Matching and Merging:
Combining Matrix Elements and Parton Showers

Leif Gellersen

Lund University

leif.gellersen@thep.lu.se

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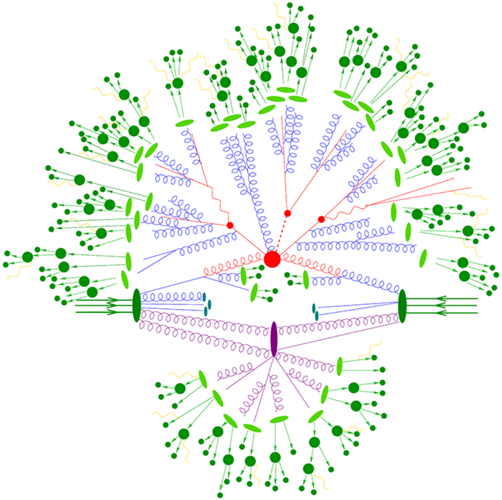
Hadron Collisions: QCD, QCD, QCD, . . .



Split the problem into many pieces

- Hard Process, resonant decays
- Parton Shower
- MPIs
- Hadronisation
- PDFs: Pick a parton from a hadron
- Hadron Decays
- Hadronic rescattering
- Beam Remnants/UE

Figure from Stefan Höche

Recap: Parton Showers

Start from hard $2 \to 2$ scattering, dress with extra partons to get exclusive $2 \to n$ cross section

$$\mathrm{d}\sigma_n^{\mathrm{ex}} = F_0^+ F_0^- |M_0|^2 \mathrm{d}\phi_0 \times \left[\prod_{i=1}^{n} \frac{\alpha_\mathrm{s}(\rho_i)}{2\pi} \frac{F_i}{F_{i-1}} P_i \frac{\mathrm{d}\rho_i}{\rho_i} \mathrm{d}z \Pi_{i-1}(\rho_{i-1}, \rho_i) \right] \Pi_n(\rho_n, \rho_{\mathrm{min}})$$

- $|M_0|^2 \mathrm{d}\phi_0$: Born-level ME and phase space
- $F_i = x_i f_i(x_i, \rho_i)$: PDF's from both sides of i-parton state, $\pm$ for $\pm p_z$ beams
- $P_i \mathrm{d}z \mathrm{d}\rho_i / \rho_i$: Differential emission rate, correct for soft/collinear splittings
- ρ, z: Splitting variables, ρ jet resolution scale, z energy/momentum fraction
- $\Pi(\rho_{i-1}, \rho_i)$: No-emission probabilities
- ρ_{min}: Minimal resolution scale / shower cut-off scale

Recap: No-emission Probabilities

$$\Pi_i(\rho_i, \rho_{i+1}) = \exp\left(- \int_{\rho_{i+1}}^{\rho_i} \frac{\mathrm{d}\rho}{\rho} \frac{\alpha_\mathrm{s}(\rho)}{2\pi} \int_{z_\mathrm{min}}^{z_\mathrm{max}} \mathrm{d}z \frac{F_{i+1}}{F_i} P_i(z) \right)$$

- Probability of not having any emissions harder than ρ_{i+1} when starting shower from ρ_i
- Introduces all order corrections in $\alpha_\mathrm{s} \rightarrow$ (N)LL Resummation
- F_{i+1}/F_i only included for ISR
- Exclusive description of final state needs no-emission probabilities

Unitarity of Parton Shower: Fixed Order Expansion

Expand to $\mathcal{O}(\alpha_s^2)$
Use $\frac{1}{2\pi}\frac{F_{i+1}}{F_i}P_i(z) = \bar{P}_i$ for ISR, $\frac{1}{2\pi\rho}P_i(z) = \bar{P}_i$ for FSR to simplify notation

$$\frac{\mathrm{d}\sigma_0^{\mathrm{ex}}}{\mathrm{d}\phi_0} = F_0^+ F_0^- |M_0|^2 \left[1 - \alpha_\mathrm{s}\int_{\rho_\mathrm{min}}^{\rho_0} \mathrm{d}\rho \mathrm{d}z \bar{P}_1 + \frac{\alpha_s^2}{2}\left(\int_{\rho_\mathrm{min}}^{\rho_0} \mathrm{d}\rho \mathrm{d}z \bar{P}_1\right)^2\right]$$

$$\frac{\mathrm{d}\sigma_1^{\mathrm{ex}}}{\mathrm{d}\phi_0} = F_0^+ F_0^- |M_0|^2 \alpha_\mathrm{s} \mathrm{d}\rho_1 \mathrm{d}z_1 \bar{P}_1 \left[1 - \alpha_\mathrm{s}\int_{\rho_1}^{\rho_0} \mathrm{d}\rho \mathrm{d}z \bar{P}_1 - \alpha_\mathrm{s}\int_{\rho_\mathrm{min}}^{\rho_1} \mathrm{d}\rho \mathrm{d}z \bar{P}_2\right]$$

$$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\phi_0} = F_0^+ F_0^- |M_0|^2 \alpha_\mathrm{s}^2 \mathrm{d}\rho_1 \mathrm{d}z_1 \bar{P}_1 \mathrm{d}\rho_2 \mathrm{d}z_2 \bar{P}_2 \Theta(\rho_1 - \rho_2)$$

