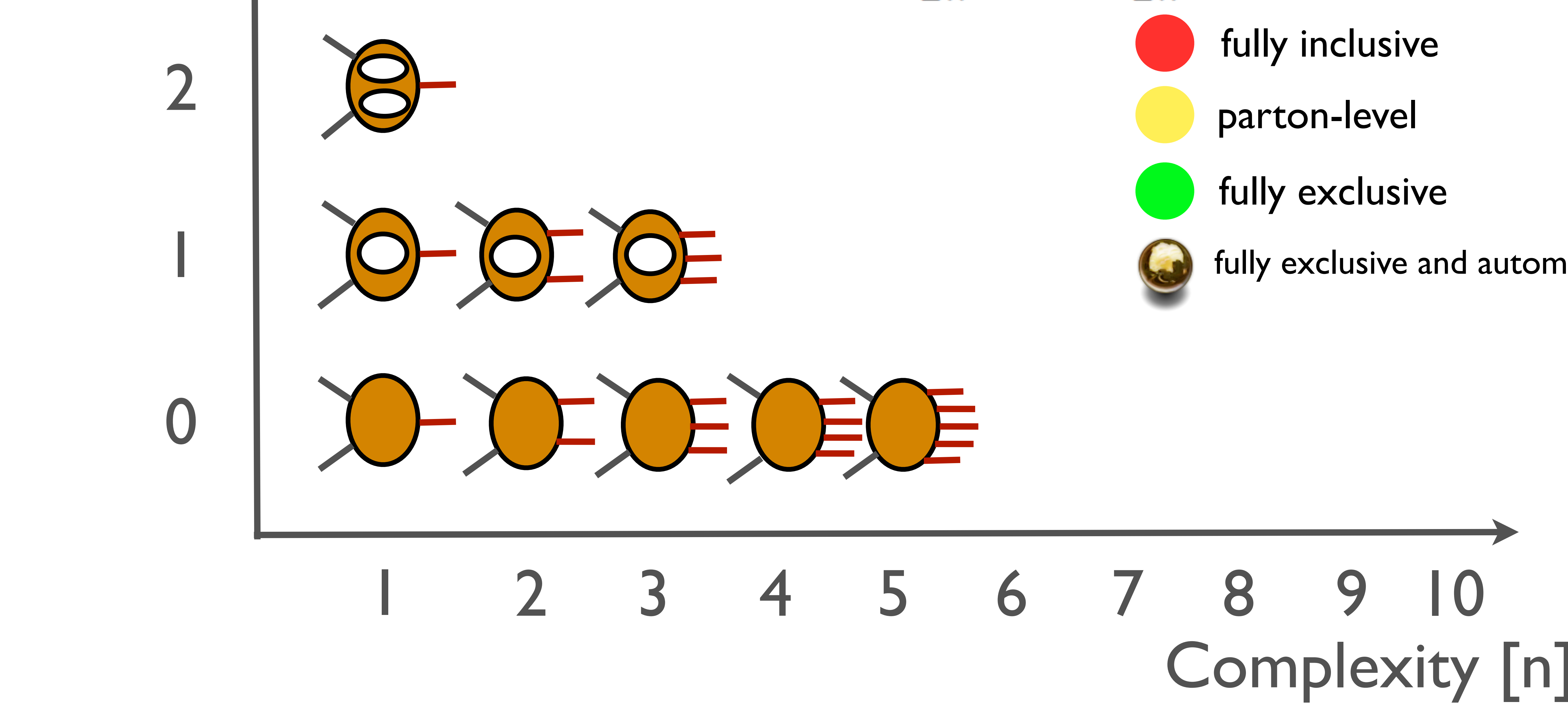
HADRON COLLIDER PHYSICS: 15 YEARS AGO

p p → n particles

in the Standard Model

via strong coupling expansion

$$d\sigma = d\sigma_0\left[1 + \frac{\alpha_s}{2\pi}\Delta_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta_2 + \ldots\right]$$



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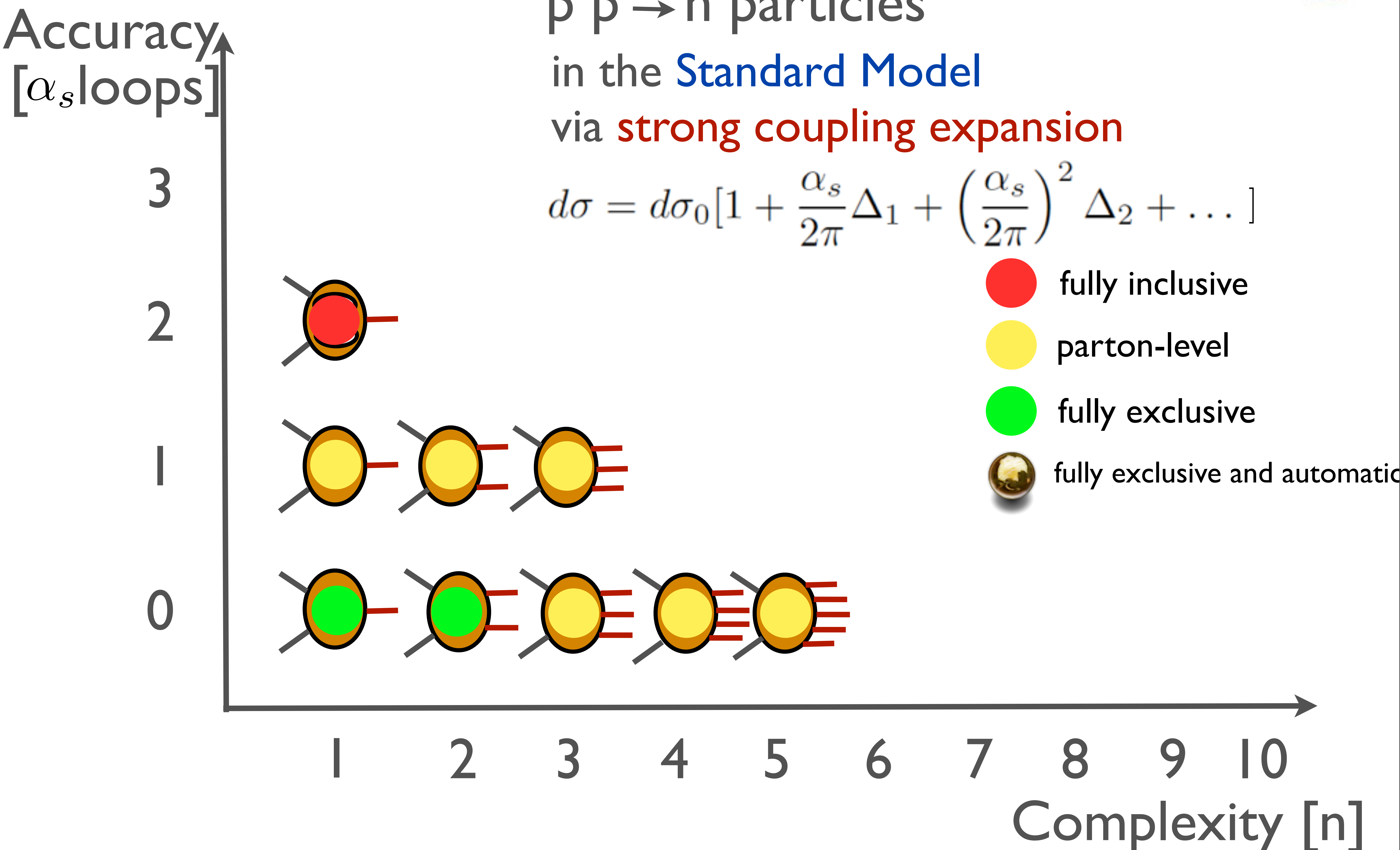

HADRON COLLIDER PHYSICS: NOW

p p → n particles

in the Standard Model

via strong coupling expansion

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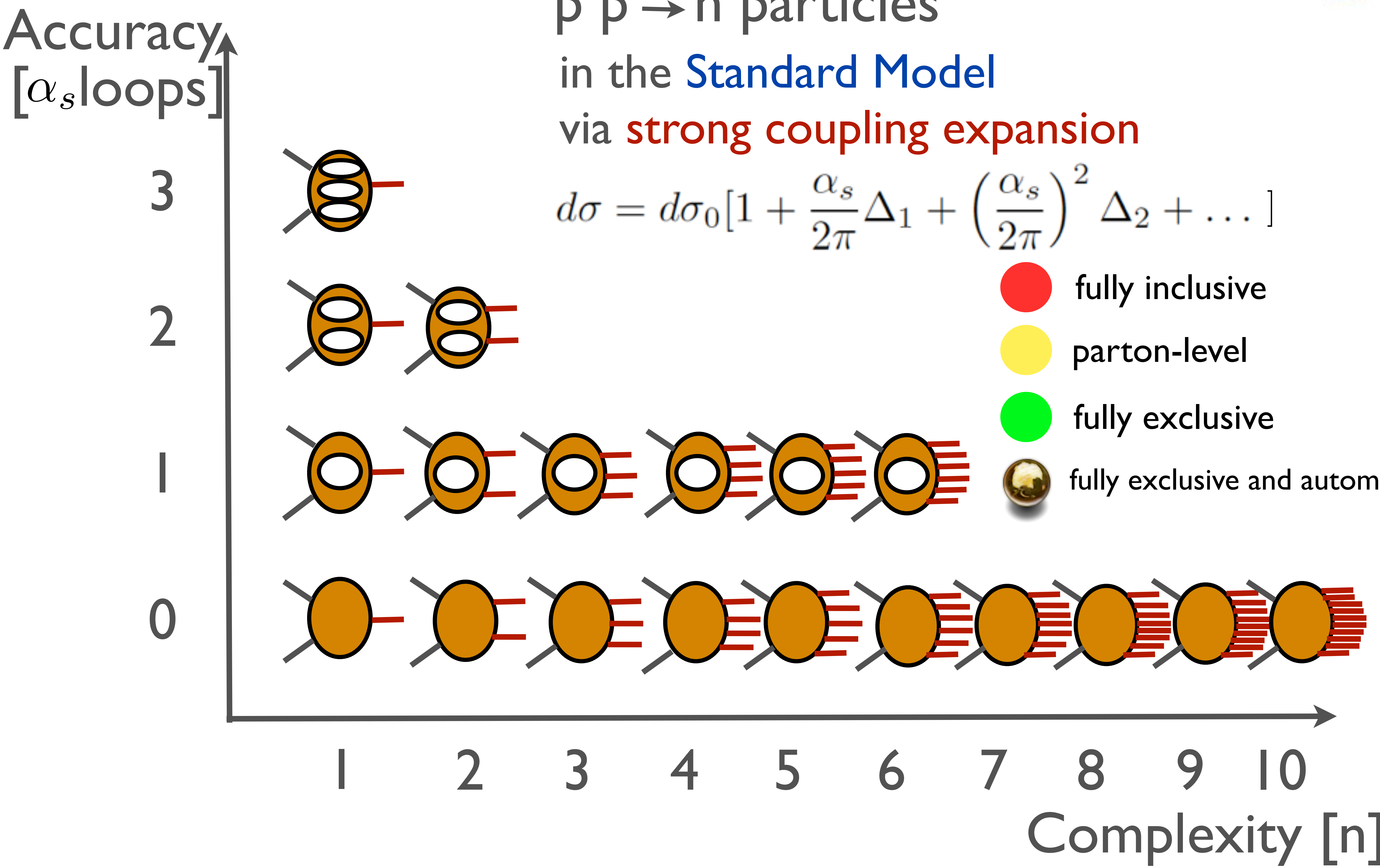

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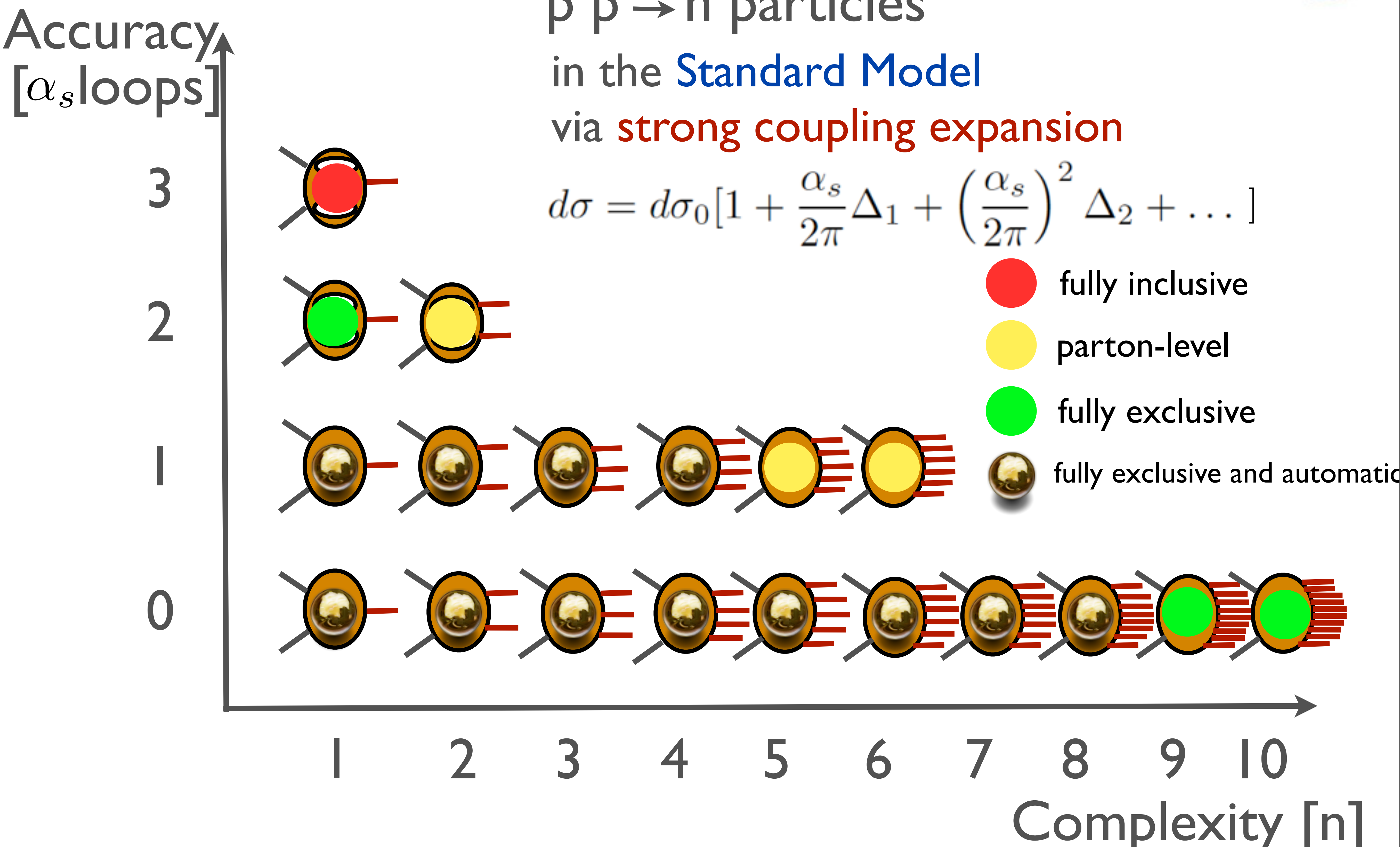

HADRON COLLIDER PHYSICS: NOW

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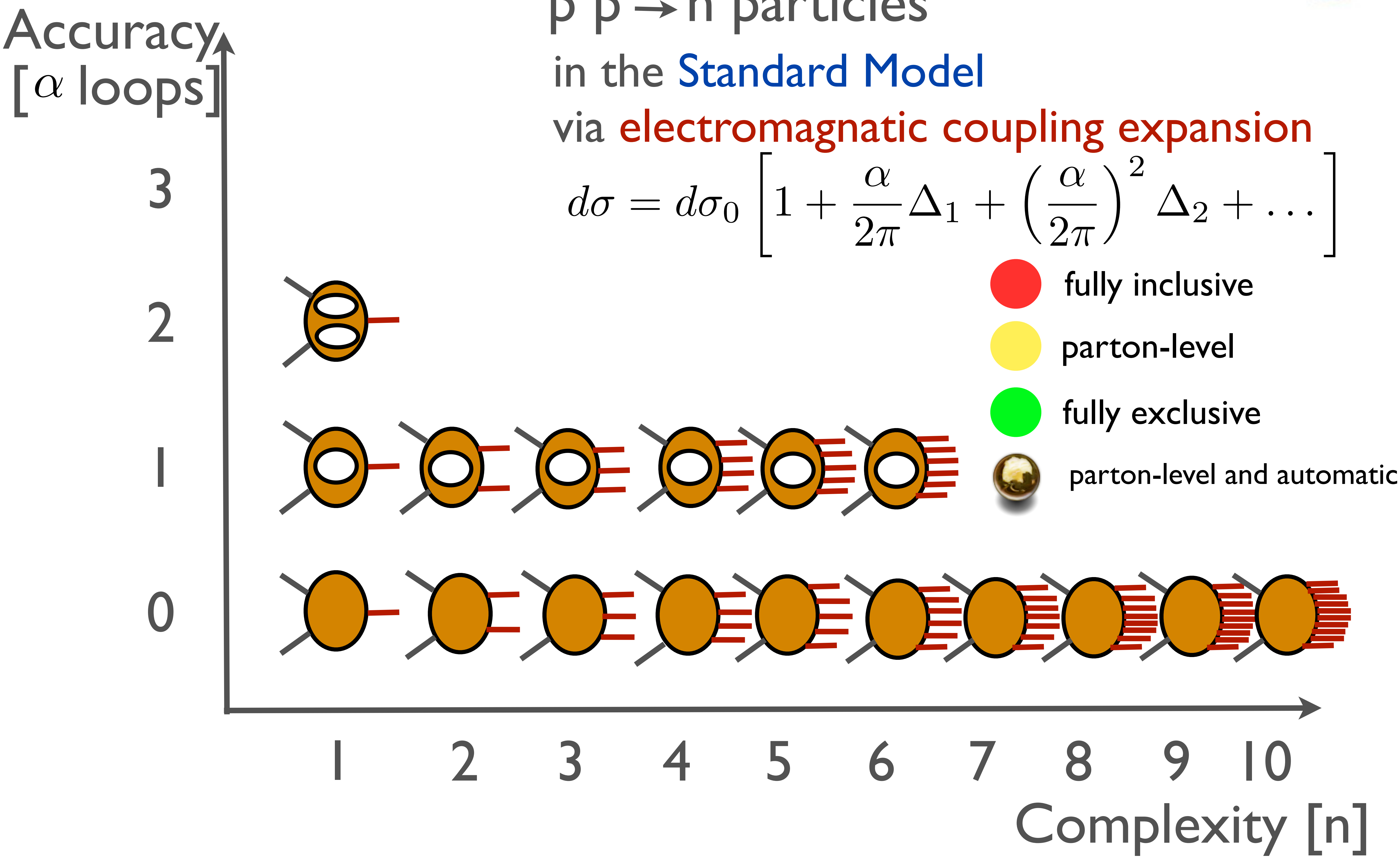

HADRON COLLIDER PHYSICS: NOW

p p → n particles

in the Standard Model

via electromagnetic coupling expansion

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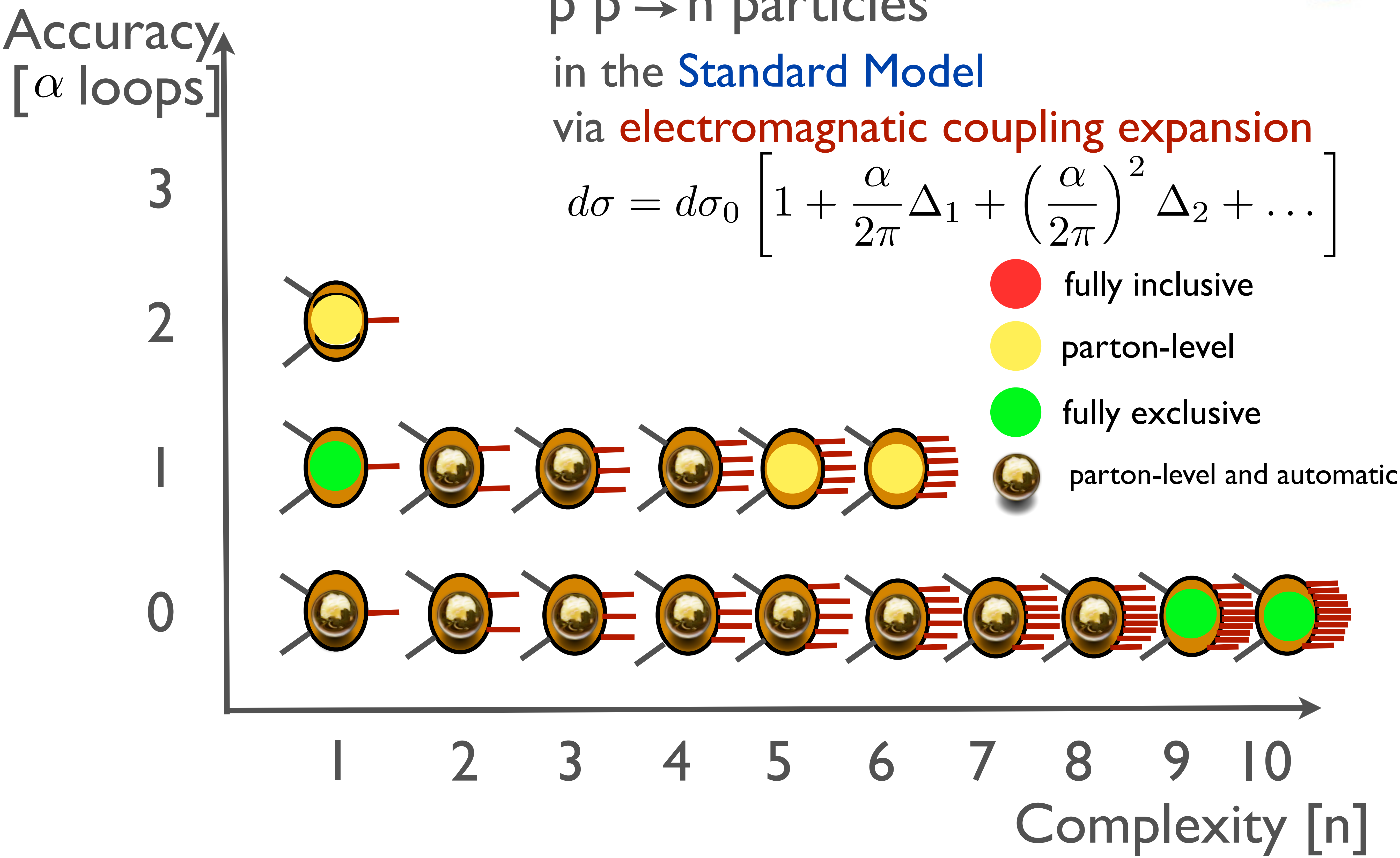

HADRON COLLIDER PHYSICS: NOW

p p → n particles
in Beyond the Standard Model
via strong coupling expansion

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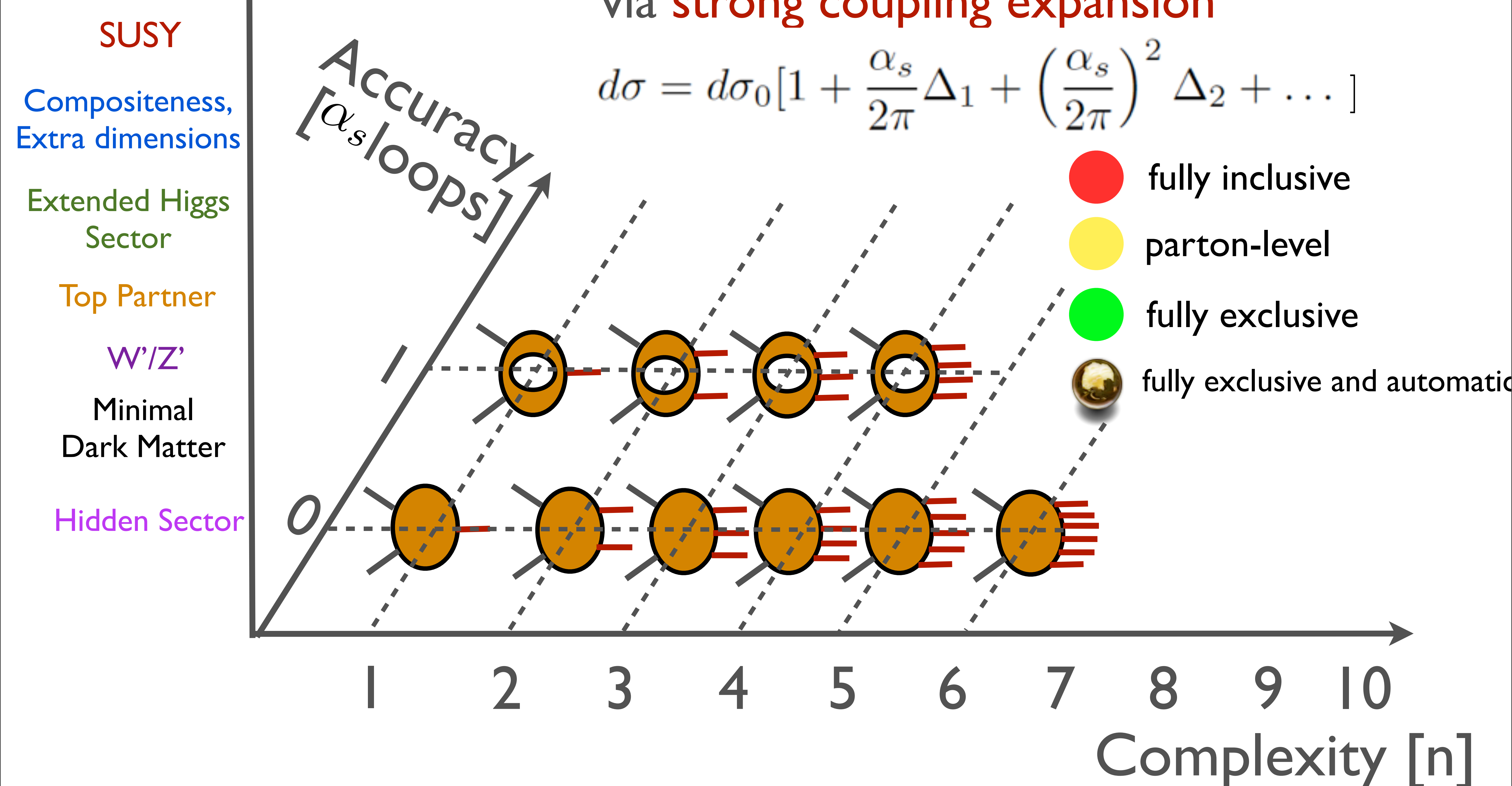

Tuesday, November 19, 19

HADRON COLLIDER PHYSICS: NOW

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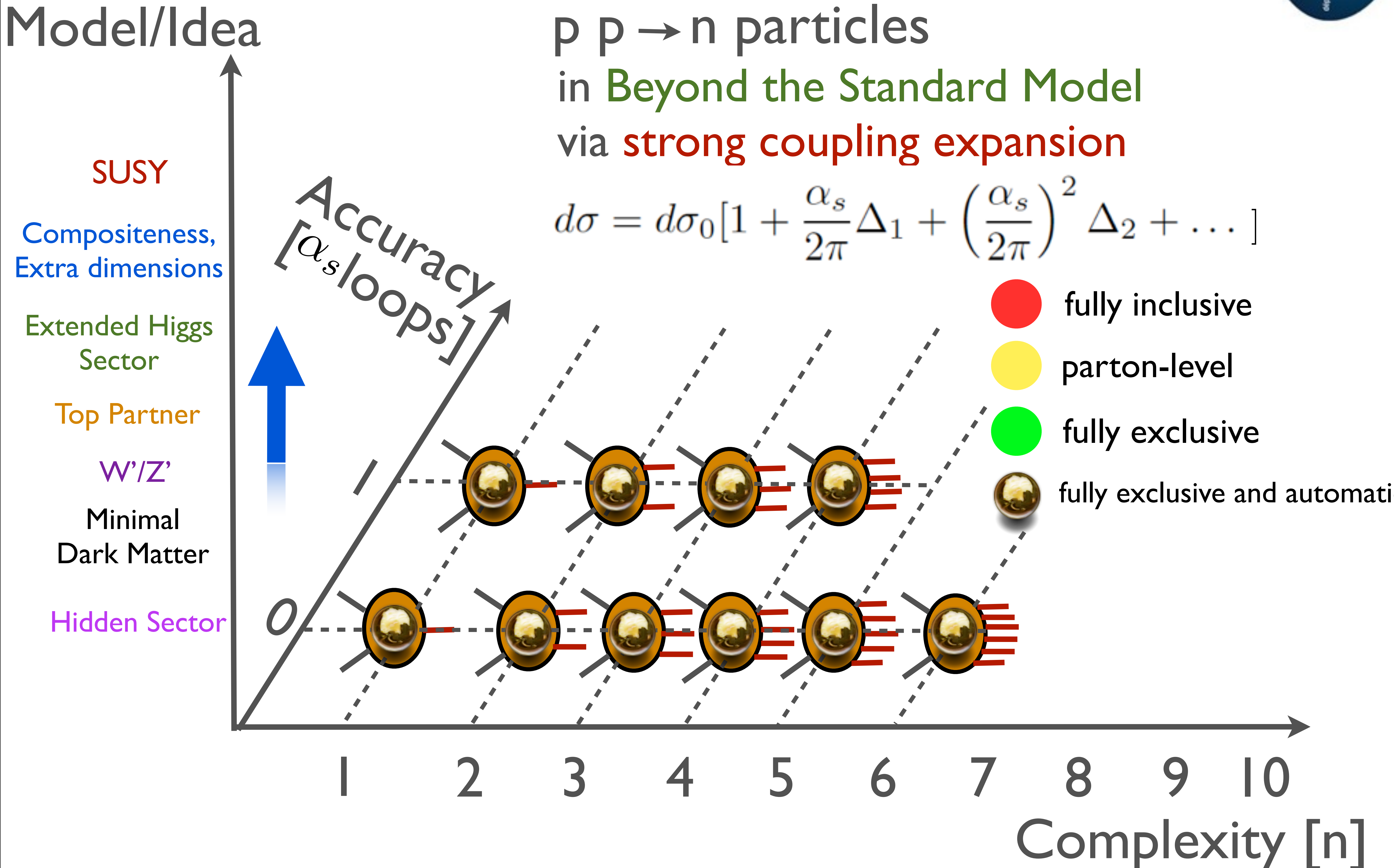
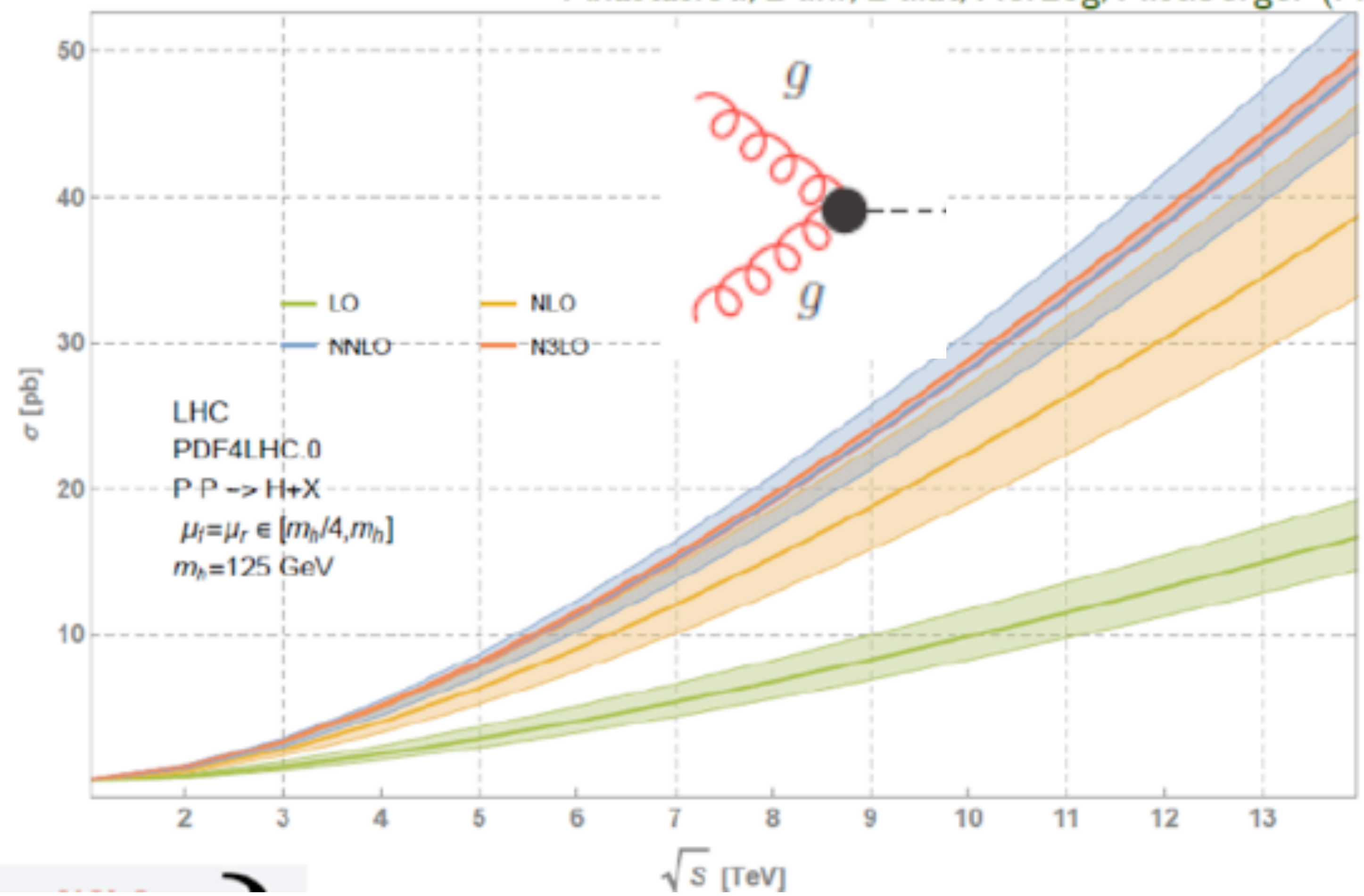


N^3LO HIGGS(+HIGGS) PRODUCTION: HIGHEST ACCURACY

- **Percent level inclusive ggF Higgs cross section**

Anastasiou, Duhr, Dulat, Herzog, Mistlberger (PRL'15)



Integral Statistics

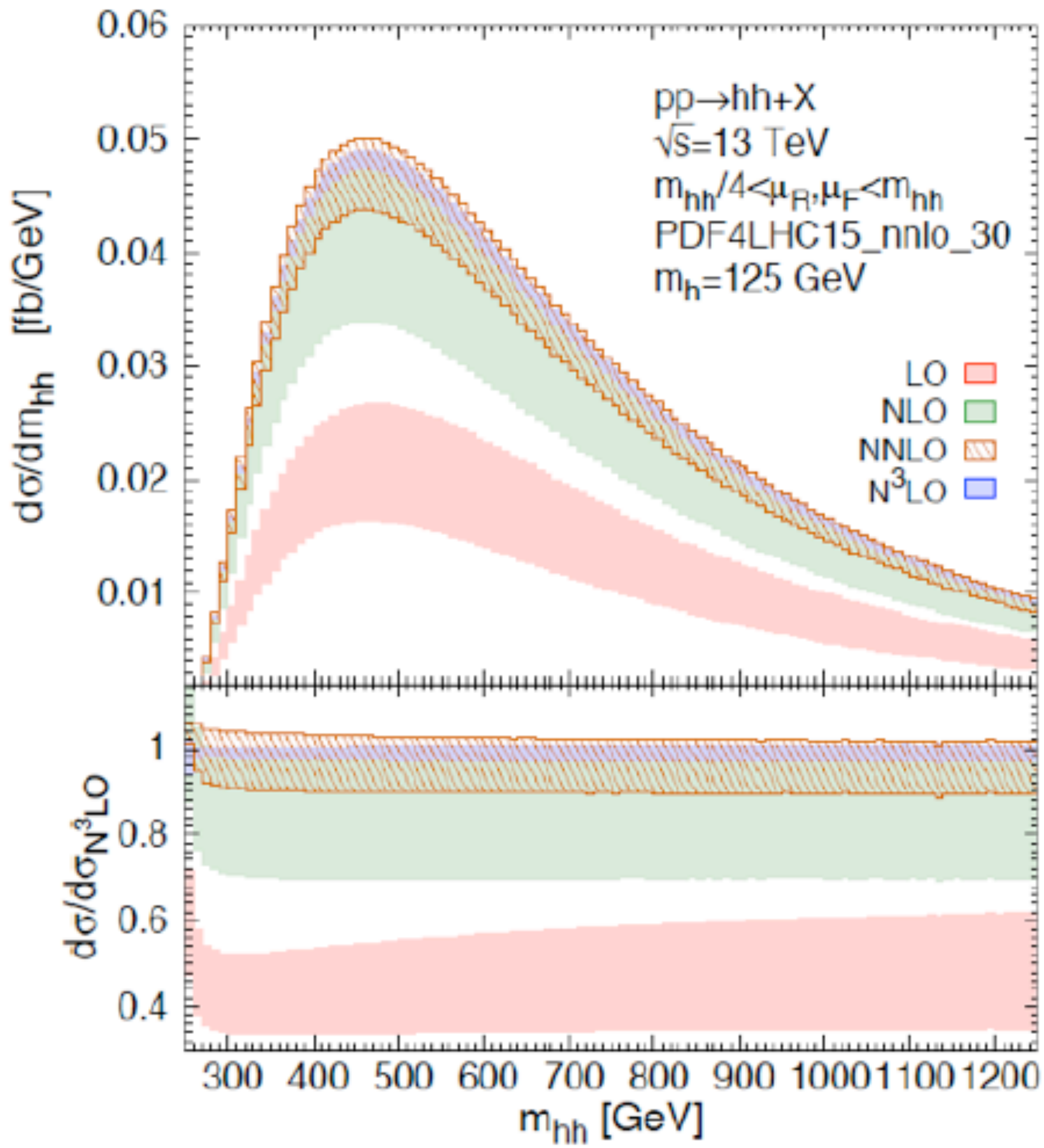
	NNLO	N3LO
#diagrams	~1.000	~100.000
#integrals	~50.000	517.531.178
#masters	27	1.028
#soft masters	5	78

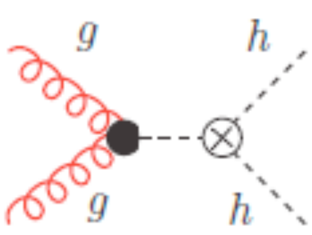
- Reverse Unitarity
- Differential equations
- Mellin Barnes Representations
- Hopf Algebra of Generalized Polylogs
- Number Theory
- Soft Expansion by Region
- Optimised Algorithm for IBP reduction and powerful computing resour

N^3LO HIGGS(+HIGGS) PRODUCTION: HIGHEST ACCURACY

- Percent level inclusive ggF Higgs cross section
- Percent level inclusive ggF Higgs+Higgs cross section

Chen, Li, HSS, Wang (1909.06808)





- Higher order -> more reliable of (differential cross sections)
- Scale uncertainties decrease
- Perturbative series is convergent
- The scale uncertainties are not reliable in LO but capture the correct missing higher order in NLO !

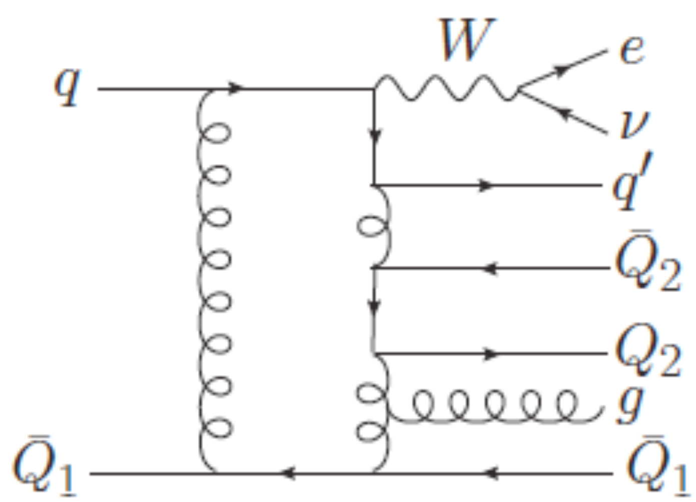
NLO W+JETS: HIGHEST JET MULTIPLICITY AT NLO

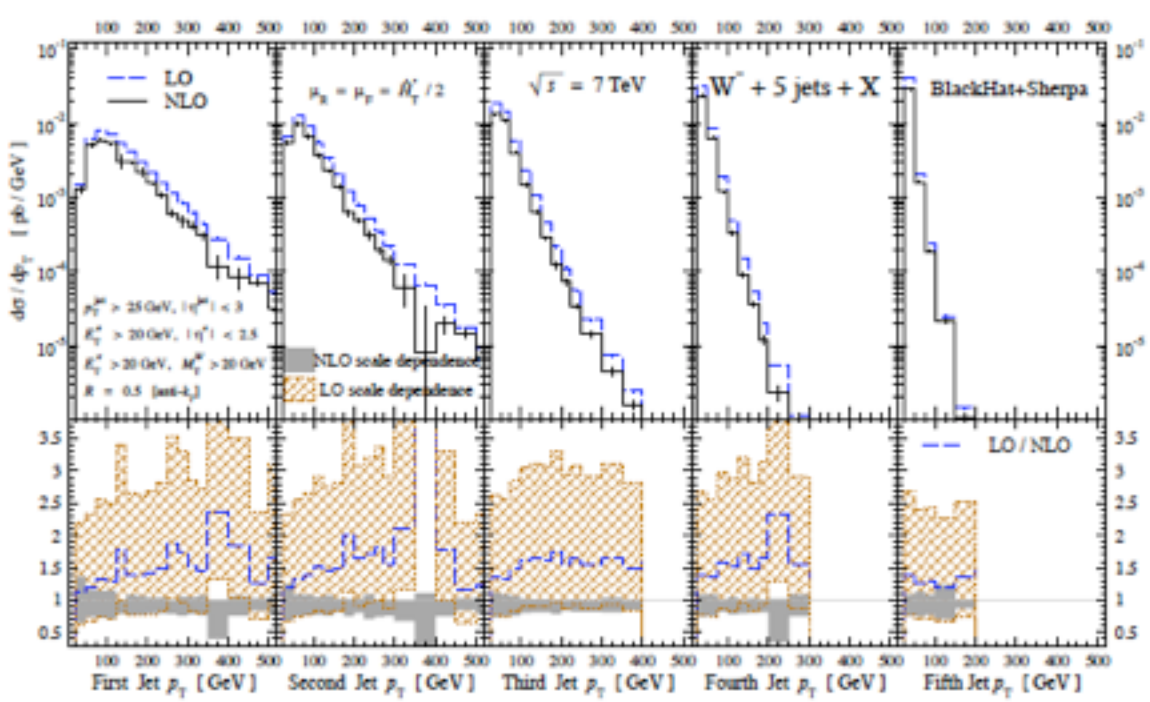
- Important (and often dominant) background at the LHC

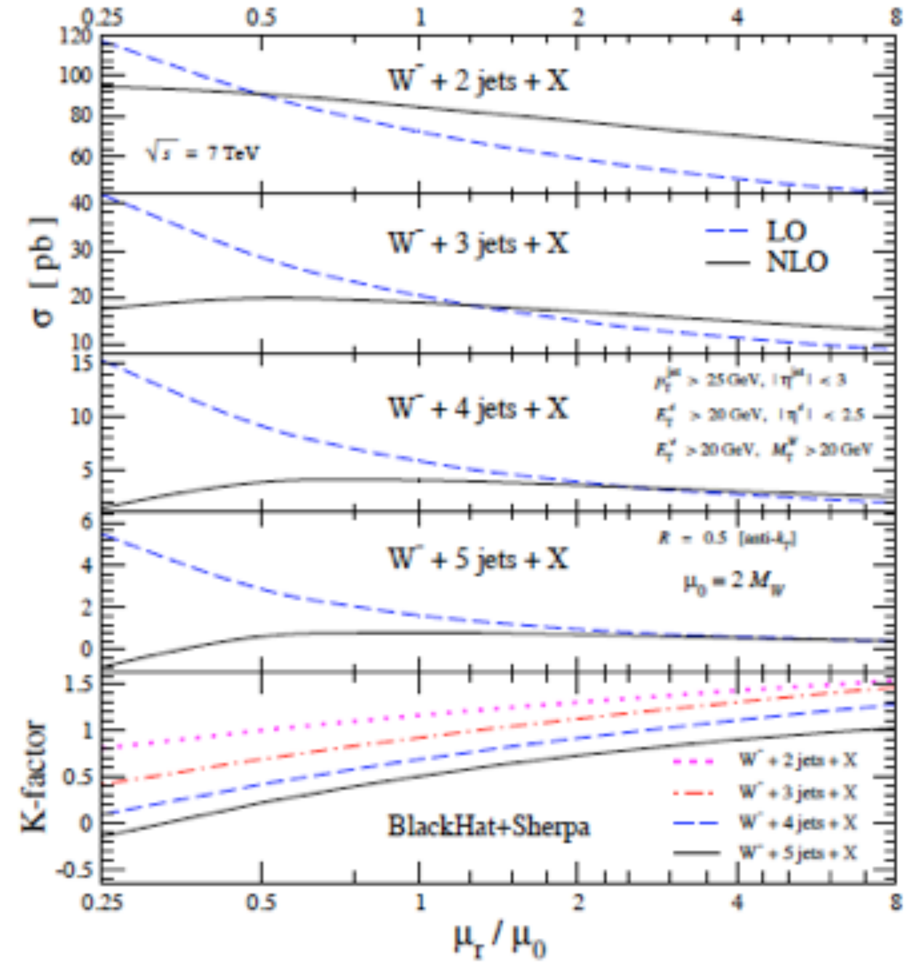
NLO W+JETS: HIGHEST JET MULTIPLICITY AT NLO

- **Important (and often dominant) background at the LHC**
- **NLO QCD correction: W+(>=n) jets, n=0,...,5**

Bern, Dixon, Febres Cordero, Hoche, Ita, Kosower, Maitre, Ozeren (PRD'13)







Technique improvements:

- Unitarity cuts
- Integrand reduction
- Recursion relations
- Local IR subtraction

