Plan for 2nd lecture

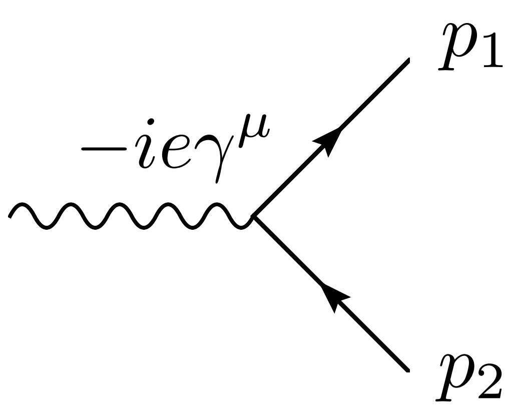
- Infrared and collinear divergences and IRsafety
- Parton model: incoherent sum of all partonic cross-sections
- Sum rules (momentum, charge, flavor conservation)
- Determination of parton densities (electron & neutrino scattering in DIS or Drell-Yan)
- Radiative corrections: failure of parton model
- Factorization of initial state divergences into scale dependent parton densities
- DGLAP evolution of parton densities $\Rightarrow$ measure gluon PDF

The soft approximation

Let's consider again the R-ratio. This is determined by $\gamma^* \to q\bar{q}$

At leading order:

$$M_0^\mu = \bar{u}(p_1)(-ie\gamma^\mu)v(p_2)$$



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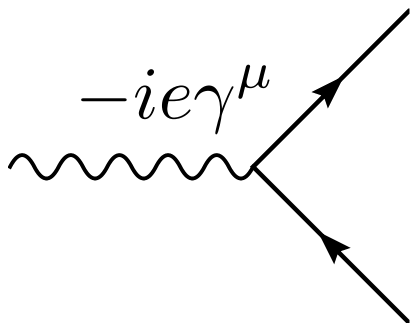
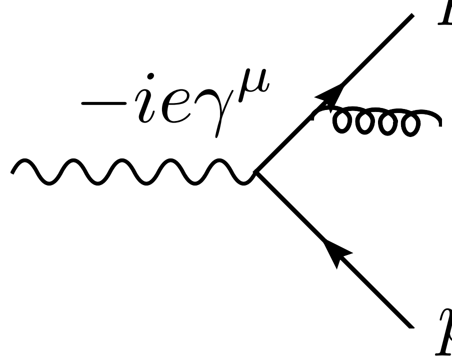


Emit one gluon:

$$\begin{aligned} M^\mu_{q\bar{q}g} &= \bar{u}(p_1)(-ig_s t^a \not{\epsilon})\frac{i(\not{p}_1 + \not{k})}{(p_1+k)^2}(-ie\gamma^\mu)v(p_2) \\ &- \bar{u}(p_1)(-ie\gamma^\mu)\frac{i(\not{p}_2 + \not{k})}{(p_2+k)^2}(-ig_s t^a \not{\epsilon})v(p_2) \end{aligned}$$



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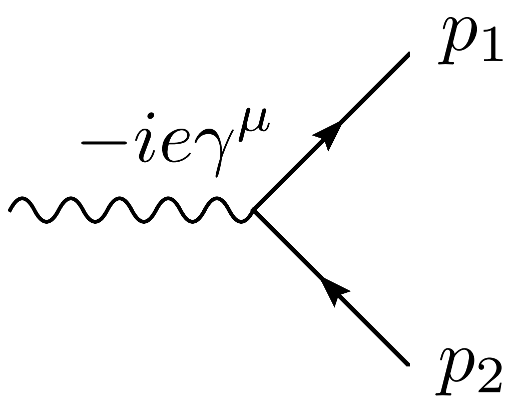

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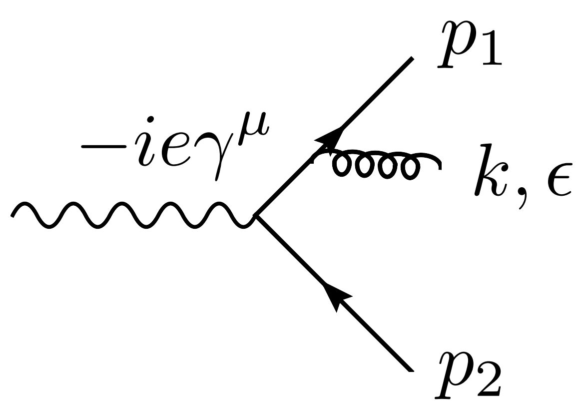

Consider the soft approximation: $k \ll p_1, p_2$

$$M^\mu_{q\bar{q}g} = \bar{u}(p_1)\left((-ie\gamma^\mu)(-ig_s t^a)v(p_2)\right)\left(\frac{p_1\epsilon}{p_1 k} - \frac{p_2\epsilon}{p_2 k}\right)$$

$\Rightarrow$ factorization of soft part

Soft divergences

The squared amplitude becomes

$$|M^{\mu}_{q\bar{q}g}|^2 = \sum_{\text{pol}} \left| \bar{u}(p_1)\left((-ie\gamma^{\mu})(-ig_s t^a)v(p_2)\right)\left(\frac{p_1\epsilon}{p_1 k} - \frac{p_2\epsilon}{p_2 k}\right)\right|^2$$

$$= \quad |M_{q\bar{q}}|^2 C_F g_s^2 \frac{2p_1p_2}{(p_1k)(p_2k)}$$

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Including phase space

$$d\phi_{q\bar{q}g}|M_{q\bar{q}g}|^2 \quad = \quad d\phi_{q\bar{q}}|M_{q\bar{q}}|^2 \frac{d^3 k}{2\omega(2\pi)^3} C_F g_s^2 \frac{2p_1 p_2}{(p_1 k)(p_2 k)}$$

$$= \quad d\phi_{q\bar{q}}|M_{q\bar{q}}|^2 \omega d\omega d\cos\theta \frac{d\phi}{2\pi}\frac{2\alpha_s C_F}{\pi}\frac{1}{\omega^2(1-\cos^2\theta)}$$

Soft divergences

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= \quad & |M_{q\bar{q}}|^2 C_F g_s^2 \frac{2p_1 p_2}{(p_1 k)(p_2 k)}
\end{aligned}
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&= d\phi_{q\bar{q}}|M_{q\bar{q}}|^2 \omega d\omega d\cos\theta \frac{d\phi}{2\pi} \frac{2\alpha_s C_F}{\pi} \frac{1}{\omega^2(1-\cos^2\theta)}
\end{aligned}
$$

The differential cross section is

$$
d\sigma_{q\bar{q}g} = d\sigma_{q\bar{q}} \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}
$$

Soft & collinear divergences

Cross section for producing a q$\bar{q}$-pair and a gluon is infinite (IR divergent)

$$d\sigma_{q\bar{q}g} = d\sigma_{q\bar{q}} \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$

<u>$\omega \to 0$:</u> soft divergence

<u>$\theta \to 0$:</u> collinear divergence

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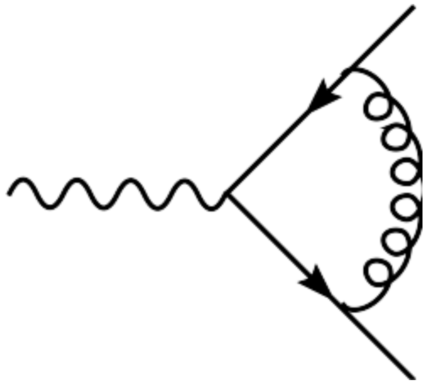
$\underline{\omega \to 0}$: soft divergence

$\underline{\theta \to 0}$: collinear divergence

But the full O(α_s) correction to R is finite, because one must include a virtual correction which cancels the divergence of the real radiation

$$d\sigma_{q\bar{q},v} \sim -d\sigma_{q\bar{q}} \frac{2\alpha_s C_F}{\pi} \frac{d\omega}{\omega} \frac{d\theta}{\sin\theta} \frac{d\phi}{2\pi}$$



NB: here we kept only soft terms, if we do the full calculation one gets a finite correction of α_s/π

Soft & collinear divergences

$\underline{\omega \rightarrow 0}$ soft divergence: the four-momentum of the emitted particle approaches zero, typical of gauge theories, even if matter (radiating particle) is massive

$\underline{\theta \rightarrow 0}$ collinear divergence: particle emitted collinear to emitter. Divergence present only if all particles involved are massless

Infrared safety (= finiteness)

So, the R-ratio is an infrared safe quantity.

In perturbation theory one can compute only IR-safe quantities, otherwise get infinities, which can not be renormalized away (why not?)

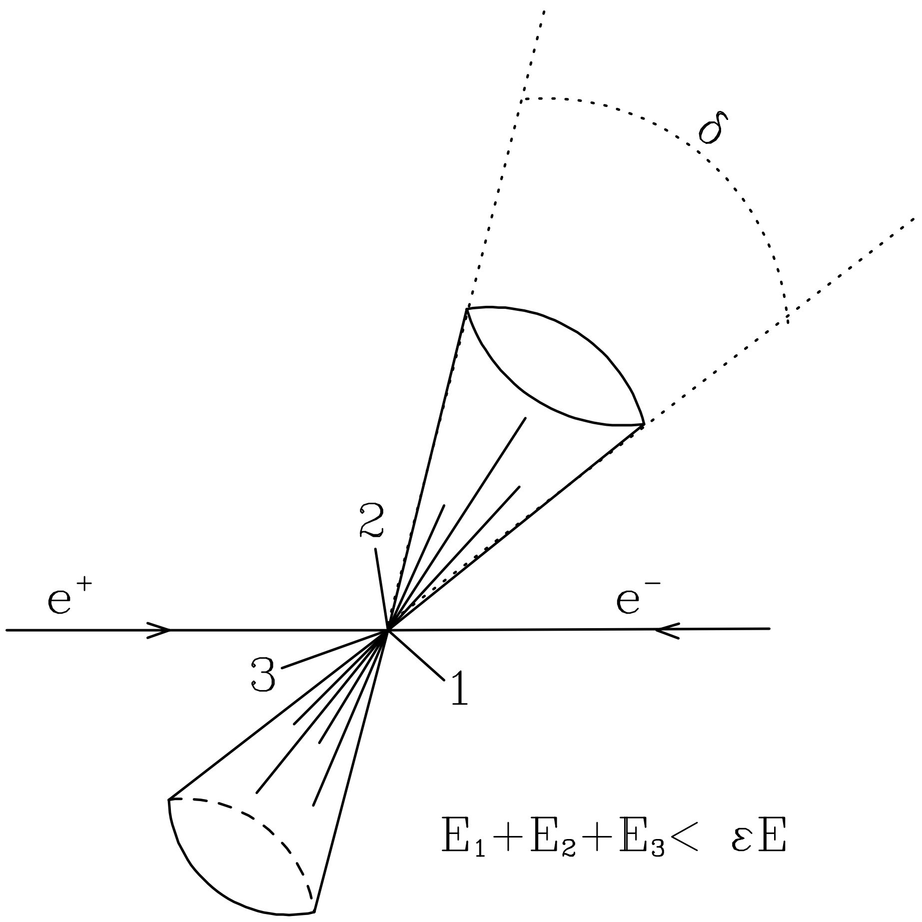
So, the natural questions are:

- are there other IR-safe quantities?
- what property of R guarantees its IR-safety?