$\Rightarrow$ Unitarity in every order of α_s, total cross-section

$$\frac{\mathrm{d}\sigma_0^{\mathrm{inc}}}{\mathrm{d}\phi_0} = \frac{\mathrm{d}\sigma_0^{\mathrm{ex}}}{\mathrm{d}\phi_0} + \int \frac{\mathrm{d}\sigma_1^{\mathrm{ex}}}{\mathrm{d}\phi_0} + \int\int \frac{\mathrm{d}\sigma_2}{\mathrm{d}\phi_0} = F_0^+ F_0^- |M_0|^2$$

But 1-jet cross section not correct for hard/wide-angle emissions

Matrix Elements vs. Parton Showers

Matrix Elements

Fixed order good for hard jets

- + Contains all terms in given order of α_s
- + Valid also for high relative $p_{\perp}^2$
- - Only feasible for a few emissions

Parton Showers

Approx. excl. multi-parton cross section

- + Always finite
- + Can produce any number of emissions
- - Is only valid in soft/collinear regions

Combine strengths of Matrix Elements and Parton Showers

Experiments measure both high and low $p_{\perp}^2$ phenomena

- Describe hard emissions by fixed order predictions
- Add further emissions and include no-emission probabilities from PS

Outline

Combine Matrix Element calculations and Parton Showers. Improve in different ways:

Matrix Element Corrections Oldest scheme, correct first emission of parton shower according to full process-dependent real emission calculation

Multi-jet Merging Improve radiation pattern of parton shower by adding higher-multiplicity matrix elements

NLO Matching Improve the perturbative precision by one higher order (NLO in α_s) cross section matched to parton showers

NLO Multi-jet Matching/Merging Combine multiple higher-multiplicity and higher-order cross sections in parton shower

Matrix Element Corrections / Tree-level Matching

Want improved parton shower with full matrix elements for hard emissions
$\Rightarrow$ First step: Use full real-emission matrix element for hardest emission, process-dependent!

$$\alpha_{\mathrm{s}} \bar{P}_i \to \alpha_s \bar{P}_i^{\mathrm{ME}} \equiv \frac{|M_i|^2 \mathrm{d}\phi_i}{|M_{i-1}|^2 \mathrm{d}\phi_{i-1} \mathrm{d}\rho \mathrm{d}z}$$

- Old, but very good! [Bengtsson, Sjöstrand (1987)]
- + Natural and efficient within PS: Use modified acceptance probability
- - Difficult to generalize beyond one emission
- Vincia parton shower exponentiates n-parton matrix elements [Giele, Kosower, Skands (2008)]

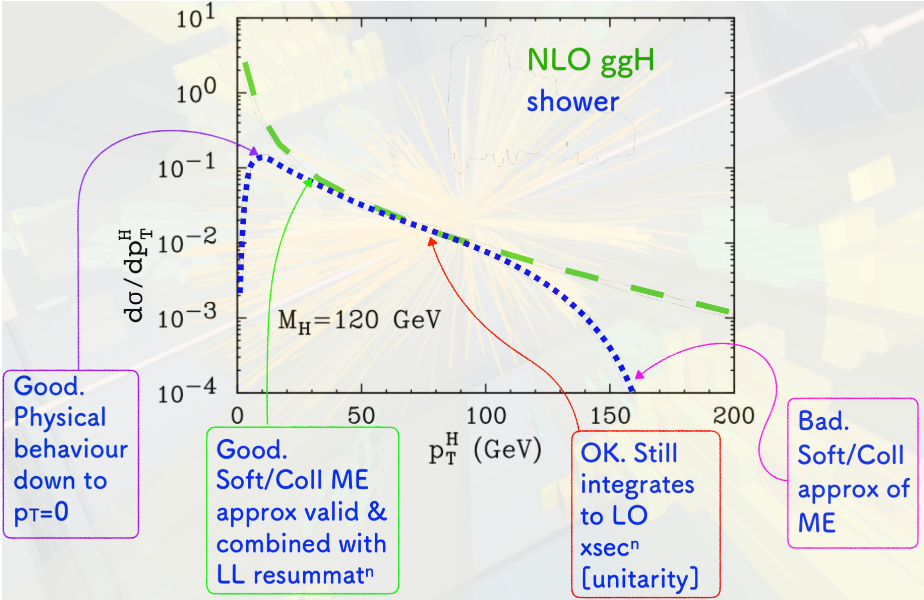
Matrix Element Corrections Preserve PS Unitarity

$$\frac{\mathrm{d}\sigma_0^{\mathrm{ex}}}{\mathrm{d}\phi_0} = F_0^+ F_0^- |M_0|^2 \left[1 - \alpha_s \int_{\rho_{\min}}^{\rho_0} \mathrm{d}\rho \mathrm{d}z \bar{P}_1^{\mathrm{ME}} + \frac{\alpha_s^2}{2} \left(\int_{\rho_{\min}}^{\rho_0} \mathrm{d}\rho \mathrm{d}z \bar{P}_1^{\mathrm{ME}} \right)^2 \right]$$