NLO W+JETS: HIGHEST JET MULTIPLICITY AT NLO

- **Important (and often dominant) background at the LHC**

- **NLO QCD correction: W+(>=n) jets, n=0,...,5**

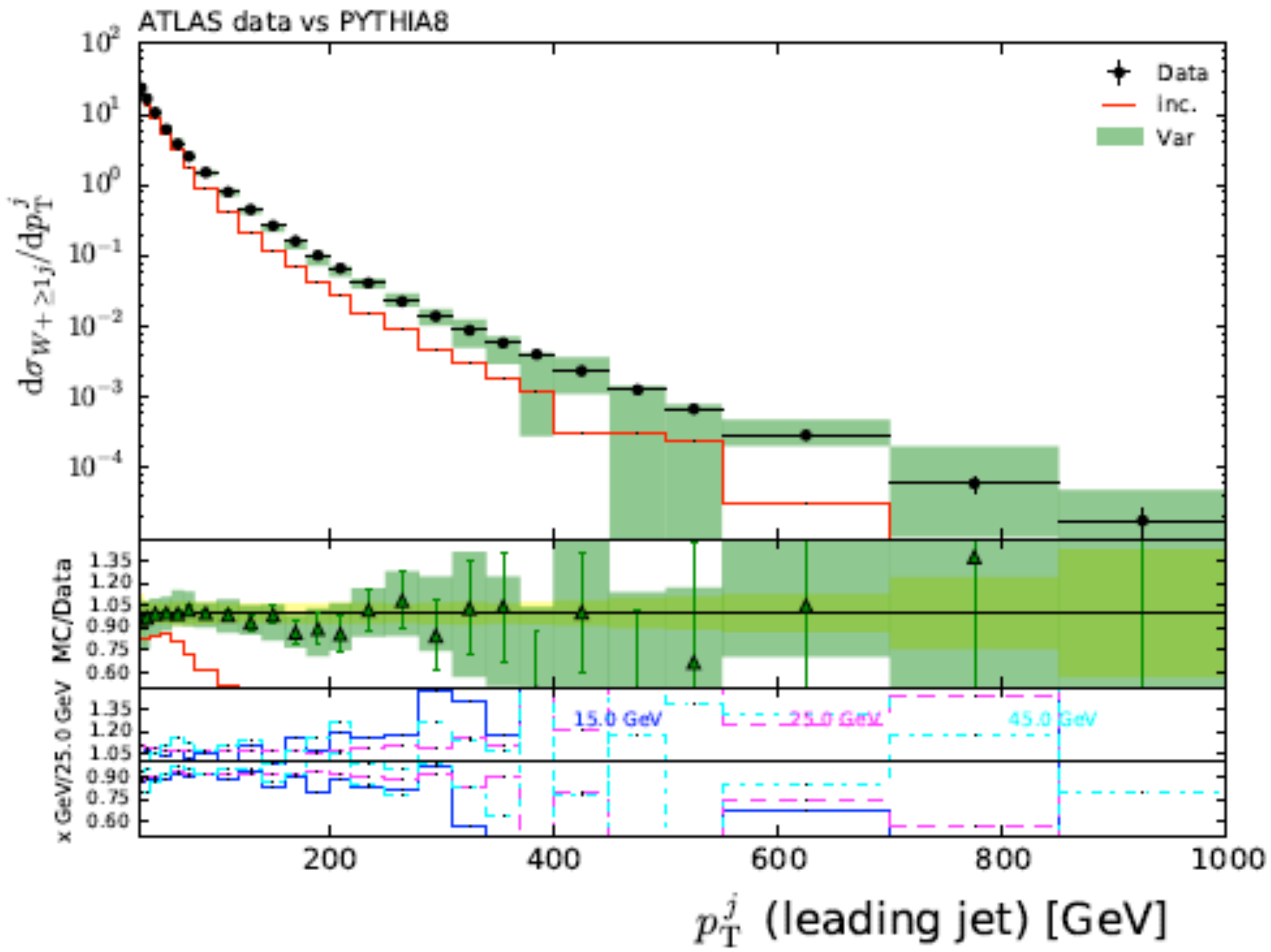
  Bern, Dixon, Febres Cordero, Hoche, Ita, Kosower, Maitre, Ozeren (PRD'13)

- **Automated NLO QCD: exclusive W+n jets, n=0,...,2**

  Frederix, Frixione, Papaefstathiou, Prestel, Torrielli (JHEP'15)

Commands:

```
./bin/mg5_aMC
MG5_aMC > import model loop_sm-no_b_mass
MG5_aMC > define p = p b b~; define j = p
MG5_aMC > define l = e+ mu+ e- mu-
MG5_aMC > define vl = ve vm ve~ vm~
MG5_aMC > generate p p > l vl [QCD] @ 0
MG5_aMC > generate p p > l vl j [QCD] @ 1
MG5_aMC > generate p p > l vl j j [QCD] @ 2
MG5_aMC > output; launch
```



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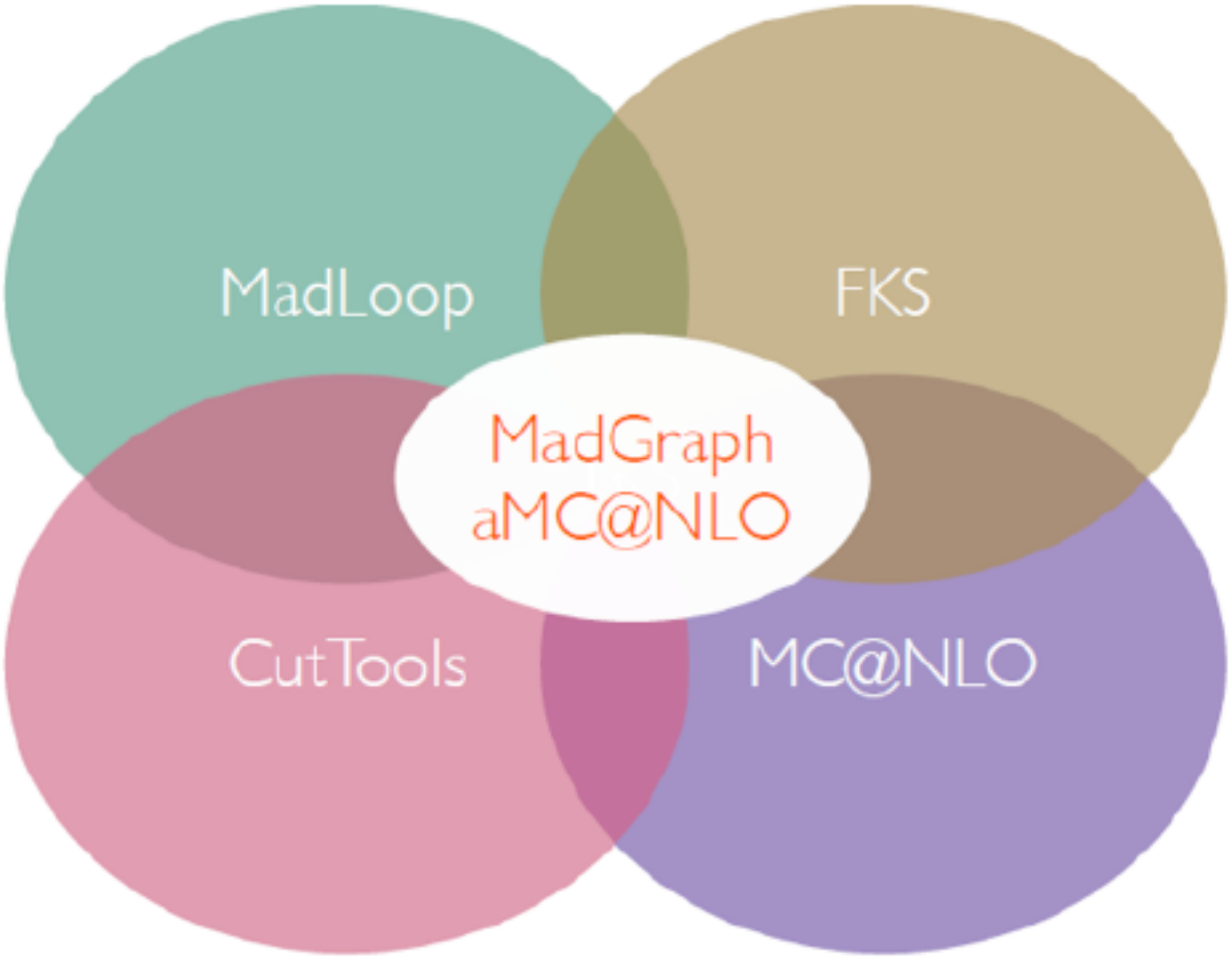
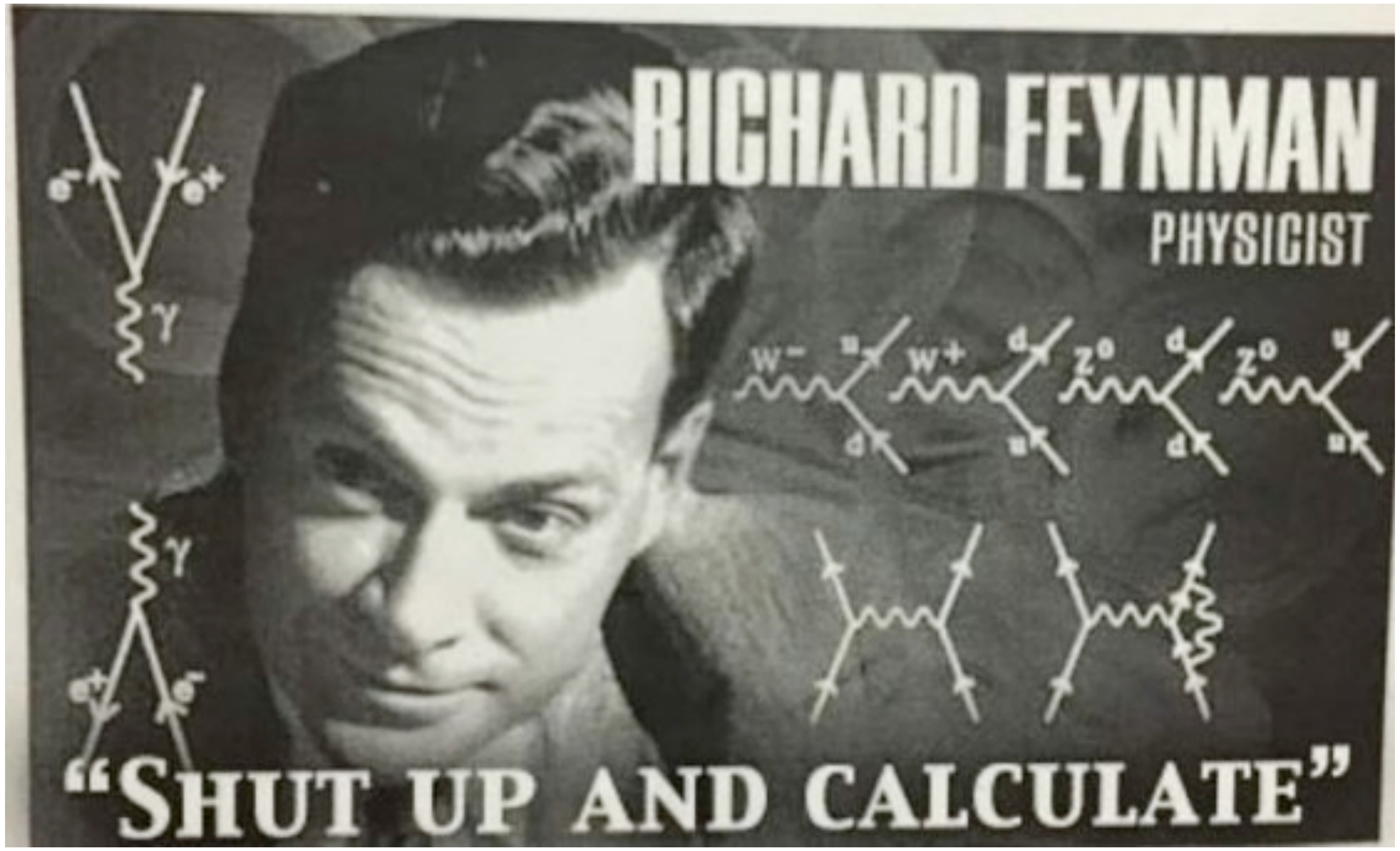
Technique improvements:
- Matured automated framework
- Methods of matching ME to PS
- Merging of multi-jet ME with PS



Alwall, Frederix, Frixione, Hirschi, Maltni, Mattelaer, HSS, Stelzer, Torrielli, Zaro (JHEP'14)

Lecture 1

NLO Basics



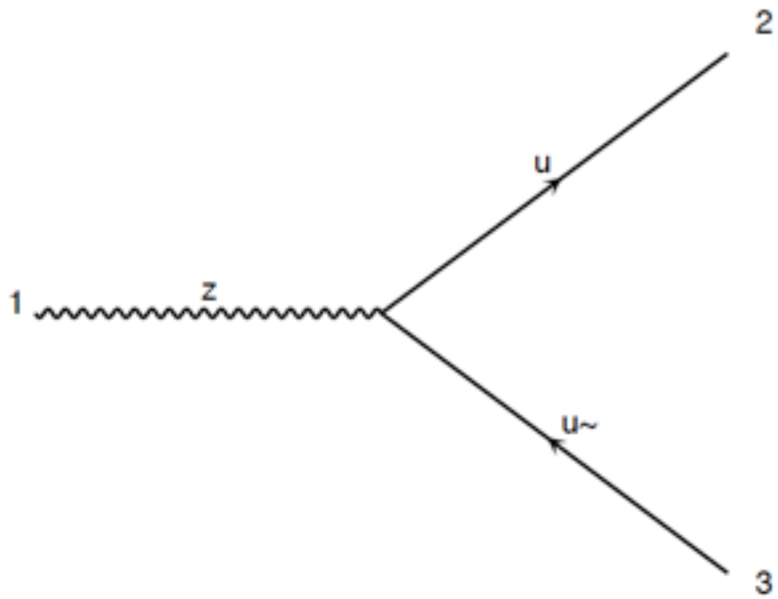
Lecture 1

NLO Basics

A NLO example

A NLO EXAMPLE: BORN

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Writing down Born amplitude according to Feynman rules**



For simplicity, we assume quarks are massless

$$\mathcal{A}_{\text{Born}} = -\delta_{c_q c_{\bar{q}}} \varepsilon_\mu(p_Z) \bar{u}(p_q).\Gamma^\mu_{Zq\bar{q}}.v(p_{\bar{q}})$$

$$\Gamma^\mu_{Zq\bar{q}} = ie\left(\frac{I_q}{\cos\theta_w \sin\theta_w} - Q_q \frac{\sin\theta_w}{\cos\theta_w}\right)\gamma^\mu P_L - ieQ_q \frac{\sin\theta_w}{\cos\theta_w}\gamma^\mu P_R$$

  - **Squaring amplitude, summing over colours and spins, and averaging the spin of the initial state**

$$\alpha = \frac{e^2}{4\pi}$$

$$\overline{\sum}|\mathcal{A}_{\text{Born}}|^2 = 8\pi\alpha m_Z^2 \left(2Q_q^2\left(\frac{\sin\theta_w}{\cos\theta_w}\right)^2 - 2\frac{I_q Q_q}{\cos^2\theta_w} + \frac{I_q^2}{\cos^2\theta_w \sin^2\theta_w}\right)$$

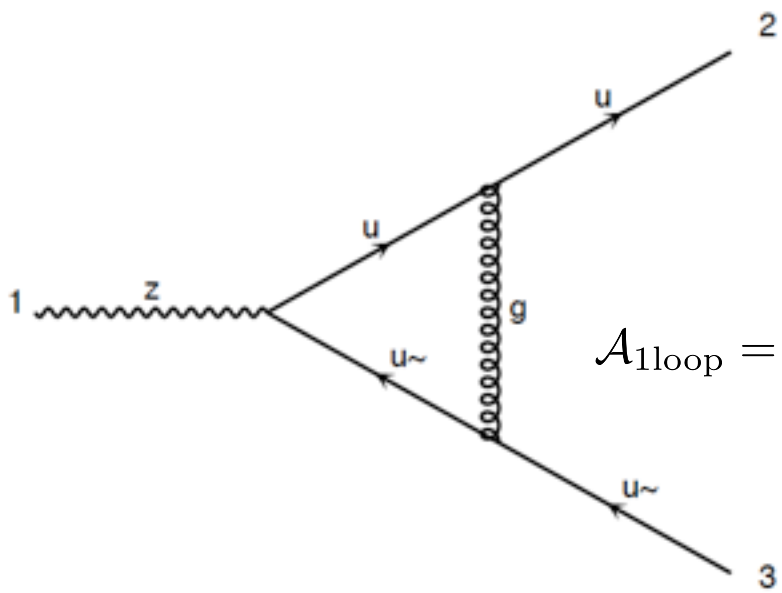
  - **Phase-space integration**

$$\Gamma_{\text{Born}}(Z \to q\bar{q}) = \frac{1}{2m_Z}\int (2\pi)^4 \delta^4(p_Z - p_q - p_{\bar{q}}) \frac{1}{(2\pi)^{3\times 2}} \frac{d^3 p_q}{2E_q} \frac{d^3 p_{\bar{q}}}{2E_{\bar{q}}} \overline{\sum}|\mathcal{A}_{\text{Born}}|^2$$
$$= \alpha m_Z \left(Q_q^2 \frac{\sin^2\theta_w}{\cos^2\theta_w} - \frac{Q_q I_q}{\cos^2\theta_w} + \frac{I_q^2}{2\cos^2\theta_w \sin^2\theta_w}\right)$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Writing down one-loop amplitude according to Feynman rules**

For simplicity, we assume quarks are massless



$$\mathcal{A}_{1\text{loop}} = ig^{\nu\rho}\varepsilon_\mu(p_Z)\bar{u}(p_q).\left(-ig_s\gamma_\nu T^a_{c_q c}\right).\int\frac{d^d\bar{l}}{(2\pi)^d}\frac{\bar{\not{l}}.\Gamma^\mu_{Zq\bar{q}}.\left(\bar{\not{l}}-\not{p}_Z\right)}{\bar{l}^2\left(\bar{l}-p_q\right)^2\left(\bar{l}-p_Z\right)^2}.\left(-ig_s\gamma_\rho T^a_{cc_{\bar{q}}}\right).v(p_{\bar{q}})$$

- **Need to evaluate two tensor integrals**

$$I_1^\mu = \int\frac{d^d\bar{l}}{(2\pi)^d}\frac{\bar{l}^\mu}{\bar{l}^2\left(\bar{l}-p_q\right)^2\left(\bar{l}-p_Z\right)^2} \qquad I_2^{\mu\nu} = \int\frac{d^d\bar{l}}{(2\pi)^d}\frac{\bar{l}^\mu\bar{l}^\nu}{\bar{l}^2\left(\bar{l}-p_q\right)^2\left(\bar{l}-p_Z\right)^2}$$

according to Lorentz structures

$$I_1^\mu = p_q^\mu B_1 + p_Z^\mu B_2 \qquad I_2^{\mu\nu} = g^{\mu\nu}B_{00} + p_q^\mu p_q^\nu B_{11} + p_Z^\mu p_Z^\nu B_{22} + \left(p_q^\mu p_Z^\nu + p_Z^\mu p_q^\nu\right)B_{12}$$

Solving the coefficients B, e.g.

$$p_q\cdot I_1 = p_q^2 B_1 + p_q\cdot p_Z B_2 = p_q\cdot p_Z B_2 \qquad p_Z\cdot I_1 = p_q\cdot p_Z B_1 + p_Z^2 B_2 = p_q\cdot p_Z B_1 + m_Z^2 B_2$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Solving the coefficients B, e.g.

$$B_2 = \frac{p_q \cdot I_1}{p_q \cdot p_Z} \qquad B_1 = \frac{p_Z \cdot I_1 - m_Z^2 B_2}{p_q \cdot p_Z}$$

$$\begin{aligned}
p_q \cdot I_1 &= \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{p_q \cdot \bar{l}}{\bar{l}^2 \left(\bar{l} - p_q\right)^2 \left(\bar{l} - p_Z\right)^2} \\
&= \frac{1}{2} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{\bar{l}^2 - \left(\bar{l} - p_q\right)^2}{\bar{l}^2 \left(\bar{l} - p_q\right)^2 \left(\bar{l} - p_Z\right)^2} \\
&= \frac{1}{2} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{1}{\left(\bar{l} - p_q\right)^2 \left(\bar{l} - p_Z\right)^2} - \frac{1}{2} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{1}{\bar{l}^2 \left(\bar{l} - p_Z\right)^2} \\
&= \frac{1}{2} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{1}{\bar{l}^2 \left(\bar{l} - p_{\bar{q}}\right)^2} - \frac{1}{2} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{1}{\bar{l}^2 \left(\bar{l} - p_Z\right)^2}
\end{aligned}$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Evaluating the scalar integrals, e.g.

$$\int \frac{d^d\bar{l}}{(2\pi)^d} \frac{1}{\bar{l}^2 \left(\bar{l}-p_{\bar{q}}\right)^2} = \int_0^1 dx \int \frac{d^d\bar{l}}{(2\pi)^d} \frac{1}{\left[x\bar{l}^2 + (1-x)\left(\bar{l}-p_{\bar{q}}\right)^2\right]^2}$$

Feynman parameterization !

$$= \int_0^1 dx \int \frac{d^d\bar{l}}{(2\pi)^d} \frac{1}{\left(\bar{l}-(1-x)p_{\bar{q}}\right)^4}$$

Using on-shell condition !

$$= \int_0^1 dx \int \frac{d^d\bar{l}}{(2\pi)^d} \frac{1}{\left(\bar{l}^2\right)^2}$$

Translational invariance !

$$= \int \frac{d^d\bar{l}}{(2\pi)^d} \frac{1}{\left(\bar{l}^2\right)^2}$$

Integration over x !

$$= \int \frac{d\bar{l}_0 d^{d-1}\vec{\bar{l}}}{(2\pi)^d} \frac{1}{\left(\bar{l}_0^2 - |\vec{\bar{l}}|^2\right)^2}$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Evaluating the scalar integrals, e.g.

$$\int \frac{d^d\bar{l}}{(2\pi)^d} \frac{1}{\bar{l}^2 \left(\bar{l}-p_{\bar{q}}\right)^2} \overset{\bar{l}_0 \to i\bar{l}_0}{=} \frac{i}{(2\pi)^d} \int d\Omega_d \int_0^{+\infty} d|\bar{l}||\bar{l}|^{d-5}$$

Wick rotation & spherical coordinate !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d} \int_0^{+\infty} d|\bar{l}||\bar{l}|^{d-5}$$

Integration over solid angle !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d} \left(\int_0^1 d|\bar{l}||\bar{l}|^{d-5} + \int_1^{+\infty} d|\bar{l}||\bar{l}|^{d-5} \right)$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Evaluating the scalar integrals, e.g.

$$\int \frac{d^d\bar{l}}{(2\pi)^d} \frac{1}{\bar{l}^2 \left(\bar{l}-p_{\bar{q}}\right)^2} \overset{\bar{l}_0\to i\bar{l}_0}{=} \frac{i}{(2\pi)^d} \int d\Omega_d \int_0^{+\infty} d|\bar{l}||\bar{l}|^{d-5}$$

Wick rotation & spherical coordinate !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d} \int_0^{+\infty} d|\bar{l}||\bar{l}|^{d-5}$$

Integration over solid angle !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d} \left(\int_0^{1} d|\bar{l}||\bar{l}|^{d-5} + \int_1^{+\infty} d|\bar{l}||\bar{l}|^{d-5} \right)$$

$|\bar{l}| \to 0$ (IR): the integral is divergent when $d \leq 4$

$|\bar{l}| \to +\infty$ (UV): the integral is divergent when $d \geq 4$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Evaluating the scalar integrals, e.g.

$$\int \frac{d^d\bar{l}}{(2\pi)^d} \frac{1}{\bar{l}^2 \left(\bar{l}-p_{\bar{q}}\right)^2} \overset{\bar{l}_0 \to i\bar{l}_0}{=} \frac{i}{(2\pi)^d} \int d\Omega_d \int_0^{+\infty} d|\bar{l}||\bar{l}|^{d-5}$$

Wick rotation & spherical coordinate !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d} \int_0^{+\infty} d|\bar{l}||\bar{l}|^{d-5}$$

Integration over solid angle !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d} \left(\int_0^1 d|\bar{l}||\bar{l}|^{d-5} + \int_1^{+\infty} d|\bar{l}||\bar{l}|^{d-5} \right)$$

$|\bar{l}| \to 0$ (IR): the integral is divergent when $d \leq 4$

$|\bar{l}| \to +\infty$(UV): the integral is divergent when $d \geq 4$

Regularisations:

$$d = 4 - 2\epsilon_{\rm IR}, \epsilon_{\rm IR} \to 0-$$
$$d = 4 - 2\epsilon_{\rm UV}, \epsilon_{\rm UV} \to 0+$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Evaluating the scalar integrals, e.g.

$$\int \frac{d^d\bar{l}}{(2\pi)^d}\frac{1}{\bar{l}^2\left(\bar{l}-p_{\bar{q}}\right)^2}=\frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d}\left(-\frac{1}{2\epsilon_{\rm IR}}+\frac{1}{2\epsilon_{\rm UV}}\right)$$

- **Squaring with Born amplitude, summing over colours and spins, and averaging the spin of the initial state**

$$\overline{\sum}2\Re\{\mathcal{A}_{\rm 1loop}\mathcal{A}^*_{\rm Born}\}=\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)\frac{\alpha_s}{\pi}\left[\frac{2}{3\epsilon_{\rm UV}}-\frac{4}{3\epsilon^2_{\rm IR}}-\frac{4}{3\epsilon_{\rm IR}}\left(1-\log\frac{m_Z^2}{4\pi^2\mu_R^2}\right)\right.$$
$$\left.-\frac{2}{3}\left(5-\pi^2-\log\frac{m_Z^2}{4\pi^2\mu_R^2}+\log^2\frac{m_Z^2}{4\pi^2\mu_R^2}\right)\right]$$

$$\alpha_s=\frac{g_s^2}{4\pi}$$

- **The UV divergence needs renormalisation**

$$\overline{\sum}2\Re\{\mathcal{A}_{\rm UV}\mathcal{A}^*_{\rm Born}\}=\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)\frac{\alpha_s}{\pi}\left[-\frac{2}{3\epsilon_{\rm UV}}+\frac{2}{3\epsilon_{\rm IR}}\right]$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Evaluating the scalar integrals, e.g.

$$\int \frac{d^d\bar{l}}{(2\pi)^d}\frac{1}{\bar{l}^2\left(\bar{l}-p_{\bar{q}}\right)^2}=\frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d}\left(-\frac{1}{2\epsilon_{\rm IR}}+\frac{1}{2\epsilon_{\rm UV}}\right)$$

  - **Squaring with Born amplitude, summing over colours and spins, and averaging the spin of the initial state**

$$\overline{\sum}2\Re\{\mathcal{A}_{\rm 1loop}\mathcal{A}^*_{\rm Born}\}=\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)\frac{\alpha_s}{\pi}\left[\frac{2}{3\epsilon_{\rm UV}}-\frac{4}{3\epsilon^2_{\rm IR}}-\frac{4}{3\epsilon_{\rm IR}}\left(1-\log\frac{m_Z^2}{4\pi^2\mu_R^2}\right)-\frac{2}{3}\left(5-\pi^2-\log\frac{m_Z^2}{4\pi^2\mu_R^2}+\log^2\frac{m_Z^2}{4\pi^2\mu_R^2}\right)\right]$$

$$\alpha_s=\frac{g_s^2}{4\pi}$$

  - **The UV divergence needs renormalisation**

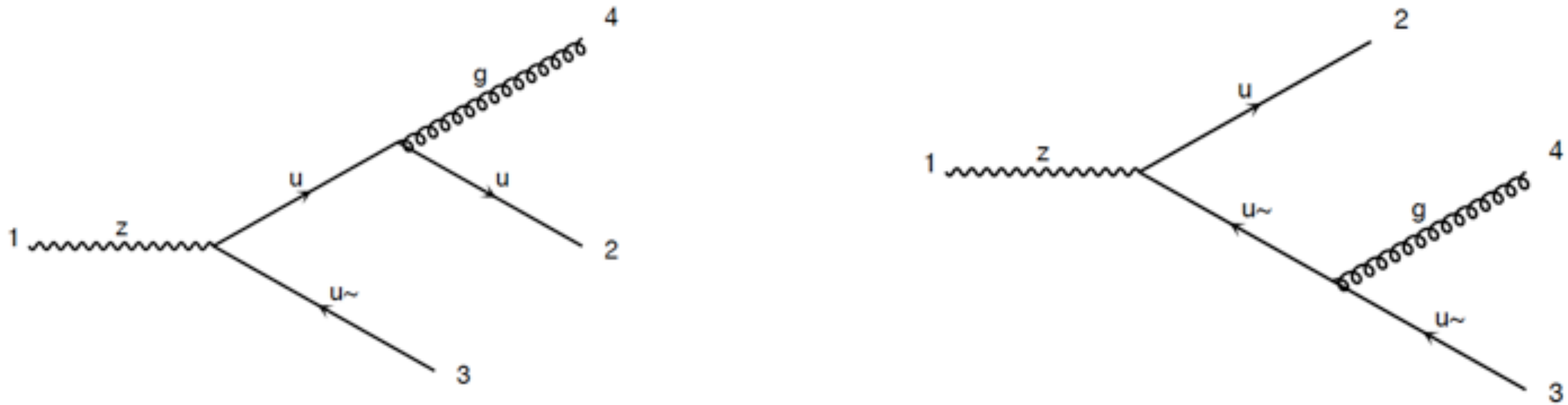
$$\overline{\sum}2\Re\{\mathcal{A}_{\rm UV}\mathcal{A}^*_{\rm Born}\}=\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)\frac{\alpha_s}{\pi}\left[-\frac{2}{3\epsilon_{\rm UV}}+\frac{2}{3\epsilon_{\rm IR}}\right]$$

  - **The virtual matrix element is:**

$$\mathcal{V}=\overline{\sum}2\Re\{\mathcal{A}_{\rm 1loop}\mathcal{A}^*_{\rm Born}\}+\overline{\sum}2\Re\{\mathcal{A}_{\rm UV}\mathcal{A}^*_{\rm Born}\}$$

A NLO EXAMPLE: REAL

- Let us calculate NLO QCD of Z -> q qbar decay
  - Writing down real amplitude according to Feynman rules



- Squaring amplitude, summing over colours and spins, and averaging the spin of the initial state

$$\overline{\sum}|\mathcal{A}_{\mathrm{real}}|^2 = \left(\overline{\sum}|\mathcal{A}_{\mathrm{Born}}|^2\right)\alpha_s \frac{8\pi(d-2)}{3m_Z^2 s_{24}s_{34}}$$

$$\times\left[(d-2)s_{24}^2 + 2(d-4)s_{24}s_{34} + (d-2)s_{34}^2 - 4m_Z^2(s_{24}+s_{34}) + 4m_Z^4\right]$$

$$s_{24} = (p_q + p_g)^2, s_{34} = (p_{\bar{q}} + p_g)^2$$

A NLO EXAMPLE: REAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **3-body phase-space integration**

$$\Gamma_{\rm real}=\frac{1}{2m_Z}\int (2\pi)^d\,\delta^d\left(p_Z-p_q-p_{\bar{q}}-p_g\right)\frac{1}{(2\pi)^{3(d-1)}}\frac{d^{d-1}\vec{p}_q}{2E_q}\frac{d^{d-1}\vec{p}_{\bar{q}}}{2E_{\bar{q}}}\frac{d^{d-1}\vec{p}_g}{2E_g}\overline{\sum}|\mathcal{A}_{\rm real}|^2$$

$$y=\frac{s_{34}}{m_Z^2},1-y-z=\frac{s_{24}}{m_Z^2}$$

$$d\Phi^{(2)}(p_Z\to p_q,p_{\bar{q}})=(2\pi)^d\delta^d(p_Z-p_q-p_{\bar{q}})\frac{1}{(2\pi)^{2(d-1)}}\frac{d^{d-1}\vec{p}_q}{2E_q}\frac{d^{d-1}\vec{p}_{\bar{q}}}{2E_{\bar{q}}}$$
$$=\frac{(4\pi)^{2\epsilon}}{8(2\pi)^2}\frac{1}{m_Z^{2\epsilon}}d\Omega_d$$

$$d\Phi^{(3)}(p_Z\to p_q,p_{\bar{q}},p_g)=(2\pi)^d\delta^d(p_Z-p_q-p_{\bar{q}}-p_g)\frac{1}{(2\pi)^{3(d-1)}}\frac{d^{d-1}\vec{p}_q}{2E_q}\frac{d^{d-1}\vec{p}_{\bar{q}}}{2E_{\bar{q}}}\frac{d^{d-1}\vec{p}_g}{2E_g}$$
$$=\frac{(4\pi)^{3\epsilon}}{32(2\pi)^4\Gamma(1-\epsilon)}(m_Z^2)^{1-2\epsilon}d\Omega_d$$
$$\times\int_0^1 dz z^{-\epsilon}\int_0^{1-z}dy y^{-\epsilon}(1-z-y)^{-\epsilon}$$
$$=d\Phi^{(2)}(p_Z\to p_q,p_{\bar{q}})\times\frac{(4\pi)^{\epsilon}}{16\pi^2\Gamma(1-\epsilon)}(m_Z^2)^{1-\epsilon}$$
$$\times\int_0^1 dz z^{-\epsilon}\int_0^{1-z}dy y^{-\epsilon}(1-z-y)^{-\epsilon}$$

A NLO EXAMPLE: REAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **3-body phase-space integration**

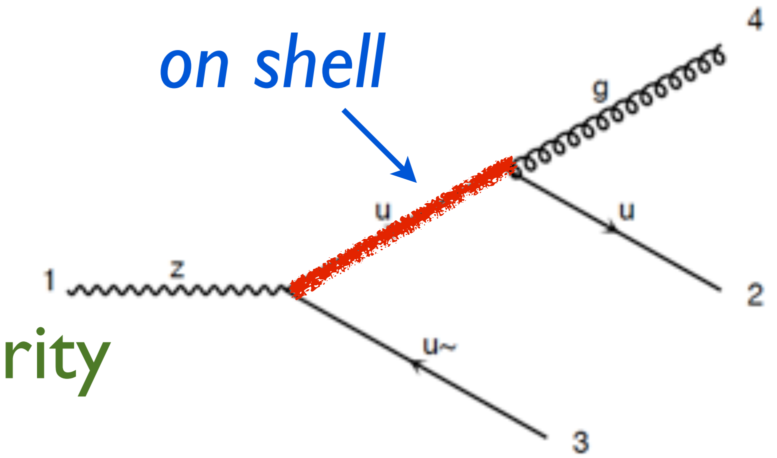
$$\overline{\sum}|\mathcal{A}_{\rm real}|^2 = \left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)\alpha_s \frac{8\pi(d-2)}{3m_Z^2 y(1-z-y)}\left[(d-2)(1-z)^2+4y^2-4y(1-z)+4z\right]$$

The integration over y is divergent when $d \leq 4\ (\epsilon \geq 0)$

$$y \to 1-z\ (s_{24}\to 0)$$

(1) $p_g \to 0\ (z\to 0)$ soft singularity

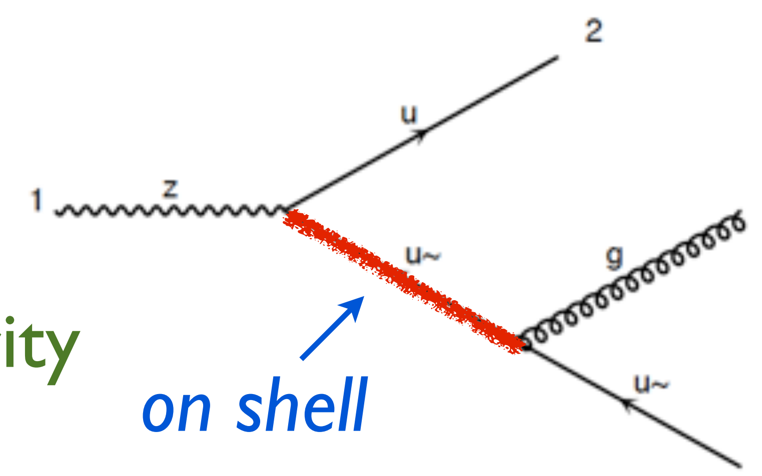
(2) $p_g || p_q$ collinear singularity



$$y \to 0\ (s_{34}\to 0)$$

(1) $p_g \to 0\ (z\to 0)$ soft singularity

(2) $p_g || p_{\bar{q}}$ collinear singularity



A NLO EXAMPLE: REAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **3-body phase-space integration**

$$\Gamma_{\rm real} = \frac{1}{2m_Z}\int d\Phi^{(3)}(p_Z\to p_q,p_{\bar q},p_g)\overline{\sum}|\mathcal{A}_{\rm real}|^2$$
$$= \frac{1}{2m_Z}\int d\Phi^{(2)}(p_Z\to p_q,p_{\bar q})\left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)$$
$$\times\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\frac{\alpha_s}{\pi}\left[\frac{4}{3\epsilon_{\rm IR}^2}+\frac{2}{3\epsilon_{\rm IR}}\left(1-2\log\frac{m_Z^2}{4\pi^2\mu_R^2}\right)\right.$$
$$\left.+\frac{1}{3}\left(2\log^2\frac{m_Z^2}{4\pi^2\mu_R^2}-2\log\frac{m_Z^2}{4\pi^2\mu_R^2}-2\pi^2+13\right)\right]$$

  - **Sum real and virtual**

$$\Gamma_{\rm virtual} = \frac{1}{2m_Z}\int d\Phi^{(2)}(p_Z\to p_q,p_{\bar q})\mathcal{V}$$

$$\Gamma_{\rm virtual}+\Gamma_{\rm real} = \frac{1}{2m_Z}\int d\Phi^{(2)}(p_Z\to p_q,p_{\bar q})\left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\frac{\alpha_s}{\pi}$$

A NLO EXAMPLE: NLO

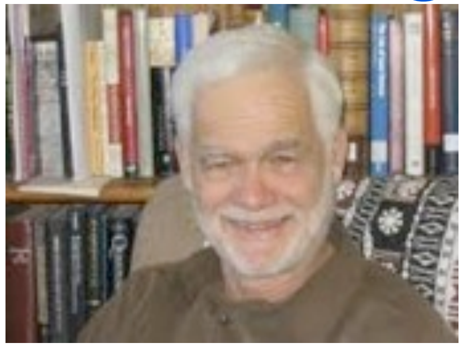
- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Sum real and virtual**

All remaining IR poles cancel (in general KLN theorem)

Kinoshita

Lee

Nauenberg



$$\Gamma_{\rm virtual}+\Gamma_{\rm real}=\frac{1}{2m_Z}\int d\Phi^{(2)}(p_Z\to p_q,p_{\bar{q}})\left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)\frac{(4\pi)^{\epsilon}}{\Gamma(1-\epsilon)}\frac{\alpha_s}{\pi}$$

$$\overset{\epsilon\to 0}{=}\Gamma_{\rm Born}(Z\to q\bar{q})\frac{\alpha_s}{\pi}$$

$$\boxed{\Gamma_{\rm NLO}(Z\to q\bar{q}+X)=\Gamma_{\rm Born}(Z\to q\bar{q})\left(1+\frac{\alpha_s}{\pi}\right)}$$

We finally get a well-known result !

ex: Filling all the gaps I did not show !

In general, NLO calculations are complex (and tedious, error-prone). Let us work with the aid of a computer and MadGraph5_aMC@NLO.

Lecture 2

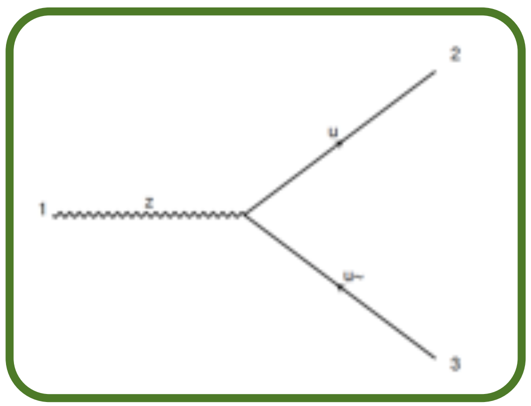
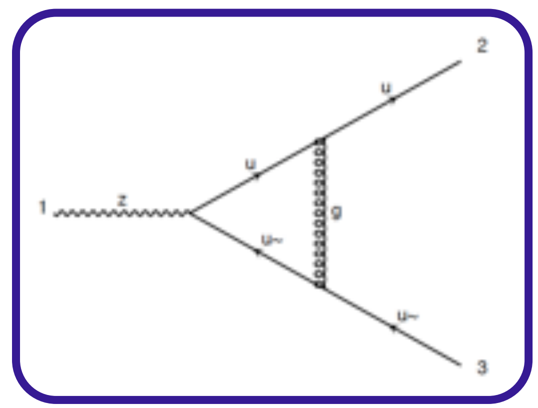
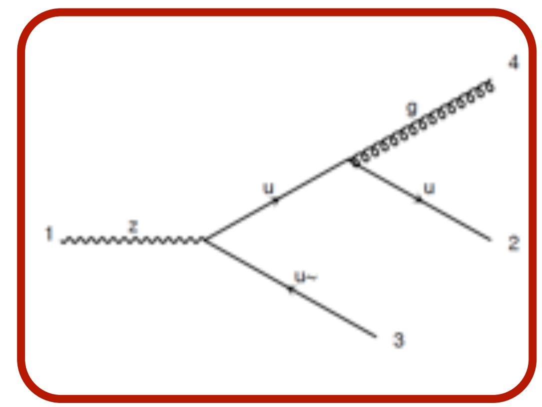
NLO Generics

NLO ANATOMY

- **Three parts need to be computed in a NLO calculation**

$$\sigma_{\rm NLO} = \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\mathcal{V} + \int d\Phi^{(n+1)}\mathcal{R}$$

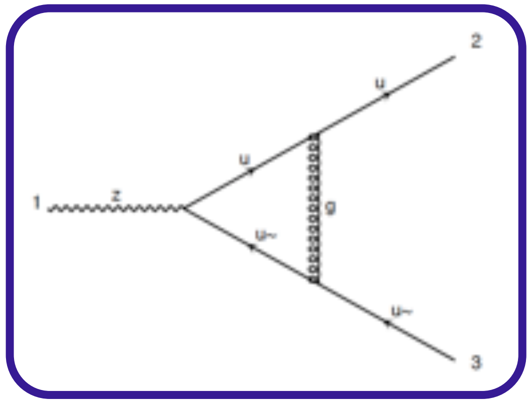
$$\mathcal{O}(\alpha_s^b) \qquad \mathcal{O}(\alpha_s^{b+1}) \qquad \mathcal{O}(\alpha_s^{b+1})$$

Born cross section	Virtual correction	Real correction
		
Finite	Divergent	Divergent
MadEvent	**MadLoop**	**MadFKS**

Lecture 2

NLO Generics

Virtual=Loop+UV



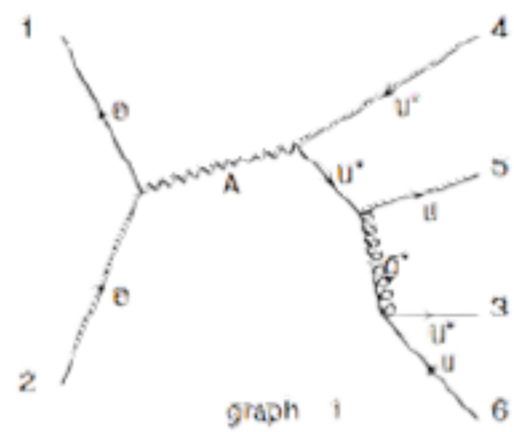
Tuesday, November 19, 19

ONE-LOOP DIAGRAM GENERATION

- No external tool for loop diagram generation:
  Reuse MG5_aMC efficient tree level diagram generation!

- Cut loops have two extra external particles

Trees ($e^+e^- \rightarrow$ u u~ u u~) ≡ Loops ($e^+e^- \rightarrow$ u u~)



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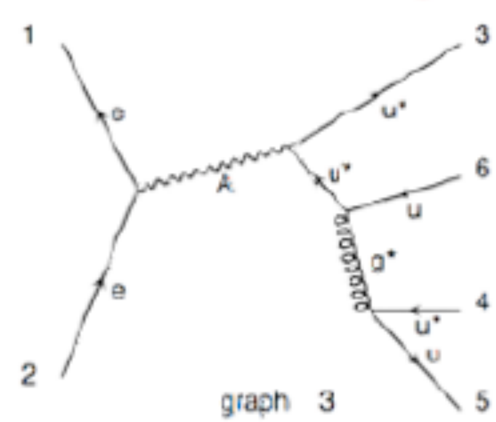

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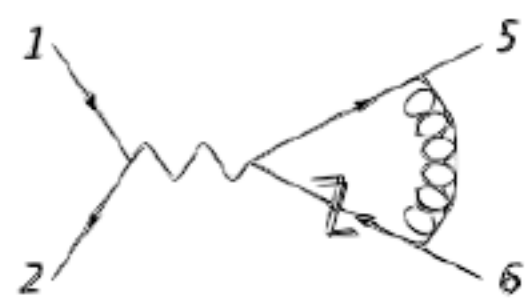
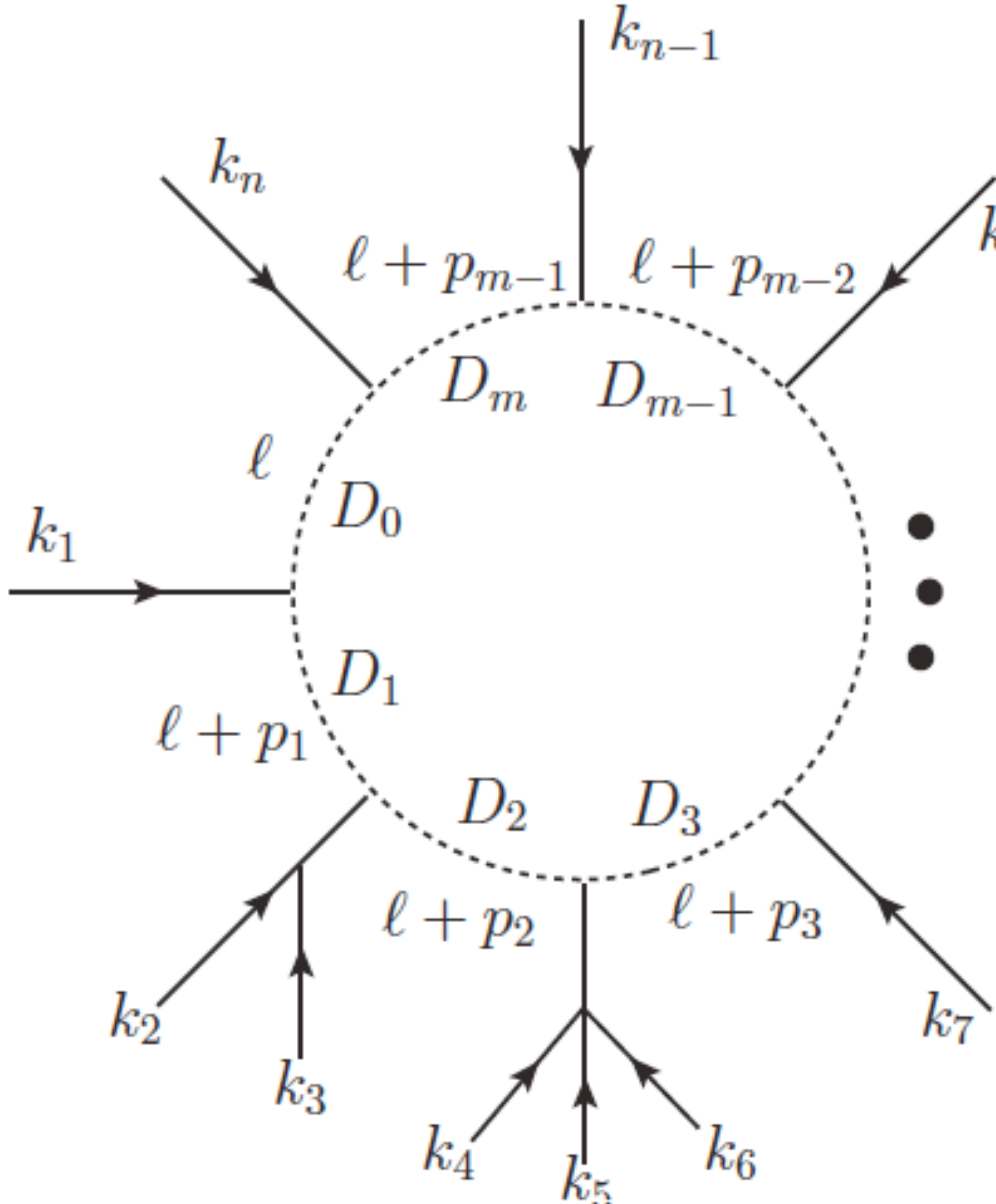


ONE-LOOP INTEGRAL EVALUATION



- Consider this m-point loop diagram with n external momenta

$$\equiv \int \frac{d^d\ell}{(2\pi)^d} \frac{\mathcal{N}(\ell)}{D_0 D_1 D_2 D_3 \cdots D_{m-2} D_{m-1}}$$

with $D_i = (\ell + p_i)^2 - m_i^2$

We will denote by $\mathcal{C}$ this integral.