Sterman-Weinberg jets

First formulation of cross-sections which are finite in perturbation theory and describe the hadronic final state

Introduce two parameters ε and δ: a pair of Sterman-Weinberg jets are two cones of opening angle δ that contain all the energy of the event excluding at most a fraction ε



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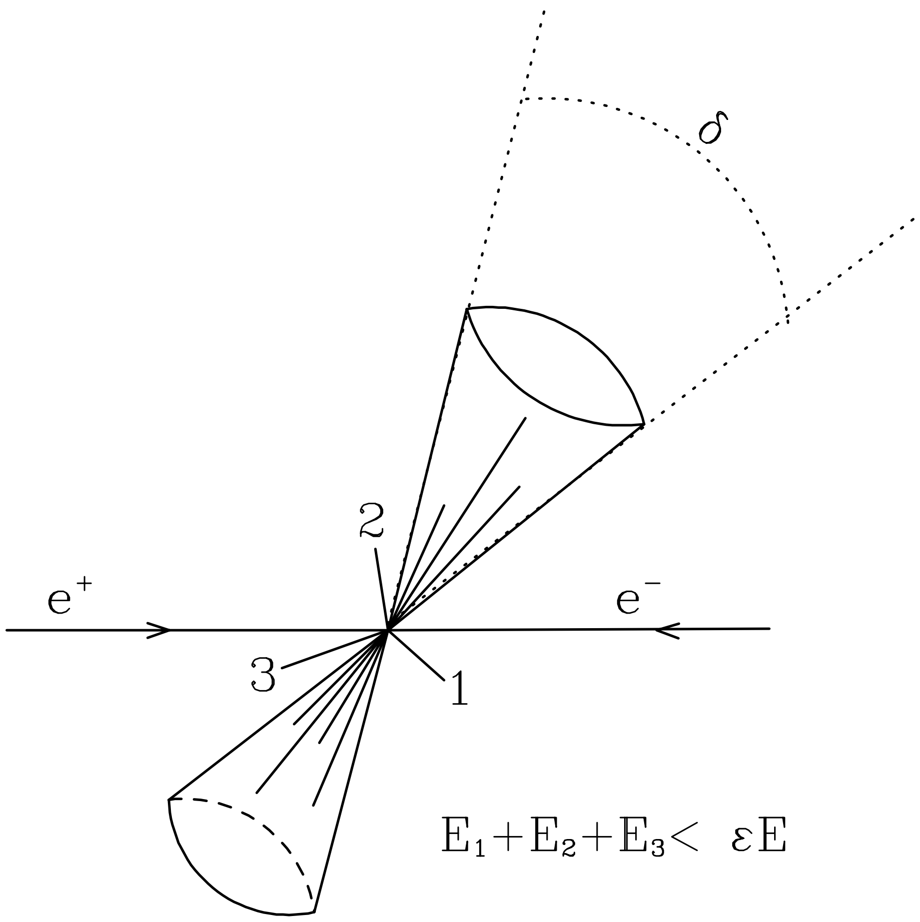

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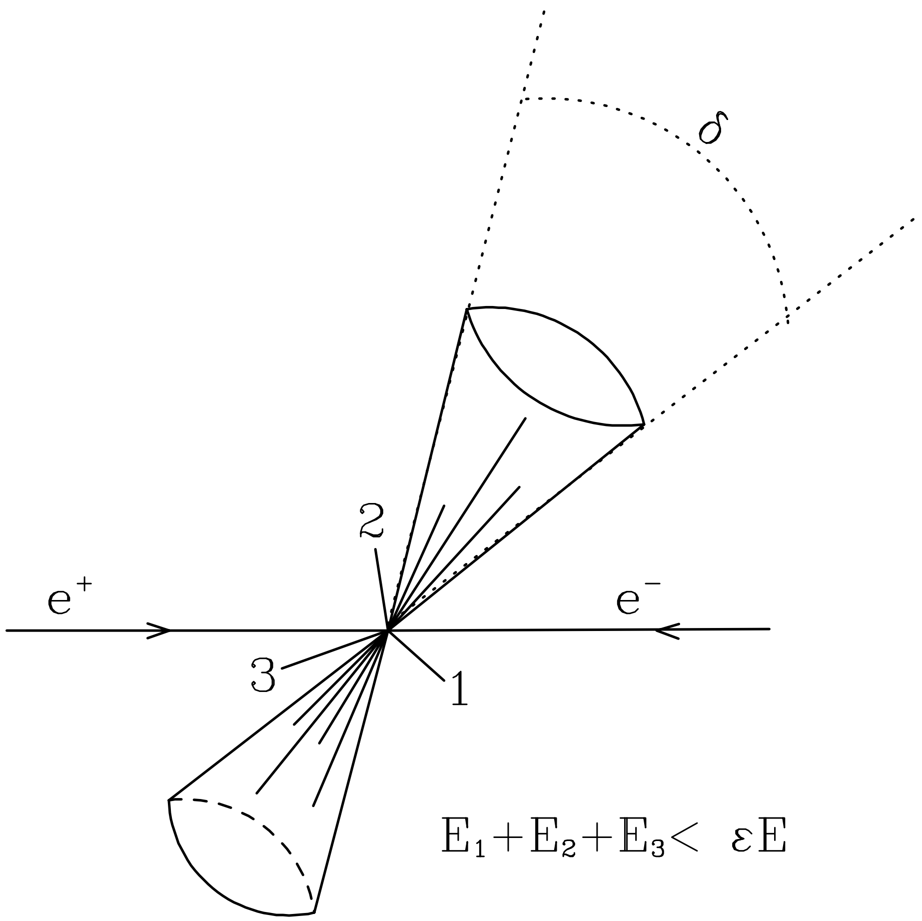

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Kinoshita-Lee-Nauenberg (KLN) theorem:
final-state infrared divergences cancel in measurable quantities (transition probabilities, cross-sections summed over indistinguishable states...)

Sterman-Weinberg jets

The Sterman-Weinberg jet cross-section up to $O(\alpha_s)$ is given by

$$\sigma_1 = \sigma_0 \left(1 + \frac{2\alpha_s C_F}{\pi} \ln \epsilon \ln \delta^2 \right)$$

Effective expansion parameter in QCD is often $\alpha_s C_F/\pi$ not α_s

α_s-expansion enhanced by a double log: left-over from real-virtual cancellation

- if more gluons are emitted, one gets for each gluon
  - a power of $\alpha_s C_F/\pi$
  - a soft logarithm $\ln\varepsilon$
  - a collinear logarithm $\ln\delta$
- if ε and/or δ become too small the above result diverges
- if the logs are large, fixed order meaningless, one needs to resum large infrared and collinear logarithms to all orders in the coupling constant

Infrared safety: definition

An observable $\mathcal{O}$ is infrared and collinear safe if

$$\mathcal{O}_{n+1}(k_1, k_2, \ldots, k_i, k_j, \ldots k_n) \to \mathcal{O}_n(k_1, k_2, \ldots k_i + k_j, \ldots k_n)$$

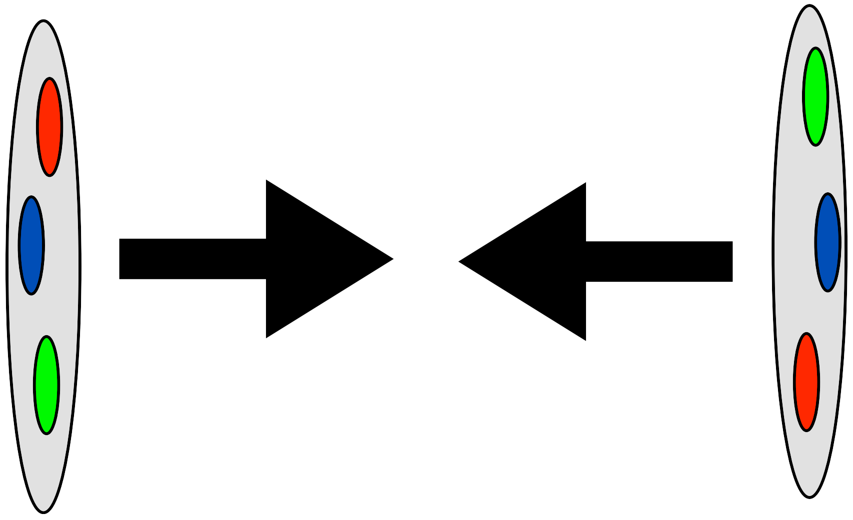
whenever one of the k_i/k_j becomes soft or k_i and k_j are collinear

i.e. the observable is insensitive to emission of soft particles or to collinear splittings

Partons in the initial state

- We talked a lot about final state QCD effects
- This is the only thing to worry about at e^+e^- colliders (LEP)
- Hera/Tevatron/LHC involve protons in the initial state
- Proton are made of QCD constituents