$$\frac{\mathrm{d}\sigma_1^{\mathrm{ex}}}{\mathrm{d}\phi_0} = F_0^+ F_0^- |M_0|^2 \alpha_{\mathrm{s}} \mathrm{d}\rho_1 \mathrm{d}z_1 \bar{P}_1^{\mathrm{ME}} \left[1 - \alpha_{\mathrm{s}} \int_{\rho_1}^{\rho_0} \mathrm{d}\rho \mathrm{d}z \bar{P}_1^{\mathrm{ME}} - \alpha_{\mathrm{s}} \int_{\rho_{\min}}^{\rho_1} \mathrm{d}\rho \mathrm{d}z \bar{P}_2 \right]$$

$$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\phi_0} = F_0^+ F_0^- |M_0|^2 \alpha_{\mathrm{s}}^2 \mathrm{d}\rho_1 \mathrm{d}z_1 \bar{P}_1^{\mathrm{ME}} \mathrm{d}\rho_2 \mathrm{d}z_2 \bar{P}_2 \Theta(\rho_1 - \rho_2)$$

- Still unitary to all orders in α_{s}
- Valid in whole shower emission phase space, down to scale $\rho_{\min}$



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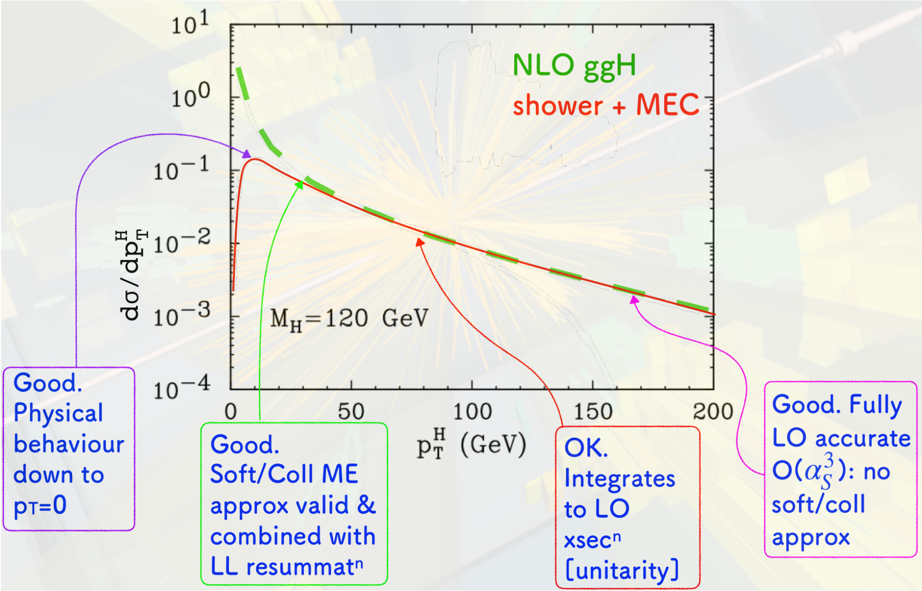

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Multi-jet Merging: The Naive (and Wrong) Way

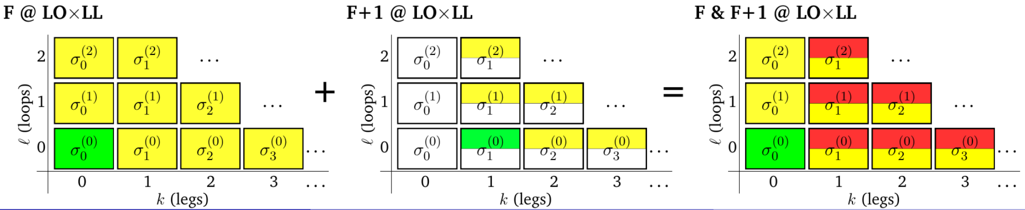
Want to improve PS emissions for more than hardest emission. Naive approach:

- Generate $[X]_{\text{ME}}$ + parton shower
- Generate $[X + 1\text{jet}]_{\text{ME}}$ + parton shower
- Generate $[X + 2\text{jet}]_{\text{ME}}$ + parton shower
- ...

And combine everything into one sample. Does not work, double counting!

- $[X]_{\text{ME}}$ + parton shower is inclusive
- $[X + 1\text{jet}]_{\text{ME}}$ + parton shower is inclusive
- ...