ONE-LOOP INTEGRAL EVALUATION

$$\mathcal{C}^{\text{1-loop}} = \sum_{i_0<i_1<i_2<i_3} d_{i_0 i_1 i_2 i_3} \text{Box}_{i_0 i_1 i_2 i_3} + \sum_{i_0<i_1<i_2} c_{i_0 i_1 i_2} \text{Triangle}_{i_0 i_1 i_2} + \sum_{i_0<i_1} b_{i_0 i_1} \text{Bubble}_{i_0 i_1} + \sum_{i_0} a_{i_0} \text{Tadpole}_{i_0} + R + \mathcal{O}(\epsilon)$$

$$\text{Box}_{i_0 i_1 i_2 i_3} = \int d^d l \frac{1}{D_{i_0} D_{i_1} D_{i_2} D_{i_3}}$$

$$\text{Triangle}_{i_0 i_1 i_2} = \int d^d l \frac{1}{D_{i_0} D_{i_1} D_{i_2}}$$

$$\text{Bubble}_{i_0 i_1} = \int d^d l \frac{1}{D_{i_0} D_{i_1}}$$

$$\text{Tadpole}_{i_0} = \int d^d l \frac{1}{D_{i_0}}$$

The a, b, c, d and R coefficients depend only on external parameters and momenta.

Reduction of the loop to these scalar coefficients can be achieved using either Tensor Integral Reduction or Reduction at the integrand level

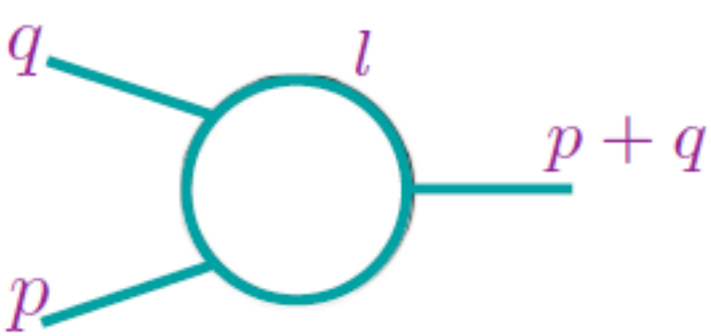
TENSOR INTEGRAL REDUCTION

- Passarino-Veltman reduction:

$$\int d^d l \frac{N(l)}{D_0 D_1 D_2 \cdots D_{m-1}} \to \sum_i \text{coeff}_i \int d^d l \frac{1}{D_0 D_1 \cdots}$$

  - Reduce a general integral to "scalar integrals" by "completing the square"

- Example:
  Application of PV to this triangle rank-1 integral



$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{(l^2 - m_1^2)((l+p)^2 - m_2^2)((l+q)^2 - m_3^2)}$$

- Implemented in codes such as:

COLLIER [A. Denner, S .Dittmaier, L. Hofer, 1604.06792]
GOLEM95 [T. Binoth, J.Guillet, G. Heinrich, E.Pilon, T.Reither, 0810.0992]

TENSOR INTEGRAL REDUCTION

$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{(l^2-m_1^2)((l+p)^2-m_2^2)((l+q)^2-m_3^2)}$$

- The only independent four vectors are p^μ and q^μ . Therefore, the integral must be proportional to those. We can set-up a system of linear equations and try to solve for C_1 and C_2

$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{(l^2-m_1^2)((l+p)^2-m_2^2)((l+q)^2-m_3^2)} = \begin{pmatrix} p^\mu & q^\mu \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

We can solve for C_1 and C_2 by contracting with p and q

$$\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} [2l\cdot p] \\ [2l\cdot q] \end{pmatrix} = G \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \equiv \begin{pmatrix} 2p\cdot p & 2p\cdot q \\ 2p\cdot q & 2q\cdot q \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

where $[2l\cdot p] = \int \frac{d^n l}{(2\pi)^n} \frac{2l\cdot p}{l^2(l+p)^2(l+q)^2}$ (For simplicity, the masses are neglected here)

- By expressing $2l.p$ and $2l.q$ as a sum of denominators we can express R_1 and R_2 as a sum of simpler integrals, *e.g.*

$$R_1 = \int \frac{d^n l}{(2\pi)^n} \frac{2l\cdot p}{l^2(l+p)^2(l+q)^2} = \int \frac{d^n l}{(2\pi)^n} \frac{(l+p)^2-l^2-p^2}{l^2(l+p)^2(l+q)^2}$$
$$= \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l+q)^2} - \int \frac{d^n l}{(2\pi)^n} \frac{1}{(l+p)^2(l+q)^2} - p^2 \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l+p)^2(l+q)^2}$$

TENSOR INTEGRAL REDUCTION

- And similarly for R_2

$$R_2 = \int \frac{d^n l}{(2\pi)^n} \frac{2l \cdot q}{l^2 (l+p)^2 (l+q)^2} = \int \frac{d^n l}{(2\pi)^n} \frac{(l+q)^2 - l^2 - q^2}{l^2 (l+p)^2 (l+q)^2}$$
$$= \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2 (l+p)^2} - \int \frac{d^n l}{(2\pi)^n} \frac{1}{(l+p)^2 (l+q)^2} - q^2 \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2 (l+p)^2 (l+q)^2}$$

- Now we can solve the equation

$$\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} [2l \cdot p] \\ [2l \cdot q] \end{pmatrix} = G \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \equiv \begin{pmatrix} 2p \cdot p & 2p \cdot q \\ 2p \cdot q & 2q \cdot q \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

by inverting the "Gram" matrix G

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = G^{-1} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

- We have re-expressed, reduced, our original integral

$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{(l^2 - m_1^2)((l+p)^2 - m_2^2)((l+q)^2 - m_3^2)} = \begin{pmatrix} p^\mu & q^\mu \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

in terms of known, simpler *scalar* integrals

INTEGRAND REDUCTION

Ossola, Papadopulos, Pittau (NPB'06)

TIR

- The decomposition to the basis scalar integrals works at the level of the integrals

$$\mathcal{C}^{\text{1-loop}} = \sum_{i_0<i_1<i_2<i_3} d_{i_0i_1i_2i_3}\text{Box}_{i_0i_1i_2i_3} + \sum_{i_0<i_1<i_2} c_{i_0i_1i_2}\text{Triangle}_{i_0i_1i_2} + \sum_{i_0<i_1} b_{i_0i_1}\text{Bubble}_{i_0i_1} + \sum_{i_0} a_{i_0}\text{Tadpole}_{i_0} + R + \mathcal{O}(\epsilon)$$

OPP

- Knowing a relation directly at the integrand level, we would be able to manipulate the reduction without doing the the integrals

$$N(l) = \sum_{i_0,i_1,i_2,i_3} (d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}) \prod_{i\neq i_0,i_1,i_2,i_3} D_i + \sum_{i_0,i_1,i_2} (c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}) \prod_{i\neq i_0,i_1,i_2} D_i + \sum_{i_0,i_1} (b_{i_0i_1} + \tilde{b}_{i_0i_1}) \prod_{i\neq i_0,i_1} D_i + \sum_{i_0} (a_{i_0} + \tilde{a}_{i_0}) \prod_{i\neq i_0} D_i + \tilde{P}(l) \prod_i D_i + \mathcal{O}(\varepsilon)$$

INTEGRAND REDUCTION

Ossola, Papadopulos, Pittau (NPB'06)

TIR

- The decomposition to the basis scalar integrals works at the level of the integrals

$$\mathcal{C}^{\text{1-loop}} = \sum_{i_0<i_1<i_2<i_3} d_{i_0i_1i_2i_3}\text{Box}_{i_0i_1i_2i_3} + \sum_{i_0<i_1<i_2} c_{i_0i_1i_2}\text{Triangle}_{i_0i_1i_2} + \sum_{i_0<i_1} b_{i_0i_1}\text{Bubble}_{i_0i_1} + \sum_{i_0} a_{i_0}\text{Tadpole}_{i_0} + R + \mathcal{O}(\epsilon)$$

OPP

- Knowing a relation directly at the integrand level, we would be able to manipulate the reduction without doing the the integrals

$$N(l) = \sum_{i_0,i_1,i_2,i_3}(d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3})\prod_{i\neq i_0,i_1,i_2,i_3} D_i + \sum_{i_0,i_1,i_2}(c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2})\prod_{i\neq i_0,i_1,i_2} D_i + \sum_{i_0,i_1}(b_{i_0i_1} + \tilde{b}_{i_0i_1})\prod_{i\neq i_0,i_1} D_i + \sum_{i_0}(a_{i_0} + \tilde{a}_{i_0})\prod_{i\neq i_0} D_i + \tilde{P}(l)\prod_i D_i + \mathcal{O}(\varepsilon)$$

Spurious term

INTEGRAND REDUCTION

- The functional form of the spurious terms is known (it depends on the rank of the integral and the number of propagators in the loop) [del Aguila, Pittau 2004]
  - for example, a box coefficient from a rank 1 numerator is

$$\tilde{d}_{i_0 i_1 i_2 i_3}(l) = \tilde{d}_{i_0 i_1 i_2 i_3}\, \epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma$$

(remember that p_i is the sum of the momentum that has entered the loop so far, so we always have $p_0 = 0$)

  - The integral is zero

$$\int d^d l \frac{\tilde{d}_{i_0 i_1 i_2 i_3}(l)}{D_0 D_1 D_2 D_3} = \tilde{d}_{i_0 i_1 i_2 i_3} \int d^d l \frac{\epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma}{D_0 D_1 D_2 D_3} = 0$$

INTEGRAND REDUCTION

- **Take Box (4-point) coefficients as an example**

$$N(l^{\pm}) = d_{0123} + \tilde{d}_{0123}(l^{\pm}) \prod_{i\neq 0,1,2,3}^{m-1} D_i(l^{\pm})$$

- Two values are enough given the functional form for the spurious term. We can immediately determine the Box coefficient

$$d_{0123} = \frac{1}{2}\left[\frac{N(l^+)}{\prod_{i\neq 0,1,2,3}^{m-1} D_i(l^+)} + \frac{N(l^-)}{\prod_{i\neq 0,1,2,3}^{m-1} D_i(l^-)}\right]$$

- By choosing other values for l, that set other combinations of 4 "denominators" to zero, we can get all the Box coefficients

INTEGRAND REDUCTION

- **In general:**

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$

$$+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}(l)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$

$$+ \sum_{i_0<i_1}^{m-1} \left[b_{i_0i_1} + \tilde{b}_{i_0i_1}(l)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i$$

$$+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i\neq i_0}^{m-1} D_i$$

$$+\tilde{P}(l) \prod_{i}^{m-1} D_i$$

$$= 0$$

To solve the OPP reduction, choosing special values for the loop momentum helps a lot

For example, choosing l such that

$$D_0(l^\pm) = D_1(l^\pm) = \\ = D_2(l^\pm) = D_3(l^\pm) = 0$$

sets all the terms in this equation to zero except the first line

There are two (complex) solutions to this equation due to the quadratic nature of the propagators

INTEGRAND REDUCTION

- **In general:**

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$
$$+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}(l)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$
$$+ \sum_{i_0<i_1}^{m-1} \left[b_{i_0i_1} + \tilde{b}_{i_0i_1}(l)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i$$
$$+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i\neq i_0}^{m-1} D_i$$
$$+\tilde{P}(l) \prod_{i}^{m-1} D_i$$
$$= 0$$

Now we choose l such that

$$D_0(l^i) = D_1(l^i) = D_2(l^i) = 0$$

sets all the terms in this equation to zero except the first and second line

Coefficient computed in a previous step

INTEGRAND REDUCTION

- **In general:**

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$

$$+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}(l)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$

$$+ \sum_{i_0<i_1}^{m-1} \left[b_{i_0i_1} + \tilde{b}_{i_0i_1}(l)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i$$

$$+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i\neq i_0}^{m-1} D_i$$

$$+\tilde{P}(l) \prod_{i}^{m-1} D_i$$

$$= 0$$

Now, choosing *l* such that

$$D_0(l^i) = D_1(l^i) = 0$$

sets all the terms in this equation to zero except the first, second and third line

Coefficient computed in a previous step

INTEGRAND REDUCTION

- **In general:**

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$

$$+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}(l)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$

$$+ \sum_{i_0<i_1}^{m-1} \left[b_{i_0i_1} + \tilde{b}_{i_0i_1}(l)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i$$

$$+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i\neq i_0}^{m-1} D_i$$

$$+\tilde{P}(l) \prod_{i}^{m-1} D_i$$

$$= 0$$

Now, choosing l such that

$$D_1(l^i) = 0$$

sets the last line to zero

Coefficient computed in a previous step

- **In general:**

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$

$$+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}(l)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$

$$+ \sum_{i_0<i_1}^{m-1} \left[b_{i_0i_1} + \tilde{b}_{i_0i_1}(l)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i$$

$$+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i\neq i_0}^{m-1} D_i$$

$$+\tilde{P}(l) \prod_{i}^{m-1} D_i$$

$$= 0$$

Now, choosing l such that

$$D_1(l^i) = 0$$

sets the last line to zero

Coefficient computed in a previous step

D-DIMENSIONAL COMPLEX

- The previous expression should in fact be written in d dimensions

$$\int \frac{d^d\bar{l}}{(2\pi)^d} \frac{N(\bar{l},\epsilon)}{\bar{D}_0\bar{D}_1\bar{D}_2\cdots\bar{D}_{m-1}}$$

$$\bar{D}_i = \left(\bar{l}+p_i\right)^2 - m_i^2, \quad p_0 = 0$$

D-DIMENSIONAL COMPLEX

- The previous expression should in fact be written in d dimensions

$$\int \frac{d^d\bar{l}}{(2\pi)^d}\frac{N(\bar{l},\epsilon)}{\bar{D}_0\bar{D}_1\bar{D}_2\cdots\bar{D}_{m-1}}$$

$$\bar{D}_i=\left(\bar{l}+p_i\right)^2-m_i^2,\quad p_0=0$$

- In numerical calculations, it is very convenient to perform the following decomposition

$$\bar{l}^\mu=l^\mu+\tilde{l}^\mu$$

$\bar{l}^\mu$: $d-\mathrm{dim}$; l^μ: $4-\mathrm{dim}$; $\tilde{l}^\mu$: $(-2\epsilon)-\mathrm{dim}$

$$\mu=\underbrace{0,1,2,3,}_{4d\ \mathrm{spacetime}}\underbrace{\cdots,3-2\epsilon}_{(-2\epsilon)d\ \mathrm{space}}$$

4d spacetime: physical; $(-2\epsilon)d$ space: abstract

$$l^\mu=0,\mu\in(-2\epsilon)d\ \mathrm{space}\qquad \tilde{l}^\mu=0,\mu\in 4d\ \mathrm{spacetime}$$

D-DIMENSIONAL COMPLEX

- The previous expression should in fact be written in d dimensions

$$\int \frac{d^d\bar{l}}{(2\pi)^d} \frac{N(\bar{l},\epsilon)}{\bar{D}_0\bar{D}_1\bar{D}_2\cdots\bar{D}_{m-1}}$$

$$\bar{D}_i = \left(\bar{l}+p_i\right)^2 - m_i^2, \quad p_0 = 0$$

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$$\mu = \underbrace{0,1,2,3,}_{4d \text{ spacetime}}\underbrace{\cdots,3-2\epsilon}_{(-2\epsilon)d \text{ space}}$$

4d spacetime: physical; $(-2\epsilon)d$ space: abstract

$$l^\mu = 0, \mu \in (-2\epsilon)d \text{ space} \qquad \tilde{l}^\mu = 0, \mu \in 4d \text{ spacetime}$$

$$N(\bar{l},\epsilon) = N(l) + \tilde{N}(l,\tilde{l},\epsilon)$$

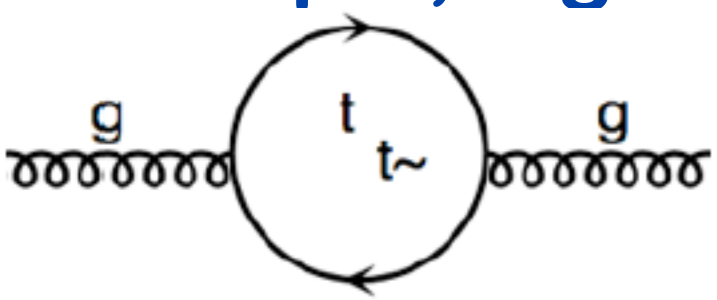
$N(l)$: Suitable for numerical calc. $\tilde{N}(l,\tilde{l},\epsilon)$: Complement with special CT R_2

D-DIMENSIONAL COMPLEX

- **Compute the remaining loop part in terms of rational functions of external momentum invariants and masses**

$$R_2 = \lim_{\epsilon \to 0} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{\tilde{N}(l, \tilde{l}, \epsilon)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}$$

- **For example, a gluon self-energy diagram:**



$$N(\bar{l}, \epsilon) = -2\pi \alpha_s \delta_{ab} \mathrm{Tr}\left[\gamma^\mu \left(\bar{\not{l}} + m_t\right) \gamma^\nu \left(\bar{\not{l}} + \not{p}_g + m_t\right)\right] \varepsilon_\mu \varepsilon_\nu$$

- **After performing some Dirac algebra, we have**

$$\tilde{N}(l, \tilde{l}, \epsilon) = 8\pi \alpha_s \delta_{ab} g^{\mu\nu} \tilde{l}^2 \varepsilon_\mu \varepsilon_\nu$$

- **Using the integration**

$$\int \frac{d^d \bar{l}}{(2\pi)^d} \frac{\tilde{l}^2}{\left(\bar{l}^2 - m_t^2\right)\left((\bar{l} + p_g)^2 - m_t^2\right)} = -\frac{i}{32\pi^2}\left(2m_t^2 - \frac{p_g^2}{3}\right) + \mathcal{O}(\epsilon)$$

- **We have R_2 term**

$$R_2 = -\frac{i\alpha_s}{4\pi}\delta_{ab}\left(2m_t^2 - \frac{p_g^2}{3}\right) g^{\mu\nu} \varepsilon_\mu \varepsilon_\nu$$

D-DIMENSIONAL COMPLEX

- It has been proven that R_2 is only UV related. Therefore, like renormalisation counterterms, they can be reexpressed into R_2 Feynman rules

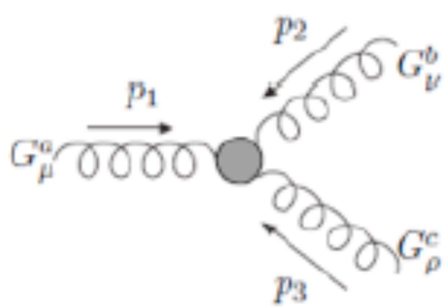
QCD R_2 Feynman Rules

$= \mathrm{Vert}(G^a_\mu, G^b_\nu)$

$= \mathrm{Vert}(Q^i_l, \bar{Q}^j_m)$

$$\mathrm{Vert}(G^a_\mu, G^b_\nu) = \frac{i g_s^2 N_c}{48\pi^2}\,\delta^{ab}\left[\frac{p^2}{2} g_{\mu\nu} + \lambda_{HV}\left(g_{\mu\nu} p^2 - p_\mu p_\nu\right) + \sum_Q \frac{p^2 - 6 m_Q^2}{N_c} g_{\mu\nu}\right]$$

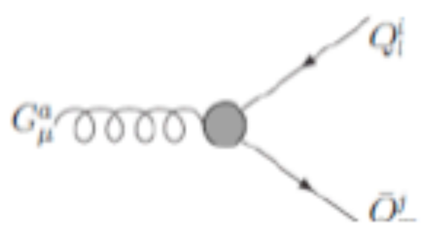
$$\mathrm{Vert}(Q^i_l, \bar{Q}^j_m) = \frac{i g_s^2}{16\pi^2}\,\frac{N_c^2 - 1}{2N_c}\,\delta^{ij}\,\delta_{lm}\,\left(-\not{p} + 2 m_{Q_l}\right)\lambda_{HV}.$$



$= \mathrm{Vert}(G^a_\mu, G^b_\nu, G^c_\rho)$

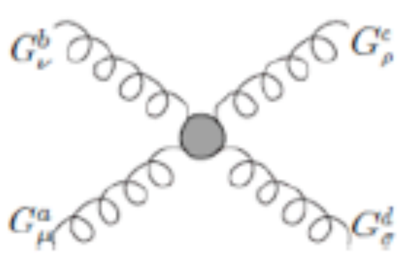
$$\mathrm{Vert}(G^a_\mu, G^b_\nu, G^c_\rho) = -\frac{g_s^3 N_c}{48\pi^2}\left(\frac{7}{4} + \lambda_{HV} + \frac{2N_f}{N_c}\right) f^{abc}\, V_{\mu\nu\rho}(p_1, p_2, p_3)$$

$$V_{\mu\nu\rho}(p_1, p_2, p_3) = g_{\mu\nu}(p_2 - p_1)_\rho + g_{\nu\rho}(p_3 - p_2)_\mu + g_{\rho\mu}(p_1 - p_3)_\nu.$$



$= \mathrm{Vert}(G^a_\mu, Q^i_l, \bar{Q}^j_m)$

$$\mathrm{Vert}(G^a_\mu, Q^i_l, \bar{Q}^j_m) = \delta_{lm}\,\frac{i g_s^3}{16\pi^2}\,T^a_{ji}\,\frac{N_c^2 - 1}{2N_c}\,\gamma_\mu\left(1 + \lambda_{HV}\right)$$



$= \mathrm{Vert}\left(G^a_\mu, G^b_\nu, G^c_\rho, G^d_\sigma\right)$

$$\mathrm{Vert}\left(G^a_\mu, G^b_\nu, G^c_\rho, G^d_\sigma\right) = \frac{i g_s^4}{48\pi^2}\left(C_1\, g_{\mu\nu} g_{\rho\sigma} + C_2\, g_{\mu\rho} g_{\nu\sigma} + C_3\, g_{\mu\sigma} g_{\nu\rho}\right),$$

$$C_1 = Tr(\{T^a, T^b\}\{T^c, T^d\})\,(5N_c + 2\lambda_{HV} N_c + 6N_f)$$
$$-\left(Tr(T^a T^c T^b T^d) + Tr(T^a T^d T^b T^c)\right)(12N_c + 4\lambda_{HV} N_c + 10N_f)$$
$$-\left(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right),\quad C_2 = C_1(b \leftrightarrow c) \qquad C_3 = C_1(b \leftrightarrow d)$$

Draggiotis, Garzelli, Papadopoulos, Pittau (JHEP'09); HSS, Zhang, Chao (JHEP'11)

D-DIMENSIONAL COMPLEX

- In integrand reduction, additional rational terms R_1 are needed !

$$N(l) = \sum_{i_0,i_1,i_2,i_3} (d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}) \prod_{i\neq i_0,i_1,i_2,i_3} D_i$$
$$+ \sum_{i_0,i_1,i_2} (c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}) \prod_{i\neq i_0,i_1,i_2} D_i$$
$$+ \sum_{i_0,i_1} (b_{i_0i_1} + \tilde{b}_{i_0i_1}) \prod_{i\neq i_0,i_1} D_i$$
$$+ \sum_{i_0} (a_{i_0} + \tilde{a}_{i_0}) \prod_{i\neq i_0} D_i$$
$$+ \tilde{P}(l) \prod_i D_i + \mathcal{O}(\varepsilon)$$

$$\frac{1}{D_i} \to \frac{1}{\bar{D}_i} = \frac{1}{D}\left(1 - \frac{\tilde{l}^2}{D_i}\right)$$

integration of this piece gives rise R_1

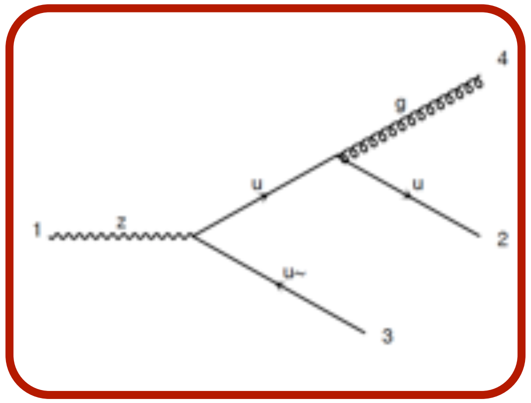
$4d$ couterparts

- **Can be included in OPP reduction**
- **Not needed in TIR reduction**

Lecture 2

NLO Generics

Real



NLO ANATOMY

- **Three parts need to be computed in a NLO calculation**

$$\sigma_{\mathrm{NLO}} = \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\mathcal{V} + \int d\Phi^{(n+1)}\mathcal{R}$$

Born cross section — Virtual correction — Real correction

$$\mathrm{Virtual} = \frac{A}{\epsilon^2} + \frac{B}{\epsilon} + V \qquad \mathrm{Real} = -\frac{A}{\epsilon^2} - \frac{B}{\epsilon} + R$$

NLO ANATOMY

- **Three parts need to be computed in a NLO calculation**

$$\sigma_{\rm NLO} = \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\mathcal{V} + \int d\Phi^{(n+1)}\mathcal{R}$$

Born cross section | Virtual correction | Real correction

$$\text{Virtual} = \frac{A}{\epsilon^2} + \frac{B}{\epsilon} + V \qquad \text{Real} = -\frac{A}{\epsilon^2} - \frac{B}{\epsilon} + R$$

NLO ANATOMY

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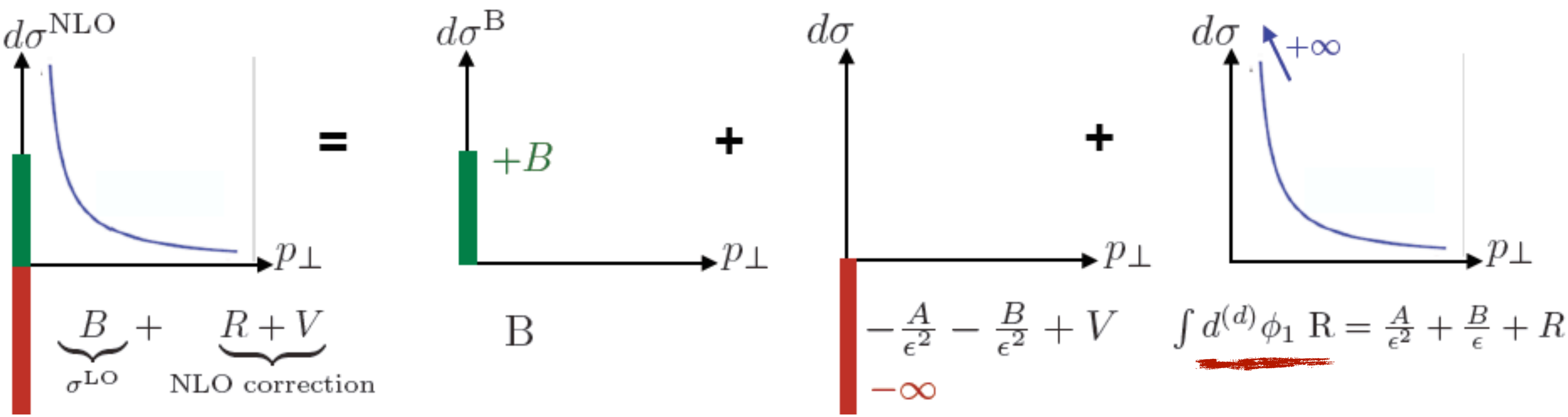
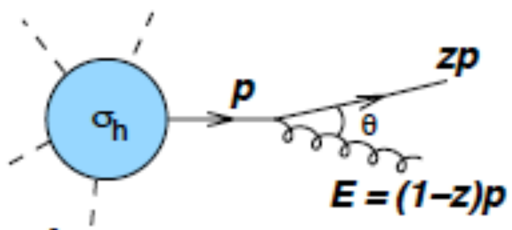
Born cross section | Virtual correction | Real correction

$$\text{Virtual} = \frac{A}{\epsilon^2} + \frac{B}{\epsilon} + V \qquad \text{Real} = -\frac{A}{\epsilon^2} - \frac{B}{\epsilon} + R$$



BRANCHING: TO BE OR NOT TO BE

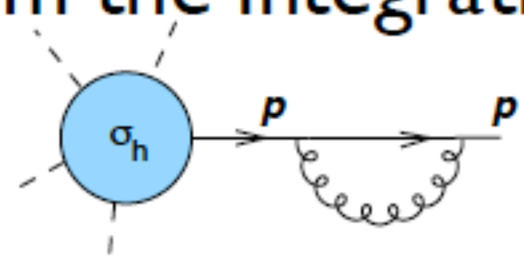
- Let us consider the branching of a gluon from a quark



$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

Where k_t is the transverse momentum of the gluon $k_t = E\sin\theta$. It diverges in the soft ($z \to 1$) and collinear ($k_t \to 0$) region

- These singularities cancel with the virtual contribution, which comes from the integration of the loop momentum



$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

- The cancelation happens if we cannot distinguish between the case of no branching, and of a soft or collinear branching

IR SAFETY

- In order to have meaningful fixed-order predictions in perturbation theory, observables must be IR-safe, i.e. not sensitive to the emission of soft/collinear partons

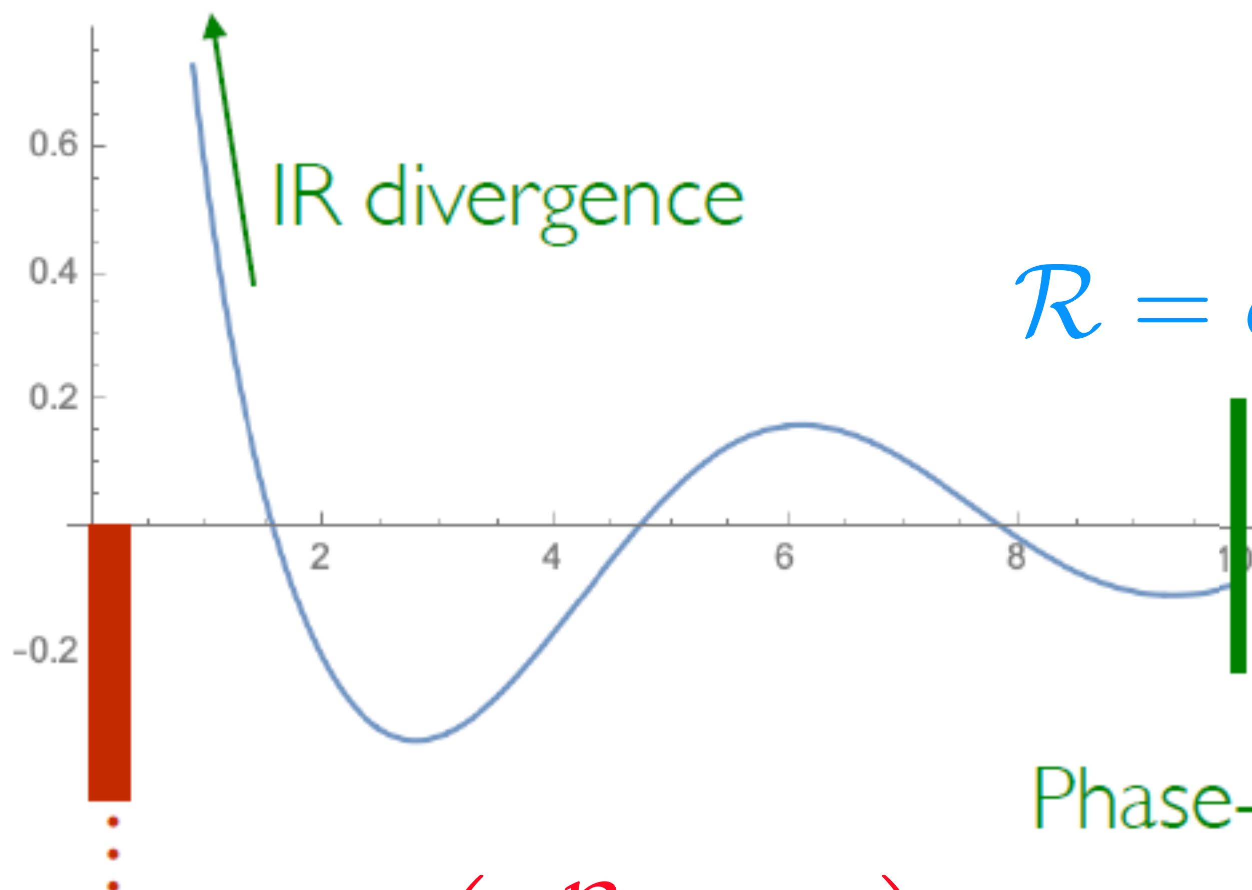
$$\lim_{p_i||p_j} \mathcal{O}(1,\cdots,i,\cdots,j-1,j,j+1,\cdots,n)=\mathcal{O}(1,\cdots,ij,\cdots,j-1,j+1,\cdots,n)$$

$$\lim_{p_i\to 0} \mathcal{O}(1,\cdots,i-1,i,i+1,\cdots,n)=\mathcal{O}(1,\cdots,i-1,i+1,\cdots,n)$$

- For example,
  - The number of gluons is NOT IR safe.
  - The leading p_T/energy particle is NOT IR safe (soft or collinear unsafe ?).
  - The colour in a given cone is NOT IR safe (soft or collinear unsafe ?).
  - The transverse energy sum is IR safe.

A TOY EXAMPLE

- **Assuming the phase space integration can be casted into a one-dimensional case** $x \in [0,1]$ **:**



$$\mathcal{R} = \delta(x)\frac{\alpha_X}{\pi}\frac{R(x)}{x}$$

$x\ (\times 10^{-1})$

Phase-space boundary

$$\delta(x)\mathcal{V} = \delta(x)\frac{\alpha_X}{\pi}\left(\frac{\mathcal{B}}{2\epsilon_{\mathrm{IR}}} + V\right)$$

$\forall\ \mathcal{O}(x)$ IR safety $\lim_{x\to 0}\mathcal{O}(x)R(x) = \mathcal{O}(0)\mathcal{B}$

A TOY EXAMPLE

$$\mathcal{O}(0)\mathcal{V} + \int_0^1 dx x^{-2\epsilon_{\rm IR}} \mathcal{O}(x)\mathcal{R}$$

Dimensionally regularise in x !

$$= \frac{\alpha_X}{\pi}\left[\mathcal{O}(0)\left(\frac{\mathcal{B}}{2\epsilon_{\rm IR}} + V\right) + \int_0^1 dx x^{-1-2\epsilon_{\rm IR}}\mathcal{O}(x)R(x)\right]$$

$$= \frac{\alpha_X}{\pi}\left[\mathcal{O}(0)\left(\frac{\mathcal{B}}{2\epsilon_{\rm IR}} + V\right) + \left(-\mathcal{O}(0)\frac{\mathcal{B}}{2\epsilon_{\rm IR}} + \int_0^1 dx \left(\frac{1}{x}\right)_+ \mathcal{O}(x)R(x)\right)\right]$$

$$= \frac{\alpha_X}{\pi}\left[\mathcal{O}(0)V + \int_0^1 dx \left(\frac{1}{x}\right)_+ \mathcal{O}(x)R(x)\right]$$

- **We have used:**

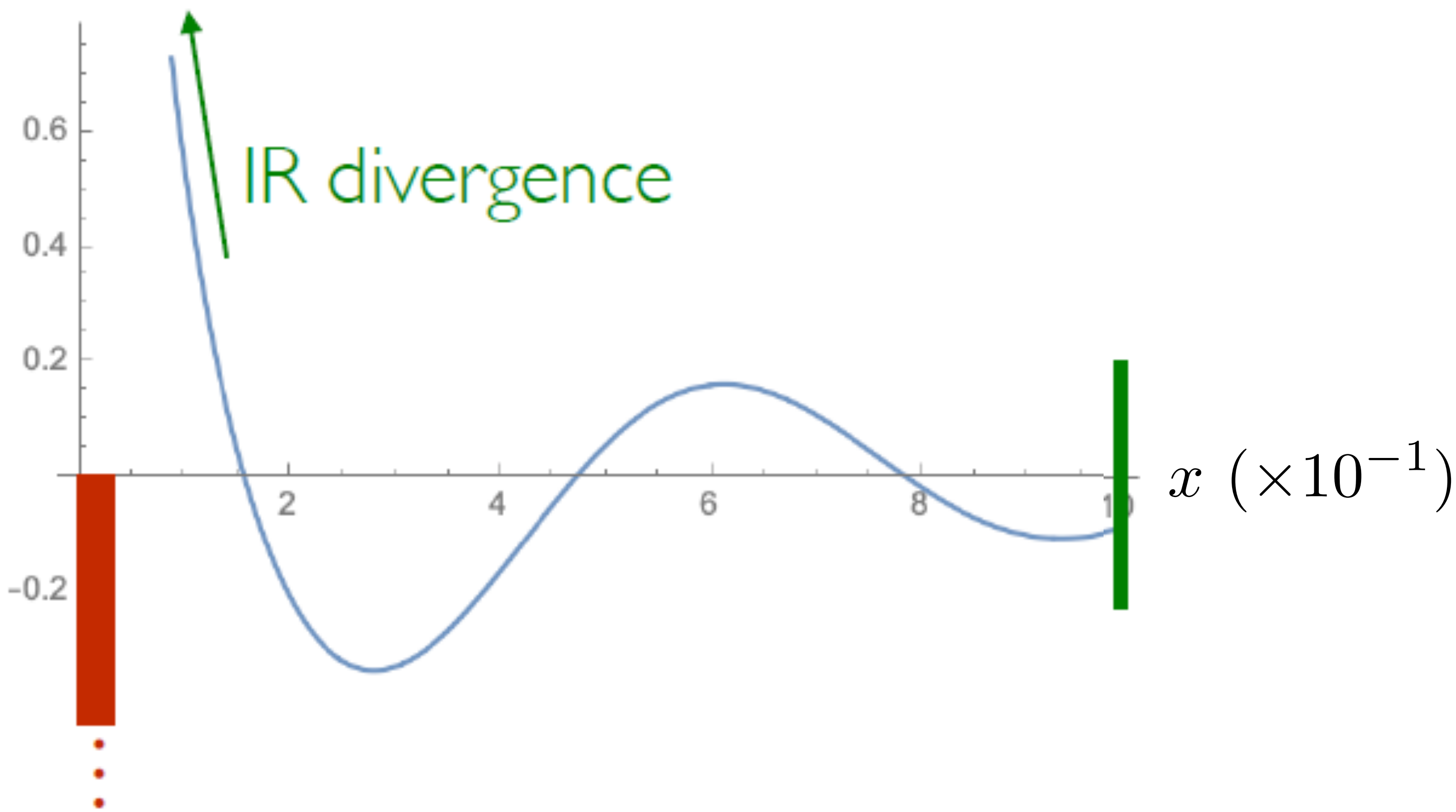
$$x^{-1-2\epsilon_{\rm IR}} = -\frac{1}{2\epsilon_{\rm IR}}\delta(x) + \left(\frac{1}{x}\right)_+ + \epsilon_{\rm IR} \text{ term}$$

$$\left(\frac{1}{x}\right)_+ f(x) \equiv \frac{f(x) - f(0)}{x} \qquad \forall\ f(x)$$

PHASE-SPACE SLICING

- **In general, the phase-space integration over real matrix element is very hard. Dedicated general approaches are developed !**
  - **Phase-space slicing**

$$\int_0^1 dx x^{-1-2\epsilon_{\rm IR}} \mathcal{O}(x) R(x)$$



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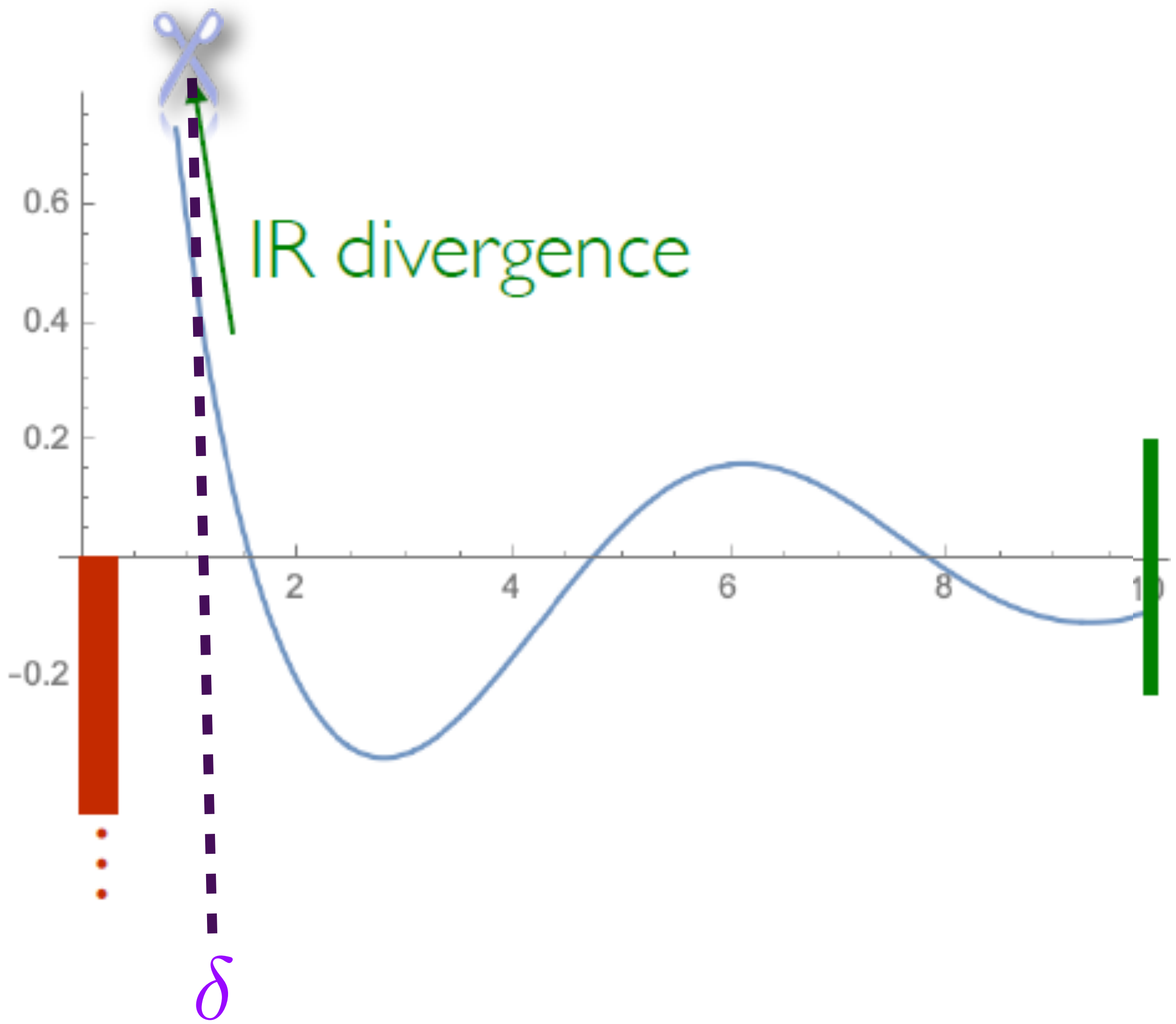

$$\int_0^1 dx x^{-1-2\epsilon_{\rm IR}}\mathcal{O}(x)R(x)$$

$$= \left(\int_0^\delta + \int_\delta^1\right) dx x^{-1-2\epsilon_{\rm IR}}\mathcal{O}(x)R(x)$$

$$\stackrel{\delta\to 0}{=} \left(-\mathcal{O}(0)\frac{\mathcal{B}}{2\epsilon_{\rm IR}}\delta^{-2\epsilon_{\rm IR}}\right) + \left(\int_\delta^1 dx x^{-1}\mathcal{O}(x)R(x) + \epsilon_{\rm IR}\ \text{term}\right)$$

$$\stackrel{\delta\to 0}{=} -\mathcal{O}(0)\mathcal{B}\left(\frac{1}{2\epsilon_{\rm IR}} - \log\delta\right) + \int_\delta^1 dx x^{-1}\mathcal{O}(x)R(x)$$

Tuesday, November 19, 19

PHASE-SPACE SLICING

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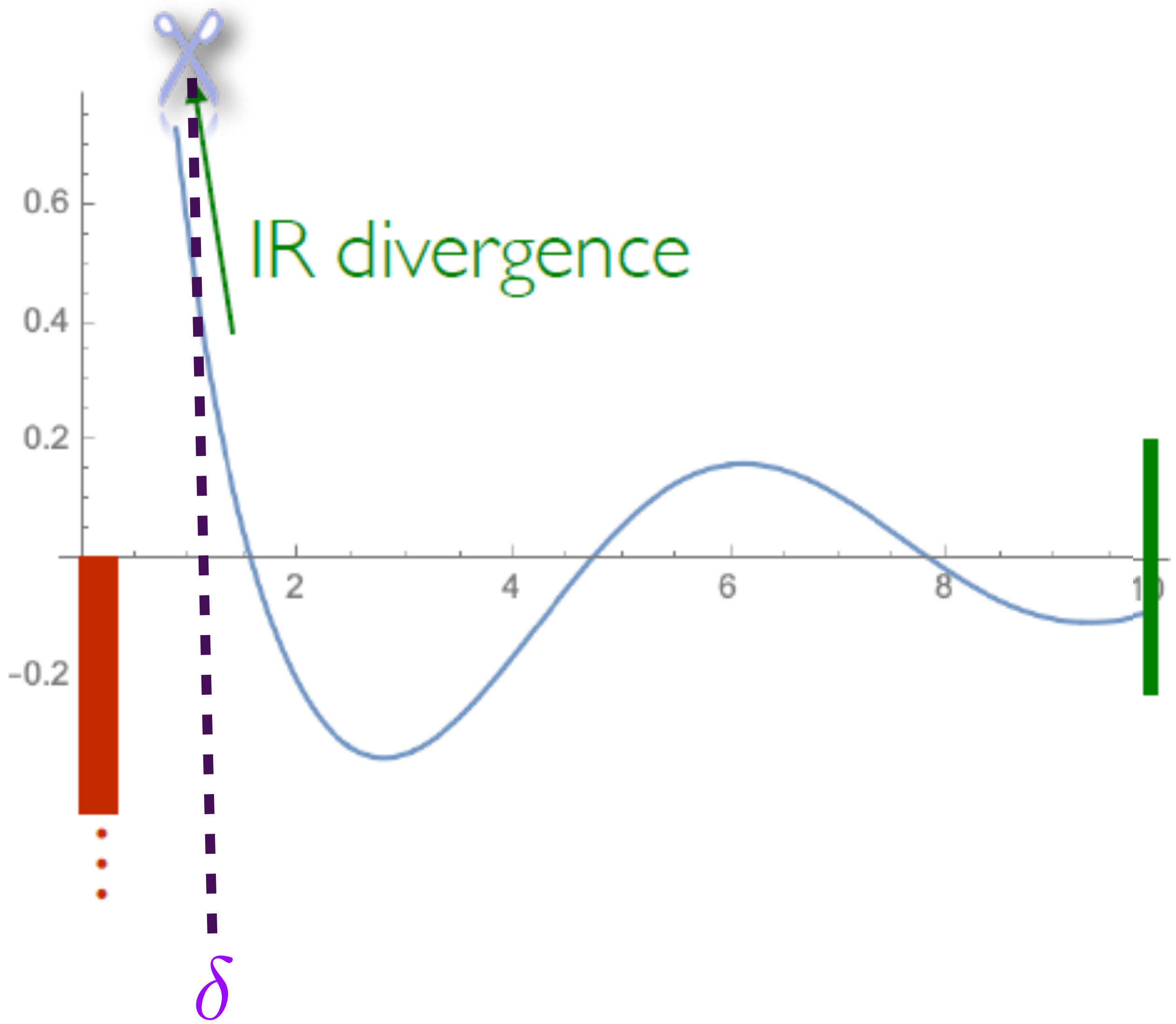

finite integral
(can be computed numerically)

$$\int_0^1 dx x^{-1-2\epsilon_{\rm IR}} \mathcal{O}(x) R(x)$$

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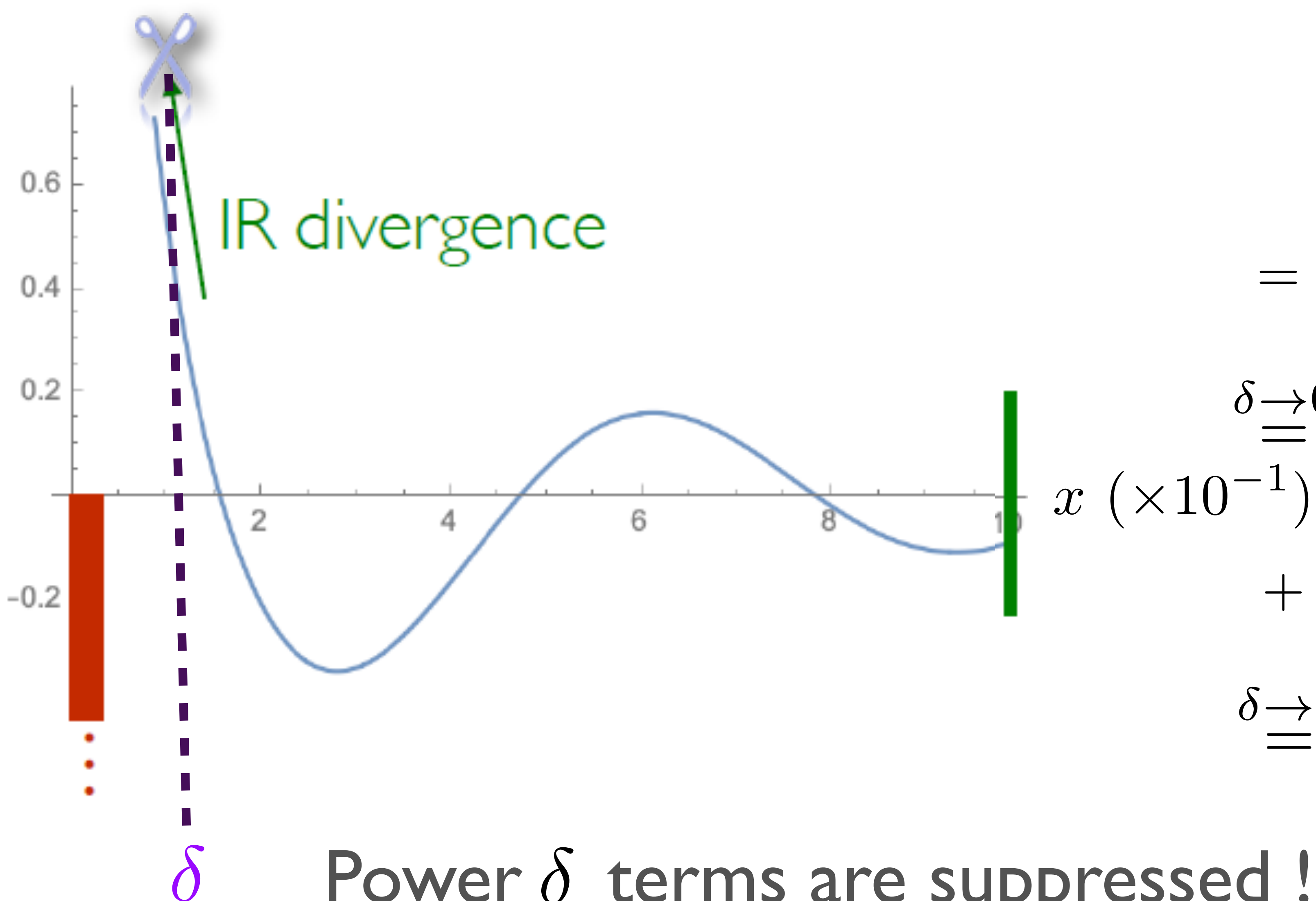

finite integral
(can be computed numerically)

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$$=\left(\int_0^\delta+\int_\delta^1\right)dx\,x^{-1-2\epsilon_{\rm IR}}\mathcal{O}(x)R(x)$$

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$$\stackrel{\delta\to 0}{=}-\mathcal{O}(0)\mathcal{B}\left(\frac{1}{2\epsilon_{\rm IR}}-\log\delta\right)+\int_\delta^1 dx\,x^{-1}\mathcal{O}(x)R(x)$$

Power δ terms are suppressed !
Large numerical cancellations !