Next we will focus mainly on aspects related to initial state effects



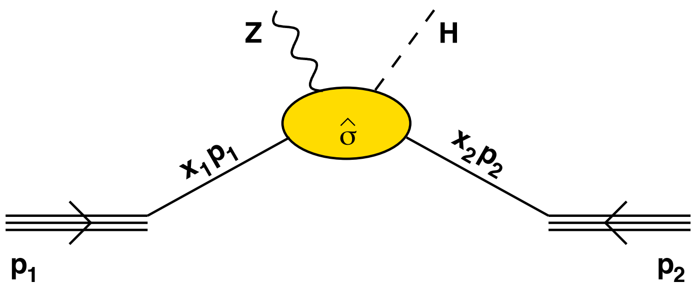
The parton model

Basic idea of the parton model: intuitive picture where in a high transverse momentum scattering partons behave as quasi free in the collision

$\Rightarrow$ cross section is the incoherent sum of all partonic cross-sections

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s) \qquad \hat{s} = x_1 x_2 s$$

NB: This formula is wrong/incomplete (see later)



$f_i^{(P_j)}(x_i)$: parton distribution function (PDF) is the probability to find parton i in hadron j with a fraction x_i of the longitudinal momentum (transverse momentum neglected), extracted from data

$\hat{\sigma}(x_1 x_2 s)$: partonic cross-section for a given scattering process, computed in perturbative QCD

Sum rules

Momentum sum rule: conservation of incoming total momentum

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

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Conservation of flavour: e.g. for a proton

$$\int_0^1 dx \left(f_u^{(p)}(x) - f_{\bar{u}}^{(p)}(x) \right) = 2$$

$$\int_0^1 dx \left(f_d^{(p)}(x) - f_{\bar{d}}^{(p)}(x) \right) = 1$$

$$\int_0^1 dx \left(f_s^{(p)}(x) - f_{\bar{s}}^{(p)}(x) \right) = 0$$

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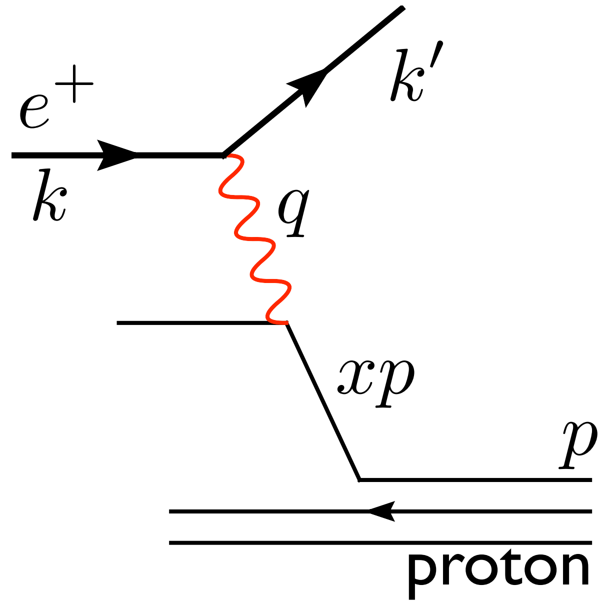
How can parton densities be extracted from data?

Deep inelastic scattering

Easier than processes with two incoming hadrons is the scattering of a lepton on a (anti)-proton



Kinematics:

$$Q^2 = -q^2 \quad s = (k+p)^2 \quad x_{Bj} = \frac{Q^2}{2p\cdot q} \quad y = \frac{p\cdot q}{k\cdot p}$$

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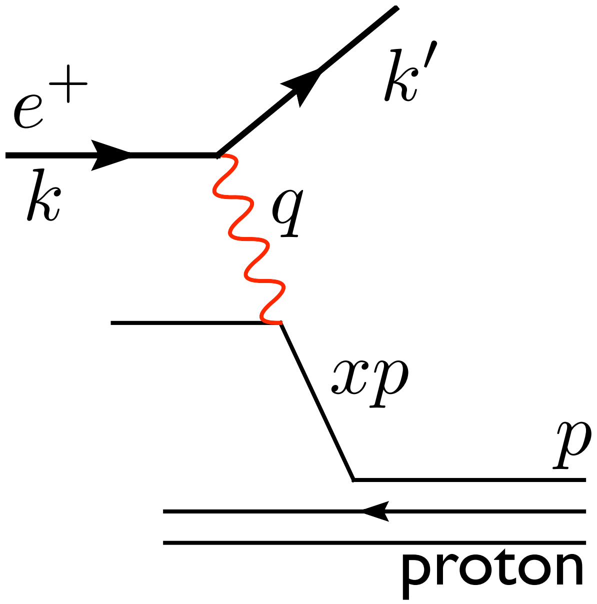

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<u>Partonic variables:</u>

$$\hat{p} = xp \quad \hat{s} = (k+\hat{p})^2 = 2k\cdot\hat{p} \quad \hat{y} = \frac{\hat{p}\cdot q}{k\cdot\hat{p}} = y \quad (\hat{p}+q)^2 = 2\hat{p}\cdot q - Q^2 = 0$$

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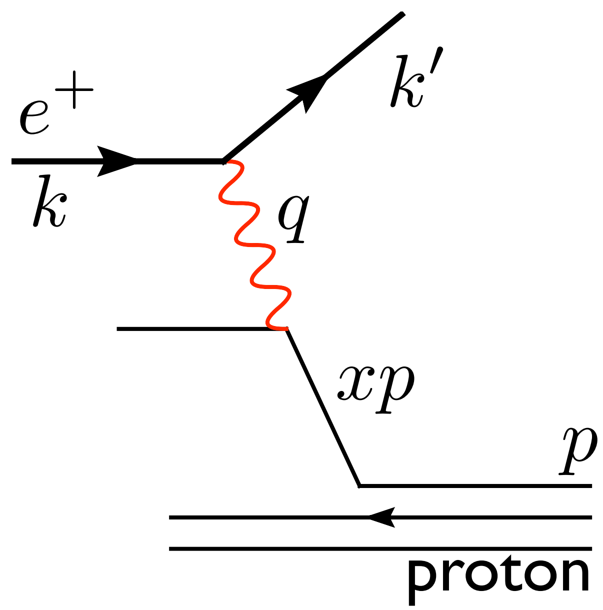

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Partonic cross section:

(just apply QED Feynman rules and add phase space)

$$\boxed{\frac{d\hat{\sigma}}{d\hat{y}} = q_l^2 \frac{\hat{s}}{Q^4} 2\,\pi\,\alpha_{em}\left(1+(1-\hat{y})^2\right)}$$

Deep inelastic scattering

Hadronic cross section:

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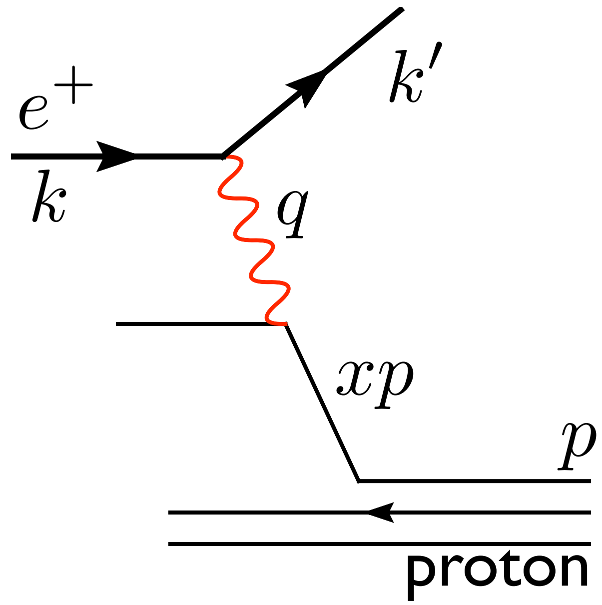
$$= \frac{2\pi\,\alpha_{em}^2 s x_{Bj}}{Q^4} \left(1 + (1-y)^2\right) \sum_l q_l^2 f_l^{(p)}(x_{Bj})$$



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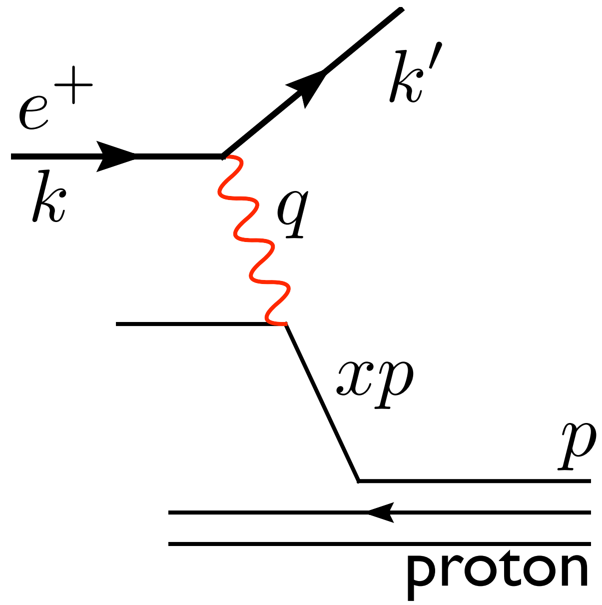

1. at fixed x_{Bj} and y the cross-section scales with s
2. the y-dependence of the cross-section is fully predicted and is typical of vector interaction with fermions ⇒Callan-Gross relation
3. can access (sums of) parton distribution functions
4. Bjorken scaling: pdfs depend on x and not on Q^2

The structure function F_2

$$\frac{d\sigma}{dydx} = \frac{2\pi\alpha_{em}^2 s}{Q^4}\left(1+\left(1-y^2\right)F_2(x)\right) \qquad F_2(x) = \sum_l x q_l^2 f_l^{(p)}(x)$$

F_2 is called structure function (describes structure/constituents of nucleus)

For electron scattering on proton

$$F_2(x) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

NB: use perturbative language of quarks and gluons despite the fact that parton distribution are non-perturbative

Question: F_2 gives only a linear combination of u and d. How can they be extracted separately?