See also Skands: Introduction to QCD



Multi-jet Merging: Exclusive Description without Double-counting

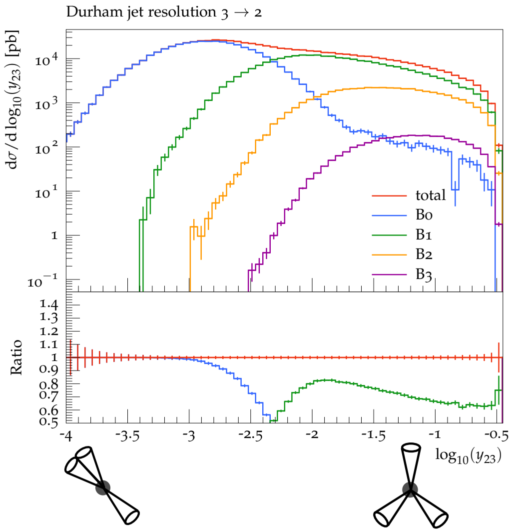
Solve double-counting issue by dividing phase space in "hard and soft region":

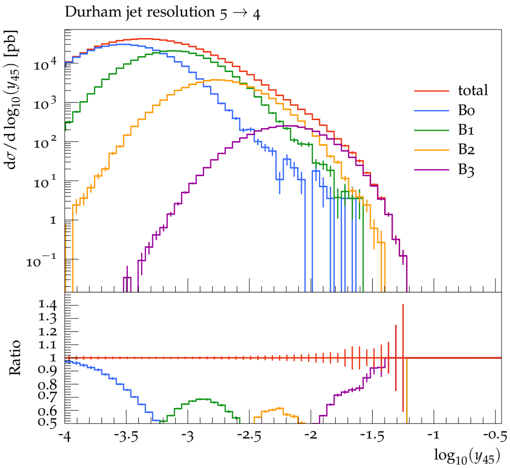
- Generating inclusive few jet samples according to exact tree-level $F_n^+ F_n^- |M_n|^2 \equiv B_n$ in "hard region"
- Using some merging scale ρ_{ms} to cut off divergences
- Making exclusive by reweighting with no-emission probabilities (and α_{s} and PDF ratios), i.e. how would PS have produced this configuration
- Using normal shower in "soft region" below ρ_{ms}

Remaining issues:

- Merging scale dependence
- Merging scale might not be defined in terms of shower evolution variable
- Might break unitarity of shower

Multi-jet Merging: $e^+e^- \to q\bar{q}$ + jets example

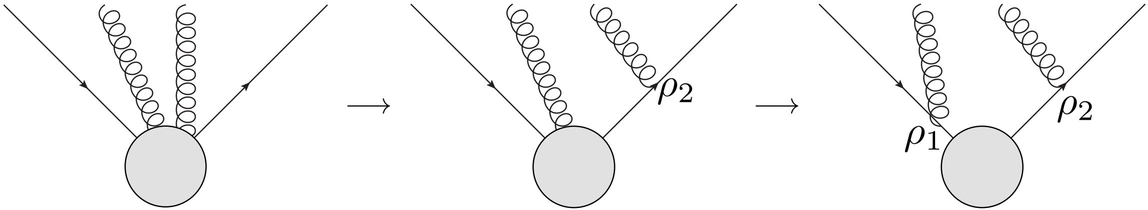




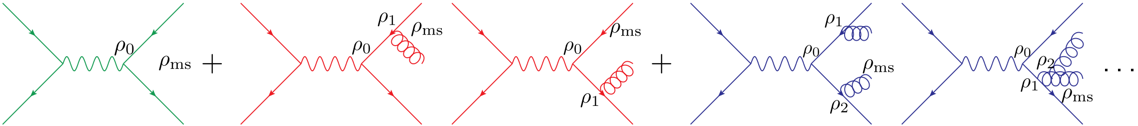
How to Reweight: Parton Shower Histories

Want to apply no-emission probabilities and scale dependent ratios $\Rightarrow$ need ρ_i. Two ways:

- Find unique history by applying sequential $2 \to 1$ jet algorithm
- Find all possible parton shower histories by $3 \to 2$ clustering, choose one according to product of splitting probabilities
    - Choose one history according to product of splitting probabilities
    - Combine partons according to parton shower kinematics



Multi-jet Merging: Illustration in FSR



Combine MEs with different multiplicities, avoid overlap by reweighting

$$\langle \mathcal{O} \rangle = \int d\phi_0 \left\{ \mathcal{O}_0 B_0 w_0 + \int d\phi_1 \mathcal{O}_1 B_1 w_1 + \int d\phi_1 \int d\phi_2 \mathcal{O}_2 B_2 w_2 \right\}$$

with the weights

$$w_0 = \Pi_0(\rho_0, \rho_{\rm ms}) \,, \;\; w_1 = \Pi_0(\rho_0, \rho_1) \frac{\alpha_s(\rho_1)}{\alpha_s(\mu_R)} \Pi_1(\rho_1, \rho_{\rm ms}) \,,$$

$$w_2 = \Pi_0(\rho_0, \rho_1) \frac{\alpha_s(\rho_1)}{\alpha_s(\mu_R)} \Pi_1(\rho_1, \rho_2) \frac{\alpha_s(\rho_2)}{\alpha_s(\mu_R)}$$

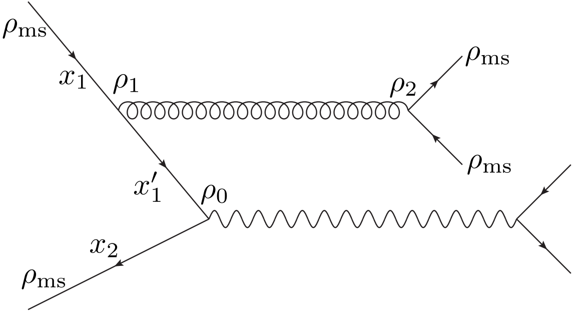
Multi-jet Merging: Illustration in ISR

Inclusive Matrix Element:

$$\frac{\mathrm{d}\sigma_2^{\mathrm{in}}}{\mathrm{d}\phi_{0+2}} = F_1(x_1, \rho_0) F_2(x_2, \rho_0) |M_2|^2$$