SUBTRACTION

- **In general, the phase-space integration over real matrix element is very hard. Dedicated general approaches are developed !**
  - **Subtraction method**
    - Find a generic simple function S has exactly same IR singularity as real matrix element

$$\lim_{p_i||p_j} \mathcal{O}(x)S = \lim_{p_i||p_j} \mathcal{O}(x)\mathcal{R} \qquad \lim_{p_i\to 0} \mathcal{O}(x)S = \lim_{p_i\to 0} \mathcal{O}(x)\mathcal{R}$$

    - ... but much easier to integrate analytically.

$$\mathcal{O}(0)\mathcal{V} + \int_0^1 dx x^{-2\epsilon_{\rm IR}} \mathcal{O}(x)\mathcal{R}$$
$$= \left(\mathcal{O}(0)\mathcal{V} + \int_0^1 dx x^{-2\epsilon_{\rm IR}} \mathcal{O}(x)S\right) + \int_0^1 dx x^{-2\epsilon_{\rm IR}} \mathcal{O}(x)\left(\mathcal{R} - S\right)$$

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$$= \left(\mathcal{O}(0)\mathcal{V} + \boxed{\int_0^1 dx x^{-2\epsilon_{\rm IR}} \mathcal{O}(x)S}\right) + \int_0^1 dx x^{-2\epsilon_{\rm IR}} \mathcal{O}(x)\left(\mathcal{R} - S\right)$$

Finite | Finite

Analytically known

SUBTRACTION

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Finite — Finite

Analytically known — Integrating numerically in 4d

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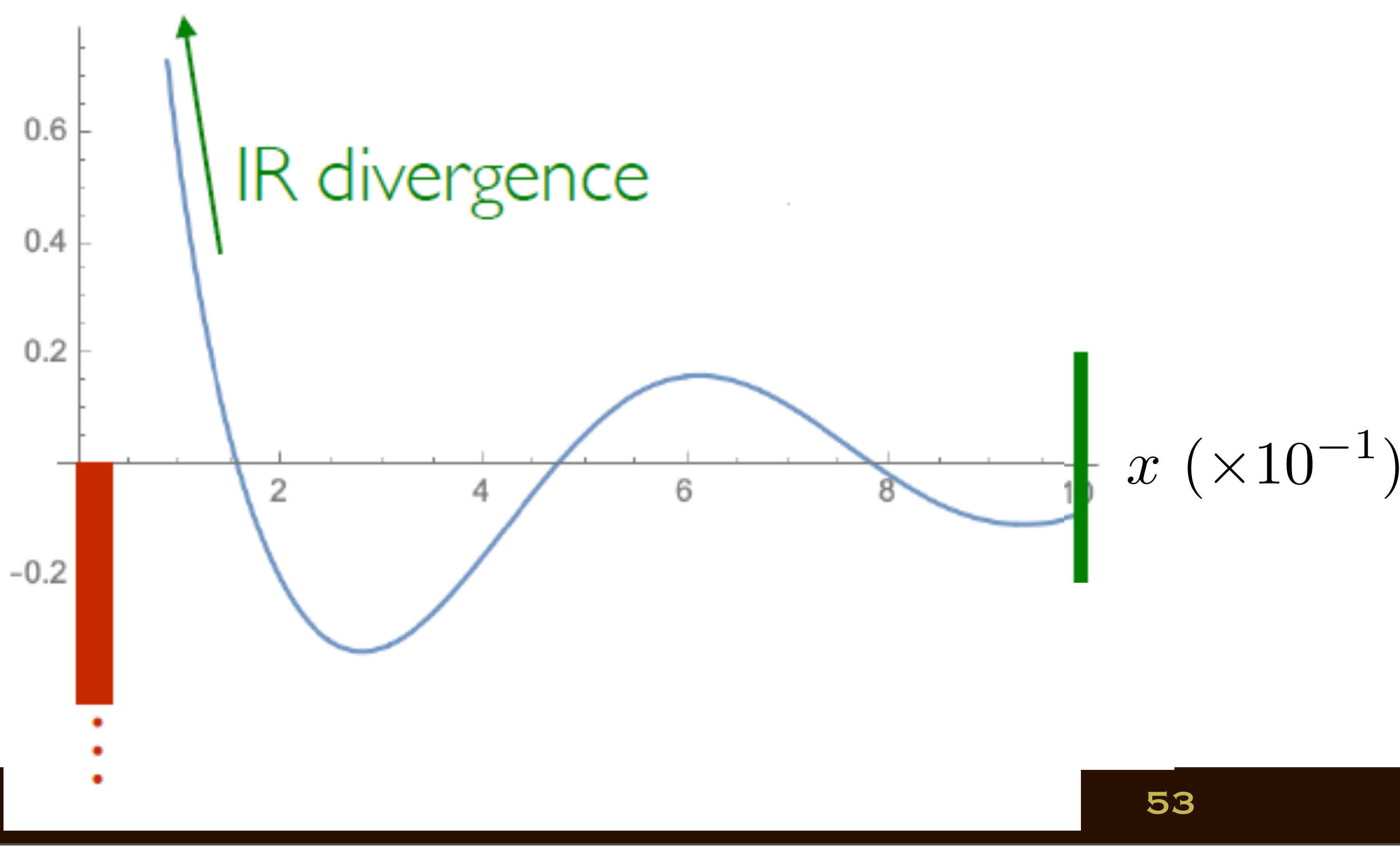

SUBTRACTION

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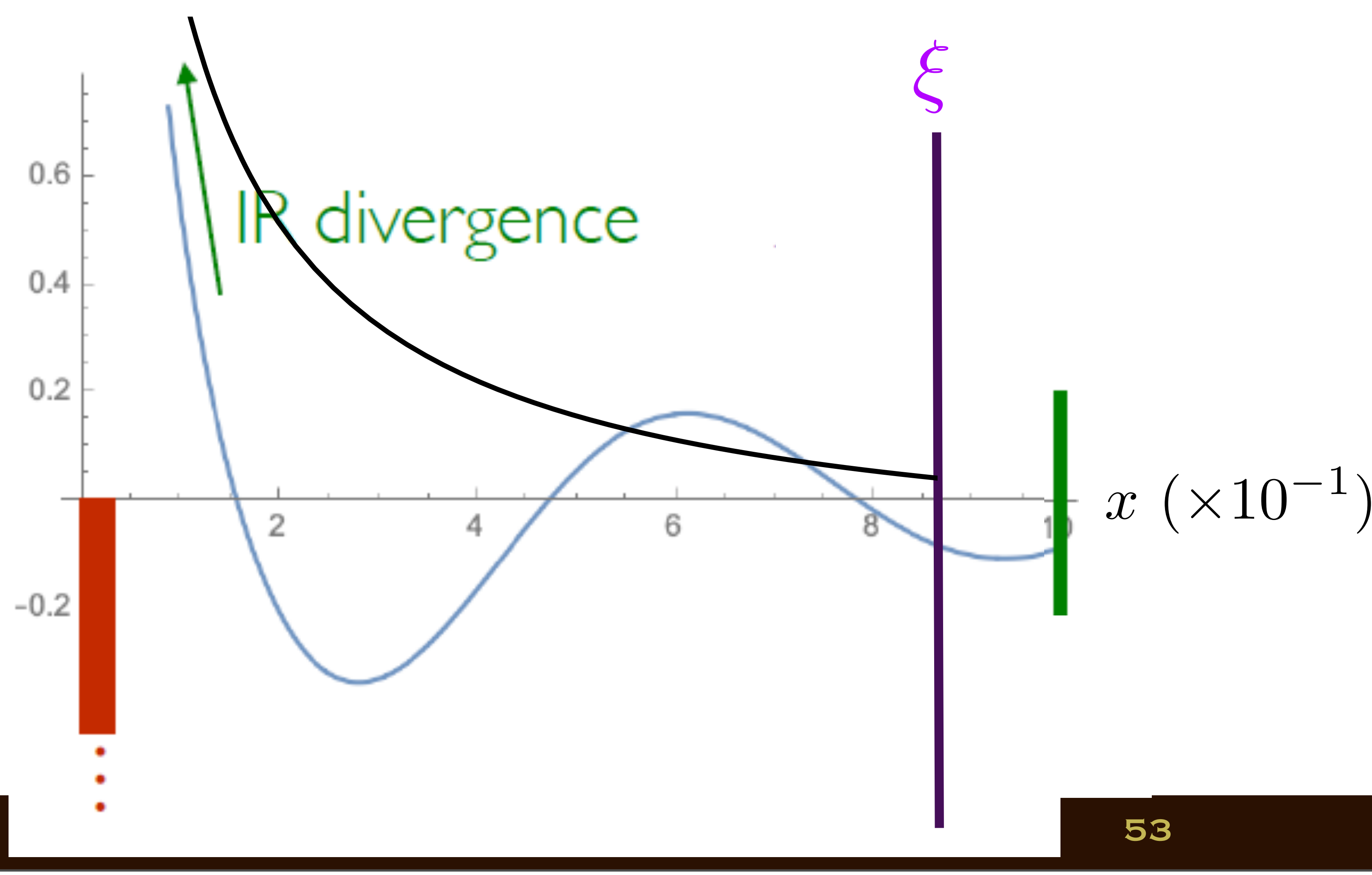
  - **Subtraction method**
    - In above toy example

$$S = \frac{\alpha_X}{\pi}\frac{\mathcal{B}\Theta(\xi - x)}{x} \qquad \forall\ \xi \in\ (0,1]$$



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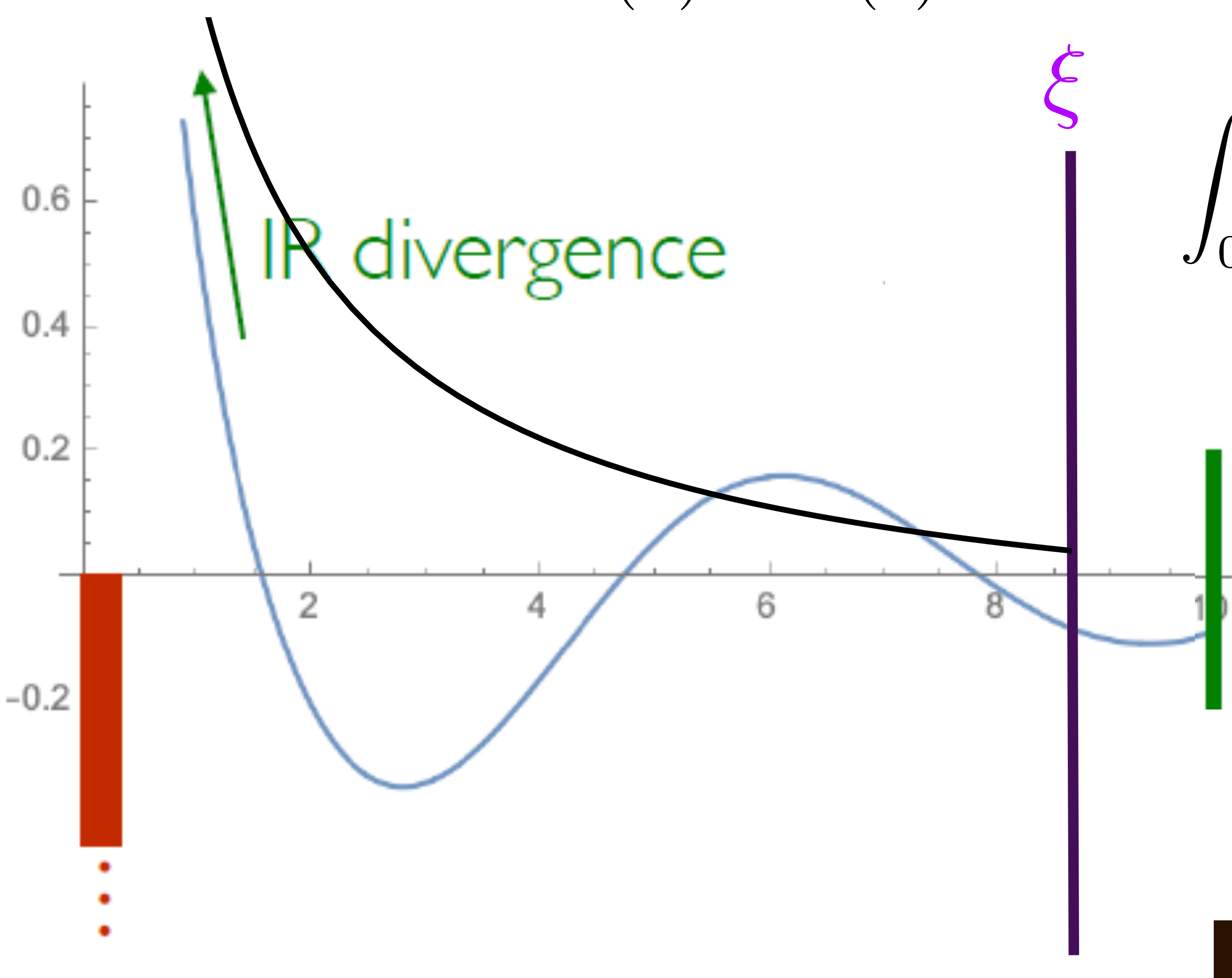

$x\ (\times 10^{-1})$

$$\int_0^1 dx x^{-2\epsilon_{\rm IR}} \mathcal{O}(x) S = -\frac{\alpha_X}{\pi}\mathcal{O}(0)\mathcal{B}\left(\frac{1}{2\epsilon_{\rm IR}} - \log\xi\right)$$

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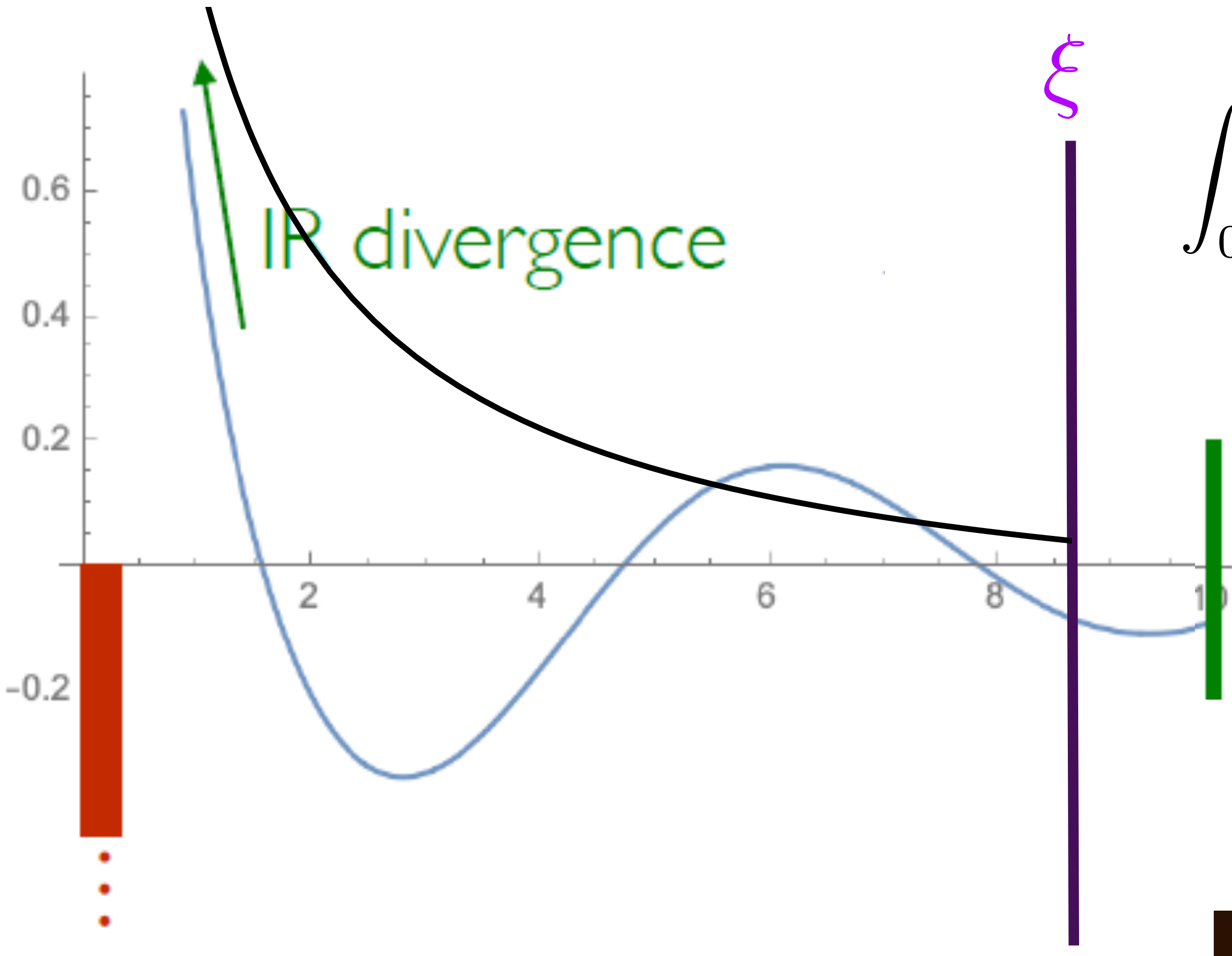


$$\int_0^1 dx x^{-2\epsilon_{\rm IR}}\mathcal{O}(x)S = -\frac{\alpha_X}{\pi}\mathcal{O}(0)\mathcal{B}\left(\frac{1}{2\epsilon_{\rm IR}} - \log\xi\right)$$

No approximation !

Numerical cancellations mitigated !

$x\ (\times 10^{-1})$

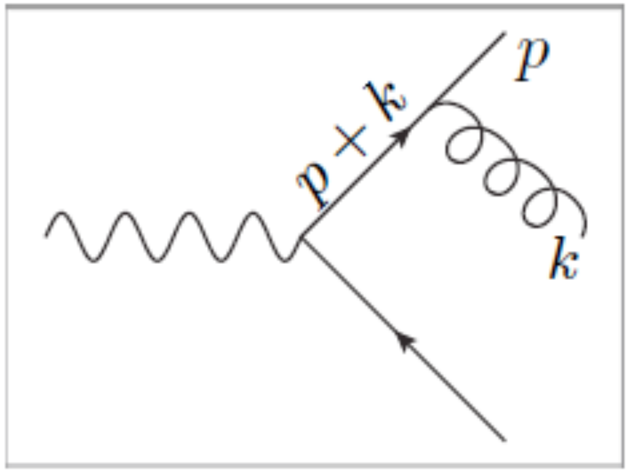
NLO SUBTRACTION

- **Master formula:**

$$\sigma_{\rm NLO} = \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\mathcal{V} + \int d\Phi^{(n+1)}\mathcal{R}$$

$$= \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\left[\mathcal{V} + \int d\Phi^{(1)}S\right] + \int d\Phi^{(n+1)}\left[\mathcal{R} - S\right]$$

- **The subtraction counterterm S should be chosen:**
  - It exactly matches the singular behaviour of real ME
  - It can be integrated numerically in a convenient way
  - It can be integrated exactly in d dimension
  - It is process independent (overall factor times Born ME)
- **In gauge theory, the singular structure is universal**



$$(p+k)^2 = 2E_pE_k(1-\cos\theta_{pk})$$

- Collinear singularity:

$$\lim_{p//k} |M_{n+1}|^2 \simeq |M_n|^2\, P^{AP}(z)$$

- Soft singularity:

$$\lim_{k\to 0} |M_{n+1}|^2 \simeq \sum_{ij} |M_n^{ij}|^2 \frac{p_i p_j}{p_i k\; p_j k}$$

TWO WIDELY-USED SUBTRACTION METHODS

Dipole subtraction

Catani, Seymour, hep-ph/9602277 & hep-ph/9605323

- Most used method
- Recoil taken by one parton $\rightarrow N^3$ scaling
- Method evolves from cancelation of soft divergences
- Proven to work for simple and complicated processes
- Automated in MadDipole, AutoDipole, Sherpa, Helac-NLO, ...

FKS subtraction

Frixione, Kunszt, Signer, hep-ph/9512328

- Less known method
- Recoil distributed among all particles $\rightarrow N^2$ scaling
- Probably (?) more efficient because less subtraction terms are needed
- Method evolves from cancelation of collinear divergences
- Proven to work for simple and complicated processes
- Automated in MadGraph5_aMC@NLO and in the Powheg box/Powhel

FKS SUBTRACTION

- **The real ME singular as**

$$\mathcal{R} \xrightarrow{\text{IR limit}} \frac{1}{\xi_i}\frac{1}{1-y_{ij}} \qquad \xi_i = \frac{E_i}{\sqrt{\hat{s}}} \qquad y_{ij} = \cos\theta_{ij}$$

- **Partition the phase space in order to have at most one soft and/or one collinear singularity**

$$\mathcal{R}d\Phi^{(n+1)} = \sum_{ij} S_{ij}\mathcal{R}d\Phi^{(n+1)} \qquad \sum_{ij} S_{ij} = 1$$

$$S_{ij} \to 1 \text{ if } p_i \cdot p_j \to 0$$

$$S_{ij} \to 0 \text{ if } p_m \cdot p_n \to 0,\ \{m,n\} \neq \{i,j\}$$

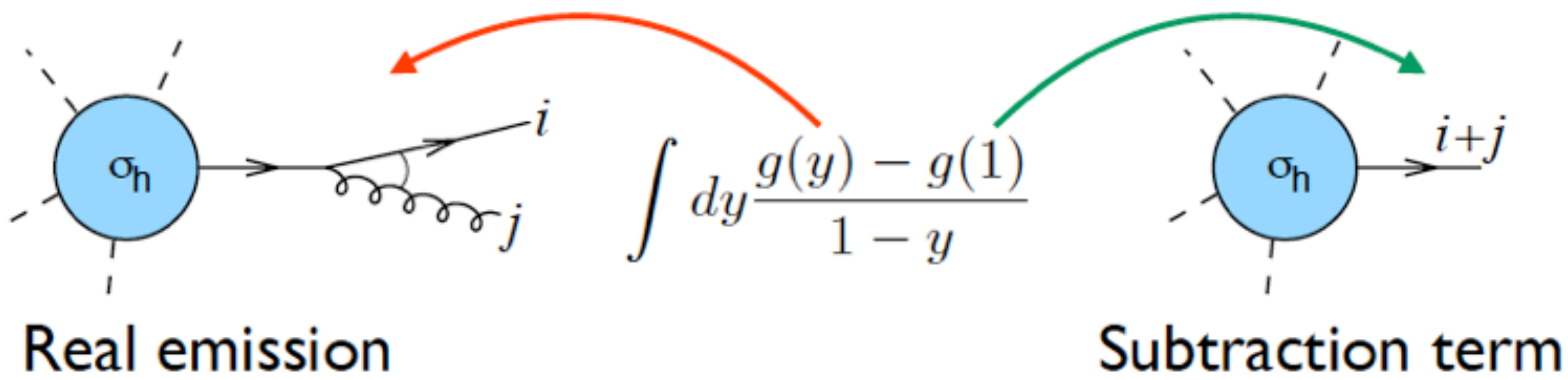
- **Use plus prescriptions to subtract the divergences**

$$d\sigma_{\tilde{R}} = \sum_{ij}\left(\frac{1}{\xi_i}\right)_+ \left(\frac{1}{1-y_{ij}}\right)_+ \xi_i\,(1-y_{ij})\,S_{ij}\mathcal{R}d\Phi^{(n+1)}$$

$$\int d\xi \left(\frac{1}{\xi}\right)_+ f(\xi) = \int d\xi \frac{f(\xi)-f(0)}{\xi} \qquad \int dy \left(\frac{1}{1-y}\right)_+ g(y) = \int dy \frac{g(y)-g(1)}{1-y}$$

FKS SUBTRACTION

- **Counterevents:**
  - Soft counterevent ($\xi_i \to 0$)
  - Collinear counterevents ($y_{ij} \to 1$)
  - Soft-collinear counterevents ($\xi_i \to 0$ and $y_{ij} \to 1$)

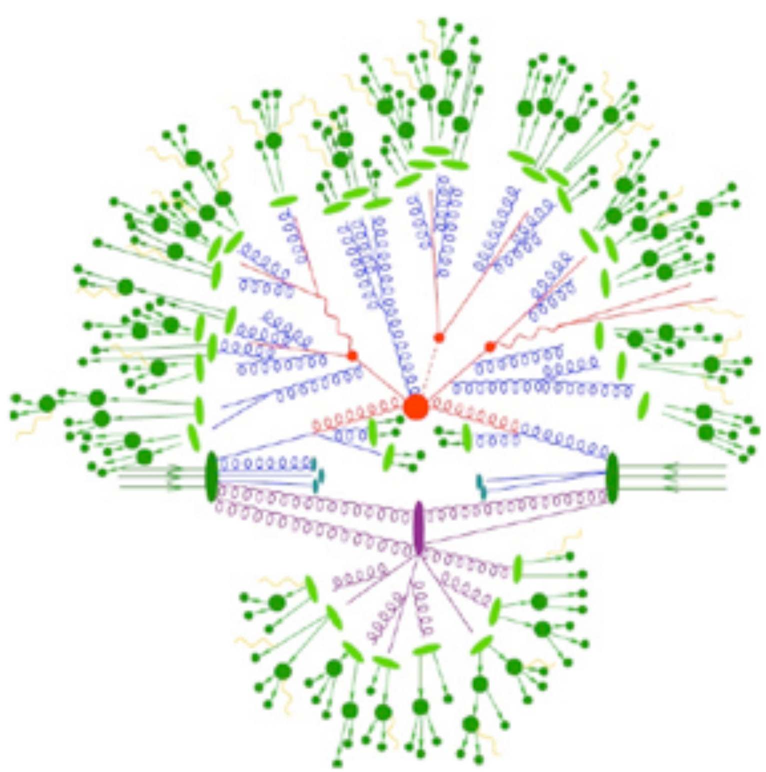


  - If i and j are on-shell in the event, for the counterevent the combined particle $i+j$ must be on shell
  - $i+j$ can be put on shell only be reshuffling the momenta of the other particles
  - It can happen that event and counterevent end up in different histogram bins
    - Use IR-safe observables and don't ask for infinite resolution!

Lecture 2

NLO Generics

NLO+PS

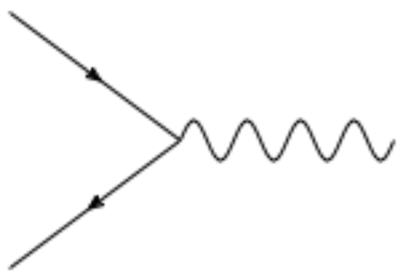


WHY MATCH TO PARTON SHOWERS ?

- Parton showers evolve hard partons by emitting extra quanta down to a more realistic final states (made of hadrons)
- They resum the large logarithms appearing in the phase-space corners, which complement with fixed order.
- A fully exclusive description of the event is available
- Only after matching to parton showers, the NLO unweighted events can be generated.
  - Higher efficiency in particular for time-consuming simulations (e.g. detector)
- NLO calculations are inclusive (though fully-differential), but provide the first reliable estimate of rates and uncertainties.

A CAVEAT IN DOUBLE COUNTING

- **Matching to parton showers:** avoid double counting



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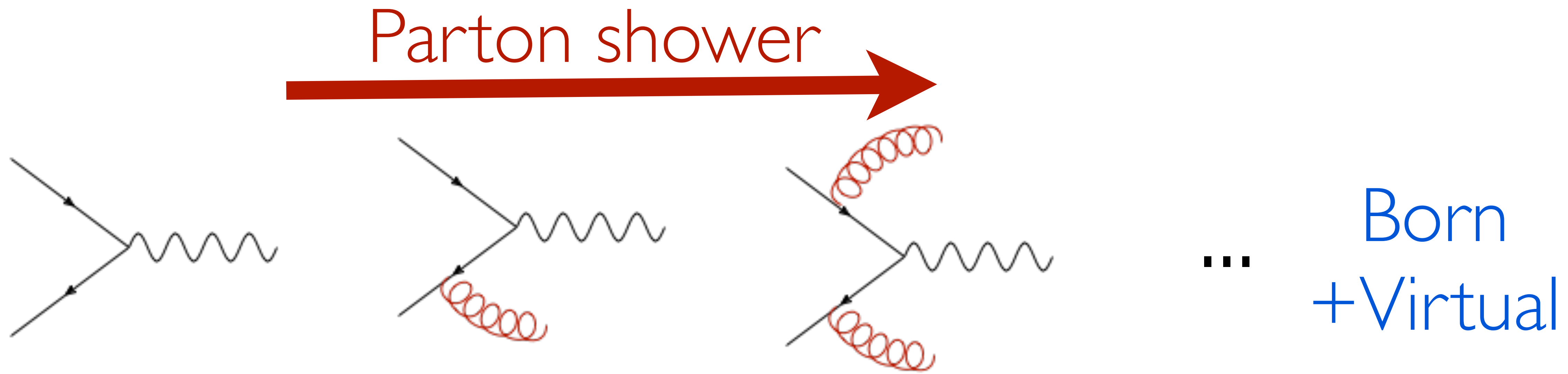

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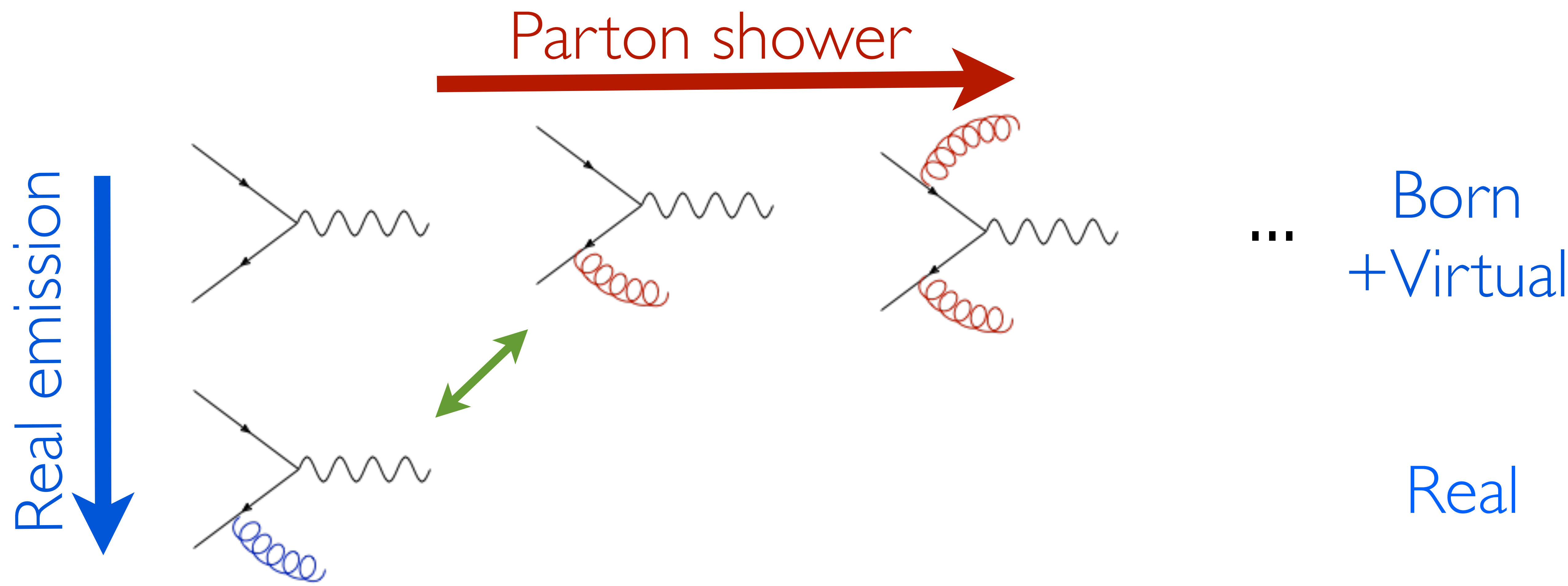

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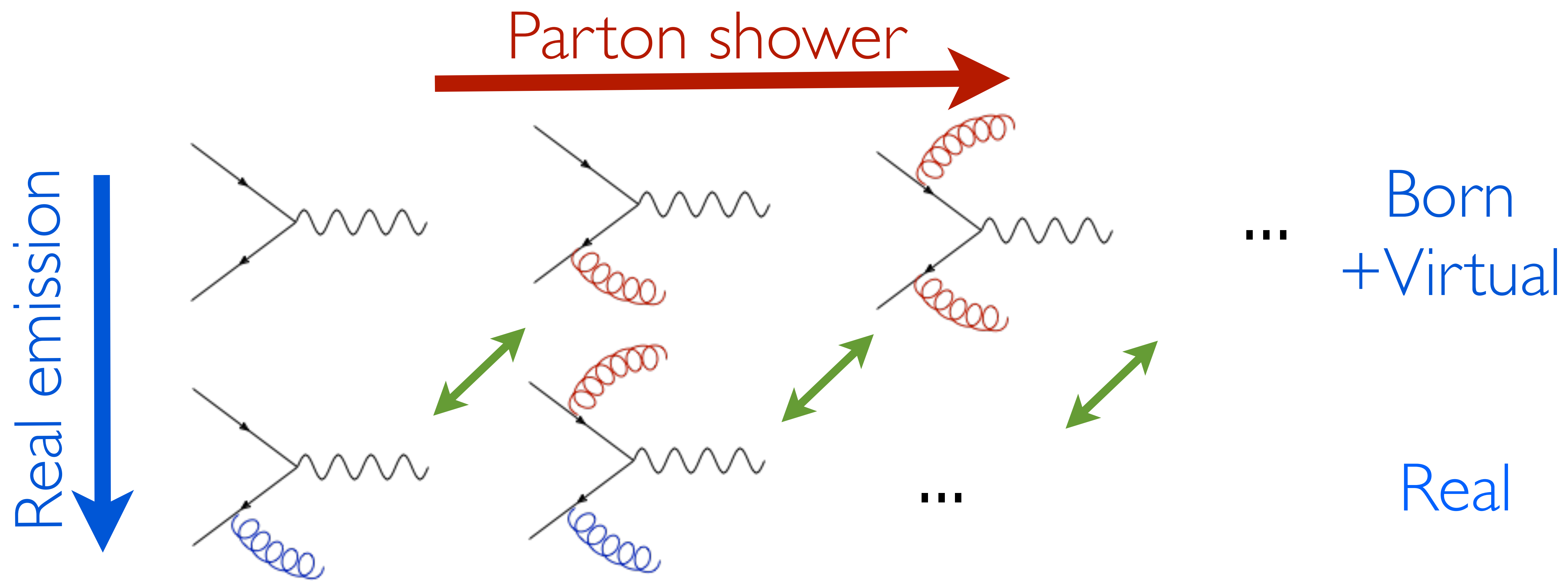

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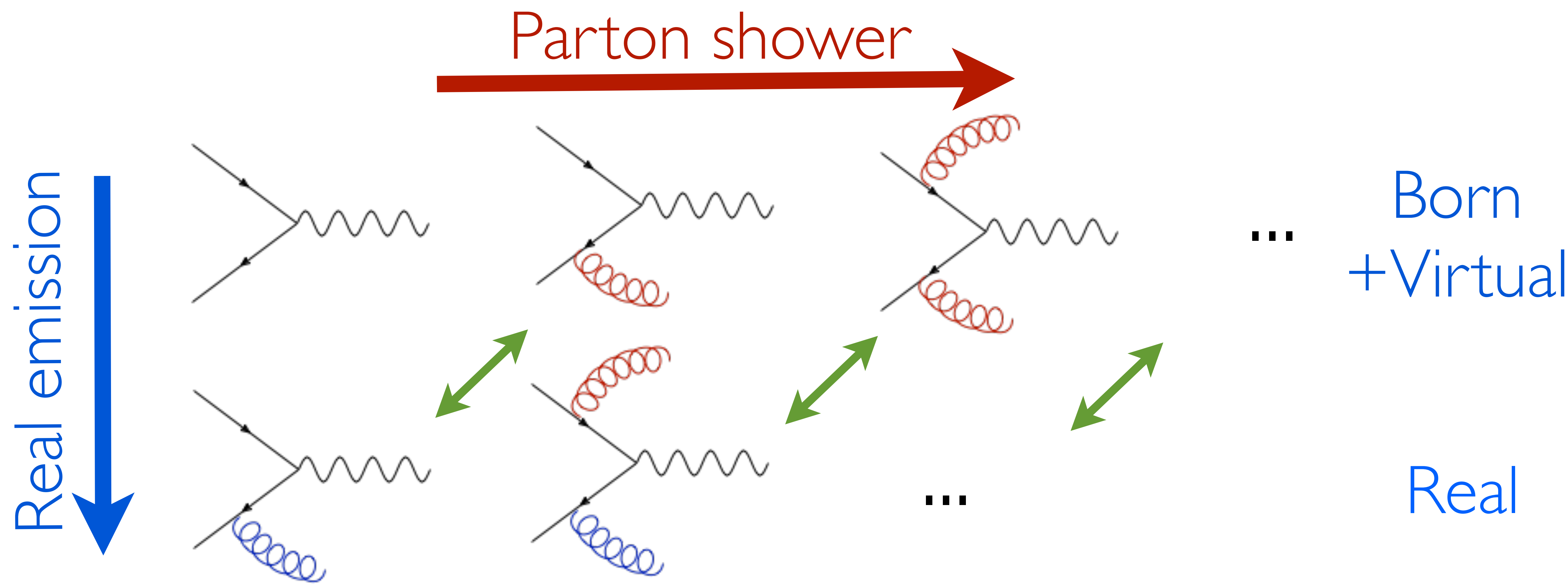

- **Double couting between real emission and parton shower**
- **Double couting between virtual corrections and the non-emission probability via the Sudakov factor in parton shower**

A CAVEAT IN DOUBLE COUNTING

- Like LO, let us wrongly generate events separately from Born, virtual and real parts, and then pass these events to a parton shower:

$$d\sigma^{\rm naive}_{\rm NLO+PS} = \left[\mathcal{B}+\mathcal{V}\right] d\Phi^{(n)} I^{(n)}_{\rm MC} + \mathcal{R} d\Phi^{(n+1)} I^{(n+1)}_{\rm MC}$$

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Parton shower operators

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- Because of unitarity of parton shower, we should get full NLO cross section after expanding PS operators

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- Because of unitarity of parton shower, we should get full NLO cross section after expanding PS operators
  - Let us check ...

$$I_{\text{MC}} = \Delta_a + \Delta_a d\Phi^{(1)} \frac{\alpha_s}{2\pi} P_{a\to bc}$$

$$\Delta_a = \exp\left(-\int d\Phi^{(1)} \frac{\alpha_s}{2\pi} P_{a\to bc}\right) = 1 - \int d\Phi^{(1)} \frac{\alpha_s}{2\pi} P_{a\to bc} + \mathcal{O}(\alpha_s^2)$$

$$I_{\text{MC}} = \left(1 - \int d\Phi^{(1)} \frac{\alpha_s}{2\pi} P_{a\to bc}\right) + d\Phi^{(1)} \frac{\alpha_s}{2\pi} P_{a\to bc} + \mathcal{O}(\alpha_s^2)$$

$$d\sigma^{\text{naive}}_{\text{NLO+PS}} = (\mathcal{B}+\mathcal{V})\, d\Phi^{(n)} + \mathcal{R} d\Phi^{(n+1)}$$

$$+\mathcal{B} d\Phi^{(n)} \left(d\Phi^{(1)} \frac{\alpha_s}{2\pi} P_{a\to bc} - \int d\Phi^{(1)} \frac{\alpha_s}{2\pi} P_{a\to bc}\right) + \mathcal{O}(\alpha_s^{b+2})$$

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$$d\sigma^{\text{naive}}_{\text{NLO+PS}} = (\mathcal{B}+\mathcal{V})\, d\Phi^{(n)} + \mathcal{R} d\Phi^{(n+1)}$$
$$+\mathcal{B} d\Phi^{(n)} \left(d\Phi^{(1)} \frac{\alpha_s}{2\pi} P_{a\to bc} - \int d\Phi^{(1)} \frac{\alpha_s}{2\pi} P_{a\to bc}\right) + \mathcal{O}(\alpha_s^{b+2}) \neq d\sigma_{\text{NLO}} + \mathcal{O}(\alpha_s^{b+2})$$

MC@NLO

Frixione, Webber JHEP'02

- **In the MC@NLO formalism, double counting can be cured by the so-called Monte Carlo counterterms**

$$\Delta = \exp\left(-\int d\Phi^{(1)} MC\right)$$

$$I_{\rm MC} = \Delta + \Delta d\Phi^{(1)} MC = 1 - \int d\Phi^{(1)} MC + d\Phi^{(1)} MC + \mathcal{O}(\alpha_s^2)$$

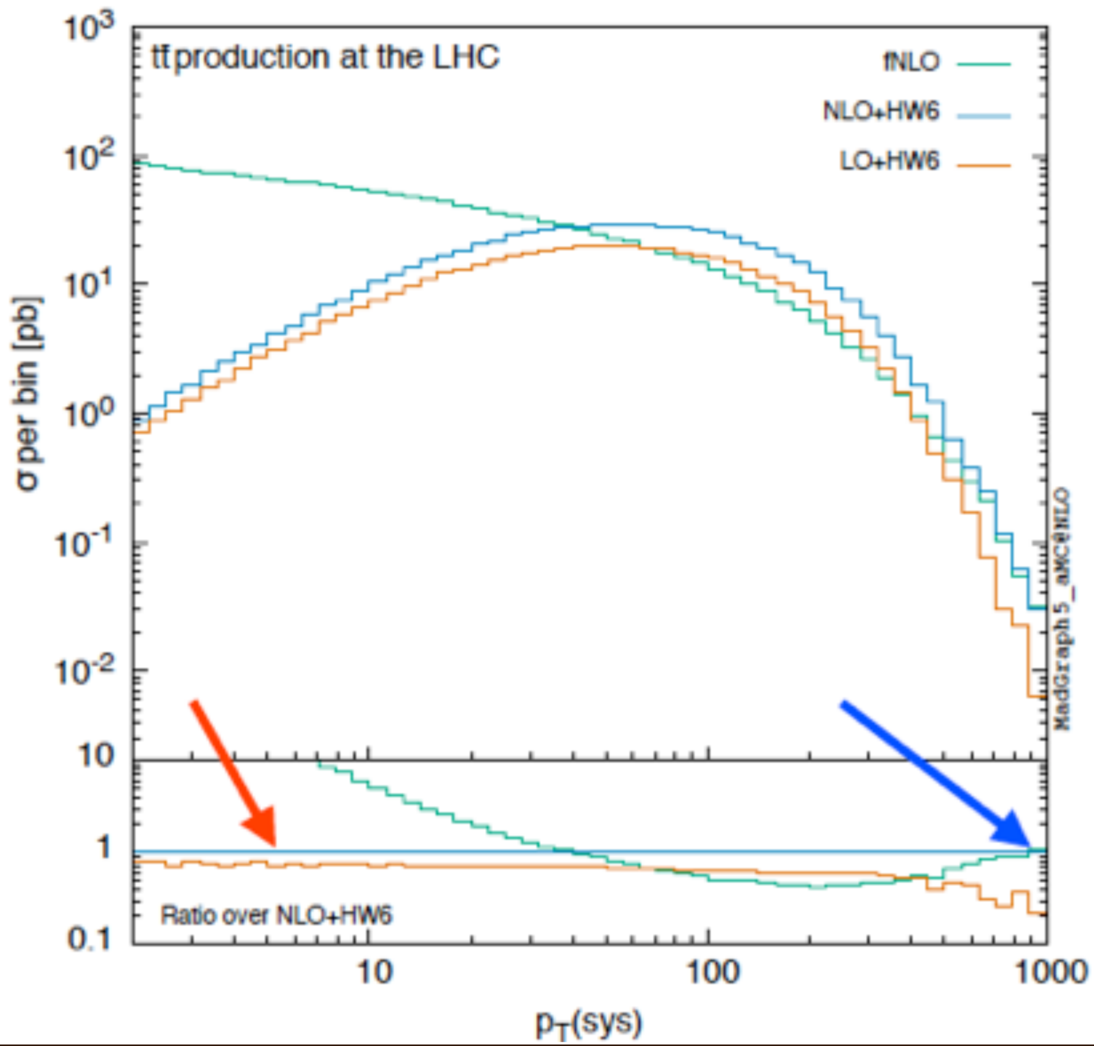
- **The MC@NLO cross section is:**

$$d\sigma^{\rm MC@NLO}_{\rm NLO+PS} = \left(\mathcal{B} + \mathcal{V} + \mathcal{B}\int d\Phi^{(1)} MC\right) d\Phi^{(n)} I^{(n)}_{\rm MC} + \left(\mathcal{R} - \mathcal{B} MC\right) d\Phi^{(n+1)} I^{(n+1)}_{\rm MC}$$

- **Expanding the Sudakov up to NLO:**

$$\begin{aligned} d\sigma^{\rm MC@NLO}_{\rm NLO+PS} &= \left(\mathcal{B} + \mathcal{V} + \mathcal{B}\int d\Phi^{(1)} MC\right) d\Phi^{(n)} + \left(\mathcal{R} - \mathcal{B} MC\right) d\Phi^{(n+1)} \\ &\quad + \mathcal{B}\left(d\Phi^{(1)} MC - \int d\Phi^{(1)} MC\right) d\Phi^{(n)} + \mathcal{O}(\alpha_s^{b+2}) \\ &= d\sigma_{\rm NLO} + \mathcal{O}(\alpha_s^{b+2}) \end{aligned}$$

- The MC counterterm has remarkable properties:
  - Avoiding double counting
  - Matching the IR singular behaviour of the real ME, making it possible to generate unweighted events (up to a sign though)
  - A smooth matching between PS and ME: in the IR (hard) region, same shape as PS (ME)



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$$d\sigma_{\rm NLO+PS}^{\rm MC@NLO} = \left(\mathcal{B}+\mathcal{V}+\mathcal{B}\int d\Phi^{(1)}MC\right)d\Phi^{(n)}I_{\rm MC}^{(n)} + \left(\mathcal{R}-\mathcal{B}MC\right)d\Phi^{(n+1)}I_{\rm MC}^{(n+1)}$$

S-event H-event

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S-event H-event

Without showering, NLO events from LHE file is NOT physical.