Isospin

Neutron is like a proton with u & d exchanged

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For electron scattering on a neutron

$$F_2^n(x) = x\left(\frac{1}{9}d_n(x) + \frac{4}{9}u_n(x)\right) = x\left(\frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)\right)$$

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F_2^n and F_2^p allow determination of u_p and d_p separately

NB: experimentally get F_2^n from deuteron: $F_2^d(x) = \frac{1}{2}\left(F_2^p(x) + F_2^n(x)\right)$

Sea quark distributions

Inside the proton there are fluctuations, and pairs of $u\bar{u}$,$d\bar{d}$,$c\bar{c}$,$s\bar{s}$ ... can be created

An infinite number of pairs can be created as long as they have very low momentum, because of momentum sum rule.

We saw before that when we say that the proton is made of uud what we mean is

$$\int_0^1 dx\,(u_p(x) - \bar{u}_p(x)) = 2 \qquad \int_0^1 dx\,\left(d_p(x) - \bar{d}_p(x)\right) = 1$$

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Photons interact in the same way with u(d) and $\bar{u}$($\bar{d}$)

How can one measure the difference?

Question: What interacts differently with particle and antiparticle?

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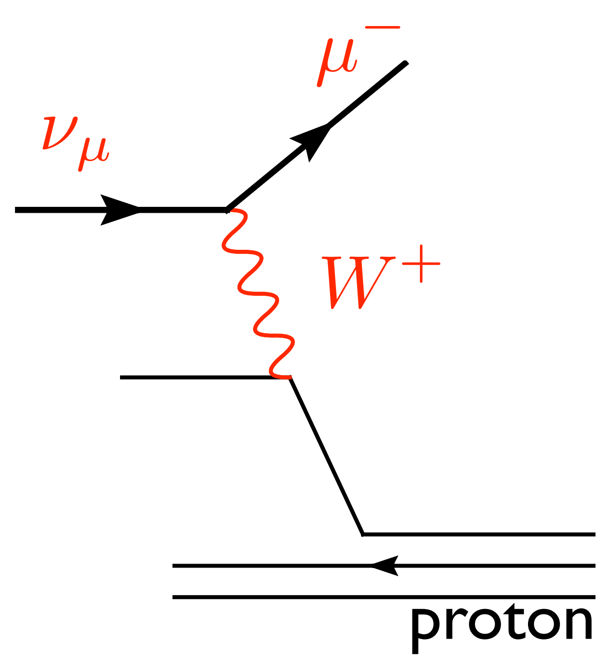

Question: What interacts differently with particle and antiparticle? W^+/W^- from neutrino scattering

Check of the momentum sum rule

$$\int_0^1 dx \sum_i x f_i^{(p)}(x) = 1$$

u_v	0.267
d_v	0.111
u_s	0.066
d_s	0.053
s_s	0.033
c_c	0.016
total	0.546

➡ *half of the longitudinal momentum is missing*

What is missing?

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What is missing?

The gluon!

$\gamma/W^{+/-}$ don't interact with gluons

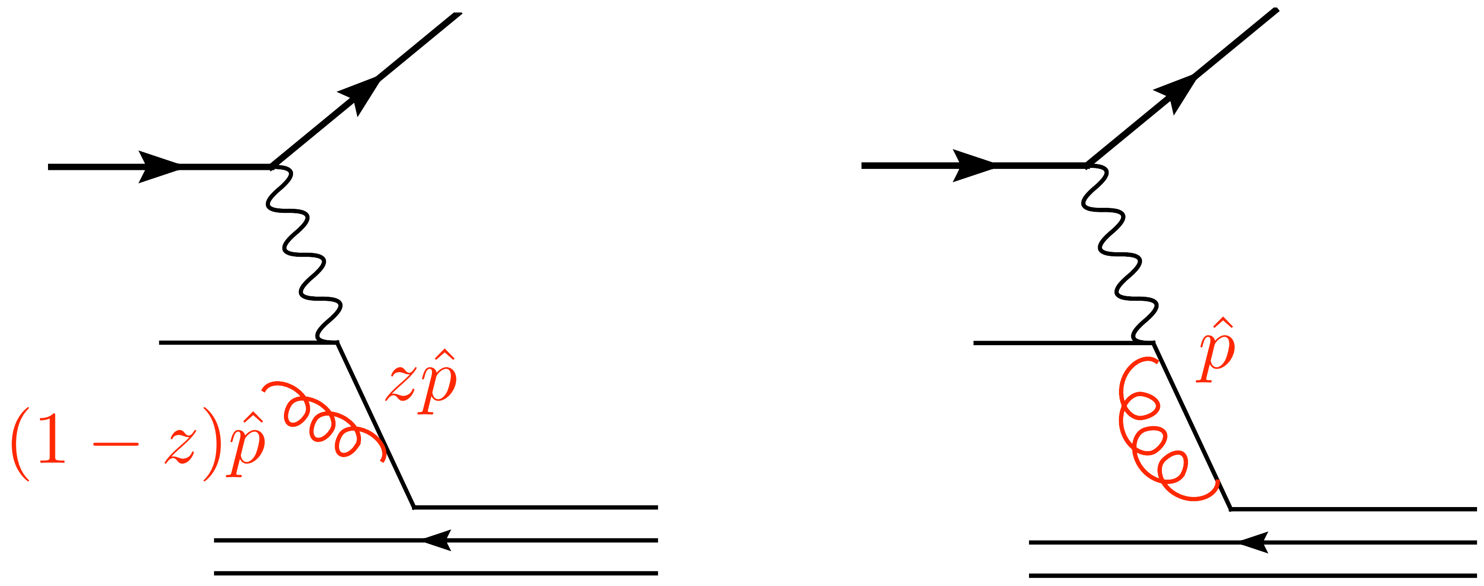
How can one measure gluon parton densities?

We need to discuss radiative effects first

Radiative corrections

To first order in the coupling:

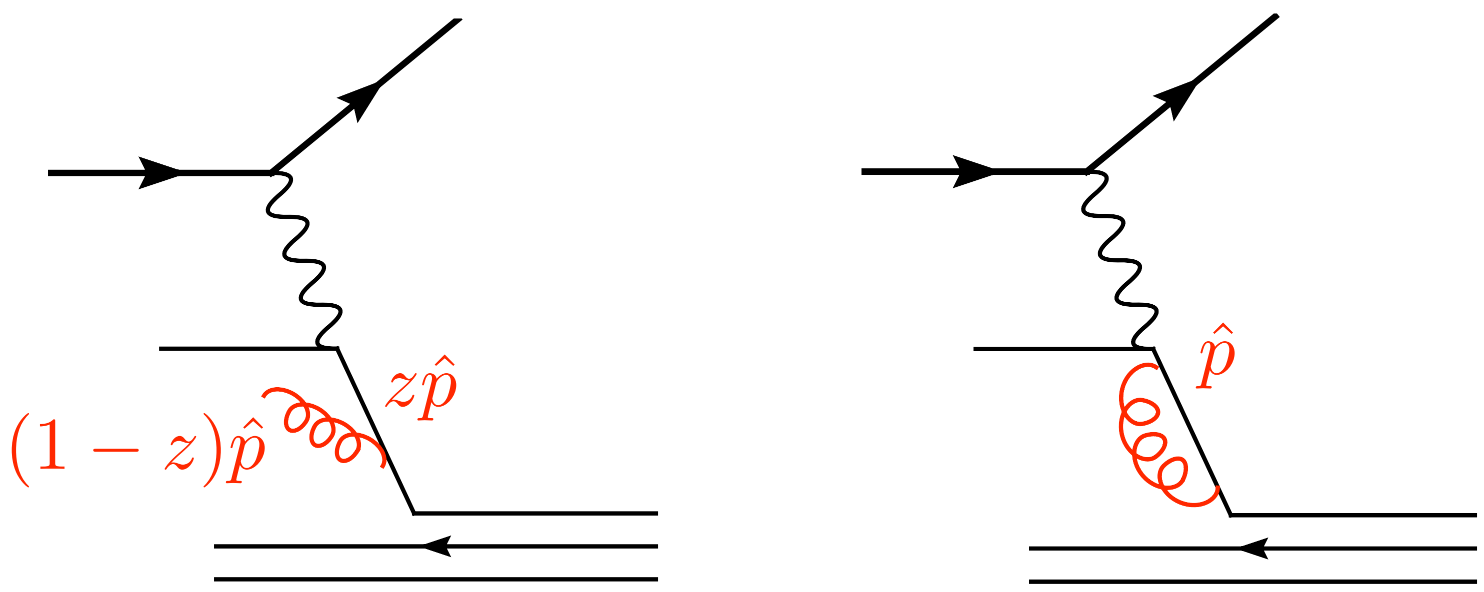
need to consider the emission of one real gluon and a virtual one



Radiative corrections

To first order in the coupling:

need to consider the emission of one real gluon and a virtual one



Adding real and virtual contributions, the partonic cross-section reads

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int dz \frac{dl_\perp^2}{l_\perp^2} \frac{1+z^2}{1-z} \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Partial cancellation between real (positive), virtual (negative), but real gluon changes the energy entering the scattering, the virtual does not

Radiative corrections

Partonic cross-section:

$$\sigma^{(1)} = \frac{C_F \alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^1 dz\, P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Soft limit: singularity at z=1 cancels between real and virtual terms

Collinear singularity: $k_\perp \to 0$ with finite z. Collinear singularity does not cancel because partonic scatterings occur at different energies

$\Rightarrow$ naive parton model does not survive radiative corrections

Similarly to what is done when renormalizing UV divergences, collinear divergences from initial state emissions are absorbed into parton distribution functions

The plus prescription

Partonic cross-section:

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^1 dz\, P(z) \left(\sigma^{(0)}(z\hat{p}) - \sigma^{(0)}(\hat{p}) \right)$$

Plus prescription makes the universal cancelation of soft singularities explicit

$$\int_0^1 dz f(z)_+ g(z) = \int_0^1 dz f(z) \left(g(z) - g(1) \right)$$

The partonic cross section becomes

$$\sigma^{(1)} = \frac{\alpha_s}{2\pi} \int_{\lambda^2}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} \int_0^1 dz\, P(z)_+ \sigma^{(0)}(z\hat{p}) \qquad P(z) = C_F \left(\frac{1+z^2}{1-z} \right)$$

Collinear singularities still there, but they factorize.