Exclusive Parton Shower:

$$\frac{\mathrm{d}\sigma_2^{\mathrm{ex}}}{\mathrm{d}\phi_0 \mathrm{d}\phi_{1,2}} = F_1'(x_1', \rho_0) F_2(x_2, \rho_0) |M_0|^2 \Pi_0(\rho_0, \rho_1) \frac{\alpha_s(\rho_1)}{2\pi} \frac{F_1(x_1, \rho_1)}{F_1'(x_1', \rho_1)} \frac{P_1}{\rho_1} \Pi_1(\rho_1, \rho_2) \frac{\alpha_s(\rho_2)}{2\pi} \frac{P_2}{\rho_2} \Pi_2(\rho_2, \rho_{\mathrm{ms}})$$

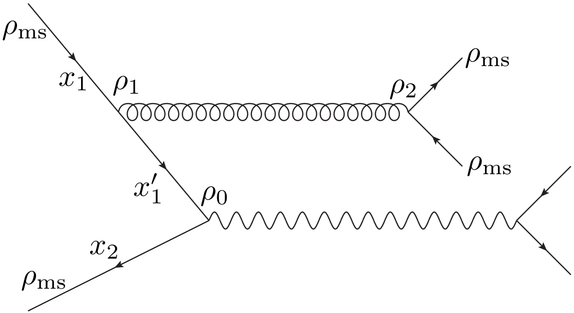


Find weight to make inclusive matrix element exclusive:

$$\frac{\mathrm{d}\sigma_2^{\mathrm{ex}}}{\mathrm{d}\phi_0 \mathrm{d}\phi_{1,2}} = w \frac{\mathrm{d}\sigma_2^{\mathrm{in}}}{\mathrm{d}\phi_{0+2}}$$

Multi-jet Merging: Merging Weight in ISR

$$
\begin{aligned}
w &= w_{\alpha_s} w_{\mathrm{pdf}} w_{\mathrm{no-em}} \\
w_{\alpha_s} &= \frac{\alpha_s(\rho_1)}{\alpha_s(\rho_0)} \frac{\alpha_s(\rho_2)}{\alpha_s(\rho_0)} \\
w_{\mathrm{pdf}} &= \frac{f(x_1', \rho_0)}{f(x_1', \rho_1)} \frac{f(x_1, \rho_1)}{f(x_1, \rho_0)} \\
w_{\mathrm{no-em}} &= \Pi_0(\rho_0, \rho_1)\Pi_1(\rho_1, \rho_2)\Pi_2(\rho_2, \rho_{\mathrm{ms}})
\end{aligned}
$$



Multi-Jet Merging Algorithm

Summary of general multi-jet merging procedure:

1. Calculate inclusive cross sections for $X + n$ partons (with kinematic cut ρ_{ms} to avoid singularities)
2. Cluster according to jet algorithm or find parton shower history to find scales for no-emission probabilities and scale dependent ratios
3. Multiply with no-emission probability
4. Multiply with scale dependent ratios
5. If $n < N$, with N highest fixed order multiplicity, multiply no-emission probability towards merging scale ρ_{ms}
6. Allow further parton shower emissions below ρ_{ms}, for $n = N$ also above

CKKW Merging [Catani, Krauss, Kuhn, Webber (2001)]

- Cluster according to $k_\perp$ jet algorithm
- Apply analytic Sudakov factors (NLL accuracy) as no-emission probabilities
- Perform "truncated showering", since parton shower evolution variable not exactly identical to merging scale cut: Start shower from ρ_0, but forbid emissions above t_{ms}. Handle hard emissions (in ρ) below t_{ms} with care!
    - $+$ Best theoretical treatment
    - - Requires dedicated PS implementation
    - - Mismatch between analytical Sudakov and parton shower
    - Implemented in Sherpa (v 1.1) [Krauss (2002)]

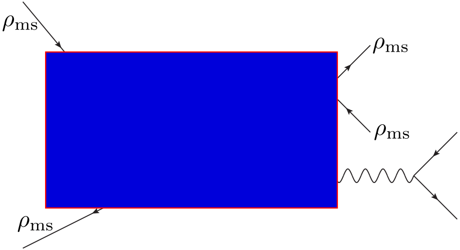
CKKW-L Merging [Lönnblad (2001)]

- Cluster back to parton shower history according to splitting probabilities in PS
- Generate of no-emission probabilities using parton shower
- Perform showering step-by-step for each step in history, starting from respective clustering scale
- Veto event if emission at larger scale than next clustering scale or ρ_{ms} in last step
- Keep PS emissions below ρ_{ms} (and between ρ_n and ρ_{ms} at highest multiplicity)
  - $+$ Agreement between Sudakov and shower by construction $\Rightarrow$ Reduced merging scale dependence
  - $+$ Use simple veto in shower if ρ_{ms} in terms of PS evolution variable
  - - Requires dedicated PS implementation
  - Implemented in Sherpa ($\geq$1.2) [Höche, Krauss, Schumann, Siegert (2009)], Pythia8 [Lönnblad, Prestel (2012)] and Herwig7 [Bellm, Gieseke, Plätzer (2018)]