POWHEG

Nason JHEP'04

- **In the POWHEG formalism, it modifies the Sudakov for the first emission.**

$$\tilde{\Delta}(Q,Q_0) = \exp\left(-\int_{Q_0}^{Q} d\Phi^{(1)}\frac{\mathcal{R}}{\mathcal{B}}\right)$$

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Where t is the scale at which R/B is evaluated

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- **Verifying there is no double counting.**

$$\tilde{\Delta}(Q,t)d\Phi^{(1)}\frac{\mathcal{R}}{\mathcal{B}}=\frac{d\tilde{\Delta}(Q,t)}{dt} \Rightarrow \int_{Q_0}^{Q}dt\tilde{\Delta}(Q,t)d\Phi^{(1)}\frac{\mathcal{R}}{\mathcal{B}}=\tilde{\Delta}(Q,Q)-\tilde{\Delta}(Q,Q_0)=1-\tilde{\Delta}(Q,Q_0)$$

t integration goes to 1

$$d\sigma^{\rm POWHEG}_{\rm NLO+PS}=\left(\mathcal{B}+\mathcal{V}+\int d\Phi^{(1)}\mathcal{R}\right)d\Phi^{(n)}\left(1-\int d\Phi^{(1)}\frac{\mathcal{R}}{\mathcal{B}}+d\Phi^{(1)}\frac{\mathcal{R}}{\mathcal{B}}+\mathcal{O}(\alpha_s^2)\right)$$

$$=d\sigma_{\rm NLO}+\mathcal{O}(\alpha_s^{b+2})$$

POWHEG

$$d\sigma^{\rm POWHEG}_{\rm NLO+PS} = \left(\mathcal{B} + \mathcal{V} + \int d\Phi^{(1)}\mathcal{R}\right) d\Phi^{(n)} \left(\tilde{\Delta}(Q,Q_0) + \tilde{\Delta}(Q,t)d\Phi^{(1)}\frac{\mathcal{R}}{\mathcal{B}}\right)$$

global K factor

modified Sudakov for 1st emission

- Note that when matching to PS one has to veto emissions harder than t (in the Powheg formalism, is has to be interpreted as transverse momentum), even for showers with a different ordering variable
  - Formula to be modified for angular-ordered PS in order to keep color coherence
- MC@NLO and Powheg are formally equivalent at NLO level. In practice, there are many differences between the two

MC@NLO VS POWHEG

- **The two methods can be cast into a single formula**

$$d\sigma_{\text{NLO+PS}} = \overline{\mathcal{B}}^s \left(\Delta^s(Q, Q_0) + \Delta^s(Q, t) d\Phi^{(1)} \frac{\mathcal{R}^s}{\mathcal{B}} \right) d\Phi^{(n)} + \mathcal{R}^f d\Phi^{(n+1)}$$

$$\overline{\mathcal{B}}^s = \mathcal{B} + \mathcal{V} + \int d\Phi^{(1)} \mathcal{R}^s$$

$$\mathcal{R} = \boxed{\mathcal{R}^s} + \boxed{\mathcal{R}^f}$$

singular finite

MC@NLO $\mathcal{R}^s = \mathcal{B}MC$

Powheg $\mathcal{R}^s = F\mathcal{R}, \mathcal{R}^f = (1-F)\mathcal{R}$

default $F=1$, but can be tuned in order to suppress non-singular part of R

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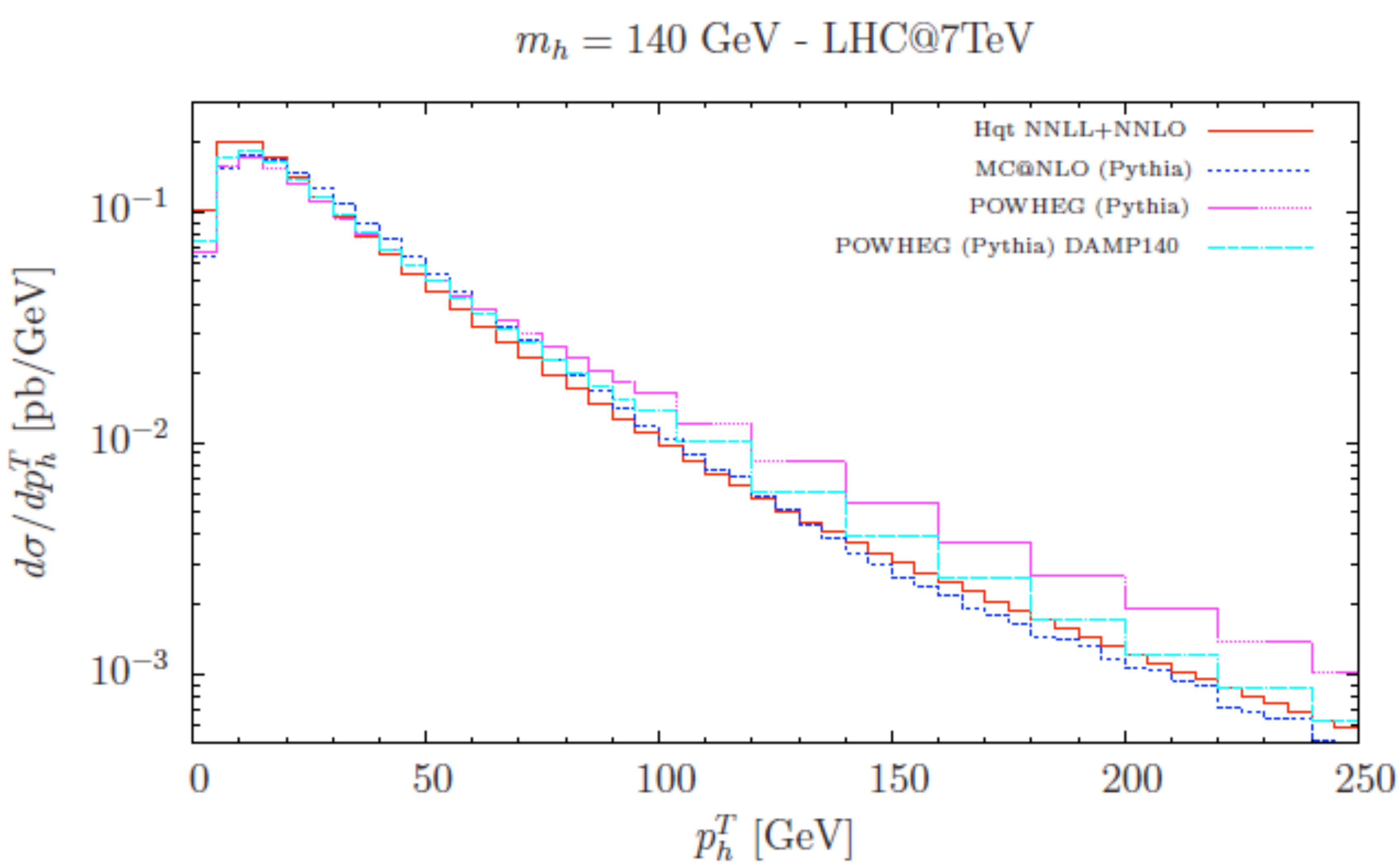
$$F = \frac{h^2}{h^2 + p_T^2}$$ $p_T \gg h$ are suppressed



MC@NLO naturally matches analytic resummation+FO curve at large p_T · to

Powheg (without damping) overshoots the FO of R

Damping recovers matching at large p_T

MC@NLO VS POWHEG

	MC@NLO	POWHEG
Parton showers are (usually) not exact in the soft limit: MC@NLO needs an artificial smoothing	☹	☺
MC@NLO does not exponentiate the non-singular part of the real emission amplitudes	☺	☹
MC@NLO does not require any tricks for treating Born zeros	☺	☹
POWHEG is independent from the parton shower (although, in general the shower should be a truncated vetoed)	☹	☺
POWHEG has (almost) no negatively weighted events	☹	☺
Automation of the methods: http://amcatnlo.cern.ch, http://powhegbox.mib.infn.it, http://www.sherpa-mc.de	☺	☺

Lecture 3

Advanced NLO Topics

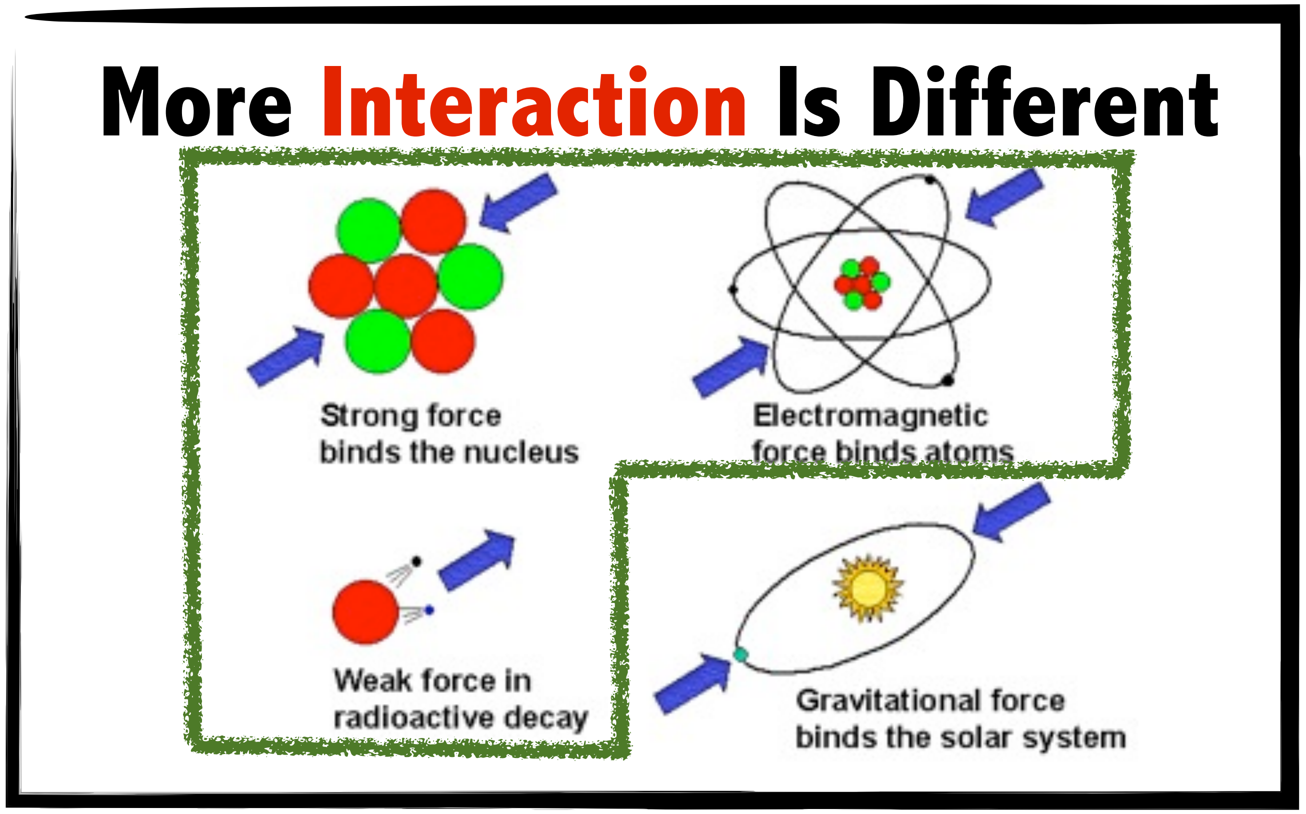
More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

Lecture 3

Advanced NLO Topics



WHY WE CARE EW CORRECTIONS ?

- LHC will run (ran) at 14 (13) TeV and future colliders at 100 TeV
  - energy reaches deeper into multi-TeV region & high integrated luminosity
  - many processes (even rare processes before) reach precision era (precent)
- NLO QCD becomes standard: automation (e.g. MG5_aMC)
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b.13	$pp \to W^+W^+jj$	$2.251 \pm 0.011 \cdot 10^{-1}$	$^{+10.5\%}_{-10.6\%}$ $^{+2.2\%}_{-1.6\%}$
b.14	$pp \to W^-W^-jj$	$1.003 \pm 0.003 \cdot 10^{-1}$	$^{+10.1\%}_{-10.4\%}$ $^{+2.5\%}_{-1.8\%}$
b.15	$pp \to W^+W^-jj$ (4f)	$1.396 \pm 0.005 \cdot 10^{1}$	$^{+5.0\%}_{-6.8\%}$ $^{+0.7\%}_{-0.6\%}$
b.16	$pp \to ZZjj$	$1.706 \pm 0.011 \cdot 10^{0}$	$^{+5.8\%}_{-7.2\%}$ $^{+0.8\%}_{-0.6\%}$
b.17	$pp \to ZW^\pm jj$	$9.139 \pm 0.031 \cdot 10^{0}$	$^{+3.1\%}_{-5.1\%}$ $^{+0.7\%}_{-0.5\%}$
b.18	$pp \to \gamma\gamma jj$	$7.501 \pm 0.032 \cdot 10^{0}$	$^{+8.8\%}_{-10.1\%}$ $^{+0.6\%}_{-1.0\%}$
b.19*	$pp \to \gamma Zjj$	$4.242 \pm 0.016 \cdot 10^{0}$	$^{+6.5\%}_{-7.3\%}$ $^{+0.6\%}_{-0.6\%}$
b.20*	$pp \to \gamma W^\pm jj$	$1.448 \pm 0.005 \cdot 10^{1}$	$^{+3.6\%}_{-5.4\%}$ $^{+0.6\%}_{-0.7\%}$

d.1	$pp \to jj$	$1.580 \pm 0.007 \cdot 10^{6}$	$^{+8.4\%}_{-9.0\%}$ $^{+0.7\%}_{-0.9\%}$
d.2	$pp \to jjj$	$7.791 \pm 0.037 \cdot 10^{4}$	$^{+2.1\%}_{-23.2\%}$ $^{+1.1\%}_{-1.3\%}$
d.7	$pp \to t\bar{t}$	$6.741 \pm 0.023 \cdot 10^{2}$	$^{+9.8\%}_{-10.9\%}$ $^{+1.8\%}_{-2.1\%}$
d.8	$pp \to t\bar{t}j$	$4.106 \pm 0.015 \cdot 10^{2}$	$^{+8.1\%}_{-12.2\%}$ $^{+2.1\%}_{-2.5\%}$
d.9	$pp \to t\bar{t}jj$	$1.795 \pm 0.006 \cdot 10^{2}$	$^{+9.3\%}_{-16.1\%}$ $^{+2.4\%}_{-2.9\%}$
d.10	$pp \to t\bar{t}t\bar{t}$	$9.201 \pm 0.028 \cdot 10^{-3}$	$^{+30.8\%}_{-25.6\%}$ $^{+5.5\%}_{-5.9\%}$
d.11	$pp \to t\bar{t}b\bar{b}$ (4f	$1.452 \pm 0.005 \cdot 10^{1}$	$^{+37.6\%}_{-27.5\%}$ $^{+2.9\%}_{-3.5\%}$

Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, HSS, Stelzer, Torrielli, Zaro JHEP'14

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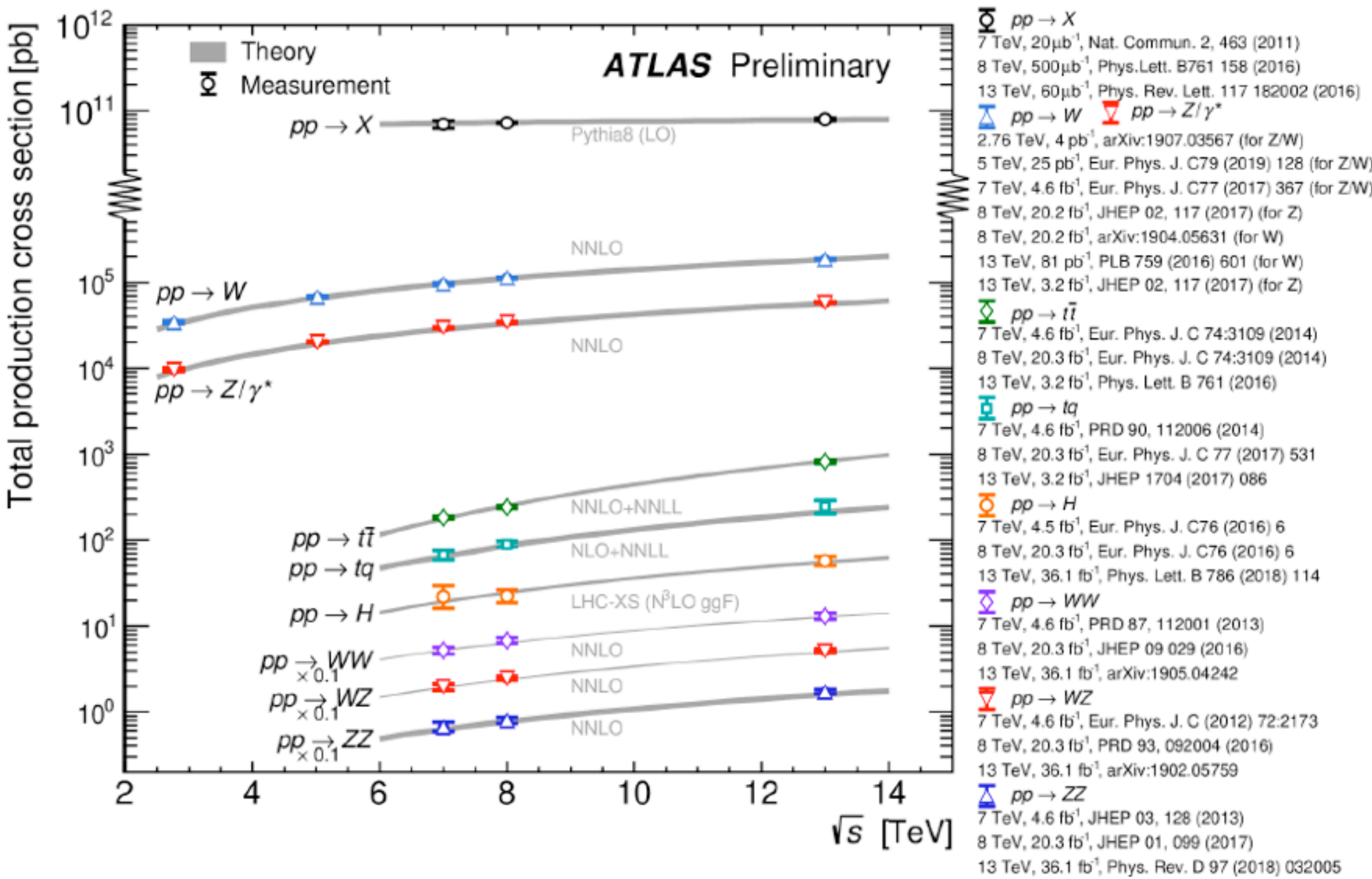

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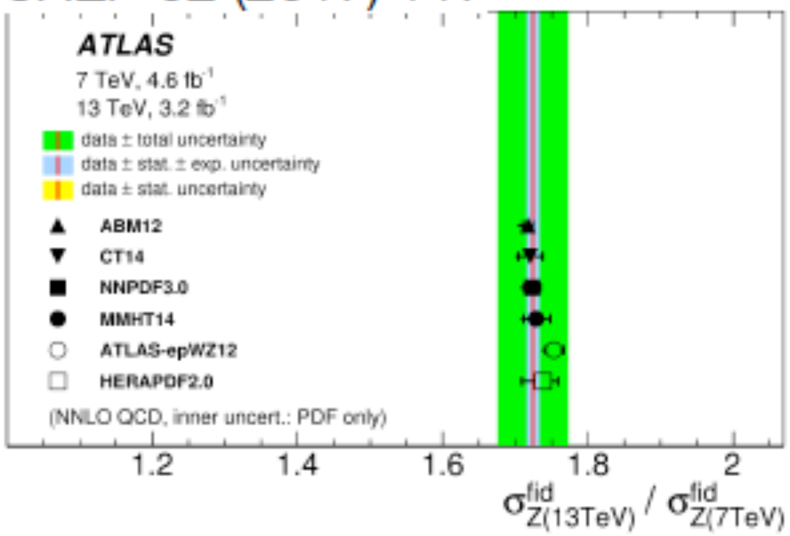

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  - First opportunity to explore TeV scale kinematics, where EWC ~ 10%
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    - cross section ratios, e.g. different center-of-mass energy, different processes

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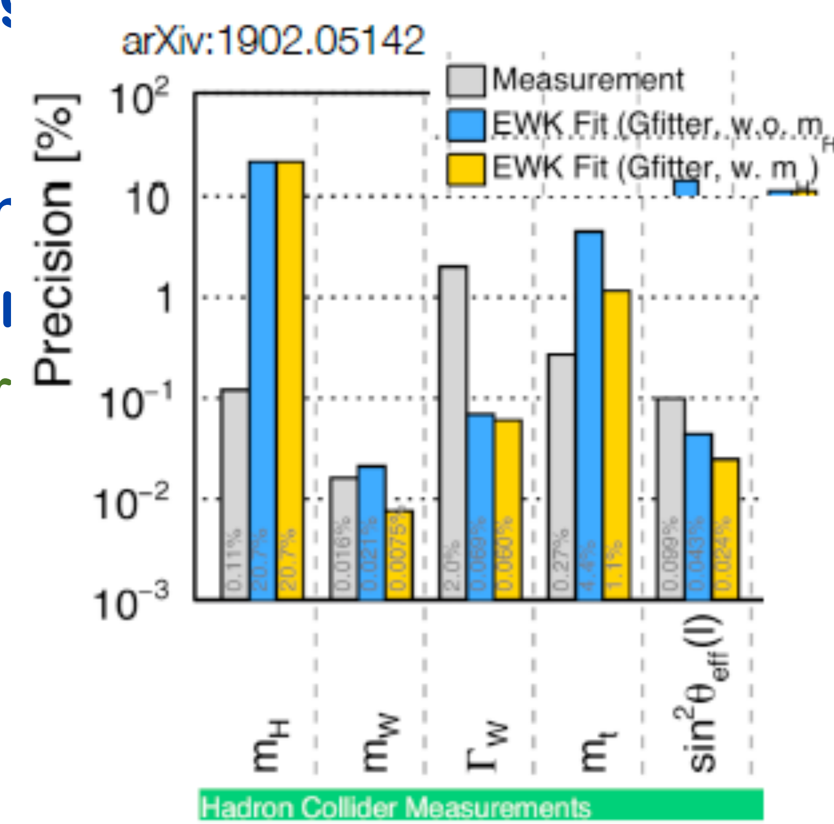

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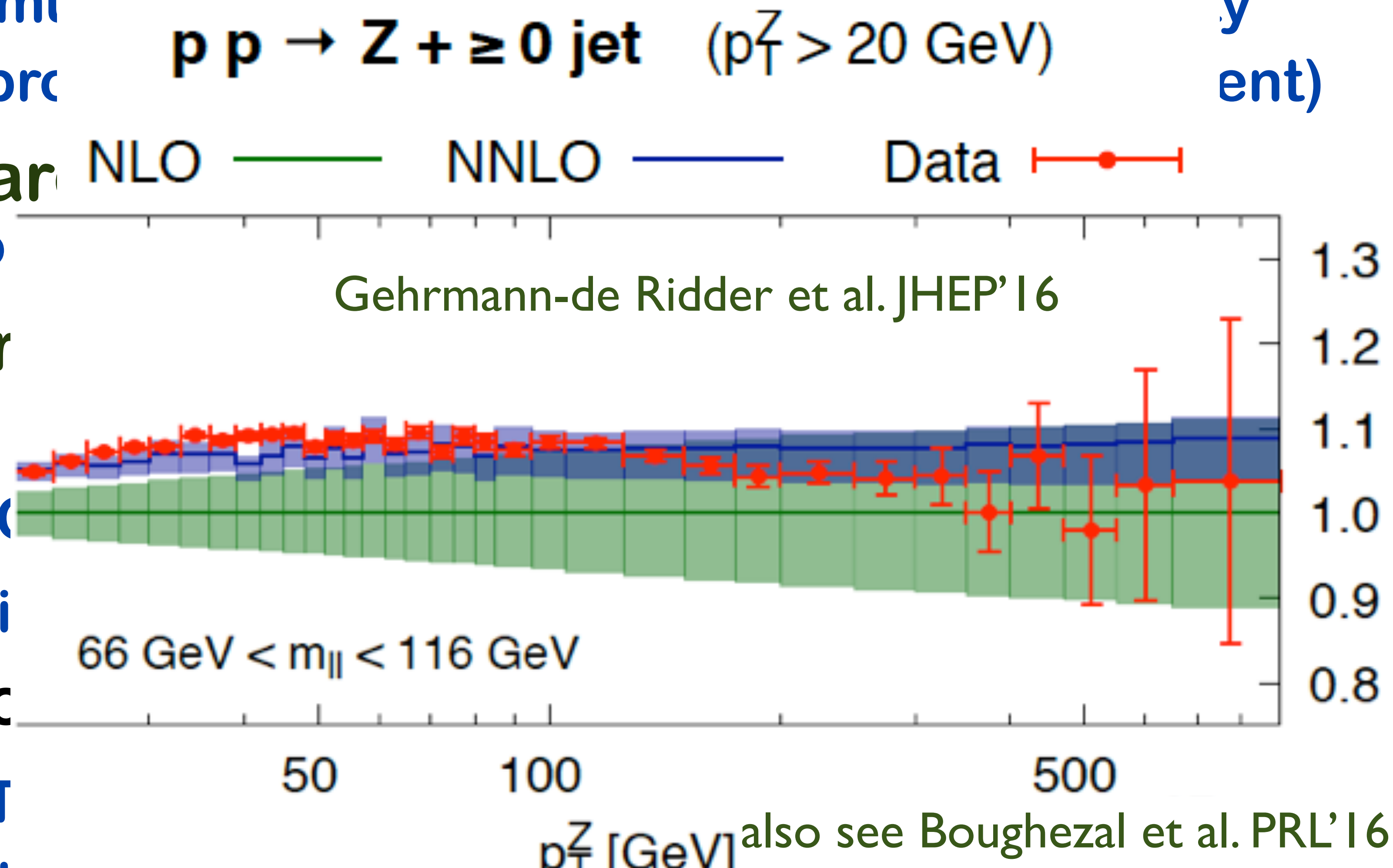

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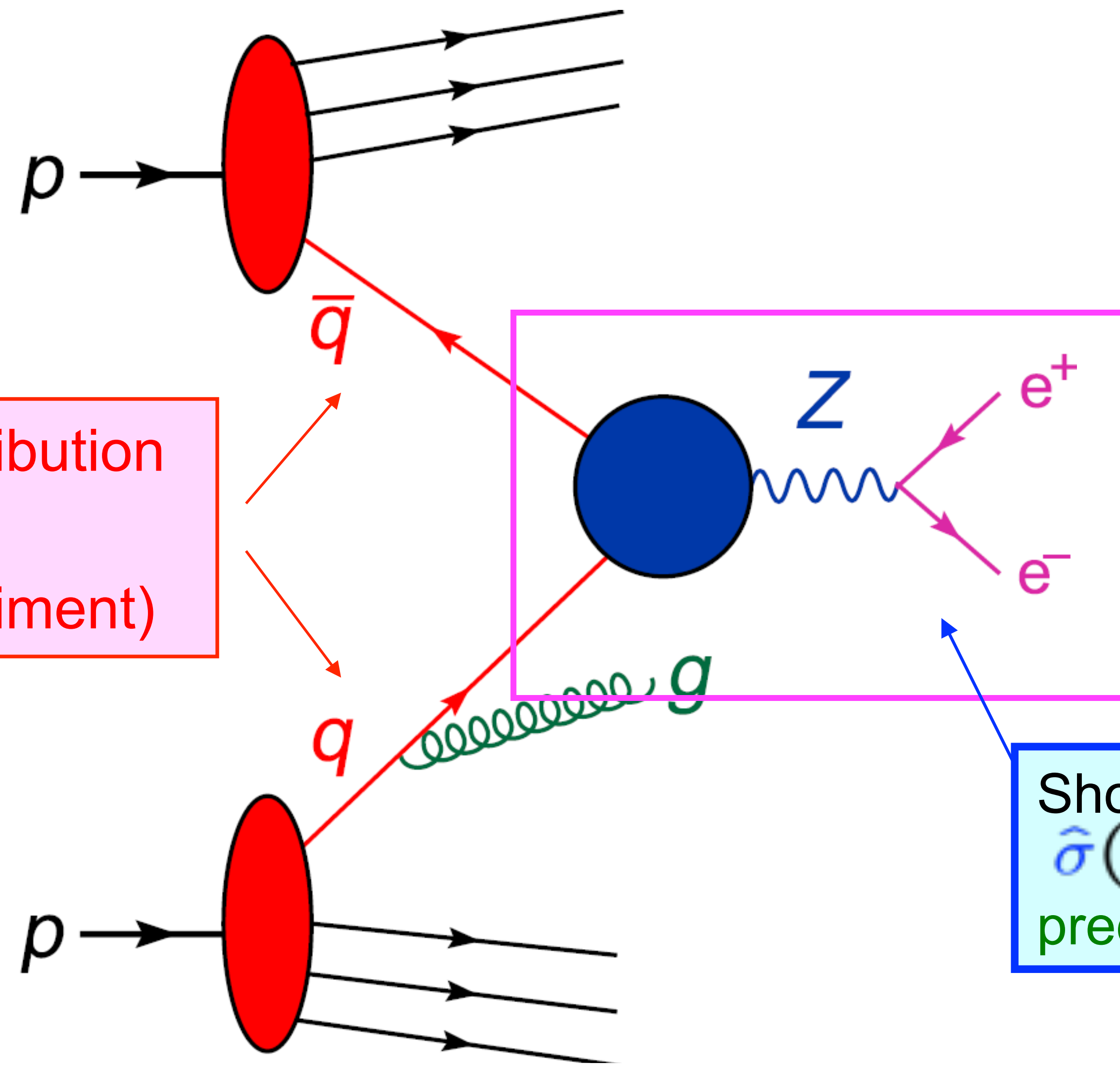

Parton distribution functions (from experiment)

The "femto-universe"
size = factorization scale μ_F ("arbitrary")

Short-distance cross section
$\hat{\sigma}(\alpha_s, \mu_F, \mu_R)$
predictable using perturbative QCD

$$\sigma(pp \to Z + X) = \int dx_1 dx_2 f(x_1, \mu_F) f(x_2, \mu_F) \hat{\sigma}(\alpha_s, \mu_F, \mu_R)$$

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LO NLO NNLO

$$\hat{\sigma}(\alpha_s,\mu_F,\mu_R)=[\alpha_s(\mu_R)]^n\left[\hat{\sigma}^{(0)}+\frac{\alpha_s}{2\pi}\sigma^{(1)}(\mu_F,\mu_R)+\left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}^{(2)}(\mu_F,\mu_R)+\cdots\right]$$

QCD

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$$+ \frac{\alpha}{2\pi}\sigma^{(0,1)}(\mu_F, \mu_R) + \left(\frac{\alpha}{2\pi}\right)^2 \hat{\sigma}^{(0,2)}(\mu_F, \mu_R) + \cdots$$

$$+ \sum_{i\geq 1}\sum_{j\geq 1} \left(\frac{\alpha_s}{2\pi}\right)^i \left(\frac{\alpha}{2\pi}\right)^j \hat{\sigma}^{(i,j)}(\mu_F, \mu_R) \Big]$$

QCD

EW

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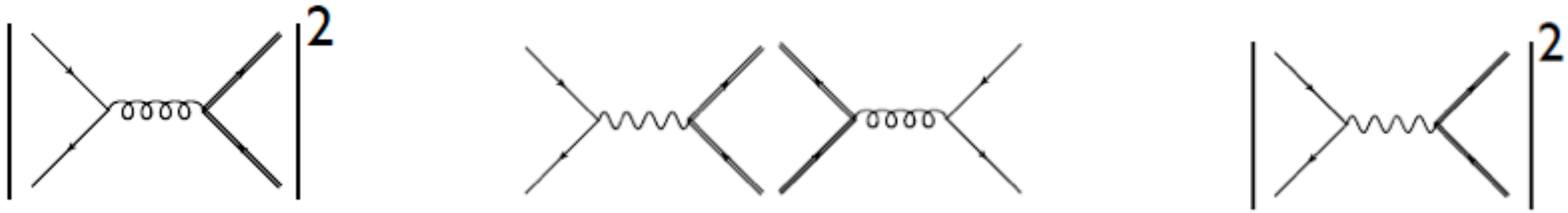

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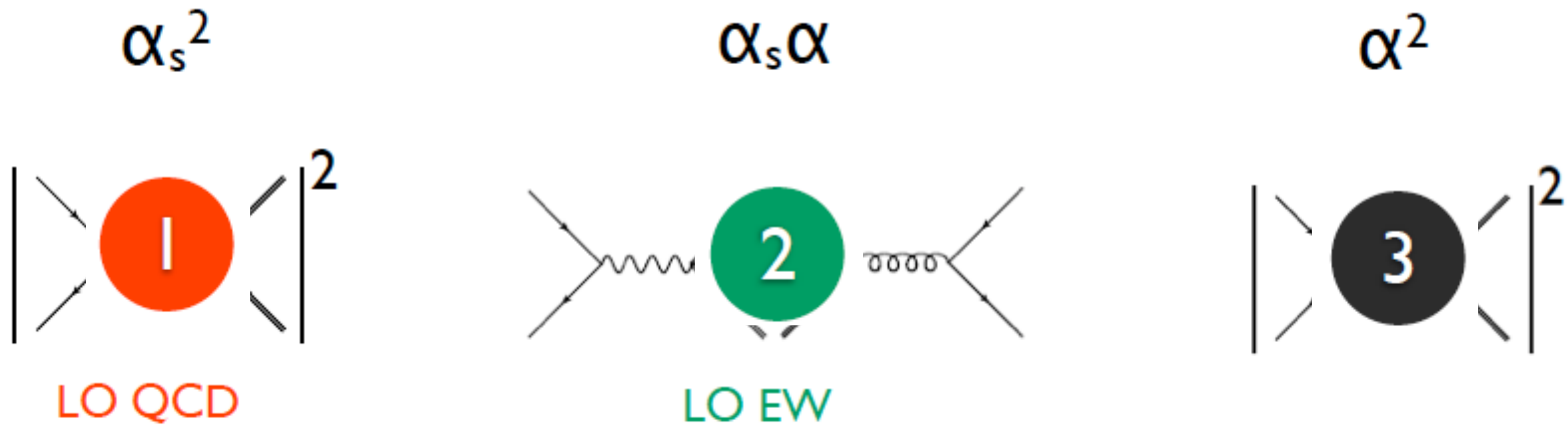

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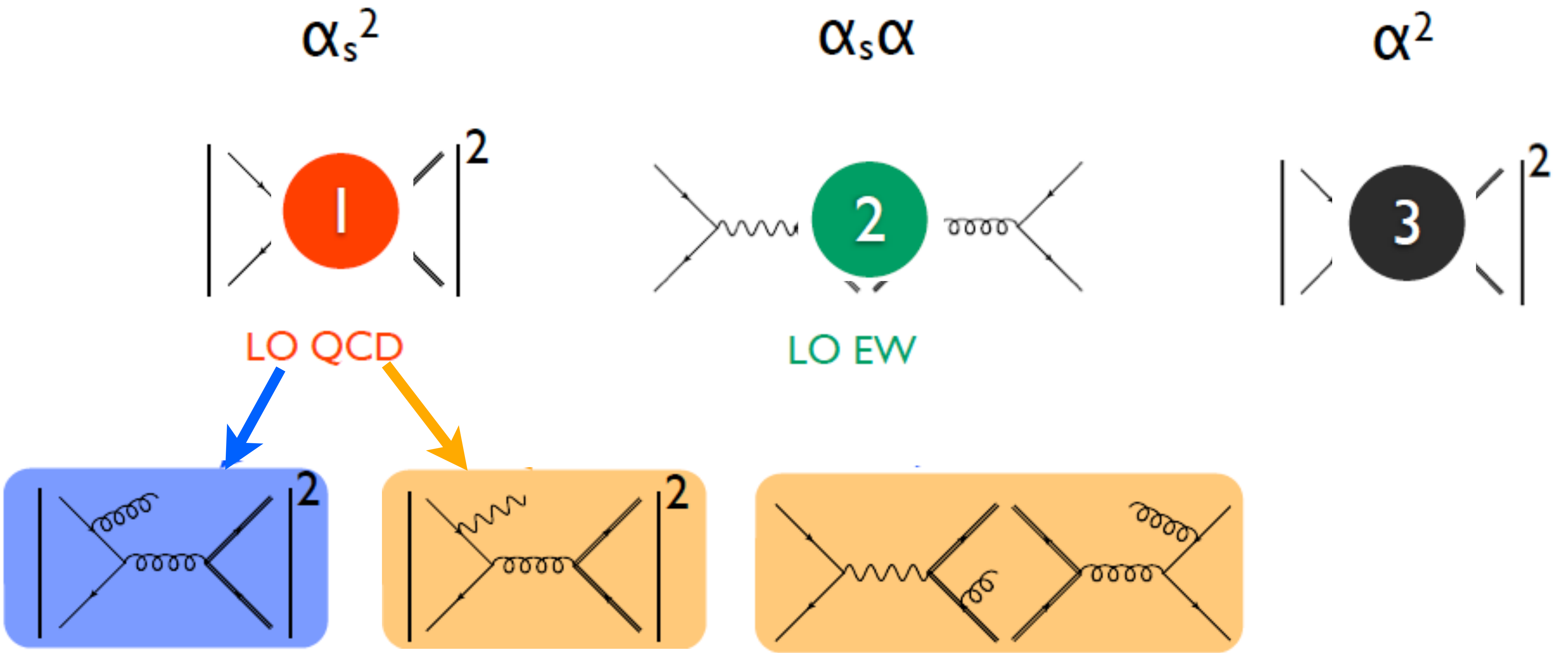

Tuesday, November 19, 19

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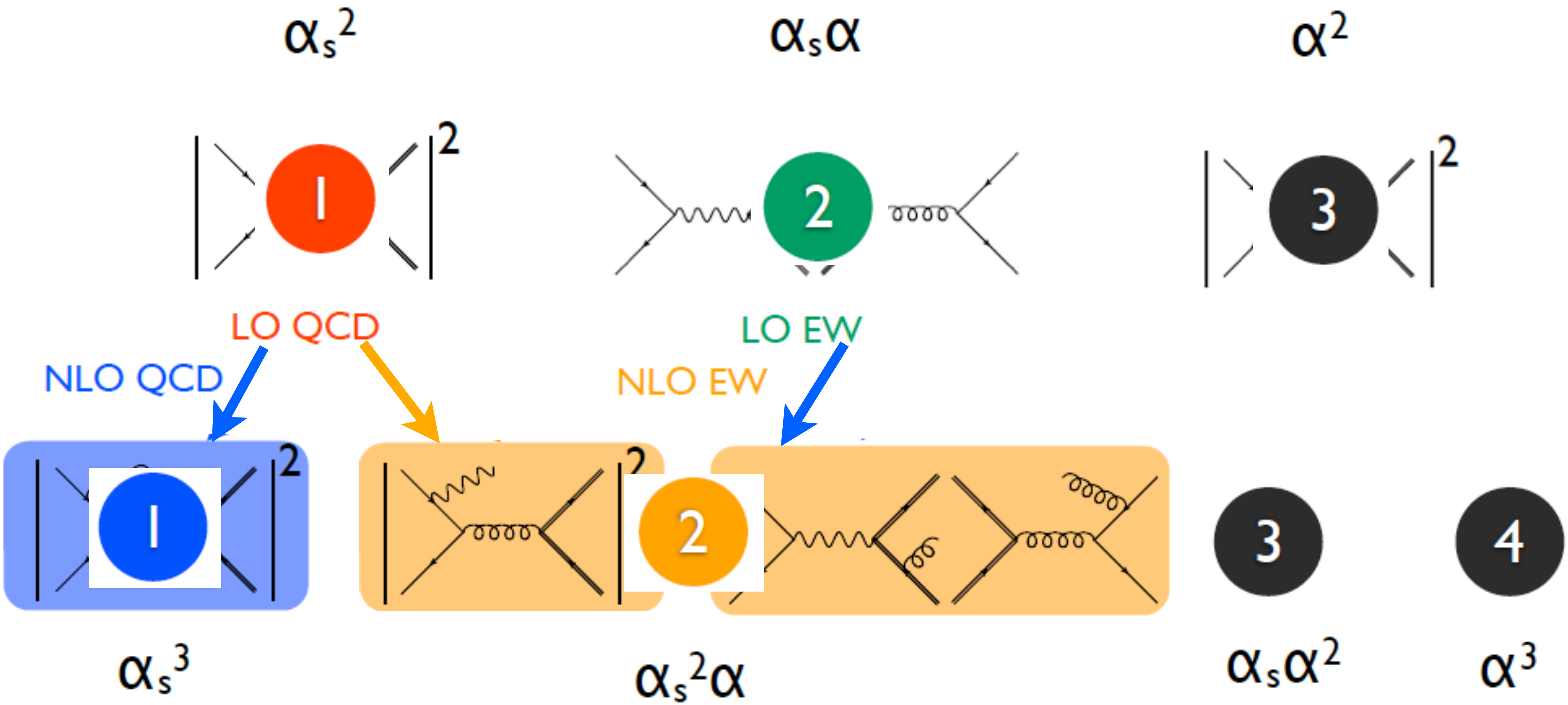

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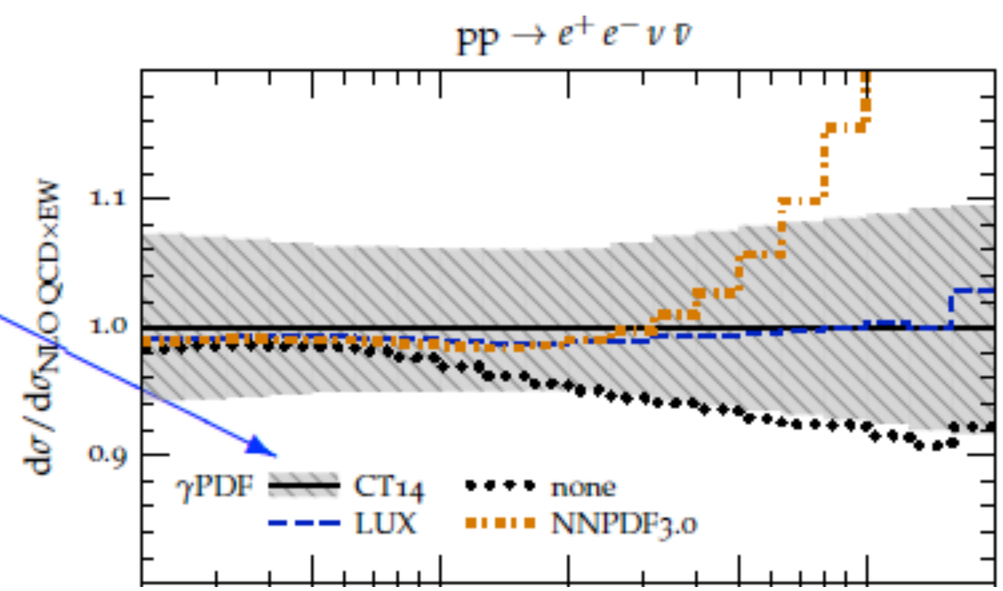
    - Photon PDF will be quite relevant, which is usually poorly determined (? LUXqed)
    - Photon and jet is not well separated (need fragmentation function or some approximations)
    - If phase space is enough, EW boson radiation will be quite often (do we need them ?)
    - The general matching between matrix element and parton shower will be difficult

relative importance of γ-induced channels wrt. NLO QCD$\times$EW

CT14qed (baseline)   no γPDF
LUXqed   NNPDF3.0qed

Kallweit et al. JHEP'17



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Shall we use different scheme/renormalization for different vertices in one diagram ?