Factorization scale

Schematically use

$$\ln \frac{Q}{\lambda^2} = \ln \frac{Q}{\mu_F^2} + \ln \frac{\mu_F}{\lambda^2}$$

$$\sigma = \sigma^{(0)} + \sigma^{(1)} = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_+\right) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_+\right) \sigma^{(0)}$$

So we define

$$f_q(x, \mu_F) = f_q(x) \times \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{\mu_F^2}{\lambda^2} P_{qq}^{(0)}\right) \qquad \hat{\sigma}(p, \mu_F) = \left(1 + \frac{\alpha_s}{2\pi} \ln \frac{Q^2}{\mu_F^2} P_{qq}^{(0)}\right) \sigma^{(0)}(p)$$

NB:

- universality, i.e. the PDF redefinition does not depend on the process
- choice of $\mu_F \sim Q$ avoids large logarithms in partonic cross-sections
- PDFs and hard cross-sections don't evolve independently

Improved parton model

<u>Naive parton model:</u>

$$\cancel{\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1) f_2^{(P_2)}(x_2) \hat{\sigma}(x_1 x_2 s)} \qquad \hat{s} = x_1 x_2 s$$

<u>After radiative corrections:</u>

$$\sigma = \int dx_1 dx_2 f_1^{(P_1)}(x_1, \mu^2) f_2^{(P_2)}(x_2, \mu^2) \hat{\sigma}(x_1 x_2 s, \mu^2)$$

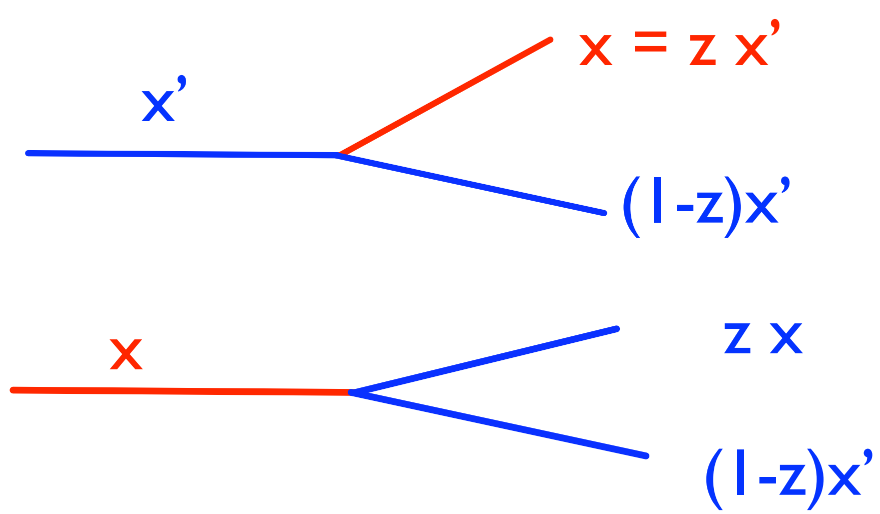
Intermediate recap

- With initial state parton collinear singularities don't cancel
- Initial state emissions with $k_\perp$ below a given scale are included in PDFs
- This procedure introduces a scale μ_F, the so-called factorization scale which factorizes the low energy (non-perturbative) dynamics from the perturbative hard cross-section
- As for the renormalization scale, the dependence of cross-sections on μ_F is due to the fact that the perturbative expansion has been truncated
- The dependence on μ_F becomes milder when including higher orders

Evolution of PDFs

A parton distribution changes when

- a different parton splits and produces it
- the parton itself splits



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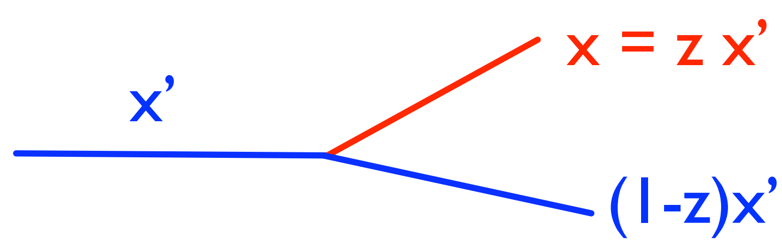
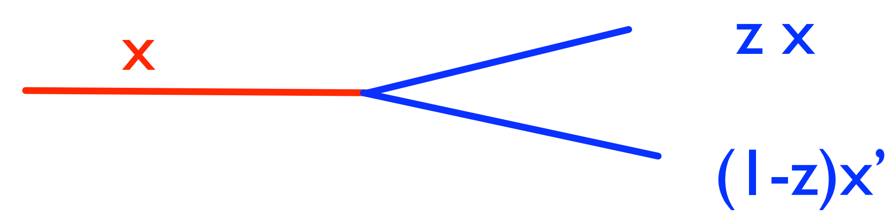


- the parton itself splits



$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_0^1 dx' \int_x^1 dz \frac{\alpha_s}{2\pi} P(z) f(x',\mu^2) \delta(zx'-x) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f(x,\mu^2)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right) - \int_0^1 dz \frac{\alpha_s}{2\pi} P(z) f\left(x,\mu^2\right)$$

$$= \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z},\mu^2\right)$$

<u>The plus prescription</u> $$\int_0^1 dz f_+(z) g(z) \equiv \int_0^1 dz f(z) \left(g(z) - g(1)\right)$$

DGLAP equation

$$\mu^2 \frac{\partial f(x,\mu^2)}{\partial \mu^2} = \int_x^1 \frac{dz}{z} \frac{\alpha_s}{2\pi} P(z) f\left(\frac{x}{z},\mu^2\right)$$

Altarelli, Parisi; Gribov-Lipatov; Dokshitzer '77

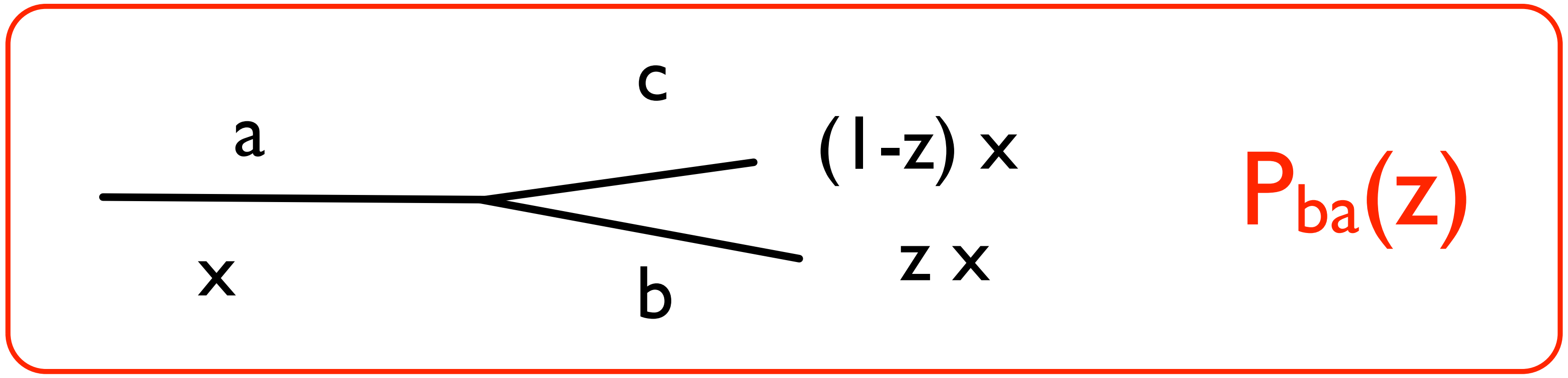
Master equation of QCD: we can not compute parton densities, but we can predict how they evolve from one scale to another

Universality of splitting functions: we can measure pdfs in one process and use them as an input for another process

Plus prescription implicit from now on

Conventions for splitting functions

There are various partons types. Standard notation:



Accounting for the different species of partons the DGLAP equations become:

$$\mu^2 \frac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \sum_j \int_x^1 \frac{dz}{z} P_{ij}(z) f_j\left(\frac{x}{z},\mu^2\right)$$

This is a system of coupled integro/differential equations

The above convolution in compact notation:

$$\mu^2 \frac{\partial f_i(x,\mu^2)}{\partial \mu^2} = \sum_j P_{ij} \otimes f_j(\mu^2)$$

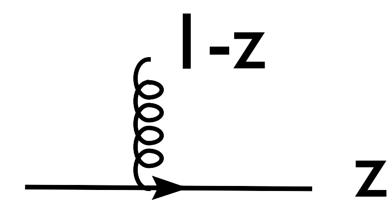
General DGLAP equation

Evolution equations in the general case:

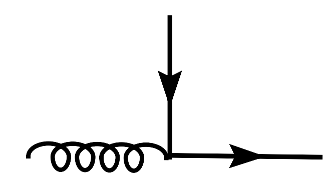
$$\mu^2 \frac{\partial f_i(z,\mu^2)}{\partial \mu^2} = \sum_j P_{ij} \otimes f_j(\mu^2) \qquad P_{ij}(x) = \frac{\alpha_s}{2\pi} P_{ij}^{(0)} + \left(\frac{\alpha_s}{2\pi}\right)^2 P_{ij}^{(2)} + \ldots$$

Leading order splitting functions:

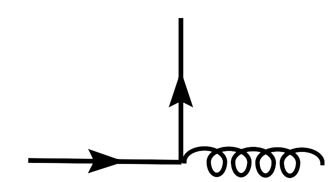
$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F \left[\left(\frac{1+z^2}{1-z} \right)_+ + \frac{3}{2}\delta(1-z) \right]$$



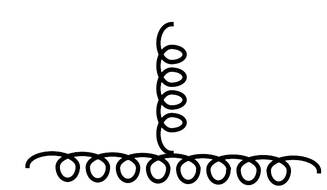
$$P_{qg}^{(0)} = P_{\bar{q}g}^{(0)} = T_R \left(z^2 + (1-z) \right)$$



$$P_{gq}^{(0)} = P_{g\bar{q}}^{(0)} = C_F \frac{1+(1-z)^2}{z}$$



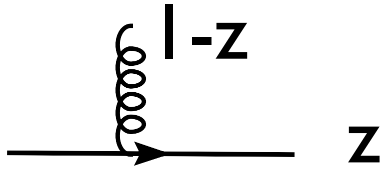
$$P_{gg}^{(0)} = 2C_A \left[z \left(\frac{1}{1-z} \right)_+ + \frac{1-z}{z} + z(1-z) + b_0 \delta(1-z) \right]$$