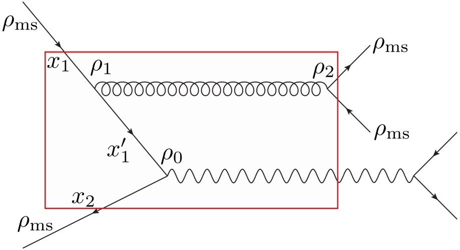
MLM [Mangano (2002)] [Mangano, Moretti, Piccinini, Treccani (2007)]

- Simplest way to estimate Sudakov suppression: Run shower on ME state without prior reclustering, starting from ρ_0
- Perform jet clustering, and reject if PS emits any jets harder than original partons or partons that are not clustered to hard partons
- No reconstructed history $\Rightarrow$ Sudakov factor corresponds to final partons only, not taking into account intermediate states
- Approximation turns out to be good enough
  - + Simplest available scheme
  - + Matching with any shower algorithm without specific implementation
  - - Sudakov suppression not exact $\Rightarrow$ mismatch with shower

Sudakov Factor: MLM vs. CKKW-L



- First shower from ρ_0 to $\rho_{\rm ms}$
- Then do jet clustering to veto if hard emissions occured
- Resulting no-emission probability: $\Pi_q^2(\rho_0, \rho_{\rm ms})\Pi_q^2(\rho_0, \rho_{\rm ms})$



- First construct parton shower history
- Then do trial shower on reconstructed history, veto event if emission above merging scale
- Resulting no-emission probability: $\Pi_q^2(\rho_0, \rho_2)\Pi_g(\rho_1, \rho_2)\Pi_q^4(\rho_2, \rho_{\rm ms})$

Unitarity in Multi-jet Merging

$$\frac{d\sigma_0^{ex}}{d\phi_0} = F_0^+ F_0^- |M_0|^2 \left[1 - \alpha_s \int_{\rho_{min}}^{\rho_0} d\rho dz \bar{P}_1 + \frac{\alpha_s^2}{2} \left(\int_{\rho_{min}}^{\rho_0} d\rho dz \bar{P}_1 \right)^2 \right]$$

$$\frac{d\sigma_1^{ex}}{d\phi_0} = F_0^+ F_0^- |M_0|^2 \alpha_s d\rho_1 dz_1 \bar{P}_1^{ME} \left[1 - \alpha_s \int_{\rho_1}^{\rho_0} d\rho dz \bar{P}_1 - \alpha_s \int_{\rho_{min}}^{\rho_1} d\rho dz \bar{P}_2 \right]$$

$$\frac{d\sigma_2}{d\phi_0} = F_0^+ F_0^- |M_0|^2 \alpha_s^2 d\rho_1 dz_1 \bar{P}_1^{ME} d\rho_2 dz_2 \bar{P}_2^{ME} \Theta(\rho_1 - \rho_2)$$

- Unitarity of parton shower broken in all multi-jet merging schemes mentioned above
- Inclusive cross-section only preserved if splitting probabilities in no-emission probability identical to full fixed order splitting probabilities

Unitary Merging: UMEPS [Lönnblad, Prestel (2012)]

Start from CKKW-L scheme, want to restore PS unitarity. Use:

$$\Pi_n(\rho_n, \rho_{\mathrm{ms}}) = 1 - \int_{\rho_{\mathrm{ms}}}^{\rho_n} \mathrm{d}\rho \mathrm{d}z \alpha_{\mathrm{s}} \bar{P}_{n+1}^{\mathrm{ME}}(\rho, z) \Pi_n(\rho_0, \rho)$$

i.e. probability of no emission is 1 - probability of at least one emission

$$\frac{\mathrm{d}\sigma_0^{\mathrm{ex}}}{\mathrm{d}\phi_0} = F_0^+ F_0^- |M_0|^2 \Pi_0(\rho_0, \rho_{\mathrm{ms}})$$

$$\frac{\mathrm{d}\sigma_1^{\mathrm{ex}}}{\mathrm{d}\phi_0} = F_1^+ F_1^- |M_1|^2 \mathrm{d}\rho_1 \mathrm{d}z_1 \Pi_0(\rho_0, \rho_1) \Pi_1(\rho_1, \rho_{\mathrm{ms}})$$

$$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\phi_0} = F_2^+ F_2^- |M_2|^2 \mathrm{d}\rho_1 \mathrm{d}z_1 \Pi_0(\rho_0, \rho_1) \mathrm{d}\rho_2 \mathrm{d}z_2 \Pi_1(\rho_1, \rho_2)$$