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  - G_μ scheme: works good for weak bosons and well measured

Shall we use different scheme/renormalization for different vertices in one diagram ?

- Use $K_{\mathrm{NLO\ QCD}} \times K_{\mathrm{NLO\ EW}}$ to capture the missing higher order ?

ENHANCE EW CORRECTIONS

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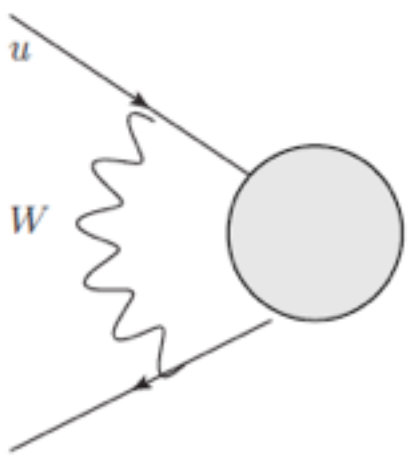

Leading Log — Next-to-Leading Log

$$\sim -c_{\rm LL}\frac{\alpha}{\pi s_w^2}\log^2\frac{Q^2}{M_W^2} + c_{\rm NLL}\frac{3\alpha}{\pi s_w^2}\log\frac{Q^2}{M_W^2} + \cdots$$

soft — collinear

e.g.

$Q = 1\ \mathrm{TeV}$ $\qquad -c_{\rm LL}\times 26\% + c_{\rm NLL}\times 16\%$

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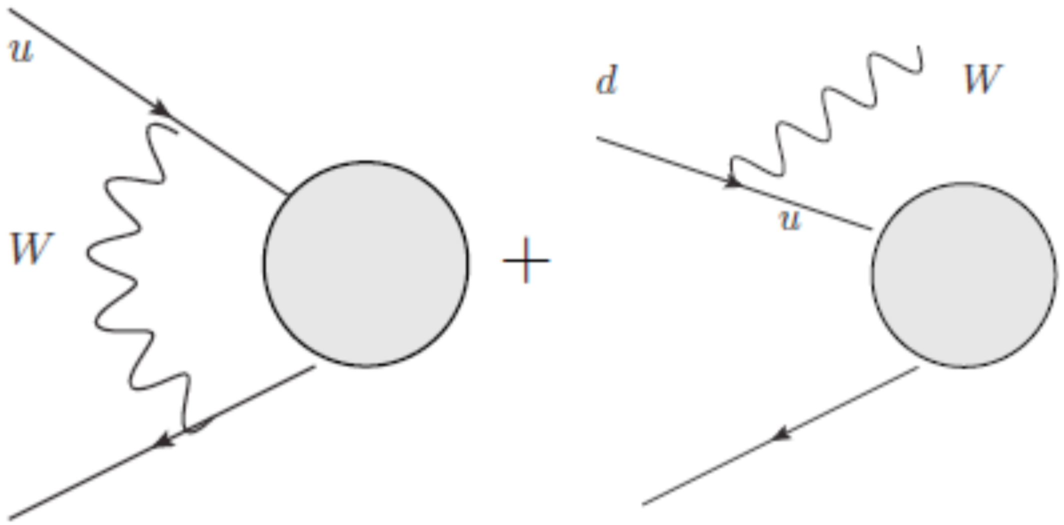

$$\sim (f_{u/p}-f_{d/p})\log\frac{Q^2}{M_W^2}$$

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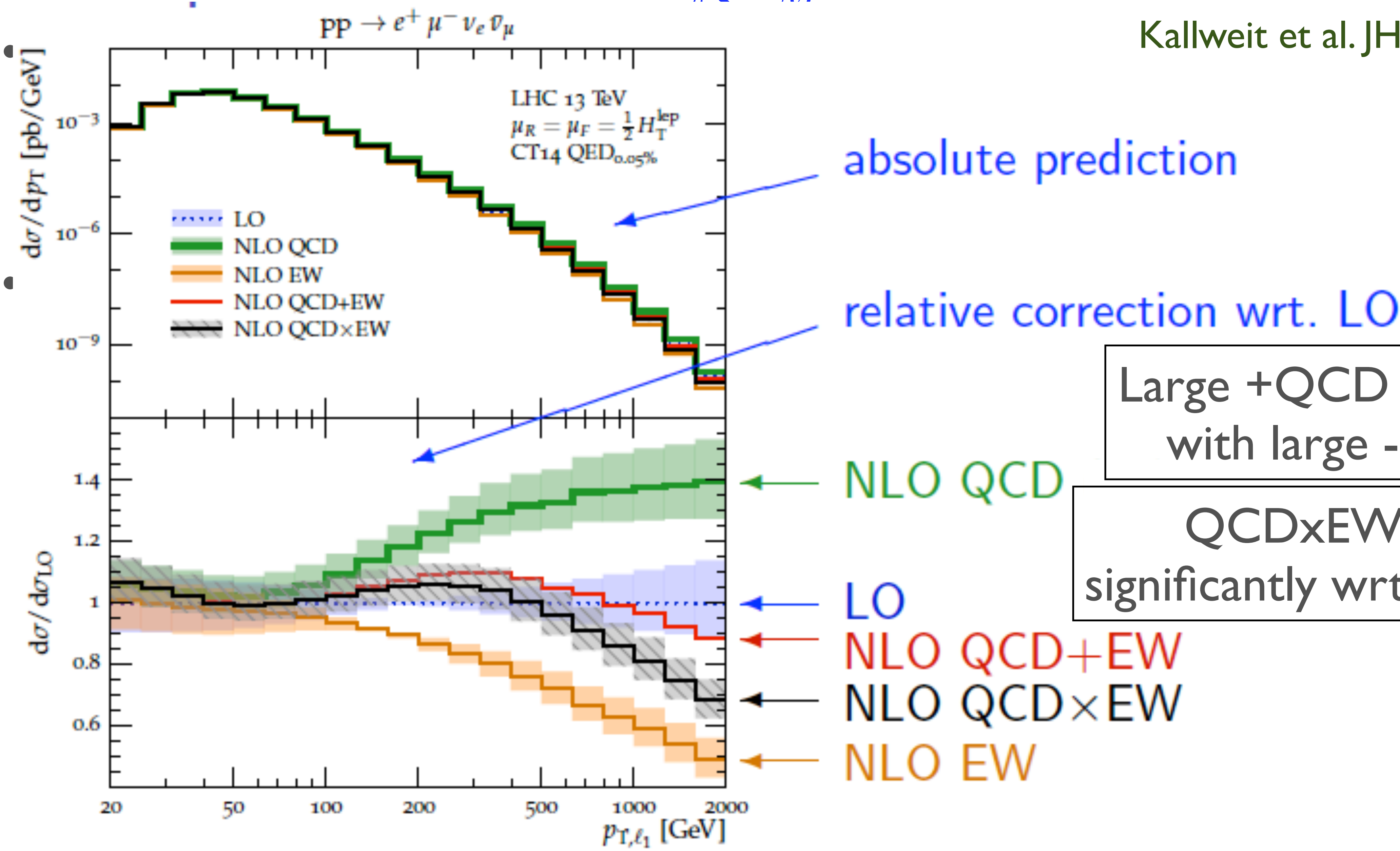
Kallweit et al. JHEP'17



Large +QCD corr. cancel with large -EW corr.

QCDxEW differs significantly wrt QCD+EW

Tuesday, November 19, 19

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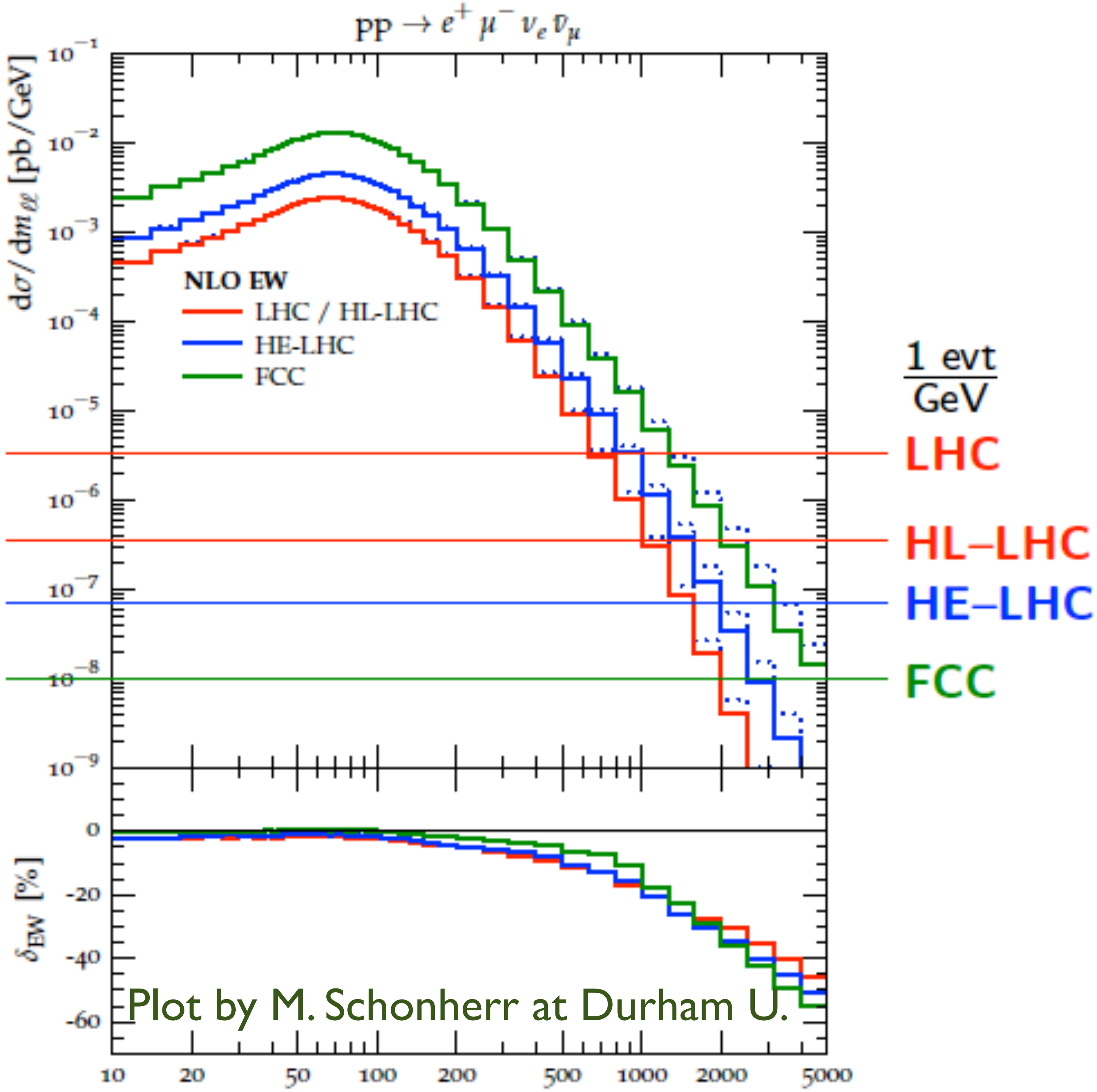
  - Even treat W/Z as inclusive as gluon/photon: initial state is not SU(2) singlet
  - However, EW Sudakov logarithms is not always relevant in Sudakov regime
    - e.g. Drell-Yan at large invariant mass receives large contributions from small t Dittmaier et al.'10

EW IN HIGH-ENERGY SCATTERINGS

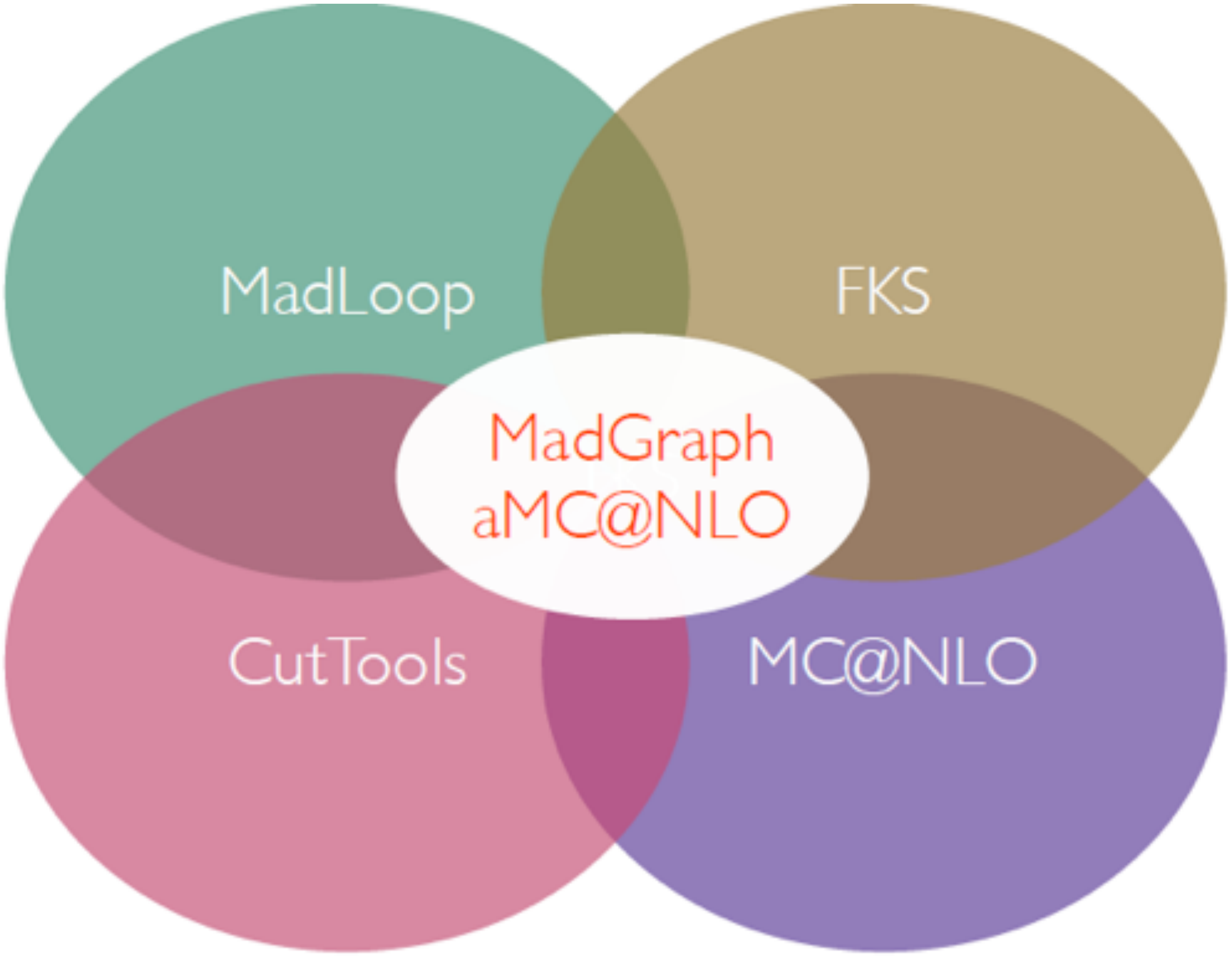


- BSM effects are expected to be enhanced in the high-energy scatterings
- -> motivated BSM search go to the tail
- EW corr. increase up to tens of percent due to EW Sudakov logs
- The EW log resummation is still not mandatory@ (HL-)LHC as

$$\alpha L \ll 1$$

MADGRAPH5_AMC@NLO IN A NUTSHELL

Alwall, Frederix, Frixione, Hirschi, Maltoni, Mattelaer, HSS, Stelzer, Torrielli, Zaro JHEP'14



4 commands for a NLO calculation

```
> ./bin/mg5_aMC
> generate process [QCD]
> output
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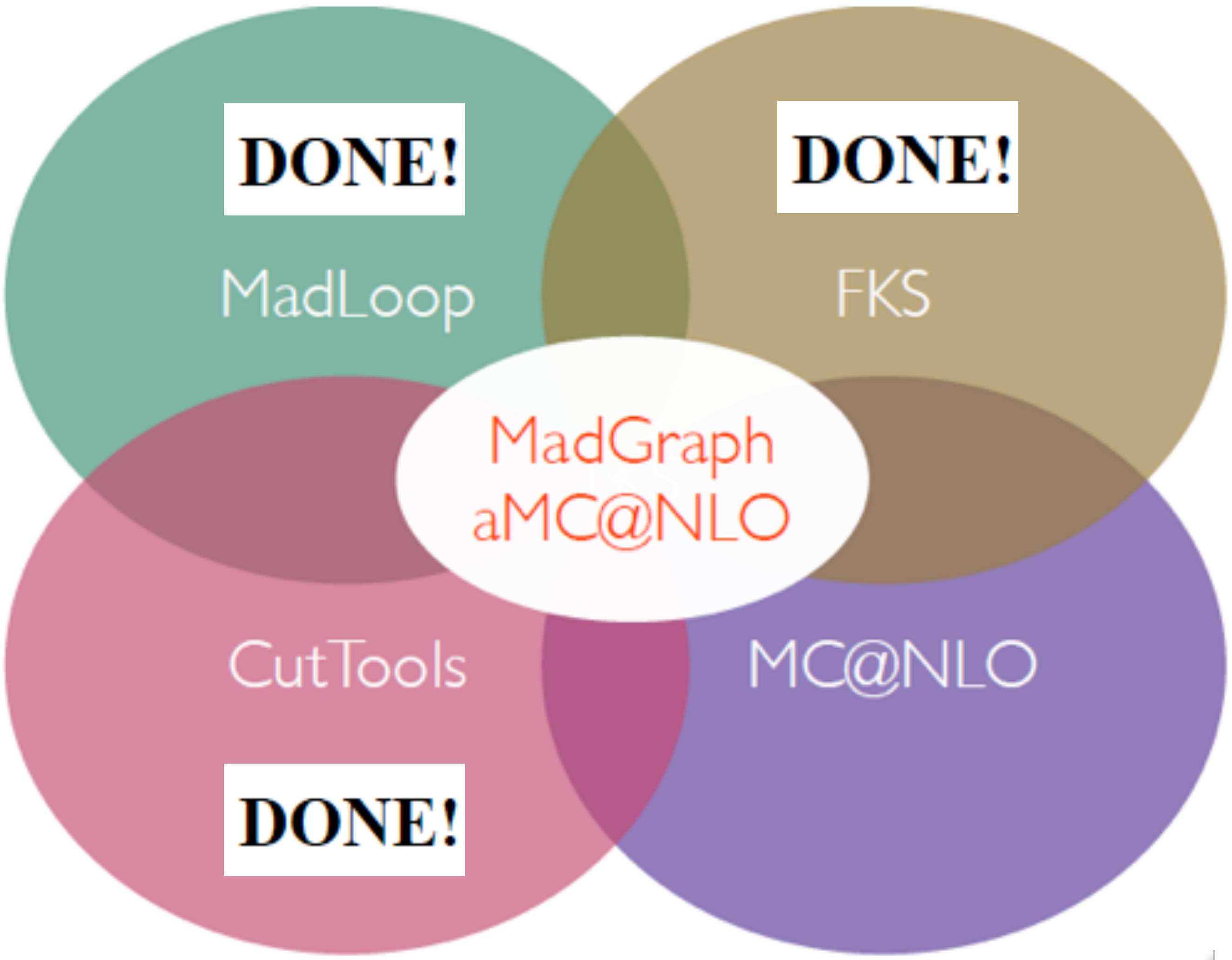

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complete automation for
QCD+EW

Frederix, Frixione, Hirschi, Pagani, HSS, Zaro JHEP'18

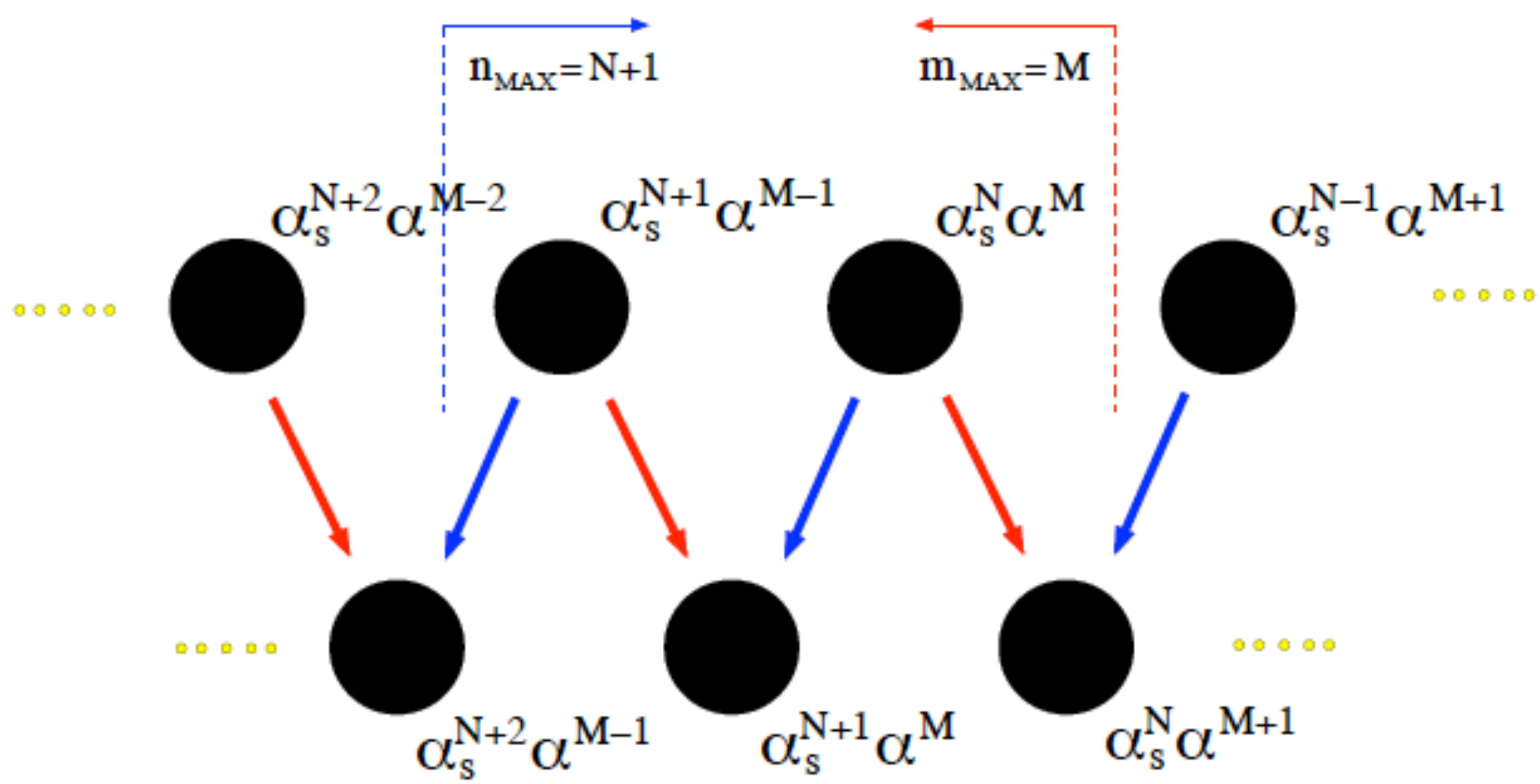
```
> ./bin/mg5_aMC
> generate process [QCD QED]
> output
> launch
```

MADGRAPH5_AMC@NLO: COMPLETE NLO

- Generation syntax for any LO and NLO (in v3.X):

Frederix, Frixione, Hirschi, Pagani, HSS, Zaro JHEP'18

MG5_aMC> generate p_1 p_2 > p_3 p_4 p_5 p_6 QCD=n_{max} QED=m_{max} [QCD QED]



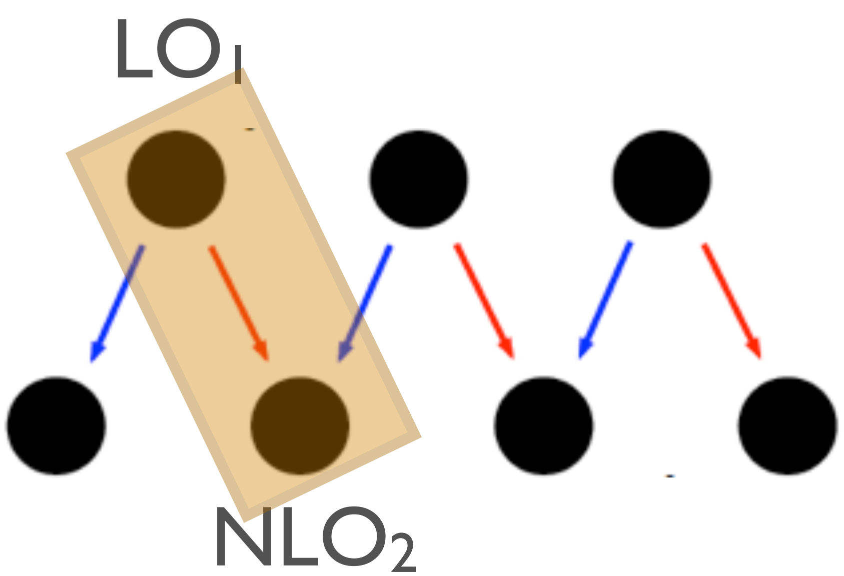
$$\text{LO}: \quad \alpha_s^n\alpha^m\,, \quad n \le n_{max}\,, \quad m \le m_{max}\,, \quad n+m=k_0\,,$$

$$\text{NLO}: \quad \alpha_s^n\alpha^m\,, \quad n \le n_{max}+1\,, \quad m \le m_{max}+1\,, \quad n+m=k_0+1$$

Tuesday, November 19, 19

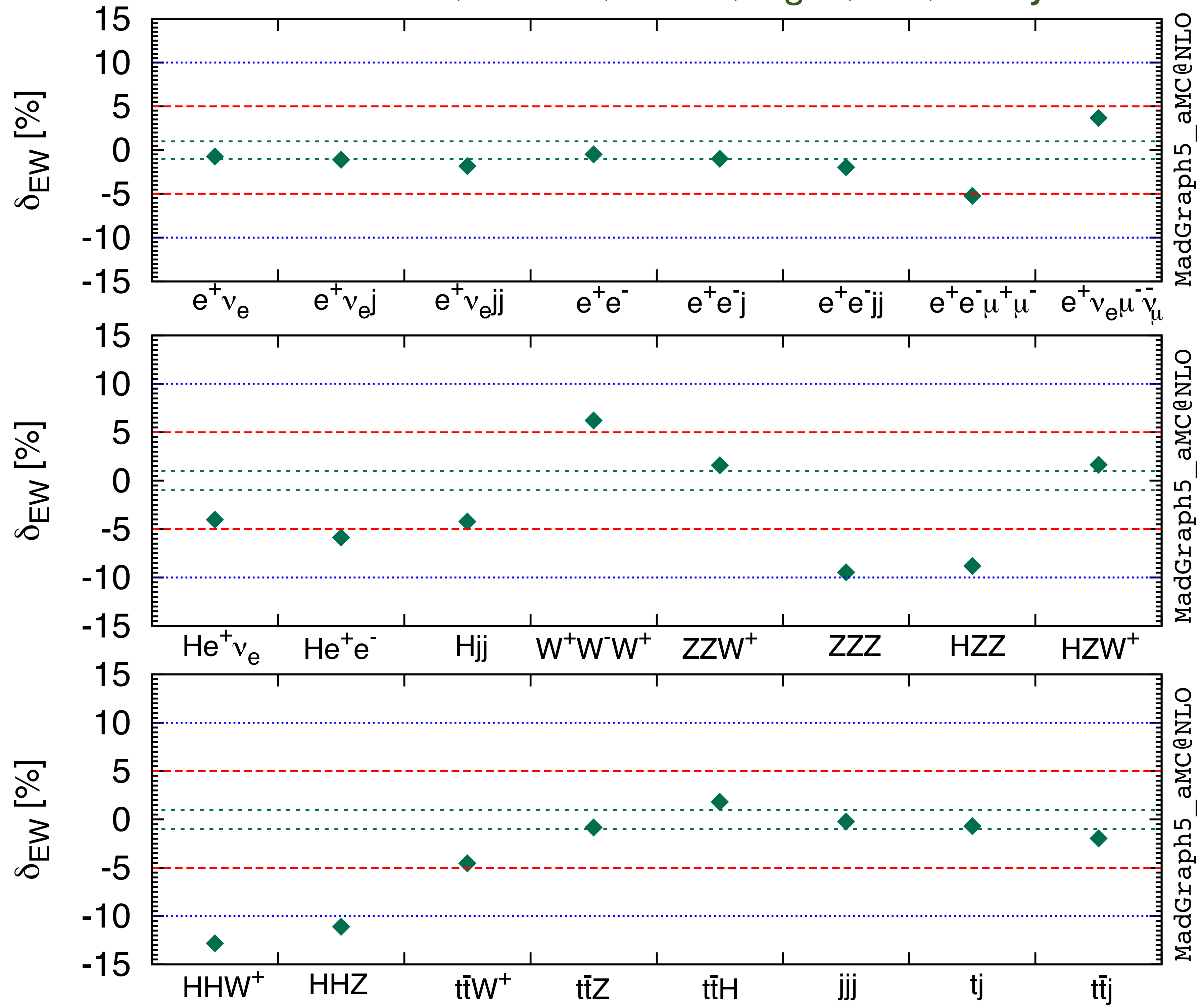
MADGRAPH5_AMC@NLO: NLO EW

- Examples:



$$\delta_{\rm EW} = \frac{\rm NLO_2}{\rm LO_1}$$

Frederix, Frixione, Hirschi, Pagani, HSS, Zaro JHEP'18



MADGRAPH5_AMC@NLO: NLO EW

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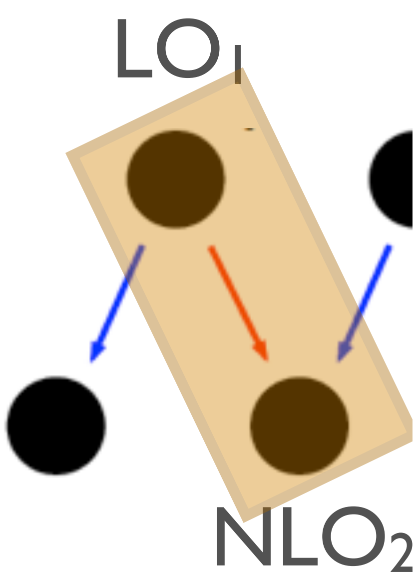
Frederix, Frixione, Hirschi, Pagani, HSS, Zaro JHEP'18



$\delta_{\mathrm{EW}} =$

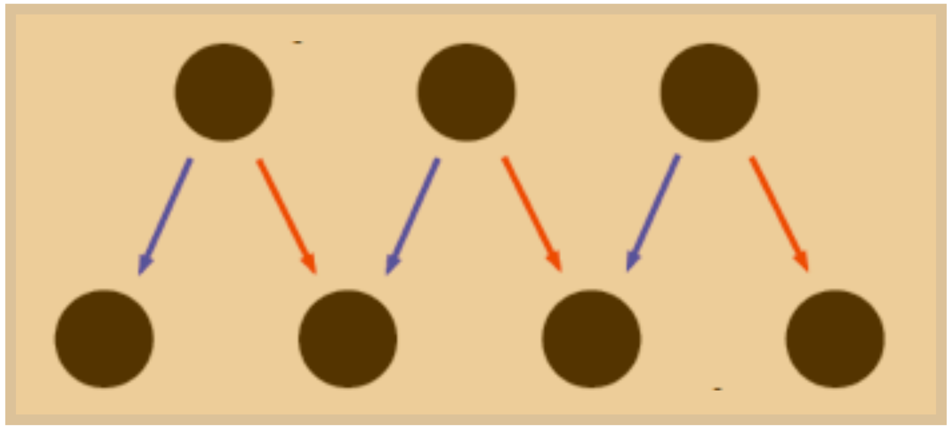
Process	Syntax	Cross section (in pb) LO	Cross section (in pb) NLO	Correction (in %)
$pp \to e^+\nu_e$	p p > e+ ve QCD=0 QED=2 [QED]	$5.2498 \pm 0.0005 \cdot 10^3$	$5.2113 \pm 0.0006 \cdot 10^3$	-0.73 ± 0.01
$pp \to e^+\nu_e j$	p p > e+ ve j QCD=1 QED=2 [QED]	$9.1468 \pm 0.0012 \cdot 10^2$	$9.0449 \pm 0.0014 \cdot 10^2$	-1.11 ± 0.02
$pp \to e^+\nu_e jj$	p p > e+ ve j j QCD=2 QED=2 [QED]	$3.1562 \pm 0.0003 \cdot 10^2$	$3.0985 \pm 0.0005 \cdot 10^2$	-1.83 ± 0.02
$pp \to e^+e^-$	p p > e+ e- QCD=0 QED=2 [QED]	$7.5367 \pm 0.0008 \cdot 10^2$	$7.4997 \pm 0.0010 \cdot 10^2$	-0.49 ± 0.02
$pp \to e^+e^- j$	p p > e+ e- j QCD=1 QED=2 [QED]	$1.5059 \pm 0.0001 \cdot 10^2$	$1.4909 \pm 0.0002 \cdot 10^2$	-1.00 ± 0.02
$pp \to e^+e^- jj$	p p > e+ e- j j QCD=2 QED=2 [QED]	$5.1424 \pm 0.0004 \cdot 10^1$	$5.0410 \pm 0.0007 \cdot 10^1$	-1.97 ± 0.02
$pp \to e^+e^-\mu^+\mu^-$	p p > e+ e- mu+ mu- QCD=0 QED=4 [QED]	$1.2750 \pm 0.0000 \cdot 10^{-2}$	$1.2083 \pm 0.0001 \cdot 10^{-2}$	-5.23 ± 0.01
$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu$	p p > e+ ve mu- vm~ QCD=0 QED=4 [QED]	$5.1144 \pm 0.0007 \cdot 10^{-1}$	$5.3019 \pm 0.0009 \cdot 10^{-1}$	$+3.67 \pm 0.02$
$pp \to He^+\nu_e$	p p > h e+ ve QCD=0 QED=3 [QED]	$6.7643 \pm 0.0001 \cdot 10^{-2}$	$6.4914 \pm 0.0012 \cdot 10^{-2}$	-4.03 ± 0.02
$pp \to He^+e^-$	p p > h e+ e- QCD=0 QED=3 [QED]	$1.4554 \pm 0.0001 \cdot 10^{-2}$	$1.3700 \pm 0.0002 \cdot 10^{-2}$	-5.87 ± 0.02
$pp \to Hjj$	p p > h j j QCD=0 QED=3 [QED]	$2.8268 \pm 0.0002 \cdot 10^0$	$2.7075 \pm 0.0003 \cdot 10^0$	-4.22 ± 0.01
$pp \to W^+W^-W^+$	p p > w+ w- w+ QCD=0 QED=3 [QED]	$8.2874 \pm 0.0004 \cdot 10^{-2}$	$8.8017 \pm 0.0012 \cdot 10^{-2}$	$+6.21 \pm 0.02$
$pp \to ZZW^+$	p p > z z w+ QCD=0 QED=3 [QED]	$1.9874 \pm 0.0001 \cdot 10^{-2}$	$2.0189 \pm 0.0003 \cdot 10^{-2}$	$+1.58 \pm 0.02$
$pp \to ZZZ$	p p > z z z QCD=0 QED=3 [QED]	$1.0761 \pm 0.0001 \cdot 10^{-2}$	$0.9741 \pm 0.0001 \cdot 10^{-2}$	-9.47 ± 0.02
$pp \to HZZ$	p p > h z z QCD=0 QED=3 [QED]	$2.1005 \pm 0.0003 \cdot 10^{-3}$	$1.9155 \pm 0.0003 \cdot 10^{-3}$	-8.81 ± 0.02
$pp \to HZW^+$	p p > h z w+ QCD=0 QED=3 [QED]	$2.4408 \pm 0.0000 \cdot 10^{-3}$	$2.4809 \pm 0.0005 \cdot 10^{-3}$	$+1.64 \pm 0.02$
$pp \to HHW^+$	p p > h h w+ QCD=0 QED=3 [QED]	$2.7827 \pm 0.0001 \cdot 10^{-4}$	$2.4259 \pm 0.0027 \cdot 10^{-4}$	-12.82 ± 0.10
$pp \to HHZ$	p p > h h z QCD=0 QED=3 [QED]	$2.6914 \pm 0.0003 \cdot 10^{-4}$	$2.3926 \pm 0.0003 \cdot 10^{-4}$	-11.10 ± 0.02
$pp \to t\bar{t}W^+$	p p > t t~ w+ QCD=2 QED=1 [QED]	$2.4119 \pm 0.0003 \cdot 10^{-1}$	$2.3025 \pm 0.0003 \cdot 10^{-1}$	-4.54 ± 0.02
$pp \to t\bar{t}Z$	p p > t t~ z QCD=2 QED=1 [QED]	$5.0456 \pm 0.0006 \cdot 10^{-1}$	$5.0033 \pm 0.0007 \cdot 10^{-1}$	-0.84 ± 0.02
$pp \to t\bar{t}H$	p p > t t~ h QCD=2 QED=1 [QED]	$3.4480 \pm 0.0004 \cdot 10^{-1}$	$3.5102 \pm 0.0005 \cdot 10^{-1}$	$+1.81 \pm 0.02$
$pp \to t\bar{t}j$	p p > t t j QCD=3 QED=0 [QED]	$3.0277 \pm 0.0003 \cdot 10^2$	$2.9683 \pm 0.0004 \cdot 10^2$	-1.96 ± 0.02
$pp \to jjj$	p p > j j j QCD=3 QED=0 [QED]	$7.9639 \pm 0.0010 \cdot 10^6$	$7.9472 \pm 0.0011 \cdot 10^6$	-0.21 ± 0.02
$pp \to tj$	p p > t j QCD=0 QED=2 [QED]	$1.0613 \pm 0.0001 \cdot 10^2$	$1.0539 \pm 0.0001 \cdot 10^2$	-0.70 ± 0.02

MADGRAPH5_AMC@NLO: COMPLETE NLO

- Examples:

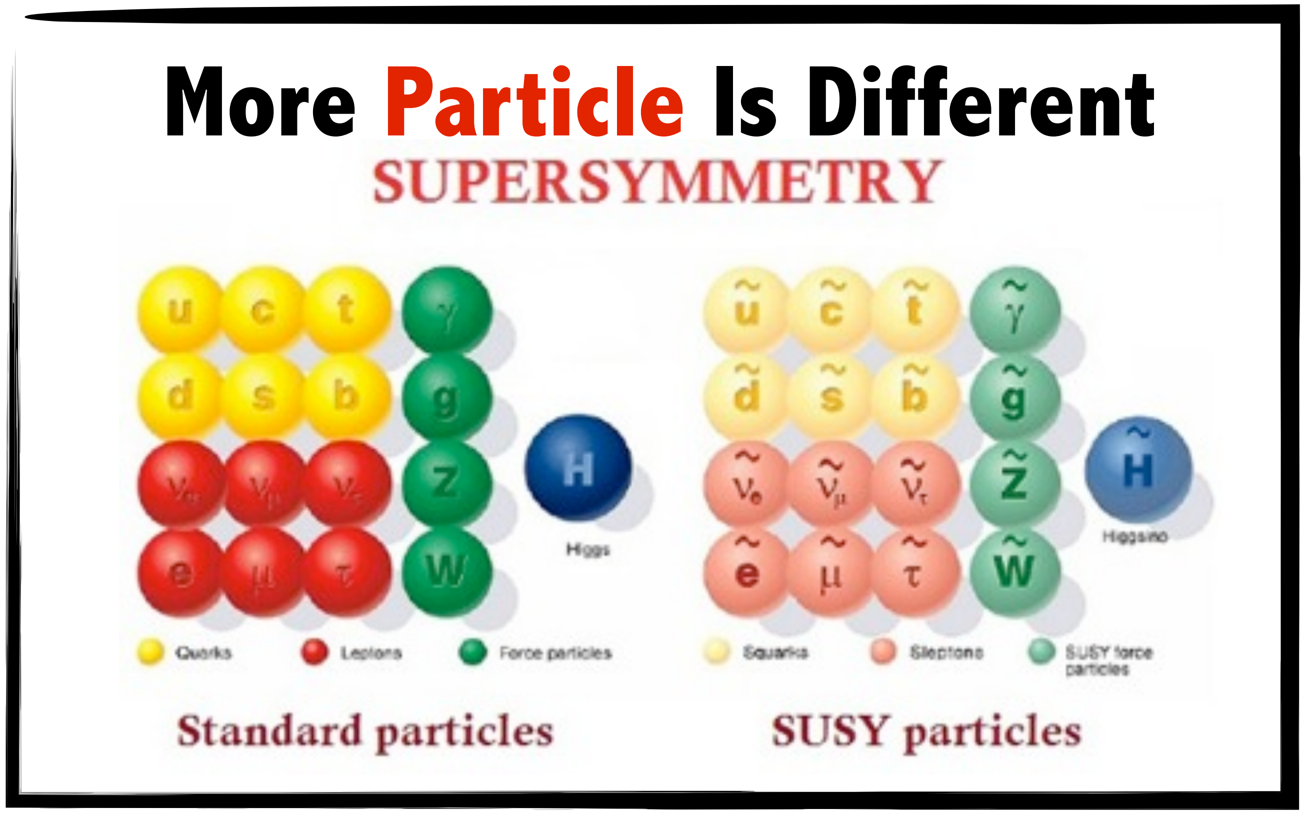
Frederix, Frixione, Hirschi, Pagani, HSS, Zaro JHEP'18

	$pp \to t\bar{t}$	$pp \to t\bar{t}Z$	$pp \to t\bar{t}W^+$	$pp \to t\bar{t}H$	$pp \to t\bar{t}j$
LO_1	$4.3803 \pm 0.0005 \cdot 10^2$ pb	$5.0463 \pm 0.0003 \cdot 10^{-1}$ pb	$2.4116 \pm 0.0001 \cdot 10^{-1}$ pb	$3.4483 \pm 0.0003 \cdot 10^{-1}$ pb	$3.0278 \pm 0.0003 \cdot 10^2$ pb
LO_2	$+0.405 \pm 0.001$ %	-0.691 ± 0.001 %	$+0.000 \pm 0.000$ %	$+0.406 \pm 0.001$ %	$+0.525 \pm 0.001$ %
LO_3	$+0.630 \pm 0.001$ %	$+2.259 \pm 0.001$ %	$+0.962 \pm 0.000$ %	$+0.702 \pm 0.001$ %	$+1.208 \pm 0.001$ %
LO_4					$+0.006 \pm 0.000$ %
NLO_1	$+46.164 \pm 0.022$ %	$+44.809 \pm 0.028$ %	$+49.504 \pm 0.015$ %	$+28.847 \pm 0.020$ %	$+26.571 \pm 0.063$ %
NLO_2	-1.075 ± 0.003 %	-0.846 ± 0.004 %	-4.541 ± 0.003 %	$+1.794 \pm 0.005$ %	-1.971 ± 0.022 %
NLO_3	$+0.552 \pm 0.002$ %	$+0.845 \pm 0.003$ %	$+12.242 \pm 0.014$ %	$+0.483 \pm 0.008$ %	$+0.292 \pm 0.007$ %
NLO_4	$+0.005 \pm 0.000$ %	-0.082 ± 0.000 %	$+0.017 \pm 0.003$ %	$+0.044 \pm 0.000$ %	$+0.009 \pm 0.000$ %
NLO_5					$+0.005 \pm 0.000$ %

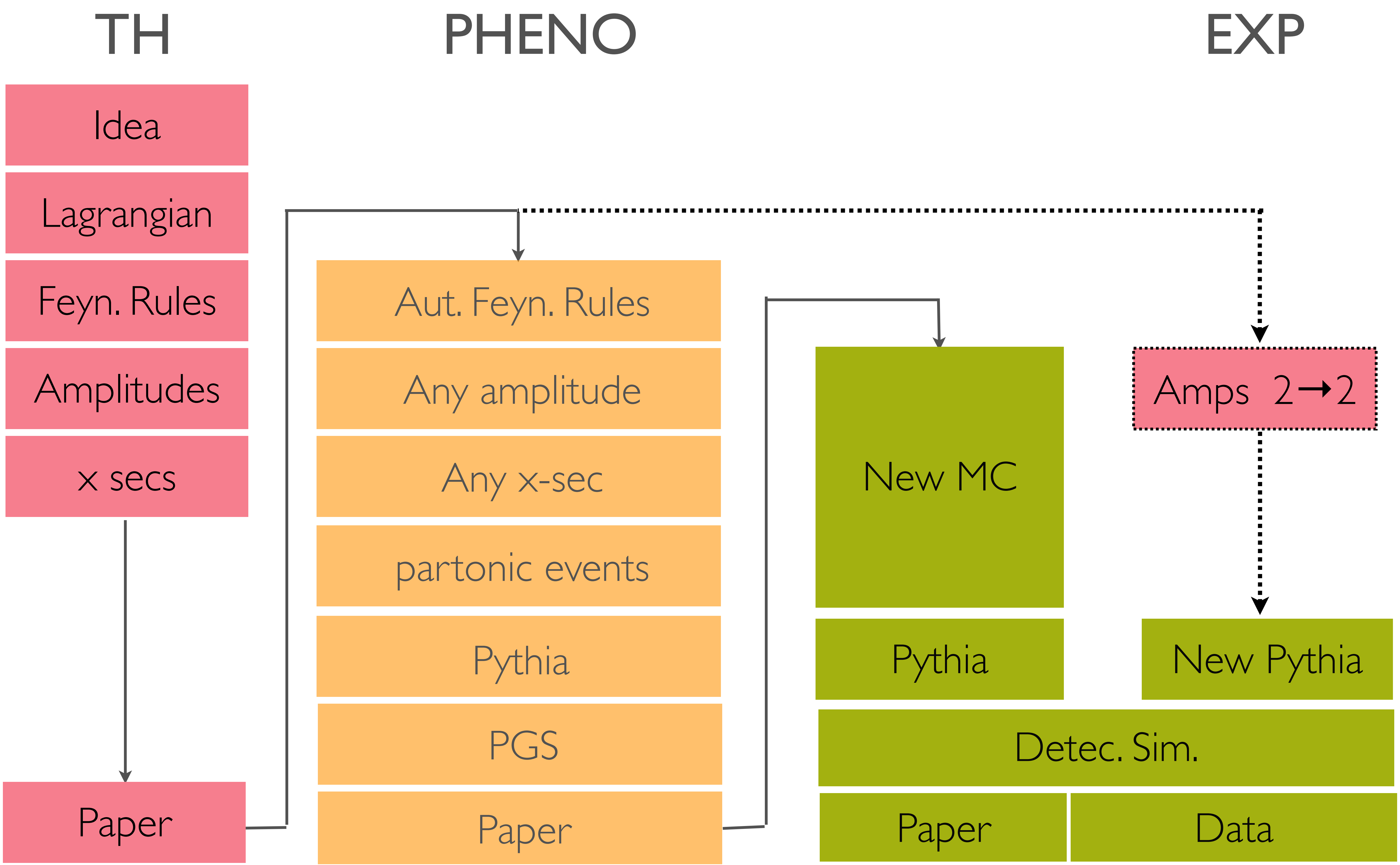


Lecture 3

Advanced NLO Topics



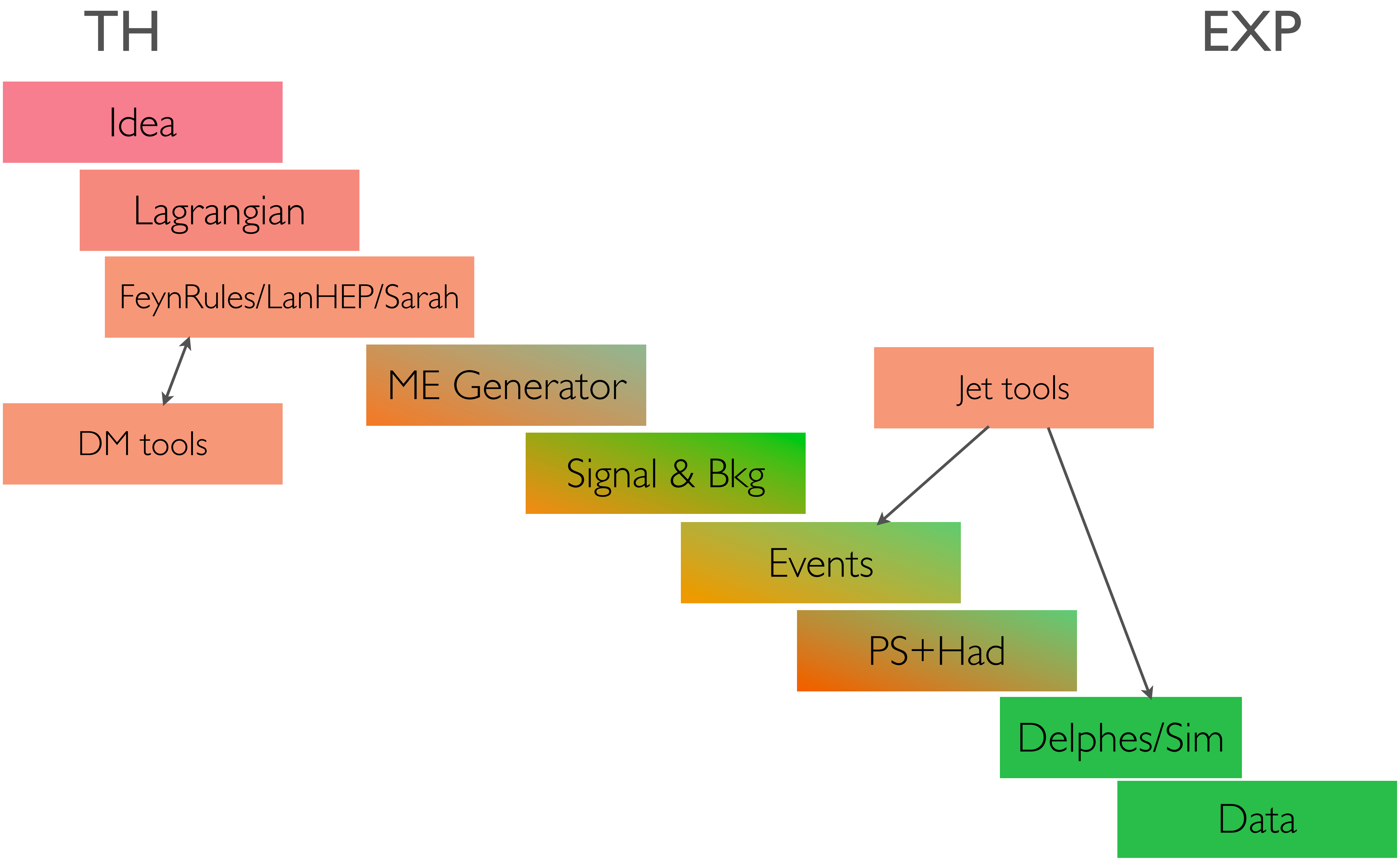
BSM TH/EXP INTERACTIONS: THE OLD WAY



BSM TH/EXP INTERACTIONS: THE OLD WAY

TH	PHENO	EXP
Idea		
Lagrangian		
	Aut. Feyn. Rules	
	Any amplitude	
	Any x-sec	
	partonic events	
		Pythia
		Detec. Sim.
		Data

BSM TH/EXP INTERACTIONS AUGMENTED

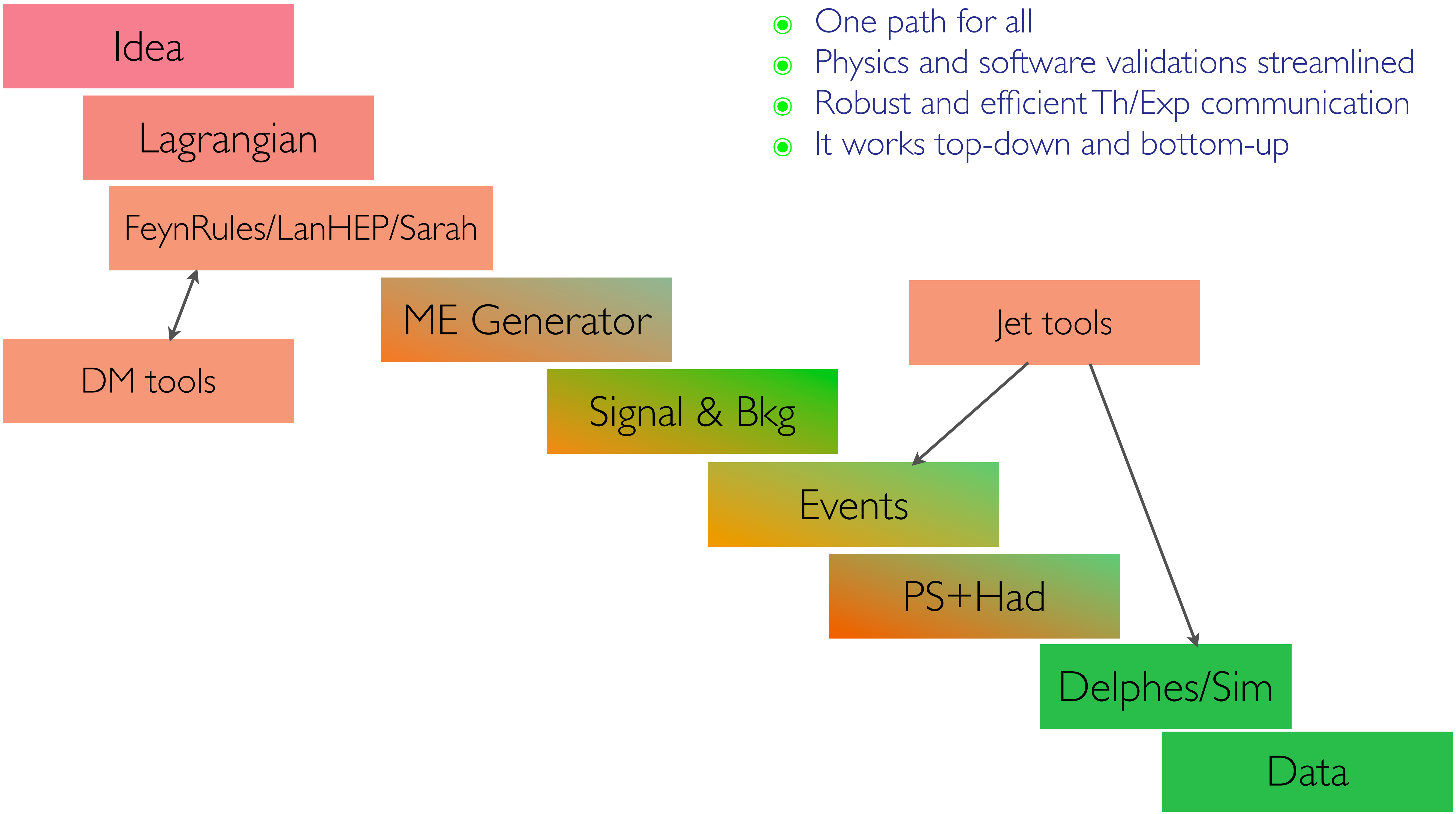


BSM TH/EXP INTERACTIONS AUGMENTED

TH

EXP

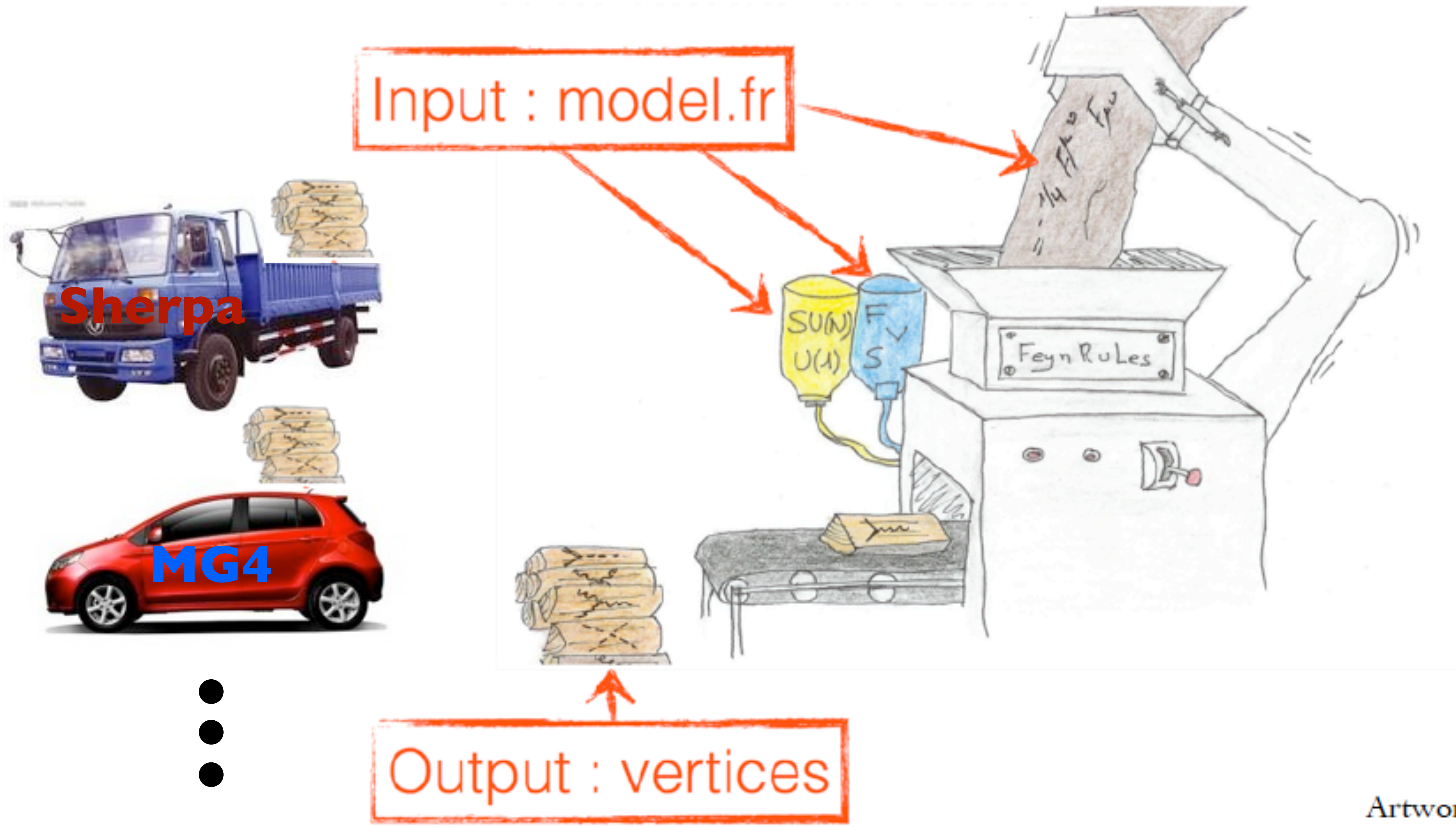
- One path for all
- Physics and software validations streamlined
- Robust and efficient Th/Exp communication
- It works top-down and bottom-up



FEYNRULES

Christensen, Duhr (CPC'09); Alloul, Christensen, Duhr, Degrande, Fuks (CPC'14)

- How to incorporate all of above information in a model file ?



Artwork by C. Degrande

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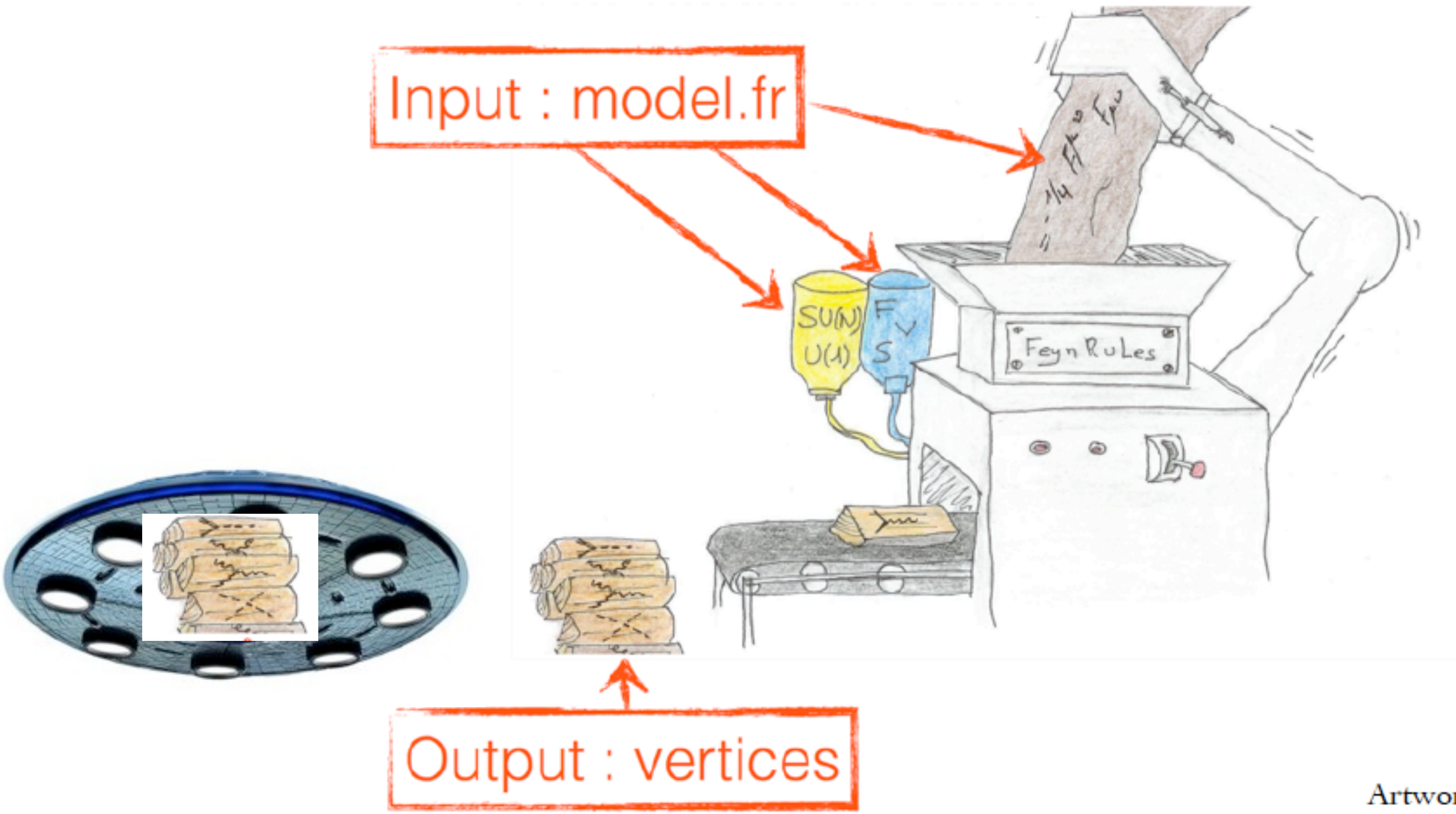


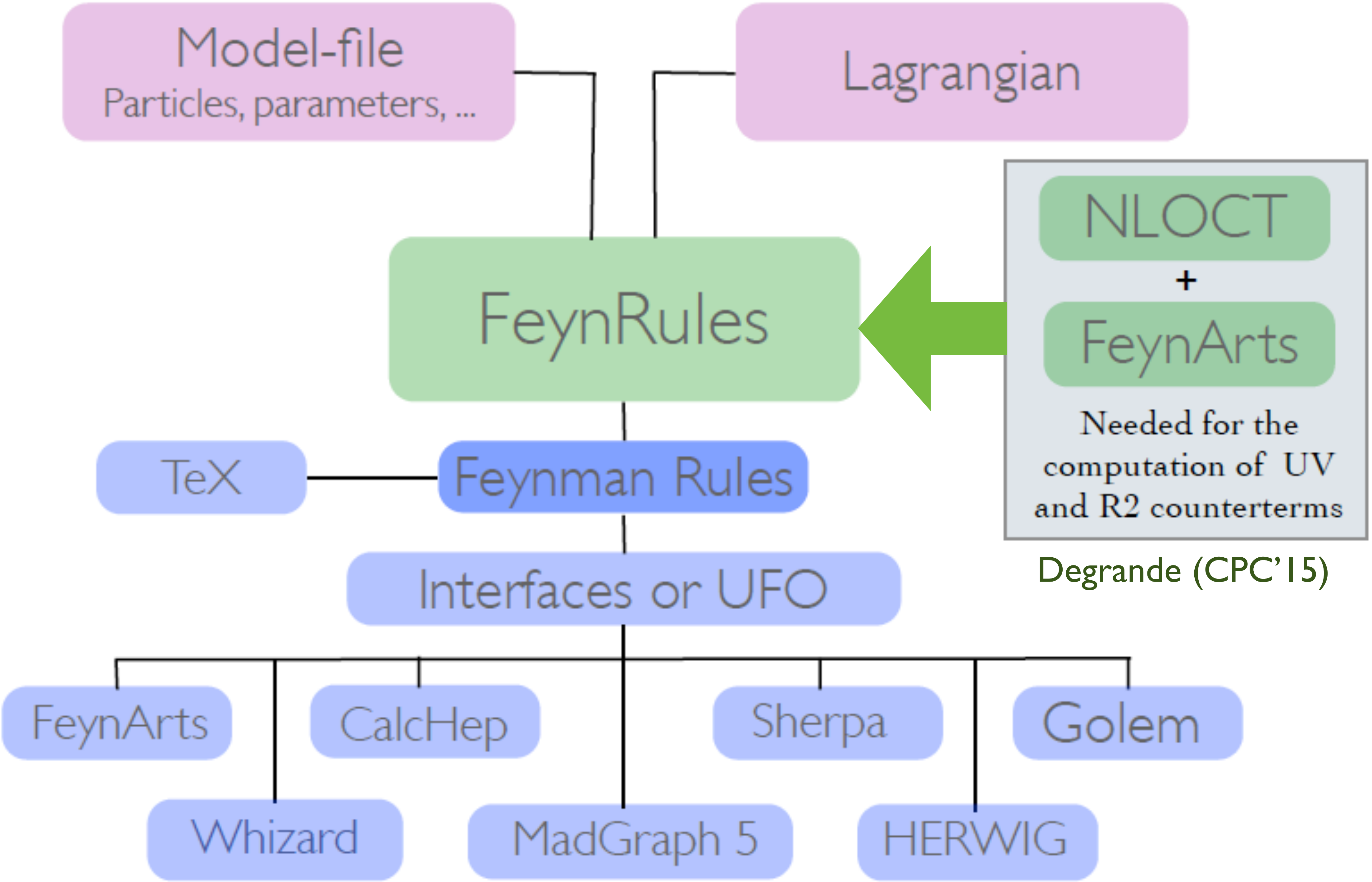
Artwork by C. Degrande

- UFO stands for Universal FeynRules Output:

Degrande, Duhr, Fuks, Grellscheid, Mattelaer, Reiter (CPC'12)

FEYNRULES: NLO

Christensen, Duhr (CPC'09); Alloul, Christensen, Duhr, Degrande, Fuks (CPC'14); Degrande (CPC'15)



Degrande (CPC'15)

A UFO MODEL

Degrande, Duhr, Fuks, Grellscheid, Mattelaer, Reiter (CPC'12)

- The UFO is a set of PYTHON files
  - Particle information (particles.py)
  - Interaction information (vertices.py, couplings.py, lorentz.py, couplings_orders.py)
  - Parameter information (parameters.py)
  - Propagator information (propagators.py)
  - Tools (function_library.py, object_library.py, write_param_card.py, decays.py)
  - NLO counterterms (CT_couplings.py, CT_parameters.py, CT_vertices.py)

For example: SUSY QCD