NB: at higher orders $P_{q_i q_j}$ arise

Properties of splitting functions

$$P_{qq}^{(0)} = P_{\bar{q}\bar{q}}^{(0)} = C_F\left[\left(\frac{1+z^2}{1-z}\right)_+ + \frac{3}{2}\delta(1-z)\right]$$



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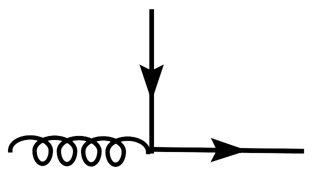

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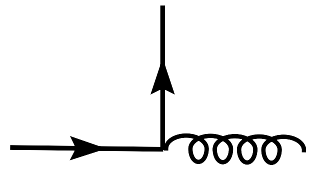
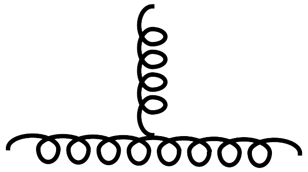


$$P_{gg}^{(0)} = 2C_A\left[z\left(\frac{1}{1-z}\right)_+ + \frac{1-z}{z} + z(1-z) + b_0\delta(1-z)\right]$$



- P_{qg} anf P_{gg} symmetric under z (1-z)
- P_{qq} and P_{gg} divergence for z=1 (soft gluon)
- P_{gq} and P_{gg} divergenge for z=0 (soft gluon)
- P_{qg} no soft divergence for gluon splitting to quarks

➠ gluon PDF grows at small x

History of splitting functions

☑ $P_{ab}^{(0)}$: Altarelly, Parisi; Gribov-Lipatov; Dokshitzer (1977)

☑ $P_{ab}^{(1)}$: Curci, Furmanski, Petronzio (1980)

☑ $P_{ab}^{(2)}$: Moch, Vermaseren, Vogt (2004)

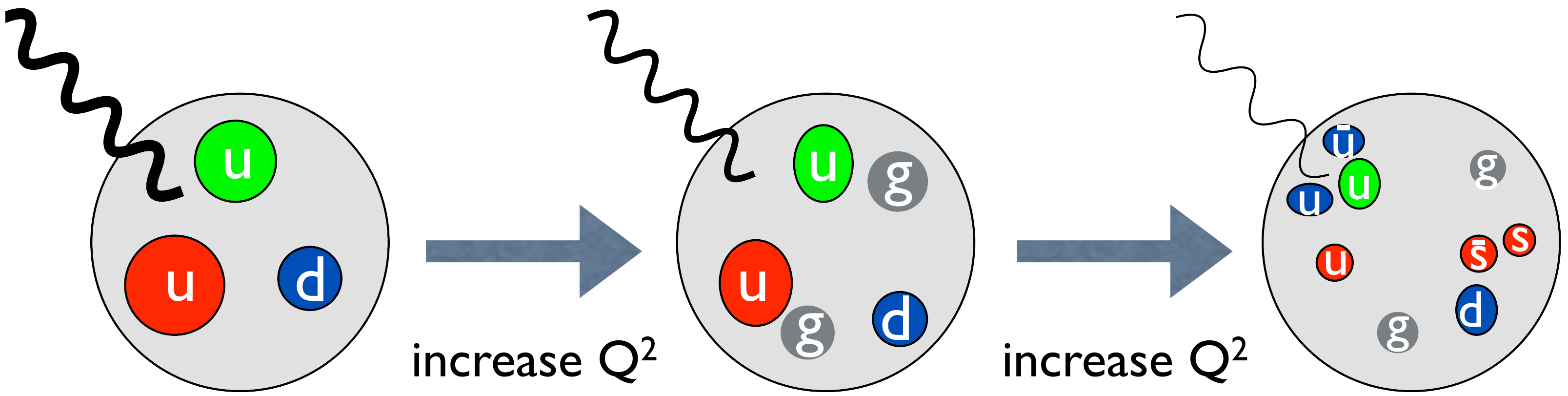
☛ $P_{ab}^{(2)}$: maybe hardest calculation ever performed in perturbative QCD

☛ Essential input for NNLO pdfs determination (state of the art today)

Evolution

So, in perturbative QCD we can not predict values for

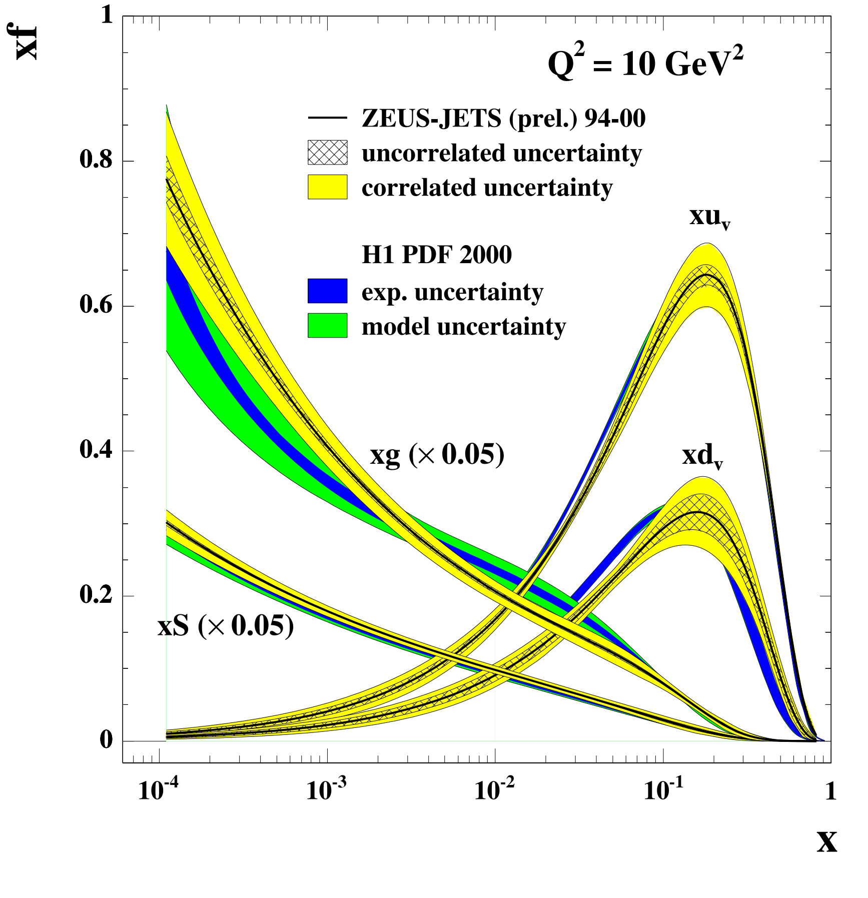
- the coupling
- the masses
- the parton densities
- ...

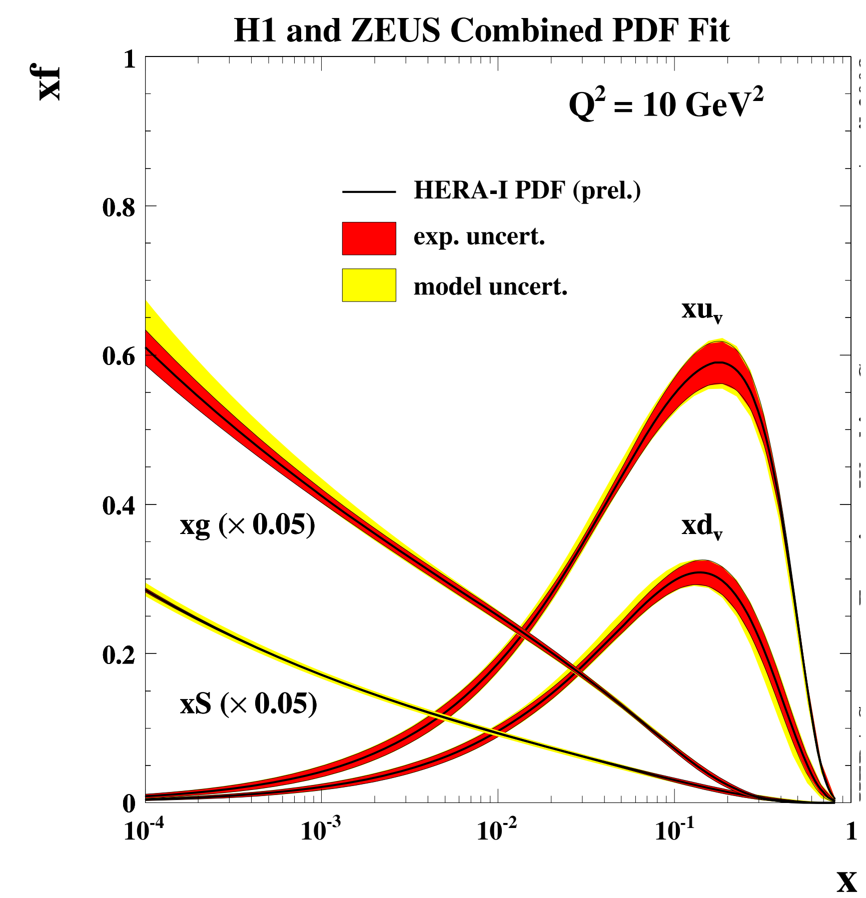


What we can predict is the evolution with the Q^2 of those quantities. These quantities must be extracted at some scale from data.