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i.e. probability of no emission is 1 - probability of at least one emission

$$\frac{\mathrm{d}\sigma_0^{\mathrm{ex}}}{\mathrm{d}\phi_0} = F_0^+ F_0^- |M_0|^2 \cancel{\Pi_0(\rho_0, \rho_{\mathrm{ms}})} - \int F_1^+ F_1^- |M_1|^2 \mathrm{d}\rho_1 \mathrm{d}z_1 \Pi_0(\rho_0, \rho_1)$$

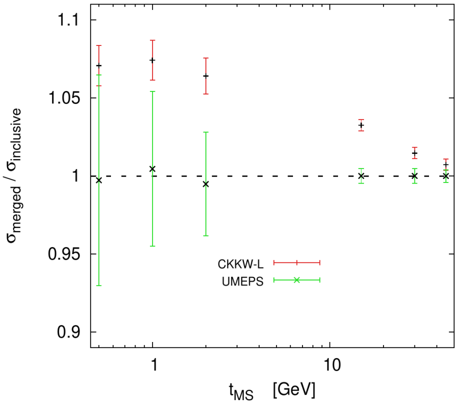
$$\frac{\mathrm{d}\sigma_1^{\mathrm{ex}}}{\mathrm{d}\phi_0} = F_1^+ F_1^- |M_1|^2 \mathrm{d}\rho_1 \mathrm{d}z_1 \Pi_0(\rho_0, \rho_1) \cancel{\Pi_1(\rho_1, \rho_{\mathrm{ms}})}$$

$$- \mathrm{d}\rho_1 \mathrm{d}z_1 \Pi_0(\rho_0, \rho_1) \int F_2^+ F_2^- |M_2|^2 \mathrm{d}\rho_2 \mathrm{d}z_2 \Pi_1(\rho_1, \rho_2)$$

$$\frac{\mathrm{d}\sigma_2}{\mathrm{d}\phi_0} = F_2^+ F_2^- |M_2|^2 \mathrm{d}\rho_1 \mathrm{d}z_1 \Pi_0(\rho_0, \rho_1) \mathrm{d}\rho_2 \mathrm{d}z_2 \Pi_1(\rho_1, \rho_2)$$

Unitary Merging: UMEPS [Lönnblad, Prestel (2012)]

- Still add CKKW-L reweighted samples
- Instead of last Sudakov, subtract +1 parton integrated sample
  ⇒ Individual multiplicities still exclusive
- Can still add normal PS below merging scale
- + Procedure does not change inclusive cross section
- - UMEPS introduces negative weights ⇒ less efficient

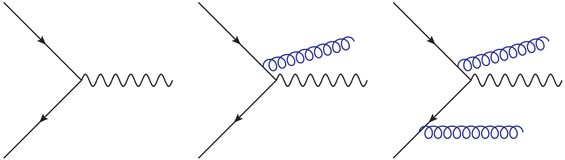


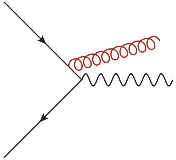
Matching of NLO Matrix Elements & Parton Showers

We want precision predictions: Combine NLO fixed order calculations with Parton showers.

- Again problem of double counting of emissions by real emission matrix element and emissions generated by parton shower
- Also double counting of virtual terms through virtual corrections and Sudakov factors

Parton Shower ⟶





Real emission

Finite Numerical NLO Cross Section

NLO prediction for observable $\mathcal{O}$ given by

$$\langle \mathcal{O} \rangle = \int \mathrm{d}\phi_n (B_n + V_n) \mathcal{O}_n(\phi_n) + \int \mathrm{d}_{n+1} B_{n+1} \mathcal{O}_{n+1}(\phi_{n+1})$$

but both V_n and B_{n+1} separately divergent, only sum is finite.
Use universal subtraction terms to get finite results: [Frixione, Kunszt, Siegner (1996)] [Catani, Seymour (1997)]

$$\begin{aligned}\langle \mathcal{O} \rangle = &\int \mathrm{d}\phi_n (B_n + V_n + B_n \otimes I_1) \mathcal{O}_n(\phi_n) \\ &+ \int \mathrm{d}\phi_{n+1} (B_{n+1} \mathcal{O}_{n+1}(\phi_{n+1}) - B_n \otimes D_1 \mathcal{O}_n(\phi_{n+1}))\end{aligned}$$

Event interpretation not yet possible, $\mathcal{O}_n$ and $\mathcal{O}_{n+1}$ contributions must be finite separately

Shower Subtraction

Want to attach shower (include factor α_s in $\bar{P}$)

$$\mathcal{O}_n(\phi_n) \to \mathcal{F}_n(\mathcal{O}, \phi_n) = \Pi(\rho_n, \rho_\mathrm{min})\mathcal{O}_n(\phi_n) + \int \mathrm{d}\phi_{+1} \Pi(\rho_n, \rho_{n+1})\bar{P}_{n+1}\mathcal{F}_{n+1}(\mathcal{O}, \phi_{n+1})$$

$$\stackrel{\mathcal{O}(\alpha_\mathrm{s})}{\to} 1 - \int \mathrm{d}\phi_{+1} \bar{P}_{n+1}\mathcal{O}_n(\phi_{n+1}) + \int \mathrm{d}\phi_{+1} \bar{P}_{n+1}\mathcal{O}_{n+1}(\phi_{n+1})$$