```
bogon:SUSYQCD_CTprm_UFO erdissshaw$ ls
CT_couplings.py        SUSYQCD_CTprm_UFO.log    couplings.py           object_library.py      propagators.py
CT_parameters.py       __init__.py              function_library.py    parameters.py          vertices.py
CT_vertices.py         coupling_orders.py       lorentz.py             particles.py           write_param_card.py
```

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For example: SUSY QCD

```
bogon:SUSYQCD_CTprm_UFO erdissshaw$ ls
CT_couplings.py      SUSYQCD_CTprm_UFO.log   couplings.py          object_library.py    propagators.py
CT_parameters.py     __init__.py             function_library.py   parameters.py        vertices.py
CT_vertices.py       coupling_orders.py      lorentz.py            particles.py         write_param_card.py
```

Particles

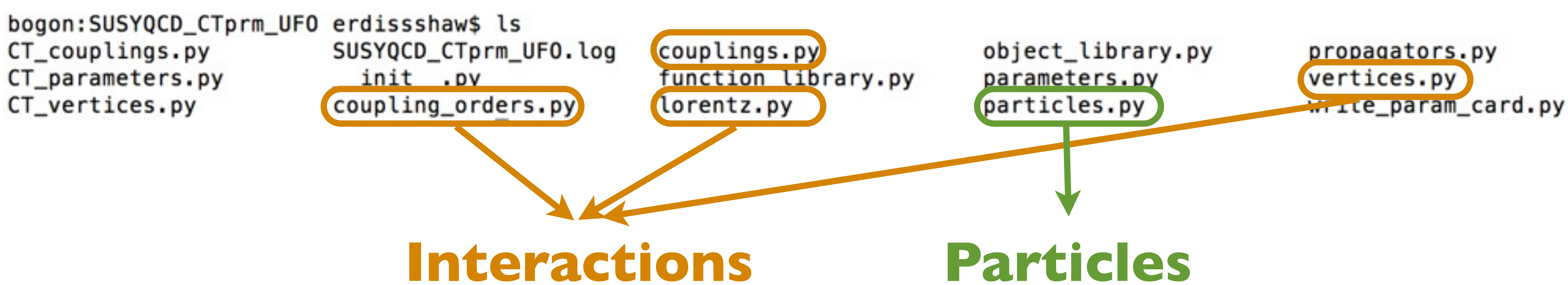
Tuesday, November 19, 19

A UFO MODEL

Degrande, Duhr, Fuks, Grellscheid, Mattelaer, Reiter (CPC'12)

- The UFO is a set of PYTHON files
  - Particle information (particles.py)
  - Interaction information (vertices.py, couplings.py, lorentz.py, couplings_orders.py)
  - Parameter information (parameters.py)
  - Propagator information (propagators.py)
  - Tools (function_library.py, object_library.py, write_param_card.py, decays.py)
  - NLO counterterms (CT_couplings.py, CT_parameters.py, CT_vertices.py)

For example: SUSY QCD



A UFO MODEL

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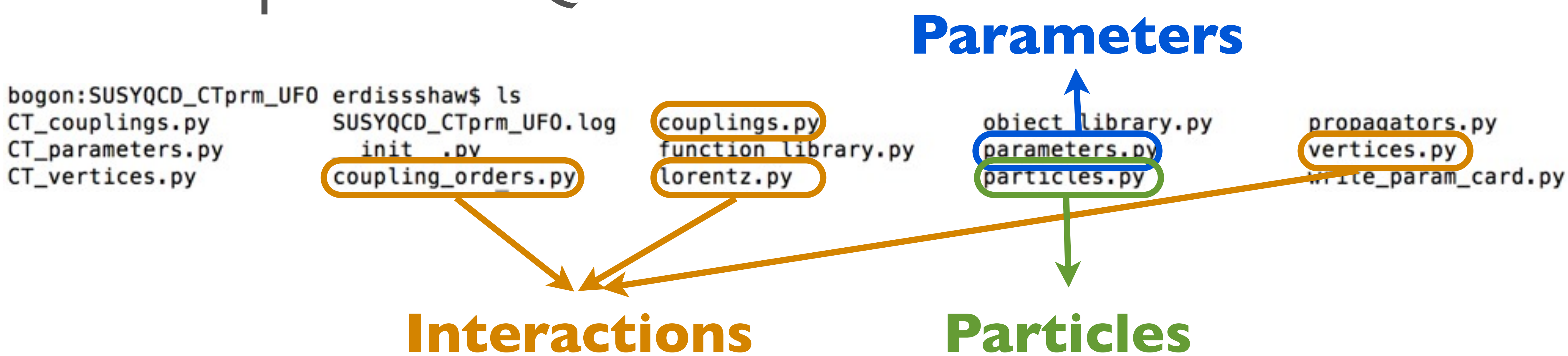
  - Tools (function_library.py, object_library.py, write_param_card.py, decays.py)
  - NLO counterterms (CT_couplings.py, CT_parameters.py, CT_vertices.py)

For example: SUSY QCD



A UFO MODEL

Degrande, Duhr, Fuks, Grellscheid, Mattelaer, Reiter (CPC'12)

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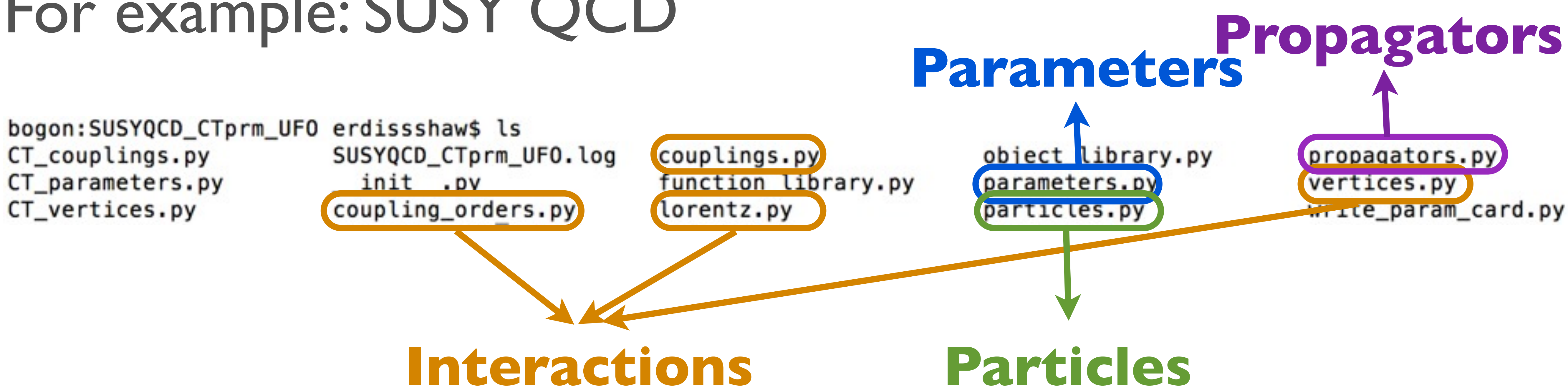
  - NLO counterterms (CT_couplings.py, CT_parameters.py, CT_vertices.py)

For example: SUSY QCD

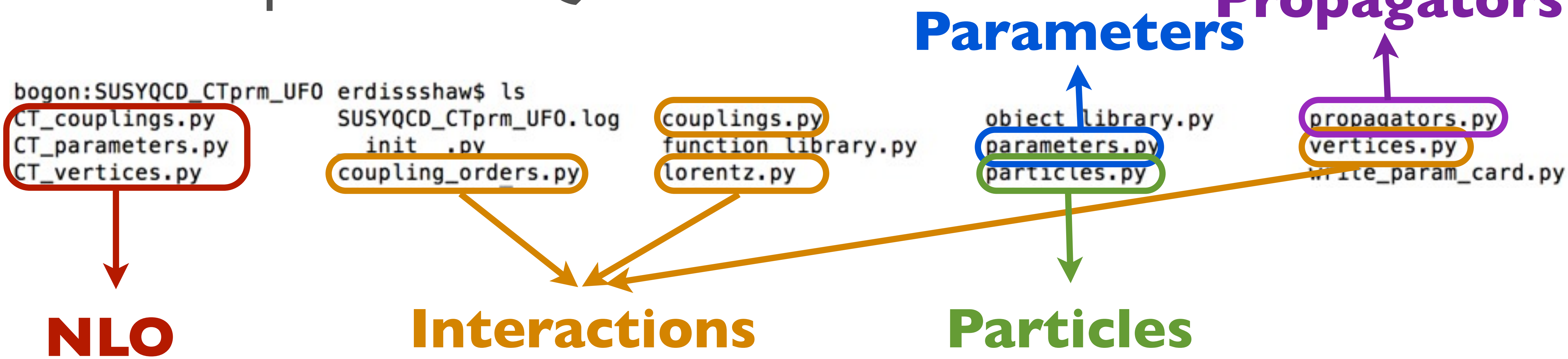


A UFO MODEL

Degrande, Duhr, Fuks, Grellscheid, Mattelaer, Reiter (CPC'12)

- The UFO is a set of PYTHON files
  - Particle information (particles.py)
  - Interaction information (vertices.py, couplings.py, lorentz.py, couplings_orders.py)
  - Parameter information (parameters.py)
  - Propagator information (propagators.py)
  - Tools (function_library.py, object_library.py, write_param_card.py, decays.py)
  - NLO counterterms (CT_couplings.py, CT_parameters.py, CT_vertices.py)

For example: SUSY QCD



A UFO MODEL

Degrande, Duhr, Fuks, Grellscheid, Mattelaer, Reiter (CPC'12)

A UFO MODEL

Degrande, Duhr, Fuks, Grellscheid, Mattelaer, Reiter (CPC'12)

- **Particles are in** particles.py
  - Instances of the particle class
  - spin, color, mass, width, PDG etc

```
go = Particle(pdg_code = 1000021,
              name = 'go',
              antiname = 'go',
              spin = 2,
              color = 8,
              mass = Param.Mgo,
              width = Param.Wgo,
              texname = 'go',
              antitexname = 'go',
              charge = 0,
              GhostNumber = 0,
              LeptonNumber = 0,
              Y = 0)
```

A UFO MODEL

Degrande, Duhr, Fuks, Grellscheid, Mattelaer, Reiter (CPC'12)

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  - Instances of the particle class
  - spin, color, mass, width, PDG etc

```
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              name = 'go',
              antiname = 'go',
              spin = 2,
              color = 8,
              mass = Param.Mgo,
              width = Param.Wgo,
              texname = 'go',
              antitexname = 'go',
              charge = 0,
              GhostNumber = 0,
              LeptonNumber = 0,
              Y = 0)
```

- **Parameters are in** parameters.py
  - External parameters are in LHA-like
  - Python-compliant formula for int. para

```
aS = Parameter(name = 'aS',
               nature = 'external',
               type = 'real',
               value = 0.1184,
               texname = '\\alpha _s',
               lhablock = 'SMINPUTS',
               lhacode = [ 3 ])

G = Parameter(name = 'G',
              nature = 'internal',
              type = 'real',
              value = '2*cmath.sqrt(aS)*cmath.sqrt(cmath.pi)',
              texname = 'G')
```

A UFO MODEL

Degrande, Duhr, Fuks, Grellscheid, Mattelaer, Reiter (CPC'12)

- **Interactions are in** vertices.py, couplings.py, lorentz.py, coupling_orders,py
  - Vertices are decomposed in a spin x color basis, coupling being coordinates
  - Example: the quartic gluon vertex can be written as

$$
\begin{aligned}
& i g_s^2 f^{a_1 a_2 b} f^{b a_3 a_4}\left(\eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3}-\eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4}\right) \\
& +i g_s^2 f^{a_1 a_3 b} f^{b a_2 a_4}\left(\eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3}-\eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4}\right) \\
& +i g_s^2 f^{a_1 a_4 b} f^{b a_2 a_3}\left(\eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4}-\eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4}\right)
\end{aligned}
\Rightarrow
\begin{aligned}
&\left(f^{a_1 a_2 b} f^{b a_3 a_4}, \quad f^{a_1 a_3 b} f^{b a_2 a_4}, f^{a_1 a_4 b} f^{b a_2 a_3}\right) \\
&\times\begin{pmatrix} i g_s^2 & 0 & 0 \\ 0 & i g_s^2 & 0 \\ 0 & 0 & i g_s^2 \end{pmatrix}
\begin{pmatrix} \eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3}-\eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4} \\ \eta^{\mu_1 \mu_4} \eta^{\mu_2 \mu_3}-\eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4} \\ \eta^{\mu_1 \mu_3} \eta^{\mu_2 \mu_4}-\eta^{\mu_1 \mu_2} \eta^{\mu_3 \mu_4} \end{pmatrix}
\end{aligned}
$$

  - vertices.py: define all Feynman rules for vertices in the model

```
V_37 = Vertex(name = 'V_37',
              particles = [ P.g, P.g, P.g, P.g ],
              color = [ 'f(-1,1,2)*f(3,4,-1)', 'f(-1,1,3)*f(2,4,-1)', 'f(-1,1,4)*f(2,3,-1)' ],
              lorentz = [ L.VVVV2, L.VVVV3, L.VVVV4 ],
              couplings = {(1,0):C.GC_20,(0,0):C.GC_20,(2,1):C.GC_20,(0,1):C.GC_19,(2,2):C.GC_19,(1,2):C.GC_19})
```

  - lorentz.py: define the Lorentz structure in the model

```
VVVV2 = Lorentz(name = 'VVVV2',
                spins = [ 3, 3, 3, 3 ],
                structure = 'Metric(1,4)*Metric(2,3)')
```

  - couplings.py: define the coupling constant in the model

```
GC_20 = Coupling(name = 'GC_20',
                 value = 'complex(0,1)*G**2',
                 order = {'QCD':2})
```

  - coupling_orders.py: define the coupling orders in the model

```
QCD = CouplingOrder(name = 'QCD',
                    expansion_order = 99,
                    hierarchy = 1,
                    perturbative_expansion = 1)
```

A UFO MODEL

Degrande, Duhr, Fuks, Grellscheid, Mattelaer, Reiter (CPC'12)

- **Interactions are in** vertices.py, couplings.py, lorentz.py, coupling_orders,py
  - Vertices are decomposed in a spin x color basis, coupling being coordinates
  - Example: the quartic gluon vertex can be written as

$$ig_s^2\, f^{a_1a_2b} f^{ba_3a_4} \left(\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}\right) + ig_s^2\, f^{a_1a_3b} f^{ba_2a_4} \left(\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}\right) + ig_s^2\, f^{a_1a_4b} f^{ba_2a_3} \left(\eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}\right) \Rightarrow \left(f^{a_1a_2b} f^{ba_3a_4},\ f^{a_1a_3b} f^{ba_2a_4},\ f^{a_1a_4b} f^{ba_2a_3}\right) \times \begin{pmatrix} ig_s^2 & 0 & 0 \\ 0 & ig_s^2 & 0 \\ 0 & 0 & ig_s^2 \end{pmatrix} \begin{pmatrix} \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} \\ \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \\ \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \end{pmatrix}$$

  - vertices.py: define all Feynman rules for vertices in the model

```
V_37 = Vertex(name = 'V_37',
              particles = [ P.g, P.g, P.g, P.g ],
              color = [ 'f(-1,1,2)*f(3,4,-1)', 'f(-1,1,3)*f(2,4,-1)', 'f(-1,1,4)*f(2,3,-1)' ],
              lorentz = [ L.VVVV2, L.VVVV3, L.VVVV4 ],
              couplings = {(1,0):C.GC_20,(0,0):C.GC_20,(2,1):C.GC_20,(0,1):C.GC_19,(2,2):C.GC_19,(1,2):C.GC_19})
```

  - lorentz.py: define the Lorentz structure in the model

```
VVVV2 = Lorentz(name = 'VVVV2',
                spins = [ 3, 3, 3, 3 ],
                structure = 'Metric(1,4)*Metric(2,3)')
```

  - couplings.py: define the coupling constant in the model

```
GC_20 = Coupling(name = 'GC_20',
                 value = 'complex(0,1)*G**2',
                 order = {'QCD':2})
```

  - coupling_orders.py: define the coupling orders in the model

```
QCD = CouplingOrder(name = 'QCD',
                    expansion_order = 99,
                    hierarchy = 1,
                    perturbative_expansion = 1)
```

Make sure > 0 for NLO QCD

UFO@NLO

UFO@NLO

- Provide renormalization scale in parameters.py

```
MU_R = Parameter(name = 'MU_R',
                 nature = 'external',
                 type = 'real',
                 value = 91.188,
                 texname = '\\text{\\mu_r}',
                 lhablock = 'LOOP',
                 lhacode = [1])
```

UFO@NLO

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```
MU_R = Parameter(name = 'MU_R',
                 nature = 'external',
                 type = 'real',
                 value = 91.188,
                 texname = '\\text{\\mu_r}',
                 lhablock = 'LOOP',
                 lhacode = [1])
```

- CT_vertices.py:UV, R2 counter term vertices

```
V_2 = CTVertex(name = 'V_2',
               type = 'R2',
               particles = [ P.g, P.g, P.g, P.g ],
               color = [ 'd(-1,1,3)*d(-1,2,4)', 'd(-1,1,3)*f(-1,2,4)', 'd(-1,1,4)*d(-1,2,3)', 'd(-1,1,4)*f(-1,2,3)', 'd(-1,2,3)*f(-1,1,4)', 'd(-1,2,4)*f(-1,1,3)', 'f(-1,1\
,2)*f(-1,3,4)', 'f(-1,1,3)*f(-1,2,4)', 'f(-1,1,4)*f(-1,2,3)', 'Identity(1,2)*Identity(3,4)', 'Identity(1,3)*Identity(2,4)', 'Identity(1,4)*Identity(2,3)' ],
               lorentz = [ L.VVVV2, L.VVVV3, L.VVVV4 ],
               loop_particles = [ [ [P.b], [P.c], [P.d], [P.s], [P.t], [P.u] ], [ [P.g] ], [ [P.go] ] ],
               couplings = {(2,0,0):C.R2GC_101_4,(2,0,1):C.R2GC_100_3,(2,0,2):C.R2GC_100_2,(0,0,0):C.R2GC_101_4,(0,0,1):C.R2GC_100_3,(0,0,2):C.R2GC_100_2,(4,0,0):C.R2GC_9\
9_171,(4,0,1):C.R2GC_99_172,(4,0,2):C.R2GC_99_173,(3,0,0):C.R2GC_99_171,(3,0,1):C.R2GC_99_172,(3,0,2):C.R2GC_99_173,(8,0,0):C.R2GC_100_1,(8,0,1):C.R2GC_100_2,(8,0,2):C.R2\
GC_100_3,(6,0,0):C.R2GC_110_22,(6,0,1):C.R2GC_112_26,(6,0,2):C.R2GC_110_23,(7,0,0):C.R2GC_111_24,(7,0,1):C.R2GC_105_11,(7,0,2):C.R2GC_111_25,(5,0,0):C.R2GC_99_171,(5,0,1)\
:C.R2GC_99_172,(5,0,2):C.R2GC_99_173,(1,0,0):C.R2GC_99_171,(1,0,1):C.R2GC_99_172,(1,0,2):C.R2GC_99_173,(11,0,0):C.R2GC_103_7,(11,0,1):C.R2GC_103_8,(11,0,2):C.R2GC_103_9,(\
10,0,0):C.R2GC_103_7,(10,0,1):C.R2GC_103_8,(10,0,2):C.R2GC_103_9,(9,0,1):C.R2GC_102_5,(9,0,2):C.R2GC_102_6,(2,1,0):C.R2GC_101_4,(2,1,1):C.R2GC_100_3,(2,1,2):C.R2GC_100_2,\
(0,1,0):C.R2GC_101_4,(0,1,1):C.R2GC_100_3,(0,1,2):C.R2GC_100_2,(4,1,0):C.R2GC_99_171,(4,1,1):C.R2GC_99_172,(4,1,2):C.R2GC_99_173,(3,1,0):C.R2GC_99_171,(3,1,1):C.R2GC_99_1\
72,(3,1,2):C.R2GC_99_173,(8,1,0):C.R2GC_100_1,(8,1,1):C.R2GC_105_11,(8,1,2):C.R2GC_100_3,(6,1,0):C.R2GC_115_29,(6,1,1):C.R2GC_115_30,(6,1,2):C.R2GC_115_31,(7,1,0):C.R2GC_\
111_24,(7,1,1):C.R2GC_100_2,(7,1,2):C.R2GC_111_25,(5,1,0):C.R2GC_99_171,(5,1,1):C.R2GC_99_172,(5,1,2):C.R2GC_99_173,(1,1,0):C.R2GC_99_171,(1,1,1):C.R2GC_99_172,(1,1,2):C.\
R2GC_99_173,(11,1,0):C.R2GC_103_7,(11,1,1):C.R2GC_103_8,(11,1,2):C.R2GC_103_9,(10,1,0):C.R2GC_103_7,(10,1,1):C.R2GC_103_8,(10,1,2):C.R2GC_103_9,(9,1,1):C.R2GC_102_5,(9,1,\
2):C.R2GC_102_6,(0,2,0):C.R2GC_101_4,(0,2,1):C.R2GC_100_3,(0,2,2):C.R2GC_100_2,(2,2,0):C.R2GC_101_4,(2,2,1):C.R2GC_100_3,(2,2,2):C.R2GC_100_2,(5,2,0):C.R2GC_99_171,(5,2,1\
):C.R2GC_99_172,(5,2,2):C.R2GC_99_173,(1,2,0):C.R2GC_99_171,(1,2,1):C.R2GC_99_172,(1,2,2):C.R2GC_99_173,(7,2,0):C.R2GC_114_27,(7,2,1):C.R2GC_104_10,(7,2,2):C.R2GC_114_28,\
(4,2,0):C.R2GC_99_171,(4,2,1):C.R2GC_99_172,(4,2,2):C.R2GC_99_173,(3,2,0):C.R2GC_99_171,(3,2,1):C.R2GC_99_172,(3,2,2):C.R2GC_99_173,(8,2,0):C.R2GC_100_1,(8,2,1):C.R2GC_10\
4_10,(8,2,2):C.R2GC_100_3,(6,2,0):C.R2GC_110_22,(6,2,2):C.R2GC_110_23,(11,2,0):C.R2GC_103_7,(11,2,1):C.R2GC_103_8,(11,2,2):C.R2GC_103_9,(10,2,0):C.R2GC_103_7,(10,2,1):C.R\
2GC_103_8,(10,2,2):C.R2GC_103_9,(9,2,1):C.R2GC_102_5,(9,2,2):C.R2GC_102_6})
```

UFO@NLO

- Provide renormalization scale in parameters.py

```
MU_R = Parameter(name = 'MU_R',
                 nature = 'external',
                 type = 'real',
                 value = 91.188,
                 texname = '\\text{\\mu_r}',
                 lhablock = 'LOOP',
                 lhacode = [1])
```