- not only is the coupling scale-dependent, but partons have a scale dependent sub-structure
- we started with the question of how one can access the gluon pdf: because of the coupled DGLAP evolution we can access the gluon pdf indirectly, through the way it changes the evolution of quark pdfs

The Hera PDF





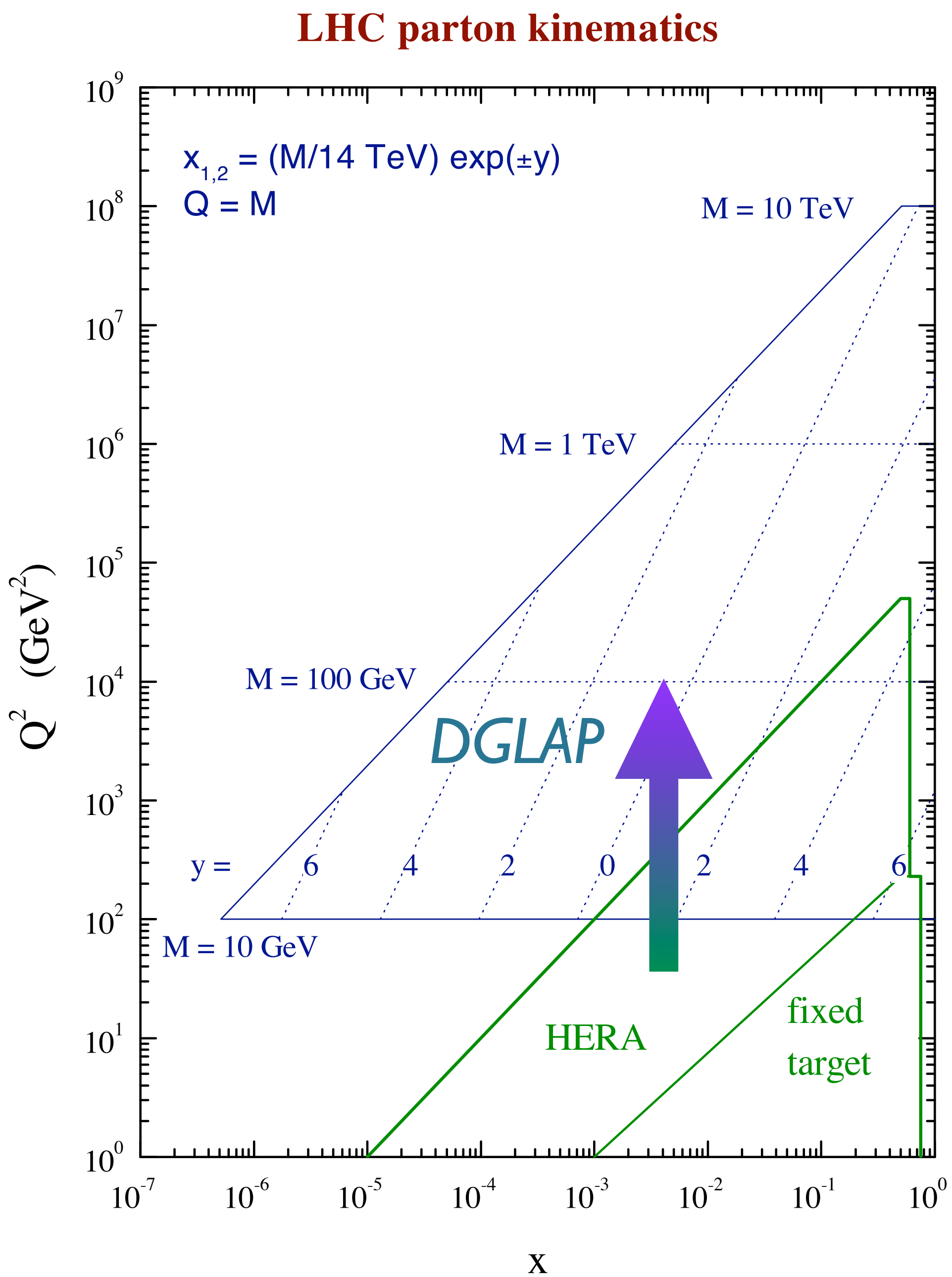
Hera structure function wg '08

Till recently: H1 and Zeus consistent within large uncertainties

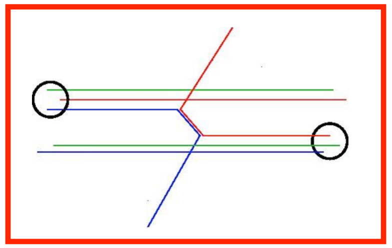
Now: single Hera fit with improved error (still more data to come)

Parton density coverage

- most of the LHC x-range covered by Hera
- need 2-3 orders of magnitude Q^2-evolution
- rapidity distributions probe extreme x-values
- 100 GeV physics at LHC: small-x, sea partons
- TeV physics: large x



➠ *Hera: key and essential input to the LHC*

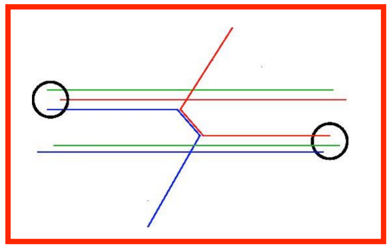


Parton densities: recent progress

<u>Recent major progress:</u>

- full NNLO evolution (previous approximate NNLO)
- improved treatment of heavy flavors near the quark mass
  [<u>Numerically</u>: e.g. (6-7)% effect on Drell-Yan at LHC]
- more systematic use of uncertainties/correlations (e.g. dynamic tolerance, combinations of PDF + α_s uncertainty)
- Neural Network (NN) PDFs

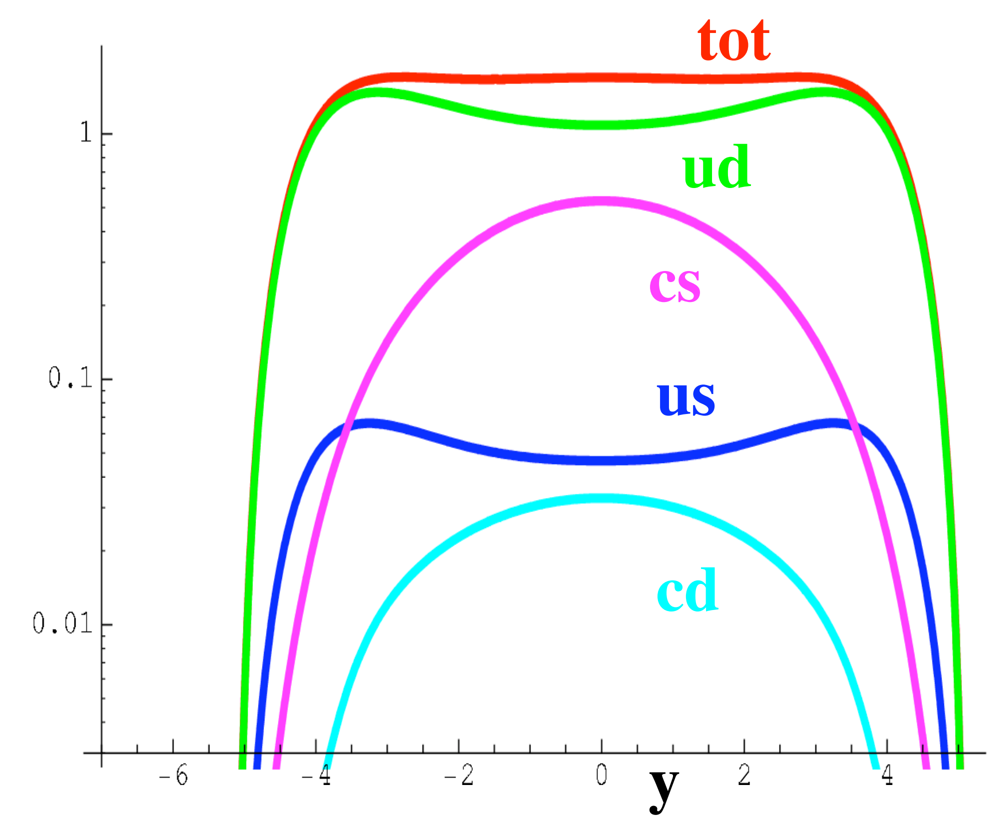
splitting functions at NNLO: Moch, Vermaseren, A. Vogt '04
[+ much related theory progress '04 -'08]
Alekhin, CTEQ, MSTW (new MSTW08), NN collaboration

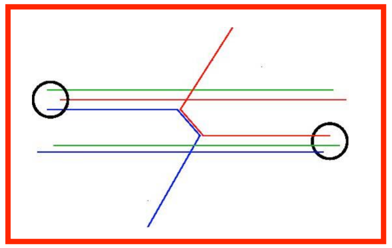


Parton densities: some open issues

- heavy quark treatment theoretically not ‘clean’ (various schemes, ad hoc procedures), but very important at the LHC