But $B_n\mathcal{F}_n$ contains $\mathcal{O}(\alpha_\mathrm{s})$ terms $\Rightarrow$ subtract shower terms to first order in α_s such that accuracy of NLO not spoiled by shower

MC@NLO [Frixione, Webber (2002)]

With shower subtraction, arrive at MC@NLO prescription

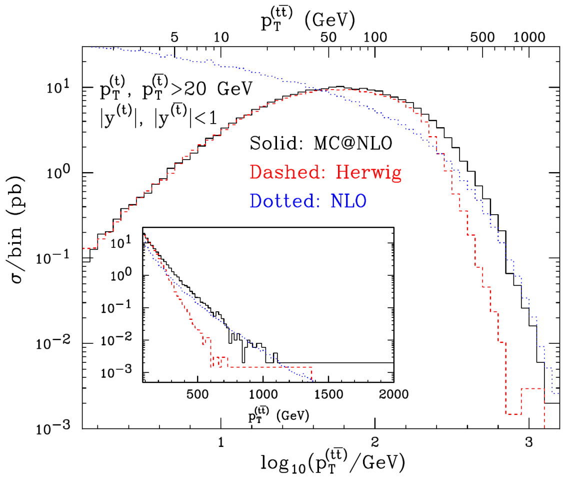
$$\langle\mathcal{O}\rangle_{\text{MC@NLO}} = \int \mathrm{d}\phi_n (B_n + V_n + B_n \otimes I_1)\mathcal{F}_n(\mathcal{O}, \phi_n) \qquad \text{Born + subtracted virtual}$$

$$+ \int \mathrm{d}\phi_{n+1} (B_n \bar{P}_{n+1} - B_n \otimes D_1)\mathcal{F}_n(\mathcal{O}, \phi_{n+1})) \qquad \text{Shower virtual - subtraction}$$

$$+ \int \mathrm{d}\phi_{n+1} (B_{n+1} - B_n \bar{P}_{n+1})\mathcal{F}_{n+1}(\mathcal{O}, \phi_{n+1}) \qquad \text{Real - shower real}$$

- Event generation possible since $\mathcal{O}_n$ and $\mathcal{O}_{n+1}$ separately finite
- Sudakov supression agrees with shower prediction
- Distribution correct only if parton shower is attached to cancel MC counterterms
- Can lead to many events with negative weights
- Needs to be implemented for each shower separately
- Automated in aMC@NLO [Frederix, Frixione, Hirschi, Maltoni, Pittau, Torrielli (2012)]

MC@NLO



- MC@NLO gives smooth transition between real emission pattern at high scales and parton shower at low scales
- Inclusive cross section correct at NLO

Plot form [Nason, Webber (2012)]

POWHEG [Nason (2004)] [Frixione, Nason, Oleari (2007)]

Positive Weight Hardest Emission Generator

$$\langle \mathcal{O} \rangle_{\mathrm{POWHEG}} = \int \mathrm{d}\phi_n (B_n + V_n + B_n \otimes I_1) \mathcal{F}_n^{\mathrm{HI}}(\mathcal{O}, \phi_n) \qquad \text{Born + subtracted virtual}$$

$$+ \int \mathrm{d}\phi_{n+1} (B_{n+1} - B_n \otimes D_1) \mathcal{F}_n^{\mathrm{HI}}(\mathcal{O}, \phi_{n+1})) \qquad \text{Shower virtual - subtraction}$$

Based on MC@NLO, modify shower to get "shower real" = "real" for hardest emission (similar to matrix element corrections)

- Less negative weights ⇒ Improved efficiency
- Hardest emission modified ⇒ Differences compared to MC@NLO, but both NLO correct
- Implementation process by process, but independent of attached shower

Combine NLO Matching and Multi-leg Merging

Goal: Combine several NLO matrix elements for same process: NLO for X, $X + 1$, $X + 2$, ...
Mostly based on parton shower unitarity
Different methods available:

- UNLOPS, based on UMEPS [Lönnblad, Prestel (2013)]
- MiNLO, based on POWHEG [Hamilton, Nason, Zanderighi (2012)] [Frederix, Hamilton (2016)]
- FxFx, based on MC@NLO [Frederix, Frixione (2012)]
- (Vincia, based on NLO MEC) [Hartgring, Laenen, Skands (2013)]
- ...

Current Developments:

- NNLO for inclusive cross section
- Improved uncertainty estimates
- Matching NNLO ME to NLO PS

Matching and Merging Summary

Goal: Combine matrix elements and parton showers. The Problem:

- Parton showers generate singular terms of higher-order matrix elements
- Same terms present in X + jet(s) matrix elements
- Combination must not double count

ME Corrections

- Oldest scheme, correct PS emissions to match full real emission ME
- Hard to iterate beyond one emission
- Developments: higher multiplicity, NLO in VINCIA

Multi-jet Merging

- Combine multiple LO ME samples by reweighting
- Separate phase space regions to deal with divergence
- Different schemes available

NLO Matching

- MC subtraction allows for NLO ME + PS
- MC@NLO and POWHEG
- Can be combined with multi-jet merging