- CT_vertices.py:UV, R2 counter term vertices

```
V_351 = CTVertex(name = 'V_351',
                 type = 'UV',
                 particles = [ P.g, P.g, P.g, P.g ],
                 color = [ 'd(-1,1,3)*d(-1,2,4)', 'd(-1,1,3)*f(-1,2,4)', 'd(-1,1,4)*d(-1,2,3)', 'd(-1,1,4)*f(-1,2,3)', 'd(-1,2,3)*f(-1,1,4)', 'd(-1,2,4)*f(-1,1,3)', 'f(-1,1,2)*f(-1,3,4)', 'f(-1,1,3)*f\
(-1,2,4)', 'f(-1,1,4)*f(-1,2,3)', 'Identity(1,2)*Identity(3,4)', 'Identity(1,3)*Identity(2,4)', 'Identity(1,4)*Identity(2,3)' ],
                 lorentz = [ L.VVVV2, L.VVVV3, L.VVVV4 ],
                 loop_particles = [ [ [P.b] ], [ [P.b], [P.c], [P.d], [P.s], [P.sbL], [P.sbR], [P.scL], [P.scR], [P.sdL], [P.sdR], [P.ssL], [P.ssR], [P.stL], [P.stR], [P.suL], [P.suR], [P.t], [P.u] ],\
 [ [P.b], [P.c], [P.d], [P.s], [P.t], [P.u] ], [ [P.c] ], [ [P.d] ], [ [P.g] ], [ [P.ghG] ], [ [P.go] ], [ [P.s] ], [ [P.sbL] ], [ [P.sbL], [P.sbR], [P.scL], [P.scR], [P.sdL], [P.sdR], [P.ssL], [P.ssR\
], [P.stL], [P.stR], [P.suL], [P.suR] ], [ [P.sbR] ], [ [P.scL] ], [ [P.scR] ], [ [P.sdL] ], [ [P.sdR] ], [ [P.ssL] ], [ [P.ssR] ], [ [P.stL] ], [ [P.stR] ], [ [P.suL] ], [ [P.suR] ], [ [P.t] ], [ [P.\
u] ] ],
                 couplings = {(2,0,5):C.UVGC_100_2,(2,0,6):C.UVGC_100_1,(0,0,5):C.UVGC_100_2,(0,0,6):C.UVGC_100_1,(4,0,5):C.UVGC_99_1085,(4,0,6):C.UVGC_99_1086,(3,0,5):C.UVGC_99_1085,(3,0,6):C.UVGC_99\
_1086,(8,0,5):C.UVGC_100_1,(8,0,6):C.UVGC_100_2,(6,0,0):C.UVGC_112_137,(6,0,3):C.UVGC_112_138,(6,0,4):C.UVGC_112_139,(6,0,5):C.UVGC_112_140,(6,0,6):C.UVGC_112_141,(6,0,7):C.UVGC_112_142,(6,0,8):C.UVGC\
_112_143,(6,0,9):C.UVGC_112_144,(6,0,11):C.UVGC_112_145,(6,0,12):C.UVGC_112_146,(6,0,13):C.UVGC_112_147,(6,0,14):C.UVGC_112_148,(6,0,15):C.UVGC_112_149,(6,0,16):C.UVGC_112_150,(6,0,17):C.UVGC_112_151,\
(6,0,18):C.UVGC_112_152,(6,0,19):C.UVGC_112_153,(6,0,20):C.UVGC_112_154,(6,0,21):C.UVGC_112_155,(6,0,22):C.UVGC_112_156,(6,0,23):C.UVGC_112_157,(7,0,0):C.UVGC_112_137,(7,0,3):C.UVGC_112_138,(7,0,4):C.\
UVGC_112_139,(7,0,5):C.UVGC_105_31,(7,0,6):C.UVGC_113_158,(7,0,7):C.UVGC_112_142,(7,0,8):C.UVGC_112_143,(7,0,9):C.UVGC_112_144,(7,0,11):C.UVGC_112_145,(7,0,12):C.UVGC_112_146,(7,0,13):C.UVGC_112_147,(\
7,0,14):C.UVGC_112_148,(7,0,15):C.UVGC_112_149,(7,0,16):C.UVGC_112_150,(7,0,17):C.UVGC_112_151,(7,0,18):C.UVGC_112_152,(7,0,19):C.UVGC_112_153,(7,0,20):C.UVGC_112_154,(7,0,21):C.UVGC_112_155,(7,0,22):\
C.UVGC_112_156,(7,0,23):C.UVGC_112_157,(5,0,5):C.UVGC_99_1085,(5,0,6):C.UVGC_99_1086,(1,0,5):C.UVGC_99_1085,(1,0,6):C.UVGC_99_1086,(11,0,5):C.UVGC_103_5,(11,0,6):C.UVGC_103_6,(10,0,5):C.UVGC_103_5,(10\
,0,6):C.UVGC_103_6,(9,0,5):C.UVGC_102_3,(9,0,6):C.UVGC_102_4,(2,1,5):C.UVGC_100_2,(2,1,6):C.UVGC_100_1,(0,1,5):C.UVGC_100_2,(0,1,6):C.UVGC_100_1,(4,1,5):C.UVGC_99_1085,(4,1,6):C.UVGC_99_1086,(3,1,5):C\
.UVGC_99_1085,(3,1,6):C.UVGC_99_1086,(8,1,0):C.UVGC_105_28,(8,1,3):C.UVGC_105_29,(8,1,4):C.UVGC_105_30,(8,1,5):C.UVGC_105_31,(8,1,6):C.UVGC_105_32,(8,1,7):C.UVGC_105_33,(8,1,8):C.UVGC_105_34,(8,1,9):C\
.UVGC_105_35,(8,1,11):C.UVGC_105_36,(8,1,12):C.UVGC_105_37,(8,1,13):C.UVGC_105_38,(8,1,14):C.UVGC_105_39,(8,1,15):C.UVGC_105_40,(8,1,16):C.UVGC_105_41,(8,1,17):C.UVGC_105_42,(8,1,18):C.UVGC_105_43,(8,\
1,19):C.UVGC_105_44,(8,1,20):C.UVGC_105_45,(8,1,21):C.UVGC_105_46,(8,1,22):C.UVGC_105_47,(8,1,23):C.UVGC_105_48,(6,1,0):C.UVGC_114_159,(6,1,3):C.UVGC_114_160,(6,1,4):C.UVGC_114_161,(6,1,5):C.UVGC_115_\
179,(6,1,6):C.UVGC_115_180,(6,1,7):C.UVGC_114_163,(6,1,8):C.UVGC_114_164,(6,1,9):C.UVGC_115_181,(6,1,11):C.UVGC_115_182,(6,1,12):C.UVGC_115_183,(6,1,13):C.UVGC_115_184,(6,1,14):C.UVGC_115_185,(6,1,15)\
:C.UVGC_115_186,(6,1,16):C.UVGC_115_187,(6,1,17):C.UVGC_115_188,(6,1,18):C.UVGC_115_189,(6,1,19):C.UVGC_115_190,(6,1,20):C.UVGC_115_191,(6,1,21):C.UVGC_115_192,(6,1,22):C.UVGC_114_177,(6,1,23):C.UVGC_\
114_178,(7,1,1):C.UVGC_110_133,(7,1,5):C.UVGC_100_1,(7,1,6):C.UVGC_111_136,(7,1,7):C.UVGC_110_134,(5,1,5):C.UVGC_99_1085,(5,1,6):C.UVGC_99_1086,(1,1,5):C.UVGC_99_1085,(1,1,6):C.UVGC_99_1086,(11,1,5):C\
.UVGC_103_5,(11,1,6):C.UVGC_103_6,(10,1,5):C.UVGC_103_5,(10,1,6):C.UVGC_103_6,(9,1,5):C.UVGC_102_3,(9,1,6):C.UVGC_102_4,(0,2,5):C.UVGC_100_2,(0,2,6):C.UVGC_100_1,(2,2,5):C.UVGC_100_2,(2,2,6):C.UVGC_10\
0_1,(5,2,5):C.UVGC_99_1085,(5,2,6):C.UVGC_99_1086,(1,2,5):C.UVGC_99_1085,(1,2,6):C.UVGC_99_1086,(7,2,0):C.UVGC_114_159,(7,2,3):C.UVGC_114_160,(7,2,4):C.UVGC_114_161,(7,2,5):C.UVGC_104_10,(7,2,6):C.UVG\
C_114_162,(7,2,7):C.UVGC_114_163,(7,2,8):C.UVGC_114_164,(7,2,9):C.UVGC_114_165,(7,2,11):C.UVGC_114_166,(7,2,12):C.UVGC_114_167,(7,2,13):C.UVGC_114_168,(7,2,14):C.UVGC_114_169,(7,2,15):C.UVGC_114_170,(\
7,2,16):C.UVGC_114_171,(7,2,17):C.UVGC_114_172,(7,2,18):C.UVGC_114_173,(7,2,19):C.UVGC_114_174,(7,2,20):C.UVGC_114_175,(7,2,21):C.UVGC_114_176,(7,2,22):C.UVGC_114_177,(7,2,23):C.UVGC_114_178,(4,2,5):C\
.UVGC_99_1085,(4,2,6):C.UVGC_99_1086,(3,2,5):C.UVGC_99_1085,(3,2,6):C.UVGC_99_1086,(8,2,0):C.UVGC_104_7,(8,2,3):C.UVGC_104_8,(8,2,4):C.UVGC_104_9,(8,2,5):C.UVGC_104_10,(8,2,6):C.UVGC_104_11,(8,2,7):C.\
UVGC_104_12,(8,2,8):C.UVGC_104_13,(8,2,9):C.UVGC_104_14,(8,2,11):C.UVGC_104_15,(8,2,12):C.UVGC_104_16,(8,2,13):C.UVGC_104_17,(8,2,14):C.UVGC_104_18,(8,2,15):C.UVGC_104_19,(8,2,16):C.UVGC_104_20,(8,2,1\
7):C.UVGC_104_21,(8,2,18):C.UVGC_104_22,(8,2,19):C.UVGC_104_23,(8,2,20):C.UVGC_104_24,(8,2,21):C.UVGC_104_25,(8,2,22):C.UVGC_104_26,(8,2,23):C.UVGC_104_27,(6,2,2):C.UVGC_110_133,(6,2,6):C.UVGC_102_3,(\
6,2,7):C.UVGC_110_134,(6,2,10):C.UVGC_110_135,(11,2,5):C.UVGC_103_5,(11,2,6):C.UVGC_103_6,(10,2,5):C.UVGC_103_5,(10,2,6):C.UVGC_103_6,(9,2,5):C.UVGC_102_3,(9,2,6):C.UVGC_102_4})
```

UFO@NLO

- Provide renormalization scale in parameters.py

```
MU_R = Parameter(name = 'MU_R',
                 nature = 'external',
                 type = 'real',
                 value = 91.188,
                 texname = '\\text{\\mu_r}',
                 lhablock = 'LOOP',
                 lhacode = [1])
```

- CT_vertices.py:UV, R2 counter term vertices

- CT_couplings.py: couplings for UV and R2 counter terms

```
UVGC_104_23 = Coupling(name = 'UVGC_104_23',
                       value = '-((FRCTdeltaxaSxstR*complex(0,1)*G**2)/aS) - 2*FRCTdeltaZxGGxstR*complex(0,1)*G**2 + (complex(0,1)*G**4*invFREps)/(32.*cmath.pi**2)',
                       order = {'QCD':4})
```

UFO@NLO

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                       order = {'QCD':4})
```

- CT_parameters.py: parameters for UV and R2

```
FRCTdeltaZxttLxtG = CTParameter(name = 'FRCTdeltaZxttLxtG',
                                type = 'complex',
                                value = {-1:'-G**2/(6.*cmath.pi**2)',0:'-G**2/(3.*cmath.pi**2) + (G**2*reglog(MT/MU_R))/(2.*cmath.pi**2)'},
                                texname = 'FRCTdeltaZxttLxtG')
```

UFO@NLO

- Provide renormalization scale in parameters.py

```
MU_R = Parameter(name = 'MU_R',
                 nature = 'external',
                 type = 'real',
                 value = 91.188,
                 texname = '\\text{\\mu_r}',
                 lhablock = 'LOOP',
                 lhacode = [1])
```

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- CT_couplings.py: couplings for UV and R2 counter terms

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```

- CT_parameters.py: parameters for UV and R2

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                                texname = 'FRCTdeltaZxttLxtG')
```

coefficient of $\frac{1}{\epsilon}$

UFO@NLO

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                 nature = 'external',
                 type = 'real',
                 value = 91.188,
                 texname = '\\text{\\mu_r}',
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                 lhacode = [1])
```

- CT_vertices.py:UV, R2 counter term vertices

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                       order = {'QCD':4})
```

- CT_parameters.py: parameters for UV and R2

```
FRCTdeltaZxttLxtG = CTParameter(name = 'FRCTdeltaZxttLxtG',
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                                value = {-1:'-G**2/(6.*cmath.pi**2)',0:'-G**2/(3.*cmath.pi**2) + (G**2*reglog(MT/MU_R))/(2.*cmath.pi**2)'},
                                texname = 'FRCTdeltaZxttLxtG')
```

coefficient of $\frac{1}{\epsilon}$ finite piece

UFO@NLO

- Provide renormalization scale in parameters.py

```
MU_R = Parameter(name = 'MU_R',
                 nature = 'external',
                 type = 'real',
                 value = 91.188,
                 texname = '\\text{\\mu_r}',
                 lhablock = 'LOOP',
                 lhacode = [1])
```

- CT_vertices.py:UV, R2 counter term vertices

- CT_couplings.py: couplings for UV and R2 counter terms

```
UVGC_104_23 = Coupling(name = 'UVGC_104_23',
                       value = '-((FRCTdeltaxaSxstR*complex(0,1)*G**2)/aS) - 2*FRCTdeltaZxGGxstR*complex(0,1)*G**2 + (complex(0,1)*G**4*invFREps)/(32.*cmath.pi**2)',
                       order = {'QCD':4})
```

- CT_parameters.py: parameters for UV and R2

```
FRCTdelFRCTdeltaZxttLxgostL = CTParameter(name = 'FRCTdeltaZxttLxgostL',
                              type = 'complex',
                              value = {0:'( 0 if 2*Mgo*MstL + MT**2>=Mgo**2 + MstL**2 and MT**2<=(Mgo + MstL)**2 else ( 0 if Mgo==MstL else ( 0 if Mgo==MT else ( 0 if MstL==MT else \
(G**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2))/(12.*cmath.pi**2*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4\
 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)) + (G**2*Mgo**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2\ *2)'},
))/MU_R**2))/(12.*cmath.pi**2*MT**2*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)) - (G**2*MstL**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**\
2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2))/(12.*cmath.pi**2*MT**2*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 \
- MT**2/MU_R**2)**2)) - (G**2*Mgo**4*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog(Mgo/MstL))/(12.*cm\
ath.pi**2*MT**4*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)) + (G**2*Mgo**2*MstL**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + \
MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog(Mgo/MstL))/(6.*cmath.pi**2*MT**4*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R*\
*2 - MT**2/MU_R**2)**2)) - (G**2*MstL**4*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog(Mgo/MstL))/(12\
.*cmath.pi**2*MT**4*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)) - (G**2*Mgo**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**\
2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog(Mgo/MstL))/(12.*cmath.pi**2*MT**2*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 \
- MT**2/MU_R**2)**2)) + (G**2*MstL**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog(Mgo/MstL))/(12.*c\
math.pi**2*MT**2*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)) ) ) ) ) + ( ( 0 if Mgo==MstL else ( 0 if Mgo==MT else ( 0 if MstL==MT el\
se (G**2*Mgo*MstL*re((-(MU_R**2*(Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2 + cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)))/(2.*\
Mgo*MstL) - (MU_R**2*(-(Mgo**2/MU_R**2) - MstL**2/MU_R**2 + MT**2/MU_R**2 + cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)))/(2.*Mgo*MstL\
))*reglog((MU_R**2*(Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2 + cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)))/(2.*Mgo*MstL))))/\
(12.*cmath.pi**2*MT**2) ) ) ) if 2*Mgo*MstL + MT**2>=Mgo**2 + MstL**2 and MT**2<(Mgo + MstL)**2 else 0 ) + ( ( 0 if Mgo==MstL else ( 0 if Mgo==MT else ( 0 if MstL==MT else (MU_R**2*G**2*\
Mgo**2*re(((MT**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2))/MU_R**2 + (-(Mgo**2/MU_R**2) + MstL**2/MU_R\
**2)*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog(Mgo/MstL) + (MstL**4/MU_R**4 + (Mgo**2*(Mgo**2/MU_\
R**2 - MT**2/MU_R**2))/MU_R**2 - (MstL**2*((2*Mgo**2)/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog((MU_R**2*(Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2 + cmath.sqrt((-4*Mgo**2*MstL**2\
)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)))/(2.*Mgo*MstL)))/cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)))/(12\
.*cmath.pi**2*MT**4) - (MU_R**2*G**2*MstL**2*re(((MT**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2))/MU_R*\
*2 + (-(Mgo**2/MU_R**2) + MstL**2/MU_R**2)*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog(Mgo/MstL) + \
(MstL**4/MU_R**4 + (Mgo**2*(Mgo**2/MU_R**2 - MT**2/MU_R**2))/MU_R**2 - (MstL**2*((2*Mgo**2)/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog((MU_R**2*(Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/M\
U_R**2 + cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)))/(2.*Mgo*MstL)))/cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL\
**2/MU_R**2 - MT**2/MU_R**2)**2)))/(12.*cmath.pi**2*MT**4) + (MU_R**2*G**2*re(((MT**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + M\
T**2/MU_R**2))/MU_R**2))/MU_R**2 + (-(Mgo**2/MU_R**2) + MstL**2/MU_R**2)*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2)\
```

UFO@NLO

- Provide renormalization scale in parameters.py

```
MU_R = Parameter(name = 'MU_R',
                 nature = 'external',
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                 value = 91.188,
                 texname = '\\text{\\mu_r}',
                 lhablock = 'LOOP',
                 lhacode = [1])
```

- CT_vertices.py:UV, R2 counter term vertices

- CT_couplings.py: couplings for UV and R2 counter terms

```
UVGC_104_23 = Coupling(name = 'UVGC_104_23',
                       value = '-((FRCTdeltaxaSxstR*complex(0,1)*G**2)/aS) - 2*FRCTdeltaZxGGxstR*complex(0,1)*G**2 + (complex(0,1)*G**4*invFREps)/(32.*cmath.pi**2)',
                       order = {'QCD':4})
```

- CT_parameters.py: parameters for UV and R2

```
FRCTdeltaZxttLxgostL = CTParameter(name = 'FRCTdeltaZxttLxgostL',
                                   type = 'complex',
                                   value = {0:'( 0 if 2*Mgo*MstL + MT**2>=Mgo**2 + MstL**2 and MT**2<=(Mgo + MstL)**2 else ( 0 if Mgo==MstL else ( 0 if Mgo==MT else ( 0 if MstL==MT else \
(G**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2))/(12.*cmath.pi**2*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4\
 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)) + (G**2*Mgo**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2\
))/MU_R**2))/(12.*cmath.pi**2*MT**2*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)) - (G**2*MstL**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**\
2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2))/(12.*cmath.pi**2*MT**2*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 \
- MT**2/MU_R**2)**2)) - (G**2*Mgo**4*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog(Mgo/MstL))/(12.*cm\
ath.pi**2*MT**4*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R**2 - MT**2/MU_R**2)**2)) + (G**2*Mgo**2*MstL**2*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + \
MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog(Mgo/MstL))/(6.*cmath.pi**2*MT**4*cmath.sqrt((-4*Mgo**2*MstL**2)/MU_R**4 + (Mgo**2/MU_R**2 + MstL**2/MU_R\
*2 - MT**2/MU_R**2)**2)) - (G**2*MstL**4*cmath.sqrt(MstL**4/MU_R**4 + (-(Mgo**2/MU_R**2) + MT**2/MU_R**2)**2 - (2*MstL**2*(Mgo**2/MU_R**2 + MT**2/MU_R**2))/MU_R**2)*reglog(Mgo/MstL))/(12\
...
*2)'},
```

Complicated mass spectrum makes the computation heavy !!!

AN ISSUE WITH RICH PARTICLE SPECTRUM

- **How to define final states at NLO without spoiling perturbative convergence ?**

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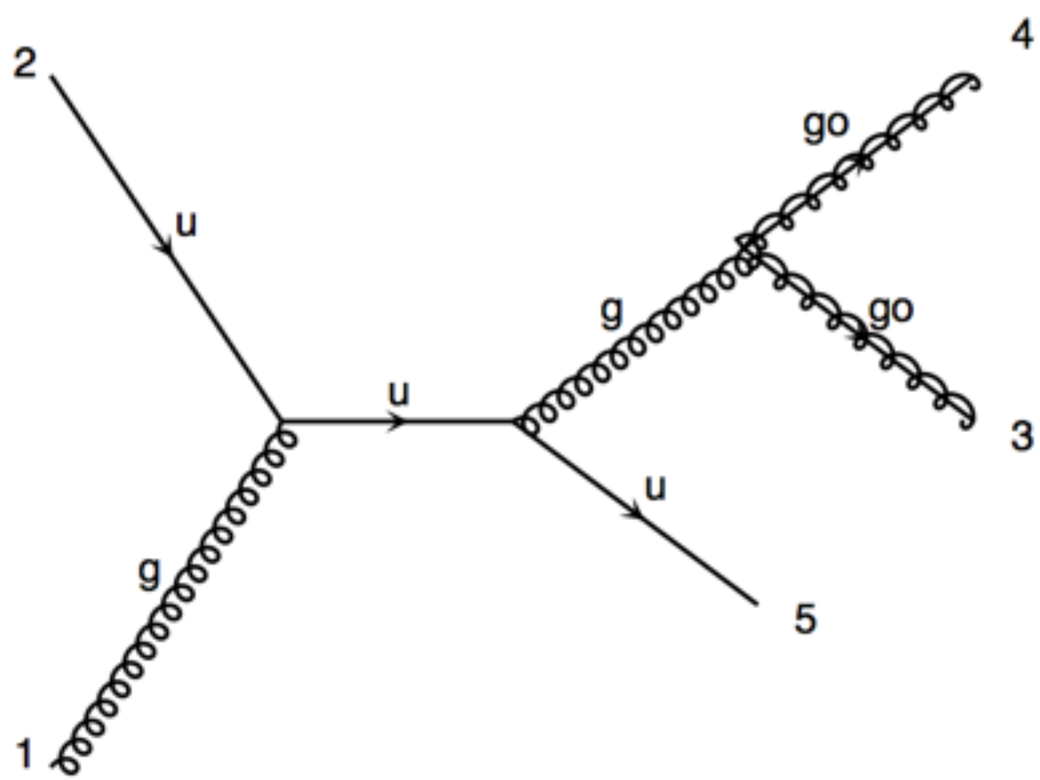
    - **Let us consider gluino pair production in SUSY**



NLO diagram for gluino-pair

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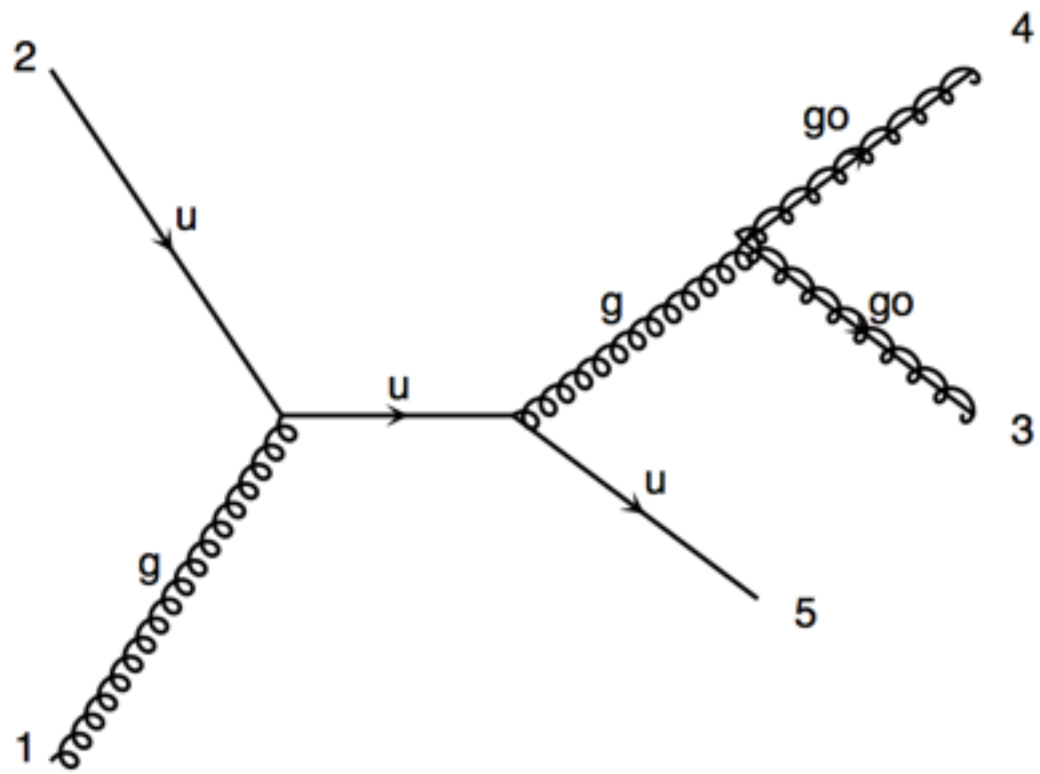

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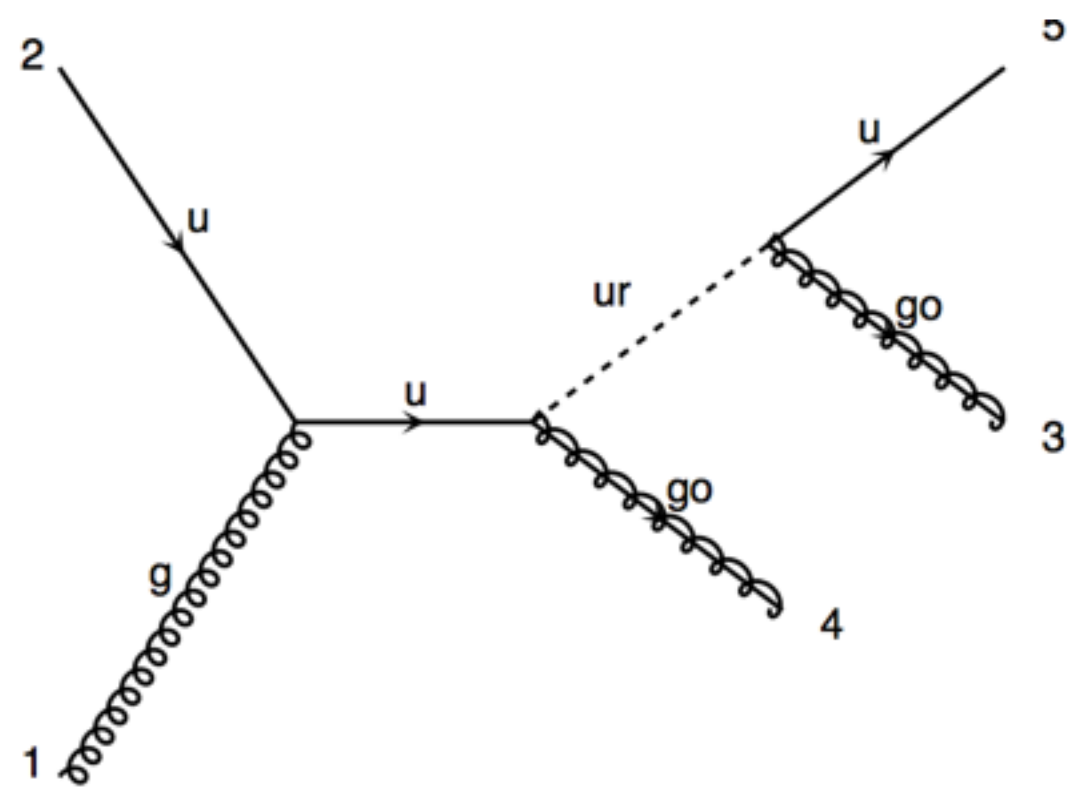

LO diagram for gluino-squark with squark decay

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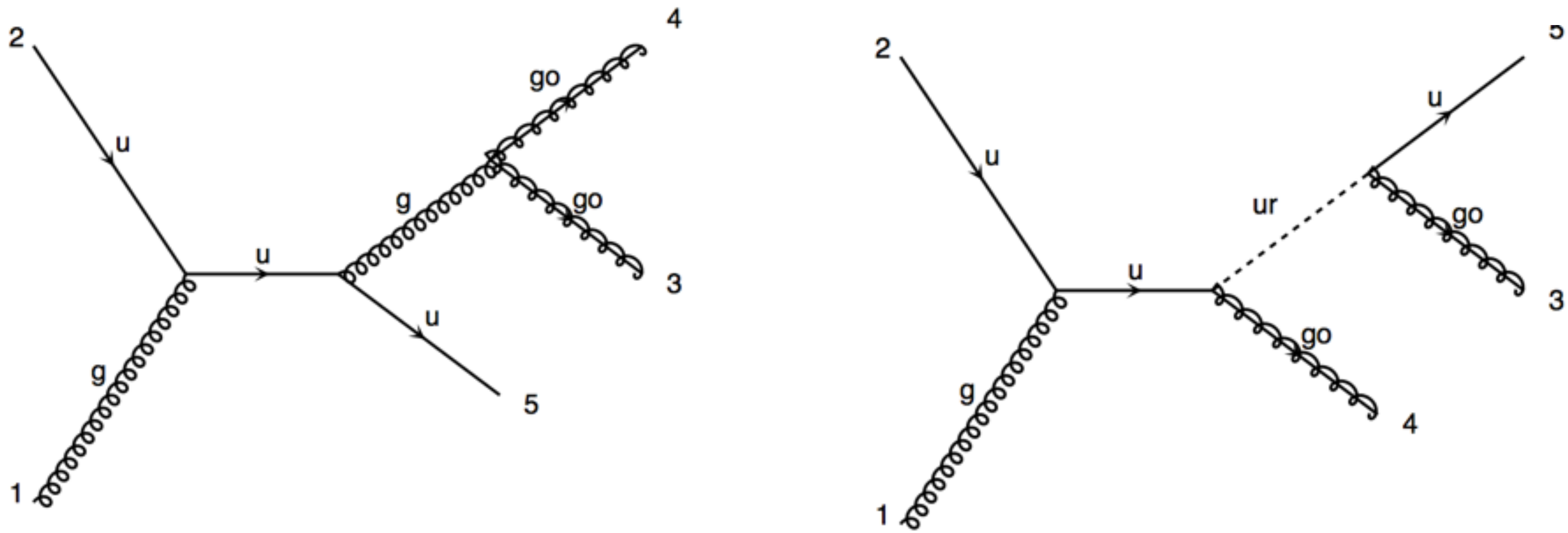

NLO diagram for gluino-pair

LO diagram for gluino-squark with squark decay

Simplified Treatments of Resonances

MadSTR

SIMPLIFIED TREATMENTS OF RESONANCES

Frixione, Fuks, Hirschi, Mawatari, HSS, Sunder and Zaro (JHEP'19)

- The formulation of the problem is:

LO: $a+b \longrightarrow \delta + X$

NLO(Real): $a+b \longrightarrow \delta+\gamma+X$ with/without $\beta \longrightarrow \delta+\gamma$

$$\mathcal{A}_{ab\to\delta\gamma X} = \underline{\mathcal{A}^{(\not\beta)}_{ab\to\delta\gamma X}} + \underline{\mathcal{A}^{(\beta)}_{ab\to\delta\gamma X}}$$

non-resonance
resonance

$$\left|\mathcal{A}_{ab\to\delta\gamma X}\right|^2 = \left|\mathcal{A}^{(\not\beta)}_{ab\to\delta\gamma X}\right|^2 + 2\Re\left(\mathcal{A}^{(\not\beta)}_{ab\to\delta\gamma X}\mathcal{A}^{(\beta)\dagger}_{ab\to\delta\gamma X}\right) + \left|\mathcal{A}^{(\beta)}_{ab\to\delta\gamma X}\right|^2$$

- No fully satisfactory solutions but a few proposals:

Diagram Removal

istr=1 • DR: **remove the resonance diagrams/amplitude**

istr=2 • DRI: **remove the resonance amplitude squared**

Diagram Subtraction

$$d\sigma^{(\mathrm{DS})}_{ab\to\delta\gamma X} \propto \left\{\left|\mathcal{A}^{(\not\beta)}_{ab\to\delta\gamma X}\right|^2 + 2\Re\left(\mathcal{A}^{(\not\beta)}_{ab\to\delta\gamma X}\mathcal{A}^{(\beta)\dagger}_{ab\to\delta\gamma X}\right) + \left|\mathcal{A}^{(\beta)}_{ab\to\delta\gamma X}\right|^2\right\} d\phi$$
$$- \; f(m^2_{\delta\gamma})\,\mathbb{P}\left(\left|\mathcal{A}^{(\beta)}_{ab\to\delta\gamma X}\right|^2 d\phi\right), \quad (18)$$

DS subtraction term

istr=6 • DS-finalresh-runBW: **P (FS momenta reshuffling), f (ratio of two BWs with running width**

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istr=5 • DS-finalresh-stdBW: **P (FS momenta reshuffling), f (ratio of two standard BWs)**

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} Not gauge invariant

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SIMPLIFIED TREATMENTS OF RESONANCES

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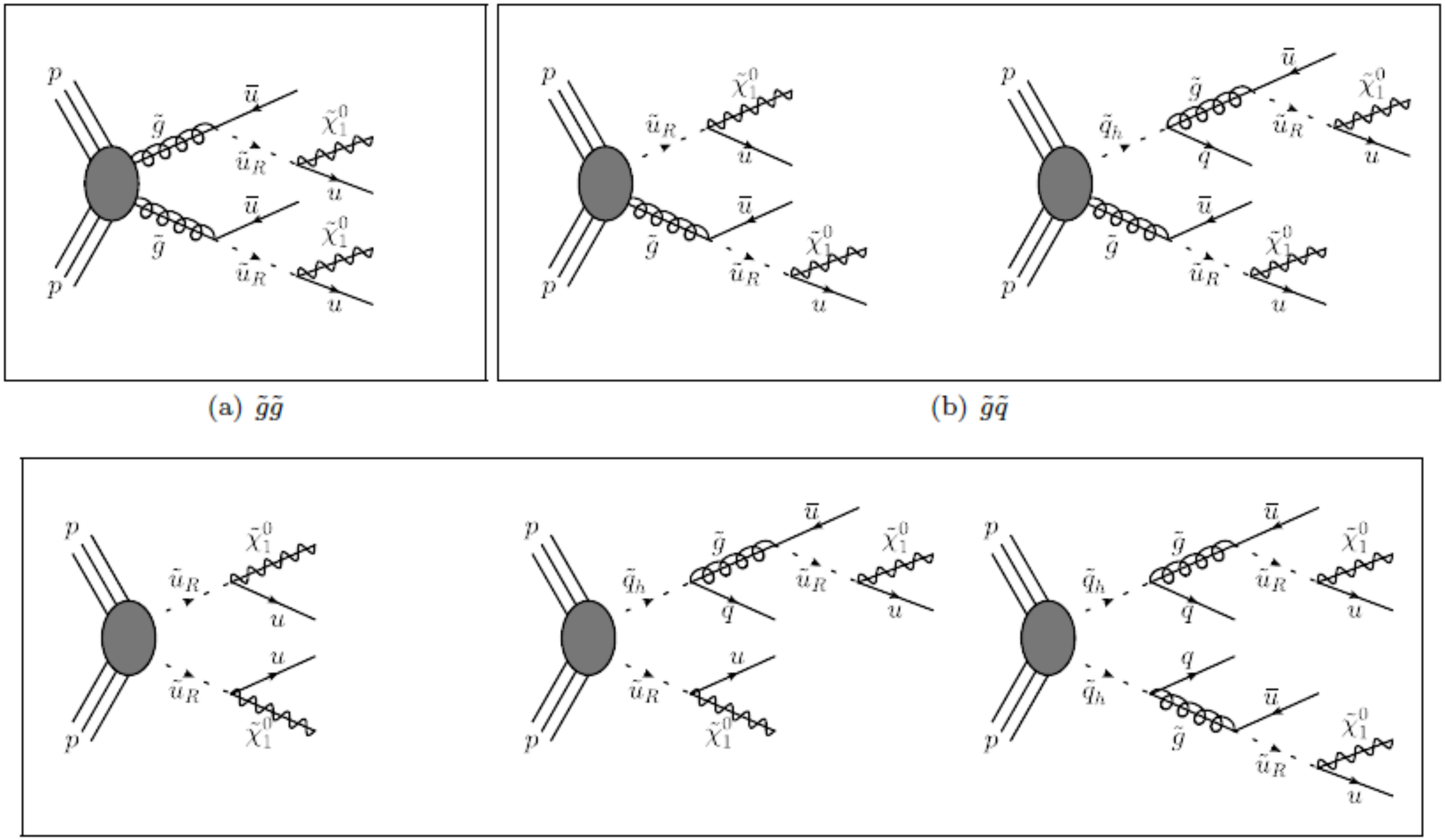
- Jets plus missing Et $pp \longrightarrow nj + \not{E}_T$



(a) $\tilde{g}\tilde{g}$

(b) $\tilde{g}\tilde{q}$

(c) $\tilde{q}\tilde{q}$

SIMPLIFIED TREATMENTS OF RESONANCES

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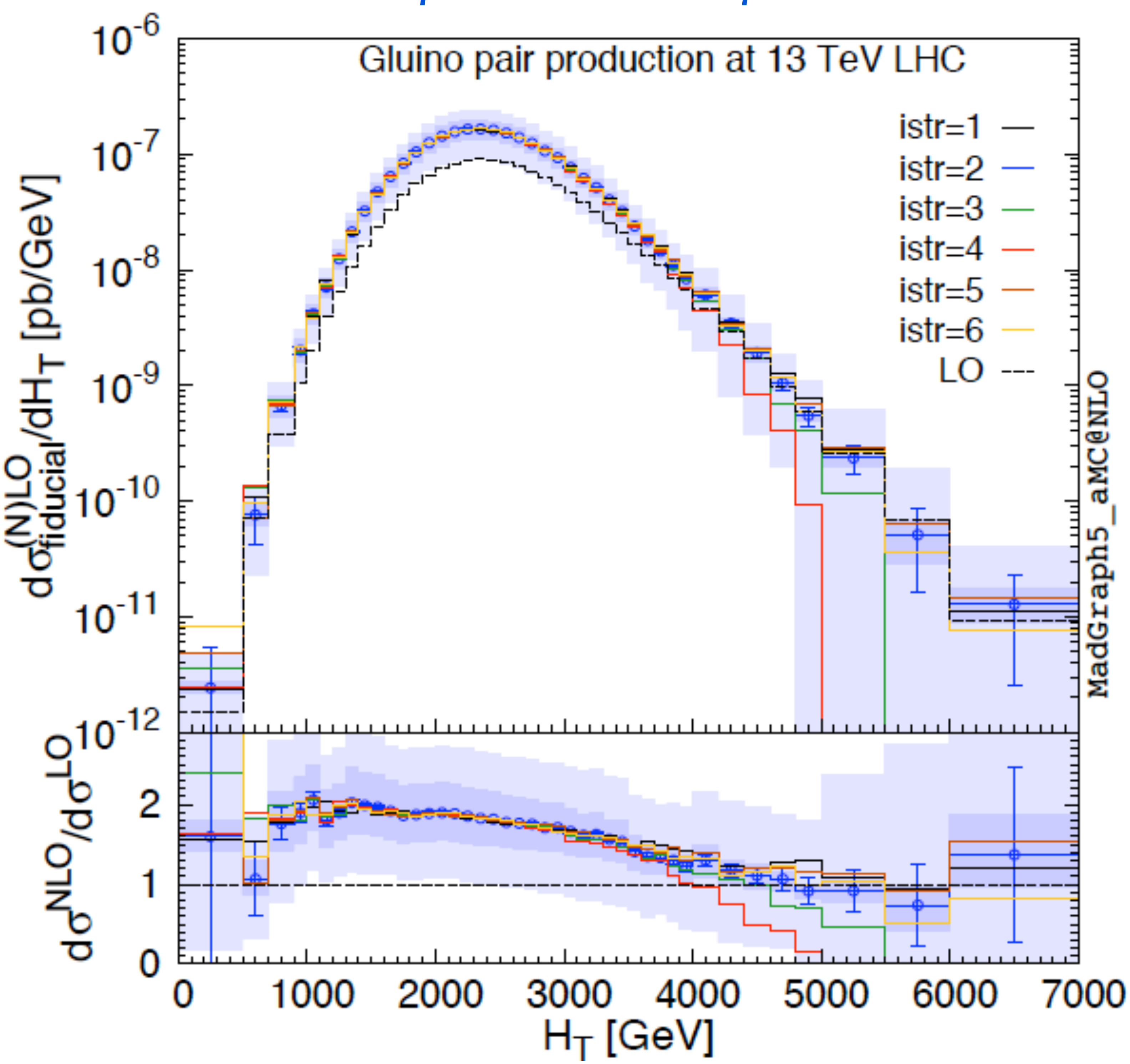
https://code.launchpad.net/~maddevelopers/mg5amcnlo/MadSTRPlugin



> ./bin/mg5_aMC --mode=MadSTR
> import model MSSMatNLO_UFO
> generate p p > go go [QCD]
> output; launch

Tuesday, November 19, 19

SIMPLIFIED TREATMENTS OF RESONANCES

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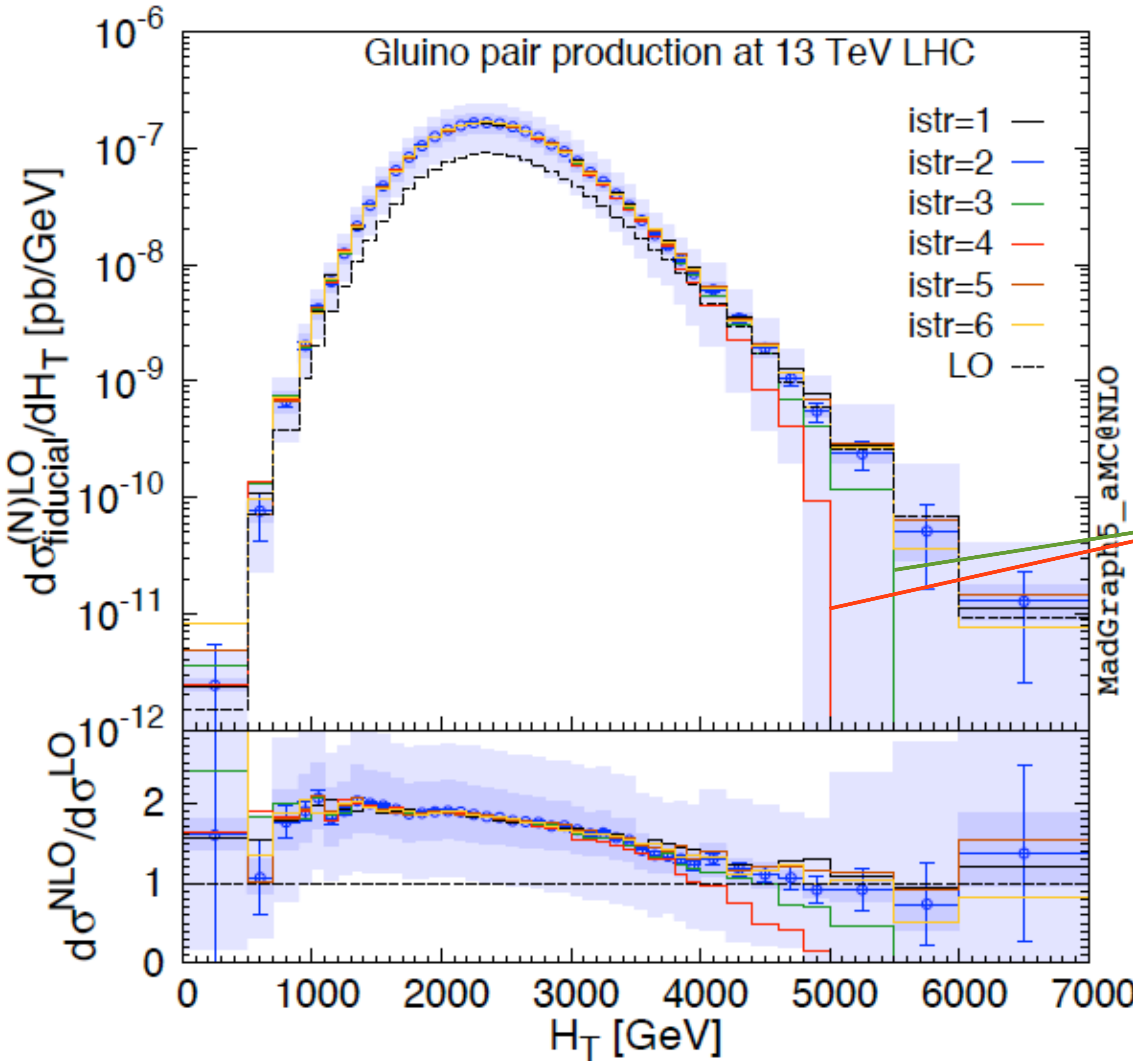

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Sensitive to large x PDF due to initial momenta reshuffling !

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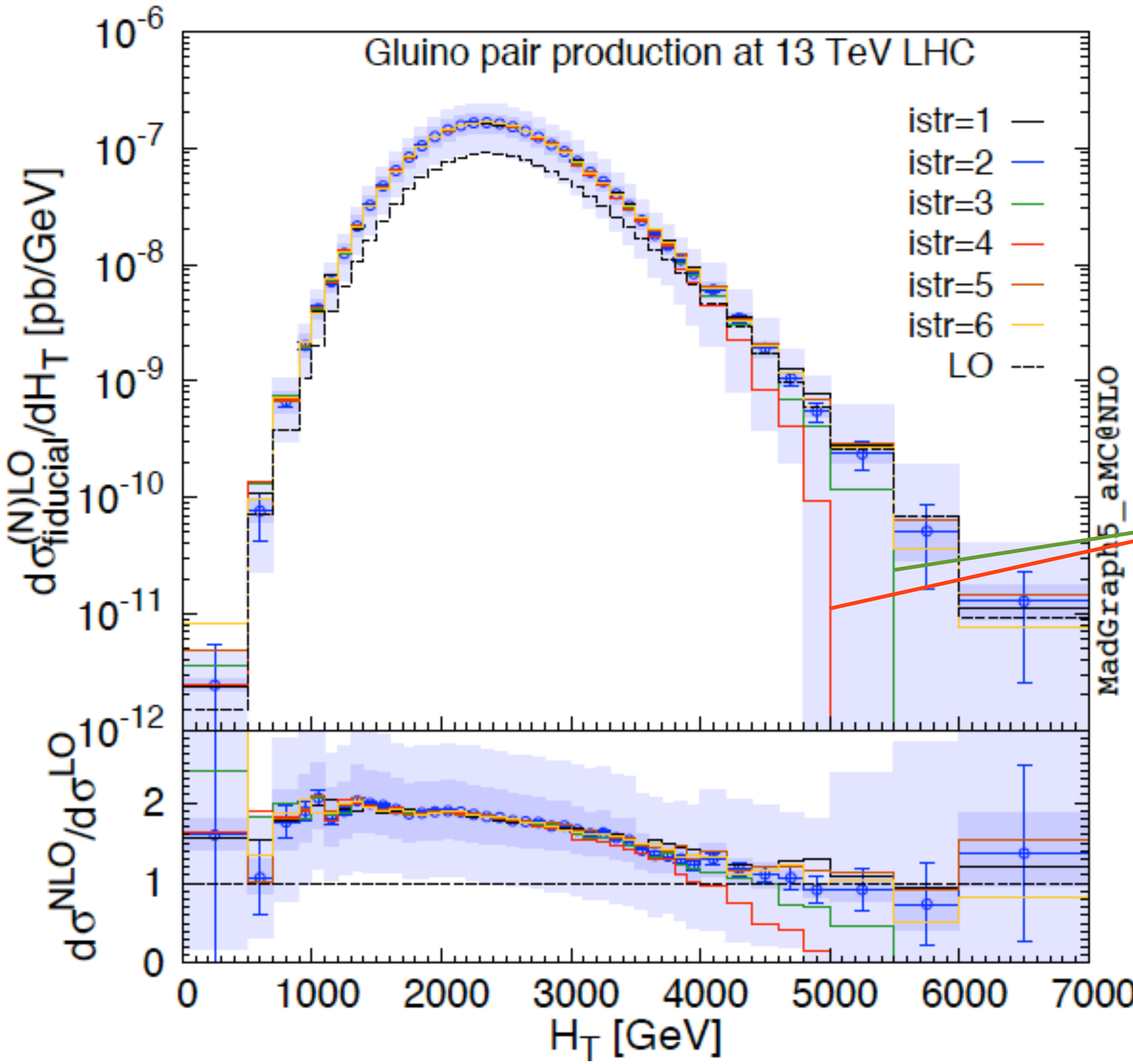

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Important to check the systematic dependences !