$d\sigma/dy(W^+)$ at LHC





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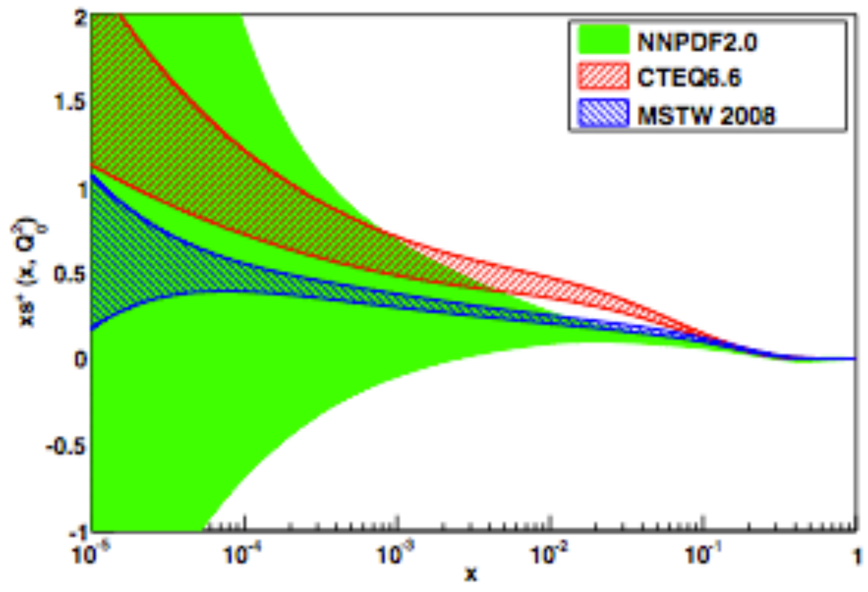
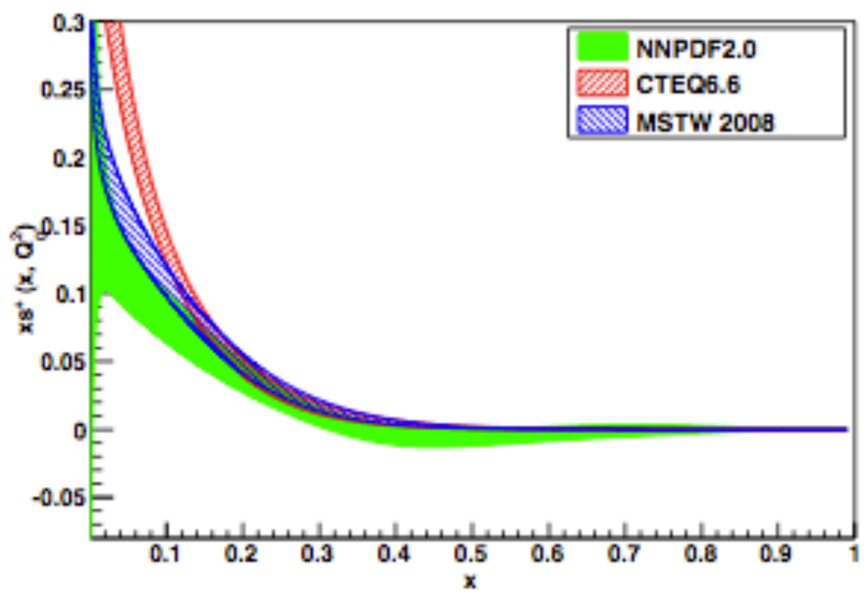
- heavy quark treatment theoretically not 'clean' (various schemes, ad hoc procedures), but very important at the LHC
- inconsistency between PDFs using different data sets

Strange pdf at $Q_0^2 = 2$ GeV^2





Parton densities: some open issues

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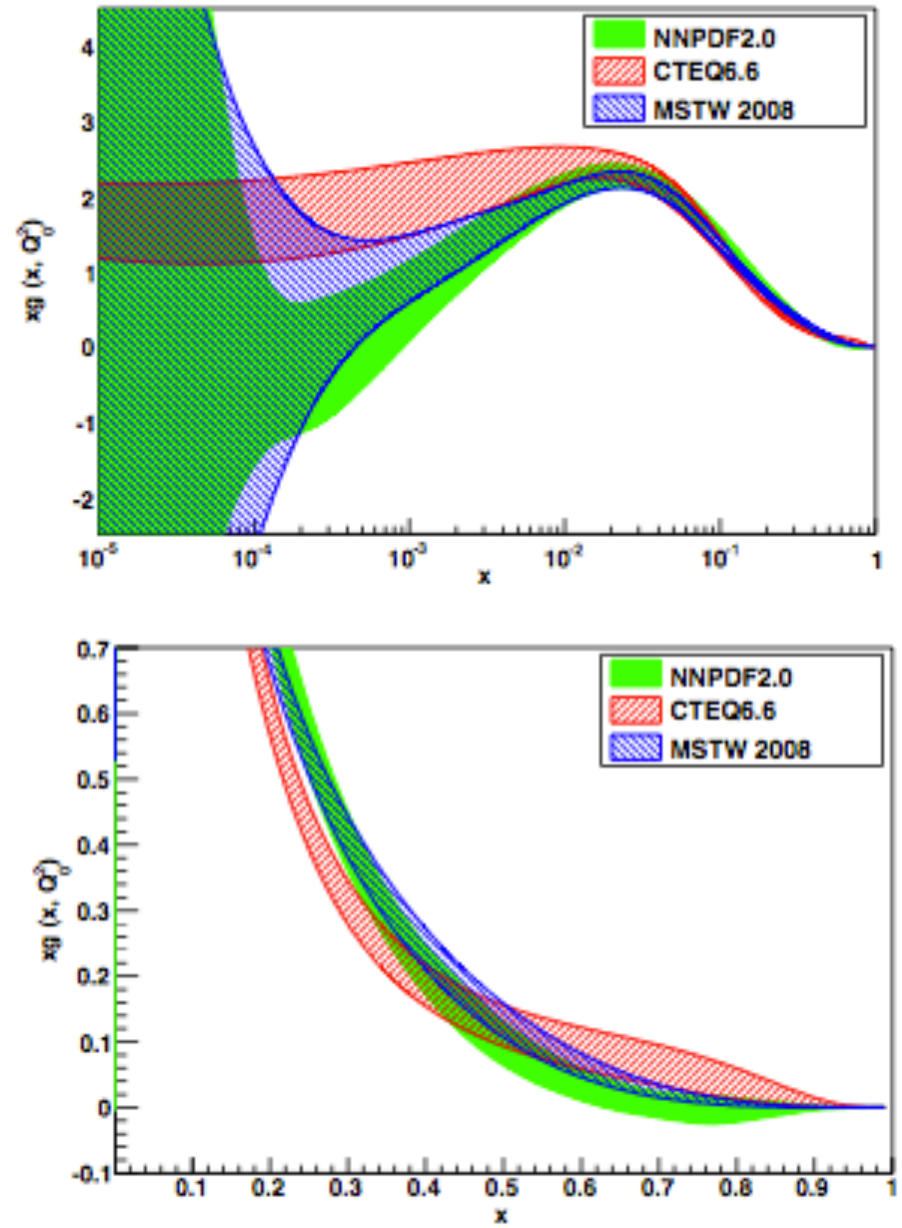
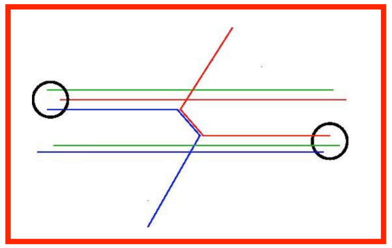
- inconsistency between PDFs using different data sets
- treatment of theory uncertainties (parameterizations, scheme for HQ, higher orders ...)

Gluon pdf at Q_0^2 = 2 GeV^2





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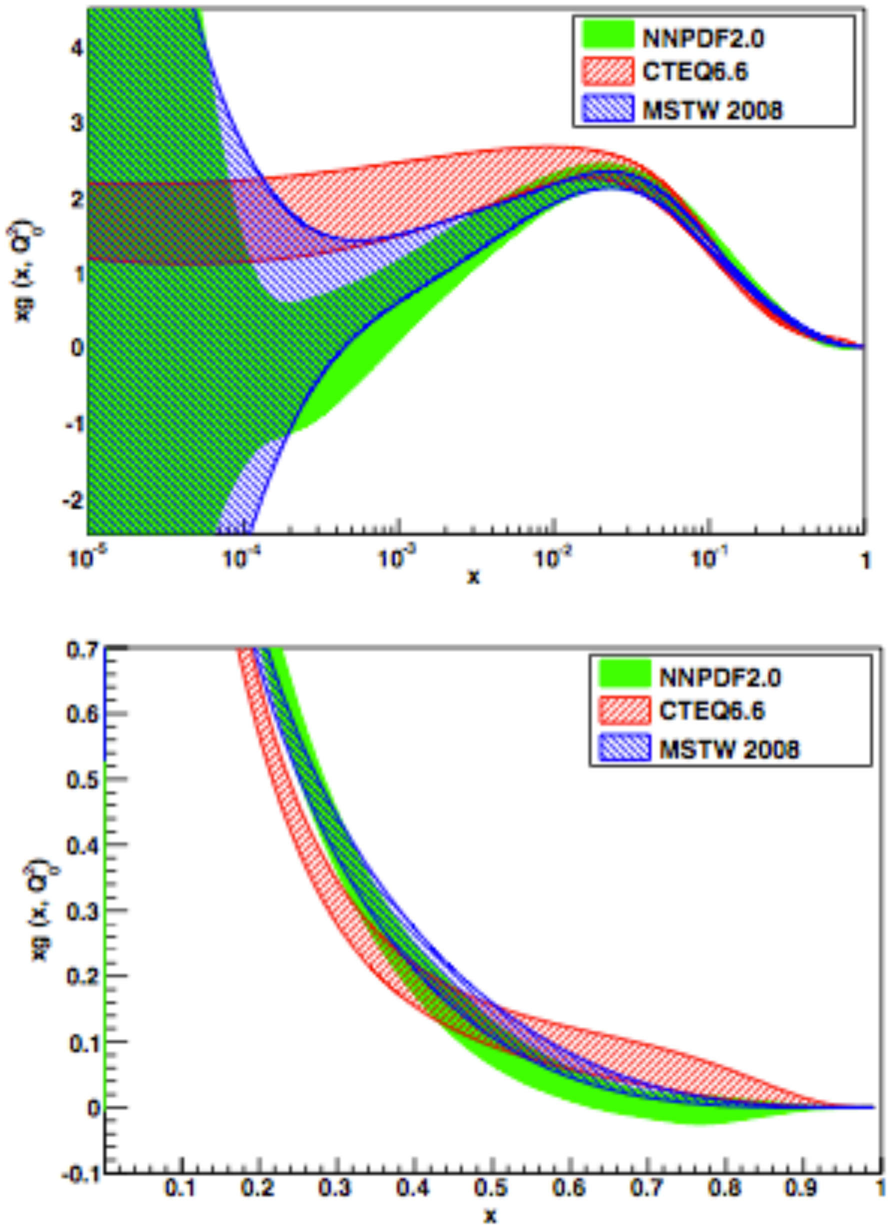

⇒ Description of PDFs reaching precision, but still some work ahead

Recap. of 2nd lecture

- There are potential infrared and collinear divergences $\Rightarrow$ not all quantities can be computed in PT, but we saw what is the property of observables guaranties IR-finiteness
- Parton model: incoherent sum of all partonic cross-sections
- Sum rules (momentum, charge, flavor conservation)
- Determination of parton densities (electron & neutrino scattering in DIS or Drell-Yan)
- Radiative corrections: failure of parton model
- Factorization of initial state divergences into scale dependent parton densities
- DGLAP evolution of parton densities $\Rightarrow$ measure gluon PDF
- Issues in today's determination of PDFs